3-24-2016

An Evaluation of Forecasting Methods that Could be Used in the Brazilian Air Force Uniform Distribution Process

Leandro Valviesse de Oliveira

AN EVALUATION OF FORECASTING METHODS THAT COULD BE USED IN THE BRAZILIAN AIR FORCE UNIFORM DISTRIBUTION PROCESS

THESIS
MARCH 2016

Leandro Valviesse de Oliveira, Captain, Brazilian Air Force

AFIT-ENS-MS-16-M-122

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT-ENS-MS-16-M-122

AN EVALUATION OF FORECASTING METHODS THAT COULD BE USED IN THE BRAZILIAN AIR FORCE UNIFORM DISTRIBUTION PROCESS

THESIS

Presented to the Faculty

Department of Operational Sciences

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics and Supply Chain Management

Leandro Valviesse de Oliveira

Captain, Brazilian Air Force

March 2016

AFIT-ENS-MS-16-M-122

AN EVALUATION OF FORECASTING METHODS THAT COULD BE USED IN THE BRAZILIAN AIR FORCE UNIFORM DISTRIBUTION PROCESS

Leandro Valviesse de Oliveira

Captain, Brazilian Air Force

Committee Membership:

Dr. Jeffrey Ogden
Chair

Dr. Michael Kretser
Member

AFIT-ENS-MS-16-M-122

Abstract

Every year the Brazilian Air Force (BAF) spends the equivalent of approximately 15 million dollars for uniforms. These purchases come from a tight budget, are executed through public procurement processes, and are tied to Brazilian acquisition regulations, which are often very strict. For this reason, lead times are unpredictable. It can take anywhere from one month to a year to replenish an item.

The purpose of this research is to analyze the forecasting process performed at a BAF military organization named Sub-directorate of Supply (SDS) with the intent of building an algorithm comprised of a selection of forecasting models in order to help SDS optimize its inventory investments.

With this in mind, monthly sales, prices, and inventory records from January of 2010 to July of 2015 were extracted from a database and converted to a standard spreadsheet format. Several forecasting models were evaluated and applied to randomly selected items from the database to create the algorithm.

In the final analysis, it was concluded that two models precisely depicted the behavior of sales in BAF's stores. These two models were then utilized to develop the forecasting tool that may prove valuable in future BAF uniform purchasing decisions.

AFIT-ENS-MS-16-M-122

My eternal love and gratitude to my wife, for your priceless love, support, understanding, and words of encouragement that have always made me reach farther.

Acknowledgments

I wish to thank my committee members who were more than generous with their expertise and precious time. A special thanks to Dr. Jeffrey Ogden, for his guidance and support throughout the course of this thesis effort. The insight and experience was certainly appreciated. I would also like to thank my reader, Dr. Michael Kretser for his valuable contribution to this research endeavor. His comments and expertise were highly appreciated.

I am forever indebted to Mrs. Annette Robb, International Military Student Officer, to Mrs. Clara Delgado for their encouraging contributions in new language adaptation; and Mr. Victor Grazier for reviewing my work. Their participation was essential to the success of this study.

Finally, I would like to thank my classmates, American and International Students, who were very supportive throughout this program. I truly valued your contributions, team efforts, and friendship.

Leandro Valviesse de Oliveira

Table of Contents

List of Figures

List of Tables

AN EVALUATION OF FORECASTING METHODS THAT COULD BE USED IN THE BRAZILIAN AIR FORCE UNIFORM DISTRIBUTION PROCESS

I. Introduction

Purpose

Every year the Brazilian Air Force (BAF) spends the equivalent of approximately 15 million dollars in uniforms for its military. These purchases come from a tight budget and are executed through public procurement processes, according to the Brazilian acquisition regulations. This compendium of regulations obliges the bidding commission to always buy from the cheapest supplier. Despite having issues in this area, this research will focus on the previous step to acquisition itself: the forecast.

The purpose of this research is to analyze a process performed by the Supply Division of a BAF military organization named Sub-directorate of Supply (SDS). This study aims to identify opportunities for improvement as well as applicable performance evaluation metrics that could be applied to the process in order to drive management actions toward quality and performance enhancements. The actual expected product of this study, however, is an algorithm comprising a selection of forecasting models that can be valuable in purchasing decisions in the future.

Background

The Sub-directorate of Supply, located in São Paulo, is the main unit in the BAF responsible for forecasting, acquiring, and distributing uniforms. The process of uniform distribution is performed through two main depots. The first is stationed inside the boundaries of SDS. The second is located in Rio de Janeiro, in a Unit named Intendancy Central Depot (ICD). From these two units, uniform items are distributed to warehouses located in 64 Air Force Bases, which will distribute either to end-users (i.e. militaries), or to other lower-level units.

The ranks in BAF are distributed as enlisted and officers. The enlisted ranks are, literally translated, Recruits, First-class Soldiers, Corporals, Third Sergeant, Second Sergeant, First Sergeant, and Sub-officer. The officer ranks are Second Lieutenant, First Lieutenant, Captain, Major, Lieutenant Colonel, and Colonel. All Recruits, First-class Soldiers, and Corporals are entitled to use their uniforms while serving the Air Force, receiving them from the respective organizations they are assigned to, free of charge. At pre-determined periods of time (which varies according to the uniform) they have the right to renew their uniforms. Either when renewing or leaving the active duty, they are expected to return every piece of uniform found in their possession.

In contrast, all military members with ranks of Third Sergeant and above are eligible to receive a military clothing allowance. These military personnel can buy their uniforms in one of the 29 Regional Uniform Stores (RUS) dispersed around the country, positioned inside BAF Units.

This research focuses on the latter case as it aims to setup a framework that can be applied to a specific sales behavior. However, the results can certainly be beneficial for both cases, serving as a starting point to a variety of cases such as previously described, related to whom is entitled to the right of receiving their uniforms free of charge.

Problem Statement

Initially, it was detected that the volume of sales and inventory levels were incompatible, with excess inventory for some items, and empty shelves for others. Furthermore, after analyzing the process for forecasting uniform sales, there was a visible sign that no scientific methods are currently being used to predict how much of each item has to be purchased to replenish the warehouse.

Additionally, there does not appear to be a procedure or metric currently in place at the Supply Division regarding its inventory policy. Ultimately, the unpredictable lead time for most of the items, due to Brazil`s acquisition regulations, further complicates the decision for managers to determine what inventory policy to use.

Research Objectives

The primary objective of this research was to identify potential flaws in the forecasting process that could be recognized as ineffective and inefficient, proposing potential solutions according to the literature on this subject. This would allow proposing specific actions for enhancing the overall performance of the Supply Division in SDS.

Secondarily, this research attempted to recommend an algorithm with a set or a combination of forecasting models that enhances the SDS acquisitions, enabling all the 29 RUSs to have the right items, at the right moment, in the right amount. The concept was that the algorithm had to be functional and easily implementable.

Research Questions

In order to address the objectives outlined for this study, some investigative questions were formulated and needed to be answered throughout the paper, as follows:

1. What metrics are currently being utilized by the Supply Division?

2. How can the forecasting process for uniform sales be improved in the SDS?

3. How to best assess accuracy of forecasting sales in the SDS?

4. Is it possible to build an algorithm where historical sales data can be evaluated and the best forecast suggested?

Methodology

The present study is mainly quantitative, although some qualitative aspects related to the context in which forecasting is being processed in SDS had to be clarified in order to provide the necessary support for the research. With this intention, SDS regulations were examined in order to identify what categories of metrics were effective in the timeframe researched. Moreover, information regarding this feature was obtained from the responses provided in an interview with the head of the Supply Division.

As far as the quantitative part, all aspects of the data collection and cleansing, as well as the tools employed, were given special care in order to preserve reliability in the results.

A deep analysis throughout a variety of forecast methods and accuracy parameters was performed with care prior to their selection. In addition, all formulas were written from scratch to ensure exactness of the calculations. Afterwards, a comparison chart was built to facilitate displaying and highlighting the best results. Further details on the methodology used in this research are stated in Chapter III.

Assumptions and Limitations of the Study

An important assumption in this study was that an accurate forecasting process results in valuable data that serves as a subsidy to allow management to make reasonable decisions on purchasing, given the current inventory policies.

The focus of this study was primarily quantitative, although it is evident that the subject covered in this paper is directly attached to qualitative aspects. In other words, forecasting is strictly related to inventory control and lead times. The latter two, despite being sensitive issues in SDS, were not treated by the present research. Thus, this research focused strictly on the calculations involving the forecasting process, not with their interaction.

The absence of metrics currently being taken regarding inventory holding cost rate on inventory policies, such as storage, obsolescence, and opportunity costs (as well as accurate information about lead times) prevented a more comprehensive approach to the problems revealed.

Organization

Chapter I offers the necessary background to understand the context of the process under study as well as provides the purpose, the problem that was brought to attention, the objectives of this research and the research questions. Chapter I also provides an overview on the theoretical background that guided the analysis and on the methodology along with some of the assumptions and limitations applied to this study.

Chapter II gives a conceptual foundation with the theoretical background that guided this research in terms of the methodology adopted for data analysis and accuracy measurements, expressing the opinions of experts concerning the concepts covered.

Chapter III gives special attention to applying the methodologies used in this research, particularly with respect to the procedure for data collection, statistical analysis, and the criteria adopted.

Chapter IV analyzes and displays the results obtained by applying the models developed in this study in an attempt to solve the problem and answer the Research Questions previously stated. Also in this chapter, further investigation exposed thoughts necessary to exhaust the possibilities and produce solid outcomes.

Chapter V presents the conclusions of this research, as well as recommendations for further investigation in the area.

II. Literature Review

Overview

This chapter explores theoretical perspectives and previous research findings that can help in developing a tailored response for the analysis to be performed in the present research. It contains aspects, such as some key methods available, the relationship between forecasting and inventory control, assessment of statistical significance, forecasting accuracy, comparison between simple and complex forecasting, and procedures and rules used for combining forecasts.

Methods

In the book "*Principles of Forecasting,*" Professor John Scott Armstrong of the Wharton School, University of Pennsylvania, addresses problems related to finance, marketing, personnel, and production, covering all types of forecasting methods: judgmental methods, such as Delphi, role-playing, and intentions studies, and quantitative methods, including econometric methods, expert systems, and extrapolation. Some methods, such as conjoint analysis, analogies, and rule-based forecasting, integrate quantitative and judgmental procedures. In each area, he identifies what is known as "if-then clauses" (e.g. "if the results are required tomorrow, then I will need two additional people to perform testing today") and summarizes evidence on these principles (JS Armstrong, 2002).

Nevertheless, before reaching a higher level of knowledge, it is important to start with basic principles, rules, and definitions. Following, they are briefly presented in order to provide the reader with an initial framework.

There are two broad categories of forecasting techniques: qualitative methods and quantitative methods. According to Hyndman & Athanasopoulos, (2014), qualitative methods are well-developed, structured approaches to obtaining good forecasts without using historical data, while quantitative methods are based on algorithms of varying complexity and can be applied when two conditions are satisfied. These conditions are: numerical information about the past is available, and it is reasonable to assume that some aspects of the past patterns will continue into the future (Hyndman & Athanasopoulos, 2014).

There is a wide range of quantitative forecasting methods, often developed within specific disciplines for specific purposes. Each method has its own properties, accuracies, and costs that must be considered when choosing a specific method. Most quantitative forecasting problems use either time series data (collected at regular intervals over time) or cross-sectional data (collected at a single point in time) (Bowerman, Connell, & Koehler, 2005).

Time series data are of interest for this study as the data collected refers to sales information encompassing 67 months. This type of data is particularly useful when one wants to forecast something that is happening over time and thus is subject to externalities. These methods can be the simplest to deploy and yet quite accurate, particularly over the short term. Quantitative forecasting methods analyze patterns in historical data in an attempt to use past patterns to predict future patterns.

The methods designed for time series can use models as simple as the moving average or as complex as the ARIMA models. In the first case, the forecast is the average of the previous determined number “x” of observations or periods, where "x" is a number that best apply for that time series. For instance, if there is monthly sales data being forecasted, a 12-month (period) moving average might be used, where always the forecast for the next month is the average over the past 12 months.

Simple averaging observations, however, may not work well enough when there is trend or seasonality in the data. In that case, other techniques, such as exponential smoothing, may be more appropriate.

With moving average, every data point has identical weight in calculating forecast. With smoothing methods, more importance is placed on the most recent data than on the historical data. If there is trend present in the data, placing more weight in recent observations will make the forecast more likely to reproduce the trend.

Moving averages and simple exponential smoothing techniques are available in Excel and easy to execute. That is part of the great advantage of time series methods: they are generally simple, cheap to run, and relatively easy to interpret (Hyndman & Athanasopoulos, 2014).

There are more complex time series techniques as well, such as Box-Jenkins models that can deal with data with trends and seasonality. The Box-Jenkins ARIMA model is a combination of the AR (autoregressive) and MA (moving average) models, with the "I" standing for "Integrated" (NIST/SEMATECH, 2013). Chapter III will provide more in-depth descriptions of the methods and their models selected to perform the data analysis in this study.

Forecasting and Inventory Control

According to Gardner (1990), “forecasting is a prerequisite to inventory decisions in practice”. This topic appears very convenient to be discussed since it has the scope of combining forecasting and inventory control. In fact, the decision over inventory strategy can be made over a tradeoff curve between service level and inventory investment. By improving forecast accuracy this curve should be shifted in such a way that both improves customer service and reduces inventory investment. To accomplish such calculations, a high number of metrics should be taken into account, most of which do not exist in the Military Organization focus of this study.

Although a combination of Forecasting and Inventory Control would be the perfect approach for this study, as will be seen in Chapter IV, the inventory policies in Sub-directorate of Supply (SDS) are complex enough for another thesis topic, due to several aspects. Here we can emphasize, as an example, the unpredictable lead times that result from the Brazilian Acquisition Law. As Arraes, K. G. G. observed, “The fact that the bidding process derived in Brazil during the Portuguese colonization might be one of the reasons why it is still so attached to formal procedures”. Due to this excess of formal procedures, a purchase, depending on its complexity, can last between one month and one year (Arraes, 2011).

Likewise, the lack of metrics at SDS, including inventory holding cost rate, on inventory policies, such as storage, obsolescence, and opportunity costs; and information about lead times will be discussed. For this reason, in this research we will keep a focus on the obvious: an accurate forecasting process will result in valuable data that will serve as a subsidy to allow management to make reasonable decisions on purchasing, given the actual inventory policies. In other words, forecasting processes will affect inventory policies, but not the other way round.

Statistical Significance Assessment

The concern with the quality of the results was a constant in this research, leading to search previous papers that address most of the common issues faced when a forecasting process has to be implemented, and statistical significance tests are no exception. According to Mayer (2012), when testing independent variables for statistical significance, achieving a satisfactory result (i.e. a significant p-value) means the statistic is consistent, that the procedures were followed properly and the right (significant) variables were selected. It does not denote that the finding is relevant. Rather, significance is a statistical term that tells how likely it is that a relationship exists (Mayer, 2012).

When there are many possible predictors (independent variables), it is necessary to develop some strategy for selecting the best predictors to use in a regression model. A common approach is to plot the forecast dependent variable against a particular predictor in order to look for any noticeable relationship. The flaw in this procedure is that it is not always possible to see the relationship from a scatterplot, especially when there are the effects of other predictors not accounted for (Chatfield, 2000). Another common approach is to do a multiple linear regression on all independent variables and disregard all variables whose p-values are greater than 0.05. To start with, statistical significance does not indicate predictive value. This is not a good strategy because the p-values can be misleading when two or more predictors are correlated.

Armstrong (2007) states that tests of statistical significance harm scientific progress in forecasting. Even when done properly, significance tests are dangerous. He concludes that tests of statistical significance are harmful to the development of scientific knowledge in a number of ways. For example, there is a bias against publishing papers that fail to reject the null hypothesis, although papers that fail to reject null hypotheses might contain important findings, while those that have significant results can be very trivial. (Armstrong, 2007)

Another reason is that they distract the researcher from the use of proper methods. Researchers might address questions that can only be answered by significance tests, rather than studying problems that are important. It leads researchers to think that they have completed the analysis, even though much remains to be done. The focus should be on those producing reasonably good predictions (e.g. good effect sizes) instead of good p-values (Kostenko & Hyndman, 2008).

Only when we know whether we are dealing with a large or a trivial effect size will we be able to interpret its meaning and, so to speak, the substantive significance of our results. The substantive significance of a result, in contrast, has nothing to do with the p-value and everything to do with the estimated effect size. Substantive significance is the size of the effect that an independent variable has on the dependent variable, and is more important than statistical significance.

Forecast Quality

Despite not being possible to evaluate the entire supply chain in this study, special preoccupation was dedicated to improving the forecast accuracy, one of the most important tasks in supply chain management, for it affects several elements in the system. The investment in inventory, for instance, is tied directly to forecast results, allowing the reduction of the safety-stock levels if a certain degree of improvement is met.

As with any analytical technique, nevertheless, one should not use it indiscriminately or assume the results are absolute truths. In fact, all forecasts are invariably wrong. It is just a matter of how wrong they are. Therefore, the effort should be to try to find a model that provides the most adequate approximation to the data behavior in a way that best accomplishes the task.

Thus, combined with the concept of effect size mentioned in the previous section, error measurement statistics play a critical role in tracking forecast accuracy, monitoring for exceptions, and benchmarking forecasting processes. Interpretation of these statistics can be risky, particularly when working with low-volume data or when trying to assess accuracy across multiple items (e.g., SKUs, locations, customers, etc.).

For forecast accuracy, one can understand it as a measurement based on forecast error, which is simply the difference between the actual response (also called dependent variable) and the predicted values to that variable (Hoover, 2009). It is not acceptable that any set of forecasts have larger errors, on average, than those produced by a naïve forecast, the crudest forecast conceivable (e.g. using the preceding actual information as a forecast). Therefore, this method (naïve forecast) can be used as a benchmark, and established as the lower bound when evaluating forecast quality (Morlidge, 2013). In other words, he states that it should be the least desired, or accepted quality level for a model to be considered for use.

In order to assess the potential of a forecast to add more value (how much improvement it is possible to make), it is necessary to identify the lower bound of forecast error. Attempts to find methods to measure forecastability have been unsuccessful on the self-referential nature of the problem: it is only possible to assess the performance of a forecasting method by examining its inputs or its outputs in comparison with an unspecifiable set of several possible methods (J. S. Armstrong, 2001).

Hoover (2009) and Armstrong (2001) have proposed alternative ways of assessing forecastability. As expected, these methods are somewhat complex and consequently more difficult to implement and interpret.

A "perfect" forecasting algorithm would describe the past signal, leaving only errors that represent pure noise and are hence unavoidable. Since the errors from a naïve forecast are a way to measure the observed amount of noise in data, there might be a mathematical relationship between the naïve forecast errors and the lowest possible errors from a forecast. Therefore, avoidability sets a theoretical lower bound to the forecast error that is independent of the forecaster and the available tool set, and it can be quantified using a common error metric such as Mean Squared Error (MSE) or Mean Absolute Error (MAE) (MORLIDGE, 2014).

Thereby, it was found that, under ordinary circumstances, ratios between forecast errors from a model and forecast errors of a naïve forecast, which gives a measurement called Relative Absolute Error (RAE), can provide benchmarks with which one can examine, if only unavoidable error has been eliminated.

The implication is that forecasting methods could expect at best to reduce forecast error by about 30% below that of the naïve forecast. Morlidge (2014) presented evidence that a 0.7 limit of forecastability was theoretically supported when data had no trend and seasonality. However, it was theoretically possible to beat an RAE of 0.7 if there was trending and other patterns present in the data, although an RAE of about 0.5 seemed to represent a practical limit on what could be achieved (MORLIDGE, 2014).

On the other hand, an RAE greater than one suggests that forecast error from the chosen method is actually worse than the naïve forecast error, an undesirable situation. Unfortunately, although it should be easy to out-perform the naïve forecast, it was found that such a result occurs about half the time with supply-chain data (MORLIDGE, 2014).

An advantage of using the naïve forecast as a benchmark is that it implicitly incorporates the notion of volatility, since the naïve forecast has the same level of variation as the variable itself (Morlidge, 2013). According to him, errors associated with the naïve forecast are also probably a better predictor of forecastability for time series purposes than the Coefficient of Variation because they measure period-to-period variation in the data.

Ultimately, the safety stock needed to meet a given service level is determined by the forecast error. If the RAE of a forecast model is 1.0, yielding the same error on average as a naïve forecast, the buffer inventory set by the naïve errors is appropriate. If a forecast model has an RAE below 1.0, however, it means that the business needs to hold less inventory than that indicated by the naïve, indicating less inventory investment is required. This is how forecasting adds value to a supply chain: the greater the level of absolute errors below those of the naïve forecast, the less stock is needed and the more value is added (Morlidge, 2014).

Smoothing the Bullwhip Effect in Seasonal Supply Chains

The bullwhip effect occurs when the end links of the supply chain make decisions that can over- or under-estimate the product demand, creating amplified fluctuations in inventory levels of the entire supply chain. An example of the comparison between Sales and Inventory will be seen in Chapter IV, demonstrating this phenomenon as a problem currently faced by SDS and deserves special mention. In this matter, Costantino, Di Gravio, Shaban, & Tronci (2014) state that “smoothing inventory decision rules have been recognized as the most powerful approach to counteract the bullwhip effect.”

Paik & Bagchi (2007) confirmed, through computer simulation, that the bullwhip effect should be mitigated by effective information flow and channel coordination, showing the important influence of all elements of a supply chain. According to his study, “in order to control the bullwhip effect, retailers need to share the actual demand information with their partners”. However, they concluded that demand forecast updating was among the most significant variables that cause the bullwhip effect.

Long lead times lead safety stocks to increase, rising the fluctuation in demand to more significant levels. Using the exponential smoothing method, for example, continually updates future demand forecasts as new demand data become available (H. L. Lee, Padmanabhan, & Whang, 1997). Still, according to these authors, the order ‘send to the supplier’ reflects the forecasted needs for replenishing stock and necessary safety stock. With long lead times, safety stocks will naturally grow, leading to a growth in order quantities over time, as the forecasting information will become outdated.

Forecasting Role in the Supply Chain (Costantino, Di Gravio, Shaban, & Tronci, 2015)

Costantino et al., (2015) evaluated the role of forecasting in the supply chain. According to them, “although forecasting is an essential component in the inventory management of supply chains, it has been recognized as a major cause of ordering and inventory instability in supply chains”. As researchers investigate the impact of the bullwhip effect problem caused by different available forecasting methods, they aim to selecting what parameters should be used under various operational conditions.

They proposed a forecasting system based on a statistical process control that can avoid frequent reactions to demand changes, counteracting the bullwhip effect, without affecting the inventory performance. This system uses two control charts integrated to decision rules to estimate the expected demand and control the inventory position. The first control chart represents a simple forecasting mechanism to predict the demand based on current variation of incoming orders/demand through a set of decision rules without over/under-reaction to demand changes. The second control chart controls the inventory position and allows order smoothing.

Their study considered the impact of the forecasting methods on the bullwhip effect investigating the effects of inventory variance in inventory costs, proving that the proper selection of forecasting methods and their parameters can help improve both ordering and inventory stability in supply chains. After all, the results confirmed the significant contribution of lead-time to the bullwhip effect and they concluded that "improved forecasting (using control charts) to control sensitivity to demand changes can reduce the contribution of longer lead-times to both the bullwhip effect and the inventory variance."

What Experts Say

Green & Armstrong (2015) state that when it comes to forecasting, some subjects such as how to choose a method, or forecastability are always controversial issues, and usually discussed with passion. This section brings together the opinion of experts in these matters to help address basic, and yet fundamental, concerns that arose during the research.

Simple versus Complex Forecasting

The supra-mentioned authors affirm that, despite the common belief among scientists that scientists should make every effort in favor of simplicity, a trend toward complexity remains popular among researchers, forecasters, and clients. The evidence is that the popularity of complexity increases as its incentive is strengthened in different ways. They reveal that researchers are rewarded for publishing in highly ranked journals, which favor complexity. Forecasters can use complex methods to provide forecasts that support decision-makers' plans. Clients may be reassured by the apparent sophistication implied by incomprehensibility. The titles and abstracts of forecasting papers in academic journals attest to the proliferation of complex methods. Not only managers, but also even practitioners and many researchers, are likely to struggle to comprehend typical forecasting papers (Green & Armstrong, 2015).

Simplicity in forecasting has the evident advantage of inspiring engagement by facilitating understanding. In addition, simplicity helps in detecting mistakes, significant omissions, irrelevant variables, unsupported conclusions, and even fraud. That said, there are still some reasons forecasters avoid simplicity. If the method is intuitive, reasonable, and simple, there is a fear that the client will probably not hire the forecaster, preferring, instead, to do their own forecasting. Moreover, complexity is often persuasive, even if its content is questionable. Researchers are aware that they can advance their careers by writing in a complex way. Clients might prefer (complex) forecasts that support their plans, developing complex methods that can be used to provide forecasts that support a desired outcome. It is all a matter of incentives, that is, how situations are rewarded and, in consequence, reinforced (Green & Armstrong, 2015).

In fact, incentives have been the cornerstone of human existence. An understanding of human behavior as it expresses itself in the sometimes-foggy mist of incentives is the key to clearly comprehending its function. Indeed, many people in different cultures and lifestyles, who might have a natural tendency to be honest, find subtle ways of, and reasons for, cheating to move forward their position, or even to support their preferences when incentives are strong enough (Levitt & Dubner, 2006).

Simplicity avoids, or minimizes, this possible misbehavior. Although the concept of simplicity in forecasting is difficult to define, simple forecasting, for the purpose of this research, will be considered as a process that is understandable and, mostly, auditable to forecast users. Specifically, the forecasting process must be understandable with respect to methods, representation of prior knowledge in models, relationships among the model elements, and relationships among models, forecasts, and decisions.

A good example illustrates the comparison between simple and complex forecasts: Bayes' method has the advantage of providing another way to incorporate prior knowledge in forecasting models. However, the method has the disadvantage of being too complex for most people to understand. Experts have been unable to find evidence that Bayesian approaches yield ex ante (based on forecasts rather than actual results) forecasts that are more accurate than forecasts from simple, evidence-based methods. The **Makridakis Competition** (also known as M-Competition), organized by the Prof. Spyros Makridakis, aims to evaluate and compare the accuracy of different forecasting methods, including tests of Bayesian forecasting for 1 to 18 period-ahead forecasts for 997 time series. As results, forecasts from simple methods, including naïve forecasts on deseasonalized data, were more accurate than Bayesian forecasts on the basis of mean absolute percentage error (MAPE). Forecasts from the benchmark deseasonalized single exponential smoothing method reduced error by 12.4 percent. Bayesian forecasts were not included in subsequent M-Competitions. The result was that simply averaging forecasts from different methods yields forecasts that reduced error, on average, by 5 percent across five studies compared to those from Bayesian approaches (Graefe, Küchenhoff, Stierle, & Riedl, 2015; S. Makridakis et al., 1982).

A simple combination of methods also provides an operational benchmark. Ahead, circumstances under which combining forecasts is beneficial in terms of results are explored.

Forecastability

As previously mentioned, popular approaches are based on comparisons of forecast accuracy with a benchmark such as the accuracy of a naïve forecast, where the actual value for a period is used as the forecast for the subsequent period (i.e. no change forecast). Metrics employed in this approach are ratios of forecast errors from a designated model to the naïve forecast errors, and include Theil's U statistic, the Relative Absolute Error or RAE, the Mean Absolute Percentage Error or MAPE, as well as the concept of forecast Value Added (Gilliland, 2013).

A benefit of using the naïve forecast as a benchmark is that it implicitly integrates the concept of volatility, since the naïve forecast has as much variation as the dependent variable itself. Errors associated with the naïve forecast are also probably a better predictor of forecastability for time series purposes than, for example, the Coefficient of Variation (CoV) because they measure period-to-period variation in the data. For instance, a series in which successive observations are highly positively correlated may drift away from the sample mean for several periods, thereby contributing to a high CoV. On the other hand, the errors from a naïve forecast will be relatively small because the successive observations are similar (Small & Wong, 2001).

Using a small set of easily calculated measures, such as RAE, Theil's U, MAPE, and others, does appear to provide an objective and rational platform for constructing a set of forecast-improvement strategies tailored to a product portfolio or segment, setting the goal to maximize the overall outcome (i.e. considering the measures altogether).

Compared to similar classifications but based on conventional error metrics, these parameters bring a number of benefits, such as allowing one to assess the forecast quality by comparing cross-model same-measure results; providing a quick and simple approach for dealing with items that are forecasted poorly and where the scope for improvement does not warrant the effort (the naïve forecast); and helping to set meaningful goals, individualized to the nature of the product and the dataset behavior within a portfolio.

Combining Forecasts

Combining forecasts can reduce errors arising from faulty assumptions, bias, or mistakes in data. This procedure refers to the averaging of independent forecasts. Sometimes also referred to as composite forecasting, this technique can be based on different datasets or different methods or both. The averaging is done using a rule that can be replicated, such as taking a simple average of the forecasts. To improve forecasting accuracy, one would combine forecasts derived from methods that differ substantially and draw from different sources of information. It is indicated, when not too costly, that it is sensible to combine forecasts from at least five methods, and to use formal mechanical procedures to combine forecasts, which should be fully described. The equal weighting rule is appealing because it is simple and easy to describe, and offers a reasonable starting point. If there is good domain knowledge, or information on which method should be most accurate, it is of good sense to use different weights. Either way, the use of trimmed mean (a method of averaging that removes the largest and smallest values before calculating the mean) is desirable if you combine forecasts resulting from five or more methods. Combining forecasts is especially useful when there is uncertainty about which method is most accurate and when it is important to avoid large errors. When compared with errors of the typical individual forecast, combining reduces errors (Js Armstrong, 2001).

Another way of combining forecasts is by decomposition. This technique provides a path to simplicity for many forecasting problems. Decomposition in forecasting consists of breaking down or separating a complex problem into simpler elements before forecasting each element. Decomposition can be used with any forecasting method. Actually, the routine is most useful when different elements of the forecasting problem are forecasted by different methods, when there is valid and reliable information about each element, when the elements are subject to different causal forces, and when they are easier to predict than the whole. The separated forecasts of the elements are then combined. Decomposition is, therefore, a strategy for simplifying problems.

The relationships among the elements of the decomposed problem should be simple. Decomposition based on additive relationships is ideal. This approach is also referred to as segmentation. Decomposition based on multiplicative relationships is somewhat more complex, bearing the risk that errors will multiply. In this approach, the elements are multiplied together to obtain a forecast of the whole. Multiplicative decomposition is often useful for simplifying complex problems (Green & Armstrong, 2015).

Forecasting and Inventory Control

If properly related, the choice of forecasting model directly affects the amount of investment needed to support any target level of customer service. Alternative forecasting models define, each, a unique tradeoff curve between inventory investment and customer service. Careful selection of the forecasting model for an inventory system can enhance the customer service provided by a fixed investment, shifting the tradeoff curve to a higher level in parallel with its respective axis that is maintaining a constant inventory investment (Gardner, 1990).

The characteristics of the time series of inventory demands should then be analyzed in order to identify alternative forecasting models. However, since it is difficult to measure the cost of delay time in any inventory system (i.e. greater lead time), it is similarly difficult to determine where the tradeoff curve should operate (i.e. what combination between inventory investment and service level is optimal for that particular system).

Tradeoff curves between inventory investment and customer service are broadly used to support decisions in inventory control. However, it is generally accepted practice to select a specific forecasting model for an inventory and thus to establish one tradeoff curve to work with (Gardner, 1990).

Additionally, little research was found showing a relation between forecasting and inventory decisions, yet not closely related to this study. For instance, Lee & Adam (1986) show that the size of forecast errors influences the choice of lot-sizing rules in material requirements planning systems for manufacturing inventories. In distribution inventories, Eppen & Martin (1988) show that forecast errors can seriously distort projections of customer service.

After presenting the key aspects of the theories that form the framework for the investigation to be carried out under the present research, it is necessary to provide the reader with appropriate details regarding the methodology that will be employed for both data collection and analysis. This is the subject of the next chapter of this paper.

III. Methodology

Overview

This chapter provides details about data collection and the methodology used for analysis. Specifics of both the statistical analyses and the criteria for ordering and selecting the best results are also provided.

Data Collection

Every research project needs data to help answer questions, to understand a specific issue or to test a hypothesis. According to Patton (2014), “When one examines and judges accomplishments and effectiveness, one is engaged in evaluation. When this examination of effectiveness is conducted systematically and empirically through careful data collection and thoughtful analysis, one is engaged in evaluation research” (Patton, 2014).

For this reason, special attention was given to this step. As the number of fields associated with each line item was considerable, it was important to determine which fields would be appropriate for this study. The selection of which fields to be collected was discussed with a software engineer, whose concern was with data integrity. Therefore, the engineer provided only audited data that was proven consistent. This gave reliability to the research, but also created constraints which prevented a more comprehensive study. As discussed in the previous chapter, for the same reasons that led to the conclusion that a simple forecast is better than a complex one, we will employ a simple data structure. The purpose is to make it as simple as possible for ease of use and understanding to ensure that the BAF Unit is able to adopt the potential recommendations without undue complication in either its integration or utilization, and without hindering the effectiveness and accuracy of the final results.

Researchers can obtain their data by getting it directly from the subjects they are studying. The resulting information is referred to as primary data. Another type of data, called secondary data, is that which has already been gathered by someone else. An advantage of using primary data is that researchers are collecting information for the specific purposes of their study. In essence, the questions the researchers ask are tailored to elicit the data that will help them with their study.

In this research, the data obtained were tailored to the research questions; that is, pulled from a large ERP database, from which only specific fields were chosen. The data needed by the present research can be considered primary, obtained directly from the organization in which the process is being analyzed. The system (ERP)'s engineer and manager have made the recorded data available.

The records were extracted from a PostgreSQL database, which is an open source object-relational database system, and was converted to a regular spreadsheet format, which allowed analyses to be performed. The data contained information about monthly sales and inventory over the course of five years (2010 to 2014) and the first seven months of 2015, as well as the items' prices.

Once received, the data have been comprehensively cleansed and organized in order to be prepared for the research. In this way, all the records have been screened for any sort of inaccuracy as well as any missing data points, in which case those data points would not be considered for this research. The analysis includes only data points for which a complete and precise set of records are available. This was a necessary effort prior to the beginning of the study itself. However, as the ERP system contains its own audit, fortunately there were no records with missing information, with the exception of those in which no information was recorded for the reasons described below.

Item Selection

The selection of the items to be studied was another matter addressed with care, at the risk of jeopardizing the entire study. The first concern was to make sure that the items were picked randomly. In order to accomplish this, since there were 240 data points with sales information, a new column was inserted, and 240 random numbers were created using the "=rand" function in Microsoft Excel®. All the cells were then ordered by the column containing the random numbers, from smallest to largest values to create a randomly ordered listing. The selection of the items occurred from the top row (with smallest random numbers) to the bottom.

Once the data was ordered, it was noticed that there were several missing data points in some of the data fields (i.e. zero sales) due to the implementation of new items, as well as item removal due to obsolescence. These missing data points differ from those observed due to lack of integrity in the data. Differently, in these cases the absence of information occurred, for instance, because an item became obsolete in January of 2012, which reduced the sales information available to 24 months only (from January of 2010 to December of 2011). In order to keep the utility of this research to its maximum, the items with missing data points were disregarded and discarded from the selection process.

Tools

The next step is to find proper tools with which analyses can be run. In this particular case, MS Excel® was the primary tool utilized, as well as SAS's statistical software solution: JMP®. Both are robust and reliable software, widely used for statistical analyses.

Methodology Used to Address the Research Questions

This section describes the methods and procedures used to address the formulated research questions.

1. What metrics are currently being utilized by the Supply Division?

To better understand the context of this study, SDS regulations were carefully examined in order to identify what categories of metrics were effective in the timeframe researched. It was then revealed that none of the documents examined disclose what types of measurements are to be used. Consequently, the information regarding this aspect of the research was obtained from the responses provided in the interview with the head of the Supply Division.

2. How can the forecasting process for uniform sales be improved in the SDS?

A proper forecast process is fundamental as a tool to optimize expenditures as well as to adjust inventory policy. Therefore, a deep analysis throughout a variety of forecast methods was necessary. Following are the methods evaluated in this research and their respective descriptions. They were chosen for being the most commonly used by practitioners as well for the ease of use.

Time Series Regression

Time series regression is a statistical method for predicting a future response based on the response history (known as autoregressive dynamics) and the transfer of dynamics from relevant predictors fitting straight lines to patterns of data. In a linear regression model, the variable of interest, called “dependent” variable, is predicted from *k* other variables, called “independent” variables, using a linear equation. Notation wise, Y indicates the dependent variable (in this study, referring to the total monthly sales of a specific item), while X_1, X_2, …, X_k represent the independent variables. Thus, the assumption is that the value of Y at time *t* in the dataset is determined by a linear equation:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \ldots + \beta_k X_{kt} + \varepsilon_t \tag{1}$$

Where β_0 is known as the intercept of the model, and is the expected value of Y when all Xs are equal to zero. β_i’s are the coefficients of the variables X_i. ε_t is the error term in time period *t*. The betas together with the mean and standard deviation of the epsilons are the parameters of the model. In this research, three types of models were created in order to verify whether there were any kinds of seasonal variations or trends in the data. The first model was created using dummy variables to represent the months of the year, assigning 1 to the observed month and 0 to all others. The formula used in this case was the following:

$$y_t = \beta_0 + \beta_1 t + \beta_2 M_1 + \beta_3 M_2 + \ldots. + \beta_{12} M_{11} + \varepsilon_t \tag{2}$$

The second model used trigonometric functions (sine and cosine) in an attempt to describe seasonal patterns. In order to test for seasonal trends, three seasonal periods (L) were used, with 4-month, 2-month and 1-month periods, for the trigonometric functions, as can be seen in the following formula:

$$y_t = \beta_0 + \beta_1 t + \beta_2 \sin\left(\frac{2\pi t}{L}\right) + \beta_3 \cos\left(\frac{2\pi t}{L}\right) + \beta_4 \sin\left(\frac{4\pi t}{L}\right) + \beta_5 \cos\left(\frac{4\pi t}{L}\right) + \varepsilon_t \tag{3}$$

Then, an autocorrelation component was added to a simplified version of the trigonometric function. This last factor was obtained by multiplying the previous residual (ε_{t-1}) to a correlation coefficient (named ϕ) between ε_t and ε_{t-1}. As in the previous model, three seasonal periods (L) were used, with 4-month, 2-month and 1-month periods, for the trigonometric functions. The formula for this model is the following:

$$y_t = \beta_0 + \beta_1 t + \beta_2 \sin\left(\frac{2\pi t}{L}\right) + \beta_3 \cos\left(\frac{2\pi t}{L}\right) + \varepsilon_t \tag{4}$$

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + a_t \tag{5}$$

Here *a* is assumed to be an error term (often called a random shock) with mean zero, which satisfies the constant variance, independence, and normality assumptions. Afterwards, a 4th-order polynomial model was developed in an attempt to yield a better prediction than the linear regression equation provides, with the following formula:

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_2 t^3 + \beta_2 t^4 + \varepsilon_t \tag{6}$$

Decomposition

Although decomposition models are strictly an intuitive approach, they are very useful when a time series data exhibits trend, seasonal, and cyclical effects, and parameters of the time series do not change over time. In practice, this method provides a forecasted point estimate "decomposing" the data into distinct components. For this study, two decomposition methods were used, namely multiplicative and additive, each one with three components: trend, seasonal, and cyclical. Following is an overview of these three components and a brief explanation of how they affect the behavior of the time series. Subsequently, the description of the models will be presented.

Trend

A trend exists when there is a long-term increase or decrease in the data. This change over time is not necessarily linear. Sometimes a trend "changing direction" will be referred to when it might go from an increasing trend to a decreasing trend or vice versa (Hyndman & Athanasopoulos, 2014).

Seasonal

Seasonality can be defined as the periodic fluctuations in a determined pattern. A common example can be found in retail sales, which tend to peak for the Christmas season and to decline quickly after the holidays. Thus, the time series of retail sales will typically show increasing sales from October through December, followed by rapidly declining sales in January (NIST/SEMATECH, 2013).

Cyclical

A cyclical pattern exists when data exhibit a rise-and-fall pattern that does not occur within a fixed period. The duration of these fluctuations is usually at least 2 years (Hyndman & Athanasopoulos, 2014).

Additive and Multiplicative Methods

The multiplicative decomposition model is useful when the modeling time series displays increasing or decreasing seasonal variation. The equation for the multiplicative decomposition model is the following:

$$y_t = TR_t * \mathrm{SN}_t * \mathrm{CR}_t \tag{7}$$

The additive decomposition model is appropriate when the time series exhibits constant seasonal variation (Bowerman et al., 2005).

The equation for additive decomposition model is the following:

$$y_t = TR_t + \mathrm{SN}_t + \mathrm{CR}_t \tag{8}$$

Exponential Smoothing

Exponential smoothing is the most effective forecasting method when components of the time series change over time. This method weighs the actual time series values unequally, with more importance placed on the most recent data rather than earlier historical data. There are several models in exponential smoothing as expressed in (Bowerman et al., 2005), each method with a unique power to make predictions.

Simple Exponential Smoothing

This smoothing assumes that the time series has no systematic trend or seasonal components. Nevertheless, it has a mean (or level), which may change over time. Given such a form of data, a practical approach is to take a weighted average of past values (Bowerman et al., 2005; NIST/SEMATECH, 2013).

The equation of the simple exponential smoothing method is the following:

$$Y_t = \beta_0 + \varepsilon_t \tag{9}$$

Holt's Trend Corrected Exponential Smoothing

This method to forecast time series involves introducing a term to take into consideration the possibility of a series exhibiting some sort of trend, which can be constant or non-constant. The equation of Holt's trend corrected exponential smoothing method is the following where the additional term represents a fixed rate of change:

$$Y_t = (\beta_0 + \beta_{1t}) + \varepsilon_t \tag{10}$$

Holt-Winters

Holt's method can be enhanced to deal with time series containing both trend and seasonal components. The Holt-Winters method has additive and multiplicative versions.

The Additive Holt-Winter method is more useful for constant seasonal variation while the multiplicative Holt-Winter method is more useful for increasing seasonal variation (Bowerman et al., 2005).

The equations of the Holt-Winter methods are the following:

Additive Holt-Winters:

$$Y_t = (\beta_0 + \beta_{1t}) + SN_t + \varepsilon_t \tag{11}$$

<u>*Multiplicative Holt-Winters*</u>:

$$Y_t = (\beta_0 + \beta_{1t}) \times SN_t \times IR_t + \varepsilon_t \qquad (12)$$

Box-Jenkins

The Box-Jenkins method, named after the statisticians George Box and Gwilym Jenkins, applies autoregressive moving average ARMA models to find the best fit of a time series model to past values of a time series. The Box-Jenkins ARMA model is a combination of the AR (autoregressive) and MA (moving average) models. ARMA models aim to describe the autocorrelations in the data (NIST/SEMATECH, 2013).

The first step in developing a Box-Jenkins model is to determine whether the time series is stationary and if there is any significant seasonality that needs to be modeled. A stationary time series is one whose properties do not depend on the time at which the series is observed. Analyzing the data in Time Series Basic Diagnostics in JMP®, we can identify the behavior of both the Sample Autocorrelation Function (SAC) and the Sample Partial Autocorrelation Function (SPAC). A nonstationary time series will exhibit a SAC function that dies down slowly, while a stationary series will exhibit a SAC that either cuts off or dies down quickly.

Box and Jenkins recommend differencing non-stationary series one or more times to achieve stationarity. Doing so produces an ARIMA model, with the "I" standing for "Integrated". In addition, if there was an increasing trend in data, a pre-differencing transformation had to be performed in order to remove it using, for example, the natural logarithm.

If the data series were already stationary, no differencing transformation was added to the potential models. As for an initial investigation of the SAC and SPAC, when the SAC died down, and the SPAC cut off, an autoregressive model was selected. When the SAC cut off, and SPAC died down, a moving average model was selected. Finally, when both died down, a mixed model was selected. This procedure enabled initial combinations of models, as a starting point from which the determination of the best models was pursued.

ARIMA models are also capable of modeling a wide range of seasonal data. A seasonal ARIMA model is formed by including additional seasonal terms in regular ARIMA models, being necessary to determine the number of periods per season (Bowerman et al., 2005; Hyndman & Athanasopoulos, 2014; NIST/SEMATECH, 2013).

The following chart may be of some help in identifying the proper ARIMA model: (NIST/SEMATECH, 2013)

Table 1. ARIMA identification

SHAPE	INDICATED MODEL
Exponential, decaying to zero	Autoregressive model. Use the partial autocorrelation plot to identify the order of the autoregressive model.
Alternating positive and negative, decaying to zero	Autoregressive model. Use the partial autocorrelation plot to help identify the order.
One or more spikes, rest are essentially zero	Moving average model, order identified by where plot cuts off.
Decay, starting after a few lags	Mixed autoregressive and moving average model.
All zero or close to zero	Data is essentially random.
High values at fixed intervals	Include seasonal autoregressive term.
No decay to zero	Series is not stationary.

