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Maximizing Strike Planning Efficiency for a Given Class of Targets

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MAXIMIZING STRIKE PLANNING EFFICIENCY
FOR A GIVEN CLASS OF TARGETS

THESIS

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Abstract

Strike planning is one of the fundamental tasks of the Turkish Air Force and involves assignment of strike aircraft to targets with a maximum level of efficiency. Therefore, planning an optimal strike plan based on the preferences of the decision maker is crucial. The efficiency of the strike plan in this research implies attacking the maximum number of targets while considering target priority and the desired level of damage on each target. Another objective is to minimize the cost of the plan.

This research develops an exact model that maximizes the efficiency of the strike plan using LINGO with Excel Spreadsheets. Given this efficiency, the aircraft and weapon costs plus the distance flown is minimized while maintaining efficiency. The model also takes into account the aircraft and weapon capacities for particular types at each base to avoid assigning aircraft to targets from a base where there is an insufficient resource in terms of the aircraft and weapon capacity.

The results show that the model developed in this research provides a great deal of cost saving (i.e., approximately 50 %) for a strike plan compared to a strike plan which does not consider the total cost.

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MAXIMIZING STRIKE PLANNING EFFICIENCY
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I. Introduction

In this chapter, Section 1.1 describes the problem statement of this research. Section 1.2 presents the research question and the objectives of this research. Finally, the scope, limitations, and the assumptions are discussed in Sections 1.3 and 1.4.

1.1 Problem Statement

The primary responsibility of the Turkish Air Force (TUAF) is to defend Turkish airspace and territory. Alert aircraft are located in particular regions in Turkey so an immediate and effective reaction capability against airspace intrusions is maintained. On the other hand, attack aircraft are assigned so as to achieve desired levels of damages on targets. The attack aircraft, desired levels of damages, and targets are determined in advance. The attack aircraft are located at bases depending on potential threats to the Turkish Republic. The main difference between attack aircraft and alert aircraft is that alert aircraft have similar capabilities and carry similar weapons no matter where they are located. Furthermore, alert aircraft require less detailed planning, as many of the tasking decisions take place in the air, since these taskings are based upon the proximity of the alert aircraft to the threat. The main problem in tasking alert aircraft is to locate alert aircraft bases to obtain an optimum level of coverage of the national airspace against possible intrusions from foreign countries.

On the other hand, assigning attack aircraft to targets requires a detailed analysis of the targets and capabilities of the attacking forces. The final assignments are announced by the Air Tasking Order (ATO). The ATO matches the targets with the squadrons and assigns aircraft with an appropriate number of weapons to achieve a desired level of damage on each target; however, there may be several alternative ways of achieving the desired level of damage by assigning different types of aircraft with different types of weapons from another base. Moreover, some of these alternative assignments might be more cost effective. In other words, there might be multiple optimal assignments with respect to damage expectancy when it is assumed that the final ATO assignment is optimal and the decision maker could select an assignment which is more cost effective.

Cost effectiveness refers to the type and number of aircraft and weapons assigned and the distance flown by these aircraft. Therefore, a smaller number of aircraft and weapons is generally more cost effective. Thus, cost effective assignments give the decision maker more options for future missions because there will be more weapons available and the turnaround time for the aircraft will be decreased due to the shorter distances between targets and the assigned bases. Therefore, more unassigned available aircraft and weapons increase flexibility in the decision making process.

However, the overall cost effectiveness of an ATO refers to the total cost of the assigned strike packages. A strike package is defined as a group of attack aircraft carrying weapons to achieve the goal of destroying a set of targets. For instance, 2 F-16s carrying 4 MK-84s is a strike package. A smaller number of weapons is not always more cost effective due to the different costs of different weapons. Total cost depends on the weapon costs and the aircraft sortie costs based upon the distance flown. Clearly, an ATO that maximizes damage expectancy, uses the least number of aircraft and weapons or the most cost effective weapon and aircraft combination, and has the shortest total distance flown is desired.

It is important to make decisions as quickly as possible in an operational environment since time is limited due to the necessity of taking immediate action against enemy attacks. Otherwise, the enemy takes preventive measures and the desired level of damage on targets may not be achievable due to these preventive measures. The TUAF must react to all possible threats immediately and hence, having robust and efficient user friendly tools that aide decision makers in assigning strike packages to targets in a cost effective way is critical to the overall security of Turkey.

1.2 Research Question

The research question is:

What type of weapons and how many weapons by type should be assigned to specified targets in order to achieve a desired level of damage on each target while minimizing the total cost of the assignment with respect to the type and the number of aircraft and weapons used, and the distance flown?

One of the main objectives of assigning weapons to targets is to achieve a user defined desired level of damage on each target. The desired level of damage could be achieved by different types of weapons and different aircraft assignments from several bases, where each assignment has a particular cost, and hence, another objective is to find the assignment with the minimum cost.

1.3 Scope and Limitations

There are five phases in the strike planning process: *target selection, weapon allocation, mission formation and assignment, mission routing and scheduling process, and contingency plans.* This research deals with the weapon allocation and the mission formation phases of strike planning. The weapon allocation phase assigns weapons to targets to achieve the desired level of damage on the targets. On the other hand, the mission formation phase constructs the actual strike packages to carry weapons to targets. Target selection, mission routing and scheduling process, and contingency plans phases are not considered in this research, and it is assumed that targets are determined by the decision maker.

All targets are individual. There is no network structure among them. Because there can be multiple targets in sequence for a strike package to attack, the planner does not have to assign different strike packages to every target. One strike package can carry different types of weapons and attack targets sequentially. However, it is not a generally accepted situation for an aircraft to carry different types of weapons to attack multiple targets. Furthermore, attacking sequentially affects the surprise effect of the attack negatively since all targets cannot be attacked at the same time. In general, the aircraft attack target groups and the target groups contain several single targets. In this research, it is assumed that every strike package attacks a single target in a target group and returns to base. Therefore, aircraft do not carry different types of weapons in a strike package for a single target. This assumption is based upon real world applications. Additionally, all aircraft in a strike package which are assigned to a single target come from the same base to avoid violating the flight leadership and common site briefing considerations. [6, 8] However, different strike packages from different bases can be assigned to a target group containing several single targets where each strike package may carry different types of weapons and attacks a single target in a target group. Thus, all strike packages may attack single targets in a target group at the same time to accomplish a surprise attack on the enemy's forces.

This research also considers only ground targets. The model in this research does not assign Combat Air Patrol (CAP) aircraft to intercept enemy aircraft nor Suppression of Enemy Air Defense (SEAD) support for the strike package.

It is imperative for the air strike assets to accomplish their mission. This research takes into consideration that the desired level of damage on each target must be achieved at the minimum cost. There are important factors affecting mission planning such as policy or strategy, personnel availability, aircraft emergency rates, and weather conditions. The scope of this research is limited by only the probability of damages (POD) and the total cost of the mission in terms of the aircraft, weapons, and distance flown.

1.4 Assumptions

There are several assumptions in this research:

- Once the aircraft are airborne, they are able to complete their missions. No emergency situations are considered in this research because the ATO planner does not consider emergency situations in real world applications. In an emergency situation, a contingency plan may be executed. Note that even a contingency plan does not consider emergency situations. However, a contingency ATO planner takes into account factors such as weather conditions, target changes, and intelligence reports before starting to prepare the contingency plans. In other words, immediate changes are not considered in the ATO process dynamically.
- All weapons are released over the target with 100 % probability. There is a probability that a weapon is not released once the pilot hits the release button due to mechanical problems. Furthermore, the pilot can miss the point that he or she is required to fire the weapon to have the maximum impact on a target depending on the altitude and the diving angle. In this research, these possible problems are not considered because the ATO planner cannot anticipate these problems before planning since planning requires approximately 2 days.
- PODs of strike packages are calculated and put into the model by the user with respect to each target since each target and strike package has a different POD. The *Joint Munitions Effectiveness Manual (JMEM)* is used to calculate PODs

and integration of the JMEM into any type of ATO planning tool requires a license from the authorities of the JMEM software package.

- In practical applications, if a strike package attacks a single target, each aircraft in a strike package carries the same type of weapons. There may be a target group to attack and every single target in the target group may have a different structure that requires different types of weapons to be destroyed at a desired level of damage. In that case, aircraft in a strike package carry different types of weapons to attack targets sequentially to have the desired level of damage on each target. However, it is not a generally accepted situation for an aircraft to carry different types of weapons to attack multiple targets since attacking sequentially affects the surprise effect of the attack negatively because of the fact that all targets cannot be attacked at the same time. Therefore, it is assumed that strike packages attack single targets and carry only one type of weapon in this research.

- There is only one type of aircraft at each base due to the structure of the TUAF. However, different strike packages from different bases can be assigned to a target group which contains several single targets. Although every aircraft in a strike package carries the same type of weapon, the strike packages do not have to carry the same types of weapons as the other strike packages. Every strike package can attack a target group with different weapon types whereas each aircraft must carry the same type of weapon in a strike package.

- The model in this research addresses a single planning period. In other words, this research deals with the *Static Strike Planning Problem*. Once a mission is accomplished, the decision maker may execute this process repeatedly with the updated number of aircraft and weapons for another mission planning cycle.

1.5 Summary

In this chapter, the problem statement, the research question, and the objectives of the study are presented. Next, the scope, the limitations and the assumptions of the study are stated. Chapter II discusses the definition and the general formulation of the *Weapon and Target Assignment (WTA) Problem*, the *Strike Planning Problem*, and the associated solution methodologies existing in the literature. Chapter III presents the exact methodology to solve the static strike planning problem, which is a variation of the WTA problem developed in this research. In Chapter IV, LINGO interfaced with Excel spreadsheets model is introduced first, and then the analysis related to the solution time and the cost efficiency is presented. Finally, conclusions and future research recommendations are made in Chapter V.

II. Literature Review

In this chapter, Section 2.1 presents the definition and the general formulation of the WTA problem. Next, the static and the dynamic WTA problem are presented to make a distinction between the static and the dynamic WTA problem. Then different solution methodologies for the static WTA problem existing in the literature are presented. In Section 2.2, the Air Tasking Order (ATO) Models to solve the static strike planning problem, which is a variation of the static WTA problem, are presented. Existing solution methodologies are discussed. Although this research deals with the static strike planning problem, the solution methodologies for the dynamic strike planning problem existing in the literature are also briefly presented since the dynamic strike planning problem is an extension of the static strike planning problem. Finally, Section 2.3 and Section 2.4 give brief descriptions of the *Assignment Problem* and *Goal Programming*, respectively. The model in this research is basically developed as an assignment problem. Goal programming is widely used in solving the WTA problem considering target priority.

2.1 Weapon and Target Assignment (WTA) Models

2.1.1 The Definition of the WTA Problem.

A WTA problem solution provides an appropriate assignment of weapons to targets to maximize the damage expectancy on each target based on target values which are determined by the decision maker. The basic idea behind the WTA problem is to maximize the overall effect on targets. [1]

However, there are some other definitions in the literature. According to Ahuja et al. [22], the WTA problem is to assign weapons to targets to minimize the total survival values of all targets. The WTA problem takes place in battlefield situations where a decision maker wishes to maximize the total damage expectancy on targets. The problem seeks solutions that have the minimum total survival values from the minimization standpoint, and it tries to achieve the maximum total damage expectancy from the maximization standpoint. [22]

Another perspective on defining the WTA problem is that of minimizing the expected damage on friendly-force assets. On modern battlefields, targets may have PODs on the attacking platforms that carry the weapons since targets may have defensive systems. This depends both on the types of targets and the attacking platforms. The POD for a target on an attacking platform affects the expected damage value (EDV) on an attacking platform, and the problem minimizes the total EDV on the attacking platforms considering different PODs for a target on different attacking platforms. [31, 32, 34, 35]

Another point of view is that the WTA problem is a resource allocation problem that tries to allocate resources to activities such that the total cost is minimized. Examples of the resource allocation problem are loan distribution, computer scheduling, production planning, and the WTA problem. In this problem type, the formulation of the problem seeks an assignment of resources to the demand points. Similarly, the WTA problem formulation tries to obtain an assignment of weapons to targets where weapons are resources and the targets are demand points. [33]

There are several different models in the literature to solve the WTA problem, but each model has a different objective function and constraints. No model can solve the WTA problem while considering all details. Varying assumptions are made in modeling the WTA problem for particular purposes and the difference between the models occurs because of these assumptions. [24]

Each model deals with the problem to a certain level of detail. However, there are major aspects among the models that are common to all references in the literature. These aspects are: *the weapon system, the target complex, the engagement, the damage, and the assignment algorithm.* In addition, each model has a different level of complexity based on the assumptions that have been made. Also, several of the references combine the engagement and the level of damage by defining a single parameter, POD_{tw}, the probability of damage for weapon w on target t. [24]

These aspects are explained below:

The Weapon System

The weapon system is divided into three categories: the *scope of the weapon system* which considers different types of weapons and their quantities, the *weapon suitability* which takes into account the usable types of weapons for each target, and the *weapon commitment policy* which deals with weapon availability uncertainties.

The scope of the weapon system considers either one type of weapon or more than one type of weapon. One type of weapon indicates that every weapon has the same

POD on the targets and has the same technical properties. Different types of weapons are distinguished by differing technical properties such as PODs and accuracies.

The weapon suitability explains whether a particular type of weapon could be used for a target or not. Since all types of weapons are not appropriate for all types of targets, inappropriate types of weapons may be eliminated to reduce the number of decision variables in the model.

The weapon commitment policy has two different aspects: the deterministic and stochastic approaches to the WTA problem. In the deterministic approach, all weapons are available, released reliably, and the damage assessment is executed perfectly. On the other hand, the stochastic approach assumes all weapons are not always available, the release of the weapons may be unreliable, and there may be an unreliable damage assessment. [24]

The Target Complex

The target complex aspect is categorized by the *types of targets*, the *values or weights assigned to targets*, and the *defense capabilities associated with target*s.

The types of targets include point and area targets. Point targets can be missile launchers, radar installations, bridges and small cities. If a particular type of weapon is capable of killing a target, it is considered a point target. If more than one type of weapon is required to kill the target, it is considered to be an area target. The key point is that the determination of whether a target is a point or an area target does not depend solely on the target. The lethal radius of the weapon must also be considered.

The expected target value is the usual measure of effectiveness when different types of weapons are compared. Although this is not the case in reality, the values of the targets are assumed to be the same from both offensive and defensive points of view in the literature.

The aspect of *defense capabilities* of a target may be categorized based on the type of defense and the treatment of the defensive systems in the model. The types of defenses may be none, terminal only, area with or without terminal defenses, either preferential or not. The treatment of the defensive systems can also be categorized as explicit or implicit cases. In the explicit case, the defensive parameters are explicitly provided. The implicit case provides specified PODs for the targets only. The explicit case is also divided into two categories depending on whether or not the assignment of the defensive weapons in a defensive system of a target against the offensive forces is known to the offense.

The Engagement

The engagement aspect can be categorized based on whether the offensive or defensive system is deterministic or probabilistic. If the offensive system is deterministic, the POD of a weapon can be obtained perfectly whereas the weapons have probabilities of not impacting at their intended level in the probabilistic approach. Likewise, the deterministic defensive systems have defense capabilities such as radars and interceptors which work perfectly. The probabilistic defensive systems similarly have probabilistic defense capabilities which are capable of identifying the enemy attack with a probability of less than 1.

The Damage

In the damage aspect, the damage may be either deterministic or probabilistic and be either partial or total. If a target is killed with a probability of less than 1, the damage becomes probabilistic and it is deterministic if there is a probability of kill of 1. The difference between the partial and the total damage is whether the entire target either is killed or not.

The Assignment Algorithm

The assignment algorithm aspect contains different solution methodologies which are generally used to find the optimum assignment of weapons to targets in accordance with the preferences of the decision maker. These solution methodologies are *game theory, graphical or manual techniques, graph theory, linear or non-linear programming, dynamic programming, heuristic approaches, exhaustive searches, or Monte Carlo techniques.*

> Lagrange-multiplier techniques are frequently incorporated with these methods since they allow constraints to be easily handled in the optimization, thereby reducing the computational effort. Double Lagrange multipliers are employed in two-sided allocations or games. [24]

The algorithms or solution methodologies essentially depend on the decision maker and the nature of the problem. The possible alternatives are:

- The solution is optimal as opposed to near optimal solutions. This may also include a proof of optimality,
- The solution is either integer or continuous,
- The optimal defensive support such as SEAD and CAP can also be provided,
- The computational complexity of the algorithm.

The ideal algorithm should return an integer solution that can be proved to be optimal. The algorithm should also provide an optimum defensive support, be capable of dealing with large instances of the general WTA problem, run in an acceptably short amount of time, be insensitive to small variations in the number of weapons and targets and the parameters in the model, and yield a global optimum solution rather than a local optimum. [24]

2.1.2 The General Formulation of the WTA Problem.

The general WTA problem can be formulated non-linearly as follows:

$$\text{Minimize} \sum_{t=1}^{|T|} \mu_t \cdot \prod_{w=1}^{|W_w|} (1 - POD_{tw})^{x_{tw}} \tag{2.1}$$

Subject to

$$\sum_{t=1}^{|T|} x_{tw} \leq |W_w|, \qquad \mathit{for\ each}\ w \,\epsilon\, W_w \tag{2.2}$$

$$x_{tw} \geq 0 \text{ and integer}, \qquad \mathit{for\ each}\ t \,\epsilon\, T\ \mathit{and}\ \mathit{for\ each}\ w \,\epsilon\, W_w \tag{2.3}$$

where

T is the set of targets and $t \epsilon T$,

W_w is the set of weapon types and $w \epsilon W_w$,

μ_t is the target priority for each target $t \epsilon T$,

POD_{tw} is the probability of damage of a single weapon $w \epsilon W_w$ on target $t \epsilon T$,

x_{tw} is the decision variable, which is the number of weapons by type $w \epsilon W_w$ assigned to target $t \epsilon T$.