Choosing the methods

A template spreadsheet containing 19 models, encompassing all the methods described above, was the foundation of the data analysis. Each item, out of 240 available items, was chosen individually at a time, following the table with the randomly ordered items, from the top to the bottom.

As the sales information was plugged into the template, it was replicated to every model by linking the cells from each model tab to the original where the numbers were pasted. Subsequently, only minor adaptations were required to calculate a forecast at each model tab as well as all other formulas to calculate the accuracy parameters (i.e. residual statistics) necessary to assess the quality of the models. The parameters will be listed and described in next section.

Finally, a tab containing a comparison chart with links to all parameters of all models was filled, and the conditional formatting tool found in MS Excel® highlighted the five best results of each parameter among all models. From all 19 models, the five with more highlighted parameters were selected to compose another comparison chart, with each item and its respective five best models.

The latter step was repeatedly performed, with the creation of one spreadsheet for each item evaluated, and the construction of the comparison chart with each item observed and its respective five best models. Every time one model appeared five times in the second comparison chart, this model was elected. The number of items evaluated was selected in order to yield to yield five elected models. More details of this procedure will be provided when describing the construction of the comparison chart.

Validity Assumptions

An important step, after having the five best models, is to verify whether the validity assumptions hold for all of them. That is, the residuals should be tested regarding normality, independence, and constant variance. Informal procedures such as diagnostic plots of residuals versus time, as pertains to time series, are recurrently used to assess the validity of these assumptions as well as to identify possible outliers. Violation of the latter two of the assumptions (independence and constant variance) required root data transformation or removal of outlying observations.

3. How to best assess accuracy of forecasting sales in the SDS?

A good approach to test the expectations of a model and to convincingly compare its forecasting performance against other models is to perform an out-of-sample validation. To do so, 12 data points of the sample data were withheld from the model estimation process for post validation, leaving 48 data points for model estimation (totaling 60 data points, or 5 years worth of data). The data which were not held out (i.e. the estimation period) were used to help select the model and to estimate its parameters. Hence, the selected model is used to make predictions for the holdout data in order to perceive how accurate they are and to determine whether their residual statistics are similar to those that the model made within the sample of data that was fitted, a process called validation. Forecasts made in the estimation period are not fully "authentic" because data on both sides of each observation are used to help determine the forecast.

The model is then tested on data within the validation period, and forecasts were generated beyond the end of the estimation and validation periods. For the study’s purposes, only estimation and validation forecasts sufficed, and no forecasts beyond those periods were calculated.

In order to assess the results obtained from all 19 models, five parameters were calculated in a way to quantify and compare them: the Sum of Squared Errors (SSE), the coefficient of determination (R^2), the Mean Absolute Percentage Error (MAPE), the Relative Absolute Error (RAE), and the Theil's U-statistic (Theil's U). Following are the descriptions of the parameters utilized.

Parameters

Sum of Squared Errors

This parameter is obtained by simply squaring each error term, and adding them. The result by itself does not say much for not having an upper boundary, but for purposes of comparison it can be very useful. Since this research compared different forecasts, the SSE helped in selecting the best fits of different forecasting methods (Bowerman et al., 2005).

The formula is as presented below:

$$\sum (y_i - \hat{y}_i)^2 \tag{13}$$

Coefficient of determination

The coefficient of determination (R^2) is a number between zero and one that indicates how well data fit a statistical model. Because this number represents a percentage, it can be easily understood. An R^2 of 1 indicates that the regression line perfectly fits the data, while an

R^2 of 0 indicates that the line does not fit the data whatsoever. It can be calculated by dividing the explained variation in data by the total variation (Bowerman et al., 2005).

The explained variation is denoted by:

$$\sum(\hat{y}_i - \bar{y})^2 \tag{14}$$

The total variation is denoted by:

$$\sum(y_i - \bar{y})^2 \tag{15}$$

The coefficient of determination (R^2) is then obtained by dividing the explained variation by the total variation.

$$R^2 = \frac{Explained\,Variation}{Total\,Variation} \tag{16}$$

Mean Absolute Percentage Error

The MAPE expresses forecasting accuracy as a percentage measure of the error. It shows how much the forecast is off (e.g. a MAPE of 20 means that the forecast is off, on average, by 20%).

Measures based on percentage errors have the disadvantage of possibly being infinite or undefined if $y_i = 0$ for any i in the period of interest (Bowerman et al., 2005).

It is defined by the formula:

$$\frac{1}{n}\sum_{t=1}^{n} 100\,\frac{|\hat{y}_i - y_i|}{y_i} \tag{17}$$

Relative Absolute Error

The Relative Absolute Error (RAE) is a metric where actual forecast error is compared to a naïve forecast error in a ratio basis, placing actual forecast error in the numerator and the naïve forecast error in the denominator. An RAE greater than one means that a naïve forecast is probably better than the method being tested. Lower RAEs are preferable, because this result shows that the forecast error is proportionally smaller than one from a naïve forecast. The RAE is represented by:

$$RAE = \frac{actual\ forecast\ error}{naïve\ forecast\ error} \tag{18}$$

Theil's U-statistic

The accuracy measure U-statistic, developed by Theil H. (1966), emphasizes the importance of large errors, squaring them as well as providing a relative basis for comparison with naïve forecasting methods, as in RAE (Small & Wong, 2001). As it calculates a ratio between a determined model and a naïve forecast, the lower the value, the better. If $U = 1$, it means that the forecasting method being used is as good as the naïve forecast. If $U > 1$, it means that the naïve forecast has better performance that the method being used.

The interpretation of the ranges of output from the statistic can be shown as follows (Makridakis, Wheelwright, & Hyndman, 1998):

U = 1: A naive forecasting method is as good as the method in question.

U < 1: The forecasting method being used is better than a naive forecast.

U > 1: The naive forecast outperforms the method in question.

Mathematically, Theil's U-statistic is defined as:

$$U = \sqrt{\frac{\sum_{t=1}^{n-1} \left(\frac{\hat{y}_{t+1} - y_{t+1}}{y_t}\right)^2}{\sum_{t=1}^{n-1} \left(\frac{y_{t+1} - y_t}{y_t}\right)^2}} \tag{19}$$

Comparison charts

These five parameters (SSE, R^2, MAPE, RAE, and Theil's U) were calculated for both estimation and validation periods, with exception of R^2, which is calculated based on estimation data only.

After calculating all parameters, a comparison chart for the estimation and for validation periods was filled automatically by linking the cells from the comparison chart to their original respective places in each of the models' tabs. In order to better assess the results, another table was built, consolidating the parameters from both estimation and validation periods. This consolidation was done by averaging the MAPE, RAE, and Theil's U obtained in estimation and validation, by adding the SSE's, and by simply repeating the R^2, calculated only for the estimation period.

Finally, the conditional formatting tool found in MS Excel® highlighted the five best results of each parameter among all models. Each of all the 19 models was assessed, and the number of highlighted consolidated parameters was counted. For each item, the five models with the highest number of highlighted consolidated parameters were selected to constitute another comparison chart, for model selection, containing only the items and their respective selected models. The number of times a model appeared was dynamically counted, and when a model reached five appearances, that model was elected.

As a tiebreaker, the rule established was to look for the models with highest number of highlights in the validation chart and in the estimation chart, respectively. That is, every time two or more models had the same number of highlighted cells in the consolidation chart, it led back, respectively, to the validation chart, and to the estimation chart. This procedure was reiterated as much as needed to elect five models.

4. Is it possible to build an algorithm where historical sales data can be evaluated and the best forecast suggested?

The goal of this research is to obtain results that can be used in practice to improve the process of forecasting in SDS, in such a way that by providing the managers with reliable numbers to work with, it enables them to review the inventory policy currently effective in SDS. For this reason, in addition to accuracy and reliability, special attention was given to the ease of use of the formulas and the display of the results.

After running all tests with randomly selected items, the researcher decided to test the elected models in other items and check the results. This time, items were tested that comprise the Air Battle Uniform (ABU): the coat, trousers, t-shirt, and hat.

Although the residual statistics (SSE, RAE, MAPE, and Theil's U) as well as the R^2 yield decent estimates of how accurate the forecast is, they may understate the magnitude of the errors that will be made when the model is used to predict the future: this is due to the possibility that the data may have been over-fitted. Specifically, by ruthlessly minimizing the sum of squared errors, the model may have accidentally fitted some of the existing "noise" in the estimation period data as being a "signal". For this reason, the present methodology intends to mitigate the possibility of over-fitting the models by using a wide combination of models and items.

As a result, another parameter was calculated, by multiplying the residuals by the price of the respective item for each of the 5 models, obtaining, basically, the cost of the residuals. This new parameter, “cost of the residuals”, was included in the consolidation comparison chart, working also as a new tiebreaker, with precedence over the others.

Once the methodology aspects of this research are delimited, the analysis and the results can be presented. In Chapter IV each research question will be addressed and all methodology presented will be put into practice.

IV. Results and Analysis

Overview

This chapter aims to stage the results of this research, obtained after applying the methodology proposed to answer the research questions formulated.

The first step was then to understand how the forecasting process is currently being performed, with the purpose of extracting essential information that could guide the analysis. Therefore, the first section of this chapter is assigned to the description of the forecasting process, obtained by an interview with the head of the Supply Division. This chapter then presents the results of the models assessed, with emphasis on those models elected, that is, those with the best results. After that, a practical use of the models and relevant aspects of accuracy evaluation are provided, as well as a comparison chart where the results can be easily displayed, and finally some dispositions regarding the findings will announce the last chapter.

Forecasting in place at SDS

First off, it is necessary to point out that inventory control, lead times, and forecasting are strictly related. In the description of the process, one can see the frequency with which these words appear. However, according to the interviewee, there are no metrics currently being utilized regarding inventory policies, such as inventory holding cost, obsolescence, cost of equipment to handle inventory, operating costs, insurance premiums, and opportunity costs. In addition, accurate information about lead times and others is not being considered at all.

Therefore, this research strived to maintain focus on the calculations involving the forecasting process, not dealing with their interaction. Thus, an important assumption in this study was that an accurate forecasting process results in valuable data that serves as reliable foundation to allow management to make reasonable decisions on purchasing, given the actual inventory policies. In other words, forecasting processes will affect inventory policies, but not the other way around.

Another premise to be considered in this paper comes from the consistency of the inventory levels. The SDS warehouse is equipped with an automated vertical storage, which uses a robot combined with RFID antennas to execute and verify all in and out movement. Each item contains an RFID tag, so, once the item is placed in its respective box, the robot carries it through a conveyor belt into a chamber where the antennas will read all tags there, while a scale weighs the box, with no human interaction. That is to say, the error margin in the inventory is zero.

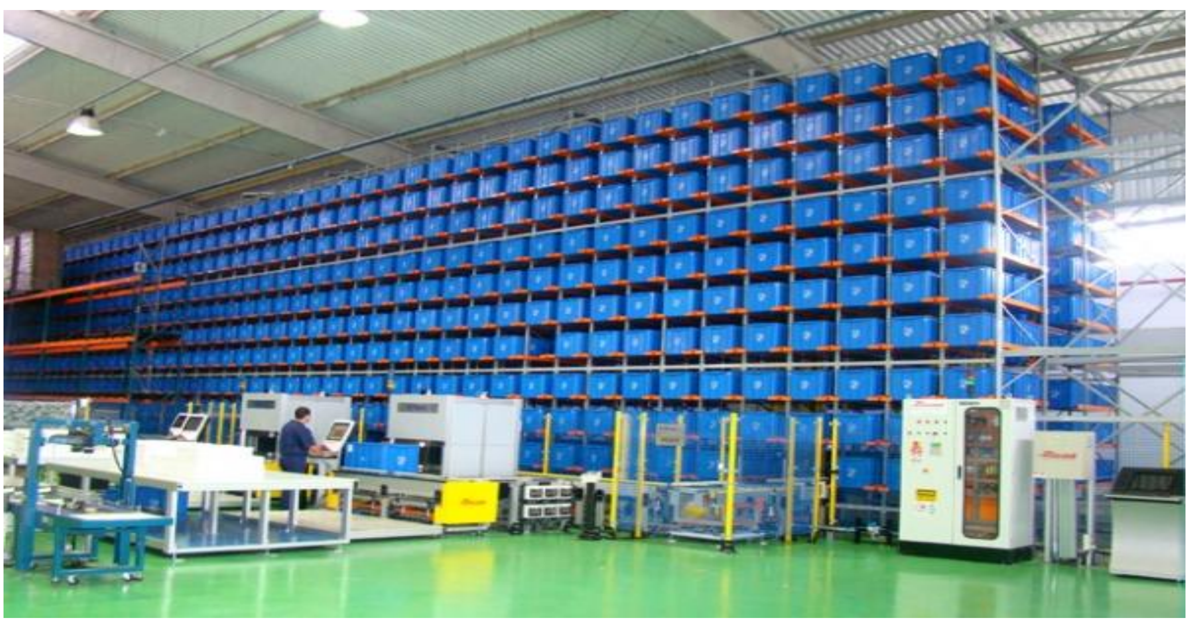


Figure 1. Automated vertical storage in SDS

Another important detail to be mentioned is that, on average, the lead time for the items is 11 months for purchases and 13 months for garment sewing, when the fabric is purchased and stored in SDS. Then a company is hired to sew the uniforms. Therefore, a piece of uniform can take between one month and one year to have its replenishment completed. For this reason, there is not a specific timeframe when the forecasting is performed in SDS. So to speak, it happens whenever an item reaches the reorder point. Regarding the forecasting itself, the Enterprise Resource Planning (ERP) system currently calculates in isolation the needs for each store for the next given number of months, using an adaptation of the Moving Average method, where the peaks are recorded and the actual forecast is either the Moving Average, or the last record, whichever is higher. This way, each forecast is calculated and multiplied by the number of periods desired, then subtracted by the inventory level at that particular store. After the forecasts for all stores are consolidated, the main warehouse inventory is then subtracted from the total amount.

As for the whole system, considering all the stores and the warehouse, it was detected that the volume of sales and the inventory level in SDS were incompatible. This led to a discussion of which metrics are currently being utilized by the Supply Management Division, specifically the ones that affect the forecasting process. Comparing the sales levels with inventory for these items, one could notice an extreme difference between them, with inventory numbers spiking into the thousands, while sales were usually in the low hundreds.

Figure 2 (below) depicts the contrast of sales level relative to the inventory held for a particular item in SDS.

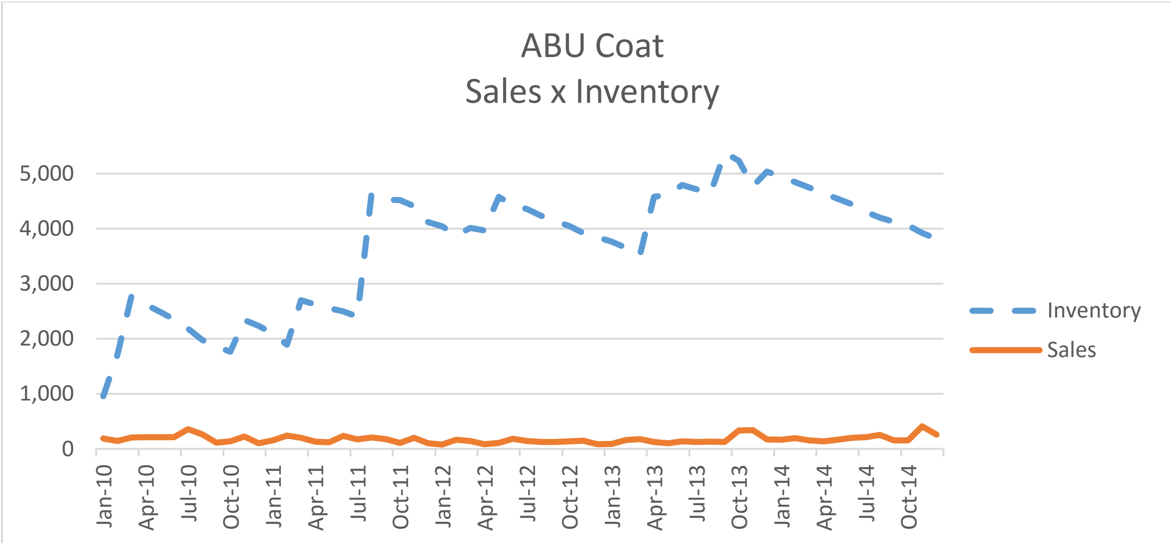


Figure 2. Example of the comparison between Sales and Inventory

Under these circumstances, one could conclude that forecasting in SDS is currently being performed based on empirical inferences and that a scientific methodology for forecasting must be implemented.

Forecasting methods

A template spreadsheet containing 19 models was built as the foundation of the data analysis. Some models were very simple, such as the Simple Linear Regression; others, more complex, such as the ARIMA. The idea was to gather models that could capture behaviors as simple as a mere trend as well as more complex jump shifts.

For all the data collected, 67 months' worth of data was available for this research. As either 12 or 4 month-periods were considered, the researcher decided to drop the seven final months as they represented an incomplete year and work with 60. From these, 48 months were considered for estimation of the forecast parameters, and 12 for validation.

After testing an item for all models, a comparison chart gathered the results of the five parameters considered for this research, four of them based on the residual statistics, and one based on the relationship between the mean and the forecast or actual values, highlighting the five best results for each parameter, as can be seen in Table 2 below.

Table 2. Example of a comparison chart for the estimation period

	Value of Interest – Estimation				
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	1,226.1710	0.0206	59.9948	0.8515	1.3014
Trend	1,141.6839	0.0881	53.8485	0.7643	0.9862
Dummy	571.8196	0.5432	41.8358	0.5938	0.9800
Trigonometric L=4 years	1,128.7502	0.0984	52.8685	0.7504	0.9997
Trigonometric L=2 years	882.2283	0.2953	40.3655	0.5729	0.5013
Trigonometric L=1 year	914.0196	0.2699	52.8933	0.7507	1.1373
Autocorrelation L=4 years	1,113.1735	0.0935	53.2665	0.7560	0.8206
Autocorrelation L=2 years	1,051.8715	0.1373	49.0765	0.6966	0.8150
Autocorrelation L=1 year	961.5000	0.2175	53.2229	0.7554	0.8556
Decomposition Multiplicative (12 months)	306.4304	0.8353	25.3237	0.3594	0.4042
Decomposition Multiplicative (4 months)	516.3118	0.4808	33.8308	0.4802	0.4660
Decomposition Additive (12 months)	277.2919	0.7396	24.3381	0.3454	0.2867
Simple Exponential Smoothing	1,284.1096	0.0228	56.7411	0.8054	1.1757
Holt's Trend	1,226.1710	0.0206	59.9948	0.8515	1.3014
Additive Holt-Winters (4 months)	1,238.2103	0.3118	53.9862	0.7663	1.3198
Additive Holt-Winters (12 months)	710.9401	0.6943	42.8238	0.6078	0.6889
Multiplicative Holt-Winters (4 months)	1,298.4788	0.5530	55.7680	0.7915	0.8332
Multiplicative Holt-Winters (12 months)	1,410.9289	0.6584	51.4371	0.7301	0.7043
ARIMA	1,026.5668	0.1478	57.4674	0.8157	0.8466

Next, another comparison chart displayed the results for the validation period, that is, for the forecast calculated without using existing data to help determine the forecast. For this table, only the parameters based on the residuals were applied, leaving the R^2 out. Table 3 below shows an example of a comparison chart for validation data.

Table 3. Example of a comparison chart for the validation period

	Value of Interest - Validation			
Model	**SSE**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	13,528.4168	82.8485	1.0956	1.4751
Trend	2,009,630.7776	1,342.1629	17.7495	29.6890
Dummy	47,847.5331	111.9778	1.4809	2.9800
Trigonometric L=4 years	111,270.8230	276.1695	3.6522	6.5183
Trigonometric L=2 years	70,196.9046	184.3638	2.4381	5.0440
Trigonometric L=1 year	31,970.5489	107.5402	1.4222	2.4224
Autocorrelation L=4 years	619,539.5023	803.9248	10.6316	16.4761
Autocorrelation L=2 years	57,632.9160	166.4128	2.2007	4.4637
Autocorrelation L=1 year	24,462.0204	93.1640	1.2321	2.0569
Decomposition Multiplicative (12 months)	4,794.6801	40.3845	0.5341	0.5971
Decomposition Multiplicative (4 months)	13,900.4601	87.9897	1.1636	1.4963
Decomposition Additive (12 months)	8,370.1682	60.8385	0.8046	1.0685
Simple Exponential Smoothing	28,044.2930	176.2001	2.3302	3.1482
Holt's Trend	15,754.8123	105.4593	1.3947	1.8678
Additive Holt-Winters (4 months)	22,783.0328	101.0333	1.3361	2.4623
Additive Holt-Winters (12 months)	58,522.7549	199.9418	2.6441	2.9906
Multiplicative Holt-Winters (4 months)	16,793.5545	115.5375	1.5279	2.0028
Multiplicative Holt-Winters (12 months)	140,936.0633	426.5862	5.6414	7.8055
ARIMA	65,741.9077	270.4882	3.5771	4.2719

The last step of this phase is a consolidation chart that combined the results from both the estimation and validation periods by averaging the MAPE, RAE, and Theil's U obtained in estimation and validation, by adding the SSE's, and by simply repeating the R^2, calculated only for the estimation period. Table 4 below shows an example of a consolidation chart.

Table 4. Example of a consolidation chart

	Value of Interest - Consolidation				
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	180,527.11	0.3494	65.1018	1.0782	1.2265
Trend	2,151,176.53	0.4485	690.9439	9.3197	15.2065
Dummy	131,689.47	0.6733	72.3911	1.1078	1.9764
Trigonometric L=4 years	237,737.93	0.5073	155.4502	2.2151	3.6831
Trigonometric L=2 years	199,954.70	0.4945	110.4334	1.6279	2.9129
Trigonometric L=1 year	186,879.25	0.3965	76.5482	1.2213	1.7335
Autocorrelation L=4 years	745,144.26	0.4266	420.6473	5.7343	8.5928
Autocorrelation L=2 years	180,731.44	0.4772	100.9769	1.4984	2.6330
Autocorrelation L=1 year	166,510.37	0.4268	67.7083	1.0892	1.4915
Decomposition Multiplicative (12 months)	41,044.96	0.9406	27.9463	0.4407	0.5265
Decomposition Multiplicative (4 months)	56,241.53	0.6760	54.9607	0.8274	1.0729
Decomposition Additive (12 months)	39,056.14	0.7748	38.7780	0.5895	0.7553
Simple Exponential Smoothing	214,062.62	0.4300	113.1985	1.7272	2.0407
Holt's Trend	193,386.21	0.3418	77.6350	1.2552	1.4307
Additive Holt-Winters (4 months)	175,049.11	0.5099	71.8665	1.1463	1.7153
Additive Holt-Winters (12 months)	132,993.37	0.8426	114.8703	1.6558	1.8376
Multiplicative Holt-Winters (4 months)	160,707.24	0.5826	77.8870	1.2146	1.4289
Multiplicative Holt-Winters (12 months)	275,363.28	0.6117	238.0084	3.3743	4.4011
ARIMA	215,150.84	0.5518	154.9664	2.2303	2.5577

The criterion established was to select the models with more highlighted parameters, meaning that more parameters of that model were among the best five. In the case when two or more models have the same number of highlighted parameters, the tiebreaker rule established was to look for the highest number of appearances in the validation chart and the estimation chart, respectively.

For each item, the respective best five models were copied to a table where the number of appearances of each model was totaled. As the goal was to elect five models, each item, out of 240 items, was picked one at a time, following the table with the randomly ordered items, from the top to the bottom. With the intention of assuring that a model was not selected just for being overly fitted to a particular distribution, each model has to appear for at least five different items so that it could be elected, that is, considered one of the five to be adopted in SDS's forecasting process.

The randomly selected items were each assessed individually, all parameters were calculated and the comparison chart was filled. For that item, the five models with the highest number of highlighted consolidated parameters were selected. Table 5 below shows the 11 items necessary to yield at least five appearances for five different models. Table 5 below shows the five best models for each of the 11 items.

Table 5. Models selection

Item 1	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)	Additive Holt-Winters (12 months)	Simple Exponential Smoothing
Item 2	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)	Simple Exponential Smoothing	Multiplicative Holt-Winters (12 months)
Item 3	Autocorrelation L=2 years	Autocorrelation L=1 year	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)

Item 4	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)	Simple Exponential Smoothing	ARIMA
Item 5	Decomposition Multiplicative (12 months)	Decomposition Additive (12 months)	Additive Holt-Winters (4 months)	Additive Holt-Winters (12 months)	Multiplicative Holt-Winters (4 months)
Item 6	Autocorrelation L=1 year	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)	Additive Holt-Winters (12 months)
Item 7	Dummy	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)	Additive Holt-Winters (12 months)
Item 8	Dummy	Decomposition Multiplicative (12 months)	Decomposition Additive (12 months)	Additive Holt-Winters (12 months)	ARIMA
Item 9	Dummy	Autocorrelation L=1 year	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)
Item 10	Dummy	Autocorrelation L=1 year	Decomposition Multiplicative (12 months)	Decomposition Additive (12 months)	Additive Holt-Winters (12 months)
Item 11	Autocorrelation L=1 year	Decomposition Multiplicative (12 months)	Decomposition Multiplicative (4 months)	Decomposition Additive (12 months)	Additive Holt-Winters (12 months)

Each row shows the five best models for that item. One example was highlighted to give a picture of the process. The Autocorrelation method with 1-year seasonal period was among the best five models for the items 3, 6, 9, 10, and 11.

Subsequently, the models selected were the Decomposition Multiplicative (12 months), and the Decomposition Additive (12 months), with 11 appearances, followed by the Decomposition Multiplicative (4 months), with 8 appearances, the Additive Holt-Winters (12 months), with 7 appearances, and the Autocorrelation (1 month), with 5 appearances. One could notice that some models appear for almost every item, while the model used as an example (Autocorrelation method with 1-year seasonal period) was the last to be selected, appearing exactly five times.

The final accountability, after 11 items tested was as displayed below in Table 6.

Table 6. Final models' accountability

Models	Appearances
Decomposition Multiplicative (12 months)	11 times
Decomposition Additive (12 months)	11 times
Decomposition Multiplicative (4 months)	8 times
Additive Holt-Winters (12 months)	7 times
Autocorrelation L=1 year	5 times
Dummy	4 times
Simple Exponential Smoothing	3 times
ARIMA	2 times
Multiplicative Holt-Winters (12 months)	1 time
Autocorrelation L=2 years	1 time
Additive Holt-Winters (4 months)	1 time
Multiplicative Holt-Winters (4 months)	1 time

Practical use of the algorithm

As the goal of this research is to obtain a practical tool that improves the process of forecasting in SDS, special attention was given to the ease of use of the formulas and the display of the results. In such a way, the researcher included another parameter that can help the decision making. By multiplying the residuals by the price of the item, the result is the cost of the models' error. In other words, this is how much that particular forecast model costs, since each residual represents how much that forecasted value deviates from the actual value.

Therefore, the consolidation chart now contains only those five elected models previously mentioned, with the cost of the error being assigned to a sixth column. Besides, only the best of each parameter is highlighted now, in order to reveal the best model, which should be used for that particular item. The previous criteria changes only in regards to the first tiebreaker, which now, is the lower cost of the error. The next two tiebreakers remain as they were. In other words, look for the highest number of highlights in the validation chart and in the estimation chart, respectively. From now on, the set with the five elected models, as well as the comparison chart and the consolidation chart, are called "algorithm".

In order to test the algorithm, this paper now shows the results from its application to four items that comprise the Air Battle Uniform (ABU): the coat, trousers, t-shirt, and hat. Subsequently, the validity assumptions test for these items will be displayed.

Coat

After applying the algorithm to this item, the results are as depicted in Table 7 below.

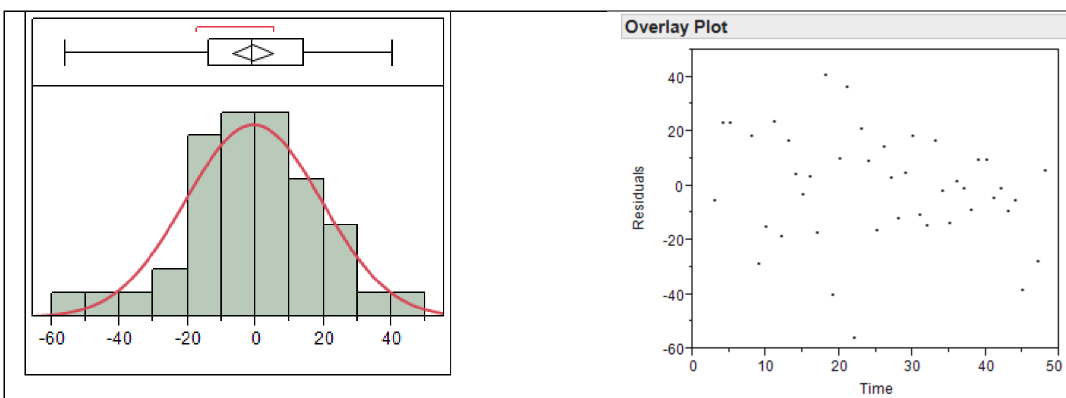
Table 7. Algorithm output for coat sales forecast

	Value of Interest – Consolidation					
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**Cost of the error**
Autocorrelation L=1 year	204,543.6679	0.2391	28.5805	0.9808	0.7784	$4,037.15
Decomposition Multiplicative (12 months)	64,027.1635	0.7972	13.3004	0.4583	0.3733	$790.30
Decomposition Multiplicative (4 months)	172,807.1323	0.6755	23.6598	0.8476	0.7528	$10,323.66

Decomposition Additive (12 months)	60,302.4162	0.7457	13.1775	0.4525	0.4090	$1,093.93
Additive Holt-Winters (12 months)	175,396.6739	0.7283	26.9339	0.9349	0.7098	$3,213.44

The table displayed above is a case of two models with the same number of highlighted parameters. Thus, since the Decomposition Multiplicative (12 months) has the smaller amount of value in its residuals, this model is chosen for this item. Additionally, MAPE and RAE for the two models are virtually the same which removes these two values for comparison purposes between these two models. Since the first tiebreaker is sufficient for choosing the model, the other comparison charts play no role in this choice.

As far as the validity assumptions, this item presented no problems of normality, despite a few outliers caused by an eventual sudden peak in sales in July of 2010 as well as October and November of 2013. Excluding the data points that refer to those outliers, all validity assumptions hold. Figure 3 below displays the histogram of the residuals, with the Shapiro-Wilk Test result, and the overlay plot of the residuals versus time.



Shapiro-Wilk W Test

W	Prob<W
0.982133	0.7314

Figure 3. Validity assumptions for coat forecasting residuals

Following is the graph demonstration (Figure 4) of the actual and forecasted values. It can be seen how well the patterns are captured, even during the validation period, that is, considering the behavior captured in the estimation period to forecasting these 12 months.

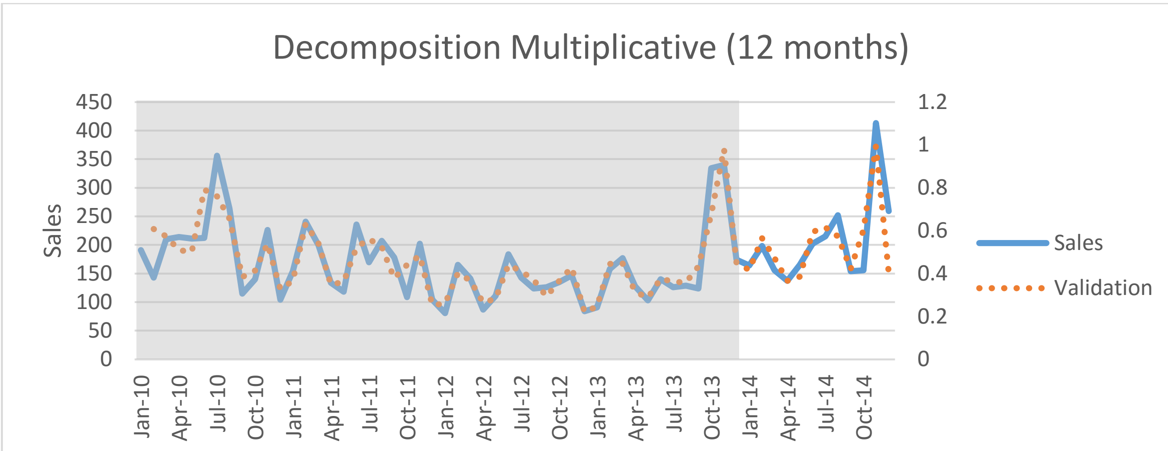


Figure 4. Graphical output of forecasted values

Trousers

The Table 8 below contains the results of the algorithm for forecasting the trousers sales.

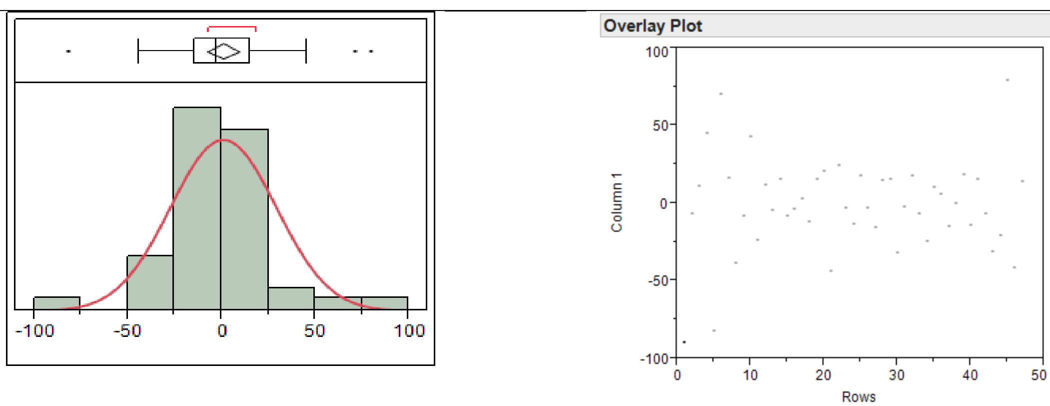
Table 8. Algorithm output for trousers sales forecast

	Value of Interest – Consolidation					
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**Cost of the error**
Autocorrelation L=1 year	198,444.5140	0.3582	22.2766	0.9378	0.7865	$4,550.50
Decomposition Multiplicative (12 months)	57,133.0881	0.8074	10.2093	0.4179	0.3783	$650.57
Decomposition Multiplicative (4 months)	247,106.3307	0.7312	27.8117	1.3174	1.0461	$14,918.88

Decomposition Additive (12 months)	51,646.0355	0.7807	10.5644	0.4342	0.4009	$760.22
Additive Holt-Winters (12 months)	167,357.4230	0.7387	18.8638	0.7630	0.7134	$256.48

As it can be seen, four out of the six parameters are best using the Decomposition Multiplicative (12 months). Therefore, no further analysis was necessary for choosing this model.

Regarding the validity assumptions, this item presented no problems of normality, despite one outlier caused by an eventual sudden drop in sales right after the first registry in February of 2010. Excluding this data point was enough to pass all validity assumptions. Below are displayed the histogram of the residuals, with the Shapiro-Wilk Test result, and the overlay plot of the residuals versus time.



Shapiro-Wilk W Test

W	Prob<W
0.953296	0.0629

Figure 5. Validity assumptions for trousers forecasting residuals

Following is the graph demonstration (Figure 6) of the actual and forecasted values:

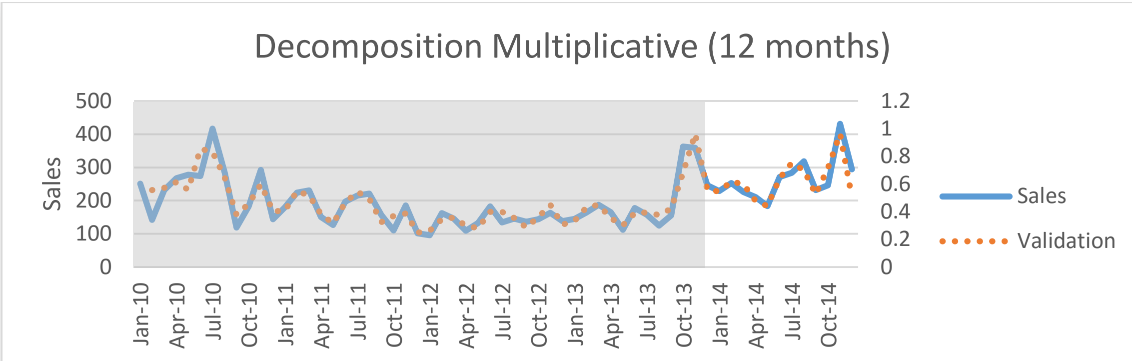


Figure 6. Graphical output of forecasted values

T-shirt

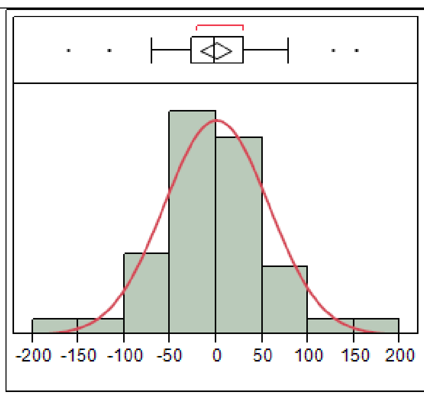
The forecasted values for t-shirts have given the parameters shown in Table 9.

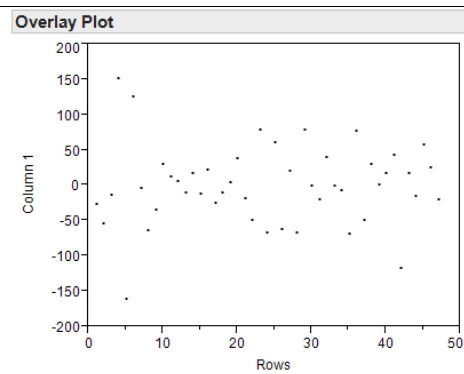
Table 9. Algorithm output for t-shirt sales forecast

	Value of Interest – Consolidation					
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**Cost of the error**
Autocorrelation L=1 year	763,615.9719	0.4119	24.8396	1.1482	0.8418	$4,102.45
Decomposition Multiplicative (12 months)	318,877.8077	0.8747	11.6025	0.5369	0.5072	$471.08
Decomposition Multiplicative (4 months)	463,181.7919	0.7425	17.1472	0.8714	0.7305	$4,935.97
Decomposition Additive (12 months)	260,493.1744	0.8070	11.0250	0.5039	0.4369	$877.23
Additive Holt-Winters (12 months)	637,393.2439	0.7285	19.2337	0.8411	0.6815	$56.40

Differing from the two previous items, the Decomposition Additive (12 months) achieved the best results in four out of the six parameters. It is noteworthy that, despite the Additive Holt-Winters (12) yielding the “cheapest error”, it can be seen that its deviation was high, and when squared, resulted in an SSE almost 2.5 times the best model, with 637,806.2593.

Regarding the validity assumptions, this item presented no problems of normality, despite one outlier caused by an eventual sudden drop in sales right after the first registry, in February of 2010. Excluding this data point was enough to pass all validity assumptions. Figure 7 below displays the histogram of the residuals, with the Shapiro-Wilk Test result, and the overlay plot of the residuals against time.





Shapiro-Wilk W Test

W	Prob<W
0.973989	0.3726

Figure 7. Validity assumptions for t-shirt forecasting residuals

Following is the graph demonstration (Figure 8) of the actual and forecasted values:

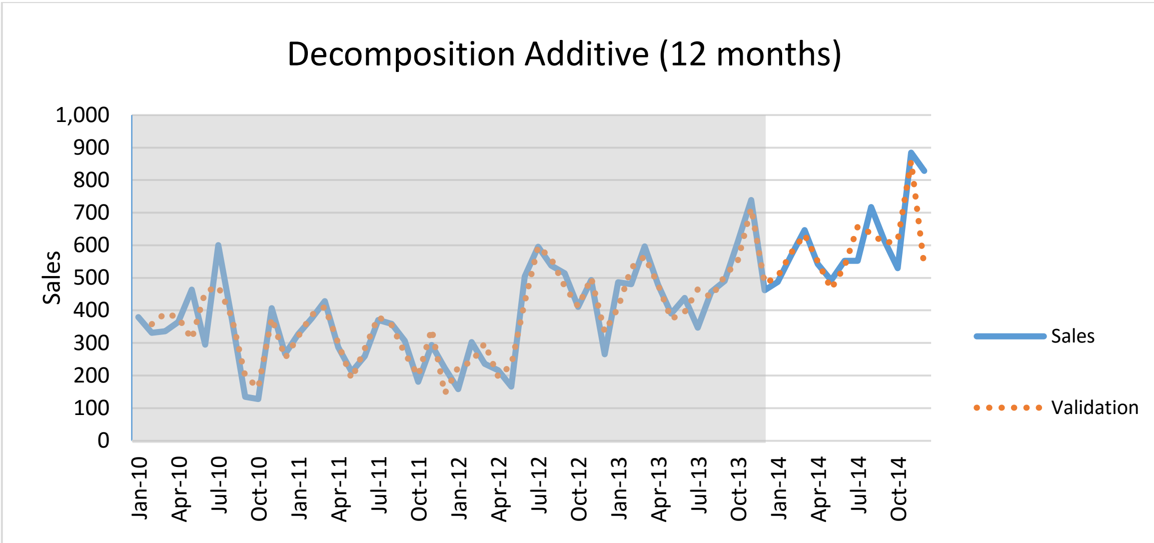


Figure 8. Graphical output of forecasted values

Hat

For this item, once again, the Decomposition Additive (12 months) achieved the best results, this time in five of the six parameters. Table 10 below shows the results for all five models.

Table 10. Algorithm output for hat sales forecast

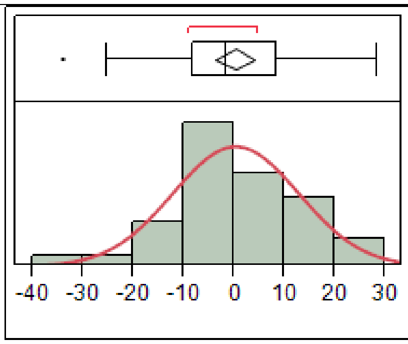
	Value of Interest – Consolidation					
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**Cost of the error**
Autocorrelation L=1 year	67,953.1065	0.2204	33.6719	0.9356	0.8139	$767.65
Decomposition Multiplicative (12 months)	19,940.1618	0.8964	14.0083	0.3907	0.3701	$138.42

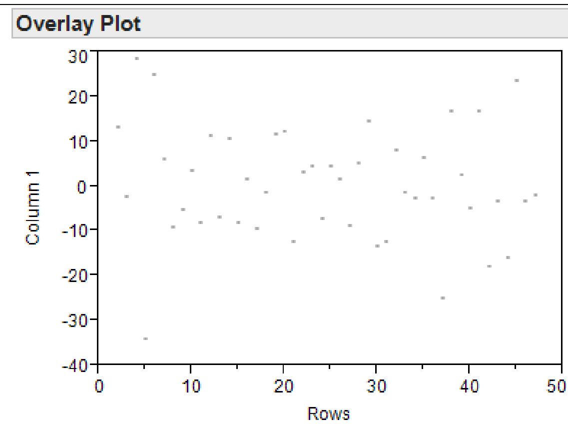
Decomposition Multiplicative (4 months)	46,364.4292	0.6434	19.6439	0.5422	0.6639	$1,373.13
Decomposition Additive (12 months)	20,305.5767	0.8706	15.1386	0.4214	0.4083	$123.28
Additive Holt-Winters (12 months)	47,578.4875	0.7236	24.8725	0.6983	0.6444	$75.69

No tiebreaker was necessary for this item, which achieved impressive results compared to the other models. Only the cost of the error was better than the other parameters, but it does not necessarily mean that it would have saved money in a scenario with any different value. For example, if most of the variation is concentrated in the first half of the forecast and a manager decides to forecast only 6 months, instead of 12, the cost of the error could have assumed a completely different amount.

Regarding the validity assumptions, this item presented no problems of normality, except for one outlier caused by an eventual sudden drop in sales right after the first registry in February of 2010. Excluding this data point was enough to pass all validity assumptions.

The Shapiro-Wilk Test result and the overlay plot of the residuals versus time are displayed in the Figure 9 below. From the histogram of the residuals, despite the boxplot showing an outlier, the normality assumption holds, with a p-value of 0.9053.





Shapiro-Wilk W Test

W	Prob<W
0.987768	0.9053

Figure 9. Validity assumptions for hat forecasting residuals

Following is the graphic demonstration (Figure 10) of the actual and forecasted values:

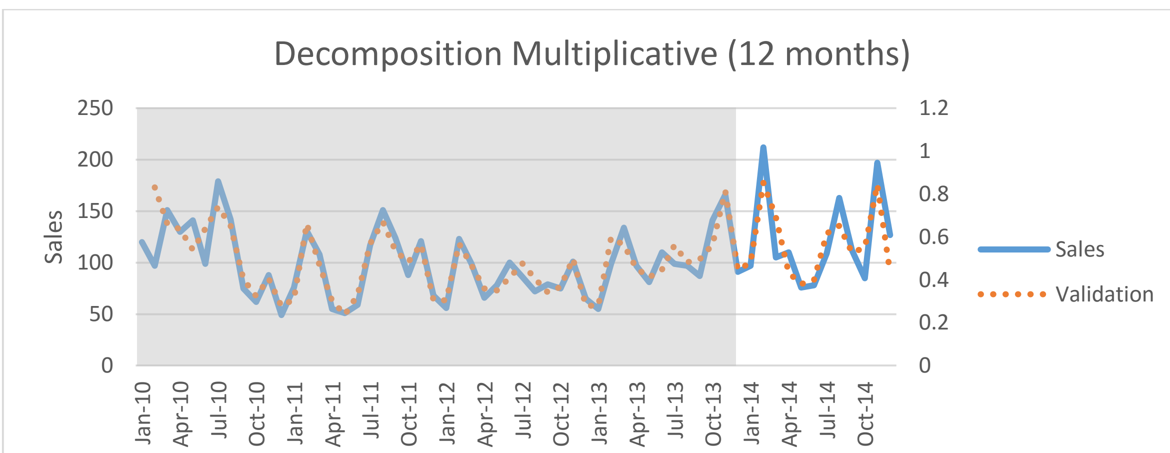


Figure 10. Graphical output of forecasted values

Final dispositions

As the Decomposition Multiplicative (12 months) model kept showing up as the best model (3 out of 4), one more item was tested. Still part of the ABU, the black buckle can be considered a secondary item, which is the reason why it was not tested in the first place. It was interesting to note that the Decomposition Additive (12 months) was the best model for this item, as can be seen in the Table 11 below.

Table 11. Algorithm output for black buckle sales forecast

	Value of Interest – Consolidation					
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**Cost of the error**
Autocorrelation L=1 year	355,552.8174	0.0430	33.8590	0.7513	0.7275	$223.90
Decomposition Multiplicative (12 months)	149,222.3174	0.7364	20.3060	0.4604	0.4943	$188.80
Decomposition Multiplicative (4 months)	230,778.2150	0.4380	25.4891	0.5746	0.6651	$678.95
Decomposition Additive (12 months)	142,223.4721	0.6991	19.6867	0.4433	0.5009	$121.70
Additive Holt-Winters (12 months)	251,636.3912	0.4512	25.5066	0.5728	0.6737	$612.25

As concerns the validity assumptions, it was observed that the residuals pass the tests for normality, independence, and constant variance, after excluding one outlier due to a jump shift detected in sales in June of 2010.

So far, five items were tested and only two models resulted as more appropriate. Either the Decomposition Multiplicative (12 months) or the Decomposition Additive (12 months) has been selected by the algorithm, both being placed first in the final model accountability with 11 appearances each.

At this point, it seemed reasonable that another item was tested to check whether the pattern holds. Indeed, after testing the algorithm for black socks, another secondary item used in the ABU, the best model was again, the Decomposition Multiplicative (12 months). The validity assumptions for this item were tested, and no problems were found. The results for this item are displayed in Table 12 below.

Table 12. Algorithm output for black sock sales forecast

	Value of Interest – Consolidation					
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**Cost of the error**
Autocorrelation L=1 year	4,136,538.6740	0.4877	93.3961	1.4251	1.1345	$506.21
Decomposition Multiplicative (12 months)	1,397,494.3000	0.8171	40.6924	0.5970	0.3624	$692.10
Decomposition Multiplicative (4 months)	3,372,916.7818	0.6806	61.4637	0.9499	0.7522	$2,012.37
Decomposition Additive (12 months)	1,424,814.2567	0.8111	47.4566	0.6871	0.4625	$821.68
Additive Holt-Winters (12 months)	5,139,910.3245	0.4875	110.1033	1.6162	0.9129	$1,992.84

After testing the methodology disclosed in the previous chapter, and exposing the findings as well as the peculiarities originated by the use of the algorithm, this paper will present in the next chapter the summary of the conclusions reached by this research. A number of recommendations will be discussed for potential improvements in the process of forecasting in SDS and for implementation of a new algorithm that enables the managers to make more judicious decisions on purchasing uniforms.

V. Discussion

Overview

This research approached an issue currently existent in a particular unit of the Brazilian Air Force: the ineffective forecasting system in SDS. The goal was to develop a practical tool that, if effective, would greatly help managers to make decisions over uniform purchases, a process that takes place year after year involving millions of dollars.

The Process Currently in Place

The first investigative question in this research argued what metrics currently are being taken by Supply Division. To answer this question, two issues had to be addressed: how the costs are considered, and how the sales forecasts are performed.

According to an interview with the head of the Supply Division in SDS, the system in place has not been satisfactory. While a great amount of inventory is held for some items, the shelves starve for others. Comparing sales with inventory levels, an extreme difference catches the eyes, with inventory reaching the thousands, while sales were usually in the low hundreds.

Considering that the highest value between the calculation of the moving average and the past sales record is taken as a predictor for the next determined number of periods, which is not true most of the time, the inventory level rises, indeed, far beyond the expected, practically reaching several years for some items.

From information obtained in the interview, no metrics are currently being utilized regarding inventory policies, such as inventory holding cost, obsolescence, and opportunity costs. The cost of equipment to handle inventory, operating costs, insurance premiums, and others are not being considered as well.

In effect, specific costs related to these metrics of holding such a high inventory level could not be calculated. Essentially, only the purchase price is considered with respect to inventory.

The acquisition price is merely one part of the costs associated with owning a good (Leenders, Flynn, & Johnson, 2010), and yet, in SDS, all other costs are considered organization-wide and do not affect decisions over the purchasing process.

Regarding the lead times, it was implied that it could take from one to thirteen months for an item to be replenished, depending on the complexity of the procurement process necessary for each item. Much of this variance comes from peculiarities of the Brazilian acquisition regulations and was not addressed by this study.

Gardner, in his Evaluating Forecast Performance in an Inventory Control System (1990), analyzed a large physical distribution system, where managers assumed that the only important impact on delay time was the amount of inventory investment. However, forecast errors are the primary element of the safety stock component of inventory investment. (Gardner, 1990)

Overall, the better the forecast exactness, the smaller the inventory investment needed to reach any particular target service level. As mentioned in Chapter IV, inventory control, lead times, and forecasting are strictly related and, hence, the implementation of metrics, at least the aforementioned essentials, is vital to improve the performance of the overall system in SDS.

However, it is necessary to establish certain goals and certain metrics that will enable the measurement of whether the goals are being achieved prior to defining a targeted service level.

Similar Studies

A similar study has been successfully performed by Downing, Chipulu, Ojiako, & Kaparis (2011), evaluating the UK Chinook helicopter, a utility and attack helicopter operated by the Royal Air Force (RAF), United Kingdom. In their paper, they concluded that non-specific formulation of forecasting techniques in the current inventory and forecasting system led several of the cost driver's demands to have been miscalculated, suggesting a possible lack of forecasting precision. They evaluate the forecast's precision by assessing its error, applying a set of parameters such as Theil's U statistic, MSE, MAE, and MAPE.

The overall conclusion from their study is that, regardless the influence of other factors in the supply chain's performance, the enhancement of the forecasting tools would greatly enhance forecasting precision of cost drivers by Boeing's UK through life customer support team, to whom the maintenance contract was awarded. Specifically, they concluded that two key recommendations have to be addressed: first, the establishment of metrics that can be easily updated and tracked; the second is the reexamination of current practice of basing forecast on monthly component repair data, considering the possibility of reducing the forecast period to two weeks in order to best fit dynamic operational changes in demands.

Similarly, they presented that, according to a report by the United States Government Accountability Office (2009), inaccurate demand forecasting was seen to be one of the reasons why military inventory estimates often failed to align with emerging requirements.

Conclusion

In practice, forecasting in SDS is currently being performed based on empirical inferences, and a scientific process for forecasting must be implemented. The methodology proposed in Chapter III attempted to achieve a proper combination of methods that can be actually used, based on past sales data.

Special attention was given to the quality of the data collected, and only registries with proven integrity were used. Several trials were made in an attempt to select appropriate models that could be used to estimate future sales, including different seasonal and cyclical patterns. Similarly, the selection of the items to run in the models was made with care, assuring that there was no bias in the process, thus assuring models were picked randomly from the database. Once defined, each item was tested for all models, and all results were recorded in a separate spreadsheet. The criteria established were satisfied strictly so results should be nothing but reliable.

As an objective tool to compare and classify the models, accuracy parameters, such as SSE, R^2, MAPE, RAE, and Theil's U, used by the experts in forecasting J. S. Armstrong, E. Gardner, and S. Morlidge, were applied to the results.

Eleven items were necessary to enable five models to be selected. That means, in order to have five models with at least five appearances each, eleven trials had to be run, as could be seen in the Table 5, in the previous chapter. These models were henceforth named "algorithm", and a new parameter has joined the comparison chart.

Since this study has the scope of obtaining a practical tool that can help decision making, the cost of the error shows how much extra inventory would have been purchased in each model.

How much the forecasted value deviates from the actual value comes from the definition of the error, which, multiplied by the price of the item, can result in a dollar figure for the sum of the errors. This figure makes sense as a new validation parameter for this specific research since it simulates a one-time purchase for a particular item in which the total quantity acquired is what matters, even with high fluctuation levels over the actual values within the period forecasted (i.e. 12-month validation period).

With this in mind, the cost of the error was added to the comparison chart and established as the first tiebreak criterion. As much as it is of a great importance, the cost of the error was not assigned a higher weight over the other parameters; however, because when it comes to past values, there is no guarantee that the pattern will hold in the future. Thus, the consistency of each predicted value has also an important role, and all parameters were treated equally in the first instance.

Going further, it was decided as reasonable to test the five elected models on different items, this time chosen by chance, not in a formal process, the Airman Battle Uniform. Interestingly, only two models were consistently selected for all items that comprise the chosen uniform. In reviewing the final accountability table, it was noticed that these two models appeared 11 times for all 11 items evaluated among the five best models (i.e. 100% of times).

Ultimately, one can conclude that these two models, named Decomposition Multiplicative (12 months) and Decomposition Additive (12 months) clearly captured the Air Force’s military consumption pattern, either for external circumstances (promotions, economy, etc.) or simply for cultural behavior. As a matter of fact, in a two-phased, several-layer selection process, these two models fit among the best five for all the items tested.

When six new items were tested, namely, coats, trousers, t-shirts, hats, buckles, and socks (all comprising the Airman Battle Uniform), in a universe of five models, these two were awarded 3 times each. It virtually discards the possibility of two overly fitted models to a few specific items. Indeed, the results obtained in the 12-month validation period for all 17 items to which these two models were applied prove their consistency.

It is important to emphasize that this study endeavored to create an algorithm that can be put to work in practice. For this reason, the ease and convenience of the algorithm was central when choosing a tool for running the models. Although JMP® was selected as the apparatus along with MS Excel®, only in extremely successful cases would it be utilized in the algorithm. In other words, only if the best results from the models calculated in MS Excel® were extremely poor, one of the ARIMA models in JMP® would be considered for use.

Indeed, no ARIMA models tested achieved satisfactory results, either with or without seasonal components. However, in case any ARIMA models were selected, it would be at the researcher's discretion to consider whether to neglect the model, given that the implementation of such a complex method departs from the scope of this research. As MS Excel® or other spreadsheet programs are tools that best approximate real life, it was preferred that the product of this study (i.e. the algorithm), used all the formulas completely hard coded in order to be easily and successfully deployed in SDS's ERP system.

Although five models were pursued to comprise the desired algorithm, it was clear that the Decomposition Multiplicative (12 months) and the Decomposition Additive (12 months), alone, depicted very well the behavior of sales in BAF's stores. As a result, it was considered that these two models, only, are eligible to be selected to compose the longed for tool.

In a final analysis, the elements of this study approximate to the one presented in the previous section in terms of methodology and even the references, with exception of the models and parameters selected to evaluate and forecast demand. As mentioned in the study related to the Royal Air Force, it was possible to conduct a review of both inventory management and forecasting tools, which departs from the content of this research. However, the similarity of methodology proposed in both cases, and the conclusion that the "current inventory and forecasting system suggests a possible lack of forecasting precision" gives support to the findings in the present report.

Recommendations

By all means, the ideal approach to the problems identified at SDS was to first establish sound metrics and policies, which would be the foundation for selecting the proper tools for each of the elements of the supply chain. As was seen in Chapter II, inventory policy and forecasting are intimately connected and this fact sets the reference for the best focus for the forecast apparatus. However, given that SDS lacks a scientific forecasting process, and given the satisfactory results obtained in this research, it would promote instant improvement implementing the algorithm proposed in this study immediately.

Under those circumstances, the first step recommended is to establish what metrics are to be implemented. Actions should be taken in order to break down costs into activities level, a process known as Activity-Based Costing (ABC), which would enable managers to consider costs from the total cost of ownership standpoint, instead of the acquisition price only. Similarly, an entire set of actions can be taken, with activity-based cost information, to improve administration on a better informed basis, which is known as Activity-Based Management (ABM) (Kaplan & Cooper, 1997).

Operationally, ABM works to enhance efficiency and assets utilization, and can increase the capacity of the resources by reducing equipment and personnel idle times, and improving or eliminating faulty activities and processes. (Kaplan & Cooper, 1997)

The inventory levels should be compatible with the expected level of sales. This denotes that an accurate forecasting system is essential to allowing the inventory policy to be fulfilled properly. According to Gardner (1990), forecasting is a prerequisite to inventory decisions in practice. Subsequently, the decision to implement a certain inventory strategy could be made utilizing a tradeoff curve between service level and inventory investment. By refining the forecast process, this curve could be shifted in such a way that both increases service level and decreases inventory investment (Gardner, 1990).

Indeed, the study and adoption of new metrics would require a much longer period than the adoption of the algorithm presented. For that reason, the solution herein presented potentially brings instant improvement to the process, which can be translated into cost savings. As will be discussed in the Future Research section, it is recommended as a next step to this initial action that there continue to be future studies in terms of generating metrics that are most adequate to SDS's reality.

In addition, it is necessary to setup a proper inventory policy, which combined with the methodology established in this research, will enable managers to forecast sales according to desired levels of customer service. Implementing a solid inventory policy, will allow combining successfully inventory control and forecasting, supporting a tradeoff curve between service level and inventory investment to be applied.

Lastly, in order make the enforcement of the policies and procedures established possible, it is essential that everything be documented. Thus, the final step recommended in this research is to create written detailed regulations or norms of all topics discussed that can be transformed into human actions.

Limitations

The absence of documentation was not exactly a limitation to this study, but to the process itself. All procedures in place at SDS regarding forecasting process come from repeated practice, not being disclosed through documents.

In reference to the lack of inventory metrics and policies, it was revealed to be a limiting factor for further research of the selected topic (i.e. forecasting). Such deficiency prevented the construction of a trade-off curve observed in the literature that could be useful to SDS.

Future Research

Finally, it is worth mentioning that a potential area for future research relates to the inventory policy, currently lacking strict regulations. The implementation of metrics, as discussed in this research, is also imperative for optimizing the overall inventory management in SDS. The results of the present research proved to be valuable for paving the way, but they have to be followed by other management actions to be entirely effective.

It is possible that the implementation of new metrics and inventory policies alter the behavior of sales in BAF's stores due to, for example, dependent demand items, or for items that may become available after implementing new policies. Nevertheless, the equations and procedures developed in this research can be adjusted as well to a potential new reality.

Appendix A. Naïve forecast sample

Date	Time	St. Dumont medal	Naive	Percentage Error	Theil's U
Jan-10	1	5			-0.8
Feb-10	2	9	5	44.44444	0.111111111
Mar-10	3	8	9	12.50000	-0.625
Apr-10	4	13	8	38.46154	-0.615384615
May-10	5	21	13	38.09524	0.523809524
Jun-10	6	10	21	110.00000	-0.8
Jul-10	7	18	10	44.44444	0.166666667
Aug-10	8	15	18	20.00000	0.533333333
Sep-10	9	7	15	114.28571	-1
Oct-10	10	14	7	50.00000	0.071428571
Nov-10	11	13	14	7.69231	0
Dec-10	12	13	13	0.00000	0.153846154
Jan-11	13	11	13	18.18182	-0.363636364
Feb-11	14	15	11	26.66667	0.533333333
Mar-11	15	7	15	114.28571	-0.142857143
Apr-11	16	8	7	12.50000	0.125
May-11	17	7	8	14.28571	-0.142857143
Jun-11	18	8	7	12.50000	-0.5
Jul-11	19	12	8	33.33333	-0.833333333
Aug-11	20	22	12	45.45455	0.727272727
Sep-11	21	6	22	266.66667	-1.166666667
Oct-11	22	13	6	53.84615	0.923076923
Nov-11	23	1	13	1,200.00000	-4
Dec-11	24	5	1	80.00000	0.4
Jan-12	25	3	5	66.66667	-2
Feb-12	26	9	3	66.66667	-0.555555556
Mar-12	27	14	9	35.71429	0.214285714
Apr-12	28	11	14	27.27273	0
May-12	29	11	11	0.00000	0
Jun-12	30	11	11	0.00000	-0.727272727
Jul-12	31	19	11	42.10526	-0.052631579
Aug-12	32	20	19	5.00000	0.55
Sep-12	33	9	20	122.22222	-1.111111111
Oct-12	34	19	9	52.63158	0.315789474
Nov-12	35	13	19	46.15385	0.076923077
Dec-12	36	12	13	8.33333	0.25
Jan-13	37	9	12	33.33333	-0.222222222
Feb-13	38	11	9	18.18182	0.272727273
Mar-13	39	8	11	37.50000	-0.5
Apr-13	40	12	8	33.33333	0.416666667
May-13	41	7	12	71.42857	-0.142857143
Jun-13	42	8	7	12.50000	-0.75
Jul-13	43	14	8	42.85714	-1
Aug-13	44	28	14	50.00000	0.5
Sep-13	45	14	28	100.00000	0.285714286
Oct-13	46	10	14	40.00000	-0.3
Nov-13	47	13	10	23.07692	-0.230769231
Dec-13	48	16	13	18.75000	0.25
Jan-14	49	12	16	33.33333	0.666666667
Feb-14	50	4	12	200.00000	-2.75
Mar-14	51	15	4	73.33333	0.133333333
Apr-14	52	13	15	15.38462	-0.076923077
May-14	53	14	13	7.14286	0.357142857
Jun-14	54	9	14	55.55556	-1.333333333
Jul-14	55	21	9	57.14286	0.428571429
Aug-14	56	12	21	75.00000	-0.5
Sep-14	57	18	12	33.33333	0.277777778
Oct-14	58	13	18	38.46154	-0.153846154
Nov-14	59	15	13	13.33333	0.4
Dec-14	60	9	15	66.66667	
Jan-15	61	10			
Feb-15	62	3			
Mar-15	63	13			
Apr-15	64	11			
May-15	65	5			
Jun-15	66	13			
Jul-15	67	16			
	AVG sales	12		MAPE estimation	MAPE validation
				70.45472	55.72395
				Theil's U estimation	Theil's U validation
				33.25584	10.63047

Appendix B. Simple Linear Regression sample

Date	Time	Sales - y	[x-avg(x)][y-avg(y)]	$[x\text{-avg}(x)]^2$	$[y\text{-avg}(y)]^2$	*Forecast*	Pred-avg(y)	$[\text{Pred-avg}(y)]^2$	Residual: [predicted(y)-y]	Percentage Error	Theil's U
Jan-10	1	5	157.6458	552.2500	45.0017	10.4660	-1.2423	1.5434	5.47	109.31973	0.3037704
Feb-10	2	9	60.9375	506.2500	7.3351	10.5189	-1.1895	1.4149	1.52	16.876136	0.2857464
Mar-10	3	8	79.7292	462.2500	13.7517	10.5717	-1.1366	1.2919	2.57	32.146476	-0.296927
Apr-10	4	13	-26.4792	420.2500	1.6684	10.6246	-1.0837	1.1745	-2.38	18.272432	-0.7940423
May-10	5	21	-181.1875	380.2500	86.3351	10.6774	-1.0309	1.0627	-10.32	49.155001	0.0347769
Jun-10	6	10	31.6042	342.2500	2.9184	10.7303	-0.9780	0.9565	0.73	7.3031553	-0.7216819
Jul-10	7	18	-110.1042	306.2500	39.5851	10.7832	-0.9252	0.8559	-7.22	40.093437	-0.2313307
Aug-10	8	15	-54.3125	272.2500	10.8351	10.8360	-0.8723	0.7609	-4.16	27.759685	0.2592609
Sep-10	9	7	72.9792	240.2500	22.1684	10.8889	-0.8194	0.6714	3.89	55.5559	-0.4368887
Oct-10	10	14	-33.2292	210.2500	5.2517	10.9418	-0.7666	0.5876	-3.06	21.844437	-0.1432397
Nov-10	11	13	-17.4375	182.2500	1.6684	10.9946	-0.7137	0.5094	-2.01	15.42581	-0.1501915
Dec-10	12	13	-16.1458	156.2500	1.6684	11.0475	-0.6608	0.4367	-1.95	15.01915	0.0077213
Jan-11	13	11	8.1458	132.2500	0.5017	11.1004	-0.6080	0.3696	0.10	0.912512	-0.3497053
Feb-11	14	15	-34.5625	110.2500	10.8351	11.1532	-0.5551	0.3081	-3.85	25.645052	0.2804072
Mar-11	15	7	44.7292	90.2500	22.1684	11.2061	-0.5022	0.2522	4.21	60.087257	0.4655677
Apr-11	16	8	31.5208	72.2500	13.7517	11.2590	-0.4494	0.2019	3.26	40.737173	0.53898
May-11	17	7	35.3125	56.2500	22.1684	11.3118	-0.3965	0.1572	4.31	61.597709	0.4806722
Jun-11	18	8	24.1042	42.2500	13.7517	11.3647	-0.3436	0.1181	3.36	42.058818	-0.0728036
Jul-11	19	12	-1.6042	30.2500	0.0851	11.4176	-0.2908	0.0845	-0.58	4.8535726	-0.8774636
Aug-11	20	22	-46.3125	20.2500	105.9184	11.4704	-0.2379	0.0566	-10.53	47.861649	0.2510592
Sep-11	21	6	19.9792	12.2500	32.5851	11.5233	-0.1850	0.0342	5.52	92.055049	-0.2373052
Oct-11	22	13	-3.2292	6.2500	1.6684	11.5762	-0.1322	0.0175	-1.42	10.952548	0.817618
Nov-11	23	1	16.0625	2.2500	114.6684	11.6290	-0.0793	0.0063	10.63	1062.9035	6.6819004
Dec-11	24	5	3.3542	0.2500	45.0017	11.6819	-0.0264	0.0007	6.68	133.63801	1.7469532
Jan-12	25	3	-4.3542	0.2500	75.8351	11.7348	0.0264	0.0007	8.73	291.15887	0.9292107
Feb-12	26	9	-4.0625	2.2500	7.3351	11.7876	0.0793	0.0063	2.79	30.97369	-0.2399447
Mar-12	27	14	5.7292	6.2500	5.2517	11.8405	0.1322	0.0175	-2.16	15.425015	0.0638117
Apr-12	28	11	-2.4792	12.2500	0.5017	11.8934	0.1850	0.0342	0.89	8.1214884	0.0860209
May-12	29	11	-3.1875	20.2500	0.5017	11.9462	0.2379	0.0566	0.95	8.6020869	0.0908269
Jun-12	30	11	-3.8958	30.2500	0.5017	11.9991	0.2908	0.0845	1.00	9.0826853	-0.6316399
Jul-12	31	19	47.3958	42.2500	53.1684	12.0520	0.3436	0.1181	-6.95	36.568625	-0.4155354
Aug-12	32	20	62.1875	56.2500	68.7517	12.1048	0.3965	0.1572	-7.90	39.475865	0.1578846
Sep-12	33	9	-23.0208	72.2500	7.3351	12.1577	0.4494	0.2019	3.16	35.085476	-0.7543824
Oct-12	34	19	69.2708	90.2500	53.1684	12.2106	0.5022	0.2522	-6.79	35.733902	-0.0387671
Nov-12	35	13	13.5625	110.2500	1.6684	12.2634	0.5551	0.3081	-0.74	5.6659652	0.02433
Dec-12	36	12	3.3542	132.2500	0.0851	12.3163	0.6080	0.3696	0.32	2.6357529	0.280763
Jan-13	37	9	-33.8542	156.2500	7.3351	12.3692	0.6608	0.4367	3.37	37.435069	0.1580024
Feb-13	38	11	-9.5625	182.2500	0.5017	12.4220	0.7137	0.5094	1.42	12.927473	0.406808
Mar-13	39	8	-53.7708	210.2500	13.7517	12.4749	0.7666	0.5876	4.47	55.936098	0.0659692
Apr-13	40	12	4.5208	240.2500	0.0851	12.5278	0.8194	0.6714	0.53	4.3979471	0.4650516
May-13	41	7	-77.6875	272.2500	22.1684	12.5806	0.8723	0.7609	5.58	79.723135	0.6619265
Jun-13	42	8	-64.8958	306.2500	13.7517	12.6335	0.9252	0.8559	4.63	57.918566	-0.1642061
Jul-13	43	14	42.3958	342.2500	5.2517	12.6864	0.9780	0.9565	-1.31	9.3832062	-1.0900559
Aug-13	44	28	317.6875	380.2500	265.4184	12.7392	1.0309	1.0627	-15.26	54.502797	-0.0431399
Sep-13	45	14	46.9792	420.2500	5.2517	12.7921	1.0837	1.1745	-1.21	8.6279801	0.2032106
Oct-13	46	10	-36.7292	462.2500	2.9184	12.8449	1.1366	1.2919	2.84	28.449486	-0.0102186
Nov-13	47	13	29.0625	506.2500	1.6684	12.8978	1.1895	1.4149	-0.10	0.7860427	-0.2345631
Dec-13	48	16	100.8542	552.2500	18.4184	12.9507	1.2423	1.5434	-3.05	19.058248	0.0627216
Jan-14	49	12			0.8403	13.0035	0.0869	0.0075	1.00	8.3628842	0.754701
Feb-14	50	4			79.5069	13.0564	0.1397	0.0195	9.06	226.4103	-0.4726806
Mar-14	51	15			4.3403	13.1093	0.1926	0.0371	-1.89	12.604815	0.0108096
Apr-14	52	13			0.0069	13.1621	0.2455	0.0603	0.16	1.2472583	-0.0603839
May-14	53	14			1.1736	13.2150	0.2983	0.0890	-0.78	5.6070757	0.3048482
Jun-14	54	9			15.3403	13.2679	0.3512	0.1233	4.27	47.420836	-0.853251
Jul-14	55	21			65.3403	13.3207	0.4041	0.1633	-7.68	36.5679	0.0654099
Aug-14	56	12			0.8403	13.3736	0.4569	0.2088	1.37	11.446724	-0.3811273
Sep-14	57	18			25.8403	13.4265	0.5098	0.2599	-4.57	25.408485	0.0266299
Oct-14	58	13			0.0069	13.4793	0.5627	0.3166	0.48	3.6872196	-0.1129074
Nov-14	59	15			4.3403	13.5322	0.6155	0.3789	-1.47	9.7853042	0.3056713
Dec-14	60	9			15.3403	13.5851	0.6684	0.4468	4.59	50.945224	
	24.50	11.71	487.0000	9,212.0000	1,251.9167	Explained Variation		25.7457			
	Average	Average	SSxy	SSxx	Total Variation						

Appendix C. Trend

	Dependent	Trend Factor Polynomial				Explained	Total	Percentage Error	
Date	y	x	x^2	x^3	x^4	Variation	Variation		Theil's U
Jan-10	5	1	1	1	1	9	45	74.96463201	0.171677489
Feb-10	9	2	4	8	16	3	7	9.537638296	0.305688604
Mar-10	8	3	9	27	81	1	14	34.38996793	-0.193931
Apr-10	13	4	16	64	256	0	2	11.93421536	-0.69450761
May-10	21	5	25	125	625	0	86	42.99332808	0.111416867
Jun-10	10	6	36	216	1296	0	3	23.39754203	-0.54273197
Jul-10	18	7	49	343	2401	1	40	30.15177623	-0.12842734
Aug-10	15	8	64	512	4096	1	11	15.41128116	0.38025499
Sep-10	7	9	81	729	6561	1	22	81.48321216	-0.19493159
Oct-10	14	10	100	1000	10000	1	5	9.746579471	-0.03581593
Nov-10	13	11	121	1331	14641	1	2	3.857100541	-0.05327034
Dec-10	13	12	144	1728	20736	0	2	5.327034108	0.082740834
Jan-11	11	13	169	2197	28561	0	1	9.778462222	-0.28950015
Feb-11	15	14	196	2744	38416	0	11	21.23001106	0.302575543
Mar-11	7	15	225	3375	50625	0	22	64.83761633	0.465091408
Apr-11	8	16	256	4096	65536	0	14	40.6954982	0.497022824
May-11	7	17	289	4913	83521	1	22	56.80260849	0.386997856
Jun-11	8	18	324	5832	104976	1	14	33.86231237	-0.19227123
Jul-11	12	19	361	6859	130321	2	0	12.81808179	-0.97986993
Aug-11	22	20	400	8000	160000	2	106	53.44745074	0.184276324
Sep-11	6	21	441	9261	194481	3	33	67.56798533	-0.51594221
Oct-11	13	22	484	10648	234256	3	2	23.81271757	0.676644987
Nov-11	1	23	529	12167	279841	4	115	879.6384826	4.733274161
Dec-11	5	24	576	13824	331776	4	45	94.66548321	1.343430989
Jan-12	3	25	625	15625	390625	4	76	223.9051649	0.249742308
Feb-12	9	26	676	17576	456976	4	7	8.324743609	-0.46336118
Mar-12	14	27	729	19683	531441	4	5	29.78750466	-0.07442564
Apr-12	11	28	784	21952	614656	3	1	9.472354707	-0.07886544
May-12	11	29	841	24389	707281	2	1	7.886544453	-0.05904504
Jun-12	11	30	900	27000	810000	2	1	5.904504311	-0.7628535
Jul-12	19	31	961	29791	923521	1	53	44.16520243	-0.47882326
Aug-12	20	32	1024	32768	1048576	1	69	45.48821016	0.111318008
Sep-12	9	33	1089	35937	1185921	0	7	24.73733502	-0.82507845
Oct-12	19	34	1156	39304	1336336	0	53	39.08266348	-0.05584765
Nov-12	13	35	1225	42875	1500625	0	2	8.162348519	0.023996715
Dec-12	12	36	1296	46656	1679616	0	0	2.599644082	0.30702801
Jan-13	9	37	1369	50653	1874161	1	7	40.93706801	0.227327232
Feb-13	11	38	1444	54872	2085136	2	1	18.59950082	0.489614329
Mar-13	8	39	1521	59319	2313441	3	14	67.32197021	0.211475829
Apr-13	12	40	1600	64000	2560000	4	0	14.09838859	0.579265381
May-13	7	41	1681	68921	2825761	5	22	99.30263682	0.878577631
Jun-13	8	42	1764	74088	3111696	6	14	76.87554271	0.034199328
Jul-13	14	43	1849	79507	3418801	7	5	1.954247295	-0.97813506
Aug-13	28	44	1936	85184	3748096	7	265	48.90675294	0.008247057
Sep-13	14	45	2025	91125	4100625	6	5	1.649411327	0.287886419
Oct-13	10	46	2116	97336	4477456	5	3	40.3040986	0.068603553
Nov-13	13	47	2209	103823	4879681	4	2	5.277196363	-0.21705357
Dec-13	16	48	2304	110592	5308416	2	18	17.63560232	0.03042392
Jan-14	12	49				0	1	4.056522674	0.632508406
Feb-14	4	50				2	80	189.7525218	-1.13351358
Mar-14	15	51				6	4	30.22702892	-0.26059572
Apr-14	13	52				15	0	30.068737	-0.50451823
May-14	14	53				30	1	46.84812097	-0.25061369
Jun-14	9	54				55	15	38.98435101	-1.97606379
Jul-14	21	55				94	65	84.68844828	-0.5435095
Aug-14	12	56				152	1	95.1141619	-1.70199357
Sep-14	18	57				235	26	113.4662377	-1.04689667
Oct-14	13	58				352	0	144.9549235	-1.90032212
Nov-14	15	59				512	4	164.6945835	-1.53565613
Dec-14	9	60				726	15	255.9426889	

Appendix D. Dummy variables

	Dependent	Trend Factor Polynomial		Cycle Using Dummy Variables											Explained Variation	Total Variation
Date	y	x	x^2	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov		
Jan-10	5	1	1	1	0	0	0	0	0	0	0	0	0	0	12.5382	45.0017
Feb-10	9	2	4	0	1	0	0	0	0	0	0	0	0	0	0.0745	7.3351
Mar-10	8	3	9	0	0	1	0	0	0	0	0	0	0	0	2.7656	13.7517
Apr-10	13	4	16	0	0	0	1	0	0	0	0	0	0	0	0.0098	1.6684
May-10	21	5	25	0	0	0	0	1	0	0	0	0	0	0	0.0462	86.3351
Jun-10	10	6	36	0	0	0	0	0	1	0	0	0	0	0	4.9335	2.9184
Jul-10	18	7	49	0	0	0	0	0	0	1	0	0	0	0	16.7511	39.5851
Aug-10	15	8	64	0	0	0	0	0	0	0	1	0	0	0	88.4873	10.8351
Sep-10	7	9	81	0	0	0	0	0	0	0	0	1	0	0	9.1765	22.1684
Oct-10	14	10	100	0	0	0	0	0	0	0	0	0	1	0	3.1851	5.2517
Nov-10	13	11	121	0	0	0	0	0	0	0	0	0	0	1	5.7665	1.6684
Dec-10	13	12	144	0	0	0	0	0	0	0	0	0	0	0	1.1824	1.6684
Jan-11	11	13	169	1	0	0	0	0	0	0	0	0	0	0	28.2112	0.5017
Feb-11	15	14	196	0	1	0	0	0	0	0	0	0	0	0	1.8863	10.8351
Mar-11	7	15	225	0	0	1	0	0	0	0	0	0	0	0	10.1471	22.1684
Apr-11	8	16	256	0	0	0	1	0	0	0	0	0	0	0	2.2424	13.7517
May-11	7	17	289	0	0	0	0	1	0	0	0	0	0	0	1.1225	22.1684
Jun-11	8	18	324	0	0	0	0	0	1	0	0	0	0	0	11.3670	13.7517
Jul-11	12	19	361	0	0	0	0	0	0	1	0	0	0	0	9.4034	0.0851
Aug-11	22	20	400	0	0	0	0	0	0	0	1	0	0	0	72.3262	105.9184
Sep-11	6	21	441	0	0	0	0	0	0	0	0	1	0	0	14.4973	32.5851
Oct-11	13	22	484	0	0	0	0	0	0	0	0	0	1	0	1.2779	1.6684
Nov-11	1	23	529	0	0	0	0	0	0	0	0	0	0	1	8.5941	114.6684
Dec-11	5	24	576	0	0	0	0	0	0	0	0	0	0	0	2.2308	45.0017
Jan-12	3	25	625	1	0	0	0	0	0	0	0	0	0	0	31.2881	75.8351
Feb-12	9	26	676	0	1	0	0	0	0	0	0	0	0	0	2.3457	7.3351
Mar-12	14	27	729	0	0	1	0	0	0	0	0	0	0	0	10.3655	5.2517
Apr-12	11	28	784	0	0	0	1	0	0	0	0	0	0	0	1.9812	0.5017
May-12	11	29	841	0	0	0	0	1	0	0	0	0	0	0	0.7149	0.5017
Jun-12	11	30	900	0	0	0	0	0	1	0	0	0	0	0	9.2022	0.5017
Jul-12	19	31	961	0	0	0	0	0	0	1	0	0	0	0	12.4504	53.1684
Aug-12	20	32	1024	0	0	0	0	0	0	0	1	0	0	0	82.6376	68.7517
Sep-12	9	33	1089	0	0	0	0	0	0	0	0	1	0	0	9.5943	7.3351
Oct-12	19	34	1156	0	0	0	0	0	0	0	0	0	1	0	3.8595	53.1684
Nov-12	13	35	1225	0	0	0	0	0	0	0	0	0	0	1	3.8945	1.6684
Dec-12	12	36	1296	0	0	0	0	0	0	0	0	0	0	0	0.1693	0.0851
Jan-13	9	37	1369	1	0	0	0	0	0	0	0	0	0	0	19.2492	7.3351
Feb-13	11	38	1444	0	1	0	0	0	0	0	0	0	0	0	0.0405	0.5017
Mar-13	8	39	1521	0	0	1	0	0	0	0	0	0	0	0	3.1163	13.7517
Apr-13	12	40	1600	0	0	0	1	0	0	0	0	0	0	0	0.0291	0.0851
May-13	7	41	1681	0	0	0	0	1	0	0	0	0	0	0	0.7341	22.1684
Jun-13	8	42	1764	0	0	0	0	0	1	0	0	0	0	0	1.4573	13.7517
Jul-13	14	43	1849	0	0	0	0	0	0	1	0	0	0	0	30.0178	5.2517
Aug-13	28	44	1936	0	0	0	0	0	0	0	1	0	0	0	124.6549	265.4184
Sep-13	14	45	2025	0	0	0	0	0	0	0	0	1	0	0	0.8083	5.2517
Oct-13	10	46	2116	0	0	0	0	0	0	0	0	0	1	0	18.3782	2.9184
Nov-13	13	47	2209	0	0	0	0	0	0	0	0	0	0	1	0.2237	1.6684
Dec-13	16	48	2304	0	0	0	0	0	0	0	0	0	0	0	4.6616	18.4184
Jan-14	12	49	2401	1	0	0	0	0	0	0	0	0	0	0	8.4170	0.8403
Feb-14	4	50	2500	0	1	0	0	0	0	0	0	0	0	0	1.9849	79.5069
Mar-14	15	51	2601	0	0	1	0	0	0	0	0	0	0	0	0.0010	4.3403
Apr-14	13	52	2704	0	0	0	1	0	0	0	0	0	0	0	4.1168	0.0069
May-14	14	53	2809	0	0	0	0	1	0	0	0	0	0	0	8.0603	1.1736
Jun-14	9	54	2916	0	0	0	0	0	1	0	0	0	0	0	0.8084	15.3403
Jul-14	21	55	3025	0	0	0	0	0	0	1	0	0	0	0	59.4318	65.3403
Aug-14	12	56	3136	0	0	0	0	0	0	0	1	0	0	0	182.7707	0.8403
Sep-14	18	57	3249	0	0	0	0	0	0	0	0	1	0	0	2.4943	25.8403
Oct-14	13	58	3364	0	0	0	0	0	0	0	0	0	1	0	47.4640	0.0069
Nov-14	15	59	3481	0	0	0	0	0	0	0	0	0	0	1	10.2367	4.3403
Dec-14	9	60	3600	0	0	0	0	0	0	0	0	0	0	0	25.0956	15.3403

Appendix E. Trigonometry (4-year cycle)

	Dependent	Trend Factor Polynomial		Cycle Using Trigonometry									Explained	Total
Date	y	x	x^2	Sin (2πt/L)	Cos (2πt/L)	Sin (4πt/L)	Cos (4πt/L)	L in Years	*Forecast*	Error	Percentage Error	Theil's U	Variation	Variation
Jan-10	5	1	1	0	1	-0.258819045	0.965925826	4	9.19977065	4.19977	83.99541297	0.23553077	6.2929	45.0017
Feb-10	9	2	4	0.130526192	0.991444861	-0.225667066	0.974204483		10.1776539	1.17765	13.08504285	0.32420028	2.3430	7.3351
Mar-10	8	3	9	0.258819045	0.965925826	-0.192825347	0.981233094		10.9178025	2.91780	36.47253123	-0.19034566	0.6249	13.7517
Apr-10	13	4	16	0.382683432	0.923879533	-0.160910494	0.986969003		11.4772347	-1.52277	11.71357899	-0.69956655	0.0534	1.6684
May-10	21	5	25	0.5	0.866025404	-0.130526192	0.991444861		11.9056349	-9.09437	43.30650054	0.10665914	0.0389	86.3351
Jun-10	10	6	36	0.608761429	0.79335334	-0.102247019	0.99475904		12.2398419	2.23984	22.39841919	-0.54990323	0.2825	2.9184
Jul-10	18	7	49	0.707106781	0.707106781	-0.076604145	0.997061585		12.5009677	-5.49903	30.55017922	-0.12808567	0.6283	39.5851
Aug-10	15	8	64	0.79335334	0.608761429	-0.054073583	0.998536954		12.694458	-2.30554	15.37028014	0.38752866	0.9724	10.8351
Sep-10	7	9	81	0.866025404	0.5	-0.035067276	0.999384954		12.8129299	5.81293	83.04185642	-0.1655449	1.2201	22.1684
Oct-10	14	10	100	0.923879533	0.382683432	-0.019926973	0.999801438		12.8411857	-1.15881	8.277245122	-0.01696651	1.2834	5.2517
Nov-10	13	11	121	0.965925826	0.258819045	-0.00892048	0.999960212		12.7624689	-0.23753	1.827162626	-0.03347226	1.1112	1.6684
Dec-10	13	12	144	0.991444861	0.130526192	-0.002239728	0.999997492		12.5648607	-0.43514	3.347225525	0.09590092	0.7336	1.6684
Jan-11	11	13	169	1	6.12574E-17	0	1		12.246712	1.24671	11.33374528	-0.28907442	0.2899	0.5017
Feb-11	15	14	196	0.991444861	-0.130526192	-0.002239728	0.999997492		11.8201814	-3.17982	21.19879089	0.28748414	0.0125	10.8351
Mar-11	7	15	225	0.965925826	-0.258819045	-0.00892048	0.999960212		11.3122622	4.31226	61.60374529	0.39472651	0.1569	22.1684
Apr-11	8	16	256	0.923879533	-0.382683432	-0.019926973	0.999801438		10.7630856	2.76309	34.53857006	0.40271552	0.8935	13.7517
May-11	7	17	289	0.866025404	-0.5	-0.035067276	0.999384954		10.2217242	3.22172	46.02463095	0.24858912	2.2100	22.1684
Jun-11	8	18	324	0.79335334	-0.608761429	-0.054073583	0.998536954		9.74012383	1.74012	21.75154787	-0.32923674	3.8738	13.7517
Jul-11	12	19	361	0.707106781	-0.707106781	-0.076604145	0.997061585		9.36610606	-2.63389	21.94911613	-1.07195394	5.4860	0.0851
Aug-11	22	20	400	0.608761429	-0.79335334	-0.102247019	0.99475904		9.1365527	-12.86345	58.470215	0.13963128	6.6141	105.9184
Sep-11	6	21	441	0.5	-0.866025404	-0.130526192	0.991444861		9.07188817	3.07189	51.19813618	-0.63786685	6.9508	32.5851
Oct-11	13	22	484	0.382683432	-0.923879533	-0.160910494	0.986969003		9.17279889	-3.82720	29.44000854	0.64767689	6.4289	1.6684
Nov-11	1	23	529	0.258819045	-0.965925826	-0.192825347	0.981233094		9.41979952	8.41980	841.9799517	4.77581771	5.2374	114.6684
Dec-11	5	24	576	0.130526192	-0.991444861	-0.225667066	0.974204483		9.77581771	4.77582	95.51635412	1.43829797	3.7346	45.0017
Jan-12	3	25	625	1.22515E-16	-1	-0.258819045	0.965925826		10.1914899	7.19149	239.7163284	0.53747258	2.3008	75.8351
Feb-12	9	26	676	-0.130526192	-0.991444861	-0.29166883	0.956519364		10.6124178	1.61242	17.91575282	-0.33474398	1.2010	7.3351
Mar-12	14	27	729	-0.258819045	-0.965925826	-0.323624898	0.94618546		10.9873042	-3.01270	21.51925603	0.01969438	0.5199	5.2517
Apr-12	11	28	784	-0.382683432	-0.923879533	-0.354131926	0.935195477		11.2757213	0.27572	2.506557219	0.04130005	0.1872	0.5017
May-12	11	29	841	-0.5	-0.866025404	-0.382683432	0.923879533		11.4543006	0.45430	4.130005063	0.04730534	0.0645	0.5017
Jun-12	11	30	900	-0.608761429	-0.79335334	-0.408831004	0.912610108		11.5203588	0.52036	4.730534218	-0.68251268	0.0353	0.5017
Jul-12	19	31	961	-0.707106781	-0.707106781	-0.432189657	0.901782735		11.4923605	-7.50764	39.51389188	-0.45225823	0.0466	53.1684
Aug-12	20	32	1024	-0.79335334	-0.608761429	-0.45243938	0.89179516		11.4070937	-8.59291	42.96453147	0.11569645	0.0907	68.7517
Sep-12	9	33	1089	-0.866025404	-0.5	-0.469323325	0.883026396		11.3139291	2.31393	25.7103228	-0.85922546	0.1556	7.3351
Oct-12	19	34	1156	-0.923879533	-0.382683432	-0.482643454	0.875816931		11.2669708	-7.73303	40.70015344	-0.08862023	0.1948	53.1684
Nov-12	13	35	1225	-0.965925826	-0.258819045	-0.492254744	0.870451186		11.3162156	-1.68378	12.95218758	-0.03853982	0.1538	1.6684
Dec-12	12	36	1296	-0.991444861	-0.130526192	-0.498059084	0.867143096		11.4989823	-0.50102	4.175147524	0.23607002	0.0438	0.0851
Jan-13	9	37	1369	-1	-1.83772E-16	-0.5	0.866025404		11.8328402	2.83284	31.47600241	0.1456725	0.0155	7.3351
Feb-13	11	38	1444	-0.991444861	0.130526192	-0.498059084	0.867143096		12.3110525	1.31105	11.91865936	0.44556401	0.3633	0.5017
Mar-13	8	39	1521	-0.965925826	0.258819045	-0.492254744	0.870451186		12.9012041	4.90120	61.26505173	0.19340546	1.4229	13.7517
Apr-13	12	40	1600	-0.923879533	0.382683432	-0.482643454	0.875816931		13.5472437	1.54724	12.89369731	0.59789154	3.3816	0.0851
May-13	7	41	1681	-0.866025404	0.5	-0.469323325	0.883026396		14.1746985	7.17470	102.4956929	0.95691178	6.0830	22.1684
Jun-13	8	42	1764	-0.79335334	0.608761429	-0.45243938	0.89179516		14.6983824	6.69838	83.72978054	0.12894597	8.9404	13.7517
Jul-13	14	43	1849	-0.707106781	0.707106781	-0.432189657	0.901782735		15.0315677	1.03157	7.368340895	-0.92175826	11.0439	5.2517
Aug-13	28	44	1936	-0.608761429	0.79335334	-0.408831004	0.912610108		15.0953844	-12.90462	46.08791283	0.02954195	11.4721	265.4184
Sep-13	14	45	2025	-0.5	0.866025404	-0.382683432	0.923879533		14.8271746	0.82717	5.908390101	0.29904824	9.7272	5.2517
Oct-13	10	46	2116	-0.382683432	0.923879533	-0.354131926	0.935195477		14.1866754	4.18668	41.86675395	0.01592163	6.1422	2.9184
Nov-13	13	47	2209	-0.258819045	0.965925826	-0.323624898	0.94618546		13.1592163	0.15922	1.224740873	-0.32649585	2.1051	1.6684
Dec-13	16	48	2304	-0.130526192	0.991444861	-0.29166883	0.956519364		11.755554	-4.24445	26.5277875	-0.12447115	0.0022	18.4184
Jan-14	12	49	2401	-2.4503E-16	1	-0.258819045	0.965925826		10.0084617	-1.991538	16.59615282	0.33055587	8.4577	0.8403
Feb-14	4	50	2500	0.130526192	0.991444861	-0.225667066	0.974204483		7.9666704	3.966670	99.16676009	-2.32821386	24.5025	79.5069
Mar-14	15	51	2601	0.258819045	0.965925826	-0.192825347	0.981233094		5.68714458	-9.312855	62.08570281	-0.65153984	52.2660	4.3403
Apr-14	13	52	2704	0.382683432	0.923879533	-0.160910494	0.986969003		3.22690235	-9.773098	75.17767427	-1.02802861	93.8915	0.0069
May-14	14	53	2809	0.5	0.866025404	-0.130526192	0.991444861		0.63562803	-13.364372	95.45979976	-0.78927424	150.8239	1.1736
Jun-14	9	54	2916	0.608761429	0.79335334	-0.102247019	0.99475904		-2.0498394	-11.049839	122.7759933	-2.86759867	223.9963	15.3403
Jul-14	21	55	3025	0.707106781	0.707106781	-0.076604145	0.997061585		-4.80838804	-25.808388	122.8970859	-0.93497963	314.1776	65.3403
Aug-14	12	56	3136	0.79335334	0.608761429	-0.054073583	0.998536954		-7.63457227	-19.634572	163.6214356	-2.37798123	422.3534	0.8403
Sep-14	18	57	3249	0.866025404	0.5	-0.035067276	0.999384954		-10.5357748	-28.535775	158.532082	-1.47373297	550.0170	25.8403
Oct-14	13	58	3364	0.923879533	0.382683432	-0.019926973	0.999801438		-13.5271935	-26.527194	204.0553346	-2.43273729	699.2777	0.0069
Nov-14	15	59	3481	0.965925826	0.258819045	-0.00892048	0.999960212		-16.6255848	-31.625585	210.8372319	-1.92285783	872.7446	4.3403
Dec-14	9	60	3600	0.991444861	0.130526192	-0.002239728	0.999997492		-19.8428674	-28.842867	320.4763048		1,073.1871	15.3403