The Objective Function (2.1) tries to minimize the total expected survival value of all targets, where $(1 - POD_{tw})$ is the survival probability of target $t \epsilon T$ if a weapon $w \epsilon W_w$ is assigned to it, taking into account that the total number of weapons assigned for a particular type should not be more than the number of weapons available of that type using Constraint (2.2).

In real world applications, there may be some additional constraints such as:

- minimum or maximum number of weapons for a particular type assigned to a target,
- minimum or maximum total number of weapons assigned to a target,
- minimum requirement on the survival value of a target. [22]

2.1.3 The Static WTA Problem.

In the static WTA problem, all weapon and target assignments should be done for a single stage in time. Additionally, all weapons and all targets to be assigned are known in advance, all weapons are assigned simultaneously, and an assignment of a weapon to a target is independent of all other assignments. [9, 22]

Some important properties of the static WTA problem are:

- It is NP-hard since the computation time will increase exponentially based upon the problem size. The most difficult aspects in solving the static WTA problem are its nonlinear nature and the several types of weapons that are available to be assigned to targets.
- The static WTA problem is discrete because fractional weapon and target assignments are not allowed.
- The static WTA problem is stochastic in nature but this should not be confused with a stochastic or non-deterministic solution technique for the WTA problem. It is stochastic because the weapon and target assignments are modeled as events with non-deterministic outcomes. [9, 23]

The static WTA problem is used to find solutions for a single period of time. However, this does not mean that the static WTA problem is solved only once and then the final weapon and target assignment is implemented. The threat evaluation and the weapon and target assignment process are executed repeatedly in the battlefield. Therefore, the real time necessities are directly related to how frequently the threat evaluation and the weapon and target assignment process are executed. [9]

2.1.4 The Dynamic WTA Problem.

The dynamic WTA problem is complicated compared to the static WTA problem because it deals with the WTA problem by considering multiple stages in time. In the dynamic WTA problem, some weapons are assigned at a single stage and the result of the assignment is evaluated. Then, the course of action for the next assignment is determined. [22]

Some important properties of the dynamic WTA problem are:

- It is NP-hard since the computation time will increase exponentially based upon the problem size. The most difficult aspects in solving the static WTA problem are its nonlinear nature, several time stages, and the several types of weapons that are available to be assigned to targets,
- The dynamic WTA problem is discrete because fractional weapon and target assignments are not allowed,
- The dynamic WTA problem is stochastic in nature because the weapon and target assignments are modeled as events with non-deterministic outcomes,

The benefit of considering the dynamic WTA problem is that it involves several time stages and the outcome of an assignment at a single stage is assessed each time and targets which have already been attacked and destroyed will not be attacked in the next time stage due to the useful information which was obtained from previous assignments and their outcomes. This is called *shoot-and-look strategy* in the literature. [16, 18, 22]

The steps in solving the dynamic WTA problem are as follows:

- Determine the targets that have not been attacked in the last stage,
- Assign the remaining weapons to the surviving targets so as to minimize the total surviving value of the targets at the end of the final stage. [17]

2.1.5 Solution Methodologies for the Static WTA Problem.

This research deals with the static strike planning problem which is a variation of the static WTA problem. Therefore, the solution methodologies reviewed in this subsection contain the solution methodologies for solving the static WTA problem.

However, reviewing the solution methods both for the static and the dynamic strike planning problem in Subsection 2.2.2 is considered useful because the strike planning problem, which is directly addressed in this research, is static and the dynamic strike planning problem is an extension of the static strike planning problem.

The methodologies to solve the WTA problem depend on the modeling assumptions and the resulting instances of the WTA problem. This is because of the NP-complete nature of the problem. [23] The multiple types of weapons available to assign and the non-linearity of the objective function make the WTA problem difficult to solve. The NP-completeness of the problem causes the computation time of any optimal algorithm to grow exponentially as the size of the problem increases. [9] Hence, the decision maker has to make some assumptions to fit the general WTA problem into his or her preferences based on battlefield requirements.

F. Johansson and G. Falkman [9] propose an *exhaustive search algorithm* to find optimal solutions for the small-sized static WTA problem. Even the static WTA problem is not an easy problem to solve even though several simplifying assumptions are made. Due to its computational complexity, the static WTA problem requires approximation algorithms to find good solutions in real time. However, it is possible to find an optimal solution to the small-sized static WTA problem since the problems have been solved optimally for problems with 8 targets and 6 weapons in approximately one second. Static WTA problems larger than this size require heuristic algorithms because the problems that contain 9 targets and 9 weapons require approximately 43.7 minutes to be solved optimally. [9]

Ahuja et al. [22] propose exact algorithms for the static WTA problem using branch-and-bound techniques and different lower bounding schemes. These branch-and-bound algorithms are the first implicit enumeration algorithms that can solve moderately sized instances (40 targets and 40 weapons) of the static WTA problem optimally in approximately 50 seconds. The lower bounding schemes that Ahuja et al. [22] propose are *the lower bounding scheme using an integer generalized network flow formulation, the minimum cost flow-based lower bounding scheme, and the maximum marginal return-based lower bounding scheme.*

The Lower Bounding Scheme Using an Integer Generalized Network Flow Formulation

Ahuja et al. [22] formulate the static WTA problem as a generalized integer network flow model on an appropriately defined network using integer linear programming (IP) methods.

> The piecewise-linear approximation of the convex objective function of this formulation gives a lower bound on the optimal solution. This integer linear program can also be viewed as an integer generalized network flow problem with convex flow costs. The generalized network flow formulation is substantially more difficult than the standard generalized network flow problem because the flow values are required to be integer numbers and the costs of flows on some arcs are convex functions. [22]

In this method, each convex function is approximated by a piecewise-linear convex function. Therefore, the optimal solution of the modified version of the formulation provides a lower bound on the optimal solution of the generalized formulation. [22]

The Minimum Cost Flow-Based Lower Bounding Scheme

This lower bound is not as effective as the bound given by the generalized integer network flow formulation. However, it requires less computational time. In this formulation, the objective function of the static WTA problem can be taken into account as maximizing the expected damage to the targets. Ahuja et al. [22] develop an upper bound on the expected damage to the targets. Subtracting this upper bound on the expected damage from the total value of the targets provides a lower bound on the optimal solution to the static WTA problem. [22]

The formulation of the static WTA problem using *the minimum cost flow-based lower bounding scheme* is essentially based upon maximizing the damage to targets as a maximum cost flow problem. Since this is a maximization problem, all arc costs in this formulation must be multiplied by -1 to obtain the minimum cost. Finally, subtracting the upper bound from the total value of the targets provides a lower bound for the static WTA problem. [22]

The Maximum Marginal Return-Based Lower Bounding Scheme

"This formulation is based on the underestimation of the survival of a target when hit by a weapon because it is assumed that every target is hit by the best weapons". [22]

This algorithm is a variation of the knapsack problem and uses a greedy approach to obtain a lower bound. This lower bounding scheme is a modified *maximum marginal return algorithm* in the literature. In this algorithm, a weapon is assigned so as to maximize the improvement in the objective function value. This algorithm is a heuristic algorithm, but it provides an optimal solution if all weapons are identical. [22]

Ahuja et al. [22] develop a branch-and-bound algorithm using the three lower bounding schemes above. This algorithm is the first exact algorithm that can solve moderately sized instances (40 targets and 40 weapons) of the WTA problem in reasonable time (for comparisons of the solution times of several instances, refer to the article [22]). A branch-and-bound algorithm basically depends on *the branch strategy, the lower bounding schemes and the search strategy.*

The branch strategy determines which weapon and target combination gives the best improvement in the objective function.

The lower bounding strategy includes three types of lower bounding schemes which are *generalized network flow, minimum cost flow*, and *maximum marginal return.*

The search strategy is an important factor depending on the number of weapons and the targets. It is better to use breadth-first search strategy for small instances (i.e. 10 weapons and 10 targets) whereas the depth-first strategy gives better results for large instances. [22]

Exact algorithms to solve the WTA problem can handle only moderately sized instances of the WTA problem. For instance, the *exhaustive search algorithm* which F. Johansson and G. Falkman propose [9] solves the small sized (9 targets and 9 weapons) static WTA problem, in which all weapons available should be assigned, in approximately 43.7 minutes. [9] Similarly, although the *branch-and-bound algorithm* can solve moderately sized instance (40 weapons and 40 targets) of the static WTA problem in approximately 50 seconds, the largest size static WTA problem for which the *branch-and-bound algorithm* can find an optimal solution consistently contains 80 targets and 80 weapons and it requires approximately 16.2 hours. [22]

In the battlefield, it is crucial to make a decision in a reasonable amount of time. However, the decision maker cannot determine the size of the weapon and the target assignment in advance. The decision maker has to be prepared to find an acceptable assignment for all possible instances of the WTA problem. Therefore, there is a demand to solve large instances of the WTA problem in a reasonable amount of time. The solution time for the WTA problem plays an important role in the real world. Heuristic algorithms can provide near optimal solutions for the WTA problem in a reasonable amount of time. However, the trade-off between the solution quality and the solution time essentially depends on the preferences of the decision maker and the necessities of the battlefield.

A *very large scale neighborhood search algorithm (VLSN)* proposed by Ahuja et al. [22] is a neighborhood search algorithm where the size of the neighborhood is very large.

The VLSN algorithm starts with a feasible solution of the optimization problem using the *minimum cost flow formulation based construction heuristic* and successively improves it by replacing the solution with an improved neighbor using the *VLSN neighborhood structure* until it converges to a locally optimal solution. [22]

The *minimum cost flow formulation based construction heuristic* solves a sequence of minimum cost flow problems to find good solutions for the static WTA problem and this solution constitutes an excellent starting feasible solution for the static WTA problem. The quality of the locally optimal solution depends on both the quality of the starting feasible solution and the structure of the neighborhood. [22]

The *VLSN neighborhood structure* is essentially based on the *multiexchange neighborhood structure* developed by Thompson and Psaraftis [20] and Thompson and Orlin [21]. The structure searches the neighborhood iteratively and the algorithm either finds a better multiexchange solution and improves the current solution or stops and declares that the current solution is locally optimal in each iteration. [22]

The VLSN search algorithm is very efficient in solving the WTA problem, since the construction heuristic gives the optimal solutions for over half of the instances up to 200 weapons and 400 targets, and the VLSN search algorithm's solutions of the remaining instances are near optimal. Moreover, the solution time even for the instance containing 200 weapons and 400 targets is approximately 2 seconds and it is less than a second for the rest of the instances. [22]

The success of the VLSN approach can be attributed to the dimension of the solution space and the starting feasible solution. For instance, the VLSN approach searches the neighborhood of size 3 billion for the instance containing 80 weapons using five-exchanges and the construction heuristic provides an excellent starting feasible solution which is very important for the quality of the final solution. [22]

Another heuristic algorithm, *an immunity-based ant colony optimization (ACO)*, is proposed by Lee et al. [32] to improve the performance in terms of the solution quality and the computational time for the static WTA problem.

The ACO algorithm imitates the behavior of real ants using artificial ants and is a constructive population-based search technique to solve optimization problems by using the principle of pheromone information. In this approach, several generations of artificial agents which are actually artificial ants executed in an evolutionary manner to search for good solutions. The artificial ants are initially randomly generated on nodes, and stochastically move from a start node to feasible neighbor nodes in the local search phase. The artificial ants collect and store information in pheromone trails during the local search phase. The pheromone can only be released when the artificial ants build solutions and is evaporated in the search process to avoid local convergence and to explore more search areas. Thus, additional pheromone is stored to update the pheromone trail so the search process can be executed in a different pheromone trail to avoid being trapped into the local optima. [33]

> Ants are capable of exploring and exploiting pheromone information, which have been left on the traversed ground. Ants then can choose paths based on the amount of pheromone. With such a concept, a multi-agent algorithm called ACO has been widely employed as a cooperative search algorithm for solving optimization problems. [32]

The success of the ACO algorithm in solving optimization problems can be attributed to the search parallelism which is based on the components of the solution. Moreover, the local search efficiency of ACO can be improved by implementing an *immune system*. The immune system eliminates the decrepit and degenerative parts but not the normal parts in the human body. [32]

The immune system in the optimization world mimics the behavior of the immune system in the human body. The approach includes two main points. The first step is to improve the current solution and the second step is to avoid making the current solution worse. At each step, the immune system approach tries not to deteriorate the current solution. The only modification in the solution that the immune system approach can make is an improvement in the current solution. [32]

Therefore, the immunity-based ACO algorithm increases the search performance of solving the static WTA problem. In this algorithm, ACO tries to find better solutions and avoid being stuck in local optima and seeks the global optima. Then, the immune system utilizes problem-specific heuristics to conduct local search and does fine-tuning in the solution space. [32]

Simulated annealing (SA) and *genetic algorithms (GA)* are also among the heuristic algorithms which are widely used to solve optimization problems in the literature and these methods yield good results in reasonable time. [32]

The SA is a representation of the annealing process of solids which heats the solid to a high temperature and then cools it down gradually. SA enables asymptotic convergence to the optimal solution escaping from local optima by using a probability function in accepting or rejecting new solutions. [35]

GA is an algorithm which handles either linear or non-linear constraints without any additional mathematical operations as matrix inverses for the objective function. The GA is an evolutionary algorithm which adopts the mechanism of natural selection to search for the best solution from candidates in the local search process. [34]

The GA starts with randomly selected chromosomes representing the initial solution. The variables are represented as *genes* in the chromosomes. The chromosomes are evaluated according to their fitness values which are evaluated using two genetic operators: the *crossover* and the *mutation*. The chromosomes with better fitness values are more likely to be selected in the next generation. After several generations, the GA converges to the global optimum. [35]

> SA has been shown to have the ability of finding the global optimum. However, due to its sequential characteristics, SA cannot be used in a parallel architecture to improve its search efficiency. GAs can be viewed as parallel search techniques that stimulate the evolution of individual structures for optimization inspired by natural evolution. However, the parallelism of search is based on the solution level, Thus, the search efficiency may not be very nice. [32]

Lee et al. [35] introduces a new gene reformation in the GA which is called *eugenic process for offspring*. The concept of eugenic is to find better solutions around the current solution before moving to the next stage of the search. These are called *local search mechanisms*. The proposed algorithm greedily reforms the current solution instead of using a random trial process, and it is called *greedy eugenics*.

The concept of eugenics is simply stated as a process which starts from an obtained feasible solution and tries to improve the current solution by local changes. If a better solution is found, then it replaces the current solution. The steps are repeated until a criterion is satisfied.

According to Lee et al. [35], the search can easily escape from local optima due to crossover and mutation operations and the parallel search methods used in the GA even though the greedy algorithms may have a high probability of being trapped in a local optima. Therefore, the greedy eugenics finds the locally best solutions and is not trapped in the local optima.

As a comparison between the GA and the GA with greedy eugenics, the 100% convergence to the optimal solution in 20.28 seconds with a standard deviation of 7.15 seconds for 10 trials is obtained using the GA with greedy eugenics as opposed to the GA, which is able to converge to the optimal solution with 40 % for 10 trials and is not able find the optimal solution within a maximum generation in any trials.

Lee et al. [35] also compare the GA in which SA is used for local search with the GA with greedy eugenics. The SA is treated as an alternative to simple eugenics in the *GA with the SA* as local search and is incorporated as the eugenic mechanism to take advantage of search strategies in which relatively worse solutions may be accepted in order to reach the global optimum rather than being trapped into local optima.

Lee, Z. J. and Lee, C. Y. [33] combine the GA and the ACO and present a *hybrid algorithm* to solve the WTA problem. This approach starts with a feasible solution using the GA to avoid having premature convergence and conducts fine-tuning in the search space using the ACO to find better solutions. In this research, the hybrid algorithm is compared with the SA, the GA, and the ACO algorithms and the hybrid algorithm converges to the global optima better among these algorithms.

Zeng et al. [31] present another heuristic algorithm which is called *discrete particle swarm optimization (DPSO)* model to solve the static WTA problem. The standard *particle swarm optimization (PSO)* is an adaptation of a simplified social behaviour. The PSO uses the personal thinking of each particle and the collaborative effect of the particles in finding the global optimal solution. Because of the continuous search of the PSO, Zeng et al. [31] develop a DPSO model for the static WTA problem.

In the DPSO model, the greedy search strategy is introduced to control the local search and converge to the global optimum. Due to the fact that the greedy search strategy has a high probability to be trapped into a local optimum, two probabilities such as *fixed probability* and *unfixed probability* are employed to the update strategy which is called *permutation* in this research.

The DPSO model converges to the global optimum quicker than the GA and the GA with greedy eugenics. For instance, the DPSO converges to a global optimum in 100 % of the tests in 0.0058 minutes whereas the GA with greedy eugenics converges to a global optimum in 100 % of the tests in 0.0087 minutes and the GA converges to a global optimum in 60 % of the tests in 0.0099 minutes.

Yücel, A. [1] proposes a *sequential method* to solve the static WTA problem. This is a heuristic approach. In this approach, the primary assignments are identified first, and the secondary assignments are executed next. The process is repeated building up a bipartite graph until no feasible assignment is left. This greedy approach is faster than the branch and bound algorithm that Yücel, A. [1] used in this paper and it allows multi-target assignments. In other words, the multiple weapons can be used against a single target. Despite the fact that the branch and bound algorithm finds the optimal solution, the sequential method consistently finds multiple assignments that are close to the optimal, with differences that may be considered operationally insignificant.