Appendix F. Trigonometry (2-year cycle)

	Dependent	Trend Factor Polynomial		Cycle Using Trigonometry									Explained	Total
Date	y	x	x^2	Sin (2πt/L)	Cos (2πt/L)	Sin (4πt/L)	Cos (4πt/L)	L in Years	*Forecast*	Error	Percentage Error	Theil's U	Variation	Variation
Jan-10	5	1	1	0	1	-0.5	0.866025404	2	8.212888073	3.21289	64.25776145	0.1538652	12.2181	45.0017
Feb-10	9	2	4	0.258819045	0.965925826	-0.378413223	0.925636771		9.769325982	0.76933	8.548066471	0.39065008	3.7597	7.3351
Mar-10	8	3	9	0.5	0.866025404	-0.258819045	0.965925826		11.5158507	3.51585	43.94813381	-0.00820154	0.0370	13.7517
Apr-10	13	4	16	0.707106781	0.707106781	-0.152758101	0.98826361		12.93438765	-0.06561	0.504710404	-0.55227554	1.5032	1.6684
May-10	21	5	25	0.866025404	0.5	-0.070091416	0.997540572		13.82041797	-7.17958	34.18848586	0.2030295	4.4609	86.3351
Jun-10	10	6	36	0.965925826	0.258819045	-0.017840249	0.99984085		14.26361946	4.26362	42.63619463	-0.35427725	6.5295	2.9184
Jul-10	18	7	49	1	6.12574E-17	0	1		14.45722747	-3.54277	19.68206959	-0.02863625	7.5564	39.5851
Aug-10	15	8	64	0.965925826	-0.258819045	-0.017840249	0.99984085		14.48454756	-0.51545	3.436349581	0.48192258	7.7074	10.8351
Sep-10	7	9	81	0.866025404	-0.5	-0.070091416	0.997540572		14.22883874	7.22884	103.2691248	-0.07632732	6.3529	22.1684
Oct-10	14	10	100	0.707106781	-0.707106781	-0.152758101	0.98826361		13.46570879	-0.53429	3.816365754	-0.06580536	3.0884	5.2517
Nov-10	13	11	121	0.5	-0.866025404	-0.258819045	0.965925826		12.07872494	-0.92128	7.086731259	-0.21139414	0.1372	1.6684
Dec-10	13	12	144	0.258819045	-0.965925826	-0.378413223	0.925636771		10.25187617	-2.74812	21.13941409	-0.19305142	2.1213	1.6684
Jan-11	11	13	169	1.22515E-16	-1	-0.5	0.866025404		8.490331489	-2.50967	22.81516828	-0.68915889	10.3555	0.5017
Feb-11	15	14	196	-0.258819045	-0.965925826	-0.612418347	0.79053385		7.419252232	-7.58075	50.53831845	0.03051913	18.3962	10.8351
Mar-11	7	15	225	-0.5	-0.866025404	-0.707106781	0.707106781		7.457786917	0.45779	6.539813104	0.08011941	18.0671	22.1684
Apr-11	8	16	256	-0.707106781	-0.707106781	-0.779482341	0.626424201		8.560835903	0.56084	7.010448791	0.39887681	9.9067	13.7517
May-11	7	17	289	-0.866025404	-0.5	-0.828849769	0.559471233		10.19101448	3.19101	45.5859212	0.50685643	2.3023	22.1684
Jun-11	8	18	324	-0.965925826	-0.258819045	-0.856967451	0.515370534		11.54799498	3.54799	44.34993728	-0.00776541	0.0257	13.7517
Jul-11	12	19	361	-1	-1.83772E-16	-0.866025404	0.5		11.93787672	-0.06212	0.517693978	-0.90848377	0.0527	0.0851
Aug-11	22	20	400	-0.965925826	0.258819045	-0.856967451	0.515370534		11.09819472	-10.90181	49.55366037	0.15112952	0.3723	105.9184
Sep-11	6	21	441	-0.866025404	0.5	-0.828849769	0.559471233		9.324849387	3.32485	55.41415644	-0.94285029	5.6810	32.5851
Oct-11	13	22	484	-0.707106781	0.707106781	-0.779482341	0.626424201		7.342898263	-5.65710	43.5161672	0.38303262	19.0570	1.6684
Nov-11	1	23	529	-0.5	0.866025404	-0.707106781	0.707106781		5.979424029	4.97942	497.9424029	0.79234122	32.8204	114.6684
Dec-11	5	24	576	-0.258819045	0.965925826	-0.612418347	0.79053385		5.792341224	0.79234	15.84682448	0.76793102	34.9990	45.0017
Jan-12	3	25	625	-2.4503E-16	1	-0.5	0.866025404		6.839655087	3.83966	127.9885029	-0.09779368	23.7040	75.8351
Feb-12	9	26	676	0.258819045	0.965925826	-0.378413223	0.925636771		8.706618953	-0.29338	3.259789413	-0.35959226	9.0103	7.3351
Mar-12	14	27	729	0.5	0.866025404	-0.258819045	0.965925826		10.76366963	-3.23633	23.11664549	0.10662375	0.8924	5.2517
Apr-12	11	28	784	0.707106781	0.707106781	-0.152758101	0.98826361		12.49273253	1.49273	13.57029573	0.2444808	0.6153	0.5017
May-12	11	29	841	0.866025404	0.5	-0.070091416	0.997540572		13.68928881	2.68929	24.44808008	0.31300148	3.9242	0.5017
Jun-12	11	30	900	0.965925826	0.258819045	-0.017840249	0.99984085		14.44301626	3.44302	31.30014781	-0.36844089	7.4785	0.5017
Jul-12	19	31	961	1	1.19447E-15	0	1		14.94715023	-4.05285	21.33078829	-0.24815809	10.4899	53.1684
Aug-12	20	32	1024	0.965925826	-0.258819045	-0.017840249	0.99984085		15.28499627	-4.71500	23.57501865	0.31699067	12.7925	68.7517
Sep-12	9	33	1089	0.866025404	-0.5	-0.070091416	0.997540572		15.3398134	6.33981	70.44237114	-0.45697673	13.1876	7.3351
Oct-12	19	34	1156	0.707106781	-0.707106781	-0.152758101	0.98826361		14.88720941	-4.11279	21.64626624	0.04267113	10.1053	53.1684
Nov-12	13	35	1225	0.5	-0.866025404	-0.258819045	0.965925826		13.81075151	0.81075	6.236550101	0.02264836	4.4202	1.6684
Dec-12	12	36	1296	0.258819045	-0.965925826	-0.378413223	0.925636771		12.2944287	0.29443	2.453572512	0.1536175	0.3435	0.0851
Jan-13	9	37	1369	3.67545E-16	-1	-0.5	0.866025404		10.84340998	1.84341	20.4823331	-0.10190481	0.7481	7.3351
Feb-13	11	38	1444	-0.258819045	-0.965925826	-0.612418347	0.79053385		10.08285668	-0.91714	8.337666569	0.22108339	2.6422	0.5017
Mar-13	8	39	1521	-0.5	-0.866025404	-0.707106781	0.707106781		10.43191732	2.43192	30.39896649	-0.01931347	1.6292	13.7517
Apr-13	12	40	1600	-0.707106781	-0.707106781	-0.779482341	0.626424201		11.84549226	-0.15451	1.287564489	0.5655164	0.0188	0.0851
May-13	7	41	1681	-0.866025404	-0.5	-0.828849769	0.559471233		13.7861968	6.78620	96.94566855	1.06481475	4.3175	22.1684
Jun-13	8	42	1764	-0.965925826	-0.258819045	-0.856967451	0.515370534		15.45370325	7.45370	93.17129066	0.26926387	14.0278	13.7517
Jul-13	14	43	1849	-1	-4.28802E-16	-0.866025404	0.5		16.15411095	2.15411	15.38650678	-0.88393179	19.7649	5.2517
Aug-13	28	44	1936	-0.965925826	0.258819045	-0.856967451	0.515370534		15.6249549	-12.37505	44.19658964	0.00579055	15.3399	265.4184
Sep-13	14	45	2025	-0.866025404	0.5	-0.828849769	0.559471233		14.16213553	0.16214	1.1581109	0.17790788	6.0211	5.2517
Oct-13	10	46	2116	-0.707106781	0.707106781	-0.779482341	0.626424201		12.49071036	2.49071	24.90710359	-0.15622379	0.6121	2.9184
Nov-13	13	47	2209	-0.5	0.866025404	-0.707106781	0.707106781		11.43776208	-1.56224	12.01721477	-0.34144575	0.0732	1.6684
Dec-13	16	48	2304	-0.258819045	0.965925826	-0.612418347	0.79053385		11.56120523	-4.43879	27.7424673	0.05744032	0.0216	18.4184
Jan-14	12	49	2401	-4.90059E-16	1	-0.5	0.866025404		12.91904505	0.919045	7.658708759	0.92471124	0.0000	0.8403
Feb-14	4	50	2500	0.258819045	0.965925826	-0.378413223	0.925636771		15.09653487	11.096535	277.4133718	0.61602788	4.7518	79.5069
Mar-14	15	51	2601	0.5	0.866025404	-0.258819045	0.965925826		17.46411151	2.464112	16.42741006	0.43358002	20.6793	4.3403
Apr-14	13	52	2704	0.707106781	0.707106781	-0.152758101	0.98826361		19.50370036	6.503700	50.02846433	0.53929097	43.3890	0.0069
May-14	14	53	2809	0.866025404	0.5	-0.070091416	0.997540572		21.0107826	7.010783	50.07701856	0.93393114	65.5147	1.1736
Jun-14	9	54	2916	0.965925826	0.258819045	-0.017840249	0.99984085		22.075036	13.075036	145.2781778	0.20996621	83.8757	15.3403
Jul-14	21	55	3025	1	5.51317E-16	0	1		22.88969593	1.889696	8.998552036	0.54943181	99.4613	65.3403
Aug-14	12	56	3136	0.965925826	-0.258819045	-0.017840249	0.99984085		23.53806793	11.538068	96.15056607	0.49195092	112.8142	0.8403
Sep-14	18	57	3249	0.866025404	-0.5	-0.070091416	0.997540572		23.90341102	5.903411	32.79672787	0.59785183	120.7086	25.8403
Oct-14	13	58	3364	0.707106781	-0.707106781	-0.152758101	0.98826361		23.76133299	10.761333	82.7794845	0.61503085	117.6068	0.0069
Nov-14	15	59	3481	0.5	-0.866025404	-0.258819045	0.965925826		22.99540104	7.995401	53.3026736	0.85264028	101.5809	4.3403
Dec-14	9	60	3600	0.258819045	-0.965925826	-0.378413223	0.925636771		21.78960418	12.789604	142.1067132		78.7290	15.3403

Appendix G. Trigonometry (1-year cycle)

	Dependent	Trend Factor Polynomial		Cycle Using Trigonometry									Explained	Total
Date	y	x	x^2	Sin (2πt/L)	Cos (2πt/L)	Sin (4πt/L)	Cos (4πt/L)	L in Years	Forecast	Error	Percentage Error	Theil's U	Variation	Variation
Jan-10	5	1	1	0	1	-0.866025404	0.5	1	10.7969502	5.7970	115.9390041	-0.11891354	0.8306	45.0017
Feb-10	9	2	4	0.5	0.866025404	-0.5	0.866025404		8.40543228	-0.5946	6.606308027	0.18342352	10.9092	7.3351
Mar-10	8	3	9	0.866025404	0.5	-0.139838063	0.990174387		9.65081167	1.6508	20.63514585	-0.14083659	4.2334	13.7517
Apr-10	13	4	16	1	6.12574E-17	0	1		11.8733073	-1.1267	8.666867052	-0.68819517	0.0272	1.6684
May-10	21	5	25	0.866025404	-0.5	-0.139838063	0.990174387		12.0534628	-8.9465	42.60255821	0.11772977	0.1191	86.3351
Jun-10	10	6	36	0.5	-0.866025404	-0.5	0.866025404		12.4723252	2.4723	24.72325233	-0.27508773	0.5837	2.9184
Jul-10	18	7	49	1.22515E-16	-1	-0.866025404	0.5		15.2491227	-2.7509	15.28265191	0.0828949	12.5372	39.5851
Aug-10	15	8	64	-0.5	-0.866025404	-1	2.83302E-16		16.4921082	1.4921	9.947387894	0.42812733	22.8845	10.8351
Sep-10	7	9	81	-0.866025404	-0.5	-0.927435205	-0.373983878		13.42191	6.4219	91.74157127	-0.54305792	2.9363	22.1684
Oct-10	14	10	100	-1	-1.83772E-16	-0.866025404	-0.5		10.1985946	-3.8014	27.15289606	-0.18319093	2.2793	5.2517
Nov-10	13	11	121	-0.866025404	0.5	-0.927435205	-0.373983878		10.435327	-2.5647	19.72825383	-0.13404989	1.6205	1.6684
Dec-10	13	12	144	-0.5	0.866025404	-1	-6.04876E-16		11.2573515	-1.7426	13.40498869	-0.15037273	0.2034	1.6684
Jan-11	11	13	169	-2.4503E-16	1	-0.866025404	0.5		9.04515457	-1.9548	17.77132212	-0.74763961	7.0925	0.5017
Feb-11	15	14	196	0.5	0.866025404	-0.5	0.866025404		6.77596429	-8.2240	54.82690477	0.07624475	24.3283	10.8351
Mar-11	7	15	225	0.866025404	0.5	-0.139838063	0.990174387		8.14367132	1.1437	16.33816176	0.35549923	12.7068	22.1684
Apr-11	8	16	256	1	1.19447E-15	0	1		10.4884946	2.4885	31.10618231	0.47387222	1.4880	13.7517
May-11	7	17	289	0.866025404	-0.5	-0.139838063	0.990174387		10.7909777	3.7910	54.15682462	0.47602398	0.8415	22.1684
Jun-11	8	18	324	0.5	-0.866025404	-0.5	0.866025404		11.3321678	3.3322	41.65209785	0.27891161	0.1415	13.7517
Jul-11	12	19	361	3.67545E-16	-1	-0.866025404	0.5		14.2312929	2.2313	18.59410749	-0.53361616	6.3653	0.0851
Aug-11	22	20	400	-0.5	-0.866025404	-1	9.49436E-16		15.5966061	-6.4034	29.10633603	0.30221525	15.1187	105.9184
Sep-11	6	21	441	-0.866025404	-0.5	-0.927435205	-0.373983878		12.6487355	6.6487	110.8122588	-0.57537538	0.8844	32.5851
Oct-11	13	22	484	-1	-4.28802E-16	-0.866025404	-0.5		9.54774774	-3.4523	26.55578665	0.68513906	4.6681	1.6684
Nov-11	1	23	529	-0.866025404	0.5	-0.927435205	-0.373983878		9.90680783	8.9068	890.6807833	5.85115995	3.2455	114.6684
Dec-11	5	24	576	-0.5	0.866025404	-1	-1.49305E-15		10.8511599	5.8512	117.0231989	1.15225814	0.7347	45.0017
Jan-12	3	25	625	-4.90059E-16	1	-0.866025404	0.5		8.76129069	5.7613	192.043023	-0.79519065	8.6851	75.8351
Feb-12	9	26	676	0.5	0.866025404	-0.5	0.866025404		6.61442806	-2.3856	26.50635493	-0.6550597	25.9479	7.3351
Mar-12	14	27	729	0.866025404	0.5	-0.139838063	0.990174387		8.10446274	-5.8955	42.11098042	-0.03059903	12.9879	5.2517
Apr-12	11	28	784	1	5.51317E-16	0	1		10.5716136	-0.4284	3.894421365	-0.00032505	1.2921	0.5017
May-12	11	29	841	0.866025404	-0.5	-0.139838063	0.990174387		10.9964244	-0.0036	0.03250513	0.05999474	0.5068	0.5017
Jun-12	11	30	900	0.5	-0.866025404	-0.5	0.866025404		11.6599422	0.6599	5.999474426	-0.39260046	0.0023	0.5017
Jul-12	19	31	961	2.38893E-15	-1	-0.866025404	0.5		14.6813949	-4.3186	22.7295005	-0.2016297	8.8391	53.1684
Aug-12	20	32	1024	-0.5	-0.866025404	-1	1.17148E-15		16.1690357	-3.8310	19.15482137	0.21717464	19.8979	68.7517
Sep-12	9	33	1089	-0.866025404	-0.5	-0.927435205	-0.373983878		13.3434928	4.3435	48.26103139	-0.95946304	2.6737	7.3351
Oct-12	19	34	1156	-1	1.10253E-15	-0.866025404	-0.5		10.3648327	-8.6352	45.44824904	-0.11335682	1.8050	53.1684
Nov-12	13	35	1225	-0.866025404	0.5	-0.927435205	-0.373983878		10.8462204	-2.1538	16.56753518	-0.00669999	0.7432	1.6684
Dec-12	12	36	1296	-0.5	0.866025404	-1	6.12574E-17		11.9129002	-0.0871	0.72583177	0.07877988	0.0418	0.0851
Jan-13	9	37	1369	-7.35089E-16	1	-0.866025404	0.5		9.94535858	0.9454	10.50398421	-0.34213071	3.1081	7.3351
Feb-13	11	38	1444	0.5	0.866025404	-0.5	0.866025404		7.92082359	-3.0792	27.99251281	0.13938054	14.3452	0.5017
Mar-13	8	39	1521	0.866025404	0.5	-0.139838063	0.990174387		9.53318592	1.5332	19.16482403	0.01533306	4.7313	13.7517
Apr-13	12	40	1600	1	2.5727E-15	0	1		12.1226645	0.1227	1.022203986	0.47248358	0.1717	0.0851
May-13	7	41	1681	0.866025404	-0.5	-0.139838063	0.990174387		12.6698029	5.6698	80.99718444	0.77937833	0.9244	22.1684
Jun-13	8	42	1764	0.5	-0.866025404	-0.5	0.866025404		13.4556483	5.4556	68.19560387	0.32492858	3.0531	13.7517
Jul-13	14	43	1849	8.57604E-16	-1	-0.866025404	0.5		16.5994287	2.5994	18.56734767	-0.69932878	23.9228	5.2517
Aug-13	28	44	1936	-0.5	-0.866025404	-1	1.61557E-15		18.2093971	-9.7906	34.96643878	0.05379221	42.2638	265.4184
Sep-13	14	45	2025	-0.866025404	-0.5	-0.927435205	-0.373983878		15.5061819	1.5062	10.75844206	0.18927496	14.4237	5.2517
Oct-13	10	46	2116	-1	8.57495E-16	-0.866025404	-0.5		12.6498494	2.6498	26.49849392	0.02535648	0.8865	2.9184
Nov-13	13	47	2209	-0.866025404	0.5	-0.927435205	-0.373983878		13.2535648	0.2536	1.950498336	-0.11980214	2.3877	1.6684
Dec-13	16	48	2304	-0.5	0.866025404	-1	-6.04876E-16		14.4425722	-1.5574	9.733923802	0.03733489	7.4761	18.4184
Jan-14	12	49	2401	-9.80119E-16	1	-0.866025404	0.5		12.5973582	0.59736	4.977985248	0.55792924	0.1020	0.8403
Feb-14	4	50	2500	0.5	0.866025404	-0.5	0.866025404		10.6951509	6.69515	167.3787722	-0.64253978	4.9351	79.5069
Mar-14	15	51	2601	0.866025404	0.5	-0.139838063	0.990174387		12.4298409	-2.57016	17.13439422	0.14277647	0.2370	4.3403
Apr-14	13	52	2704	1	2.81773E-15	0	1		15.1416471	2.14165	16.47420823	0.1393164	4.9505	0.0069
May-14	14	53	2809	0.866025404	-0.5	-0.139838063	0.990174387		15.8111131	1.81111	12.9365225	0.55137759	8.3778	1.1736
Jun-14	9	54	2916	0.5	-0.866025404	-0.5	0.866025404		16.7192862	7.71929	85.76984661	-0.11273398	14.4599	15.3403
Jul-14	21	55	3025	1.10263E-15	-1	-0.866025404	0.5		19.9853942	-1.01461	4.831456157	0.46274716	49.9669	65.3403
Aug-14	12	56	3136	-0.5	-0.866025404	-1	-1.49305E-15		21.7176903	9.71769	80.98075269	0.09473356	77.4580	0.8403
Sep-14	18	57	3249	-0.866025404	-0.5	-0.927435205	-0.373983878		19.1368027	1.13680	6.31557064	0.18904433	38.6901	25.8403
Oct-14	13	58	3364	-1	-2.94025E-15	-0.866025404	-0.5		16.4027979	3.40280	26.1753682	0.16375699	12.1531	0.0069
Nov-14	15	59	3481	-0.866025404	0.5	-0.927435205	-0.373983878		17.1288409	2.12884	14.1922727	0.62934506	17.7424	4.3403
Dec-14	9	60	3600	-0.5	0.866025404	-1	-6.04876E-16		18.440176	9.44018	104.890844		30.5092	15.3403

Appendix H. Autocorrelation (4-year cycle)

	Dependent	Trend Factor Polynomial		Cycle Using Trigonometry							Explained	Total
Date	y	x	x^2	SIN(2*PI*t/L)	COS(2*PI*t/L)	L in Years	Forecast	Error	Percentage Error	Theil's U	Variation	Variation
Jan-10	5	1	1	0	1	4	-			0.15883745		
Feb-10	9	2	4	0.130526192	0.991444861		9.79	0.7942	8.824303044	0.31153522	3.66396	7.33507
Mar-10	8	3	9	0.258819045	0.965925826		10.80	2.8038	35.04771244	-0.2361036	0.81815	13.75174
Apr-10	13	4	16	0.382683432	0.923879533		11.11	-1.8888	14.52945222	-0.6885875	0.35660	1.66840
May-10	21	5	25	0.5	0.866025404		12.05	-8.9516	42.62684697	0.15510475	0.11562	86.33507
Jun-10	10	6	36	0.608761429	0.79335334		13.26	3.2572	32.57199709	-0.5914982	2.39899	2.91840
Jul-10	18	7	49	0.707106781	0.707106781		12.09	-5.9150	32.8610087	-0.1039241	0.14189	39.58507
Aug-10	15	8	64	0.79335334	0.608761429		13.13	-1.8706	12.47088624	0.38486244	2.01934	10.83507
Sep-10	7	9	81	0.866025404	0.5		12.77	5.7729	82.47052316	-0.3216117	1.13338	22.16840
Oct-10	14	10	100	0.923879533	0.382683432		11.75	-2.2513	16.08058705	-0.0370127	0.00163	5.25174
Nov-10	13	11	121	0.965925826	0.258819045		12.48	-0.5182	3.985980912	-0.061339	0.59829	1.66840
Dec-10	13	12	144	0.991444861	0.130526192		12.20	-0.7974	6.133897272	0.07753447	0.24429	1.66840
Jan-11	11	13	169	1	6.12574E-17		12.01	1.0079	9.163164977	-0.3140786	0.08977	0.50174
Feb-11	15	14	196	0.991444861	-0.130526192		11.55	-3.4549	23.03243033	0.31936194	0.02663	10.83507
Mar-11	7	15	225	0.965925826	-0.258819045		11.79	4.7904	68.43470138	0.36837445	0.00674	22.16840
Apr-11	8	16	256	0.923879533	-0.382683432		10.58	2.5786	32.2327647	0.43201999	1.27625	13.75174
May-11	7	17	289	0.866025404	-0.5		10.46	3.4562	49.3737137	0.30039579	1.56794	22.16840
Jun-11	8	18	324	0.79335334	-0.608761429		10.10	2.1028	26.28463146	-0.2487692	2.57783	13.75174
Jul-11	12	19	361	0.707106781	-0.707106781		10.01	-1.9902	16.58461611	-0.9745376	2.88486	0.08507
Aug-11	22	20	400	0.608761429	-0.79335334		10.31	-11.6945	53.15659644	0.2435811	1.96780	105.91840
Sep-11	6	21	441	0.5	-0.866025404		11.36	5.3588	89.31307125	-0.6152082	0.12218	32.58507
Oct-11	13	22	484	0.382683432	-0.923879533		9.31	-3.6912	28.39422678	0.69845902	5.75800	1.66840
Nov-11	1	23	529	0.258819045	-0.965925826		10.08	9.0800	907.9967243	3.60115808	2.65158	114.66840
Dec-11	5	24	576	0.130526192	-0.991444861		8.60	3.6012	72.02316155	1.22059199	9.65454	45.00174
Jan-12	3	25	625	1.22515E-16	-1		9.10	6.1030	203.4319979	-0.0239976	6.78797	75.83507
Feb-12	9	26	676	-0.130526192	-0.991444861		8.93	-0.0720	0.799921344	-0.4703132	7.73021	7.33507
Mar-12	14	27	729	-0.258819045	-0.965925826		9.77	-4.2328	30.2344178	-0.0334827	3.76807	5.25174
Apr-12	11	28	784	-0.382683432	-0.923879533		10.53	-0.4688	4.261436544	-0.0571494	1.38554	0.50174
May-12	11	29	841	-0.5	-0.866025404		10.37	-0.6286	5.714942139	-0.0352477	1.78751	0.50174
Jun-12	11	30	900	-0.608761429	-0.79335334		10.61	-0.3877	3.524773737	-0.7376216	1.20134	0.50174
Jul-12	19	31	961	-0.707106781	-0.707106781		10.89	-8.1138	42.70440776	-0.4129899	0.67596	53.16840
Aug-12	20	32	1024	-0.79335334	-0.608761429		12.15	-7.8468	39.23404257	0.17968849	0.19790	68.75174
Sep-12	9	33	1089	-0.866025404	-0.5		12.59	3.5938	39.93077562	-0.822697	0.78400	7.33507
Oct-12	19	34	1156	-0.923879533	-0.382683432		11.60	-7.4043	38.96985897	0.0072322	0.01268	53.16840
Nov-12	13	35	1225	-0.965925826	-0.258819045		13.14	0.1374	1.057013291	0.05686774	2.04227	1.66840
Dec-12	12	36	1296	-0.991444861	-0.130526192		12.74	0.7393	6.160671786	0.32741803	1.06285	0.08507
Jan-13	9	37	1369	-1	-1.83772E-16		12.93	3.9290	43.65573744	0.20568558	1.49007	7.33507
Feb-13	11	38	1444	-0.991444861	0.130526192		12.85	1.8512	16.82881994	0.48561532	1.30608	0.50174
Mar-13	8	39	1521	-0.965925826	0.258819045		13.34	5.3418	66.77210711	0.14788235	2.66811	13.75174
Apr-13	12	40	1600	-0.923879533	0.382683432		13.18	1.1831	9.858823472	0.56797111	2.17482	0.08507
May-13	7	41	1681	-0.866025404	0.5		13.82	6.8157	97.36647655	0.75681237	4.44080	22.16840
Jun-13	8	42	1764	-0.79335334	0.608761429		13.30	5.2977	66.22108199	-0.0708318	2.52604	13.75174
Jul-13	14	43	1849	-0.707106781	0.707106781		13.43	-0.5667	4.047531453	-0.9932523	2.97567	5.25174
Aug-13	28	44	1936	-0.608761429	0.79335334		14.09	-13.9055	49.66261322	0.05846676	5.69364	265.41840
Sep-13	14	45	2025	-0.5	0.866025404		15.64	1.6371	11.6933513	0.26472777	15.43497	5.25174
Oct-13	10	46	2116	-0.382683432	0.923879533		13.71	3.7062	37.06188844	-0.0111351	3.99143	2.91840
Nov-13	13	47	2209	-0.258819045	0.965925826		12.89	-0.1114	0.856543224	-0.2447057	1.39315	1.66840
Dec-13	16	48	2304	-0.130526192	0.991444861		12.82	-3.1812	19.88233614	0.04056519	1.23319	18.41840
Jan-14	12	49	2401	-2.4503E-16	1		12.65	0.64904	5.408691897	0.62392001	0.07162	0.84028
Feb-14	4	50	2500	0.130526192	0.991444861		11.49	7.48704	187.1760036	-1.2892788	2.04383	79.50694
Mar-14	15	51	2601	0.258819045	0.965925826		9.84	-5.15712	34.38076857	-0.1706245	9.44814	4.34028
Apr-14	13	52	2704	0.382683432	0.923879533		10.44	-2.55937	19.68744155	-0.3778272	6.13074	0.00694
May-14	14	53	2809	0.5	0.866025404		9.09	-4.91175	35.0839535	-0.055	14.65680	1.17361
Jun-14	9	54	2916	0.608761429	0.79335334		8.23	-0.77000	8.555551002	-1.6123182	21.96484	15.34028
Jul-14	21	55	3025	0.707106781	0.707106781		6.49	-14.51086	69.09935112	-0.2467638	41.31315	65.34028
Aug-14	12	56	3136	0.79335334	0.608761429		6.82	-5.18204	43.18366628	-1.143603	37.19422	0.84028
Sep-14	18	57	3249	0.866025404	0.5		4.28	-13.72324	76.24020317	-0.5099479	74.64793	25.84028
Oct-14	13	58	3364	0.923879533	0.382683432		3.82	-9.17906	70.60816832	-1.0215186	82.73228	0.00694
Nov-14	15	59	3481	0.965925826	0.258819045		1.72	-13.27974	88.53160988	-0.5592961	125.35956	4.34028
Dec-14	9	60	3600	0.991444861	0.130526192		0.61	-8.38944	93.21601279		151.44029	15.34028

Appendix I. Autocorrelation (2-year cycle)

	Dependent	Trend Factor Polynomial		Cycle Using Trigonometry							Explained	Total
Date	y	x	x^2	SIN(2*PI*t/L)	COS(2*PI*t/L)	L in Years	Forecast	Error	Percentage Error	Theil's U	Variation	Variation
Jan-10	5	1	1	0	1	2	-			0.34033105		
Feb-10	9	2	4	0.258819045	0.965925826		10.70	1.7017	18.90728035	0.3766354	1.0134	7.3351
Mar-10	8	3	9	0.5	0.866025404		11.39	3.3897	42.37148298	-0.1614737	0.1015	13.7517
Apr-10	13	4	16	0.707106781	0.707106781		11.71	-1.2918	9.936846016	-0.6602471	0.0000	1.6684
May-10	21	5	25	0.866025404	0.5		12.42	-8.5832	40.8724401	0.15552983	0.5019	86.3351
Jun-10	10	6	36	0.965925826	0.258819045		13.27	3.2661	32.66126467	-0.5361558	2.4267	2.9184
Jul-10	18	7	49	1	6.12574E-17		12.64	-5.3616	29.78643443	-0.0964544	0.8651	39.5851
Aug-10	15	8	64	0.965925826	-0.258819045		13.26	-1.7362	11.57452644	0.39750037	2.4195	10.8351
Sep-10	7	9	81	0.866025404	-0.5		12.96	5.9625	85.17865155	-0.2616456	1.5729	22.1684
Oct-10	14	10	100	0.707106781	-0.707106781		12.17	-1.8315	13.0822783	-0.0472793	0.2117	5.2517
Nov-10	13	11	121	0.5	-0.866025404		12.34	-0.6619	5.091616625	-0.0902899	0.3966	1.6684
Dec-10	13	12	144	0.258819045	-0.965925826		11.83	-1.1738	9.028991194	0.02386781	0.0139	1.6684
Jan-11	11	13	169	1.22515E-16	-1		11.31	0.3103	2.820741539	-0.3994981	0.1584	0.5017
Feb-11	15	14	196	-0.258819045	-0.965925826		10.61	-4.3945	29.29652622	0.22177717	1.2162	10.8351
Mar-11	7	15	225	-0.5	-0.866025404		10.33	3.3267	47.52367914	0.17270643	1.9090	22.1684
Apr-11	8	16	256	-0.707106781	-0.707106781		9.21	1.2089	15.11181221	0.22593412	6.2469	13.7517
May-11	7	17	289	-0.866025404	-0.5		8.81	1.8075	25.82104184	0.0512663	8.4150	22.1684
Jun-11	8	18	324	-0.965925826	-0.258819045		8.36	0.3589	4.485801275	-0.4776428	11.2189	13.7517
Jul-11	12	19	361	-1	-1.83772E-16		8.18	-3.8211	31.84285196	-1.1367946	12.4572	0.0851
Aug-11	22	20	400	-0.965925826	0.258819045		8.36	-13.6415	62.00697591	0.14201713	11.2216	105.9184
Sep-11	6	21	441	-0.866025404	0.5		9.12	3.1244	52.07294729	-0.8063543	6.6768	32.5851
Oct-11	13	22	484	-0.707106781	0.707106781		8.16	-4.8381	37.21635145	0.61583	12.5774	1.6684
Nov-11	1	23	529	-0.5	0.866025404		9.01	8.0058	800.5790062	3.59939357	7.3037	114.6684
Dec-11	5	24	576	-0.258819045	0.965925826		8.60	3.5994	71.98787134	1.28961728	9.6655	45.0017
Jan-12	3	25	625	-2.4503E-16	1		9.45	6.4481	214.9362128	0.30939729	5.1087	75.8351
Feb-12	9	26	676	0.258819045	0.965925826		9.93	0.9282	10.31324304	-0.3315121	3.1689	7.3351
Mar-12	14	27	729	0.5	0.866025404		11.02	-2.9836	21.31149133	0.07316871	0.4788	5.2517
Apr-12	11	28	784	0.707106781	0.707106781		12.02	1.0244	9.312381789	0.12815607	0.0999	0.5017
May-12	11	29	841	0.866025404	0.5		12.41	1.4097	12.81560741	0.17598481	0.4919	0.5017
Jun-12	11	30	900	0.965925826	0.258819045		12.94	1.9358	17.59848148	-0.5127903	1.5068	0.5017
Jul-12	19	31	961	1	1.19447E-15		13.36	-5.6407	29.68785769	-0.30315	2.7257	53.1684
Aug-12	20	32	1024	0.965925826	-0.258819045		14.24	-5.7598	28.79924766	0.27418214	6.4101	68.7517
Sep-12	9	33	1089	0.866025404	-0.5		14.48	5.4836	60.92936346	-0.5857695	7.7023	7.3351
Oct-12	19	34	1156	0.707106781	-0.707106781		13.73	-5.2719	27.74697507	0.07211344	4.0794	53.1684
Nov-12	13	35	1225	0.5	-0.866025404		14.37	1.3702	10.53965645	0.1347755	7.0853	1.6684
Dec-12	12	36	1296	0.258819045	-0.965925826		13.75	1.7521	14.6006792	0.36827169	4.1769	0.0851
Jan-13	9	37	1369	3.67545E-16	-1		13.42	4.4193	49.10289178	0.21084757	2.9273	7.3351
Feb-13	11	38	1444	-0.258819045	-0.965925826		12.90	1.8976	17.25116443	0.42995973	1.4144	0.5017
Mar-13	8	39	1521	-0.5	-0.866025404		12.73	4.7296	59.1194628	0.0286235	1.0429	13.7517
Apr-13	12	40	1600	-0.707106781	-0.707106781		12.23	0.2290	1.908233308	0.44166565	0.2711	0.0851
May-13	7	41	1681	-0.866025404	-0.5		12.30	5.3000	75.71411071	0.54535709	0.3501	22.1684
Jun-13	8	42	1764	-0.965925826	-0.258819045		11.82	3.8175	47.71874564	-0.2633803	0.0119	13.7517
Jul-13	14	43	1849	-1	-4.28802E-16		11.89	-2.1070	15.05030348	-1.1090928	0.0341	5.2517
Aug-13	28	44	1936	-0.965925826	0.258819045		12.47	-15.5273	55.45463936	-0.007735	0.5843	265.4184
Sep-13	14	45	2025	-0.866025404	0.5		13.78	-0.2166	1.546998665	0.23007526	4.3060	5.2517
Oct-13	10	46	2116	-0.707106781	0.707106781		13.22	3.2211	32.21053594	0.0524739	2.2883	2.9184
Nov-13	13	47	2209	-0.5	0.866025404		13.52	0.5247	4.036453843	-0.1185504	3.2993	1.6684
Dec-13	16	48	2304	-0.258819045	0.965925826		14.46	-1.5412	9.632220524	0.21816662	7.5653	18.4184
Jan-14	12	49	2401	-4.90059E-16	1		15.49	3.4907	29.08888221	1.00614866	6.6255	0.8403
Feb-14	4	50	2500	0.258819045	0.965925826		16.07	12.0738	301.8445971	0.35769751	9.9674	79.5069
Mar-14	15	51	2601	0.5	0.866025404		16.43	1.4308	9.538600277	0.34487675	12.3491	4.3403
Apr-14	13	52	2704	0.707106781	0.707106781		18.17	5.1732	39.79347078	0.37158591	27.6306	0.0069
May-14	14	53	2809	0.866025404	0.5		18.83	4.8306	34.50440612	0.76457964	34.9748	1.1736
Jun-14	9	54	2916	0.965925826	0.258819045		19.70	10.7041	118.9346099	-0.1089354	46.0695	15.3403
Jul-14	21	55	3025	1	5.51317E-16		20.02	-0.9804	4.668659402	0.45123646	50.4514	65.3403
Aug-14	12	56	3136	0.965925826	-0.258819045		21.48	9.4760	78.96637976	0.26586869	73.2616	0.8403
Sep-14	18	57	3249	0.866025404	-0.5		21.19	3.1904	17.72457931	0.49859684	68.4551	25.8403
Oct-14	13	58	3364	0.707106781	-0.707106781		21.97	8.9747	69.03648495	0.51956622	82.0487	0.0069
Nov-14	15	59	3481	0.5	-0.866025404		21.75	6.7544	45.02907251	0.86671363	78.1048	4.3403
Dec-14	9	60	3600	0.258819045	-0.965925826		22.00	13.0007	144.4522722		82.5197	15.3403

Appendix J. Autocorrelation (1-year cycle)

	Dependent	Trend Factor Polynomial		Cycle Using Trigonometry							Explained	Total
Date	y	x	x^2	SIN(2*PI*t/L)	COS(2*PI*t/L)	L in Years	Forecast	Error	Percentage Error	Theil's U	Variation	Variation
Jan-10	5	1	1	0	1	1	-			0.1341263		
Feb-10	9	2	4	0.5	0.866025404		9.67	0.6706	7.451460894	0.22263708	4.1522	7.3351
Mar-10	8	3	9	0.866025404	0.5		10.00	2.0037	25.04667117	-0.2416761	2.9057	13.7517
Apr-10	13	4	16	1	6.12574E-17		11.07	-1.9334	14.87237453	-0.6641298	0.4118	1.6684
May-10	21	5	25	0.866025404	-0.5		12.37	-8.6337	41.11279512	0.16820858	0.4329	86.3351
Jun-10	10	6	36	0.5	-0.866025404		13.53	3.5324	35.32380204	-0.3498351	3.3271	2.9184
Jul-10	18	7	49	1.22515E-16	-1		14.50	-3.4984	19.43528228	-0.0302491	7.8026	39.5851
Aug-10	15	8	64	-0.5	-0.866025404		14.46	-0.5445	3.629887478	0.45196142	7.5470	10.8351
Sep-10	7	9	81	-0.866025404	-0.5		13.78	6.7794	96.84887547	-0.2082538	4.2894	22.1684
Oct-10	14	10	100	-1	-1.83772E-16		12.54	-1.4578	10.41268803	-0.1586465	0.6954	5.2517
Nov-10	13	11	121	-0.866025404	0.5		10.78	-2.2211	17.08500665	-0.2889128	0.8638	1.6684
Dec-10	13	12	144	-0.5	0.866025404		9.24	-3.7559	28.89128362	-0.2159286	6.0723	1.6684
Jan-11	11	13	169	-2.4503E-16	1		8.19	-2.8071	25.51883874	-0.6430009	12.3581	0.5017
Feb-11	15	14	196	0.5	0.866025404		7.93	-7.0730	47.15339786	0.09238801	14.2986	10.8351
Mar-11	7	15	225	0.866025404	0.5		8.39	1.3858	19.79743125	0.23968494	11.0391	22.1684
Apr-11	8	16	256	1	1.19447E-15		9.68	1.6778	20.97243261	0.52029044	4.1231	13.7517
May-11	7	17	289	0.866025404	-0.5		11.16	4.1623	59.46176415	0.65529246	0.2981	22.1684
Jun-11	8	18	324	0.5	-0.866025404		12.59	4.5870	57.33809038	0.18810067	0.7721	13.7517
Jul-11	12	19	361	3.67545E-16	-1		13.50	1.5048	12.5400444	-0.6963767	3.2273	0.0851
Aug-11	22	20	400	-0.5	-0.866025404		13.64	-8.3565	37.98418098	0.31368654	3.7448	105.9184
Sep-11	6	21	441	-0.866025404	-0.5		12.90	6.9011	115.018397	-0.1820345	1.4227	32.5851
Oct-11	13	22	484	-1	-4.28802E-16		11.91	-1.0922	8.401593089	0.71309585	0.0398	1.6684
Nov-11	1	23	529	-0.866025404	0.5		10.27	9.2702	927.0246011	4.02362632	2.0681	114.6684
Dec-11	5	24	576	-0.5	0.866025404		9.02	4.0236	80.47252645	1.00781381	7.2077	45.0017
Jan-12	3	25	625	-4.90059E-16	1		8.04	5.0391	167.9689691	-0.3670469	13.4635	75.8351
Feb-12	9	26	676	0.5	0.866025404		7.90	-1.1011	12.23489657	-0.6162359	14.5121	7.3351
Mar-12	14	27	729	0.866025404	0.5		8.45	-5.5461	39.61516377	-0.0943164	10.5915	5.2517
Apr-12	11	28	784	1	5.51317E-16		9.68	-1.3204	12.00390247	0.0317188	4.1159	0.5017
May-12	11	29	841	0.866025404	-0.5		11.35	0.3489	3.171879794	0.17132621	0.1292	0.5017
Jun-12	11	30	900	0.5	-0.866025404		12.88	1.8846	17.13262085	-0.4597415	1.3836	0.5017
Jul-12	19	31	961	2.38893E-15	-1		13.94	-5.0572	26.61661093	-0.3079912	4.9930	53.1684
Aug-12	20	32	1024	-0.5	-0.866025404		14.15	-5.8518	29.2591654	0.23322236	5.9528	68.7517
Sep-12	9	33	1089	-0.866025404	-0.5		13.66	4.6644	51.8271902	-0.6974428	3.8264	7.3351
Oct-12	19	34	1156	-1	1.10253E-15		12.72	-6.2770	33.03676602	-0.0964797	1.0296	53.1684
Nov-12	13	35	1225	-0.866025404	0.5		11.17	-1.8331	14.1008801	-0.1571251	0.2932	1.6684
Dec-12	12	36	1296	-0.5	0.866025404		9.96	-2.0426	17.02188099	0.01436617	3.0659	0.0851
Jan-13	9	37	1369	-7.35089E-16	1		9.17	0.1724	1.915489757	-0.2030354	6.4310	7.3351
Feb-13	11	38	1444	0.5	0.866025404		9.17	-1.8273	16.61198321	0.17386425	6.4295	0.5017
Mar-13	8	39	1521	0.866025404	0.5		9.91	1.9125	23.90633474	-0.0772391	3.2250	13.7517
Apr-13	12	40	1600	1	2.5727E-15		11.38	-0.6179	5.149274033	0.50614681	0.1064	0.0851
May-13	7	41	1681	0.866025404	-0.5		13.07	6.0738	86.76802386	0.97271718	1.8644	22.1684
Jun-13	8	42	1764	0.5	-0.866025404		14.81	6.8090	85.11275291	0.2472792	9.6143	13.7517
Jul-13	14	43	1849	8.57604E-16	-1		15.98	1.9782	14.13024027	-0.8329411	18.2320	5.2517
Aug-13	28	44	1936	-0.5	-0.866025404		16.34	-11.6612	41.64705698	0.06388655	21.4414	265.4184
Sep-13	14	45	2025	-0.866025404	-0.5		15.79	1.7888	12.77731056	0.35838772	16.6504	5.2517
Oct-13	10	46	2116	-1	8.57495E-16		15.02	5.0174	50.17428105	0.07938052	10.9501	2.9184
Nov-13	13	47	2209	-0.866025404	0.5		13.79	0.7938	6.106193764	-0.2633006	4.3492	1.6684
Dec-13	16	48	2304	-0.5	0.866025404		12.58	-3.4229	21.39317105	-0.0088275	0.7547	18.4184
Jan-14	12	49	2401	-9.80119E-16	1		11.86	-0.1412	1.176995264	0.6665664	1.1192	0.8403
Feb-14	4	50	2500	0.5	0.866025404		12.00	7.9988	199.9699211	-0.4968084	0.8425	79.5069
Mar-14	15	51	2601	0.866025404	0.5		13.01	-1.9872	13.2482238	0.09353812	0.0092	4.3403
Apr-14	13	52	2704	1	2.81773E-15		14.40	1.4031	10.7928605	0.17745499	2.2094	0.0069
May-14	14	53	2809	0.866025404	-0.5		16.31	2.3069	16.47796356	0.64857296	11.4938	1.1736
Jun-14	9	54	2916	0.5	-0.866025404		18.08	9.0800	100.8891271	-0.1706912	26.6602	15.3403
Jul-14	21	55	3025	1.10263E-15	-1		19.46	-1.5362	7.315338378	0.37442644	42.8647	65.3403
Aug-14	12	56	3136	-0.5	-0.866025404		19.86	7.8630	65.52462733	0.14800614	48.2509	0.8403
Sep-14	18	57	3249	-0.866025404	-0.5		19.78	1.7761	9.867075807	0.32428104	47.0515	25.8403
Oct-14	13	58	3364	-1	-2.94025E-15		18.84	5.8371	44.90045216	0.21173889	35.0510	0.0069
Nov-14	15	59	3481	-0.866025404	0.5		17.75	2.7526	18.35070365	0.51181843	23.3863	4.3403
Dec-14	9	60	3600	-0.5	0.866025404		16.68	7.6773	85.30307138		14.1422	15.3403

Appendix K. Decomposition Multiplicative (12 months)

Date	Month	x	Sales	12 Period Moving Average	CMAt =TRt*CLt	SNt*IRt = Yt/(TRt*CLt)	Ave SNt	Norm SNt	Deseasonalized Yt (Yt/SNt)	TRt	TRtSNt	CLtIRt	CLt	IRt	Forecast	Error	Percentage Error	Explained Variation	Total Variation	Theil's U
Jan-10	1	1	5				0.72657	0.7319728	6.830855347	11.07	8.10072	0.61723								-0.321916
Feb-10	2	2	9				0.85538	0.8617448	10.44392694	11.10	9.56215	0.94121	0.77288	1.21779	7.3904	1.6096	17.8842	18.64438	7.33507	0.265294
Mar-10	3	3	8				0.9389	0.9458791	8.457740846	11.13	10.5234	0.76021	0.9871	0.77015	10.3876	-2.3876	29.8456	1.74422	13.75174	0.236926
Apr-10	4	4	13				0.9182	0.9250251	14.05367312	11.15	10.3185	1.25987	1.44356	0.87275	14.8954	-1.8954	14.5801	10.15746	1.66840	-0.511148
May-10	5	5	21				0.80663	0.8126263	25.8421359	11.18	9.08854	2.3106	1.57947	1.4629	14.3551	6.6449	31.6425	7.00524	86.33507	0.158460
Jun-10	6	6	10				0.75792	0.7635564	13.09661022	11.21	8.56211	1.16794	1.55659	0.75032	13.3277	-3.3277	33.2767	2.62225	2.91840	-0.255290
Jul-10	7	7	18	12.17	12.42	1.44966443	1.3341	1.3440216	13.39264203	11.24	15.1105	1.19122	1.02228	1.16527	15.4471	2.5529	14.1828	13.97839	39.58507	0.229745
Aug-10	8	8	15	12.67	12.92	1.161290323	1.86656	1.8804419	7.976848319	11.27	21.1964	0.70767	0.90277	0.78389	19.1354	-4.1354	27.5695	55.16159	10.83507	0.022172
Sep-10	9	9	7	13.17	13.13	0.533333333	0.7596	0.7652457	9.147388934	11.30	8.64831	0.80941	0.84786	0.95464	7.3326	-0.3326	4.7511	19.14723	22.16840	0.114816
Oct-10	10	10	14	13.08	12.88	1.087378641	1.19478	1.2036716	11.63107965	11.33	13.6384	1.02652	1.08545	0.94571	14.8037	-0.8037	5.7408	9.58137	5.25174	-0.136307
Nov-10	11	11	13	12.67	12.08	1.075862069	0.79971	0.8056608	16.13582355	11.36	9.15226	1.42041	1.21191	1.17205	11.0917	1.9083	14.6792	0.38023	1.66840	0.100638
Dec-10	12	12	13	11.50	11.42	1.138686131	0.95306	0.960154	13.53949407	11.39	10.9354	1.1888	1.30844	0.90856	14.3083	-1.3083	10.0638	6.75981	1.66840	0.016527
Jan-11	1	13	11	11.33	11.08	0.992481203	0.72657	0.7319728	15.02788176	11.42	8.35806	1.3161	1.3418	0.98084	11.2148	-0.2148	1.9532	0.24353	0.50174	-0.322894
Feb-11	2	14	15	10.83	11.13	1.348314607	0.85538	0.8617448	17.4065449	11.45	9.86511	1.52051	1.16047	1.31025	11.4482	3.5518	23.6789	0.06769	10.83507	0.237025
Mar-11	3	15	7	11.42	11.38	0.615384615	0.9389	0.9458791	7.40052324	11.48	10.856	0.64481	0.97231	0.66317	10.5554	-3.5554	50.7912	1.32930	22.16840	-0.056614
Apr-11	4	16	8	11.33	11.29	0.708487085	0.9182	0.9250251	8.648414225	11.51	10.6437	0.75162	0.71438	1.05212	7.6037	0.3963	4.9537	16.84797	13.75174	0.064099
May-11	5	17	7	11.25	10.75	0.651162791	0.80663	0.8126263	8.614045299	11.54	9.37423	0.74673	0.80143	0.93174	7.5128	-0.5128	7.3256	17.60257	22.16840	-0.124088
Jun-11	6	18	8	10.25	9.92	0.806722689	0.75792	0.7635564	10.47728818	11.57	8.83054	0.90595	0.80758	1.1218	7.1314	0.8686	10.8577	20.94849	13.75174	0.241748
Jul-11	7	19	12	9.58	9.25	1.297297297	1.3341	1.3440216	8.928428022	11.59	15.583	0.77007	0.89418	0.8612	13.9340	-1.9340	16.1165	4.95353	0.08507	-0.346151
Aug-11	8	20	22	8.92	8.67	2.538461538	1.86656	1.8804419	11.69937753	11.62	21.8575	1.00652	0.81648	1.23276	17.8462	4.1538	18.8809	37.67331	105.91840	0.079085
Sep-11	9	21	6	8.42	8.71	0.688995215	0.7596	0.7652457	7.840619086	11.65	8.91734	0.67285	0.86796	0.77521	7.7399	-1.7399	28.9978	15.74870	32.58507	-0.836026
Oct-11	10	22	13	9.00	9.13	1.424657534	1.19478	1.2036716	10.80028825	11.68	14.0615	0.92451	0.56778	1.62829	7.9838	5.0162	38.5858	13.87181	1.66840	0.279697
Nov-11	11	23	1	9.25	9.42	0.10619469	0.79971	0.8056608	1.241217196	11.71	9.4355	0.10598	0.49134	0.2157	4.6361	-3.6361	363.6066	50.01697	114.66840	-1.626617
Dec-11	12	24	5	9.58	9.71	0.515021459	0.95306	0.960154	5.207497721	11.74	11.273	0.44354	0.29925	1.48219	3.3734	1.6266	32.5323	69.47139	45.00174	0.363130
Jan-12	1	25	3	9.83	10.13	0.296296296	0.72657	0.7319728	4.098513208	11.77	8.61539	0.34821	0.55896	0.62297	4.8156	-1.8156	60.5216	47.50910	75.83507	-0.192911
Feb-12	2	26	9	10.42	10.33	0.870967742	0.85538	0.8617448	10.44392694	11.80	10.1681	0.88512	0.82821	1.06872	8.4213	0.5787	6.4304	10.80481	7.33507	-0.254688
Mar-12	3	27	14	10.25	10.38	1.34939759	0.9389	0.9458791	14.80104648	11.83	11.1885	1.25128	1.04641	1.19578	11.7078	2.2922	16.3728	0.00000	5.25174	0.100378
Apr-12	4	28	11	10.50	10.75	1.023255814	0.9182	0.9250251	11.89156956	11.86	10.9689	1.00283	1.13095	0.88672	12.4053	-1.4053	12.7753	0.48574	0.50174	-0.019230
May-12	5	29	11	11.00	11.50	0.956521739	0.80663	0.8126263	13.5363569	11.89	9.65992	1.13873	1.11683	1.01961	10.7885	0.2115	1.9230	0.84615	0.50174	-0.026396
Jun-12	6	30	11	12.00	12.29	0.894915254	0.75792	0.7635564	14.40627125	11.92	9.09898	1.20893	1.17702	1.02711	10.7096	0.2904	2.6396	0.99739	0.50174	-0.131221
Jul-12	7	31	19	12.58	12.83	1.480519481	1.3341	1.3440216	14.1366777	11.95	16.0555	1.18339	1.09349	1.08222	17.5566	1.4434	7.5970	34.20188	53.16840	0.152810
Aug-12	8	32	20	13.08	13.17	1.518987342	1.86656	1.8804419	10.63579776	11.98	22.5186	0.88815	1.01709	0.87323	22.9034	-2.9034	14.5169	125.32926	68.75174	0.036816
Sep-12	9	33	9	13.25	13.00	0.692307692	0.7596	0.7652457	11.76092863	12.00	9.18637	0.97971	1.05987	0.92437	9.7363	-0.7363	8.1812	3.88887	7.33507	-0.164226
Oct-12	10	34	19	12.75	12.79	1.48534202	1.19478	1.2036716	15.78503667	12.03	14.4847	1.31173	1.20969	1.08435	17.5220	1.4780	7.7791	33.79836	53.16840	-0.056261
Nov-12	11	35	13	12.83	12.67	1.026315789	0.79971	0.8056608	16.13582355	12.06	9.71874	1.33762	1.22763	1.0896	11.9310	1.0690	8.2228	0.04959	1.66840	0.084810
Dec-12	12	36	12	12.50	12.38	0.96969697	0.95306	0.960154	12.49799453	12.09	11.6105	1.03354	1.1285	0.91585	13.1025	-1.1025	9.1877	1.94378	0.08507	0.013648
Jan-13	1	37	9	12.25	12.04	0.747404844	0.72657	0.7319728	12.29553962	12.12	8.87272	1.01434	1.0328	0.98213	9.1638	-0.1638	1.8197	6.47480	7.33507	-0.152144
Feb-13	2	38	11	11.83	12.17	0.904109589	0.85538	0.8617448	12.7647996	12.15	10.471	1.05052	0.91975	1.14218	9.6307	1.3693	12.4482	4.31655	0.50174	0.252853
Mar-13	3	39	8	12.50	12.71	0.629508197	0.9389	0.9458791	8.457740846	12.18	11.521	0.69438	0.9358	0.74202	10.7814	-2.7814	34.7673	0.85923	13.75174	-0.342018
Apr-13	4	40	12	12.92	12.54	0.956810631	0.9182	0.9250251	12.97262134	12.21	11.2941	1.0625	0.82024	1.29536	9.2639	2.7361	22.8012	5.97546	0.08507	0.140583
May-13	5	41	7	12.17	12.17	0.575342466	0.80663	0.8126263	8.614045299	12.24	9.94561	0.70383	0.87345	0.8058	8.6870	-1.6870	24.0999	9.12850	22.16840	-0.070110
Jun-13	6	42	8	12.17	12.33	0.648648649	0.75792	0.7635564	10.47728818	12.27	9.36742	0.85402	0.80163	1.06536	7.5092	0.4908	6.1346	17.63247	13.75174	0.253350
Jul-13	7	43	14	12.50	12.63	1.108910891	1.3341	1.3440216	10.41649936	12.30	16.528	0.84705	0.96967	0.87354	16.0268	-2.0268	14.4772	18.64917	5.25174	-0.048691
Aug-13	8	44	28	12.75	12.46	2.247491639	1.86656	1.8804419	14.89011686	12.33	23.1797	1.20795	1.17854	1.02495	27.3183	0.6817	2.4345	243.67194	265.41840	-0.121854
Sep-13	9	45	14	12.17	12.46	1.123745819	0.7596	0.7652457	18.29477787	12.36	9.4554	1.48064	1.11979	1.32224	10.5881	3.4119	24.3709	1.25497	5.25174	0.510703
Oct-13	10	46	10	12.75	12.79	0.781758958	1.19478	1.2036716	8.307914035	12.39	14.9079	0.67079	1.15039	0.5831	17.1498	-7.1498	71.4984	29.60998	2.91840	-0.196563
Nov-13	11	47	13	12.83	13.13	0.99047619	0.79971	0.8056608	16.13582355	12.41	10.002	1.29974	1.10322	1.17814	11.0344	1.9656	15.1202	0.45423	1.66840	-0.019659
Dec-13	12	48	16	13.42	13.46	1.188854489	0.95306	0.960154	16.66399271	12.44	11.9481	1.33913	1.31774	1.01623	15.7444	0.2556	1.5973	16.29005	18.41840	-0.174667
Jan-14	1	49	12	13.50	13.79	0.870090634	0.72657	0.7319728	16.39405283	12.47	9.13006	1.31434	1.00824	1.30359	9.2053	2.7947	23.2890	13.77406	0.84028	0.549847
Feb-14	2	50	4	14.08	13.42	0.298136646	0.85538	0.8617448	4.641745308	12.50	10.774	0.37126	0.98368	0.37742	10.5982	-6.5982	164.9542	5.37544	79.50694	-1.028093
Mar-14	3	51	15	12.75	12.92	1.161290323	0.9389	0.9458791	15.85826409	12.53	11.8536	1.26544	0.91851	1.37771	10.8876	4.1124	27.4158	4.11699	4.34028	0.102287
Apr-14	4	52	13	13.08	13.21	0.984227129	0.9182	0.9250251	14.05367312	12.56	11.6193	1.11882	1.25087	0.89444	14.5343	-1.5343	11.8024	2.61676	0.00694	-0.179406
May-14	5	53	14	13.33	13.42	1.043478261	0.80663	0.8126263	17.2280906	12.59	10.2313	1.36835	1.1404	1.19989	11.6677	2.3323	16.6591	1.55986	1.17361	0.168761
Jun-14	6	54	9	13.50	13.21	0.681388013	0.75792	0.7635564	11.7869492	12.62	9.63585	0.93401	1.17921	0.79207	11.3627	-2.3627	26.2517	2.41495	15.34028	-0.650529
Jul-14	7	55	21	12.92			1.3341	1.3440216	15.62474904	12.65	17.0005	1.23526	0.89087	1.38657	15.1452	5.8548	27.8798	4.96653	65.34028	0.786968
Aug-14	8	56	12				1.86656	1.8804419	6.381478655	12.68	23.8408	0.50334	1.19653	0.42066	28.5263	-16.5263	137.7194	243.66167	0.84028	-0.634985
Sep-14	9	57	18				0.7596	0.7652457	23.52185726	12.71	9.72443	1.85101	1.06743	1.73407	10.3802	7.6198	42.3323	6.43375	25.84028	0.458087
Oct-14	10	58	13				1.19478	1.2036716	10.80028825	12.74	15.331	0.84795	1.38579	0.61189	21.2456	-8.2456	63.4274	69.37060	0.00694	-0.352412
Nov-14	11	59	15				0.79971	0.8056608	18.61825794	12.77	10.2852	1.4584	1.01297	1.43973	10.4187	4.5813	30.5423	6.24009	4.34028	0.479282
Dec-14	12	60	9				0.95306	0.960154	9.373495898	12.80	12.2856	0.73256	1.31774	0.55593	16.1892	-7.1892	79.8804	10.70970	15.34028	

Appendix L. Decomposition Multiplicative (4 months)

Date	Month	x	Sales	4 Period Moving Average	CMAt =TRt*CLt	SNt*IRt = Yt/(TRt*CLt)	Ave SNt	Norm SNt	Deseasonalized Yt (Yt/SNt)	TRt	TRtSNt	CLtIRt	CLt	IRt	*Forecast*	Error	Percentage Error	Explained Variation	Total Variation	Theil's U
Jan-10	1	1	5				0.85164	0.86648	5.770442348	10.70	9.27407	0.53914								-0.34910
Feb-10	2	2	9				0.92901	0.9452	9.52180051	10.74	10.1544	0.88632	0.71442	1.24061	7.2545	1.7455	19.3945	19.83669	7.33507	0.20262
Mar-10	3	3	8	9	10.75	0.744186047	1.01586	1.03357	7.740188248	10.78	11.1451	0.7178	0.88143	0.81437	9.8236	-1.8236	22.7950	3.55223	13.75174	0.45227
Apr-10	4	4	13	13	12.88	1.009708738	1.13497	1.15475	11.25785464	10.82	12.498	1.04016	1.32966	0.78228	16.6181	-3.6181	27.8318	24.10608	1.66840	-0.59168
May-10	1	5	21	13	14.25	1.473684211	0.85164	0.86648	24.23585786	10.86	9.41279	2.23101	1.41384	1.57798	13.3081	7.6919	36.6279	2.55936	86.33507	0.30782
Jun-10	2	6	10	16	15.75	0.634920635	0.92901	0.9452	10.57977834	10.90	10.3057	0.97034	1.59759	0.60737	16.4643	-6.4643	64.6432	22.61941	2.91840	-0.38827
Jul-10	3	7	18	16	14.25	1.263157895	1.01586	1.03357	17.41542356	10.94	11.3106	1.59143	1.24815	1.27503	14.1173	3.8827	21.5704	5.80325	39.58507	-0.00965
Aug-10	4	8	15	13	13.00	1.153846154	1.13497	1.15475	12.98983227	10.98	12.6829	1.18269	1.169	1.01172	14.8263	0.1737	1.1580	9.72173	10.83507	0.22409
Sep-10	1	9	7	14	12.88	0.54368932	0.85164	0.86648	8.078619287	11.02	9.5515	0.73287	1.08479	0.67558	10.3614	-3.3614	48.0199	1.81425	22.16840	-0.40432
Oct-10	2	10	14	12	12.00	1.166666667	0.92901	0.9452	14.81168968	11.06	10.457	1.33881	1.06816	1.25338	11.1698	2.8302	20.2159	0.29005	5.25174	0.02281
Nov-10	3	11	13	12	12.25	1.06122449	1.01586	1.03357	12.5778059	11.10	11.476	1.1328	1.16063	0.97602	13.3194	-0.3194	2.4569	2.59554	1.66840	0.08163
Dec-10	4	12	13	13	12.88	1.009708738	1.13497	1.15475	11.25785464	11.14	12.8678	1.01028	1.09275	0.92453	14.0612	-1.0612	8.1631	5.53601	1.66840	0.03825
Jan-11	1	13	11	13	12.25	0.897959184	0.85164	0.86648	12.69497317	11.18	9.69022	1.13517	1.18647	0.95676	11.4972	-0.4972	4.5199	0.04458	0.50174	-0.35088
Feb-11	2	14	15	12	10.88	1.379310345	0.92901	0.9452	15.86966752	11.22	10.6083	1.41398	1.05015	1.34646	11.1403	3.8597	25.7311	0.32263	10.83507	0.21324
Mar-11	3	15	7	10	9.75	0.717948718	1.01586	1.03357	6.772664717	11.26	11.6415	0.6013	0.87606	0.68637	10.1987	-3.1987	45.6950	2.27914	22.16840	0.05449
Apr-11	4	16	8	9	8.38	0.955223881	1.13497	1.15475	6.927910545	11.30	13.0526	0.6129	0.64213	0.95449	8.3815	-0.3815	4.7683	11.06807	13.75174	-0.02783
May-11	1	17	7	8	8.13	0.861538462	0.85164	0.86648	8.078619287	11.34	9.82893	0.71218	0.68953	1.03285	6.7774	0.2226	3.1801	24.31420	22.16840	0.12373
Jun-11	2	18	8	9	10.50	0.761904762	0.92901	0.9452	8.463822675	11.38	10.7597	0.74352	0.82402	0.90231	8.8661	-0.8661	10.8268	8.07804	13.75174	0.18338
Jul-11	3	19	12	12	12.13	0.989690722	1.01586	1.03357	11.61028237	11.42	11.807	1.01635	1.1406	0.89106	13.4671	-1.4671	12.2255	3.09313	0.08507	-0.62716
Aug-11	4	20	22	12	12.63	1.742574257	1.13497	1.15475	19.051754	11.46	13.2375	1.66195	1.09341	1.51996	14.4741	7.5259	34.2088	7.64931	105.91840	0.24912
Sep-11	1	21	6	13	11.88	0.505263158	0.85164	0.86648	6.924530817	11.50	9.96765	0.60195	1.15178	0.52262	11.4806	-5.4806	91.3431	0.05187	32.58507	-1.02893
Oct-11	2	22	13	11	8.38	1.552238806	0.92901	0.9452	13.75371185	11.54	10.911	1.19146	0.62564	1.90437	6.8264	6.1736	47.4893	23.83335	1.66840	0.42883
Nov-11	3	23	1	6	5.88	0.170212766	1.01586	1.03357	0.967523531	11.58	11.9724	0.08353	0.54917	0.15209	6.5748	-5.5748	557.4846	26.35269	114.66840	-1.63152
Dec-11	4	24	5	6	5.00	1	1.13497	1.15475	4.329944091	11.62	13.4224	0.37251	0.25096	1.48435	3.3685	1.6315	32.6304	69.55317	45.00174	0.39914
Jan-12	1	25	3	5	6.13	0.489795918	0.85164	0.86648	3.462265409	11.66	10.1064	0.29684	0.49431	0.60052	4.9957	-1.9957	66.5225	45.05976	75.83507	-0.21743
Feb-12	2	26	9	8	8.50	1.058823529	0.92901	0.9452	9.52180051	11.70	11.0623	0.81357	0.75461	1.07814	8.3477	0.6523	7.2475	11.29371	7.33507	-0.30788
Mar-12	3	27	14	9	10.25	1.365853659	1.01586	1.03357	13.54532943	11.74	12.1379	1.15341	0.92513	1.24676	11.2291	2.7709	19.7922	0.22968	5.25174	0.19773
Apr-12	4	28	11	11	11.50	0.956521739	1.13497	1.15475	9.525877	11.78	13.6072	0.80839	1.01183	0.79894	13.7682	-2.7682	25.1656	4.24313	0.50174	-0.11115
May-12	1	29	11	12	12.38	0.888888889	0.85164	0.86648	12.69497317	11.82	10.2451	1.07369	0.95434	1.12505	9.7773	1.2227	11.1152	3.72879	0.50174	0.22294
Jun-12	2	30	11	13	14.13	0.778761062	0.92901	0.9452	11.63775618	11.86	11.2136	0.98095	1.19964	0.8177	13.4523	-2.4523	22.2940	3.04157	0.50174	-0.24515
Jul-12	3	31	19	15	15.00	1.266666667	1.01586	1.03357	18.38294709	11.90	12.3033	1.5443	1.32512	1.1654	16.3034	2.6966	14.1928	21.11443	53.16840	-0.11837
Aug-12	4	32	20	15	15.75	1.26984127	1.13497	1.15475	17.31977636	11.94	13.7921	1.45011	1.28705	1.12669	17.7511	2.2489	11.2447	36.51445	68.75174	0.24029
Sep-12	1	33	9	17	16.00	0.5625	0.85164	0.86648	10.38679623	11.98	10.3838	0.86674	1.32955	0.6519	13.8058	-4.8058	53.3976	4.39929	7.33507	-0.60372
Oct-12	2	34	19	15	14.25	1.333333333	0.92901	0.9452	20.10157885	12.02	11.3649	1.67181	1.19372	1.40051	13.5665	5.4335	28.5974	3.45275	53.16840	0.09738
Nov-12	3	35	13	13	13.25	0.981132075	1.01586	1.03357	12.5778059	12.06	12.4688	1.0426	1.19099	0.87541	14.8502	-1.8502	14.2324	9.87142	1.66840	0.06480
Dec-12	4	36	12	13	12.25	0.979591837	1.13497	1.15475	10.39186582	12.10	13.9769	0.85856	0.91882	0.93441	12.8423	-0.8423	7.0195	1.28597	0.08507	0.03014
Jan-13	1	37	9	11	10.63	0.847058824	0.85164	0.86648	10.38679623	12.14	10.5225	0.85531	0.88968	0.96137	9.3617	-0.3617	4.0184	5.50688	7.33507	-0.17993
Feb-13	2	38	11	10	10.00	1.1	0.92901	0.9452	11.63775618	12.18	11.5162	0.95517	0.81456	1.17262	9.3807	1.6193	14.7212	5.41801	0.50174	0.20526
Mar-13	3	39	8	10	9.75	0.820512821	1.01586	1.03357	7.740188248	12.22	12.6343	0.6332	0.81191	0.77989	10.2578	-2.2578	28.2231	2.10391	13.75174	-0.23893
Apr-13	4	40	12	10	9.13	1.315068493	1.13497	1.15475	10.39186582	12.26	14.1618	0.84735	0.71238	1.18947	10.0886	1.9114	15.9287	2.62367	0.08507	0.06510
May-13	1	41	7	9	9.50	0.736842105	0.85164	0.86648	8.078619287	12.30	10.6612	0.65659	0.72987	0.8996	7.7813	-0.7813	11.1608	15.42192	22.16840	0.21059
Jun-13	2	42	8	10	12.25	0.653061224	0.92901	0.9452	8.463822675	12.34	11.6676	0.68566	0.81201	0.8444	9.4741	-1.4741	18.4267	4.99164	13.75174	0.23988
Jul-13	3	43	14	14	15.13	0.925619835	1.01586	1.03357	13.54532943	12.38	12.7997	1.09377	1.2437	0.87945	15.9191	-1.9191	13.7075	17.73015	5.25174	-0.51691
Aug-13	4	44	28	16	16.25	1.723076923	1.13497	1.15475	24.24768691	12.42	14.3467	1.95167	1.44725	1.34854	20.7632	7.2368	25.8456	81.99097	265.41840	0.02638
Sep-13	1	45	14	17	16.38	0.854961832	0.85164	0.86648	16.15723857	12.46	10.7999	1.2963	1.36469	0.94989	14.7386	-0.7386	5.2757	9.18253	5.25174	0.17075
Oct-13	2	46	10	16	14.75	0.677966102	0.92901	0.9452	10.57977834	12.50	11.8189	0.8461	1.04836	0.80707	12.3905	-2.3905	23.9048	0.46533	2.91840	-0.02516
Nov-13	3	47	13	13	13.00	1	1.01586	1.03357	12.5778059	12.54	12.9652	1.00268	0.98328	1.01973	12.7484	0.2516	1.9352	1.08179	1.66840	-0.03815
Dec-13	4	48	16	13	12.00	1.333333333	1.13497	1.15475	13.85582109	12.58	14.5315	1.10105	1.06692	1.03199	15.5040	0.4960	3.0999	14.40721	18.41840	-0.06633
Jan-14	1	49	12	11	11.50	1.043478261	0.85164	0.86648	13.84906163	12.62	10.9387	1.09703	0.84408	1.29967	10.9387	1.0613	8.8446	3.91255	0.84028	0.66418
Feb-14	2	50	4	12	11.38	0.351648352	0.92901	0.9452	4.231911338	12.66	11.9702	0.33416	0.85785	0.38954	11.9702	-7.9702	199.2547	0.89582	79.50694	-0.46734
Mar-14	3	51	15	11	11.25	1.333333333	1.01586	1.03357	14.51285297	12.70	13.1307	1.14237	0.78663	1.45222	13.1307	1.8693	12.4623	0.04579	4.34028	0.11443
Apr-14	4	52	13	12	12.13	1.072164948	1.13497	1.15475	11.25785464	12.74	14.7164	0.88337	1.09652	0.80561	14.7164	-1.7164	13.2031	3.23904	0.00694	-0.22482
May-14	1	53	14	13	13.50	1.037037037	0.85164	0.86648	16.15723857	12.78	11.0774	1.26384	0.96323	1.31208	11.0774	2.9226	20.8760	3.38303	1.17361	0.22296
Jun-14	2	54	9	14	14.13	0.637168142	0.92901	0.9452	9.52180051	12.82	12.1215	0.74248	1.19524	0.6212	12.1215	-3.1215	34.6834	0.63228	15.34028	-0.85599
Jul-14	3	55	21	14	14.50	1.448275862	1.01586	1.03357	20.31799415	12.86	13.2961	1.57941	1.0424	1.51517	13.2961	7.7039	36.6852	0.14398	65.34028	0.13816
Aug-14	4	56	12	15	15.50	0.774193548	1.13497	1.15475	10.39186582	12.90	14.9013	0.8053	1.32985	0.60556	14.9013	-2.9013	24.1772	3.93862	0.84028	-0.56533
Sep-14	1	57	18	16	15.25	1.180327869	0.85164	0.86648	20.77359245	12.94	11.2161	1.60484	1.15646	1.38771	11.2161	6.7839	37.6884	2.89200	25.84028	-0.04040
Oct-14	2	58	13	15	14.13	0.920353982	0.92901	0.9452	13.75371185	12.98	12.2728	1.05925	1.25946	0.84104	12.2728	0.7272	5.5937	0.41454	0.00694	-0.11834
Nov-14	3	59	15	14	13.04	1.150159744	1.01586	1.03357	14.51285297	13.02	13.4616	1.11428	0.92337	1.20676	13.4616	1.5384	10.2561	0.29693	4.34028	0.40574
Dec-14	4	60	9	12	12.33	0.72972973	1.13497	1.15475	7.793899363	13.06	15.0861	0.59657	1.32985	0.4486	15.0861	-6.0861	67.6236	4.70655	15.34028	

Appendix M. Decomposition Additive (12 months)

Date	Month	x	Sales	12 Period Moving Average	CMAt =TRt+CLt	SNt+IRt = Yt - (TRt+CLt)	Ave SNt	Norm SNt	Deseasonalized Yt-SNt	TRt	TRt+SNt	CLt+IRt	CLt	IRt	*Forecast*	Error	Percentage Error	Explained Variation	Total Variation	Theil's U
Jan-10	1	1	5				-3.01	-2.900174	7.9002	10.90	8.00	-3.00								-0.34444
Feb-10	2	2	9				-2.01	-1.900174	10.9002	10.94	9.04	-0.04	-1.76	1.72	7.28	1.7222	19.1358	19.62982	7.3351	0.26929
Mar-10	3	3	8				-0.84	-0.733507	8.7335	10.97	10.24	-2.24	0.19	-2.42	10.42	-2.4236	30.2951	1.65051	13.7517	0.17318
Apr-10	4	4	13				-0.95	-0.837674	13.8377	11.00	10.17	2.83	4.22	-1.39	14.39	-1.3854	10.6571	7.16678	1.6684	-0.50080
May-10	5	5	21				-2.21	-2.098090	23.0981	11.04	8.94	12.06	5.55	6.51	14.49	6.5104	31.0020	7.73535	86.3351	0.18039
Jun-10	6	6	10				-2.94	-2.827257	12.8273	11.07	8.25	1.75	5.54	-3.79	13.79	-3.7882	37.8819	4.32582	2.9184	-0.31910
Jul-10	7	7	18	12.17	12.42	5.58	3.97	4.078993	13.9210	11.11	15.19	2.81	-0.38	3.19	14.81	3.1910	17.7276	9.61431	39.5851	0.23611
Aug-10	8	8	15	12.67	12.92	2.08	9.45	9.558160	5.4418	11.14	20.70	-5.70	-1.45	-4.25	19.25	-4.2500	28.3333	56.87674	10.8351	-0.04884
Sep-10	9	9	7	13.17	13.13	-6.13	-2.82	-2.712674	9.7127	11.18	8.46	-1.46	-2.20	0.73	6.27	0.7326	10.4663	29.60418	22.1684	0.04067
Oct-10	10	10	14	13.08	12.88	1.13	2.10	2.214410	11.7856	11.21	13.42	0.58	0.86	-0.28	14.28	-0.2847	2.0337	6.63778	5.2517	-0.10739
Nov-10	11	11	13	12.67	12.08	0.92	-1.82	-1.712674	14.7127	11.24	9.53	3.47	1.96	1.50	11.50	1.5035	11.5652	0.04486	1.6684	0.06036
Dec-10	12	12	13	11.50	11.42	1.58	-0.24	-0.129340	13.1293	11.28	11.15	1.85	2.64	-0.78	13.78	-0.7847	6.0363	4.31139	1.6684	0.05716
Jan-11	1	13	11	11.33	11.08	-0.08	-3.01	-2.900174	13.9002	11.31	8.41	2.59	3.33	-0.74	11.74	-0.7431	6.7551	0.00121	0.5017	-0.36869
Feb-11	2	14	15	10.83	11.13	3.88	-2.01	-1.900174	16.9002	11.35	9.45	5.55	1.50	4.06	10.94	4.0556	27.0370	0.58353	10.8351	0.22824
Mar-11	3	15	7	11.42	11.38	-4.38	-0.84	-0.733507	7.7335	11.38	10.65	-3.65	-0.22	-3.42	10.42	-3.4236	48.9087	1.65051	22.1684	-0.04018
Apr-11	4	16	8	11.33	11.29	-3.29	-0.95	-0.837674	8.8377	11.42	10.58	-2.58	-2.86	0.28	7.72	0.2813	3.5156	15.91678	13.7517	0.06120
May-11	5	17	7	11.25	10.75	-3.75	-2.21	-2.098090	9.0981	11.45	9.35	-2.35	-1.86	-0.49	7.49	-0.4896	6.9940	17.79785	22.1684	-0.22073
Jun-11	6	18	8	10.25	9.92	-1.92	-2.94	-2.827257	10.8273	11.49	8.66	-0.66	-2.20	1.55	6.45	1.5451	19.3142	27.59897	13.7517	0.30946
Jul-11	7	19	12	9.58	9.25	2.75	3.97	4.078993	7.9210	11.52	15.60	-3.60	-1.12	-2.48	14.48	-2.4757	20.6308	7.65829	0.0851	-0.22917
Aug-11	8	20	22	8.92	8.67	13.33	9.45	9.558160	12.4418	11.55	21.11	0.89	-1.86	2.75	19.25	2.7500	12.5000	56.87674	105.9184	0.08791
Sep-11	9	21	6	8.42	8.71	-2.71	-2.82	-2.712674	8.7127	11.59	8.88	-2.88	-0.94	-1.93	7.93	-1.9340	32.2338	14.24538	32.5851	-0.56366
Oct-11	10	22	13	9.00	9.13	3.88	2.10	2.214410	10.7856	11.62	13.84	-0.84	-4.22	3.38	9.62	3.3819	26.0150	4.36926	1.6684	0.26896
Nov-11	11	23	1	9.25	9.42	-8.42	-1.82	-1.712674	2.7127	11.66	9.94	-8.94	-5.45	-3.50	4.50	-3.4965	349.6528	52.01014	114.6684	-0.54861
Dec-11	12	24	5	9.58	9.71	-4.71	-0.24	-0.129340	5.1293	11.69	11.56	-6.56	-7.11	0.55	4.45	0.5486	10.9722	52.66324	45.0017	0.28194
Jan-12	1	25	3	9.83	10.13	-7.13	-3.01	-2.900174	5.9002	11.73	8.83	-5.83	-4.42	-1.41	4.41	-1.4097	46.9907	53.26972	75.8351	-0.12963
Feb-12	2	26	9	10.42	10.33	-1.33	-2.01	-1.900174	10.9002	11.76	9.86	-0.86	-1.25	0.39	8.61	0.3889	4.3210	9.59279	7.3351	-0.24923
Mar-12	3	27	14	10.25	10.38	3.63	-0.84	-0.733507	14.7335	11.79	11.06	2.94	0.70	2.24	11.76	2.2431	16.0218	0.00236	5.2517	0.09896
Apr-12	4	28	11	10.50	10.75	0.25	-0.95	-0.837674	11.8377	11.83	10.99	0.01	1.39	-1.39	12.39	-1.3854	12.5947	0.45844	0.5017	-0.01610
May-12	5	29	11	11.00	11.50	-0.50	-2.21	-2.098090	13.0981	11.86	9.76	1.24	1.06	0.18	10.82	0.1771	1.6098	0.78396	0.5017	0.01105
Jun-12	6	30	11	12.00	12.29	-1.29	-2.94	-2.827257	13.8273	11.90	9.07	1.93	2.05	-0.12	11.12	-0.1215	1.1048	0.34434	0.5017	-0.16888
Jul-12	7	31	19	12.58	12.83	6.17	3.97	4.078993	14.9210	11.93	16.01	2.99	1.13	1.86	17.14	1.8576	9.7770	29.52866	53.1684	0.10088
Aug-12	8	32	20	13.08	13.17	6.83	9.45	9.558160	10.4418	11.97	21.52	-1.52	0.39	-1.92	21.92	-1.9167	9.5833	104.21007	68.7517	0.06337
Sep-12	9	33	9	13.25	13.00	-4.00	-2.82	-2.712674	11.7127	12.00	9.29	-0.29	0.98	-1.27	10.27	-1.2674	14.0818	2.07640	7.3351	-0.26466
Oct-12	10	34	19	12.75	12.79	6.21	2.10	2.214410	16.7856	12.03	14.25	4.75	2.37	2.38	16.62	2.3819	12.5365	24.10537	53.1684	-0.00895
Nov-12	11	35	13	12.83	12.67	0.33	-1.82	-1.712674	14.7127	12.07	10.36	2.64	2.47	0.17	12.83	0.1701	1.3088	1.25782	1.6684	0.06036
Dec-12	12	36	12	12.50	12.38	-0.38	-0.24	-0.129340	12.1293	12.10	11.97	0.03	0.81	-0.78	12.78	-0.7847	6.5394	1.15861	0.0851	0.03414
Jan-13	1	37	9	12.25	12.04	-3.04	-3.01	-2.900174	11.9002	12.14	9.24	-0.24	0.17	-0.41	9.41	-0.4097	4.5525	5.28361	7.3351	-0.19136
Feb-13	2	38	11	11.83	12.17	-1.17	-2.01	-1.900174	12.9002	12.17	10.27	0.73	-0.99	1.72	9.28	1.7222	15.6566	5.90760	0.5017	0.25063
Mar-13	3	39	8	12.50	12.71	-4.71	-0.84	-0.733507	8.7335	12.21	11.47	-3.47	-0.72	-2.76	10.76	-2.7569	34.4618	0.90514	13.7517	-0.32682
Apr-13	4	40	12	12.92	12.54	-0.54	-0.95	-0.837674	12.8377	12.24	11.40	0.60	-2.02	2.61	9.39	2.6146	21.7882	5.39594	0.0851	0.15191
May-13	5	41	7	12.17	12.17	-5.17	-2.21	-2.098090	9.0981	12.28	10.18	-3.18	-1.35	-1.82	8.82	-1.8229	26.0417	8.32563	22.1684	-0.12550
Jun-13	6	42	8	12.17	12.33	-4.33	-2.94	-2.827257	10.8273	12.31	9.48	-1.48	-2.36	0.88	7.12	0.8785	10.9809	21.03879	13.7517	0.39280
Jul-13	7	43	14	12.50	12.63	1.38	3.97	4.078993	9.9210	12.34	16.42	-2.42	0.72	-3.14	17.14	-3.1424	22.4454	29.52866	5.2517	-0.24405
Aug-13	8	44	28	12.75	12.46	15.54	9.45	9.558160	18.4418	12.38	21.94	6.06	2.65	3.42	24.58	3.4167	12.2024	165.76563	265.4184	-0.08569
Sep-13	9	45	14	12.17	12.46	1.54	-2.82	-2.712674	16.7127	12.41	9.70	4.30	1.90	2.40	11.60	2.3993	17.1379	0.01159	5.2517	0.37748
Oct-13	10	46	10	12.75	12.79	-2.79	2.10	2.214410	7.7856	12.45	14.66	-4.66	0.62	-5.28	15.28	-5.2847	52.8472	12.79056	2.9184	-0.18368
Nov-13	11	47	13	12.83	13.13	-0.13	-1.82	-1.712674	14.7127	12.48	10.77	2.23	0.39	1.84	11.16	1.8368	14.1293	0.29718	1.6684	-0.06784
Dec-13	12	48	16	13.42	13.46	2.54	-0.24	-0.129340	16.1293	12.52	12.39	3.61	2.732	0.88	15.12	0.8819	5.5122	11.62621	18.4184	-0.16189
Jan-14	1	49	12	13.50	13.79	-1.79	-3.01	-2.900174	14.9002	12.55	9.65	2.35	-0.24	2.59	9.41	2.5903	21.5856	12.29866	0.8403	0.52315
Feb-14	2	50	4	14.08	13.42	-9.42	-2.01	-1.900174	5.9002	12.58	10.68	-6.68	-0.41	-6.28	10.28	-6.2778	156.9444	6.96373	79.5069	-0.97743
Mar-14	3	51	15	12.75	12.92	2.08	-0.84	-0.733507	15.7335	12.62	11.88	3.12	-0.79	3.91	11.09	3.9097	26.0648	3.33570	4.3403	0.09236
Apr-14	4	52	13	13.08	13.21	-0.21	-0.95	-0.837674	13.8377	12.65	11.82	1.18	2.57	-1.39	14.39	-1.3854	10.6571	2.15723	0.0069	-0.16747
May-14	5	53	14	13.33	13.42	0.58	-2.21	-2.098090	16.0981	12.69	10.59	3.41	1.23	2.18	11.82	2.1771	15.5506	1.19629	1.1736	0.22297
Jun-14	6	54	9	13.50	13.21	-4.21	-2.94	-2.827257	11.8273	12.72	9.89	-0.89	2.23	-3.12	12.12	-3.1215	34.6836	0.63225	15.3403	-0.72492
Jul-14	7	55	21	12.92			3.97	4.078993	16.9210	12.76	16.83	4.17	-2.36	6.52	14.48	6.5243	31.0681	2.43057	65.3403	0.51984
Aug-14	8	56	12				9.45	9.558160	2.4418	12.79	22.35	-10.35	0.57	-10.92	22.92	-10.9167	90.9722	100.00000	0.8403	-0.78328
Sep-14	9	57	18				-2.82	-2.712674	20.7127	12.82	10.11	7.89	-1.51	9.40	8.60	9.3993	52.2184	18.62762	25.8403	0.29360
Oct-14	10	58	13				2.10	2.214410	10.7856	12.86	15.07	-2.07	3.21	-5.28	18.28	-5.2847	40.6517	28.81602	0.0069	-0.34642
Nov-14	11	59	15				-1.82	-1.712674	16.7127	12.89	11.18	3.82	-0.68	4.50	10.50	4.5035	30.0231	5.85707	4.3403	0.43535
Dec-14	12	60	9				-0.24	-0.129340	9.1293	12.93	12.80	-3.80	2.732	-6.53	15.53	-6.5302	72.5579	6.83060	15.3403	