2.2 Air Tasking Order (ATO) Optimization Models

2.2.1 The Definition of the Strike Planning Problem.

The strike planning problem is a substantial problem in which there is a set of targets and a set of combat resources that may be assigned to targets. There may also be some target defenders. The objective of solving the strike planning problem is to maximize the strike planning efficiency in terms of the levels of damage to targets and the total cost of the strike plan while limiting the damage to strike forces caused by the defenders. [13]

The strike planning problem used for preparing an ATO that is a result of a complex process of target selection and weapons allocation covering several types of missions is a variation of the WTA problem. Strike planning has five phases: *target selection, weapon allocation, mission formation and assignment, mission routing and scheduling process, and contingency plans.* This research deals with the weapon allocation and the mission formation phases of the strike planning. The weapon allocation phase assigns weapons to targets to achieve the desired levels of damages on the targets. On the other hand, the mission formation phase constructs the actual strike packages to carry weapons to targets. [7]

A strike package can be defined as a group of attack aircraft carrying weapons to achieve the goal of destroying a set of targets. Strike packages are constructed in several steps. First, the mission planner should go through a weapon allocation process to determine the type and number of aircraft and weapons to achieve the desired levels of damages on the targets. Next, all aircraft attacking the targets in the same vicinity are grouped together considering aircraft speed restrictions and tactics. Finally, the mission planner should add SEAD or CAP aircraft into the group if necessary. [7, 8, 11]

As a result, the weapon allocation and mission formation phases of the strike planning turn out to be a variation of the WTA problem where the objective function is to maximize the number of targets destroyed based on target priority subject to the constraints such as aircraft and weapon availabilities, weapon effects, weapon suitability, distance to target, and speed depending on whether the problem is formulated as dynamic or static. [7]

This research directly addresses the static strike planning problem. Therefore, the solution methodologies reviewed in this chapter are the ones to solve the static strike planning problem. There are many solution methods proposed by different authors to solve the static strike planning problem. These solution methods are presented in the next subsection. However, the solution methodologies for the dynamic strike planning problem existing in the literature are also briefly presented in Subsection 2.2.3 since the dynamic strike planning problem is an extension of the static strike planning problem.

2.2.2 Solution Methodologies for the Static Strike Planning Problem.

Da Silva Castro, D. R. [7] proposes a *mixed integer linear program (MILP)* to assign heterogeneous strike packages to targets considering target priority and aircraft availability.

In this model, the strike packages are not necessarily homogeneous. The strike packages are allowed to contain different types of aircraft and weapons. However, the POD for each different strike package on the targets should be obtained prior to solving the model using the Equation (3.1) which is presented in Chapter 3.

There are three model objectives: minimizing the value of the targets which are not assigned, minimizing the effects of imperfect matching (incomplete damage, long-distance flight etc.) of targets to strike packages and maximizing the value of unused aircraft. These components of the multi-objective function can be combined into a *weighted sum* or the model can be solved using *goal programming* which is discussed in Section 2.4.

The optimal solution time for a strike planning problem with 100 targets, 3 types of aircraft, 2 possible aircraft configurations, 3 types of weapons, 20 different strike packages, 7 bases and 156 available aircraft is less than 2 seconds using GAMS with CPLEX and less than 3 seconds using GAMS with XA.

Da Silva Castro, D. R. [7] also develops an MILP for the static strike planning problem by using penalties which are sensitive to changes in the input data such as the number of targets, aircraft availabilities for the strike packages and the PODs of the strike packages on the targets due to adverse weather conditions. These penalty values force the model to produce a new ATO which is similar to the the previous ATO. The new ATO should be similar to the previous ATO to avoid having the same computational effort and saving time in assigning the strike packages to the targets.

Weaver, P. R. [29] presents a fast and accurate automated decision aid for the decision maker in the battlefield to change the current strike plan according to the emergence of *time sensitive targets* in accordance with the adaptation of the methodology that Da Silva Castro, D. R. [7] developed.

A *time sensitive target* has a high priority and requires immediate response because it poses a danger to friendly forces. [36]

Similar to Da Silva Castro's research [7], Weaver's research also considers maximizing the achievement of target destruction goals based on the target priority, minimizing the attrition risk, disrupting the current ATO as little as possible, and minimizing the distance flown on the newly assigned missions. However, this research deals with SEAD support as well to increase the number of strike options which is a future research recommendation of Da Silva Castro, D. R. [7].

Zacherl, B. [36] improves the automated decision aid that Weaver, P. R. [29] developed to revise the current ATO adding a prevention capability for overkilling of high-value targets at the risk of leaving lower priority targets unstruck. In this new IP model, the size of the problem is greatly reduced because the strike packages are limited to a reasonable number of missions. A *greedy heuristic algorithm* is also developed to find fast solutions compared to the IP model.

This greedy heuristic can solve a problem instance containing 20 targets and 11 missions in less than 2 seconds whereas the model that Weaver, P. R. [29] developed solves in 205 seconds. The greedy heuristic also yields near optimal solutions compared to the IP model in Zacherl's [36] research.

Dolan, M. H. [8] presents an IP model to solve the strike planning problem. In this model, the objective function maximizes the weighted sum of the destroyed targets based on target priority, less penalty values for the targets not destroyed and the distance penalty values. This model assigns the strike packages to targets based on the strike package preferences of the decision maker and on the aircraft capabilities.

This model also takes into account that all aircraft of the same type that are assigned to the same target should come from the same base to prevent violating flight leadership and common site briefing considerations. However, the aircraft may come from different bases if different types of aircraft are required for a strike package. The model considers the aircraft availabilities as well.

Dolan, M. H. [8] reduces the number of decision variables using a special feature of the modeling language (GAMS) which ensures that the decision variables and the constraints are considered only for valid base, aircraft and target combinations. In addition, targets which have similar characteristics in terms of the resistance to weapons can be grouped into a single one to reduce the number of decision variables in the model.

The solution time differs depending on the different solvers (XA and ZOOM) compatible with GAMS and XA finds solutions quicker than ZOOM. In addition, the *optimality tolerance* affects the solution time significantly. For instance, the solution time for a strike planning problem with 50 targets and zero optimality tolerance is more than one hour as opposed to the solution time of a strike planning problem with 50 targets and 0.25 optimality tolerance which is 3 minutes. Also, XA finds a solution for a strike planning problem with 100 targets and 0.25 optimality tolerance in 2 minutes and 42 seconds whereas ZOOM does not find a solution.

B. J. Griggs [11] proposes an MILP model to find an optimal allocation of strike packages to targets considering SEAD and CAP aircraft based on the fact that the targets are prioritized depending on the target values. The number of decision variables in this model is very high. For instance, a strike planning problem with less than 10 types of aircraft, 20 types of weapons, 2 different PODs, 40 types of targets, and 30 sectors has 1,920,000 decision variables and only 60 of these decision variables are binary. A *sector* is basically defined as the location of a target in the enemy's territory. Solving a strike planning problem with 1,920,000 decision variables using MILP takes more time compared to a linear programming (LP). The binary variables are first converted to continuous decision variables to solve the problem using LP. However, the solution may contain fractional assignments in this case. To overcome the problem, the fractional values of the decision variables are fixed and the strike planning problem is solved again using LP. This yields a *near optimal* solution in less time compared to MILP. The model also takes into account uncertainties of weather conditions using a *decision tree* after having a solution using the MILP.

Li et al. [13] proposes a MILP approach for a strike planning problem with SEAD and without SEAD taking into account combat resource availabilities and distance between the targets and the location of the strike forces. The computational results show that the LP relaxation of the strike planning problem without SEAD is a very good approximation of the optimal solution unless the asset availability constraint is not so restrictive. In these instances, the near optimal solutions are obtained in a short period of time. The solution time for a strike planning problem with SEAD is greater than the solution time for a strike problem without SEAD and the solution time depends heavily on the asset availability constraints.

Bardak, F. S. [3] presents an IP model to assign SEAD assets to targets. In this model, the objective function minimizes the total aircraft attrition in the strike packages. The aircraft sortie costs, the weapon costs, target priority, and weapon and aircraft availabilities for a particular base are also considered in this research.

Tikveṣ, Ṣ. [25] compares different solution methodologies such as exhaustive search using branch and bound, a greedy algorithm, a genetic algorithm, and network flow based solution techniques for the static strike planning problem. The preference of choosing a particular methodology mainly depends on the size and complexity of the problem. The results in Tikveṣ's research show that strike planning problems in an ascending order in terms of the problem size and complexity can be solved efficiently using a greedy algorithm, an exhaustive search using branch and bound, or a genetic algorithm.

2.2.3 Solution Methodologies for the Dynamic Strike Planning Problem.

As mentioned in Subsection 2.2.2, presenting the solution methodologies for the dynamic strike planning problem briefly is considered useful since the dynamic strike planning problem is an extension of the static strike planning problem. Therefore, the solution methodologies for the dynamic strike planning problem are briefly presented in this subsection.

Crawford, K. R. [6] improves the model which Dolan, M. H. [8] has developed to solve the dynamic strike planning problem. This model incorporates the time dimension into the strike planning problem, so it allows multiple assignments for an aircraft to attack targets in a single optimization model.

Da Silva Castro, D. R. [7] presents a MILP to solve the dynamic strike planning problem as well as the static strike planning problem. However, the strike packages should contain single types of aircraft and weapons. In other words, the addition of the time dimension into the model restricts the model to use only homogeneous strike packages over a multi-period time horizon.

Da Silva Castro, D. R. [7] improves this dynamic model which is sensitive to the changes in strike planning using penalties in order for the model to be persistent with the changes.

Barth, C. D. [4] develops a comprehensive *composite mission variable decomposition (CMVD)* for the dynamic strike planning problem. Barth's research deals with target selection as well as weapon allocation.

There are also some scheduling algorithms in the literature to solve the dynamic strike planning problem. These are also briefly presented below.

Van Hove, J. C. [26] presents a decomposition approach to increase the upper bound on the problem size for which it is reasonable to obtain optimal solutions.

Koewler, D. A. [12] proposes a scheduling algorithm to assign combat resources to targets in a dynamic environment. In this approach, the problem is divided into two parts such as *combat planning data structure* and *combat planning scheduling data structure*.

The combat planning data structure allows the decision maker to input planning information, and the combat planning scheduling data structure builds a schedule developing objects instead of using decision variables and equations.

Finally, Calhoun, K. M. [5] develops a *tabu search (TS)* algorithm to schedule air combat resources to targets.

The next two sections present the Assignment Problem and Goal Programming, respectively, since the solution methodology is developed in this research as an assignment problem and goal programming is widely used in solving the WTA problem considering target priority.

2.3 Assignment Problem

The assignment problem is a transportation problem where each supply node and demand node has a supply or demand equal to 1. [27] The supply nodes become weapons and the demand nodes become targets in the WTA problem. If the supply nodes can be assigned to more than one demand node but a demand node must be assigned to exactly one supply node, then the assignment problem is called the *generalized assignment problem.* [10]

The generalized assignment problem regarding the WTA problem can be formulated as follows:

$$\text{Minimize} \sum_{t=1}^{|T|} \sum_{w=1}^{|W|} c_{tw} \cdot x_{tw} \tag{2.4}$$

Subject to

$$\sum_{t=1}^{|T|} x_{tw} \leq b_w, \qquad \textit{for each } w \,\epsilon\, W \tag{2.5}$$

$$\sum_{w=1}^{|W|} x_{tw} = 1, \qquad \textit{for each } t \,\epsilon\, T \tag{2.6}$$

where

T is the set of targets and $t \,\epsilon\, T$

W is the set of weapons and $w \,\epsilon\, W$

x_{tw} is 1 if the weapon $w \,\epsilon\, W$ is assigned to target $t \,\epsilon\, T$, 0 otherwise

c_{tw} is the cost of assigning weapon $w \,\epsilon\, W$ to target $t \,\epsilon\, T$

b_w is the weapon capacity for weapon $w \epsilon W$

In this formulation, the Objective Function (2.4) minimizes the total cost of the weapon and target assignment. Constraint (2.5) ensures the capacity of each type of weapon is not exceeded and Constraint (2.6) ensures each target is attacked by only one weapon.

2.4 Goal Programming

In some situations, the decision maker may encounter multiple objectives or goals. Goal programming is a method which allows the decision maker to formulate the problem with multiple goals as an LP. The key point in goal programming is that the deviation variables which represent how well the goals are satisfied are used to convert each goal into a constraint for the LP. [27, 30]

The general goal programming model can be formulated as follows:

$$\text{Minimize} \sum_{i=1}^{|I|} w_i^+ \cdot d_i^+ + w_i^- \cdot d_i^- \tag{2.7}$$

Subject to

$$\sum_{j=1}^{|J|} a_{ij} \cdot x_j + d_i^- - d_i^+ = b_i, \qquad \textit{for each } i \,\epsilon\, I \tag{2.8}$$

$$x_j, d_i^-, d_i^+ >= 0, \qquad \textit{for each } i \,\epsilon\, I \textit{ and for each } j \,\epsilon\, J \tag{2.9}$$

where

I is the set of goals and $i \,\epsilon\, I$

J is the set of decision variables and $j \epsilon J$

x_j is the j^{th} decision variable where $j \epsilon J$

d_i^- is the deviation below goal $i \epsilon I$

d_i^+ is the deviation above goal $i \epsilon I$

w_i^- is the weight for deviation below goal $i \epsilon I$

w_i^+ is the weight for deviation above goal $i \epsilon I$

a_{ij} is the coefficient associated the j^{th} decision variable in goal $i \epsilon I$

b_i is the right hand side of goal $i \epsilon I$

In this formulation of the Objective Function (2.7), the analyst tries to achieve each goal as close as possible by assigning weights w_i^- and w_i^+ to deviations d_i^- and d_i^+, respectively, for goal $i \epsilon I$ to minimize the weighted sum of the deviations. [10]

However, it is difficult to determine the weights of the deviations from the goals in most situations. In such a case, *preemptive goal programming* in which the goals are ranked from most important to least important is used. The objective function for preemptive goal programming follows:

$$\text{Minimize} \sum_{i=1}^{|I|} P_i \cdot (d_i^- + d_i^+) \tag{2.10}$$

where

$P_i >> P_{i+1} >> ... >> P_{|I|}$

In this formulation, P_i for each goal $i \epsilon I$, represents the goal priority. From the perspective of this research, the decision maker does not have to determine each weight for each deviation from destroying the targets in solving the WTA problem. Instead, the targets are ranked with respect to associated priorities and the weapons are assigned to targets based upon these target priorities.

2.5 Summary

This chapter reviewed the WTA problem, and the strike planning problem with the associated ATO models in terms of the problem definition and the solution methodologies existing in the literature. The Assignment Problem and Goal Programming were also discussed in this chapter because the model in this research is developed as an assignment model and goal programming is widely used in the literature to solve the WTA problem considering target priority. The next chapter presents the exact solution methodology developed in this research to solve the static strike planning problem which is a variation of the WTA problem.

III. Methodology

In this chapter, Section 3.1 introduces the problem and the objectives of the mathematical model developed in this research. The definitions, the sets and indices, the parameters and the decision variables used in the methodology are defined and the solution steps in the methodology such as *Preprocessing, Phase I, and Phase II* are explained in detail in Section 3.2.

3.1 Introduction

The Turkish Air Force (TUAF) has to defend Turkish airspace and territory. The TUAF has two types of primary missions: *an immediate responding to airspace intrusions* and *attacking ground targets which are determined based on the intelligence reports*. In this research, *attacking ground targets* is only considered and it is called *the Static Strike Planning Problem*.

The problem considered in this research is static because it is assumed that it is possible to assign weapons to targets in a single stage in time (see Section 1.4). The decision maker may implement the strike package and target assignment process repeatedly and take into account that some of the targets may have been destroyed in previous attacks. This differs from the *Dynamic Strike Planning Problem*. In the dynamic strike planning problem, the mathematical model sequentially seeks solutions for multiple time periods whereas the mathematical model for the static strike planning problem seeks solutions for a single time period.

The research question is:

What type of weapons and how many weapons by type should be assigned to specified targets in order to achieve a desired level of damage on each target while minimizing the total cost of the assignment with respect to the type and number of aircraft and weapons used, and the distance flown?

The objectives of this research based on the research question above are:

- to achieve a desired level of damage on each target,
- to avoid assigning weapons to targets using the strike packages if the desired level of damage is not achievable,
- to avoid having a higher level of damage on each target than the desired level of damage,
- to minimize the total cost of the weapon and target assignment.

In the battlefield, it may not be beneficial to assign a strike package to a target if the assignment does not achieve the desired level of damage on the target. The desired level of damage is determined by the decision maker, and it is based on the fact that a target will be out of order only if a certain level of damage is achieved. Therefore, there may be limited military value to assign a strike package to a target if the desired level of damage on the target cannot be achieved. If such an assignment is made, it leads to a waste of resources in terms of aircraft, weapons, and the personnel who are responsible for carrying out the mission.

This research allows the achieved level of damage for each attacked target to exceed the desired level of damage. Therefore, the desired levels of damages on the targets are lower bounds. However, it is assumed there is no need to exceed the desired level of damage for a target. The difference between the desired level of damage and the achieved level of damage on a target should be as small as possible to conserve resources.

In addition, some targets have higher priority compared to the other targets. The decision maker may wish to have the desired levels of damages on the targets in such a way that the desired level of damage for a target that has the highest priority is achieved first, and the rest of the levels of damages on the remaining targets are achieved sequentially from the highest to the lowest in terms of the target priority. This research carries out this objective, as well. If there is insufficient resource available in terms of aircraft and weapons, the model finds a solution in such a way that as many targets as possible are destroyed to a desired level of damage based on the target priority. In other words, if it is not possible to achieve the desired level of damage for each target due to resource constraints, the targets which are not destroyed to their desired levels of damages should be the ones which have low priority. Furthermore, the model developed in this research does not allow an assignment when the achievable level of damage is below the desired level of damage for a target even when this target has a higher priority than another target whose desired level of damage can be achieved.

The other objective of this research is to minimize the total cost of the final assignment which achieves the desired levels of damages on the targets. There may be several ways to assign strike packages to targets considering the bases in a country because the locations and the types and numbers of aircraft and weapons at the bases vary. The total cost of the mission basically depends on the types of aircraft and weapons and the distance flown. This research deals with minimizing the total cost of the strike package and target assignment as well. The model finds a solution that achieves the best assignment in terms of the desired level of damage on each target without considering the total cost in Phase I. It finds a new solution in Phase II that minimizes the total cost while achieving the same levels of damages on the targets determined in Phase I.