Appendix N. Simple Exponential Smoothing

Date	Time	Sales	Smoothed Estimate (l)	Forecast	Error	Percentage Error	Explained Variation	Total Variation	Theil's U
	0		10.87500						
Jan-10	1	5	10.82879	10.87500	-5.87500	117.50000	0.69444	45.00174	0.36576
Feb-10	2	9	10.81440	10.82879	-1.82879	20.31988	0.77360	7.33507	0.31271
Mar-10	3	8	10.79227	10.81440	-2.81440	35.18006	0.79911	13.75174	-0.27597
Apr-10	4	13	10.80963	10.79227	2.20773	16.98256	0.83918	1.66840	-0.78387
May-10	5	21	10.88979	10.80963	10.19037	48.52556	0.80766	86.33507	0.04237
Jun-10	6	10	10.88279	10.88979	-0.88979	8.89787	0.67002	2.91840	-0.71172
Jul-10	7	18	10.93877	10.88279	7.11721	39.54007	0.68153	39.58507	-0.22562
Aug-10	8	15	10.97071	10.93877	4.06123	27.07487	0.59223	10.83507	0.26471
Sep-10	9	7	10.93948	10.97071	-3.97071	56.72448	0.54408	22.16840	-0.43722
Oct-10	10	14	10.96355	10.93948	3.06052	21.86085	0.59113	5.25174	-0.14546
Nov-10	11	13	10.97957	10.96355	2.03645	15.66496	0.55470	1.66840	-0.15542
Dec-10	12	13	10.99546	10.97957	2.02043	15.54175	0.53109	1.66840	-0.00035
Jan-11	13	11	10.99550	10.99546	0.00454	0.04123	0.50818	0.50174	-0.36405
Feb-11	14	15	11.02700	10.99550	4.00450	26.69666	0.50813	10.83507	0.26847
Mar-11	15	7	10.99532	11.02700	-4.02700	57.52855	0.46422	22.16840	0.4279
Apr-11	16	8	10.97176	10.99532	-2.99532	37.44154	0.50838	13.75174	0.49647
May-11	17	7	10.94052	10.97176	-3.97176	56.73947	0.54254	22.16840	0.42007
Jun-11	18	8	10.91739	10.94052	-2.94052	36.75653	0.58953	13.75174	-0.13533
Jul-11	19	12	10.92591	10.91739	1.08261	9.02172	0.62559	0.08507	-0.92284
Aug-11	20	22	11.01301	10.92591	11.07409	50.33678	0.61219	105.91840	0.22786
Sep-11	21	6	10.97358	11.01301	-5.01301	83.55023	0.48347	32.58507	-0.33774
Oct-11	22	13	10.98952	10.97358	2.02642	15.58782	0.53986	1.66840	0.76842
Nov-11	23	1	10.91095	10.98952	-9.98952	998.95223	0.51669	114.66840	5.91095
Dec-11	24	5	10.86445	10.91095	-5.91095	118.21896	0.63582	45.00174	1.57289
Jan-12	25	3	10.80260	10.86445	-7.86445	262.14848	0.71213	75.83507	0.60087
Feb-12	26	9	10.78842	10.80260	-1.80260	20.02884	0.82036	7.33507	-0.35684
Mar-12	27	14	10.81368	10.78842	3.21158	22.93988	0.84625	5.25174	-0.01331
Apr-12	28	11	10.81514	10.81368	0.18632	1.69384	0.80041	0.50174	-0.01681
May-12	29	11	10.81660	10.81514	0.18486	1.68051	0.79779	0.50174	-0.01667
Jun-12	30	11	10.81804	10.81660	0.18340	1.66730	0.79519	0.50174	-0.74381
Jul-12	31	19	10.88240	10.81804	8.18196	43.06295	0.79262	53.16840	-0.47987
Aug-12	32	20	10.95411	10.88240	9.11760	45.58802	0.68217	68.75174	0.09771
Sep-12	33	9	10.93874	10.95411	-1.95411	21.71236	0.56885	7.33507	-0.8957
Oct-12	34	19	11.00215	10.93874	8.06126	42.42767	0.59227	53.16840	-0.10515
Nov-12	35	13	11.01786	11.00215	1.99785	15.36808	0.49870	1.66840	-0.07555
Dec-12	36	12	11.02559	11.01786	0.98214	8.18447	0.47675	0.08507	0.1688
Jan-13	37	9	11.00966	11.02559	-2.02559	22.50655	0.46614	7.33507	0.00107
Feb-13	38	11	11.00958	11.00966	-0.00966	0.08779	0.48815	0.50174	0.2736
Mar-13	39	8	10.98591	11.00958	-3.00958	37.61976	0.48826	13.75174	-0.12676
Apr-13	40	12	10.99388	10.98591	1.01409	8.45077	0.52190	0.08507	0.33282
May-13	41	7	10.96247	10.99388	-3.99388	57.05549	0.51044	22.16840	0.42321
Jun-13	42	8	10.93917	10.96247	-2.96247	37.03088	0.55631	13.75174	-0.3826
Jul-13	43	14	10.96324	10.93917	3.06083	21.86308	0.59161	5.25174	-1.21691
Aug-13	44	28	11.09725	10.96324	17.03676	60.84556	0.55516	265.41840	-0.10367
Sep-13	45	14	11.12008	11.09725	2.90275	20.73393	0.37342	5.25174	0.08001
Oct-13	46	10	11.11127	11.12008	-1.12008	11.20081	0.34604	2.91840	-0.18887
Nov-13	47	13	11.12613	11.11127	1.88873	14.52868	0.35648	1.66840	-0.37491
Dec-13	48	16	11.16446	11.12613	4.87387	30.46170	0.33896	18.41840	-0.05222
Jan-14	49	12	11.17104	11.16446	0.83554	6.96280	3.07022	0.84028	0.59704
Feb-14	50	4	11.11463	11.16446	-7.16446	179.11159	3.07022	79.50694	-0.95888
Mar-14	51	15	11.14519	11.16446	3.83554	25.57024	3.07022	4.34028	-0.12237
Apr-14	52	13	11.15978	11.16446	1.83554	14.11951	3.07022	0.00694	-0.21812
May-14	53	14	11.18212	11.16446	2.83554	20.25383	3.07022	1.17361	0.1546
Jun-14	54	9	11.16496	11.16446	-2.16446	24.04959	3.07022	15.34028	-1.09284
Jul-14	55	21	11.24232	11.16446	9.83554	46.83589	3.07022	65.34028	-0.03979
Aug-14	56	12	11.24828	11.16446	0.83554	6.96280	3.07022	0.84028	-0.56963
Sep-14	57	18	11.30138	11.16446	6.83554	37.97520	3.07022	25.84028	-0.10197
Oct-14	58	13	11.31474	11.16446	1.83554	14.11951	3.07022	0.00694	-0.29504
Nov-14	59	15	11.34373	11.16446	3.83554	25.57024	3.07022	4.34028	0.1443
Dec-14	60	9	11.32530	11.16446	-2.16446	24.04959	3.07022	15.34028	

Appendix O. Holt's Trend Corrected

Date	Time	Sales	Estimate	Growth	*Forecast*	Error	Percentage Error	Explained Variation	Total Variation	Theil's U
	0		10.41312057	0.052865827						
Jan-10	1	5	10.46598639	0.052865827	10.46599	-5.46599	109.31973	1.543426	45.00174	0.30377
Feb-10	2	9	10.51885222	0.052865827	10.51885	-1.51885	16.87614	1.414865	7.33507	0.28575
Mar-10	3	8	10.57171805	0.052865827	10.57172	-2.57172	32.14648	1.291894	13.75174	-0.29693
Apr-10	4	13	10.62458388	0.052865827	10.62458	2.37542	18.27243	1.174513	1.66840	-0.79404
May-10	5	21	10.6774497	0.052865827	10.67745	10.32255	49.15500	1.062721	86.33507	0.03478
Jun-10	6	10	10.73031553	0.052865827	10.73032	-0.73032	7.30316	0.956519	2.91840	-0.72168
Jul-10	7	18	10.78318136	0.052865827	10.78318	7.21682	40.09344	0.855906	39.58507	-0.23133
Aug-10	8	15	10.83604718	0.052865827	10.83605	4.16395	27.75969	0.760883	10.83507	0.25926
Sep-10	9	7	10.88891301	0.052865827	10.88891	-3.88891	55.55590	0.671450	22.16840	-0.43689
Oct-10	10	14	10.94177884	0.052865827	10.94178	3.05822	21.84444	0.587606	5.25174	-0.14324
Nov-10	11	13	10.99464467	0.052865827	10.99464	2.00536	15.42581	0.509352	1.66840	-0.15019
Dec-10	12	13	11.04751049	0.052865827	11.04751	1.95249	15.01915	0.436687	1.66840	0.00772
Jan-11	13	11	11.10037632	0.052865827	11.10038	-0.10038	0.91251	0.369612	0.50174	-0.34971
Feb-11	14	15	11.15324215	0.052865827	11.15324	3.84676	25.64505	0.308126	10.83507	0.28041
Mar-11	15	7	11.20610798	0.052865827	11.20611	-4.20611	60.08726	0.252230	22.16840	0.46557
Apr-11	16	8	11.2589738	0.052865827	11.25897	-3.25897	40.73717	0.201924	13.75174	0.53898
May-11	17	7	11.31183963	0.052865827	11.31184	-4.31184	61.59771	0.157207	22.16840	0.48067
Jun-11	18	8	11.36470546	0.052865827	11.36471	-3.36471	42.05882	0.118080	13.75174	-0.0728
Jul-11	19	12	11.41757128	0.052865827	11.41757	0.58243	4.85357	0.084543	0.08507	-0.87746
Aug-11	20	22	11.47043711	0.052865827	11.47044	10.52956	47.86165	0.056595	105.91840	0.25106
Sep-11	21	6	11.52330294	0.052865827	11.52330	-5.52330	92.05505	0.034236	32.58507	-0.23731
Oct-11	22	13	11.57616877	0.052865827	11.57617	1.42383	10.95255	0.017467	1.66840	0.81762
Nov-11	23	1	11.62903459	0.052865827	11.62903	-10.62903	1062.90346	0.006288	114.66840	6.6819
Dec-11	24	5	11.68190042	0.052865827	11.68190	-6.68190	133.63801	0.000699	45.00174	1.74695
Jan-12	25	3	11.73476625	0.052865827	11.73477	-8.73477	291.15887	0.000699	75.83507	0.92921
Feb-12	26	9	11.78763207	0.052865827	11.78763	-2.78763	30.97369	0.006288	7.33507	-0.23994
Mar-12	27	14	11.8404979	0.052865827	11.84050	2.15950	15.42501	0.017467	5.25174	0.06381
Apr-12	28	11	11.89336373	0.052865827	11.89336	-0.89336	8.12149	0.034236	0.50174	0.08602
May-12	29	11	11.94622956	0.052865827	11.94623	-0.94623	8.60209	0.056595	0.50174	0.09083
Jun-12	30	11	11.99909538	0.052865827	11.99910	-0.99910	9.08269	0.084543	0.50174	-0.63164
Jul-12	31	19	12.05196121	0.052865827	12.05196	6.94804	36.56863	0.118080	53.16840	-0.41554
Aug-12	32	20	12.10482704	0.052865827	12.10483	7.89517	39.47586	0.157207	68.75174	0.15788
Sep-12	33	9	12.15769286	0.052865827	12.15769	-3.15769	35.08548	0.201924	7.33507	-0.75438
Oct-12	34	19	12.21055869	0.052865827	12.21056	6.78944	35.73390	0.252230	53.16840	-0.03877
Nov-12	35	13	12.26342452	0.052865827	12.26342	0.73658	5.66597	0.308126	1.66840	0.02433
Dec-12	36	12	12.31629035	0.052865827	12.31629	-0.31629	2.63575	0.369612	0.08507	0.28076
Jan-13	37	9	12.36915617	0.052865827	12.36916	-3.36916	37.43507	0.436687	7.33507	0.158
Feb-13	38	11	12.422022	0.052865827	12.42202	-1.42202	12.92747	0.509352	0.50174	0.40681
Mar-13	39	8	12.47488783	0.052865827	12.47489	-4.47489	55.93610	0.587606	13.75174	0.06597
Apr-13	40	12	12.52775365	0.052865827	12.52775	-0.52775	4.39795	0.671450	0.08507	0.46505
May-13	41	7	12.58061948	0.052865827	12.58062	-5.58062	79.72314	0.760883	22.16840	0.66193
Jun-13	42	8	12.63348531	0.052865827	12.63349	-4.63349	57.91857	0.855906	13.75174	-0.16421
Jul-13	43	14	12.68635114	0.052865827	12.68635	1.31365	9.38321	0.956519	5.25174	-1.09006
Aug-13	44	28	12.73921696	0.052865827	12.73922	15.26078	54.50280	1.062721	265.41840	-0.04314
Sep-13	45	14	12.79208279	0.052865827	12.79208	1.20792	8.62798	1.174513	5.25174	0.20321
Oct-13	46	10	12.84494862	0.052865827	12.84495	-2.84495	28.44949	1.291894	2.91840	-0.01022
Nov-13	47	13	12.89781444	0.052865827	12.89781	0.10219	0.78604	1.414865	1.66840	-0.23456
Dec-13	48	16	12.95068027	0.052865827	12.95068	3.04932	19.05825	1.543426	18.41840	0.06272
Jan-14	49	12	13.0035461	0.052865827	13.00355	-1.00355	8.36288	0.007548	0.84028	0.7547
Feb-14	50	4	13.05641193	0.052865827	13.05641	-9.05641	226.41030	0.019529	79.50694	-0.47268
Mar-14	51	15	13.10927775	0.052865827	13.10928	1.89072	12.60481	0.037099	4.34028	0.01081
Apr-14	52	13	13.16214358	0.052865827	13.16214	-0.16214	1.24726	0.060259	0.00694	-0.06038
May-14	53	14	13.21500941	0.052865827	13.21501	0.78499	5.60708	0.089008	1.17361	0.30485
Jun-14	54	9	13.26787524	0.052865827	13.26788	-4.26788	47.42084	0.123347	15.34028	-0.85325
Jul-14	55	21	13.32074106	0.052865827	13.32074	7.67926	36.56790	0.163276	65.34028	0.06541
Aug-14	56	12	13.37360689	0.052865827	13.37361	-1.37361	11.44672	0.208794	0.84028	-0.38113
Sep-14	57	18	13.42647272	0.052865827	13.42647	4.57353	25.40848	0.259902	25.84028	0.02663
Oct-14	58	13	13.47933854	0.052865827	13.47934	-0.47934	3.68722	0.316600	0.00694	-0.11291
Nov-14	59	15	13.53220437	0.052865827	13.53220	1.46780	9.78530	0.378887	4.34028	0.30567
Dec-14	60	9	13.5850702	0.052865827	13.58507	-4.58507	50.94522	0.446763	15.34028	

Appendix P. Additive Holt-Winters (4 months)

Date	Time	Sales	Estimate	Growth	Seasonal	*Forecast*	Error	Regression Estimates	Detrended	Averages / Seasons	Percentage Error	Explained Variation	Total Variation	Theil's U
	-3				-0.078551					-0.079				
	-2				2.326667					2.327				
	-1				2.731884					2.732				
	0		12.81522	-0.15522	5.803768					5.804				
Jan-10	1	5	11.04343	-0.15522	-0.078551	12.58145	-7.58145	12.66000	-7.66	1	151.62899	0.76233	45.00174	0.84298
Feb-10	2	9	9.98948	-0.15522	2.326667	13.21488	-4.21488	12.50478	-3.50	2	46.83197	2.26968	7.33507	0.50735
Mar-10	3	8	8.86064	-0.15522	2.731884	12.56615	-4.56615	12.34957	-4.35	3	57.07688	0.73585	13.75174	0.18865
Apr-10	4	13	8.38362	-0.15522	5.803768	14.50919	-1.50919	12.19435	0.81	4	11.60915	7.84480	1.66840	-0.9885
May-10	5	21	10.96841	-0.15522	-0.078551	8.14985	12.85015	12.03913	8.96	1	61.19118	12.66278	86.33507	0.14952
Jun-10	6	10	10.14369	-0.15522	2.326667	13.13985	-3.13985	11.88391	-1.88	2	31.39855	2.04925	2.91840	-0.528
Jul-10	7	18	11.11423	-0.15522	2.731884	12.72035	5.27965	11.72870	6.27	3	29.33138	1.02418	39.58507	0.09793
Aug-10	8	15	10.58314	-0.15522	5.803768	16.76278	-1.76278	11.57348	3.43	4	11.75189	25.54747	10.83507	0.22329
Sep-10	9	7	9.71375	-0.15522	-0.078551	10.34937	-3.34937	11.41826	-4.42	1	47.84820	1.84677	22.16840	-0.3021
Oct-10	10	14	10.00946	-0.15522	2.326667	11.88520	2.11480	11.26304	2.74	2	15.10574	0.03128	5.25174	-0.0296
Nov-10	11	13	9.94249	-0.15522	2.731884	12.58613	0.41387	11.10783	1.89	3	3.18362	0.77053	1.66840	0.19931
Dec-10	12	13	9.23480	-0.15522	5.803768	15.59104	-2.59104	10.95261	2.05	4	19.93112	15.07545	1.66840	-0.1538
Jan-11	13	11	9.50581	-0.15522	-0.078551	9.00103	1.99897	10.79739	0.20	1	18.17248	7.32951	0.50174	-0.3021
Feb-11	14	15	10.05909	-0.15522	2.326667	11.67726	3.32274	10.64217	4.36	2	22.15158	0.00097	10.83507	0.37572
Mar-11	15	7	8.70218	-0.15522	2.731884	12.63576	-5.63576	10.48696	-3.49	3	80.51087	0.86012	22.16840	0.90725
Apr-11	16	8	7.19281	-0.15522	5.803768	14.35073	-6.35073	10.33174	-2.33	4	79.38412	6.98226	13.75174	-0.0051
May-11	17	7	7.04633	-0.15522	-0.078551	6.95904	0.04096	10.17652	-3.18	1	0.58507	22.55574	22.16840	0.17397
Jun-11	18	8	6.63145	-0.15522	2.326667	9.21778	-1.21778	10.02130	-2.02	2	15.22222	6.20287	13.75174	-0.349
Jul-11	19	12	7.07154	-0.15522	2.731884	9.20811	2.79189	9.86609	2.13	3	23.26572	6.25110	0.08507	-0.7733
Aug-11	20	22	8.89505	-0.15522	5.803768	12.72009	9.27991	9.71087	12.29	4	42.18142	1.02365	105.91840	0.12097
Sep-11	21	6	8.17237	-0.15522	-0.078551	8.66128	-2.66128	9.55565	-3.56	1	44.35469	9.28453	32.58507	-0.4427
Oct-11	22	13	8.58353	-0.15522	2.326667	10.34382	2.65618	9.40043	3.60	2	20.43213	1.86189	1.66840	0.78155
Nov-11	23	1	6.26188	-0.15522	2.731884	11.16019	-10.16019	9.24522	-8.25	3	1016.01928	0.30046	114.66840	6.91043
Dec-11	24	5	4.63317	-0.15522	5.803768	11.91043	-6.91043	9.09000	-4.09	4	138.20859	0.04084	45.00174	0.27988
Jan-12	25	3	4.17956	-0.15522	-0.078551	4.39940	-1.39940	8.93478	-5.93	1	46.64670	53.42049	75.83507	-0.883
Feb-12	26	9	4.58918	-0.15522	2.326667	6.35101	2.64899	8.77957	0.22	2	29.43321	28.70091	7.33507	-0.7594
Mar-12	27	14	5.89119	-0.15522	2.731884	7.16585	6.83415	8.62435	5.38	3	48.81538	20.63418	5.25174	0.03855
Apr-12	28	11	5.62089	-0.15522	5.803768	11.53974	-0.53974	8.46913	2.53	4	4.90674	0.02842	0.50174	-0.5103
May-12	29	11	6.66249	-0.15522	-0.078551	5.38712	5.61288	8.31391	2.69	1	51.02620	39.95777	0.50174	-0.1969
Jun-12	30	11	6.96913	-0.15522	2.326667	8.83394	2.16606	8.15870	2.84	2	19.69148	8.26215	0.50174	-0.8595
Jul-12	31	19	8.82981	-0.15522	2.731884	9.54580	9.45420	8.00348	11.00	3	49.75894	4.67655	53.16840	-0.2906
Aug-12	32	20	9.85196	-0.15522	5.803768	14.47836	5.52164	7.84826	12.15	4	27.60820	7.67305	68.75174	0.03091
Sep-12	33	9	9.56492	-0.15522	-0.078551	9.61819	-0.61819	7.69304	1.31	1	6.86876	4.36871	7.33507	-0.8071
Oct-12	34	19	10.95851	-0.15522	2.326667	11.73637	7.26363	7.53783	11.46	2	38.22961	0.00079	53.16840	0.02817
Nov-12	35	13	10.68918	-0.15522	2.731884	13.53518	-0.53518	7.38261	5.62	3	4.11675	3.33736	1.66840	0.33367
Dec-12	36	12	9.60904	-0.15522	5.803768	16.33773	-4.33773	7.22739	4.77	4	36.14774	21.43131	0.08507	0.03127
Jan-13	37	9	9.37380	-0.15522	-0.078551	9.37527	-0.37527	7.07217	1.93	1	4.16968	5.44318	7.33507	0.06058
Feb-13	38	11	9.10232	-0.15522	2.326667	11.54525	-0.54525	6.91696	4.08	2	4.95684	0.02660	0.50174	0.33445
Mar-13	39	8	8.16265	-0.15522	2.731884	11.67899	-3.67899	6.76174	1.24	3	45.98738	0.00086	13.75174	0.2264
Apr-13	40	12	7.62123	-0.15522	5.803768	13.81120	-1.81120	6.60652	5.39	4	15.09330	4.42203	0.08507	0.03229
May-13	41	7	7.38340	-0.15522	-0.078551	7.38746	-0.38746	6.45130	0.55	1	5.53519	18.66992	22.16840	0.22212
Jun-13	42	8	6.89664	-0.15522	2.326667	9.55485	-1.55485	6.29609	1.70	2	19.43557	4.63751	13.75174	-0.5658
Jul-13	43	14	7.70664	-0.15522	2.731884	9.47331	4.52669	6.14087	7.86	3	32.33350	4.99533	5.25174	-1.0461
Aug-13	44	28	10.67410	-0.15522	5.803768	13.35519	14.64481	5.98565	22.01	4	52.30289	2.71214	265.41840	-0.1271
Sep-13	45	14	11.27790	-0.15522	-0.078551	10.44033	3.55967	5.83043	8.17	1	25.42624	1.60784	5.25174	0.24638
Oct-13	46	10	10.38718	-0.15522	2.326667	13.44935	-3.44935	5.67522	4.32	2	34.49346	3.03113	2.91840	-0.0036
Nov-13	47	13	10.23968	-0.15522	2.731884	12.96385	0.03615	5.52000	7.48	3	0.27806	1.57633	1.66840	-0.0086
Dec-13	48	16	10.10829	-0.15522	5.803768	15.88823	0.11177	5.36478	10.64	4	0.69859	17.47150	18.41840	-0.1328
Jan-14	49	12				9.87452	2.12548	5.20957	6.79	1	17.71231	9.25464	0.84028	0.67704
Feb-14	50	4				12.12452	-8.12452	5.05435	-1.05	2	203.11308	0.62749	79.50694	-0.6564
Mar-14	51	15				12.37452	2.62548	4.89913	10.10	3	17.50318	0.29392	4.34028	0.15275
Apr-14	52	13				15.29119	-2.29119	4.74391	8.26	4	17.62454	5.63836	0.00694	-0.3651
May-14	53	14				9.25365	4.74635	4.58870	9.41	1	33.90247	13.41767	1.17361	0.17883
Jun-14	54	9				11.50365	-2.50365	4.43348	4.57	2	27.81837	1.99661	15.34028	-1.0274
Jul-14	55	21				11.75365	9.24635	4.27826	16.72	3	44.03022	1.35260	65.34028	0.12716
Aug-14	56	12				14.67032	-2.67032	4.12304	7.88	4	22.25267	3.07530	0.84028	-0.7806
Sep-14	57	18				8.63278	9.36722	3.96783	14.03	1	52.04009	18.35165	25.84028	-0.1176
Oct-14	58	13				10.88278	2.11722	3.81261	9.19	2	16.28628	4.13668	0.00694	-0.2975
Nov-14	59	15				11.13278	3.86722	3.65739	11.34	3	25.78144	3.18224	4.34028	0.33663
Dec-14	60	9				14.04945	-5.04945	3.50217	5.50	4	56.10501	1.28320	15.34028	

Appendix Q. Additive Holt-Winters (12 months)

Date	Time	Sales	Estimate	Growth	Seasonal	*Forecast*	Error	Regression Estimates	Detrended	Averages / Seasons	Percentage Error	Explained Variation	Total Variation	Theil's U
	-11				-2.86609					-2.86609				
	-10				1.28913					1.28913				
	-9				-0.30565					-0.30565				
	-8				1.59957					1.59957				
	-7				2.25478					2.25478				
	-6				0.16000					0.16				
	-5				6.81522					6.81522				
	-4				12.47043					12.4704				
	-3				0.37565					0.37565				
	-2				5.53087					5.53087				
	-1				1.68609					1.68609				
	0		12.81522	-0.15522	3.34130					3.3413				
Jan-10	1	5	11.39906	-0.15522	-2.86609	9.79391	-4.79	12.66	-7.66	1	95.87826	3.66501	45.00174	0.70659
Feb-10	2	9	10.31456	-0.15522	1.28913	12.53297	-3.53	12.50478261	-3.50	2	39.25523	0.68003	7.33507	0.20597
Mar-10	3	8	9.67177	-0.15522	-0.30565	9.85369	-1.85	12.34956522	-4.35	3	23.17117	3.43969	13.75174	-0.2355
Apr-10	4	13	10.01207	-0.15522	1.59957	11.11612	1.88	12.19434783	0.81	4	14.49141	0.35072	1.66840	-0.6837
May-10	5	21	12.19476	-0.15522	2.25478	12.11163	8.89	12.03913043	8.96	5	42.32555	0.16265	86.33507	0.10474
Jun-10	6	10	11.46100	-0.15522	0.16000	12.19954	-2.20	11.88391304	-1.88	6	21.99540	0.24128	2.91840	0.0121
Jul-10	7	18	11.27395	-0.15522	6.81522	18.12100	-0.12	11.72869565	6.27	7	0.67220	41.12223	39.58507	0.47718
Aug-10	8	15	8.85953	-0.15522	12.47043	23.58917	-8.59	11.57347826	3.43	8	57.26113	141.15428	10.83507	0.13866
Sep-10	9	7	8.15722	-0.15522	0.37565	9.07996	-2.08	11.41826087	-4.42	9	29.71374	6.90834	22.16840	-0.0667
Oct-10	10	14	8.12487	-0.15522	5.53087	13.53287	0.47	11.26304348	2.74	10	3.33664	3.32893	5.25174	-0.2389
Nov-10	11	13	8.84929	-0.15522	1.68609	9.65574	3.34	11.10782609	1.89	11	25.72509	4.21314	1.66840	-0.0742
Dec-10	12	13	8.94780	-0.15522	3.34130	12.03538	0.96	10.9526087	2.05	12	7.42016	0.10696	1.66840	-0.3903
Jan-11	13	11	10.12707	-0.15522	-2.86609	5.92649	5.07	10.7973913	0.20	1	46.12277	33.42966	0.50174	-0.3399
Feb-11	14	15	10.95532	-0.15522	1.28913	11.26098	3.74	10.64217391	4.36	2	24.92681	0.20013	10.83507	0.23296
Mar-11	15	7	9.88096	-0.15522	-0.30565	10.49445	-3.49	10.48695652	-3.49	3	49.92075	1.47351	22.16840	0.47504
Apr-11	16	8	8.85109	-0.15522	1.59957	11.32531	-3.33	10.33173913	-2.33	4	41.56634	0.14671	13.75174	0.49383
May-11	17	7	7.65673	-0.15522	2.25478	10.95065	-3.95	10.17652174	-3.18	5	56.43789	0.57408	22.16840	-0.0484
Jun-11	18	8	7.59054	-0.15522	0.16000	7.66151	0.34	10.02130435	-2.02	6	4.23109	16.37676	13.75174	0.28132
Jul-11	19	12	6.84337	-0.15522	6.81522	14.25054	-2.25	9.866086957	2.13	7	18.75454	6.46284	0.08507	-0.2368
Aug-11	20	22	7.43553	-0.15522	12.47043	19.15858	2.84	9.710869565	12.29	8	12.91553	55.50624	105.91840	0.07527
Sep-11	21	6	6.84474	-0.15522	0.37565	7.65596	-1.66	9.555652174	-3.56	9	27.59936	16.42172	32.58507	-0.1299
Oct-11	22	13	6.89458	-0.15522	5.53087	12.22039	0.78	9.400434783	3.60	10	5.99697	0.26221	1.66840	0.57119
Nov-11	23	1	4.78625	-0.15522	1.68609	8.42545	-7.43	9.245217391	-8.25	11	742.54538	10.77730	114.66840	2.97234
Dec-11	24	5	3.84922	-0.15522	3.34130	7.97234	-2.97	9.09	-4.09	12	59.44675	13.95766	45.00174	-0.4344
Jan-12	25	3	4.26533	-0.15522	-2.86609	0.82792	2.17	8.934782609	-5.93	1	72.40282	118.38349	75.83507	-1.2003
Feb-12	26	9	5.05722	-0.15522	1.28913	5.39924	3.60	8.779565217	0.22	2	40.00846	39.80468	7.33507	-1.0449
Mar-12	27	14	7.37544	-0.15522	-0.30565	4.59635	9.40	8.624347826	5.38	3	67.16895	50.58036	5.25174	-0.1557
Apr-12	28	11	7.79368	-0.15522	1.59957	8.81979	2.18	8.469130435	2.53	4	19.82011	8.34369	0.50174	-0.1006
May-12	29	11	7.92957	-0.15522	2.25478	9.89325	1.11	8.313913043	2.69	5	10.06137	3.29453	0.50174	-0.2787
Jun-12	30	11	8.58071	-0.15522	0.16000	7.93436	3.07	8.158695652	2.84	6	27.86948	14.24289	0.50174	-0.3418
Jul-12	31	19	9.41430	-0.15522	6.81522	15.24071	3.76	8.003478261	11.00	7	19.78572	12.47771	53.16840	0.09103
Aug-12	32	20	8.80417	-0.15522	12.47043	21.72952	-1.73	7.84826087	12.15	8	8.64759	100.42414	68.75174	0.00123
Sep-12	33	9	8.64248	-0.15522	0.37565	9.02460	-0.02	7.693043478	1.31	9	0.27337	7.20241	7.33507	-0.5535
Oct-12	34	19	9.79764	-0.15522	5.53087	14.01813	4.98	7.537826087	11.46	10	26.22036	5.33517	53.16840	-0.088
Nov-12	35	13	10.08208	-0.15522	1.68609	11.32851	1.67	7.382608696	5.62	11	12.85760	0.14426	1.66840	0.09755
Dec-12	36	12	9.59329	-0.15522	3.34130	13.26816	-1.27	7.227391304	4.77	12	10.56803	2.43307	0.08507	-0.2023
Jan-13	37	9	10.07672	-0.15522	-2.86609	6.57199	2.43	7.072173913	1.93	1	26.97789	26.38203	7.33507	0.0234
Feb-13	38	11	9.86610	-0.15522	1.28913	11.21063	-0.21	6.916956522	4.08	2	1.91481	0.24771	0.50174	0.12775
Mar-13	39	8	9.34126	-0.15522	-0.30565	9.40523	-1.41	6.76173913	1.24	3	17.56534	5.30430	13.75174	-0.1518
Apr-13	40	12	9.50547	-0.15522	1.59957	10.78561	1.21	6.606521739	5.39	4	10.11991	0.85142	0.08507	0.38375
May-13	41	7	8.13899	-0.15522	2.25478	11.60503	-4.61	6.451304348	0.55	5	65.78616	0.01067	22.16840	0.02054
Jun-13	42	8	7.94595	-0.15522	0.16000	8.14377	-0.14	6.296086957	1.70	6	1.79713	12.70611	13.75174	0.07574
Jul-13	43	14	7.63135	-0.15522	6.81522	14.60595	-0.61	6.140869565	7.86	7	4.32825	8.39621	5.25174	-0.5752
Aug-13	44	28	9.59443	-0.15522	12.47043	19.94657	8.05	5.985652174	22.01	8	28.76225	67.86855	265.41840	-0.1495
Sep-13	45	14	10.54003	-0.15522	0.37565	9.81486	4.19	5.830434783	8.17	9	29.89384	3.58523	5.25174	0.42255
Oct-13	46	10	8.82881	-0.15522	5.53087	15.91568	-5.92	5.675217391	4.32	10	59.15679	17.70176	2.91840	-0.264
Nov-13	47	13	9.36808	-0.15522	1.68609	10.35968	2.64	5.52	7.48	11	20.31016	1.81887	1.66840	-0.2651
Dec-13	48	16	10.11922	-0.15522	3.34130	12.55416	3.45	5.364782609	10.64	12	21.53649	0.71543	18.41840	-0.3064
Jan-14	49	12				7.09791	4.90	5.209565217	6.79	1	40.85074	33.85791	0.84028	0.59149
Feb-14	50	4				11.09791	-7.10	5.054347826	-1.05	2	177.44779	3.30787	79.50694	-1.413
Mar-14	51	15				9.34791	5.65	4.899130435	10.10	3	37.68059	12.73601	4.34028	-0.1268
Apr-14	52	13				11.09791	1.90	4.743913043	8.26	4	14.63145	3.30787	0.00694	-0.1848
May-14	53	14				11.59791	2.40	4.588695652	9.41	5	17.15777	1.73911	1.17361	0.02485
Jun-14	54	9				9.34791	-0.35	4.433478261	4.57	6	3.86568	12.73601	15.34028	-0.5725
Jul-14	55	21				15.84791	5.15	4.27826087	16.72	7	24.53375	8.59220	65.34028	0.44514
Aug-14	56	12				21.34791	-9.35	4.123043478	7.88	8	77.89926	71.08589	0.84028	-0.7418
Sep-14	57	18				9.09791	8.90	3.967826087	14.03	9	49.45605	14.58289	25.84028	0.061
Oct-14	58	13				14.09791	-1.10	3.812608696	9.19	10	8.44547	1.39534	0.00694	-0.3771
Nov-14	59	15				10.09791	4.90	3.657391304	11.34	11	32.68059	7.94538	4.34028	0.17319
Dec-14	60	9				11.59791	-2.60	3.502173913	5.50	12	28.86568	1.73911	15.34028	

Appendix R. Multiplicative Holt-Winters (4 months)

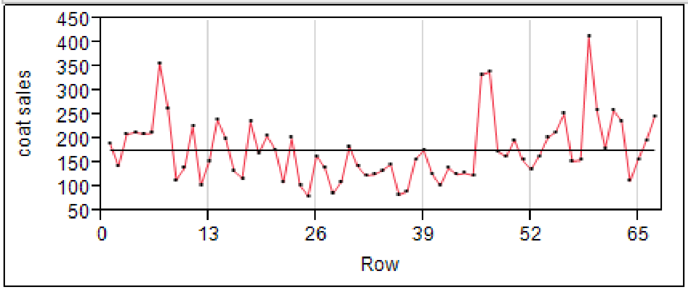
Date	Time	Sales	Estimate	Growth	Seasonal	*Forecast*	Error	Regression Estimates	Detrended	Averages / Seasons	Percentage Error	Explained Variation	Total Variation	Theil's U
	-3				1.05620					1.05620074				
	-2				1.32599					1.32598812				
	-1				1.41008					1.41008073				
	0		12.8152	-0.1552	1.84359					1.84359251				
Jan-10	1	5	8.85111	-0.1552	1.05620	13.37150	-8.37	12.66000	0.3949447	1	167.43003	2.76613	45.00174	0.50613
Feb-10	2	9	7.77876	-0.1552	1.32599	11.53065	-2.5307	12.50478	0.7197246	2	28.11835	0.03157	7.33507	0.30553
Mar-10	3	8	6.68641	-0.1552	1.41008	10.74981	-2.7498	12.34957	0.6477961	3	34.37257	0.91877	13.75174	-0.1199
Apr-10	4	13	6.7812	-0.1552	1.84359	12.04086	0.95914	12.19435	1.0660677	4	7.37802	0.11057	1.66840	-1.077
May-10	5	21	12.9965	-0.1552	1.05620	6.99837	14.0016	12.03913	1.744312	1	66.67442	22.18374	86.33507	0.33464
Jun-10	6	10	10.2945	-0.1552	1.32599	17.02737	-7.0274	11.88391	0.8414737	2	70.27370	28.29215	2.91840	-0.3703
Jul-10	7	18	11.4012	-0.1552	1.41008	14.29716	3.70284	11.72870	1.5346975	3	20.57131	6.70205	39.58507	0.3185
Aug-10	8	15	9.7516	-0.1552	1.84359	20.73296	-5.733	11.57348	1.2960667	4	38.21973	81.44387	10.83507	0.20905
Sep-10	9	7	8.16969	-0.1552	1.05620	10.13570	-3.1357	11.41826	0.6130531	1	44.79573	2.47317	22.16840	-0.4818
Oct-10	10	14	9.23685	-0.1552	1.32599	10.62709	3.37291	11.26304	1.2430033	2	24.09220	1.16908	5.25174	-0.0139
Nov-10	11	13	9.1478	-0.1552	1.41008	12.80584	0.19416	11.10783	1.170346	3	1.49357	1.20451	1.66840	0.27528
Dec-10	12	13	8.05977	-0.1552	1.84359	16.57866	-3.5787	10.95261	1.1869318	4	27.52819	23.72013	1.66840	-0.2039
Jan-11	13	11	9.1108	-0.1552	1.05620	8.34879	2.65121	10.79739	1.0187646	1	24.10190	11.28652	0.50174	-0.2841
Feb-11	14	15	10.0881	-0.1552	1.32599	11.87500	3.125	10.64217	1.4094865	2	20.83332	0.02778	10.83507	0.46708
Mar-11	15	7	7.5452	-0.1552	1.41008	14.00620	-7.0062	10.48696	0.6674959	3	100.08853	5.28018	22.16840	0.80345
Apr-11	16	8	5.924	-0.1552	1.84359	13.62412	-5.6241	10.33174	0.774313	4	70.30155	3.67025	13.75174	-0.1134
May-11	17	7	6.18145	-0.1552	1.05620	6.09299	0.90701	10.17652	0.6878578	1	12.95732	31.53211	22.16840	-0.0013
Jun-11	18	8	6.0296	-0.1552	1.32599	7.99072	0.00928	10.02130	0.7982993	2	0.11604	13.82067	13.75174	-0.4646
Jul-11	19	12	7.14101	-0.1552	1.41008	8.28335	3.71665	9.86609	1.2162877	3	30.97205	11.73049	0.08507	-0.7601
Aug-11	20	22	9.3633	-0.1552	1.84359	12.87895	9.12105	9.71087	2.2655026	4	41.45930	1.37035	105.91840	0.16934
Sep-11	21	6	7.513	-0.1552	1.05620	9.72558	-3.7256	9.55565	0.6279006	1	62.09298	3.93132	32.58507	-0.5406
Oct-11	22	13	8.53333	-0.1552	1.32599	9.75634	3.24366	9.40043	1.3829148	2	24.95125	3.81029	1.66840	0.83183
Nov-11	23	1	4.69278	-0.1552	1.41008	11.81381	-10.814	9.24522	0.108164	3	1081.38103	0.01113	114.66840	3.36542
Dec-11	24	5	3.66033	-0.1552	1.84359	8.36542	-3.3654	9.09000	0.550055	4	67.30836	11.17508	45.00174	0.14042
Jan-12	25	3	3.18567	-0.1552	1.05620	3.70210	-0.7021	8.93478	0.3357664	1	23.40339	64.09974	75.83507	-1.6606
Feb-12	26	9	4.83586	-0.1552	1.32599	4.01834	4.98166	8.77957	1.0251077	2	55.35177	59.13598	7.33507	-0.8222
Mar-12	27	14	7.20252	-0.1552	1.41008	6.60008	7.39992	8.62435	1.6233112	3	52.85654	26.09421	5.25174	0.14231
Apr-12	28	11	6.52797	-0.1552	1.84359	12.99236	-1.9924	8.46913	1.2988346	4	18.11234	1.64872	0.50174	-0.3881
May-12	29	11	8.31512	-0.1552	1.05620	6.73091	4.26909	8.31391	1.3230834	1	38.80991	24.77474	0.50174	-0.0164
Jun-12	30	11	8.22516	-0.1552	1.32599	10.81993	0.18007	8.15870	1.3482547	2	1.63698	0.78926	0.50174	-0.6928
Jul-12	31	19	10.6671	-0.1552	1.41008	11.37927	7.62073	8.00348	2.3739678	3	40.10910	0.10828	53.16840	-0.0327
Aug-12	32	20	10.6736	-0.1552	1.84359	19.37958	0.62042	7.84826	2.5483353	4	3.10210	58.84804	68.75174	0.10547
Sep-12	33	9	9.55857	-0.1552	1.05620	11.10950	-2.1095	7.69304	1.1698881	1	23.43887	0.35860	7.33507	-0.7257
Oct-12	34	19	11.7704	-0.1552	1.32599	12.46874	6.53126	7.53783	2.5206206	2	34.37506	0.57822	53.16840	0.1778
Nov-12	35	13	10.4638	-0.1552	1.41008	16.37829	-3.3783	7.38261	1.7608952	3	25.98683	21.80848	1.66840	0.53884
Dec-12	36	12	8.48271	-0.1552	1.84359	19.00488	-7.0049	7.22739	1.6603501	4	58.37396	53.23953	0.08507	-0.017
Jan-13	37	9	8.42053	-0.1552	1.05620	8.79550	0.2045	7.07217	1.2725931	1	2.27218	8.48458	7.33507	-0.0045
Feb-13	38	11	8.27992	-0.1552	1.32599	10.95971	0.04029	6.91696	1.5902948	2	0.36625	0.56043	0.50174	0.31423
Mar-13	39	8	6.94674	-0.1552	1.41008	11.45648	-3.4565	6.76174	1.1831276	3	43.20605	0.06343	13.75174	0.0651
Apr-13	40	12	6.65577	-0.1552	1.84359	12.52079	-0.5208	6.60652	1.816387	4	4.33995	0.66009	0.08507	-0.0112
May-13	41	7	6.56157	-0.1552	1.05620	6.86589	0.13411	6.45130	1.0850519	1	1.91590	23.44929	22.16840	0.07068
Jun-13	42	8	6.22705	-0.1552	1.32599	8.49475	-0.4947	6.29609	1.2706305	2	6.18435	10.32713	13.75174	-0.6798
Jul-13	43	14	7.92517	-0.1552	1.41008	8.56178	5.43822	6.14087	2.2798074	3	38.84446	9.90083	5.25174	-0.9768
Aug-13	44	28	11.3346	-0.1552	1.84359	14.32463	13.6754	5.98565	4.6778528	4	48.84061	6.84500	265.41840	-0.0783
Sep-13	45	14	12.1769	-0.1552	1.05620	11.80767	2.19233	5.83043	2.4011931	1	15.65953	0.00987	5.25174	0.42432
Oct-13	46	10	9.86871	-0.1552	1.32599	15.94054	-5.9405	5.67522	1.762047	2	59.40543	17.91160	2.91840	0.06968
Nov-13	47	13	9.47602	-0.1552	1.41008	13.69681	-0.6968	5.52000	2.3550725	3	5.36010	3.95405	1.66840	0.09106
Dec-13	48	16	9.01224	-0.1552	1.84359	17.18377	-1.1838	5.36478	2.9824135	4	7.39855	29.98038	18.41840	-0.197
Jan-14	49	12				8.84830	3.1517	5.20957	2.3034552	1	26.26415	16.55159	0.84028	0.38338
Feb-14	50	4				8.60061	-4.6006	5.05435	0.7913978	2	115.01526	18.62834	79.50694	-1.6611
Mar-14	51	15				8.35564	6.64436	4.89913	3.0617678	3	44.29576	20.80300	4.34028	-0.3422
Apr-14	52	13				7.86761	5.13239	4.74391	2.7403538	4	39.47990	25.49295	0.00694	-0.4467
May-14	53	14				8.19254	5.80746	4.58870	3.0509759	1	41.48186	22.31738	1.17361	-0.0873
Jun-14	54	9				7.77734	1.22266	4.43348	2.0300088	2	13.58506	26.41263	15.34028	-1.5022
Jul-14	55	21				7.48016	13.5198	4.27826	4.9085366	3	64.38019	29.55561	65.34028	-0.2513
Aug-14	56	12				6.72298	5.27702	4.12304	2.9104714	4	43.97515	38.36173	0.84028	-0.8719
Sep-14	57	18				7.53678	10.4632	3.96783	4.5364892	1	58.12902	28.94322	25.84028	-0.3359
Oct-14	58	13				6.95408	6.04592	3.81261	3.4097389	2	46.50708	35.55245	0.00694	-0.6458
Nov-14	59	15				6.60468	8.39532	3.65739	4.1012839	3	55.96878	39.84113	4.34028	-0.2281
Dec-14	60	9				5.57835	3.42165	3.50217	2.5698324	4	38.01831	53.85086	15.34028	