3.2 Mathematical Formulation

3.2.1 Definitions.

aircraft configuration: formation of aircraft based on the type of aircraft and the number of aircraft (e.g., 2 x F-16, 4 x F-4)

weapon configuration: formation of weapon based on the type of weapon and the number of weapons (e.g., 2 x MK-84, 4 x GBU-10)

strike package: particular type of aircraft configuration carrying a particular type of weapon configuration

3.2.2 Sets and Indices.

T={$1, 2, ..., \tau$} is the set of targets and $t \in T$

B={$1, 2, ..., \beta$} is the set of bases and $b \in B$

A_B={$1, 2, ..., \alpha_\beta$} is the set of aircraft configurations at base $b \in B$ and $a_b \in A_B$

W_B={$1, 2, ..., \omega_\beta$} is the set of weapon configurations at base $b \in B$ and $w_b \in W_B$

3.2.3 Parameters.

$acap_b$: available aircraft capacity at base $b \in B$

$wcap_{w_b}$: available weapon capacity that is needed to constitute $w_b \in W_B$ configuration at base $b \in B$

$cost_{a_b}$: cost of a single aircraft per Nautical Mile (NM) in the aircraft configuration $a_b \in A_B$ at base $b \in B$

$cost_{w_b}$: cost of a single weapon in the weapon configuration $w_b \in W_B$ at base $b \in B$

$dist_{tb}$: distance from base $b \in B$ to target $t \in T$ in Nautical Mile (NM) units

$obts_t$: the slack value that is obtained in Phase I

$reqpod_t$: desired level of damage for target $t \in T$

$pod_{tba_bw_b}$: POD of the strike package at base $b \in B$ containing aircraft configuration $a_b \in A_B$ and weapon configuration $w_b \in W_B$ on target $t \in T$

μ_t: priority of target $t \in T$ and $\mu_t \in Z^+$

n_{a_b}: number of aircraft in aircraft configuration $a_b \in A_B$

m_{w_b}: number of weapons in weapon configuration $w_b \in W_B$

3.2.4 Decision Variables.

$x_{tba_bw_b}$: 1, if a strike package at base $b \in B$ containing aircraft configuration $a_b \in A_B$ and weapon configuration $w_b \in W_B$ is assigned to target $t \in T$

0, otherwise

ξ_t: 1, if no strike package is assigned to target $t \in T$

0, otherwise

s_t: slack variable for the level of damage on target $t \in T$

3.2.5 Preprocessing.

There are two parameters that should be calculated before solving the model: *The POD of the strike package at base $b \in B$ containing aircraft configuration $a_b \in A_B$ and weapon configuration $w_b \in W_B$ on target $t \in T$ ($pod_{tba_bw_b}$), and the distance from base $b \in B$ to target $t \in T$ ($dist_{tb}$)*

Calculating the PODs for the strike packages prior to solving the model makes the mathematical formulation linear. After calculating these values, the strike planning problem, which is a variation of the WTA problem, can be solved as an assignment problem with additional constraints in Phase I and Phase II.

The problem here is that the number of decision variables associated with each POD increases combinatorially since the WTA problem is an NP-hard problem [23]. Therefore, the decision maker should determine the possible and reasonable (i.e., feasible) aircraft and weapon configurations.

The POD for the WTA problem can be obtained using:

$$Prob\,of\ Damage_{twm} = 1 - (1 - Prob\,of\ Damage_{tw})^{m} \tag{3.1}$$

where

t is a target,

w is a weapon type, and

m is the number of weapons.

$Prob\,of\ Damage_{twm}$ is the POD using m weapons of type w on target t

$Prob\,of\ Damage_{tw}$ is the unitary POD for a single weapon w on target t. [7, 19]

The PODs for the strike packages in this model can be calculated using Equation (3.1). The parameters that should be known before manipulating this equation are the number of aircraft in the strike package and the unitary POD for an aircraft in the strike package carrying particular type and number of weapons.

The distance from base $b \epsilon B$ to target $t \epsilon T$ ($dist_{tb}$) is calculated using the *Great Circle Distance (GCD)* equation. The GCD equation is:

$$\Delta\widehat{\sigma} = arctan(\frac{\sqrt{(cos\phi_f.sin\Delta\lambda)^2 + (cos\phi_s.sin\phi_f - sin\phi_s.cos\phi_f.cos\Delta\lambda)^2}}{sin\phi_s.sin\phi_f - cos\phi_s.cos\phi_f.cos\Delta\lambda}) \tag{3.2}$$

$$GCD = r.\Delta\widehat{\sigma} \tag{3.3}$$

where

$\phi_s,\ \lambda_s$: standpoint (lattitude, longitude),

$\phi_f,\ \lambda_f$: forepoint (lattitude, longitude),

$\Delta\widehat{\sigma}$: (spherical) angular difference/distance,

$\Delta\lambda$: the longitude difference between the standpoint and the forepoint, and

r : the average radius of the earth which is 3440.07 NM.

The coordinates are first converted to decimal degrees using $(Sign(Deg+(Min+Sec/60)/60))$. The *Sign* becomes 1 if the latitude is North (N) and -1 if the latitude is South (S). Likewise, the sign becomes 1 if the longitude is East (E) and -1 if the longitude is West (W). The decimal degrees should also be converted to radians multiplying by $(\pi/180)$.

The mathematical models for Phase I and Phase II are now presented and described.

3.2.6 *Phase I.*

$$\text{Minimize} \sum_{T} \{ (\sum_{B} \sum_{A_B} \sum_{W_B} x_{tba_bw_b}.pod_{tba_bw_b} - reqpod_t) + \mu_t.s_t \} \tag{3.4}$$

Subject to

$$\sum_{B} \sum_{A_B} \sum_{W_B} x_{tba_bw_b}.pod_{tba_bw_b} + s_t \geq reqpod_t \quad \textit{for each } t \,\epsilon\, T \tag{3.5}$$

$$\sum_{T} \sum_{A_B} \sum_{W_B} x_{tba_bw_b}.n_{a_b} \leq acap_b \quad \textit{for each } b \,\epsilon\, B \tag{3.6}$$

$$\sum_{T} \sum_{A_B} x_{tba_bw_b}.n_{a_b}.m_{w_b} \leq wcap_{w_b} \quad \textit{for each } b \,\epsilon\, B \textit{ and } w_b \,\epsilon\, W_B \tag{3.7}$$

$$\sum_{B} \sum_{A_B} \sum_{W_B} x_{tba_bw_b} \leq 1 \quad \textit{for each } t \,\epsilon\, T \tag{3.8}$$

$$s_t = reqpod_t.\xi_t \quad \textit{for each } t \,\epsilon\, T \tag{3.9}$$

$$x_{tba_bw_b} \,\epsilon\, \{0, 1\} \quad \textit{for each } t \,\epsilon\, T,\ b \,\epsilon\, B,\ a_b \,\epsilon\, A_B,\ w_b \,\epsilon\, W_B \tag{3.10}$$

$$\xi_t \in \{0, 1\} \qquad \textit{for each } t \in T \qquad (3.11)$$

$$s_t \geq 0 \qquad \textit{for each } t \in T \qquad (3.12)$$

The Objective Function (3.4) in Phase I minimizes the objective function value using the slack variables (s_t) and the decision variables ($x_{tba_bw_b}$). Each target has a priority (μ_t) based on its importance and a desired level of damage ($reqpod_t$) determined by the decision maker. The target which has the highest priority is the most important target and it needs to be attacked first.

As long as the slack variable s_t is not equal to zero, a value with respect to ($\mu_t.s_t$) is added to Objection Function (3.4). Even though the slack variables can take on any value greater than or equal to zero according to Constraint (3.12), Constraint (3.9) forces each slack variable to be either zero or the desired level of damage.

The purpose of Constraint (3.9) is to avoid assigning strike packages to targets when no available strike packages can achieve the desired level of damage on a target. There may be some instances of the strike planning problem such that no strike package has enough POD to satisfy the desired level of damage on a target because the desired level of damage for the target is too high or the PODs of the weapons are too low. This may occur even if there are enough resources in terms of the aircraft and the weapons because of the lower PODs of the strike packages with respect to the desired level of damage of the target. In this case, it may be undesirable to assign a strike package to a target, because it is not possible to achieve the desired level of damage on the target. The aircraft and the weapons that are not spent on these targets can be used to achieve the desired level of damage on other targets, which have lower priorities. This increases the number of targets that are destroyed to their desired levels of damage.

Since this is a minimization problem, the Objective Function (3.4) tries to make the slack variables zero starting from the most important target because the most important target increases the Objective Function (3.4) value most when not attacked due to its large target priority (μ_t). Therefore, the formulation seeks solutions sequentially starting from the most important target to the least important target.

The Objective Function (3.4) also tries not to exceed the desired level of damage on a target. Since it is minimizing the objective function value, the difference between the achieved level of damage and the desired level of damage should be as small as possible. The negative values of this difference mean that the desired level of damage is not achieved. In addition, Constraint (3.9) forces the slack variable (s_t) to be either equal to zero or to the desired level of damage ($reqpod_t$). Constraint (3.5) allows the slack variable (s_t) to take on any value greater than or equal to zero. If the slack variable (s_t) is not equal to zero according to Constraint (3.5), it should be equal to the desired level of damage according to Constraint (3.9). This means that the desired level of damage cannot be achieved and there is no need to assign a strike package that is not capable of achieving the desired level of damage on the target. Thus, this process makes the decision variables ($x_{tba_bw_b}$) zero in Constraint (3.5) so as not to unnecessarily spend resources. Therefore, the difference between the achieved level of damage and the desired level of damage in the Objective Function (3.4) becomes as small as possible to minimize overachievement of the level of damage.

Briefly, the model basically attempts to assign strike packages to the targets sequentially based on the target priority, and it avoids assigning strike packages to targets if the strike packages are not able to achieve the desired levels of damage on these targets.

Constraint (3.5) ensures the desired level of damage for a target $t \epsilon T$ is met or exceeded. If the decision variables ($x_{tba_bw_b}$) do not satisfy the desired level of damage due to insufficient resources or low PODs ($pod_{tba_bw_b}$), the slack variable (s_t) is used to make the solution feasible.

Constraint (3.6) ensures the available aircraft capacities for each base are not exceeded. Since the decision variables ($x_{tba_bw_b}$) can only be zero or one, multiplying this decision variable by the number of aircraft (n_{a_b}) in an aircraft configuration ($a_b \epsilon A_B$) determines the number of aircraft used for target $t \epsilon T$. The total number of aircraft used at base $b \epsilon B$ should be less than or equal to the aircraft capacity at base $b \epsilon B$.

Constraint (3.7) ensures the available weapon capacities for each base are not exceeded. The decision variables ($x_{tba_bw_b}$) should be multiplied by the number of weapons (m_{w_b}) in a weapon configuration ($w_b \epsilon W_B$) and the number of aircraft (n_{a_b}) in an aircraft configuration ($a_b \epsilon A_B$) to find the total number of weapons used for target

$t \epsilon T$. The total number of weapons used at base $b \epsilon B$ should be less than or equal to the weapon capacity for a particular type depending on the weapon configuration ($w_b \epsilon W_B$) at base $b \epsilon B$.

Constraint (3.8) ensures that only one strike package containing an aircraft configuration ($a_b \epsilon A_B$) and a weapon configuration ($w_b \epsilon W_B$) from base $b \epsilon B$ can be assigned to target $t \epsilon T$.

There may be numerous types of possible strike packages based on the different types of aircraft and the weapons, but it is not realistic to include all possible strike packages in a model. These possible strike packages increase the number of decision variables combinatorially since the WTA problem is an NP-hard problem and the strike planning problem is a variation of the WTA problem [23].

In real world applications, every base has different types of aircraft and there are some aircraft configurations that are commonly used among these aircraft combinations. There are also weapon configurations that are commonly used based on the types of weapons. These particular types of aircraft and weapons are mainly based on the special characteristics of the flight. It is undesirable for the pilots to fly with a configuration on which they have no training.

Some configurations are undesirable because of the characteristics of the mission. Each mission requires different types of supplementary aircraft such as *escort aircraft* and *CAP*. Each different strike package for a mission necessitates special training depending on the formation of the attacking aircraft and the supplementary aircraft.

In addition, the mathematical model maintains a linear formulation and the calculation of the PODs for different strike packages requires multiplication. If this multiplication is executed in the model, the mathematical formulation becomes nonlinear which is more challenging to solve since solvers for nonlinear formulations do not guarantee a global optimum.

Another way of obtaining the PODs for different strike packages is to calculate the PODs for all possible strike packages during preprocessing. In this model, preprocessing is accomplished in order to calculate the POD for all strike packages depending on the available aircraft configuration ($a_b \epsilon A_B$) and the weapon configuration ($w_b \epsilon W_B$) specified by the decision maker. The number of POD calculations is based on the possible number of strike packages used in the model and this number increases as the number of aircraft configurations ($a_b \epsilon A_B$) and weapon configurations ($w_b \epsilon W_B$) increase combinatorially. Therefore, there is no need to include all possible strike packages in the model if they are unrealistic in real world applications.

In this model, the possible aircraft and weapon configurations should be specified by the decision maker before solving the model and the POD should be calculated during preprocessing so the mathematical formulation remains linear.

Constraint (3.9) ensures the slack variable (s_t) should be either equal to zero or to the desired level of damage ($reqpod_t$) for target $t \epsilon T$ as mentioned above.

Constraints (3.10) and (3.11) ensure the decision variables ($x_{tba_bw_b}$) and (ξ_t) are either zero or one. The requirement for the decision variable ($x_{tba_bw_b}$) to be binary means the model should either assign a strike package to a target $t \epsilon T$ or not. The binary decision variable (ξ_t) makes the slack variable (s_t) either equal to zero or to the desired level of damage for a target $t \epsilon T$ so as not to waste resources in terms of aircraft and weapons.

Briefly, Phase I assigns the strike packages to the targets while satisfying the objectives:

- to achieve the desired level of damage on each target,
- to avoid assigning weapons to targets using the strike packages if the desired level of damage is not achievable,
- to avoid having a higher level of damage on each target than the desired level of damage.

However, this does not guarantee that the final strike package and target assignment is the most cost effective. Phase II assigns the strike packages to targets with a cost that is less than or equal to the cost of the assignment in Phase I while maintaining the levels of damage that are achieved in Phase I.

3.2.7 Phase II.

$$\text{Minimize} \sum_{T}\sum_{B}\sum_{A_B}\sum_{W_B} x_{tba_bw_b}.(cost_{a_b}.n_{a_b}.dist_{tb} + cost_{w_b}.m_{w_b}.n_{a_b}) \tag{3.13}$$

Subject to

$$\sum_{B}\sum_{A_B}\sum_{W_B} x_{tba_bw_b}.pod_{tba_bw_b} \geq reqpod_t - obts_t \quad \textit{for each } t \,\epsilon\, T \tag{3.14}$$

$$\sum_{T}\sum_{A_B}\sum_{W_B} x_{tba_bw_b}.n_{a_b} \leq acap_b \qquad \textit{for each } b \,\epsilon\, B \tag{3.15}$$

$$\sum_{T}\sum_{A_B} x_{tba_bw_b}.n_{a_b}.m_{w_b} \leq wcap_{w_b} \qquad \textit{for each } b \,\epsilon\, B \textit{ and } w_b \,\epsilon\, W_B \tag{3.16}$$

$$\sum_{B}\sum_{A_B}\sum_{W_B} x_{tba_bw_b} \leq 1 \qquad \textit{for each } t \,\epsilon\, T \tag{3.17}$$

$$x_{tba_bw_b} \,\epsilon\, \{0,1\} \qquad \textit{for each } t \,\epsilon\, T,\ b \,\epsilon\, B,\ a_b \,\epsilon\, A_B,\ w_b \,\epsilon\, W_B \tag{3.18}$$

The Objective Function (3.13) in Phase II minimizes the total cost of the final strike package and the target assignment based on the aircraft cost, the weapon cost and the distance flown. The cost of a single aircraft ($cost_{a_b}$) in the aircraft configuration ($a_b \, \epsilon \, A_B$) basically depends on the distance flown ($dist_{tb}$) from base $b \, \epsilon \, B$ to target $t \, \epsilon \, T$. The number of aircraft (n_{a_b}) in a configuration also needs to be considered to have the total *distance-based* cost of the strike package.

The total cost of the final strike package and the target assignment also includes the weapon costs. The single weapon cost ($cost_{w_b}$) and the number of weapons (m_{w_b}) in the weapon configuration ($w_b \, \epsilon \, W_B$) should be multiplied by the number of aircraft (n_{a_b}) in the aircraft configuration ($a_b \, \epsilon \, A_B$) to have the *weapon-based* cost of the strike package.

The Objective Function (3.13) in Phase II minimizes the sum of the *distance-based* and the *weapon-based* costs of the strike package and the target assignment.

Constraint (3.14) ensures that the strike package assignment for the target $t \, \epsilon \, T$ should maintain the achieved level of damage in Phase I. If the target $t \, \epsilon \, T$ is attacked in Phase I, the slack variable (s_t) is zero. If the target $t \, \epsilon \, T$ is not attacked in Phase I, the slack variable (s_t) equals the desired level of damage ($reqpod_t$) value. The obtained slack ($obts_t$) for target $t \, \epsilon \, T$ in Phase I has the same value as the slack variable (s_t) for target $t \, \epsilon \, T$ in the optimal solution of Phase I. The new right hand side ($reqpod_t$-$obts_t$) ensures the model finds feasible solutions that maintain the levels of damages achieved in Phase I.

Constraints (3.15), (3.16), (3.17), and (3.18) play the same role in Phase II as they played in Phase I.

Briefly, Phase II assigns the strike packages to the targets while satisfying the objective:

- to minimize total cost of the strike package and target assignments.