Appendix S. Multiplicative Holt-Winters (12 months)

Date	Time	Sales	Estimate	Growth	Seasonal	*Forecast*	Error	Regression Estimates	Detrended	Averages / Seasons	Percentage Error	Explained Variation	Total Variation	Theil's U
	-11				0.75552					0.75551722				
	-10				1.18615					1.1861534				
	-9				1.03043					1.03043267				
	-8				1.23890					1.23890057				
	-7				1.21008					1.21007627				
	-6				1.06466					1.06466454				
	-5				1.85119					1.85119011				
	-4				2.69694					2.69693935				
	-3				1.20301					1.20300874				
	-2				1.72715					1.72714643				
	-1				1.34862					1.34861942				
	0		12.8152	-0.1552	1.59494					1.59493761				
Jan-10	1	5	9.03504	-0.1552	1.10682	9.564848	-4.56	12.66000	0.394945	1	91.29696	4.59453	45.00174	0.30657
Feb-10	2	9	7.31585	-0.1552	1.65356	10.532836	-1.5328	12.50478	0.719725	2	17.03151	1.38179	7.33507	-0.0691
Mar-10	3	8	6.63689	-0.1552	1.32741	7.378550	0.62145	12.34957	0.647796	3	7.76813	18.74703	13.75174	-0.6212
Apr-10	4	13	7.19318	-0.1552	1.62638	8.030149	4.96985	12.19435	1.066068	4	38.22962	13.52904	1.66840	-0.9603
May-10	5	21	12.1257	-0.1552	1.19937	8.516471	12.4835	12.03913	1.744312	5	59.44538	10.18799	86.33507	0.13069
Jun-10	6	10	9.4457	-0.1552	1.56547	12.744590	-2.7446	11.88391	0.841474	6	27.44590	1.07383	2.91840	-0.0802
Jul-10	7	18	11.1106	-0.1552	1.37075	17.198442	0.80156	11.72870	1.534698	7	4.45310	30.14130	39.58507	0.80811
Aug-10	8	15	10.2169	-0.1552	1.60295	29.545958	-14.546	11.57348	1.296067	8	96.97305	318.18084	10.83507	0.34028
Sep-10	9	7	8.26052	-0.1552	1.14725	12.104261	-5.1043	11.41826	0.613053	9	72.91801	0.15676	22.16840	-0.0001
Oct-10	10	14	8.46239	-0.1552	1.57864	13.999052	0.00095	11.26304	1.243003	10	0.00677	5.24739	5.25174	-0.1283
Nov-10	11	13	8.80878	-0.1552	1.38630	11.203214	1.79679	11.10783	1.170346	11	13.82143	0.25515	1.66840	0.06168
Dec-10	12	13	8.42187	-0.1552	1.59416	13.801887	-0.8019	10.95261	1.186932	12	6.16836	4.38297	1.66840	-0.1423
Jan-11	13	11	8.82997	-0.1552	1.16184	9.149670	1.85033	10.79739	1.018765	1	16.82118	6.54676	0.50174	-0.0596
Feb-11	14	15	9.02733	-0.1552	1.59093	14.344229	0.65577	10.64217	1.409486	2	4.37181	6.94794	10.83507	0.31846
Mar-11	15	7	7.24258	-0.1552	1.32414	11.776926	-4.7769	10.48696	0.667496	3	68.24180	0.00470	22.16840	0.50382
Apr-11	16	8	6.20537	-0.1552	1.54900	11.526733	-3.5267	10.33174	0.774313	4	44.08417	0.03298	13.75174	0.03205
May-11	17	7	6.0394	-0.1552	1.16143	7.256376	-0.2564	10.17652	0.687858	5	3.66251	19.81993	22.16840	0.17308
Jun-11	18	8	5.51943	-0.1552	1.56997	9.211534	-1.2115	10.02130	0.798299	6	15.14418	6.23401	13.75174	-0.5809
Jul-11	19	12	6.9407	-0.1552	1.38408	7.352984	4.64702	9.86609	1.216288	7	38.72513	18.96907	0.08507	-0.9269
Aug-11	20	22	9.94726	-0.1552	1.64713	10.876795	11.1232	9.71087	2.265503	8	50.56002	0.69146	105.91840	0.23791
Sep-11	21	6	7.82009	-0.1552	1.10306	11.233943	-5.2339	9.55565	0.627901	9	87.23238	0.22505	32.58507	-0.15
Oct-11	22	13	7.92726	-0.1552	1.58033	12.100064	0.89994	9.40043	1.382915	10	6.92259	0.15345	1.66840	0.75188
Nov-11	23	1	4.76698	-0.1552	1.21019	10.774405	-9.7744	9.24522	0.108164	11	977.44053	0.87222	114.66840	2.3519
Dec-11	24	5	3.93988	-0.1552	1.59115	7.351901	-2.3519	9.09000	0.550055	12	47.03803	18.97850	45.00174	0.27943
Jan-12	25	3	3.3307	-0.1552	1.07310	4.397162	-1.3972	8.93478	0.335766	1	46.57206	53.45323	75.83507	-1.316
Feb-12	26	9	4.2495	-0.1552	1.65993	5.051950	3.94805	8.77957	1.025108	2	43.86722	44.30743	7.33507	-0.9532
Mar-12	27	14	7.28033	-0.1552	1.31574	5.421406	8.57859	8.62435	1.623311	3	61.27567	39.52545	5.25174	0.00263
Apr-12	28	11	7.0348	-0.1552	1.58708	11.036831	-0.0368	8.46913	1.298835	4	0.33483	0.45092	0.50174	-0.2736
May-12	29	11	8.31662	-0.1552	1.11005	7.990126	3.00987	8.31391	1.323083	5	27.36249	13.82507	0.50174	0.16483
Jun-12	30	11	7.50723	-0.1552	1.63110	12.813178	-1.8132	8.15870	1.348255	6	16.48343	1.22068	0.50174	-0.8022
Jul-12	31	19	10.3737	-0.1552	1.39212	10.175780	8.82422	8.00348	2.373968	7	46.44326	2.34872	53.16840	-0.1668
Aug-12	32	20	11.2344	-0.1552	1.61568	16.831132	3.16887	7.84826	2.548335	8	15.84434	26.24307	68.75174	0.16105
Sep-12	33	9	9.81254	-0.1552	1.08149	12.221004	-3.221	7.69304	1.169888	9	35.78894	0.26283	7.33507	-0.4154
Oct-12	34	19	10.5061	-0.1552	1.65737	15.261742	3.73826	7.53783	2.520621	10	19.67504	12.62672	53.16840	-0.0249
Nov-12	35	13	9.91925	-0.1552	1.38005	12.526615	0.47339	7.38261	1.760895	11	3.64142	0.66958	1.66840	0.272
Dec-12	36	12	8.7679	-0.1552	1.57910	15.536017	-3.536	7.22739	1.660350	12	29.46681	14.65116	0.08507	0.02019
Jan-13	37	9	8.4887	-0.1552	1.07834	9.242227	-0.2422	7.07217	1.272593	1	2.69141	6.08168	7.33507	0.31478
Feb-13	38	11	7.61033	-0.1552	1.62598	13.833001	-2.833	6.91696	1.590295	2	25.75455	4.51421	0.50174	0.16445
Mar-13	39	8	6.74825	-0.1552	1.35124	9.809000	-1.809	6.76174	1.183128	3	22.61250	3.60747	13.75174	-0.192
Apr-13	40	12	7.02197	-0.1552	1.59832	10.463652	1.53635	6.60652	1.816387	4	12.80290	1.54923	0.08507	0.05187
May-13	41	7	6.70676	-0.1552	1.07322	7.622435	-0.6224	6.45130	1.085052	5	8.89193	16.69456	22.16840	0.38375
Jun-13	42	8	5.8561	-0.1552	1.58750	10.686249	-2.6862	6.29609	1.270630	6	33.57812	1.04466	13.75174	-0.758
Jul-13	43	14	7.68733	-0.1552	1.42083	7.936332	6.06367	6.14087	2.279807	7	43.31191	14.22799	5.25174	-1.1308
Aug-13	44	28	11.789	-0.1552	1.71334	12.169482	15.8305	5.98565	4.677853	8	56.53756	0.21266	265.41840	-0.0506
Sep-13	45	14	12.2353	-0.1552	1.08373	12.581867	1.41813	5.83043	2.401193	9	10.12952	0.76306	5.25174	0.7158
Oct-13	46	10	9.61585	-0.1552	1.50642	20.021159	-10.021	5.67522	1.762047	10	100.21159	69.10307	2.91840	0.00561
Nov-13	47	13	9.32805	-0.1552	1.41680	13.056128	-0.0561	5.52000	2.355072	11	0.43175	1.81655	1.66840	-0.1166
Dec-13	48	16	9.24343	-0.1552	1.71595	14.484794	1.51521	5.36478	2.982413	12	9.47004	7.70873	18.41840	-0.1827
Jan-14	49	12	9.93422	-0.1552	1.10213	9.076057	2.92394	5.20957	2.303455	1	24.36619	14.75028	0.84028	0.39489
Feb-14	50	4	6.74233	-0.1552	1.37120	8.738674	-4.7387	5.05435	0.791398	2	118.46685	17.45562	79.50694	-1.5964
Mar-14	51	15	8.29227	-0.1552	1.47486	8.614227	6.38577	4.89913	3.061768	3	42.57182	18.51098	4.34028	-0.3166
Apr-14	52	13	7.89788	-0.1552	1.70560	8.251085	4.74891	4.74391	2.740354	4	36.53011	21.76765	0.00694	-0.43
May-14	53	14	9.85702	-0.1552	1.14924	8.410525	5.58948	4.58870	3.050976	5	39.92482	20.30531	1.17361	-0.0882
Jun-14	54	9	8.36406	-0.1552	1.32749	7.764992	1.23501	4.43348	2.030009	6	13.72231	26.53975	15.34028	-1.4778
Jul-14	55	21	10.7792	-0.1552	1.54495	7.699676	13.3003	4.27826	4.908537	7	63.33488	27.21700	65.34028	-0.2326
Aug-14	56	12	9.09436	-0.1552	1.64842	7.115909	4.88409	4.12304	2.910471	8	40.70076	33.64879	0.84028	-0.8559
Sep-14	57	18	11.8052	-0.1552	1.20485	7.729508	10.2705	3.96783	4.536489	9	57.05829	26.90662	25.84028	-0.3386
Oct-14	58	13	10.8583	-0.1552	1.30821	6.905212	6.09479	3.81261	3.409739	10	46.88298	36.13758	0.00694	-0.6289
Nov-14	59	15	10.2794	-0.1552	1.53226	6.824399	8.1756	3.65739	4.101284	11	54.50400	37.11572	4.34028	-0.1968
Dec-14	60	9	8.13582	-0.1552	1.56814	6.047288	2.95271	3.50217	2.569832	12	32.80792	47.18837	15.34028	

Appendix T. ARIMA

Time Series coat sales



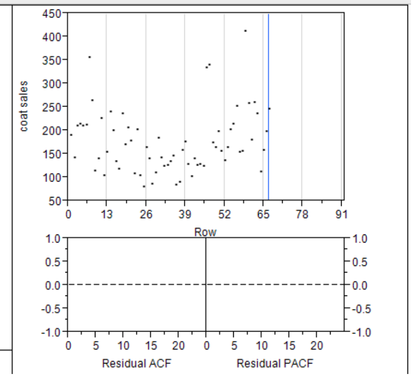
Mean	176.92537
Std	66.413969
N	67
Zero Mean ADF	-1.56643
Single Mean ADF	-5.545976
Trend ADF	-5.570628

Time Series Basic Diagnostics

Lag	AutoCorr	Ljung-Box Q	p-Value	Lag	Partial
0	1.0000	.	.	0	1.0000
1	0.3370	7.9527	0.0048*	1	0.3370
2	-0.0347	8.0382	0.0180*	2	-0.1672
3	0.2620	12.9983	0.0046*	3	0.3853
4	0.2372	17.1265	0.0018*	4	-0.0322
5	-0.0520	17.3279	0.0039*	5	-0.0717
6	-0.0030	17.3286	0.0081*	6	0.0249
7	0.2103	20.7373	0.0042*	7	0.1309
8	0.2087	24.1507	0.0022*	8	0.1455
9	0.0805	24.6670	0.0034*	9	0.0300
10	-0.0370	24.7782	0.0058*	10	-0.1703
11	-0.0095	24.7856	0.0098*	11	-0.0573
12	0.2582	30.3912	0.0024*	12	0.3287
13	0.2140	34.3136	0.0011*	13	0.1001
14	-0.0691	34.7306	0.0016*	14	-0.1357
15	0.0147	34.7499	0.0027*	15	-0.1268
16	0.1139	35.9263	0.0030*	16	-0.1274
17	-0.1014	36.8773	0.0035*	17	0.0120
18	-0.2924	44.9461	0.0004*	18	-0.1317
19	-0.1291	46.5525	0.0004*	19	-0.1214
20	0.0039	46.5540	0.0007*	20	-0.1218
21	-0.1313	48.2871	0.0006*	21	-0.0772
22	-0.2784	56.2501	<.0001*	22	-0.1447
23	-0.1495	58.5976	<.0001*	23	0.0288
24	-0.0807	59.2972	<.0001*	24	-0.1366
25	-0.1642	62.2649	<.0001*	25	-0.0734

Model Comparison

Model	DF	Variance	AIC	SBC	RSquare	-2LogLH	Weights	MAPE	MAE
Seasonal ARIMA(1, 1, 1)(1, 1, 1)12	49	2939.1076	609.99144	619.93636	0.058	599.99144	0.654781	27.346205	45.778012
Seasonal ARIMA(0, 1, 1)(0, 1, 1)12	51	3962.4203	611.27231	617.23926	-0.03	605.27231	0.345110	28.354012	47.827387
Seasonal ARIMA(1, 1, 0)(1, 1, 0)12	51	5984.1152	627.38561	633.35256	-0.36	621.38561	0.000109	33.728352	56.610923
Seasonal ARIMA(1, 1, 1)(1, 1, 1)4	57	2984.4911	696.12182	706.75750	0.155	686.12182	0.000000	26.881994	45.635867
Seasonal ARIMA(0, 1, 1)(0, 1, 1)4	59	3482.6012	697.84752	704.22893	0.080	691.84752	0.000000	28.616313	47.891559
Seasonal ARIMA(1, 1, 1)(1, 1, 1)3	58	3703.0026	716.72854	727.44421	0.023	706.72854	0.000000	30.180762	49.199748
Seasonal ARIMA(0, 1, 1)(0, 1, 1)3	60	3839.5561	718.90654	725.33594	-0.07	712.90654	0.000000	31.034801	49.336592
Seasonal ARIMA(1, 1, 0)(1, 1, 0)4	59	7174.3541	730.54141	736.92282	-0.50	724.54141	0.000000	38.951144	66.555937
Seasonal ARIMA(0, 0, 1)(0, 0, 1)12	64	3247.6613	737.80473	744.41881	0.263	731.80473	0.000000	25.283540	41.837169
Seasonal ARIMA(1, 0, 1)(1, 0, 1)12	62	3312.2783	740.87573	751.89920	0.274	730.87573	0.000000	25.576997	42.072899
Seasonal ARIMA(1, 0, 1)(1, 0, 1)4	62	3443.1686	741.43097	752.45443	0.273	731.43097	0.000000	24.616823	40.225997
Seasonal ARIMA(0, 0, 1)(0, 0, 1)4	64	3599.6547	742.18629	748.80037	0.219	736.18629	0.000000	26.119418	43.120136
Seasonal ARIMA(1, 0, 0)(1, 0, 0)12	64	3560.0769	743.10329	749.71736	0.201	737.10329	0.000000	26.915711	44.412519
Seasonal ARIMA(1, 0, 1)(1, 0, 1)3	62	3573.5203	743.69254	754.71600	0.248	733.69254	0.000000	25.962002	42.281264
Seasonal ARIMA(1, 0, 0)(1, 0, 0)4	64	3890.245	747.20464	753.81872	0.157	741.20464	0.000000	27.506661	44.646248
Seasonal ARIMA(1, 1, 0)(1, 1, 0)3	60	8775.2349	753.94340	760.37280	-0.81	747.9434	0.000000	42.648696	73.028499



Appendix U. Comparison Chart item 1 - St Dumont medal 20yrs

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	1,226.1710	0.0206	59.9948	0.8515	1.3014	210.6398	36.6245	0.6572	0.4199
Trend	1,141.6839	0.0881	53.8485	0.7643	0.9862	2,508.7139	99.8999	1.7928	1.2357
Dummy	571.8196	0.5432	41.8358	0.5938	0.9800	493.1458	55.8593	1.0024	0.4588
Trigonometric L=4	1,128.7502	0.0984	52.8685	0.7504	0.9997	4,904.3092	137.6401	2.4700	1.7930
Trigonometric L=2	882.2283	0.2953	40.3655	0.5729	0.5013	907.3103	80.2515	1.4402	0.6604
Trigonometric L=1	914.0196	0.2699	52.8933	0.7507	1.1373	321.2249	45.1715	0.8106	0.4072
Autocorrelation L=4	1,113.1735	0.0935	53.2665	0.7560	0.8206	871.0768	60.9310	1.0934	0.8572
Autocorrelation L=2	1,051.8715	0.1373	49.0765	0.6966	0.8150	720.8023	74.4652	1.3363	0.5769
Autocorrelation L=1	961.5000	0.2175	53.2229	0.7554	0.8556	325.6162	47.8180	0.8581	0.4063
Decomp Multiplicative (12)	306.4304	0.8353	25.3237	0.3594	0.4042	587.7561	54.3462	0.9753	0.5688
Decomp Multiplicative (4)	516.3118	0.4808	33.8308	0.4802	0.4660	243.1016	39.2790	0.7049	0.4364
Decomp Additive (12)	277.2919	0.7396	24.3381	0.3454	0.2867	418.7493	48.5815	0.8718	0.5418
Simple Exponential Smoothing	1,284.1096	0.0228	56.7411	0.8054	1.1757	249.7593	35.4651	0.6364	0.5313
Holt's Trend	1,226.1710	0.0206	59.9948	0.8515	1.3014	210.6398	36.6245	0.6572	0.4199
Additive Holt-Winters (4)	1,238.2103	0.3118	53.9862	0.7663	1.3198	336.7695	44.5141	0.7988	0.5290
Additive Holt-Winters (12)	710.9401	0.6943	42.8238	0.6078	0.6889	341.0256	42.7929	0.7679	0.5859
Multiplicative Holt-Winters (4)	1,298.4788	0.5530	55.7680	0.7915	0.8332	575.6635	48.9250	0.8780	0.8052
Multiplicative Holt-Winters (12)	1,410.9289	0.6584	51.4371	0.7301	0.7043	546.0436	47.5726	0.8537	0.7830
ARIMA	1,026.5668	0.1478	57.4674	0.8157	0.8466	275.6418	39.5569	0.7099	0.5469

	Value of Interest - Consolidation				
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	1,436.8108	0.0206	48.3096	0.7544	0.8607
Trend	3,650.3978	0.0881	76.8742	1.2785	1.1110
Dummy	1,064.9654	0.5432	48.8476	0.7981	0.7194
Trigonometric L=4	6,033.0593	0.0984	95.2543	1.6102	1.3963
Trigonometric L=2	1,789.5386	0.2953	60.3085	1.0065	0.5808
Trigonometric L=1	1,235.2445	0.2699	49.0324	0.7807	0.7722
Autocorrelation L=4	1,984.2503	0.0935	57.0987	0.9247	0.8389
Autocorrelation L=2	1,772.6738	0.1373	61.7708	1.0164	0.6959
Autocorrelation L=1	1,287.1162	0.2175	50.5205	0.8068	0.6310
Decomp Multiplicative (12)	894.1865	0.8353	39.8349	0.6674	0.4865
Decomp Multiplicative (4)	759.4134	0.4808	36.5549	0.5925	0.4512
Decomp Additive (12)	696.0412	0.7396	36.4598	0.6086	0.4142
Simple Exponential Smoothing	1,533.8689	0.0228	46.1031	0.7209	0.8535
Holt's Trend	1,436.8108	0.0206	48.3096	0.7544	0.8607
Additive Holt-Winters (4)	1,574.9798	0.3118	49.2502	0.7825	0.9244
Additive Holt-Winters (12)	1,051.9657	0.6943	42.8084	0.6879	0.6374
Multiplicative Holt-Winters (4)	1,874.1423	0.5530	52.3465	0.8348	0.8192
Multiplicative Holt-Winters (12)	1,956.9725	0.6584	49.5048	0.7919	0.7436
ARIMA	1,302.2086	0.1478	48.5121	0.7628	0.6967

Appendix V. Comparison Chart item 2 - PT short officer women

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	28,472.3395	0.0526	134.8655	1.1840	0.8186	14,472.6279	158.0538	1.6725	1.1053
Trend	22,093.5696	0.2649	114.9526	1.0092	0.7719	513,960.0405	1,159.0538	12.2650	7.3214
Dummy	17,882.5944	0.4050	111.5646	0.9794	0.6095	33,720.9631	333.3829	3.5278	2.2341
Trigonometric L=4	22,255.2064	0.2595	113.7783	0.9988	0.7667	190,114.9250	762.8648	8.0726	4.6743
Trigonometric L=2	22,880.4255	0.2387	116.3031	1.0210	0.7061	21,326.1018	263.8966	2.7925	1.7478
Trigonometric L=1	21,673.6068	0.2788	113.3974	0.9955	0.6691	30,216.8666	324.1395	3.4300	2.1472
Autocorrelation L=4	22,186.7217	0.2571	116.2624	1.0207	0.7727	216,354.3512	802.8698	8.4959	4.9660
Autocorrelation L=2	24,212.2885	0.1737	121.1421	1.0635	0.7779	23,096.9800	280.5892	2.9692	1.7251
Autocorrelation L=1	25,042.4615	0.1536	130.7155	1.1475	0.7708	29,181.2550	317.7842	3.3628	1.9283
Decomp Multiplicative (12)	12,795.3501	0.6892	71.8980	0.6312	0.5027	14,950.2114	140.5090	1.4869	1.7854
Decomp Multiplicative (4)	11,187.6168	0.5532	72.8699	0.6397	0.5664	16,441.3471	144.5274	1.5294	1.2098
Decomp Additive (12)	11,192.7982	0.6601	78.2526	0.6870	0.5235	10,447.2016	123.1067	1.3027	1.3555
Simple Exponential Smoothing	30,522.6667	0.0156	119.2689	1.0470	0.7840	17,631.0000	95.0890	1.0062	0.9546
Holt's Trend	30,308.0350	0.1217	123.8192	1.0870	0.8213	132,832.1263	639.9075	6.7714	3.9252
Additive Holt-Winters (4)	23,423.1477	0.3326	106.0724	0.9312	0.7781	134,172.5746	641.0189	6.7832	3.9365
Additive Holt-Winters (12)	21,052.7888	0.4058	112.5067	0.9877	0.6719	137,486.3557	652.8187	6.9081	4.1620
Multiplicative Holt-Winters (4)	27,340.5097	0.2895	103.7549	0.9109	0.8155	22,970.7002	287.1071	3.0381	1.7787
Multiplicative Holt-Winters (12)	23,238.9241	0.1815	118.0873	1.0367	0.6759	14,879.7643	185.0960	1.9587	1.2278
ARIMA	25,455.8633	0.2459	118.2601	1.0382	0.7498	19,972.1847	170.8681	1.8081	1.4170

	Value of Interest - Consolidation				
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	42,944.9674	0.0526	146.4597	1.4282	0.9619
Trend	536,053.6101	0.2649	637.0032	6.6371	4.0466
Dummy	51,603.5575	0.4050	222.4738	2.2536	1.4218
Trigonometric L=4	212,370.1315	0.2595	438.3215	4.5357	2.7205
Trigonometric L=2	44,206.5273	0.2387	190.0999	1.9068	1.2270
Trigonometric L=1	51,890.4733	0.2788	218.7684	2.2128	1.4082
Autocorrelation L=4	238,541.0729	0.2571	459.5661	4.7583	2.8694
Autocorrelation L=2	47,309.2685	0.1737	200.8657	2.0163	1.2515
Autocorrelation L=1	54,223.7165	0.1536	224.2498	2.2551	1.3495
Decomp Multiplicative (12)	27,745.5615	0.6892	106.2035	1.0590	1.1441
Decomp Multiplicative (4)	27,628.9639	0.5532	108.6986	1.0845	0.8881
Decomp Additive (12)	21,639.9998	0.6601	100.6796	0.9948	0.9395
Simple Exponential Smoothing	48,153.6667	0.0156	107.1790	1.0266	0.8693
Holt's Trend	163,140.1613	0.1217	381.8633	3.9292	2.3732
Additive Holt-Winters (4)	157,595.7223	0.3326	373.5456	3.8572	2.3573
Additive Holt-Winters (12)	158,539.1445	0.4058	382.6627	3.9479	2.4170
Multiplicative Holt-Winters (4)	50,311.2099	0.2895	195.4310	1.9745	1.2971
Multiplicative Holt-Winters (12)	38,118.6885	0.1815	151.5916	1.4977	0.9518
ARIMA	45,428.0480	0.2459	144.5641	1.4232	1.0834

Appendix W. Comparison Chart item 3 - Blue skirt

	Value of Interest - Estimation					Value of Interest - Validation			
Model	SSE	R^2	MAPE	RAE	Theil's U	SSE	MAPE	RAE	Theil's U
Simple Linear Regression	502.4440	0.2137	44.3521	0.9358	0.8439	573.4400	42.7711	1.0975	1.0723
Trend	250.3510	0.6082	34.8023	0.7343	0.7996	28,644.2823	372.4648	9.5574	9.9782
Dummy	410.5986	0.3574	43.0572	0.9085	0.8662	415.1920	49.4728	1.2695	1.3639
Trigonometric L=4	253.9009	0.6026	34.7324	0.7329	0.8396	20,414.2258	334.1456	8.5741	8.9292
Trigonometric L=2	360.7186	0.4355	37.6946	0.7954	0.7642	381.5300	45.7100	1.1729	1.3032
Trigonometric L=1	443.0863	0.3066	42.0275	0.8868	0.8609	376.8282	47.2329	1.2120	1.2853
Autocorrelation L=4	263.1681	0.5543	35.2713	0.7442	0.7978	7,817.3647	212.9838	5.4651	5.6789
Autocorrelation L=2	343.8804	0.4047	37.8876	0.7994	0.7720	278.7449	39.8351	1.0222	1.0686
Autocorrelation L=1	382.1020	0.3196	40.9533	0.8641	0.8213	247.5666	37.1916	0.9543	0.9664
Decomp Multiplicative (12)	140.8522	0.7400	27.6862	0.5842	0.5453	173.8730	24.8787	0.6384	0.8324
Decomp Multiplicative (4)	154.8056	0.7140	26.7728	0.5649	0.5858	588.1420	43.4878	1.1159	1.1224
Decomp Additive (12)	153.2024	0.7721	28.1778	0.5946	0.6064	123.8891	20.5572	0.5275	0.6560
Simple Exponential Smoothing	440.2020	0.3577	40.9174	0.8634	0.8775	611.9805	69.9417	1.7947	2.0565
Holt's Trend	439.3703	0.3526	41.5026	0.8757	0.8683	639.8634	71.6709	1.8391	2.0997
Additive Holt-Winters (4)	414.8199	0.3967	41.0970	0.8672	0.8510	636.2813	71.2490	1.8282	2.1362
Additive Holt-Winters (12)	400.2739	0.4636	44.3323	0.9354	0.8264	581.7514	67.4500	1.7308	2.0476
Multiplicative Holt-Winters (4)	434.6556	0.3560	39.7718	0.8392	0.8235	406.5443	56.2093	1.4423	1.6269
Multiplicative Holt-Winters (12)	442.9849	0.4266	41.1808	0.8689	0.8689	359.9325	44.9914	1.1545	1.2545
ARIMA	409.8865	0.3032	44.4795	0.9385	0.8220	449.5028	49.0825	1.2594	1.1845

	Value of Interest - Consolidation				
Model	SSE	R2	MAPE	RAE	Theil's U
Simple Linear Regression	1,075.8840	0.2137	43.5616	1.0167	0.9581
Trend	28,894.6334	0.6082	203.6335	5.1459	5.3889
Dummy	825.7906	0.3574	46.2650	1.0890	1.1150
Trigonometric L=4	20,668.1268	0.6026	184.4390	4.6535	4.8844
Trigonometric L=2	742.2486	0.4355	41.7023	0.9841	1.0337
Trigonometric L=1	819.9145	0.3066	44.6302	1.0494	1.0731
Autocorrelation L=4	8,080.5328	0.5543	124.1275	3.1047	3.2384
Autocorrelation L=2	622.6253	0.4047	38.8613	0.9108	0.9203
Autocorrelation L=1	629.6686	0.3196	39.0725	0.9092	0.8938
Decomp Multiplicative (12)	314.7252	0.7400	26.2824	0.6113	0.6888
Decomp Multiplicative (4)	742.9477	0.7140	35.1303	0.8404	0.8541
Decomp Additive (12)	277.0915	0.7721	24.3675	0.5610	0.6312
Simple Exponential Smoothing	1,052.1825	0.3577	55.4296	1.3290	1.4670
Holt's Trend	1,079.2337	0.3526	56.5867	1.3574	1.4840
Additive Holt-Winters (4)	1,051.1011	0.3967	56.1730	1.3477	1.4936
Additive Holt-Winters (12)	982.0253	0.4636	55.8912	1.3331	1.4370
Multiplicative Holt-Winters (4)	841.1999	0.3560	47.9906	1.1408	1.2252
Multiplicative Holt-Winters (12)	802.9174	0.4266	43.0861	1.0117	1.0617
ARIMA	859.3893	0.3032	46.7810	1.0990	1.0033

Appendix X. Comparison Chart item 4 – Collar insignia

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	178,083.1390	0.0370	62.9399	1.0587	1.1896	34,272.4972	55.3809	0.9350	1.2474
Trend	156,003.6560	0.1564	50.8393	0.8551	1.0640	311,469.7628	203.3136	3.4324	4.7369
Dummy	17,922.5581	0.9031	22.0108	0.3702	0.4062	81,466.1691	92.2770	1.5579	2.3407
Trigonometric L=4	151,985.9733	0.1782	51.2252	0.8616	1.0632	301,529.0498	163.4760	2.7599	4.4615
Trigonometric L=2	120,672.5655	0.3475	41.6812	0.7011	0.8170	82,637.4551	97.4041	1.6444	2.0654
Trigonometric L=1	67,486.8556	0.6351	43.2646	0.7277	0.7318	84,934.5527	96.4357	1.6281	2.3917
Autocorrelation L=4	155,937.1321	0.1479	49.3485	0.8301	1.0398	152,658.6544	146.3363	2.4705	3.1890
Autocorrelation L=2	160,277.3269	0.1028	52.5260	0.8835	1.1061	87,800.4021	109.5055	1.8487	2.3639
Autocorrelation L=1	113,866.7868	0.3818	43.1271	0.7254	0.8513	131,849.2485	120.3927	2.0325	2.9232
Decomp Multiplicative (12)	38,320.2694	0.8911	25.4562	0.4282	0.5006	17,760.7445	28.4347	0.4800	0.8991
Decomp Multiplicative (4)	37,076.1157	0.6112	19.6826	0.3311	0.4847	26,615.1702	45.4218	0.7668	1.5681
Decomp Additive (12)	8,848.9984	0.9454	14.4849	0.2436	0.2630	12,431.1584	27.9786	0.4723	1.1359
Simple Exponential Smoothing	185,860.3333	0.0050	61.6664	1.0373	1.1742	34,172.1875	33.1378	0.5594	0.9899
Holt's Trend	181,755.0947	0.0035	63.5501	1.0689	1.1913	31,748.5822	39.3238	0.6639	1.0086
Additive Holt-Winters (4)	118,953.2570	0.7534	54.6122	0.9186	0.8710	33,136.4129	57.0529	0.9632	1.7487
Additive Holt-Winters (12)	21,049.5766	0.9677	23.2701	0.3914	0.3776	52,752.4121	79.2899	1.3386	1.8174
Multiplicative Holt-Winters (4)	81,319.6495	0.7911	38.7763	0.6522	0.6774	43,529.4136	71.3742	1.2050	1.6011
Multiplicative Holt-Winters (12)	43,928.9885	0.8380	35.8296	0.6027	0.6844	139,668.7630	142.4098	2.4042	3.0761
ARIMA	99,552.7665	0.7578	40.8374	0.6869	0.6769	15,734.5149	44.1594	0.7455	1.0260

	Value of Interest - Consolidation				
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	212,355.6362	0.0370	59.1604	0.9968	1.2185
Trend	467,473.4189	0.1564	127.0765	2.1438	2.9004
Dummy	99,388.7272	0.9031	57.1439	0.9640	1.3735
Trigonometric L=4	453,515.0231	0.1782	107.3506	1.8107	2.7624
Trigonometric L=2	203,310.0206	0.3475	69.5427	1.1728	1.4412
Trigonometric L=1	152,421.4083	0.6351	69.8501	1.1779	1.5617
Autocorrelation L=4	308,595.7865	0.1479	97.8424	1.6503	2.1144
Autocorrelation L=2	248,077.7290	0.1028	81.0158	1.3661	1.7350
Autocorrelation L=1	245,716.0353	0.3818	81.7599	1.3790	1.8873
Decomp Multiplicative (12)	56,081.0139	0.8911	26.9454	0.4541	0.6999
Decomp Multiplicative (4)	63,691.2859	0.6112	32.5522	0.5489	1.0264
Decomp Additive (12)	21,280.1568	0.9454	21.2318	0.3580	0.6994
Simple Exponential Smoothing	220,032.5208	0.0050	47.4021	0.7984	1.0821
Holt's Trend	213,503.6769	0.0035	51.4370	0.8664	1.0999
Additive Holt-Winters (4)	152,089.6700	0.7534	55.8326	0.9409	1.3099
Additive Holt-Winters (12)	73,801.9887	0.9677	51.2800	0.8650	1.0975
Multiplicative Holt-Winters (4)	124,849.0631	0.7911	55.0753	0.9286	1.1392
Multiplicative Holt-Winters (12)	183,597.7515	0.8380	89.1197	1.5034	1.8802
ARIMA	115,287.2814	0.7578	42.4984	0.7162	0.8514

Appendix Y. Comparison Chart item 5 – Plastic clip

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	6,759,823.7251	0.1900	55.1553	1.5113	1.2825	7,277,784.8491	218.1342	3.7857	4.2057
Trend	4,215,836.0706	0.4948	34.3461	0.9411	0.7517	59,845,267.5788	597.2427	10.3650	15.3005
Dummy	3,152,880.4701	0.6222	33.0500	0.9056	0.8073	26,895,801.0432	401.8407	6.9738	7.6933
Trigonometric L=4	4,134,000.6737	0.5046	36.2451	0.9932	0.8062	37,955,375.9207	471.1913	8.1774	11.3057
Trigonometric L=2	3,181,495.5696	0.6188	29.7605	0.8155	0.7451	33,482,395.2554	450.3551	7.8158	8.9800
Trigonometric L=1	3,903,635.0926	0.5322	35.3545	0.9688	0.8184	26,507,894.6664	402.7836	6.9902	7.8401
Autocorrelation L=4	3,695,848.2475	0.5629	30.9847	0.8490	0.7262	4,265,895.0047	148.5500	2.5780	3.9897
Autocorrelation L=2	3,943,796.6564	0.5498	32.9493	0.9029	0.7563	18,997,062.7351	350.0563	6.0751	6.8395
Autocorrelation L=1	3,577,021.5740	0.5878	30.3386	0.8313	0.7600	15,887,265.3786	311.4760	5.4056	5.7800
Decomp Multiplicative (12)	1,073,057.1883	0.8980	18.9967	0.5205	0.4366	701,006.8879	58.9151	1.0225	1.9763
Decomp Multiplicative (4)	1,277,406.4263	0.7585	18.9138	0.5183	0.5042	7,452,873.5745	220.9341	3.8342	4.2308
Decomp Additive (12)	999,121.6692	0.8492	19.9646	0.5471	0.5083	903,187.8203	64.5308	1.1199	2.2934
Simple Exponential Smoothing	5,515,115.1855	0.7868	38.7348	1.0614	0.8743	3,167,946.9982	144.8734	2.5142	2.8303
Holt's Trend	5,427,110.2215	0.8117	38.6266	1.0584	0.8632	4,366,712.5398	170.7601	2.9635	3.3921
Additive Holt-Winters (4)	5,288,576.8612	0.8869	43.4186	1.1897	1.0565	1,121,967.9768	84.4273	1.4652	1.8031
Additive Holt-Winters (12)	3,585,316.3740	0.9680	34.6073	0.9483	1.2525	3,004,938.3270	125.9986	2.1867	2.3464
Multiplicative Holt-Winters (4)	7,286,573.5038	0.9896	45.6213	1.2501	0.9448	2,573,966.2651	92.0774	1.5980	2.0527
Multiplicative Holt-Winters (12)	6,420,055.6414	0.7721	45.5458	1.2480	0.9582	2,500,668.0163	88.8064	1.5412	2.0073
ARIMA	4,745,252.7924	0.5962	40.0016	1.0961	0.8132	6,363,030.1069	205.0718	3.5590	4.0605

	Value of Interest - Consolidation				
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	14,037,608.5742	0.1900	136.6447	2.6485	2.7441
Trend	64,061,103.6494	0.4948	315.7944	5.6530	8.0261
Dummy	30,048,681.5133	0.6222	217.4453	3.9397	4.2503
Trigonometric L=4	42,089,376.5943	0.5046	253.7182	4.5853	6.0560
Trigonometric L=2	36,663,890.8249	0.6188	240.0578	4.3156	4.8625
Trigonometric L=1	30,411,529.7590	0.5322	219.0691	3.9795	4.3292
Autocorrelation L=4	7,961,743.2523	0.5629	89.7673	1.7135	2.3580
Autocorrelation L=2	22,940,859.3914	0.5498	191.5028	3.4890	3.7979
Autocorrelation L=1	19,464,286.9527	0.5878	170.9073	3.1184	3.2700
Decomp Multiplicative (12)	1,774,064.0762	0.8980	38.9559	0.7715	1.2065
Decomp Multiplicative (4)	8,730,280.0008	0.7585	119.9239	2.1763	2.3675
Decomp Additive (12)	1,902,309.4894	0.8492	42.2477	0.8335	1.4008
Simple Exponential Smoothing	8,683,062.1837	0.7868	91.8041	1.7878	1.8523
Holt's Trend	9,793,822.7613	0.8117	104.6933	2.0110	2.1277
Additive Holt-Winters (4)	6,410,544.8379	0.8869	63.9229	1.3275	1.4298
Additive Holt-Winters (12)	6,590,254.7010	0.9680	80.3029	1.5675	1.7995
Multiplicative Holt-Winters (4)	9,860,539.7689	0.9896	68.8493	1.4240	1.4988
Multiplicative Holt-Winters (12)	8,920,723.6576	0.7721	67.1761	1.3946	1.4827
ARIMA	11,108,282.8993	0.5962	122.5367	2.3275	2.4368

Appendix Z. Comparison Chart item 6 – White Air Force t-shirt

	Value of Interest - Estimation					Value of Interest - Validation			
Model	SSE	R^2	MAPE	RAE	Theil's U	SSE	MAPE	RAE	Theil's U
Simple Linear Regression	166,998.7028	0.3494	47.3551	1.0607	0.9778	13,528.4168	82.8485	1.0956	1.4751
Trend	141,545.7532	0.4485	39.7250	0.8898	0.7239	2,009,630.7776	1,342.1629	17.7495	29.6890
Dummy	83,841.9459	0.6733	32.8043	0.7348	0.9728	47,847.5331	111.9778	1.4809	2.9800
Trigonometric L=4	126,467.1089	0.5073	34.7310	0.7779	0.8478	111,270.8230	276.1695	3.6522	6.5183
Trigonometric L=2	129,757.8041	0.4945	36.5031	0.8176	0.7818	70,196.9046	184.3638	2.4381	5.0440
Trigonometric L=1	154,908.7043	0.3965	45.5561	1.0204	1.0445	31,970.5489	107.5402	1.4222	2.4224
Autocorrelation L=4	125,604.7601	0.4266	37.3697	0.8370	0.7096	619,539.5023	803.9248	10.6316	16.4761
Autocorrelation L=2	123,098.5275	0.4772	35.5409	0.7961	0.8022	57,632.9160	166.4128	2.2007	4.4637
Autocorrelation L=1	142,048.3589	0.4268	42.2526	0.9464	0.9262	24,462.0204	93.1640	1.2321	2.0569
Decomp Multiplicative (12)	36,250.2861	0.9406	15.5081	0.3474	0.4560	4,794.6801	40.3845	0.5341	0.5971
Decomp Multiplicative (4)	42,341.0756	0.6760	21.9317	0.4912	0.6494	13,900.4601	87.9897	1.1636	1.4963
Decomp Additive (12)	30,685.9768	0.7748	16.7176	0.3744	0.4421	8,370.1682	60.8385	0.8046	1.0685
Simple Exponential Smoothing	186,018.3287	0.4300	50.1969	1.1243	0.9333	28,044.2930	176.2001	2.3302	3.1482
Holt's Trend	177,631.4068	0.3418	49.8106	1.1157	0.9937	15,754.8123	105.4593	1.3947	1.8678
Additive Holt-Winters (4)	152,266.0794	0.5099	42.6997	0.9564	0.9684	22,783.0328	101.0333	1.3361	2.4623
Additive Holt-Winters (12)	74,470.6241	0.8426	29.7989	0.6674	0.6847	58,522.7549	199.9418	2.6441	2.9906
Multiplicative Holt-Winters (4)	143,913.6869	0.5826	40.2366	0.9012	0.8549	16,793.5545	115.5375	1.5279	2.0028
Multiplicative Holt-Winters (12)	134,427.2221	0.6117	49.4305	1.1072	0.9968	140,936.0633	426.5862	5.6414	7.8055
ARIMA	149,408.9338	0.5518	39.4446	0.8835	0.8435	65,741.9077	270.4882	3.5771	4.2719