3.3 Summary

This chapter explained the solution methodology developed in this research for the static strike planning problem which is a variation of the WTA problem. The problem statement and the objectives of this research are also discussed in detail in this chapter. The next chapter analyzes the solution methodology presented in this chapter in terms of the solution time and cost efficiency.

IV. Data and Analysis

In this chapter, the process of building the required *data* to solve the strike planning problem and solver type are presented in Section 4.1. Next, the *Optimality Tolerance Analysis* is executed and a particular optimality tolerance for the *Resource Capacity Analysis* is determined based on the optimality tolerance analysis in Section 4.2. Next, the effect of changing the aircraft and weapon capacities are analyzed in Section 4.3. Finally, the *Cost Efficiency Analysis*, which is one of the main objectives in this research (see Section 3.2), is performed in Section 4.4.

4.1 Data and Implementation

4.1.1 Solver Type.

There are several software packages which solve optimization problems with exact methods. The differences among these solvers are: *type of optimization problem, problem definition, solution methodology, analysis of results, diagnosis of errors, graphical interfaces, and limitations on decision variables and constraints.* [2]

The strike planning problem in this research is formulated as an MILP where the integer variables are binary and the solution methodology is an *exact method*. In addition, the number of decision variables for a representative model of the Turkish Air Force is significant. For instance, the possible strike packages for a single target in this research is 420 including 8 bases, 2 different types of aircraft, 8 different types of weapons, and 35 different aircraft and weapon configurations where each of these strike packages is represented by a binary decision variable. If the decision maker would like to solve the strike planning problem for even 20 targets, the number of binary decision variables required is approximately 8400, which increases the computational effort to solve the problem. This is not surprising because the WTA problem is NP-hard and the strike planning problem is a variation of the WTA problem. [23]. Therefore, the solver used to solve the strike planning problem should have the ability to deal with a large number of decision variables.

In addition, the decision maker should input different parameters to build different problems depending on the number of targets and the aircraft and weapon capacity, and see the final assignments clearly rather than searching for them among numerous decision variables in a solution report.

Under these considerations, two types of solvers are compared in this research: *Microsoft Excel Solver* and *LINGO*.

Microsoft Excel Solver is a commercial tool used to solve optimization problems using an Excel spreadsheet structure. It is easy to build a linear model in an Excel spreadsheet due to its matrix structure. Also, the Excel spreadsheet allows the user to input parameters easily. However, it has a limitation on the number of decision variables and constraints, and it does not give consistent and precise results when solving *binary integer models*.

LINGO is a comprehensive tool that is designed to make the formulation of the optimization problems more straightforward and solves them more efficiently. The main purpose of LINGO is to allow the user to build the model quickly, solve it, and interpret the results to verify the formulation. It is a powerful language in formulating and solving optimization problems because it is integrated with a set of robust built-in solvers capable of efficiently solving most classes of optimization models. [14, 15]

LINGO has also the ability to import data from Excel spreadsheets to solve an optimization problem and export the resulting output data back to the spreadsheet using an *Object Linking and Embedding (OLE)* function. Therefore, the user can build a data structure in an Excel spreadsheet, then formulate and solve the problem in LINGO after retrieving the data from the Excel spreadsheet. [14]

Therefore, the solver type used in this research is *LINGO interfaced with Excel spreadsheets* in order to solve a representative strike planning problem consistently and precisely in LINGO and get the benefit of the spreadsheet structure of Excel to input data required for the problem and display the results in an easy way.

There are also several versions of LINGO to solve optimization problems with differing limitations on the number of decision variables and constraints. The version of LINGO used in this research is the *Extended LINGO* version, which is capable of handling an unlimited number of decision variables and constraints.

Finally, all of the analyses in this research was performed on a computer with an Intel Core (2) Duo P8400 @ 2.26 GHz Processor, 3 GB RAM and Windows Vista Home Premium 64-Bit operating system.

4.1.2 Inputting the Data.

There are constant parameters used in this analysis such as the *number of bases*, the *type of aircraft at each base*, and the *particular type of strike package*s in terms of the type and the number of aircraft and weapons.

Table 4.1: Types of Aircraft at the Bases

BASE 1	BASE 2	BASE 3	BASE 4	BASE 5	BASE 6	BASE 7	BASE 8
F-4	F-16	F-4	F-16	F-16	F-4	F-4	F-16

Table 4.2: Weapon Configurations for Different Types of Aircraft

	F-4	F-16
MK-82	2, 4, 8	2, 4, 8
MK-84	2, 4, 6	2, 4, 6
GBU-10	2, 4	2, 4
GBU-12	2, 4	2, 4
AGM-65A	2, 4, 6	2, 4, 6
AGM-65G	2, 4	2, 4
MK-20	N/A	2, 4, 8
MK-83	N/A	2, 4

Table 4.3: Allowable Number of Aircraft in a Strike Package

F-4	F-16
2, 4, 6	2, 4, 6

Table 4.1 shows the number of bases and type of aircraft at each base. The allowable number of weapons on a particular type of aircraft and the allowable number of aircraft in a strike package are shown in Table 4.2 and in Table 4.3, respectively.

These parameters should be specified in accordance with the capabilities of the Turkish Air Force to decrease the number of decision variables as explained in

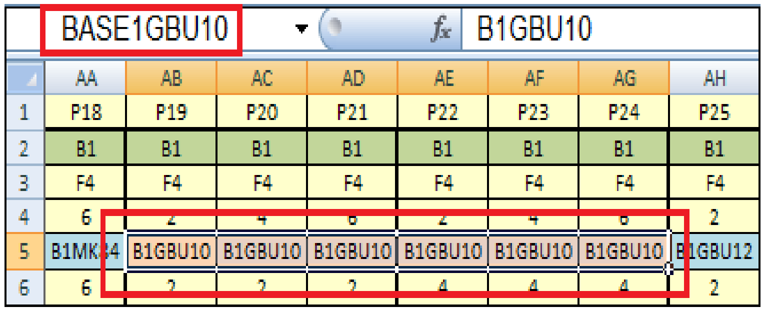


Figure 4.1: Range Name Illustration for a Weapon Type at a Base

Section 3.2. The user should give a range name to the interval of cells in an Excel spreadsheet for each type of weapon at each base and the associated number of aircraft and weapons to construct sets for LINGO. The sets allow the user to group a large number of similar decision variables and constraints. This results in quick and easy model building. This is illustrated in Figure 4.1.

A strike package is defined by the type and number of aircraft and weapons, and it is created using cells on an Excel spreadsheet in this research. In Figure 4.1, Row 1 shows the number of strike packages and Row 2 shows the number of bases from where the strike packages take-off. Row 3 shows the type of aircraft in the strike packages and Row 4 shows the number of aircraft in the strike packages. Similarly, Row 5 shows the type of weapon in the strike packages and Row 4 shows the number of weapons in the strike packages. For instance, Column AH in Figure 4.1 implies a strike package from Base 1 containing 2 F-4s carrying 2 GBU-12s.

The user also needs to define the range names in an Excel spreadsheet for PODs, strike package and target assignments which are either 0 or 1 implying that a strike package is assigned to a target or not, and distances to targets for each strike

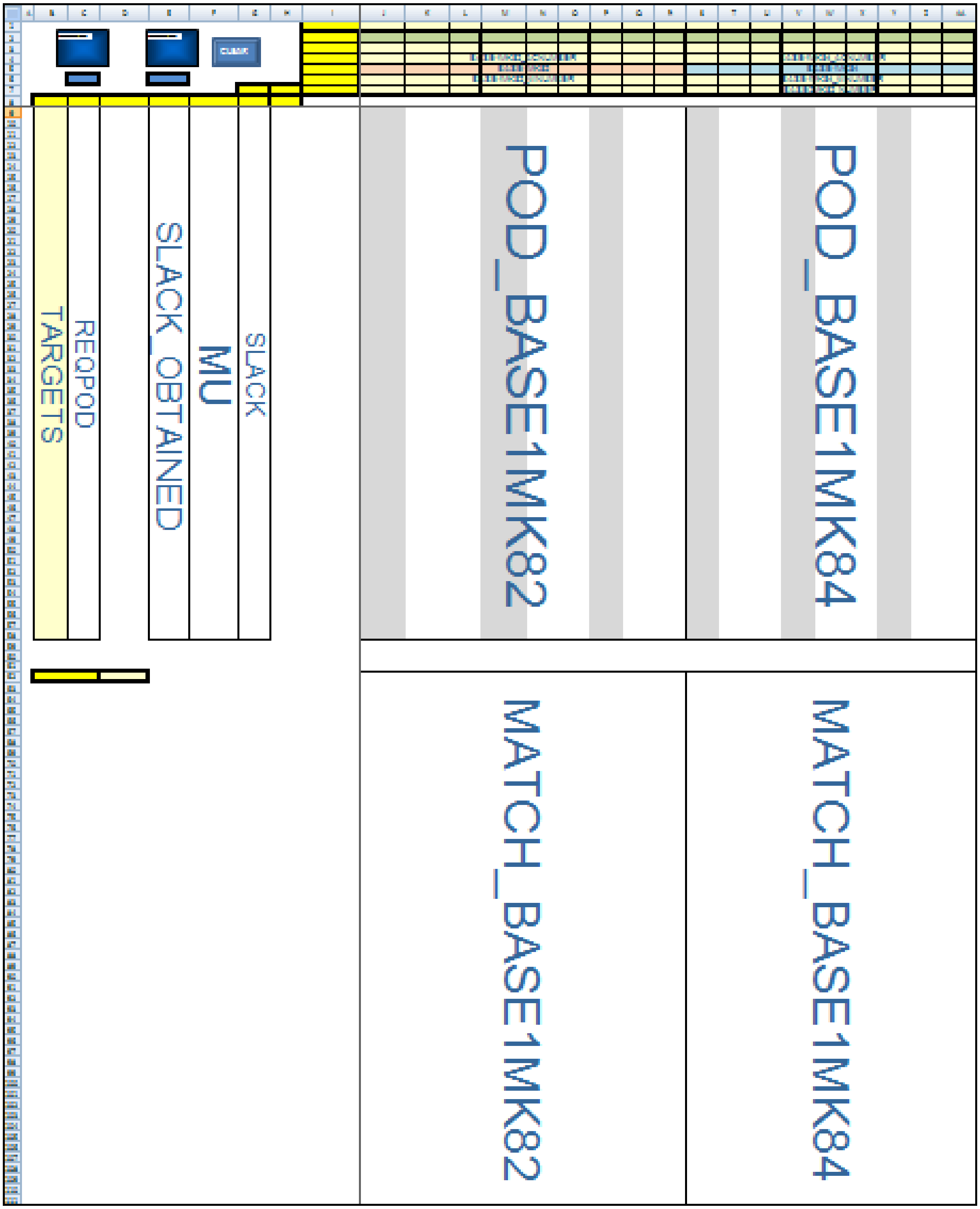


Figure 4.2: General Range Name Illustration

```
ASSIGNMENTSBASE1MK82(TARGETS, BASE1MK82): POD_BASE1MK82, MATCH_BASE1MK82, DISTANCE_BASE1MK82;
ASSIGNMENTSBASE1MK84(TARGETS, BASE1MK84): POD_BASE1MK84, MATCH_BASE1MK84, DISTANCE_BASE1MK84;
ASSIGNMENTSBASE1GBU10(TARGETS, BASE1GBU10): POD_BASE1GBU10, MATCH_BASE1GBU10, DISTANCE_BASE1GBU10;
ASSIGNMENTSBASE1GBU12(TARGETS, BASE1GBU12): POD_BASE1GBU12, MATCH_BASE1GBU12, DISTANCE_BASE1GBU12;
ASSIGNMENTSBASE1AGM65A(TARGETS, BASE1AGM65A): POD_BASE1AGM65A, MATCH_BASE1AGM65A, DISTANCE_BASE1AGM65A;
ASSIGNMENTSBASE1AGM65G(TARGETS, BASE1AGM65G): POD_BASE1AGM65G, MATCH_BASE1AGM65G, DISTANCE_BASE1AGM65G;
ASSIGNMENTSBASE1MK20(TARGETS, BASE1MK20): POD_BASE1MK20, MATCH_BASE1MK20, DISTANCE_BASE1MK20;
ASSIGNMENTSBASE1MK83(TARGETS, BASE1MK83): POD_BASE1MK83, MATCH_BASE1MK83, DISTANCE_BASE1MK83;
```

Figure 4.3: Defining Sets in LINGO

package in accordance with each type of weapon and the number of targets, since the model should consider the aircraft and the weapon capacities for each type at every base, and each base has a different number of aircraft and weapons for each type. Each strike package and target assignment in the model is multiplied by the number of aircraft and weapons in the associated strike package. If the strike package and target assignment is 1, then the numbers of aircraft and weapons are added to the used number of aircraft and weapons at the associated base.

Moreover, a strike package and target assignment is basically determined by POD of the strike package and the distance to the target from the base where the strike package takes-off.

POD of a strike package on a target is used to satisfy the desired level of damage on the target considering the overachievement and underachievement of the desired level of damage on the target which are explained in Chapter 3 and every strike package has a separate POD on a target even though the PODs may be the same for different strike packages on the same target.

The model in this research assigns only one strike package for a target. Then, the numbers of aircraft and weapons in the strike package are added to the numbers of aircraft and weapons used at the associated base. However, a strike package with the same type and number of aircraft and weapons from the same base can be assigned to several targets as long as the aircraft and weapon capacities at the associated base are not exceeded.

The total numbers of aircraft and weapons used of particular types at a base are calculated based on the strike packages assigned to several targets. Therefore, the user should define the range names in accordance with each type of weapon at each base and the number of targets to find out the number of weapons of particular types used at a base. Then, the number of aircraft used at a base can be calculated by summing all aircraft used in the strike packages carrying different available types of weapons at the base.

The user may want to define a range name for each strike package. The numbers of aircraft and weapons of particular types used can be calculated by summing the numbers of aircraft and weapons used in each strike package but this takes more time to construct the model compared to defining the range names for each type of weapon, since increasing the number of columns in a range name saves time in constructing the model. However, the biggest number of columns that can be included in a range name should not exceed the number of columns for each type of weapon because the model should consider the weapon capacities for each type.

On the other hand, the distance to target from a base where a strike package takes-off is used in the Objective Function (3.13) to minimize the total cost. Therefore, the distance between each strike package and target assignment in the model is considered and the user should also define range names for the distances in accordance with each type of weapon, because the distances are added to the Objective Function (3.13) after multiplication by the strike package and target assignments. The strike package and target assignments are given range names in accordance with each type of weapon to satisfy the weapon capacities for each type, and LINGO does not allow the user to manipulate data sets with different sizes. Therefore, the user should give range names to distances in accordance with each type of weapon, as well.

The other range names which should be defined are costs for each type of aircraft and weapons, the aircraft and the weapon capacities for each type at each base, the targets and associated desired levels of damages, priority values (μ), slack variables, and the obtained slack values.

Briefly, the dimension of the range names of PODs, the assignments, and the distances should be number of targets by the number of strike packages for a particular type of weapon at a particular base. This differs in accordance with each type of weapon since the number of allowable strike packages for different types of weapons are not the same. Figure 4.2 illustrates the range names in Excel for a target and weapon combination and Figure 4.3 shows the method of defining sets in LINGO.

Therefore, the data structure in an Excel spreadsheet should represent a model as generally as possible because adding a new possible strike package, a base or more targets may be a cumbersome effort to build the data structure in an Excel spread-

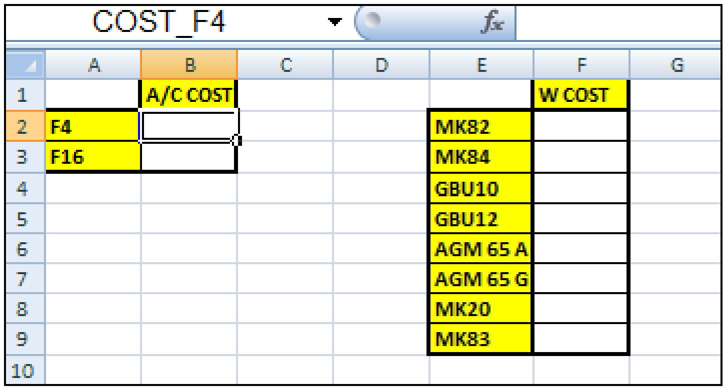


Figure 4.4: Aircraft and Weapon Cost Input Spreadsheet

sheet, which is required to solve the strike planning in LINGO, due to the fact that adding more data is time consuming. However, building smaller data structures from a more generalized one by deleting rows and columns only is easy because deleting rows and columns does not affect the range names but adding a new cell to a range requires defining the range name again.

After building the general framework of the data structure defining the constant parameters in the Excel spreadsheets as discussed above, the user inputs different aircraft and weapon costs and capacities, using the Excel spreadsheets as shown in Figures 4.4, 4.5, and 4.6. Also, the used aircraft and weapon capacities are calculated after the strike planning problem is solved and this allows the decision maker to compare the used aircraft and weapons with the aircraft and weapon capacities.

Each strike package in the model should be defined distinctively in terms of the type and number of aircraft and weapons since each strike package has a different POD on a target and these PODs should be calculated in the preprocessing stage. Therefore, each strike package from the same base has the same base coordinates.

		USED	ACAP
F4	BASE 1	0	120
F16	BASE 2	0	120
F4	BASE 3	0	120
F16	BASE 4	0	120
F16	BASE 5	0	120
F4	BASE 6	0	120
F4	BASE 7	0	120
F16	BASE 8	0	120

ASSIGNMENT CAPACITY

Figure 4.5: Aircraft Capacity Input and Used Aircraft Display Spreadsheet

	USED	WCAP	USED	WCAP	USED	WCAP	USED	WCAP	USED	WCAP	USED	WCAP	USED	WCAP	USED	WCAP
	MK82	MK82	MK84	MK84	GBU10	GBU10	GBU12	GBU12	AGM 65A	AGM 65A	AGM 65G	AGM 65G	MK20	MK20	MK83	MK83
BASE 1	0	30	0	30	0	30	0	30	0	30	0	30	0	30	0	30
BASE 2	0	30	0	30	0	30	0	30	0	30	0	30		0		0
BASE 3	0	30	0	30	0	30	0	30	0	30	0	30	0	30	0	30
BASE 4	0	30	0	30	0	30	0	30	0	30	0	30		0		0
BASE 5	0	30	0	30	0	30	0	30	0	30	0	30		0		0
BASE 6	0	30	0	30	0	30	0	30	0	30	0	30	0	30	0	30
BASE 7	0	30	0	30	0	30	0	30	0	30	0	30	0	30	0	30
BASE 8	0	30	0	30	0	30	0	30	0	30	0	30		0		0

ASSIGNMENT CAPACITY TYPE DISTANCE BASE COORDINATES COST Sayfa1

Figure 4.6: Weapon Capacity Input and Used Weapon Display Spreadsheet

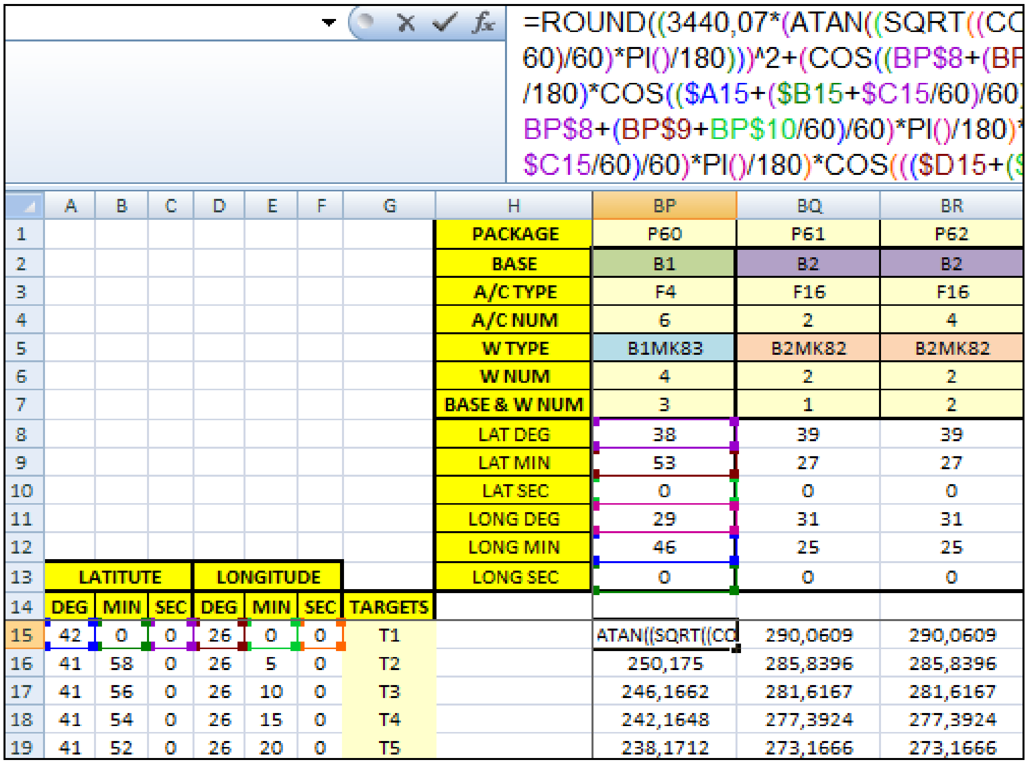
A1 f_x BASE

	A	B	C	D	E	F	G	H	I
1	BASE	B1	B2	B3	B4	B5	B6	B7	B8
2	LAT DEG								
3	LAT MIN								
4	LAT SEC								
5	LONG DEG								
6	LONG MIN								
7	LONG SEC								

Figure 4.7: Base Coordinates Input Spreadsheet

However, the user should not input the base coordinates for all strike packages in the data structure. The user inputs the base coordinates in a small spreadsheet in Figure 4.7 and all strike packages (as shown in Figure 4.8) take their associated base coordinates from this spreadsheet. The user also inputs the target coordinates in a spreadsheet in Figure 4.8 to calculate the distances between each target and each base.

Finally, the desired level of damage and the priority value of each target, and the POD for each strike package and target combination should be specified using the assignment spreadsheet in Figure 4.9. However, the user does not have to input all PODs into the assignment spreadsheet since the only necessary POD to input is the *unitary POD*. A strike package which carries a particular type and number of weapons with the smallest number of aircraft has the unitary POD, since increasing the number of aircraft in a strike package increases the POD and this can be calculated using Equation (3.1). For instance, a strike package with 2 F-16s carrying 2 GBU-12s has a unitary POD for a particular target. Then, the POD for a strike package with 4 F-16s carrying 2 GBU-12s can be calculated using Equation (3.1). 14 of 420 PODs



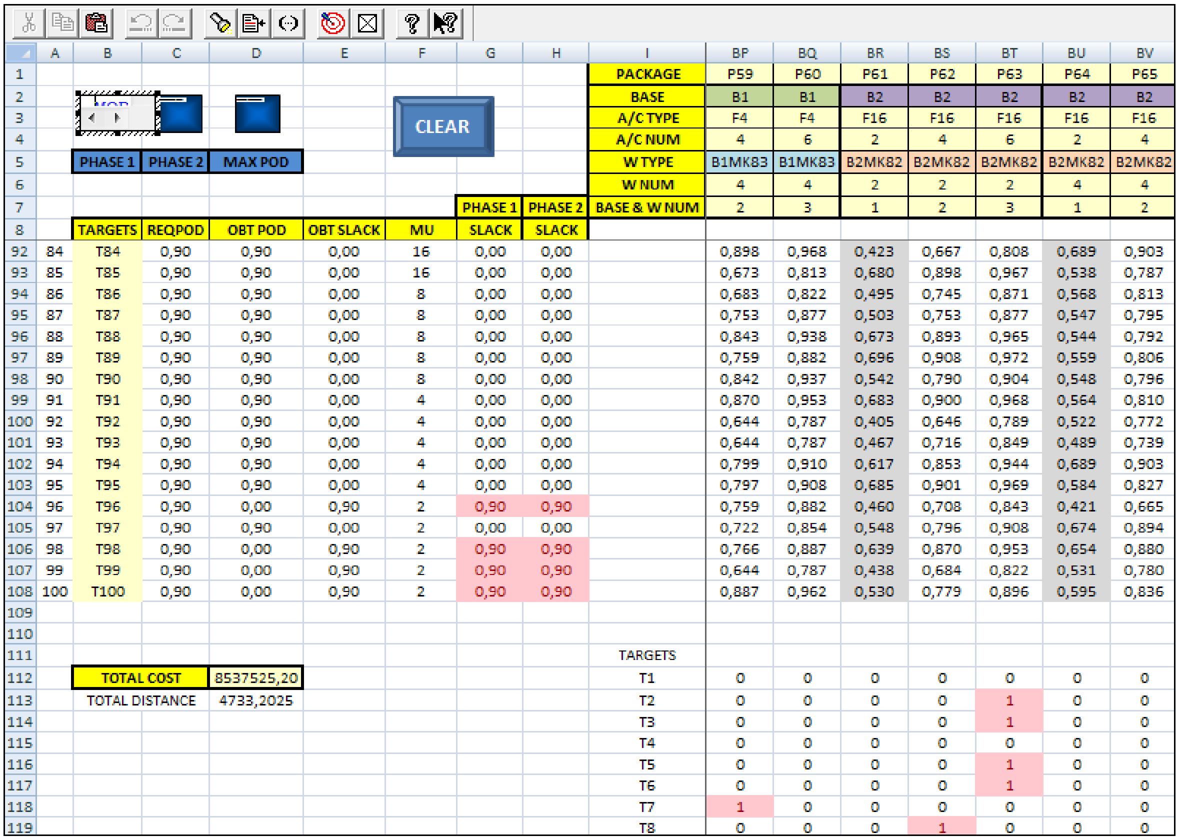
=ROUND((3440,07*(ATAN((SQRT((CC
60)/60)*PI()/180)))^2+(COS((BP$8+(BF
/180)*COS(($A15+($B15+$C15/60)/60
BP$8+(BP$9+BP$10/60)/60)*PI()/180)*
$C15/60)/60)*PI()/180)*COS((($D15+($

	A	B	C	D	E	F	G	H	BP	BQ	BR
1								PACKAGE	P60	P61	P62
2								BASE	B1	B2	B2
3								A/C TYPE	F4	F16	F16
4								A/C NUM	6	2	4
5								W TYPE	B1MK83	B2MK82	B2MK82
6								W NUM	4	2	2
7								BASE & W NUM	3	1	2
8								LAT DEG	38	39	39
9								LAT MIN	53	27	27
10								LAT SEC	0	0	0
11								LONG DEG	29	31	31
12								LONG MIN	46	25	25
13	LATITUTE			LONGITUDE				LONG SEC	0	0	0
14	DEG	MIN	SEC	DEG	MIN	SEC	TARGETS				
15	42	0	0	26	0	0	T1		ATAN((SQRT((CC	290,0609	290,0609
16	41	58	0	26	5	0	T2		250,175	285,8396	285,8396
17	41	56	0	26	10	0	T3		246,1662	281,6167	281,6167
18	41	54	0	26	15	0	T4		242,1648	277,3924	277,3924
19	41	52	0	26	20	0	T5		238,1712	273,1666	273,1666

Figure 4.8: Target Coordinates Input and Distance Calculation Spreadsheet

for a target should be input into the data structure in the preprocessing stage since there are 420 possible strike packages in this analysis and 14 of them is sufficient to calculate all PODs for a target. These 14 strike packages are constructed from only 2 different bases since each base has only one type of aircraft and there are 2 types of aircraft in this analysis. The PODs for the other strike packages can be exported from the cells of these 14 strike packages using Excel spreadsheet structure.

Note that the user also has to input the same number of unitary PODs into the model to solve the strike planning problem non-linearly as solving the strike planning problem linearly. However, the non-linear model finds out the PODs for all strike packages while it solves the problem and this increases the solution time whereas



	A	B	C	D	E	F	G	H	I	BP	BQ	BR	BS	BT	BU	BV
1									PACKAGE	P59	P60	P61	P62	P63	P64	P65
2						CLEAR			BASE	B1	B1	B2	B2	B2	B2	B2
3									A/C TYPE	F4	F4	F16	F16	F16	F16	F16
4									A/C NUM	4	6	2	4	6	2	4
5		PHASE 1	PHASE 2	MAX POD					W TYPE	B1MK83	B1MK83	B2MK82	B2MK82	B2MK82	B2MK82	B2MK82
6									W NUM	4	4	2	2	2	4	4
7							PHASE 1	PHASE 2	BASE & W NUM	2	3	1	2	3	1	2
8		TARGETS	REQPOD	OBT POD	OBT SLACK	MU	SLACK	SLACK								
92	84	T84	0,90	0,90	0,00	16	0,00	0,00		0,898	0,968	0,423	0,667	0,808	0,689	0,903
93	85	T85	0,90	0,90	0,00	16	0,00	0,00		0,673	0,813	0,680	0,898	0,967	0,538	0,787
94	86	T86	0,90	0,90	0,00	8	0,00	0,00		0,683	0,822	0,495	0,745	0,871	0,568	0,813
95	87	T87	0,90	0,90	0,00	8	0,00	0,00		0,753	0,877	0,503	0,753	0,877	0,547	0,795
96	88	T88	0,90	0,90	0,00	8	0,00	0,00		0,843	0,938	0,673	0,893	0,965	0,544	0,792
97	89	T89	0,90	0,90	0,00	8	0,00	0,00		0,759	0,882	0,696	0,908	0,972	0,559	0,806
98	90	T90	0,90	0,90	0,00	8	0,00	0,00		0,842	0,937	0,542	0,790	0,904	0,548	0,796
99	91	T91	0,90	0,90	0,00	4	0,00	0,00		0,870	0,953	0,683	0,900	0,968	0,564	0,810
100	92	T92	0,90	0,90	0,00	4	0,00	0,00		0,644	0,787	0,405	0,646	0,789	0,522	0,772
101	93	T93	0,90	0,90	0,00	4	0,00	0,00		0,644	0,787	0,467	0,716	0,849	0,489	0,739
102	94	T94	0,90	0,90	0,00	4	0,00	0,00		0,799	0,910	0,617	0,853	0,944	0,689	0,903
103	95	T95	0,90	0,90	0,00	4	0,00	0,00		0,797	0,908	0,685	0,901	0,969	0,584	0,827
104	96	T96	0,90	0,00	0,90	2	0,90	0,90		0,759	0,882	0,460	0,708	0,843	0,421	0,665
105	97	T97	0,90	0,90	0,00	2	0,00	0,00		0,722	0,854	0,548	0,796	0,908	0,674	0,894
106	98	T98	0,90	0,00	0,90	2	0,90	0,90		0,766	0,887	0,639	0,870	0,953	0,654	0,880
107	99	T99	0,90	0,00	0,90	2	0,90	0,90		0,644	0,787	0,438	0,684	0,822	0,531	0,780
108	100	T100	0,90	0,00	0,90	2	0,90	0,90		0,887	0,962	0,530	0,779	0,896	0,595	0,836
109																
110																
111									TARGETS							
112		TOTAL COST		8537525,20					T1	0	0	0	0	0	0	0
113		TOTAL DISTANCE		4733,2025					T2	0	0	0	0	1	0	0
114									T3	0	0	0	0	1	0	0
115									T4	0	0	0	0	0	0	0
116									T5	0	0	0	0	1	0	0
117									T6	0	0	0	0	1	0	0
118									T7	1	0	0	0	0	0	0
119									T8	0	0	0	1	0	0	0

Figure 4.9: Assignment Spreadsheet

the PODs in this research are calculated in the preprocessing stage. The number of PODs requiring to input into the model also decreases as the number of the same type of targets increase in the model since a POD is determined by a strike package and a target combination and a strike package has the same POD on the same type of targets.

4.1.3 Solving the Model.

The LINGO application can be inserted in the Excel spreadsheet as an object and the LINGO code is pasted onto the LINGO object. Once the user selects the LINGO object on the Excel spreadsheet, the LINGO toolbar is displayed in place of the Excel spreadsheet toolbar. Then, the user can start the model by clicking the Solve button on the LINGO toolbar. Note that the LINGO application must be opened before attempting to solve the model in the Excel spreadsheet; otherwise, an error message indicating that the LINGO application cannot be inserted to the spreadsheet is displayed. The model can also be run using LINGO only. Similarly, the Excel spreadsheet must be opened as long as the Excel file location in which there are sets and associated attributes such as number of aircraft and weapons, PODs, distances, etc. is not specified in the OLE command, which imports the data from the spreadsheet and exports it back to the spreadsheet again.

After clicking on the Solve button, LINGO compiles the model, solves it, and exports the solution of the final assignment into the Excel spreadsheet. Compiling the model in LINGO and exporting the solutions to the Excel spreadsheet takes approximately 6 and 20 seconds for a target set of 100, respectively, and the overall solution time, which is analyzed in the following sections, include these times. These times decrease as the number of targets in the model decreases. Note that Phase I should be solved first in order to solve Phase II since Phase II uses the slack values obtained in Phase I to assign the strike packages to the same targets as in Phase I while minimizing the cost.

4.2 Optimality Tolerance Analysis

The optimality tolerance used in this research is the *relative optimality tolerance* which is a value r between 0 and 1. It implies that the branch-and-bound solver should only search for integer solutions with objective function values at least $100 \cdot r\%$ better than the best integer solution found so far. This guarantees that the solution is within $100 \cdot r$ % of the optimal solution. Moreover, the relative optimality tolerance greatly decreases the solution time. For instance, the alternative of getting the near optimal solution, which is within a few percentage points of the *true optimal* solution, in several *minutes* on large integer models as opposed to the true optimal solution in several *days* makes the use of an optimality tolerance a beneficial trade-off tool between running time and solution quality where the true optimal solution is defined as the theoretical objective bound. [14]

Table 4.4: Optimality Gap with Optimality Tolerance of 0.0001 %

	ACAP	WCAP	TARGETS	OPT GAP (%)	TIME (secs)
PHASE I	60	30	100	0.000265	18000
PHASE II	60	30	100	0.67	18000

Table 4.4 shows the optimality gap between the true optimal solution and the best integer solution after 5 hours. The solver was interrupted after 5 hours because the planning of an ATO takes approximately 2 days and this planning contains *target selection, weapon and target allocation, mission formation and assignment, mission routing and scheduling, and contingency plans.* [7] Therefore, the ATO planner has only a couple of hours to determine the weapon and target allocation, and the mission formation.

ACAP and *WCAP* in Table 4.4 display the *aircraft capacity* and the *weapon capacity* at each base, respectively. In fact, the aircraft and the weapon capacities

Table 4.5: Optimality Gap of Phase I with Optimality Tolerance of 0.0001 % in 60 seconds

	ACAP	WCAP	TARGETS	OPT GAP (%)	TIME (secs)
PHASE I	60	30	100	0.000530	60

are based on the particular types of aircraft and weapons but all bases have the same number of aircraft and weapons of each type in this analysis. The aircraft capacity of 60 and the weapon capacity of 30 increases the solution time significantly as they play a *strictly binding constraint* role on the attack of all 100 targets in this instance of the formulation. *Resource Capacity Analysis* is discussed in Section 4.3. In this strictly bounded instance, the effect of the optimality tolerance can be seen clearly because finding the true optimal solution requires a large amount of time where the true optimal solution is defined as the theoretical objective bound.

The targets in T are divided into 20 target groups in this analysis and each target group has a different priority value, and every single target in a target group has the same priority. For example, if $|T| = 100$, then every target group in T contains 5 single targets. This ensures that the model in this research assigns the weapons to targets in accordance with the target group priority.