	Value of Interest - Consolidation				
Model	SSE	R2	MAPE	RAE	Theil's U
Simple Linear Regression	180,527.1196	0.3494	65.1018	1.0782	1.2265
Trend	2,151,176.5308	0.4485	690.9439	9.3197	15.2065
Dummy	131,689.4790	0.6733	72.3911	1.1078	1.9764
Trigonometric L=4	237,737.9319	0.5073	155.4502	2.2151	3.6831
Trigonometric L=2	199,954.7087	0.4945	110.4334	1.6279	2.9129
Trigonometric L=1	186,879.2532	0.3965	76.5482	1.2213	1.7335
Autocorrelation L=4	745,144.2623	0.4266	420.6473	5.7343	8.5928
Autocorrelation L=2	180,731.4435	0.4772	100.9769	1.4984	2.6330
Autocorrelation L=1	166,510.3793	0.4268	67.7083	1.0892	1.4915
Decomp Multiplicative (12)	41,044.9662	0.9406	27.9463	0.4407	0.5265
Decomp Multiplicative (4)	56,241.5356	0.6760	54.9607	0.8274	1.0729
Decomp Additive (12)	39,056.1450	0.7748	38.7780	0.5895	0.7553
Simple Exponential Smoothing	214,062.6218	0.4300	113.1985	1.7272	2.0407
Holt's Trend	193,386.2190	0.3418	77.6350	1.2552	1.4307
Additive Holt-Winters (4)	175,049.1122	0.5099	71.8665	1.1463	1.7153
Additive Holt-Winters (12)	132,993.3790	0.8426	114.8703	1.6558	1.8376
Multiplicative Holt-Winters (4)	160,707.2414	0.5826	77.8870	1.2146	1.4289
Multiplicative Holt-Winters (12)	275,363.2854	0.6117	238.0084	3.3743	4.4011
ARIMA	215,150.8414	0.5518	154.9664	2.2303	2.5577

Appendix AA. Comparison Chart item 7 – Shoulder badge

	Value of Interest - Estimation					Value of Interest - Validation			
Model	SSE	R^2	MAPE	RAE	Theil's U	SSE	MAPE	RAE	Theil's U
Simple Linear Regression	243,985.8787	0.0400	44.4208	0.7677	0.7280	61,535.6017	30.8518	0.7713	0.8369
Trend	242,071.0198	0.0475	43.9598	0.7598	0.7260	94,865.3318	50.3575	1.2589	1.3031
Dummy	72,609.6100	0.7143	25.2008	0.4356	0.4104	64,142.1332	27.9183	0.6979	0.9033
Trigonometric L=4	233,360.5436	0.0818	42.8020	0.7398	0.7065	1,144,429.2561	154.2551	3.8562	4.2655
Trigonometric L=2	209,238.8388	0.1767	43.0581	0.7442	0.7140	40,620.6100	31.3289	0.7832	0.6911
Trigonometric L=1	139,236.2732	0.4521	39.2847	0.6790	0.6377	47,274.7886	28.6876	0.7172	0.9311
Autocorrelation L=4	237,747.7722	0.0501	42.7604	0.7391	0.7231	67,705.1222	41.6210	1.0405	1.0189
Autocorrelation L=2	232,128.7038	0.0707	42.0242	0.7263	0.7134	51,884.3969	32.4377	0.8109	0.7888
Autocorrelation L=1	189,717.0652	0.2465	41.6858	0.7205	0.6777	67,297.1982	33.3620	0.8340	0.9612
Decomp Multiplicative (12)	80,636.8468	0.7870	23.6300	0.4084	0.3714	56,011.4216	31.0192	0.7754	0.8293
Decomp Multiplicative (4)	111,357.6590	0.4894	30.5312	0.5277	0.4977	48,514.2020	27.7332	0.6933	0.8776
Decomp Additive (12)	74,191.8372	0.8084	25.5850	0.4422	0.3870	37,628.4740	25.4533	0.6363	0.7169
Simple Exponential Smoothing	258,519.3333	0.0172	43.2735	0.7479	0.7430	99,124.4167	27.6457	0.6911	1.0301
Holt's Trend	248,645.4262	0.0898	48.5777	0.8396	0.7416	57,878.0164	33.6052	0.8401	0.8630
Additive Holt-Winters (4)	199,100.3015	0.2761	37.1747	0.6425	0.6719	46,069.2778	29.4472	0.7361	0.8977
Additive Holt-Winters (12)	78,846.1174	0.7685	24.7234	0.4273	0.4118	58,357.1120	27.5891	0.6897	0.8630
Multiplicative Holt-Winters (4)	200,826.4337	0.2854	37.8603	0.6544	0.6817	56,785.0645	33.2777	0.8319	0.8654
Multiplicative Holt-Winters (12)	139,377.8479	0.5819	31.4893	0.5442	0.5510	56,849.7257	33.2995	0.8325	0.8650
ARIMA	178,445.3154	0.3667	37.8404	0.6540	0.6987	80,727.4228	27.0357	0.6759	0.9746

	Value of Interest - Consolidation				
Model	SSE	R2	MAPE	RAE	Theil's U
Simple Linear Regression	305,521.4804	0.0400	37.6363	0.7695	0.7824
Trend	336,936.3516	0.0475	47.1587	1.0093	1.0145
Dummy	136,751.7432	0.7143	26.5596	0.5667	0.6568
Trigonometric L=4	1,377,789.7997	0.0818	98.5286	2.2980	2.4860
Trigonometric L=2	249,859.4488	0.1767	37.1935	0.7637	0.7026
Trigonometric L=1	186,511.0618	0.4521	33.9861	0.6981	0.7844
Autocorrelation L=4	305,452.8944	0.0501	42.1907	0.8898	0.8710
Autocorrelation L=2	284,013.1007	0.0707	37.2310	0.7686	0.7511
Autocorrelation L=1	257,014.2635	0.2465	37.5239	0.7772	0.8195
Decomp Multiplicative (12)	136,648.2684	0.7870	27.3246	0.5919	0.6004
Decomp Multiplicative (4)	159,871.8611	0.4894	29.1322	0.6105	0.6877
Decomp Additive (12)	111,820.3112	0.8084	25.5192	0.5393	0.5519
Simple Exponential Smoothing	357,643.7500	0.0172	35.4596	0.7195	0.8865
Holt's Trend	306,523.4426	0.0898	41.0915	0.8398	0.8023
Additive Holt-Winters (4)	245,169.5793	0.2761	33.3110	0.6893	0.7848
Additive Holt-Winters (12)	137,203.2294	0.7685	26.1562	0.5585	0.6374
Multiplicative Holt-Winters (4)	257,611.4982	0.2854	35.5690	0.7431	0.7736
Multiplicative Holt-Winters (12)	196,227.5736	0.5819	32.3944	0.6883	0.7080
ARIMA	259,172.7381	0.3667	32.4381	0.6649	0.8366

Appendix AB. Comparison Chart item 8 – 2nd Sgt hat badge

	Value of Interest - Estimation					Value of Interest - Validation			
Model	SSE	R^2	MAPE	RAE	Theil's U	SSE	MAPE	RAE	Theil's U
Simple Linear Regression	122,118.2863	0.0086	53.6540	0.9253	0.8519	49,626.6597	54.7633	1.2286	1.2214
Trend	113,882.7282	0.0754	50.0680	0.8634	0.8447	116,821.7106	139.4782	3.1293	2.6099
Dummy	16,162.3791	0.8688	20.9824	0.3618	0.4021	10,732.4962	19.9366	0.4473	0.5266
Trigonometric L=4	114,235.3604	0.0726	49.8737	0.8601	0.8547	184,230.1943	171.0864	3.8384	3.2616
Trigonometric L=2	118,858.4312	0.0350	50.4712	0.8704	0.8368	45,863.3173	66.3979	1.4897	1.3496
Trigonometric L=1	92,990.8352	0.2450	42.8972	0.7398	0.7073	36,983.4121	34.8051	0.7809	0.8688
Autocorrelation L=4	114,471.0670	0.0610	48.1823	0.8309	0.8363	50,069.6148	76.0806	1.7069	1.5470
Autocorrelation L=2	115,595.2920	0.0473	46.5050	0.8020	0.8315	41,771.2196	62.1794	1.3950	1.2626
Autocorrelation L=1	92,940.2630	0.2278	44.2721	0.7635	0.7229	38,869.9456	38.9958	0.8749	0.8989
Decomp Multiplicative (12)	6,350.8080	0.9738	14.4510	0.2492	0.2103	6,299.9209	14.3844	0.3227	0.3311
Decomp Multiplicative (4)	16,208.3192	0.5957	20.2874	0.3499	0.3489	36,813.7337	42.0547	0.9435	1.2297
Decomp Additive (12)	5,430.8035	0.9302	14.3396	0.2473	0.2091	2,674.2769	11.7703	0.2641	0.2058
Simple Exponential Smoothing	123,228.1667	0.0004	54.9055	0.9468	0.8595	53,131.7500	47.0149	1.0548	1.1620
Holt's Trend	133,463.4292	0.0672	63.9944	1.1036	0.9106	48,091.0753	68.5539	1.5380	1.4181
Additive Holt-Winters (4)	78,805.0983	0.6467	48.5222	0.8368	0.7936	42,329.2215	77.7588	1.7446	1.8239
Additive Holt-Winters (12)	20,295.3361	0.9198	26.6217	0.4591	0.4448	7,401.2726	28.8329	0.6469	0.5897
Multiplicative Holt-Winters (4)	45,420.3431	0.6944	38.7750	0.6687	0.6260	53,287.9568	84.7765	1.9020	1.7521
Multiplicative Holt-Winters (12)	32,560.7622	0.7982	29.7915	0.5138	0.5559	47,458.6367	54.1467	1.2148	1.2036
ARIMA	23,575.4462	0.9368	30.8236	0.5316	0.4932	12,046.6397	22.6002	0.5071	0.5754

	Value of Interest - Consolidation				
Model	SSE	R2	MAPE	RAE	Theil's U
Simple Linear Regression	171,744.9461	0.0086	54.2086	1.0770	1.0366
Trend	230,704.4388	0.0754	94.7731	1.9964	1.7273
Dummy	26,894.8753	0.8688	20.4595	0.4046	0.4644
Trigonometric L=4	298,465.5546	0.0726	110.4800	2.3493	2.0581
Trigonometric L=2	164,721.7484	0.0350	58.4345	1.1800	1.0932
Trigonometric L=1	129,974.2473	0.2450	38.8511	0.7603	0.7881
Autocorrelation L=4	164,540.6818	0.0610	62.1315	1.2689	1.1916
Autocorrelation L=2	157,366.5116	0.0473	54.3422	1.0985	1.0470
Autocorrelation L=1	131,810.2087	0.2278	41.6339	0.8192	0.8109
Decomp Multiplicative (12)	12,650.7290	0.9738	14.4177	0.2860	0.2707
Decomp Multiplicative (4)	53,022.0530	0.5957	31.1711	0.6467	0.7893
Decomp Additive (12)	8,105.0805	0.9302	13.0549	0.2557	0.2074
Simple Exponential Smoothing	176,359.9167	0.0004	50.9602	1.0008	1.0108
Holt's Trend	181,554.5045	0.0672	66.2742	1.3208	1.1643
Additive Holt-Winters (4)	121,134.3198	0.6467	63.1405	1.2907	1.3088
Additive Holt-Winters (12)	27,696.6087	0.9198	27.7273	0.5530	0.5173
Multiplicative Holt-Winters (4)	98,708.2999	0.6944	61.7757	1.2853	1.1891
Multiplicative Holt-Winters (12)	80,019.3989	0.7982	41.9691	0.8643	0.8797
ARIMA	35,622.0859	0.9368	26.7119	0.5193	0.5343

Appendix AC. Comparison Chart item 9 – Blue jacket for men

	Value of Interest - Estimation					Value of Interest - Validation			
Model	SSE	R^2	MAPE	RAE	Theil's U	SSE	MAPE	RAE	Theil's U
Simple Linear Regression	8,583.1092	0.0154	65.2415	1.7233	1.1768	2,669.0393	43.7833	1.0471	0.9893
Trend	8,234.8861	0.0553	64.0876	1.6928	1.1955	45,341.8294	186.5106	4.4605	4.0121
Dummy	1,719.3357	0.8028	24.9549	0.6592	0.5321	1,222.7126	31.4393	0.7519	0.7964
Trigonometric L=4	6,406.8947	0.2650	51.0190	1.3476	1.0354	373,008.2932	584.2174	13.9718	11.6300
Trigonometric L=2	4,258.1488	0.5115	38.0557	1.0052	0.7450	1,061.7627	31.0383	0.7423	0.6808
Trigonometric L=1	2,483.8237	0.7151	30.3559	0.8018	0.6249	1,275.8856	42.5682	1.0180	0.9510
Autocorrelation L=4	4,714.2522	0.4328	37.4115	0.9882	0.7820	5,082.3913	58.3545	1.3956	1.5071
Autocorrelation L=2	4,734.7360	0.4229	38.1670	1.0081	0.7837	1,447.6790	37.8833	0.9060	0.8650
Autocorrelation L=1	2,522.1517	0.7199	23.7504	0.6273	0.5809	950.5044	33.4478	0.7999	0.7991
Decomp Multiplicative (12)	1,281.2826	0.9072	14.7148	0.3887	0.4163	462.9488	19.1323	0.4576	0.4160
Decomp Multiplicative (4)	1,324.0467	0.7589	16.2179	0.4284	0.4169	2,615.9737	43.9042	1.0500	0.9841
Decomp Additive (12)	1,221.0226	0.8754	15.0942	0.3987	0.3821	522.7751	21.2621	0.5085	0.4759
Simple Exponential Smoothing	6,370.5491	0.9777	43.7009	1.1543	0.9956	4,618.8080	48.3498	1.1563	1.2317
Holt's Trend	6,598.4848	0.9697	45.0791	1.1907	1.0089	5,868.9148	57.4765	1.3746	1.3976
Additive Holt-Winters (4)	5,645.4845	0.9936	43.7804	1.1564	0.9124	3,982.1497	47.2925	1.1310	1.1451
Additive Holt-Winters (12)	2,148.7396	0.8984	27.6833	0.7312	0.6688	2,335.6231	42.7848	1.0232	0.9156
Multiplicative Holt-Winters (4)	6,544.1526	0.9967	46.9108	1.2391	0.9303	7,360.7715	65.8260	1.5743	1.5981
Multiplicative Holt-Winters (12)	3,064.4539	0.5808	34.6338	0.9148	0.7757	2,260.7062	46.7154	1.1172	0.9825
ARIMA	4,334.2485	0.5730	38.2070	1.0092	0.7351	1,289.7386	34.3048	0.8204	0.7591

	Value of Interest - Consolidation				
Model	SSE	R2	MAPE	RAE	Theil's U
Simple Linear Regression	11,252.1485	0.0154	54.5124	1.3852	1.0831
Trend	53,576.7155	0.0553	125.2991	3.0766	2.6038
Dummy	2,942.0483	0.8028	28.1971	0.7055	0.6643
Trigonometric L=4	379,415.1880	0.2650	317.6182	7.6597	6.3327
Trigonometric L=2	5,319.9115	0.5115	34.5470	0.8737	0.7129
Trigonometric L=1	3,759.7093	0.7151	36.4621	0.9099	0.7880
Autocorrelation L=4	9,796.6435	0.4328	47.8830	1.1919	1.1445
Autocorrelation L=2	6,182.4151	0.4229	38.0252	0.9571	0.8244
Autocorrelation L=1	3,472.6561	0.7199	28.5991	0.7136	0.6900
Decomp Multiplicative (12)	1,744.2315	0.9072	16.9236	0.4231	0.4161
Decomp Multiplicative (4)	3,940.0204	0.7589	30.0610	0.7392	0.7005
Decomp Additive (12)	1,743.7977	0.8754	18.1782	0.4536	0.4290
Simple Exponential Smoothing	10,989.3570	0.9777	46.0253	1.1553	1.1137
Holt's Trend	12,467.3997	0.9697	51.2778	1.2826	1.2033
Additive Holt-Winters (4)	9,627.6342	0.9936	45.5365	1.1437	1.0288
Additive Holt-Winters (12)	4,484.3626	0.8984	35.2341	0.8772	0.7922
Multiplicative Holt-Winters (4)	13,904.9241	0.9967	56.3684	1.4067	1.2642
Multiplicative Holt-Winters (12)	5,325.1601	0.5808	40.6746	1.0160	0.8791
ARIMA	5,623.9871	0.5730	36.2559	0.9148	0.7471

Appendix AD. Comparison Chart item 10 – ABU Blouse

	Value of Interest - Estimation					Value of Interest - Validation			
Model	SSE	R^2	MAPE	RAE	Theil's U	SSE	MAPE	RAE	Theil's U
Simple Linear Regression	16,834.3778	0.0173	131.6995	2.0100	1.4977	6,121.0272	50.5994	1.0981	1.1316
Trend	16,646.8607	0.0283	128.2131	1.9568	1.5081	13,583.6249	83.0486	1.8023	1.5736
Dummy	3,667.3264	0.7859	43.2009	0.6593	0.5553	2,816.2674	35.1654	0.7631	0.8919
Trigonometric L=4	15,068.4286	0.1204	102.8529	1.5697	1.1301	268,767.7474	417.1848	9.0536	7.1026
Trigonometric L=2	7,803.5545	0.5445	69.6710	1.0633	0.8070	4,695.4063	25.1560	0.5459	0.9041
Trigonometric L=1	4,884.4140	0.7149	57.4867	0.8774	0.7067	3,026.8218	38.9244	0.8447	1.0542
Autocorrelation L=4	10,339.9083	0.3984	79.6561	1.2157	0.8748	3,935.2342	40.1252	0.8708	0.9832
Autocorrelation L=2	10,163.2518	0.4021	78.2933	1.1949	0.8485	4,798.8331	42.6196	0.9249	1.0438
Autocorrelation L=1	6,270.3484	0.6240	51.3895	0.7843	0.6359	2,479.0178	33.7642	0.7327	0.8826
Decomp Multiplicative (12)	1,776.1344	0.9096	28.0654	0.4283	0.4598	1,289.8922	23.1463	0.5023	0.5252
Decomp Multiplicative (4)	2,073.7732	0.7126	32.8602	0.5015	0.4886	6,305.7494	53.7903	1.1673	1.3122
Decomp Additive (12)	1,978.9871	0.9175	28.8880	0.4409	0.4500	1,024.9370	21.7210	0.4714	0.4774
Simple Exponential Smoothing	17,318.8333	0.0110	118.6167	1.8103	1.2711	8,125.2500	47.0498	1.0211	1.1775
Holt's Trend	13,137.5871	0.9995	75.8045	1.1569	0.9916	8,874.4770	45.0760	0.9782	1.2179
Additive Holt-Winters (4)	10,727.6408	0.9985	74.7459	1.1408	0.8021	7,525.4320	48.4634	1.0517	1.3129
Additive Holt-Winters (12)	3,745.7468	0.7709	44.2838	0.6759	0.5381	3,643.0433	36.9760	0.8024	0.9301
Multiplicative Holt-Winters (4)	8,912.7125	0.9421	67.8780	1.0359	0.8709	7,656.8308	46.6060	1.0114	1.1508
Multiplicative Holt-Winters (12)	8,922.7600	0.6929	64.5787	0.9856	0.8214	6,022.3201	49.3047	1.0700	1.1164
ARIMA	6,529.8188	0.7340	73.4996	1.1217	0.5480	2,436.7480	34.2938	0.7442	0.9536

	Value of Interest - Consolidation				
Model	SSE	R2	MAPE	RAE	Theil's U
Simple Linear Regression	22,955.4051	0.0173	91.1495	1.5540	1.3147
Trend	30,230.4856	0.0283	105.6309	1.8795	1.5408
Dummy	6,483.5939	0.7859	39.1831	0.7112	0.7236
Trigonometric L=4	283,836.1760	0.1204	260.0188	5.3117	4.1163
Trigonometric L=2	12,498.9608	0.5445	47.4135	0.8046	0.8555
Trigonometric L=1	7,911.2358	0.7149	48.2055	0.8610	0.8805
Autocorrelation L=4	14,275.1425	0.3984	59.8907	1.0432	0.9290
Autocorrelation L=2	14,962.0849	0.4021	60.4564	1.0599	0.9462
Autocorrelation L=1	8,749.3662	0.6240	42.5768	0.7585	0.7592
Decomp Multiplicative (12)	3,066.0266	0.9096	25.6059	0.4653	0.4925
Decomp Multiplicative (4)	8,379.5226	0.7126	43.3252	0.8344	0.9004
Decomp Additive (12)	3,003.9241	0.9175	25.3045	0.4561	0.4637
Simple Exponential Smoothing	25,444.0833	0.0110	82.8332	1.4157	1.2243
Holt's Trend	22,012.0641	0.9995	60.4402	1.0676	1.1047
Additive Holt-Winters (4)	18,253.0727	0.9985	61.6046	1.0962	1.0575
Additive Holt-Winters (12)	7,388.7901	0.7709	40.6299	0.7391	0.7341
Multiplicative Holt-Winters (4)	16,569.5433	0.9421	57.2420	1.0237	1.0109
Multiplicative Holt-Winters (12)	14,945.0801	0.6929	56.9417	1.0278	0.9689
ARIMA	8,966.5667	0.7340	53.8967	0.9330	0.7508

Appendix AE. Comparison Chart item 11 – Blue hat for men

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	15,838.5353	0.0029	52.7267	1.1055	0.6799	5,992.5083	53.1060	0.9004	1.2405
Trend	8,234.8861	0.7008	38.9553	0.8167	0.8452	70,544.1074	205.1681	3.4787	5.6387
Dummy	1,719.3357	1.1110	45.3691	0.9512	0.9631	11,235.9773	52.4756	0.8897	1.0437
Trigonometric L=4	27,061.2182	0.8159	49.3381	1.0344	0.8138	429,566.6783	556.1340	9.4295	15.9337
Trigonometric L=2	27,058.3966	0.9512	42.9349	0.9002	0.9475	11,314.3184	47.4485	0.8045	1.1569
Trigonometric L=1	28,145.8315	1.0629	44.5095	0.9332	0.8866	9,400.4283	43.9021	0.7444	0.9949
Autocorrelation L=4	26,149.1783	0.9045	44.0010	0.9225	0.9360	15,207.8721	67.5936	1.1461	1.8807
Autocorrelation L=2	26,504.5980	0.8932	46.1894	0.9684	0.9175	8,184.2944	45.0449	0.7638	1.0158
Autocorrelation L=1	27,311.9959	1.0418	44.4033	0.9310	0.9263	7,884.9352	40.5796	0.6880	0.9192
Decomp Multiplicative (12)	5,701.3031	0.6367	24.0411	0.5040	0.5177	1,946.8608	36.3505	0.6163	1.3337
Decomp Multiplicative (4)	6,746.3136	0.4731	27.3152	0.5727	0.5954	6,402.7271	58.3789	0.9898	1.3530
Decomp Additive (12)	5,594.2274	0.6378	26.8019	0.5619	0.4947	1,795.7828	34.9325	0.5923	1.3046
Simple Exponential Smoothing	25,604.7837	0.9763	48.0019	1.0064	0.9987	5,350.1223	60.2940	1.0223	1.4849
Holt's Trend	25,235.3253	0.9582	46.5861	0.9767	1.0060	5,118.0623	54.4823	0.9238	1.2618
Additive Holt-Winters (4)	21,677.2073	1.0071	49.4690	1.0372	0.9280	6,441.9200	63.0073	1.0683	1.4902
Additive Holt-Winters (12)	9,744.7167	0.5986	34.7246	0.7280	0.6887	3,806.1216	47.4323	0.8042	0.9265
Multiplicative Holt-Winters (4)	21,217.0710	0.7927	46.2959	0.9706	0.9706	19,850.0581	70.4504	1.1945	1.6931
Multiplicative Holt-Winters (12)	19,056.9149	0.6365	50.6747	1.0624	0.7654	14,158.6474	55.3426	0.9384	1.2424
ARIMA	14,080.4788	0.1432	47.3516	0.9928	0.5911	5,701.8865	53.2391	0.9027	1.2384

	Value of Interest - Consolidation				
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**
Simple Linear Regression	21,831.0436	0.0029	52.9164	1.0030	0.9602
Trend	78,778.9935	0.7008	122.0617	2.1477	3.2420
Dummy	12,955.3131	1.1110	48.9224	0.9205	1.0034
Trigonometric L=4	456,627.8965	0.8159	302.7360	5.2320	8.3738
Trigonometric L=2	38,372.7150	0.9512	45.1917	0.8523	1.0522
Trigonometric L=1	37,546.2598	1.0629	44.2058	0.8388	0.9407
Autocorrelation L=4	41,357.0503	0.9045	55.7973	1.0343	1.4083
Autocorrelation L=2	34,688.8924	0.8932	45.6172	0.8661	0.9667
Autocorrelation L=1	35,196.9311	1.0418	42.4915	0.8095	0.9227
Decomp Multiplicative (12)	7,648.1639	0.6367	30.1958	0.5602	0.9257
Decomp Multiplicative (4)	13,149.0407	0.4731	42.8471	0.7813	0.9742
Decomp Additive (12)	7,390.0102	0.6378	30.8672	0.5771	0.8996
Simple Exponential Smoothing	30,954.9060	0.9763	54.1480	1.0144	1.2418
Holt's Trend	30,353.3876	0.9582	50.5342	0.9502	1.1339
Additive Holt-Winters (4)	28,119.1273	1.0071	56.2382	1.0527	1.2091
Additive Holt-Winters (12)	13,550.8383	0.5986	41.0785	0.7661	0.8076
Multiplicative Holt-Winters (4)	41,067.1291	0.7927	58.3732	1.0826	1.3318
Multiplicative Holt-Winters (12)	33,215.5623	0.6365	53.0086	1.0004	1.0039
ARIMA	19,782.3652	0.1432	50.2953	0.9477	0.9147

Appendix AF. Comparison Chart - Algorithm – ABU Coat

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Autocorrelation L=1	143,664.6818	0.2391	27.7642	0.8114	0.7745	60,878.9861	29.3968	1.1503	0.7823
Decomp Multiplicative (12)	42,722.4338	0.7972	12.5549	0.3669	0.3856	21,304.7297	14.0459	0.5496	0.3610
Decomp Multiplicative (4)	47,900.0403	0.6755	15.7933	0.4616	0.4368	124,907.0920	31.5263	1.2336	1.0688
Decomp Additive (12)	36,872.2926	0.7457	12.7519	0.3727	0.3657	23,430.1236	13.6031	0.5323	0.4523
Additive Holt-Winters (12)	124,011.4992	0.7283	24.0267	0.7022	0.7156	51,385.1746	29.8411	1.1676	0.7040

	Value of Interest - Consolidation					
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**	**$ Error**
Autocorrelation L=1	204,543.6679	0.2391	28.5805	0.9808	0.7784	$4,037.15
Decomp Multiplicative (12)	64,027.1635	0.7972	13.3004	0.4583	0.3733	$790.30
Decomp Multiplicative (4)	172,807.1323	0.6755	23.6598	0.8476	0.7528	$10,323.66
Decomp Additive (12)	60,302.4162	0.7457	13.1775	0.4525	0.4090	$1,093.93
Additive Holt-Winters (12)	175,396.6739	0.7283	26.9339	0.9349	0.7098	$3,213.44

Appendix AG. Comparison Chart - Algorithm – ABU Hat

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Autocorrelation L=1	37,224.0091	0.2204	26.5511	0.8185	0.7963	30,729.0974	40.7927	1.0527	0.8315
Decomp Multiplicative (12)	12,969.2681	0.8964	11.6419	0.3589	0.3995	6,970.8937	16.3746	0.4225	0.3407
Decomp Multiplicative (4)	11,863.3891	0.6434	14.0654	0.4336	0.4231	34,501.0401	25.2223	0.6509	0.9047
Decomp Additive (12)	11,985.4236	0.8706	12.2635	0.3781	0.4157	8,320.1532	18.0138	0.4648	0.4009
Additive Holt-Winters (12)	30,475.1226	0.7236	22.4851	0.6932	0.6436	17,103.3649	27.2600	0.7034	0.6453

	Value of Interest - Consolidation					
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**	**$ Error**
Autocorrelation L=1	67,953.1065	0.2204	33.6719	0.9356	0.8139	$767.65
Decomp Multiplicative (12)	19,940.1618	0.8964	14.0083	0.3907	0.3701	$138.42
Decomp Multiplicative (4)	46,364.4292	0.6434	19.6439	0.5422	0.6639	$1,373.13
Decomp Additive (12)	20,305.5767	0.8706	15.1386	0.4214	0.4083	$123.28
Additive Holt-Winters (12)	47,578.4875	0.7236	24.8725	0.6983	0.6444	$75.69

Appendix AH. Comparison Chart - Algorithm – ABU Trousers

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Autocorrelation L=1	147,741.3532	0.3582	23.5397	0.7851	0.7607	50,703.1608	21.0135	1.0904	0.8124
Decomp Multiplicative (12)	43,979.3477	0.8074	12.0671	0.4025	0.4005	13,153.7404	8.3516	0.4334	0.3562
Decomp Multiplicative (4)	44,307.0878	0.7312	13.5641	0.4524	0.4554	202,799.2429	42.0593	2.1825	1.6367
Decomp Additive (12)	37,815.9238	0.7807	12.2982	0.4102	0.3863	13,830.1117	8.8306	0.4582	0.4154
Additive Holt-Winters (12)	141,276.4346	0.7387	23.2832	0.7765	0.8120	26,080.9885	14.4443	0.7495	0.6149

	Value of Interest - Consolidation					
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**	**$ Error**
Autocorrelation L=1	198,444.5140	0.3582	22.2766	0.9378	0.7865	$4,550.50
Decomp Multiplicative (12)	57,133.0881	0.8074	10.2093	0.4179	0.3783	$650.57
Decomp Multiplicative (4)	247,106.3307	0.7312	27.8117	1.3174	1.0461	$14,918.88
Decomp Additive (12)	51,646.0355	0.7807	10.5644	0.4342	0.4009	$760.22
Additive Holt-Winters (12)	167,357.4230	0.7387	18.8638	0.7630	0.7134	$256.48

Appendix AI. Comparison Chart - Algorithm – ABU T-shirt

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Autocorrelation L=1	544,063.4041	0.4119	29.6661	0.9198	0.7240	219,552.5678	20.0132	1.3767	0.9596
Decomp Multiplicative (12)	173,120.6473	0.8747	13.8288	0.4288	0.3795	145,757.1604	9.3761	0.6450	0.6349
Decomp Multiplicative (4)	193,023.7446	0.7425	16.3079	0.5056	0.4626	270,158.0473	17.9865	1.2373	0.9983
Decomp Additive (12)	147,940.4664	0.8070	13.4708	0.4177	0.3401	112,552.7080	8.5792	0.5901	0.5337
Additive Holt-Winters (12)	481,045.1449	0.7285	25.5104	0.7910	0.6604	156,348.0989	12.9570	0.8913	0.7026

	Value of Interest - Consolidation					
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**	**$ Error**
Autocorrelation L=1	763,615.9719	0.4119	24.8396	1.1482	0.8418	$4,102.45
Decomp Multiplicative (12)	318,877.8077	0.8747	11.6025	0.5369	0.5072	$471.08
Decomp Multiplicative (4)	463,181.7919	0.7425	17.1472	0.8714	0.7305	$4,935.97
Decomp Additive (12)	260,493.1744	0.8070	11.0250	0.5039	0.4369	$877.23
Additive Holt-Winters (12)	637,393.2439	0.7285	19.2337	0.8411	0.6815	$56.40

Appendix AJ. Comparison Chart - Algorithm – Black buckle

	Value of Interest - Estimation					Value of Interest - Validation			
Model	SSE	R^2	MAPE	RAE	Theil's U	SSE	MAPE	RAE	Theil's U
Autocorrelation L=1	265,463.1593	0.0430	36.8093	0.7401	0.7257	90,089.6582	30.9087	0.7626	0.7292
Decomp Multiplicative (12)	95,122.8254	0.7364	17.7826	0.3575	0.4748	54,099.4921	22.8294	0.5633	0.5138
Decomp Multiplicative (4)	119,714.3125	0.4380	23.7674	0.4779	0.5054	111,063.9025	27.2108	0.6714	0.8248
Decomp Additive (12)	93,992.2748	0.6991	18.5827	0.3736	0.4827	48,231.1973	20.7908	0.5130	0.5191
Additive Holt-Winters (12)	154,746.5648	0.4512	24.7457	0.4975	0.5680	96,889.8264	26.2674	0.6481	0.7794

	Value of Interest - Consolidation					
Model	SSE	R2	MAPE	RAE	Theil's U	$ Error
Autocorrelation L=1	355,552.8174	0.0430	33.8590	0.7513	0.7275	$223.90
Decomp Multiplicative (12)	149,222.3174	0.7364	20.3060	0.4604	0.4943	$188.80
Decomp Multiplicative (4)	230,778.2150	0.4380	25.4891	0.5746	0.6651	$678.95
Decomp Additive (12)	142,223.4721	0.6991	19.6867	0.4433	0.5009	$121.70
Additive Holt-Winters (12)	251,636.3912	0.4512	25.5066	0.5728	0.6737	$612.25

Appendix AK. Comparison Chart - Algorithm – Black sock

	Value of Interest - Estimation					Value of Interest - Validation			
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**SSE**	**MAPE**	**RAE**	**Theil's U**
Autocorrelation L=1	1,489,612.1770	0.4877	100.0266	1.2945	0.4607	2,646,926.4970	86.7657	1.5557	1.8084
Decomp Multiplicative (12)	605,944.5324	0.8171	53.1833	0.6883	0.3652	791,549.7676	28.2014	0.5057	0.3596
Decomp Multiplicative (4)	746,475.1510	0.6806	60.9956	0.7894	0.3795	2,626,441.6307	61.9317	1.1104	1.1248
Decomp Additive (12)	662,507.9047	0.8111	65.6562	0.8497	0.5039	762,306.3520	29.2570	0.5246	0.4212
Additive Holt-Winters (12)	1,307,495.5068	0.4875	143.5098	1.8572	0.4613	3,832,414.8177	76.6968	1.3752	1.3646

	Value of Interest - Consolidation					
Model	**SSE**	**R2**	**MAPE**	**RAE**	**Theil's U**	**$ Error**
Autocorrelation L=1	4,136,538.6740	0.4877	93.3961	1.4251	1.1345	$506.21
Decomp Multiplicative (12)	1,397,494.3000	0.8171	40.6924	0.5970	0.3624	$692.10
Decomp Multiplicative (4)	3,372,916.7818	0.6806	61.4637	0.9499	0.7522	$2,012.37
Decomp Additive (12)	1,424,814.2567	0.8111	47.4566	0.6871	0.4625	$821.68
Additive Holt-Winters (12)	5,139,910.3245	0.4875	110.1033	1.6162	0.9129	$1,992.84

Appendix AL. Thesis quad chart

AN EVALUATION OF FORECASTING METHODS THAT COULD BE USED IN THE BRAZILIAN AIR FORCE UNIFORM DISTRIBUTION PROCESS

INTRODUCTION

Every year the Brazilian Air Force (BAF) spends the equivalent of approximately 15 millions of dollars for uniforms. The Sub-directorate of Supply (SDS) is the main unit in the BAF responsible for forecasting, purchasing, and distributing uniforms.

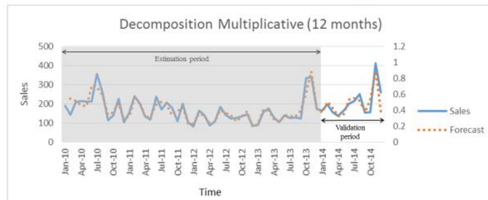
THE PROBLEM

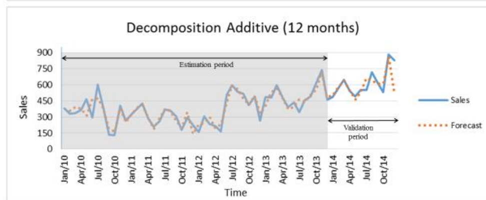
Uniform purchases in BAF are executed through public procurement processes, according to Brazilian acquisition regulations. For this reason, lead times are unpredictable, taking anywhere from one month to a year to replenish an item. It was also detected that the volume of sales and inventory levels were incompatible.

METHODOLOGY

Monthly sales, prices, and inventory records from January of 2010 to July of 2015 were extracted from a database and converted to a standard spreadsheet format. Several forecasting models were evaluated and applied to randomly selected items from the database to create the algorithm.

Capt. Leandro Valviesse de Oliveira
Advisor: Dr. Jeffrey Ogden
Reader: Dr. Michael Kretser
Department of Operational Sciences (ENS)
Air Force Institute of Technology





	Value of Interest – Consolidation					
Model	**SSE**	**R^2**	**MAPE**	**RAE**	**Theil's U**	**Cost of the error**
Autocorrelation L=1 year	204,543.6679	0.2391	28.5805	0.9808	0.7784	$4,037.15
Decomposition Multiplicative (12 months)	64,027.1635	0.7972	13.3004	0.4583	0.3733	$790.30
Decomposition Multiplicative (4 months)	172,807.1323	0.6755	23.6598	0.8476	0.7528	$10,323.66
Decomposition Additive (12 months)	60,302.4162	0.7457	13.1775	0.4525	0.4090	$1,093.93
Additive Holt-Winters (12 months)	175,396.6739	0.7283	26.9339	0.9349	0.7098	$3,213.44

RESEARCH OBJECTIVES

1. Understand what metrics are currently being utilized by the Supply Division at SDS;
2. Study how the forecasting process for uniform sales can be improved by the SDS;
3. Establish how to best assess accuracy of forecasting sales in SDS;
4. Assess the possibility to build an algorithm where historical sales data can be evaluated and the best forecast suggested.

Results and Conclusions

1. All costs in SDS are considered organization-wide and do not affect decisions over the purchasing process;
2. Forecasting system in place at SDS has been currently performed based on empirical inferences;
3. Decomposition Multiplicative (12-month season) and Decomposition Additive (12-month season) forecasting models clearly captured the behavior of sales in BAF's stores;
4. Successfully built an algorithm for improving forecast system, optimizing the overall inventory management in SDS;
5. Areas for further research include inventory management policies, implementation of supply chain-related metrics.

DEPARTMENT OF OPERATIONAL SCIENCES

Bibliography

Armstrong, J. (2001). Combining forecasts. *Principles of Forecasting: A Handbook for Researchers and Practitioners*, 1–19.

Armstrong, J. (2002). *Principles of Forecasting: A Handbook for Researchers and Practitioners*. Kluwer Academic Publishers.

Armstrong, J. (2007). Significance tests harm progress in forecasting. *International Journal of Forecasting*.

Armstrong, J. S. (2001). Evaluating Forecasting Methods. In *Principles of Forecasting: A Handbook for Researchers and Practitioners*. Kluwer Academic Publishers.

Arraes, K. G. G. (2011). A Comparative Study between U.S. and Brazilian Acquisition Regulations and Practices, 135.

Bowerman, B. L., Connell, R. T. O., & Koehler, A. B. (2005). Forecasting, time series, and regression: an applied approach. Belmont, CA : Thomson Brooks/Cole,.

Chatfield, C. (2000). *Time series Forecasting*. CRC Press.

Costantino, F., Di Gravio, G., Shaban, A., & Tronci, M. (2014). Smoothing Inventory Decision Rules in Seasonal Supply Chains. *Expert Systems with Applications*, *44*, 304–319.

Costantino, F., Di Gravio, G., Shaban, A., & Tronci, M. (2015). SPC Forecasting System to Mitigate the Bullwhip Effect and Inventory Variance in Supply Chains. *Expert Systems with Applications*, *42*(3), 1773–1787.

Downing, M., Chipulu, M., Ojiako, U., & Kaparis, K. (2011). Forecasting in Airforce Supply Chains. *The International Journal of Logistics Management*, *22*(1), 127–144.

Eppen, G. D., & Martin, R. K. (1988). Determining Safety Stock in the Presence of Stochastic Lead Time and Demand. *Management Science*, *34*(11), 1380–1390.

Gardner, E. S. (1990). Evaluating Forecast Performance in an Inventory Control System. *Management Science*, *36*(4), 490–499.

Graefe, A., Küchenhoff, H., Stierle, V., & Riedl, B. (2015). Limitations of Ensemble Bayesian Model Averaging for forecasting social science problems. *International Journal of Forecasting*, *31*(3), 943–951.

Green, K. C., & Armstrong, J. S. (2015). Simple Versus Complex Forecasting: The Evidence. *Journal of Business Research*, *68*(8), 1678–1685.

Hoover, J. (2009). How To Track Forecast Accuracy To Guide Forecast. *Foresight*, *summer*(14), 17–24.

Hyndman, R. J., & Athanasopoulos, G. (2014). *Forecasting: Principles and Practice*.

Kaplan, R. S., & Cooper, R. (1997). *Cost and effect: using integrated cost system to drive profitability and performance.* Harvard Business Press.

Kostenko, A. V, & Hyndman, R. J. (2008). Forecasting without significance tests?, (1995), 1–5.

Lee, H. L., Padmanabhan, V., & Whang, S. (1997). The Bullwhip Effect in Supply Chains. *Sloan Management Review*, *38*(3), 93–102.

Lee, T. S., & Adam, E. E. (1986). Forecasting Error Evaluation in Material Requirements Planning (MRP) Production-Inventory Systems. *Management Science*, *32*(9), 1186–1205.

Leenders, M. R., Flynn, A., & Johnson, P. F. (2010). *Purchasing and supply management*. McGraw-Hill Education.

Levitt, S. D., & Dubner, S. J. (2006). *Freakonomics: A Rogue Economist Explores the Hidden Side of Everything (Revised and Expanded Edition)*. William Morrow.

Makridakis, S., Andersen, A., Carbone, R., Fildes, R., Hibon, M., Lewandowski, R., … Winkler, R. (1982). The Accuracy of Extrapolative (Time Series) Methods - Results of a Forecasting

Competition. *Journal of Forecasting*, *1*(2), 111–153.

Makridakis, S. G., Wheelwright, S. C., & Hyndman, R. J. (1998). *Forecasting: Methods and Applications (Book).* New York: John Wiley & Sons Inc.

Mayer, T. (2012). Ziliak and McCloskey's Criticisms of Significance Tests: An Assessment. *Econ Journal Watch*.

Morlidge, S. (2013). How Good Is a "Good" Forecast?: Forecast Errors and Their Avoidability. *Foresight: The International Journal of Applied Forecasting*

MORLIDGE, S. (2014). Do Forecasting Methods Reduce Avoidable Error? Evidence from Forecasting Competitions. *Foresight: The International Journal of Applied Forecasting*, (32), 34–39.

Morlidge, S. (2014). Using Relative Error Metrics to Improve Forecast Quality in the Supply Chain. *Foresight: The International Journal of Applied Forecasting*

NIST/SEMATECH. (2013). e-Handbook of Statistical Methods. Retrieved August 1, 2016, from http://www.itl.nist.gov/div898/handbook/

Paik, S.-K., & Bagchi, P. K. (2007). Understanding the Causes of the Bullwhip Effect in a Supply Chain. *International Journal of Retail & Distribution Management*, *35*(4), 308–324.

Patton, M. Q. (2014). *Qualitative Research & Evaluation Methods: Integrating Theory and Practice*. SAGE Publications.

Small, G. R., & Wong, R. (2001). The Validity of Forecasting, (August 2001), 1–14.

United States Government Accountability Office. (2009). Defense Inventory: Army Needs to Evaluate Impact of Recent Actions to Improve Demand Forecasts for Spare Parts. *United States Government Accountability Office - Report*, *GAO-09-199*(January), 1–41.

REPORT DOCUMENTATION PAGE			*Form Approved* *OMB No. 0704–0188*

1. REPORT DATE (DD–MM–YYYY)	2. REPORT TYPE	3. DATES COVERED (*From* — *To*)
24-03-2016	Master's Thesis	September 2014 – March 2016

4. TITLE AND SUBTITL-E
An Evaluation of Forecasting Methods that Could be Used in the Brazilian Air Force Uniform Distribution Process

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Oliveira, Leandro, Valviesse, Captain, Brazilian Air Force

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Institute of Technology
Graduate School of Engineering and Management (AFIT/EN)
2950 Hobson Way
WPAFB OH 45433-7765

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT-ENS-MS-16-M-122

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Intentionally left blank

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Distribution Statement A. Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES
This work is declared a work of the U.S. Government and is not subject to copyright protection in the United States

14. ABSTRACT
Every year the Brazilian Air Force (BAF) spends the equivalent of approximately 15 millions of dollars for uniforms. These purchases come from a tight budget, are executed through public procurement processes, and are tied to Brazilian acquisition regulations, which are often very strict. For this reason, lead times are unpredictable. It can take anywhere from one month to a year to replenish an item. The purpose of this research is to analyze the forecasting process performed at a BAF military organization named Sub-directorate of Supply (SDS) with the intent of building an algorithm comprised of a selection of forecasting models in order to help SDS optimize its inventory investments. With this in mind, monthly sales, prices, and inventory records from January of 2010 to July of 2015 were extracted from a database and converted to a standard spreadsheet format. Several forecasting models were evaluated and applied to randomly selected items from the database to create the algorithm. In the final analysis, it was concluded that two models precisely depicted the behavior of sales in BAF's stores. These two models were then utilized to develop the forecasting tool that may prove valuable in future BAF uniform purchasing decisions.

15. SUBJECT TERMS
Forecasting, Inventory Control, Lead Time, Process Metrics, Supply Chain

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Jeffrey A. Ogden, PhD
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U	UU	128	19b. TELEPHONE NUMBER (Include Area Code) (937) 255-3636 X 4653 jeffrey.ogden@afit.edu

Standard Form 298 (Rev. 8–98)
Prescribed by ANSI Std. Z39.18