The *optimality gap* is defined as the gap between the best integer solution found and the true optimal solution. Phase I of this instance, consisting of 60 aircraft and 30 weapons for each type at each base, converges to the optimal solution with an optimality gap of 0.000530 % in 1 minute as shown in Table 4.5. This is a very small gap and close to the near optimal solution which was found in 5 hours. The

Table 4.6: Optimality Gap Changes of Phase II with Optimality Tolerance of 0.0001%

	ACAP	WCAP	TARGETS	OPT GAP (%)	TIME (secs)
PHASE II	60	30	100	1.97	60
PHASE II	60	30	100	1.1	300
PHASE II	60	30	100	0.87	600
PHASE II	60	30	100	0.69	3600
PHASE II	60	30	100	0.67	18000

Table 4.7: Solution Times of Phase II with Different Optimality Tolerances

	ACAP	WCAP	TARGETS	OPT TOL (%)	TIME (secs)
PHASE II	60	30	100	2	116
PHASE II	60	30	100	1	357
PHASE II	60	30	100	0.9	429
PHASE II	60	30	100	0.8	727
PHASE II	60	30	100	0.7	1674

optimality gap difference between these two solutions is 0.000265 % and the solution time difference compensates for this optimality gap.

Table 4.6 shows that the optimality gap decreases as the elapsed running time increases. In this example, the model is allowed to run 5 hours and the optimality gap percentages and the associated time values are snapshot values along with the running process. The optimality gap difference between the near optimal solution in 1 minute and the near optimal solution in 5 hours is 1.3 % where the total cost of the weapon and target assignment in 1 minute is \$ $1.42182 \cdot 10^7$ and the total cost of the weapon and target assignment in 5 hours is \$ $1.41529 \cdot 10^7$. The cost difference between these two near optimal solution is \$ 65,300 which is very small compared to the total cost in 5 hours. The acceptable trade-off between the solution time and the solution quality depends on the decision maker.

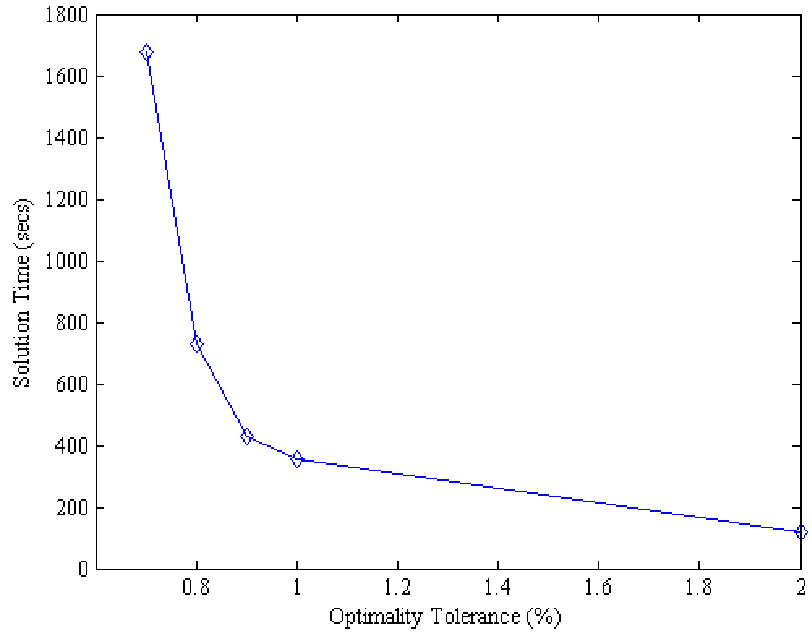


Figure 4.10: Solution Times of Phase II with Different Optimality Tolerances

Table 4.7 and Figure 4.10 are built according to the optimality gap percentages and elapsed running times in Table 4.6. In this case, the *time* for an instance implies the solution time with the optimality tolerance associated with it rather than the elapsed running time in Table 4.6.

It is clear in Table 4.7 and Figure 4.10 that the solution time decreases as the optimality tolerance increases, and the decision maker can make a trade-off between the solution quality and the solution time.

The optimality tolerance of 0.7 % is selected for the following analyses since the optimality tolerance of 0.7 % finds a solution for the instance of the strike planning problem, consisting of 100 targets, 60 aircraft at each base, and 30 weapons for each type at each base, in less than half an hour. If the optimality tolerance is set to 0.6%, the solution is not found after 5 hours.

However, the optimality tolerance of 0.7 % is not a generally acceptable optimality tolerance for the strike planning problem. The optimality of tolerance of 0.7 % is selected for an instance of the strike planning problem containing 100 targets, 60 aircraft at each base, and 30 weapons for each type at each base in this analysis. A generally acceptable optimality tolerance for the strike planning problem can be found applying *Design of Experiments (DOE)* on the optimality tolerance.

4.3 Resource Capacity Analysis

4.3.1 Increasing the Capacity.

In this subsection, the analysis investigates the affect of increasing the number of aircraft only, the affect of increasing the number of weapons only, and the affect of increasing both the number of aircraft and weapons on the solution time.

Increasing the aircraft capacity at each base yields erratic results for Phase II in terms of the solution time as shown in Table 4.8 and Figure 4.11. The solution times for Phase I are approximately the same for each aircraft and weapon capacity combination as can be seen in Table 4.8 and Figure 4.11. In addition, the solutions with increasing number of aircraft are not necessarily true optimal solutions as the optimality tolerance is set to 0.7 % in this analysis. In other words, there is a gap between the true optimal solution and the best integer solution that is within 0.7% of the true optimal solution. Although LINGO does have the ability to find the true

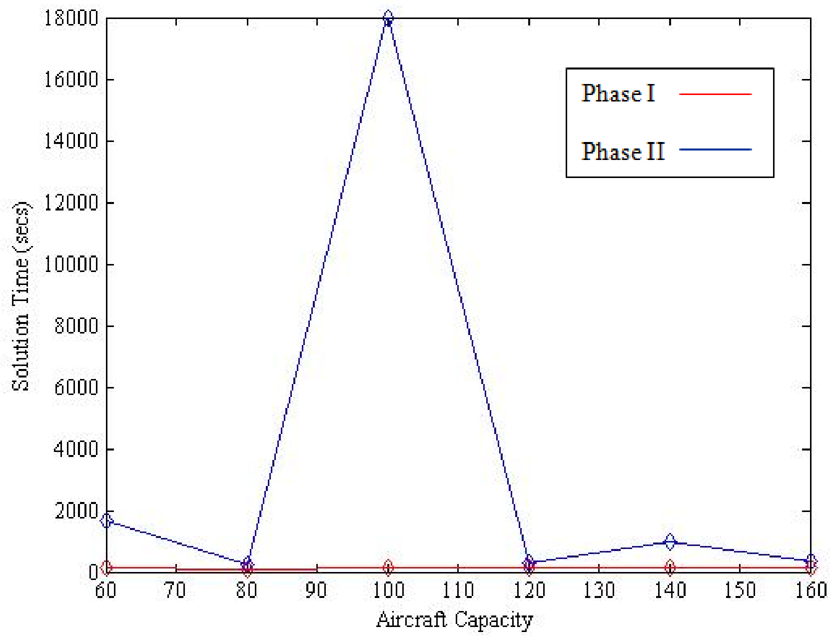


Figure 4.11: Solution Times (in seconds) with Increasing Aircraft Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

optimal solution with optimality tolerance other than 0, it does not find the true optimal solutions in this instance.

Therefore, increasing the aircraft capacity while maintaining the given weapon capacities does not consistently decrease the solution time.

Table 4.8: Solution Times (in seconds) with Increasing Aircraft Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

ACAP	WCAP	PHASE I	PHASE II	TRUE OPTIMAL
60	30	128	1674	NO
80	30	104	247	NO
100	30	119	> 18000	NO
120	30	130	299	NO
140	30	130	968	NO
160	30	133	363	NO

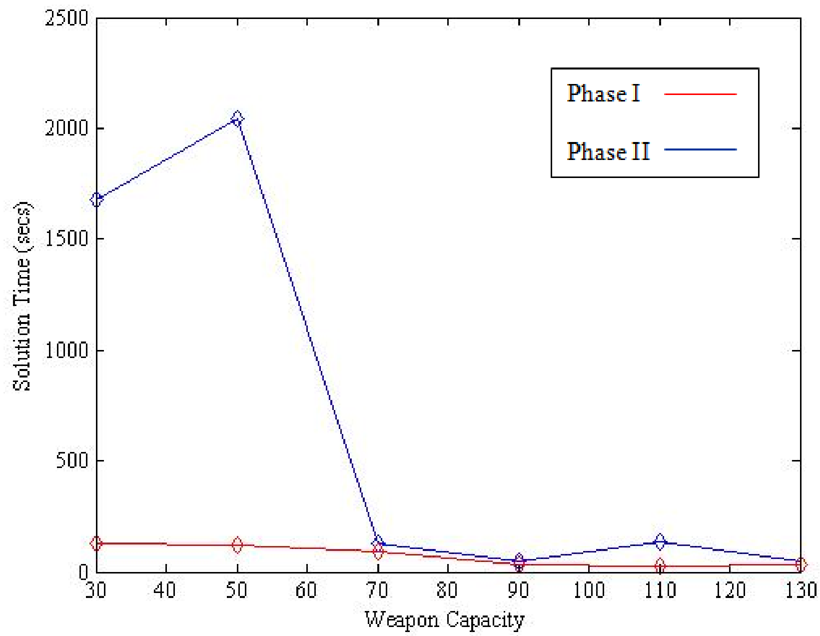


Figure 4.12: Solution Times (in seconds) with Increasing Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

Increasing the weapon capacity for all types of weapons at each base yields erratic results for Phase II, which are shown in Table 4.9 and Figure 4.12. This resembles the affect of increasing the aircraft capacity. However, the solution time tended to decrease for both Phase I and Phase II compared to the solution time for a combination of the aircraft capacity of 60 at each base and the weapon capacity of 30 for each type of weapon at each base as the increment in the weapon capacity increases. The solution time for Phase I decreases as the weapon capacity increases whereas it stays approximately constant as the aircraft capacity increases. Moreover, some of the solutions with increased weapon capacities are true optimal solutions (i.e., the objective function bound equals the best integer solution). Therefore, the

Table 4.9: Solution Times (in seconds) with Increasing Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

ACAP	WCAP	PHASE I	PHASE II	TRUE OPTIMAL
60	30	128	1674	NO
60	50	122	2043	NO
60	70	89	130	NO
60	90	30	45	YES
60	110	29	132	NO
60	130	31	50	YES

Table 4.10: Solution Times (in seconds) with Increasing Aircraft and Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

ACAP	WCAP	PHASE I	PHASE II	TRUE OPTIMAL
60	30	128	1674	NO
80	50	23	158	YES
100	70	25	68	YES
120	90	24	32	YES
140	110	25	36	YES
160	130	27	37	YES

weapon capacity had more affect than the aircraft capacity on the solution time for both Phase I and Phase II.

In Table 4.10 and Figure 4.13, the combined affect of increasing both the aircraft capacity and the weapon capacity show that the solution times for both Phase I and Phase II decrease significantly. The reason for a significant decrease in solution time for Phase I and Phase II after increasing both the aircraft and the weapon capacity is that the weapons are carried by the aircraft and increasing the capacities of only one of them does not significantly increase the possible strike package combinations because building different strike packages depends on both the type of aircraft and the type of weapon. In other words, increasing one of them does not relax the formulation significantly, since it only allows an increase in either the aircraft capacity

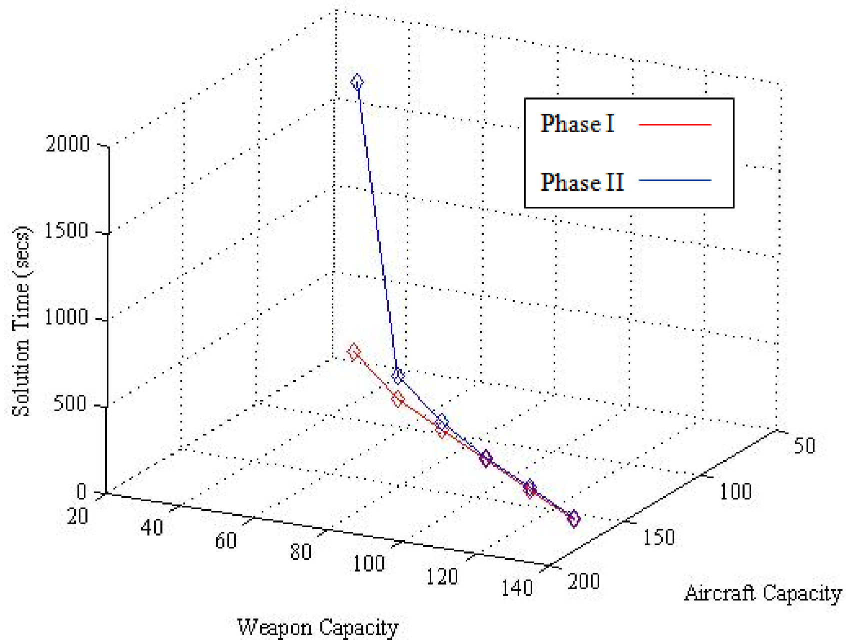


Figure 4.13: Solution Times (in seconds) with Increasing Aircraft and Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

or the weapon capacity, and this generates several more alternative strike package combinations to consider without increasing the resource capacity for several binding constraints resulting in erratic solution times.

More importantly, the solutions found with an increased number of resources in terms of both the aircraft and the weapons are true optimal solutions even though the optimality tolerance is set to 0.7 %.

Finally, the solution times for Phase I with different aircraft and weapon capacities are significantly shorter than the solution times for Phase II because Phase I considers only increasing the number of targets attacked in accordance with target priority. Phase I avoids exceeding the levels of damage beyond the desired levels of damage in order to attack as many targets as possible and does not consider the

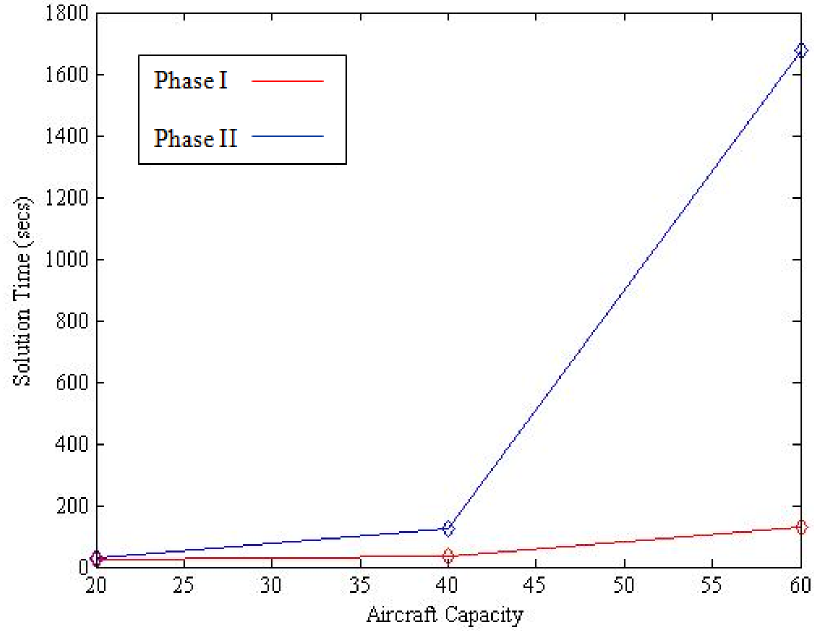


Figure 4.14: Solution Times (in seconds) with Decreasing Aircraft Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

cost. Therefore, the addition of minimizing the cost of attacking the same number of targets as in Phase I in Phase II model increases the solution time. However, the difference between solution times for Phase I and Phase II are not significant when there are sufficient resources to build different strike packages with different costs.

4.3.2 Decreasing the Capacity.

In this subsection, the analysis contains the affect of decreasing the number of aircraft only, the affect of decreasing the number of weapons only, and the affect of decreasing both the number of aircraft and weapons on the solution time and the number of targets attacked.

Table 4.11: Solution Times (in seconds) with Decreasing Aircraft Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

ACAP	WCAP	PHASE I TIME	PHASE II TIME	# TARGETS ATTACKED
60	30	128	1674	100
40	30	32	126	75
20	30	26	28	37

Table 4.12: Solution Times (in seconds) with Decreasing Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

ACAP	WCAP	PHASE I TIME	PHASE II TIME	# TARGETS ATTACKED
60	30	128	1674	100
60	20	120	169	95
60	10	15	20	54

Table 4.11 and Figure 4.14 show that the solution times for both Phase I and Phase II decrease significantly as the aircraft capacity decreases because the number of targets attacked decreases due to insufficient aircraft capacity to attack 100 targets. The model developed in this research solves the strike planning problem preemptively as long as the priority values are specified in the proper way. It is determined that the priority value for a target that has the higher priority should be at least twice as much as the priority value of a target that has the lower priority. Since the model solves the strike planning problem preemptively starting from the target that has the highest priority level to the target that has the lowest priority level, the model with decreased number of aircraft deals with fewer decision variables. Therefore, the decreased number of decision variables decreases the solution time significantly. For example, the Phase II solution time in Table 4.11 decreases by approximately 2 orders of magnitude when aircraft capacity decreases from 60 to 20.

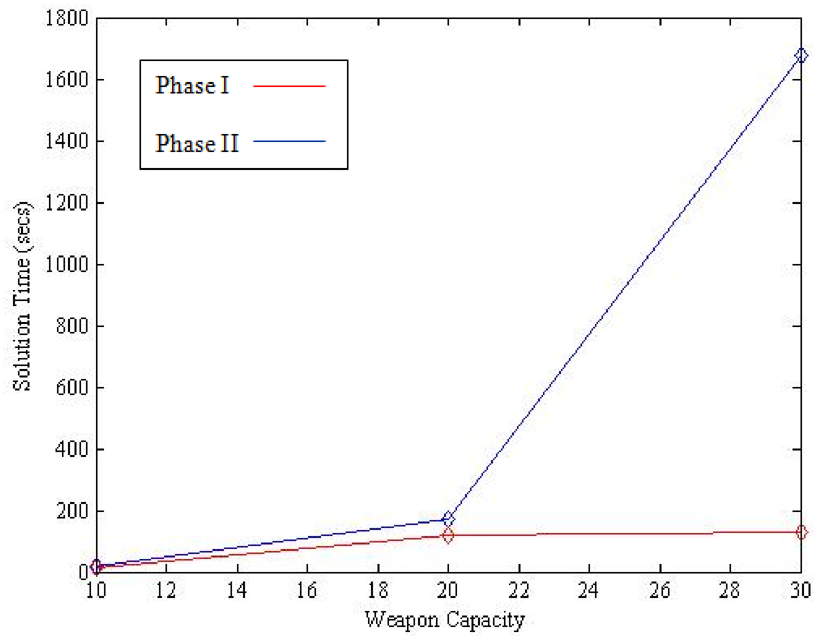


Figure 4.15: Solution Times (in seconds) with Decreasing Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

Table 4.13: Solution Times (in seconds) with Decreasing Aircraft and Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

ACAP	WCAP	PHASE I TIME	PHASE II TIME	# TARGETS ATTACKED
60	30	128	1674	100
40	20	97	46	75
20	10	19	17	40

Decreasing the weapon capacity also decreases the solution time significantly similar to the reduced aircraft capacity as can be seen in Table 4.12 and Figure 4.15. For example, the Phase II solution time in Table 4.12 decreases by approximately 2 orders of magnitude when weapon capacity decreases from 30 to 10.

Table 4.13 and Figure 4.16 illustrates the combined affect of decreasing both the aircraft and the weapon capacity on the solution time and the number of targets attacked.

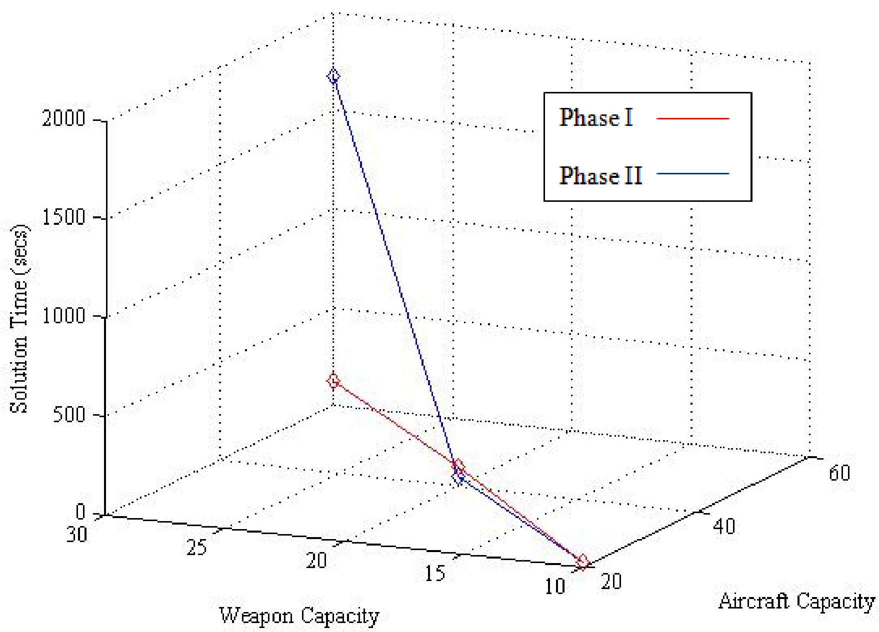


Figure 4.16: Solution Times (in seconds) with Decreasing Aircraft and Weapon Capacities for 100 Target Instance with 0.7 % Optimality Tolerance

As a result, the solution time significantly decreases as the aircraft or weapon capacity decreases. Decreasing the aircraft or weapon capacity reduces the number of targets which can be attacked and decreases the solution time. However, these results show the affect of resource capacities for an instance of the strike planning problem that contains 8 bases, 2 different types of aircraft, 8 different types of weapons, and 35 different types of aircraft and weapon configurations. For a general result, the DOE should be performed.

4.4 Cost Efficiency Analysis

The main objective of this research is to attack as many targets as possible with the minimum cost while considering the target priority. The cost efficiency, which implies minimizing the total cost of the strike plan, and the solution times are analyzed in this research with different scenarios containing different *resource packages (RP)*, *target sets (TS)*, and usable bases.

A resource package consists of a number of bases with particular aircraft and weapon capacities and a target package consists of a number of targets located at particular coordinates. Five different resource packages and 5 different target sets, which are notional but represent real world situations for the TUAF, are constructed and these are described below.

There are also 8 different notional bases used in this research and the resource packages are constructed based upon these notional bases. Table C.1 shows the coordinates and Figure C.1 illustrates the locations of the bases in Appendix C.

Resource Package 1

Resource Package 1 is a generalized resource package consisting of different numbers of aircraft and weapons of particular types at each base. All aircraft are able to fly and there is no restriction on aircraft and weapons besides their associated capacities at each base (see Figure D.1 in Appendix D, Table E.1 in Appendix E, and Table F.1 in Appendix F for locations of available bases for Resource Package 1 and associated aircraft and weapon capacities).

Resource Package 2

Resource Package 2 assumes all F-4s are under inspection due to maintenance problems. Therefore, the only aircraft that can fly during the intended strike planning period are F-16s (see Figure D.2 in Appendix D, Table E.1 in Appendix E, and Table F.2 in Appendix F for locations of available bases for Resource Package 2 and associated aircraft and weapon capacities).

Resource Package 3

Resource Package 3 assumes Turkey has been attacked from both the west and east. The bases located in west and east Turkey are unusable. Therefore, the strike planner cannot assign aircraft from these bases. The available bases are Base 2, Base 3, Base 4, and Base 7. (see Figure D.3 in Appendix D, Table E.1 in Appendix E, and Table F.3 in Appendix F for locations of available bases for Resource Package 3 and associated aircraft and weapon capacities).

Resource Package 4

Resource Package 4 assumes MK series weapons experience some mechanical problems. Therefore, the strike packages carrying these weapons cannot be assigned to targets. Aircraft availabilities and weapon availabilities for other weapon types are subject to aircraft and weapon capacities at each base (see Figure D.4 in Appendix D, Table E.1 in Appendix E, and Table F.4 in Appendix F for locations of available bases for Resource Package 4 and associated aircraft and weapon capacities).

Resource Package 5

Resource Package 5 assumes the only usable bases are the ones located in the northern part of Turkey because there is an epidemic disease in the southern part of Turkey. All cities in the south have been evacuated and a serious terrorist attack at different locations is expected according to intelligence reports. Therefore, a strike plan against terrorist targets should be performed using the usable bases (see Figure D.5 in Appendix D, Table E.1 in Appendix E, and Table F.5 in Appendix F for locations of available bases for Resource Package 5 and associated aircraft and weapon capacities).

Finally, there are 5 different notional target sets used to construct different scenarios for the *Cost Efficiency Analysis*. The number of targets in the target sets are shown in Table 4.14. The biggest target set considered for the cost efficiency analysis contains 50 targets and the other target sets contain the same targets in terms of target location and the desired level of damage with a decrease of 10 targets having the least target priorities in the previous target set. For instance, Target Set 1 consists of 50 targets and Target Set 2 consists of the 40 targets having the highest priority in Target Set 1.

Table 4.14: Number of Targets Contained in Target Sets

	TS 1	TS 2	TS 3	TS 4	TS 5
# Targets	50	40	30	20	10

Table 4.15 shows the cost efficiency performance of Phase II in the methodology of this research compared to Phase I and the number of targets attacked in Phase I and Phase II in parentheses, respectively, for 25 different scenarios. Obviously, Phase II provides a great deal of savings in terms of cost for all scenarios.

In Table 4.15, the cost saving percentage implies the difference between costs of both Phase I and Phase II divided by the cost of Phase I.

Note that the cost saving percentages are different for the scenarios in which the same targets are attacked because Phase I does not consider the cost of the final assignment of the strike packages to targets and therefore it ends up with an arbitrary cost whereas Phase II finds the assignment with the minimum cost.

Table 4.15: Cost Saving Percentages Between Phase I and Phase II and the Number of Targets Attacked in Phase I and Phase II

	RP 1	RP 2	RP 3	RP 4	RP 5
TS 1	47.75 % (37, 37)	54.23 % (16, 16)	35.75 % (18, 18)	38.01 % (33, 33)	15.69 % (24, 24)
TS 2	47.31 % (37, 37)	53.64 % (16, 16)	34.00 % (18, 18)	39.66 % (33, 33)	14.76 % (24, 24)
TS 3	64.70 % (30, 30)	53.39 % (16, 16)	44.73 % (18, 18)	50.58 % (30, 30)	14.88 % (24, 24)
TS 4	65.59 % (20, 20)	54.70 % (16, 16)	44.63 % (18, 18)	56.83 % (20, 20)	54.60 % (20, 20)
TS 5	64.44 % (10, 10)	69.58 % (10, 10)	52.99 % (10, 10)	63.91 % (10, 10)	66.77 % (10, 10)

For instance, the cost saving percentages are different for the scenarios consisting of Resource Package 4 and Target Set 1, and Resource Package 4 and Target Set 2 even though the same targets are attacked , but Target Set 2 only contains the 40 highest priority targets in Target Set 1. Since Phase I only considers attacking the maximum number of targets satisfying the desired level of damage on each target, there are usually multiple ways of attacking the same targets considering the target priority, and Phase I can select any of them, and therefore, Phase I ends up with inconsistent total costs as opposed to Phase II, which always finds the optimal assignment of the strike packages from Phase I with the minimum cost.

The LINGO Interfaced with Excel Spreadsheet Models for different scenarios to analyze the cost efficiency are included in Appendix B.

For a deeper analysis of the cost efficiency, the scenario consisting of Resource Package 2 and Target Set 4 was selected and 30 different test cases with different target locations but the same desired levels of damages were examined. The results of these test cases are analysed using the *Central Limit Theorem*. According to the Central Limit Theorem, the distribution of a population can be approximated by a normal distribution if 30 or more samples are taken from that population. [28]

The mean cost saving of 56.1 % with a standard deviation of 1.28 % is achieved in these 30 different test cases. The cost savings for these 30 different test cases are shown in Table G.1 in Appendix G.

As a result, Phase II provides a great deal of cost saving after attacking the maximum number of targets considering target priority in Phase I.

4.5 Summary

This chapter presented the solver used in this research. Inputting the necessary data and solving the model was explained in detail. Optimality tolerance was analyzed to find a reasonable optimality tolerance for the resource capacity and cost efficiency analyses in this research since the mathematical formulation of the model in this research is an MILP and it requires a great amount of time (i.e., more than 25 hours) to solve to optimality when the constraints are strictly binding. The resource capacity analysis was performed for this reason. Finally, cost efficiency, which is one of the main objectives of this research, was analyzed. The next chapter presents the conclusions of this research and discusses possible recommendations for future research of the strike planning problem.

V. Conclusions and Recommendations

5.1 Summary of the Research

The first chapter in this research introduces the problem statement and the research objectives. The scope, limitations, and assumptions are also discussed in Chapter I.

The formulation of the general WTA problem, the static and the dynamic WTA problem, and the existing solution methodologies for the static WTA problem are presented in Chapter II. The ATO model to solve the static strike planning problem and the associated solution methodologies in the literature are presented in Chapter II, as well. Although this research directly addresses the static strike planning problem, it is also considered useful to briefly present the solution methodologies for the dynamic strike planning problem in Chapter II since the dynamic strike planning problem is an extension of the static strike planning problem.

Chapter III explains the solution methodology developed in this research and discusses the objectives of this research in detail. The definitions, the sets and indices, the parameters and the decision variables used in the methodology are defined and the solution steps in the methodology such as *Preprocessing, Phase I, and Phase II* are also explained in this chapter.

In Chapter IV, LINGO interfaced with Excel Spreadsheets, which is selected as the solver type to solve the strike planning problem in this research, and the reasons for this selection are discussed first. Next, inputting the necessary data to solve the model is explained and illustrated in detail. Then, the optimality tolerance was analysed to find a reasonable optimality tolerance for the resource capacity and the cost efficiency analyses in this research since the mathematical formulation of the model in this research is an MILP and it requires a great amount of time (i.e., more than 25 hours) to solve to optimality when the constraints are strictly binding. The resource capacity analysis was also performed for this reason. Finally, the cost efficiency which is one of the main objectives of this research was analyzed.

Finally, this chapter presents the conclusion of this research and discusses recommendations for future research of the strike planning problem.

5.2 Conclusions

This research deals with maximizing the strike planning efficiency for a given class of targets. The strike planning efficiency implies minimizing the total cost of assigning strike packages to targets in terms of the aircraft and the weapon costs, and the distance flown.

The solution methodology developed in this research finds an optimal strike plan attacking the maximum number of targets in Phase I and minimizing the total cost in Phase II.

The solution methodology also avoids assigning strike packages to targets if the desired levels of damage are not achievable and avoids having a higher level of damage on a target than the associated desired level of damage to save resources for possible future assignments using PODs only, which can be obtained using JMEM, rather than forcing the decision maker to give preferences to the strike packages. The aircraft and weapon capacities for particular types at each base are also considered in this research.

Moreover, the solution methodology developed in this research is an MILP to solve the strike planning problem optimally considering the target priority and the desired level of damage on each target. Since the mathematical formulation of the model is an MILP, it requires a great deal of time (i.e., more than 25 hours) to solve to optimality when the constraints are strictly binding. Therefore, the optimality tolerance analysis is performed to determine a reasonable optimality tolerance for resource capacity and cost efficiency analyses. The optimality tolerance of 0.7 % is selected for an instance of the strike planning problem, which contains 8 bases, 2 different types of aircraft, 8 different types of weapons, and 35 different aircraft and weapon configurations, in this research for the resource capacity and the cost efficiency analyses because the ATO planner has a couple of hours to prepare a strike plan even though the ATO process requires approximately 2 days to complete, but the 2-day time period includes target selection, weapon allocation, mission formation and assignment, mission routing and scheduling process, and contingency plans. However, the optimality tolerance of 0.7 % is not a generally acceptable optimality tolerance for the strike planning problem. The optimality tolerance analysis in this research shows a way of determining a generally acceptable optimality tolerance for the strike planning problem.

The resource capacity analysis shows that increasing either aircraft capacity at each base or weapon capacity for a particular type of weapon at each base does not decrease the solution time. However, increasing both the aircraft capacity and the weapon capacity significantly and consistently decreases the solution time.

Finally, cost efficiency is one of the main objectives in this research besides achieving the desired level of damage on each target and avoiding assigning weapons to targets if the desired level of damage is not achievable. The solution methodology maintains significant cost savings between Phase I and Phase II as illustrated in Chapter IV for 25 different scenarios and 30 different test cases for one of these scenarios.

5.3 Future Research Recommendations

LINGO interfaced with Excel spreadsheets model developed in this research is flexible in terms of inputting the cost of aircraft and weapons, target and base coordinates, types of targets, and the desired levels of damage on targets. However, there is still a need to develop a tool that will allow the user to build more flexible scenarios with different numbers of bases and allowable strike packages. In this research, building a different scenario with different numbers of bases and allowable strike packages may be time consuming because the user has to give range names for possible strike packages in the Excel spreadsheets to transfer the data to LINGO. The user also needs to adapt the LINGO code for different scenarios. A flexible tool having a graphical user interface may be very beneficial to the strike planning tool in this research.

The model in this research directly addresses a single strike planning period. The exact solution methodology developed in this research to solve the static strike planning problem can be extended to solve the dynamic strike planning problem considering multiple strike planning periods. Preparing a strike plan considering multiple strike planning periods increases the flexibility in the decision making process since the aircraft capacities at each base differ in time and the decision maker can consider different aircraft capacities at each base based on the turnaround times of the aircraft.

The model developed in this research does not consider the defensive systems of the targets. Therefore, the model does not assign any SEAD or CAP support for the strike packages. Adding defensive systems into the model by using different objective functions and constraints can solve more realistic strike planning problems.

The optimality tolerance of 0.7 % determined in Chapter IV is not a generally acceptable optimality tolerance for the strike planning problem. The optimality tolerance of 0.7 % is selected for an instance of the strike planning problem containing 100 targets, 60 aircraft at each base, and 30 weapons for each type at each base in this analysis. A generally acceptable optimality tolerance for the strike planning problem can be found applying *Design of Experiments (DOE)* on the optimality tolerance.

Finally, the affect of changing the aircraft and weapon capacities for particular types at each base analyzed in this research is valid for an instance of the strike planning problem containing 8 bases, 2 different types of aircraft, 8 different types of weapons, and 35 different types of aircraft and weapon configurations. For a general result, the DOE should be performed.

Appendix A. LINGO Codes for Phase I and Phase II

The CD associated with this thesis includes the LINGO Codes for Different Instances for Phase I and Phase II.

Appendix B. LINGO Interfaced with Excel Spreadsheet Models

The CD associated with this thesis includes the LINGO Interfaced with Excel Spreadsheet Models for the Instances with Different Number of Targets.

Appendix C. Notional Base Locations

There are 8 different notional bases used in this research and the resource packages are constructed based upon these notional bases. Table C.1 shows the coordinates and Figure C.1 illustrates the locations of the bases.

Table C.1: Notional Base Coordinates

	LATITUDE			LONGITUDE		
	DEG	MIN	SEC	DEG	MIN	SEC
BASE 1	40	08	0	29	09	0
BASE 2	39	27	0	31	25	0
BASE 3	40	34	0	34	22	0
BASE 4	39	40	0	37	46	0
BASE 5	39	19	0	40	57	0
BASE 6	37	43	0	38	54	0
BASE 7	37	38	0	33	27	0
BASE 8	38	16	0	28	52	0

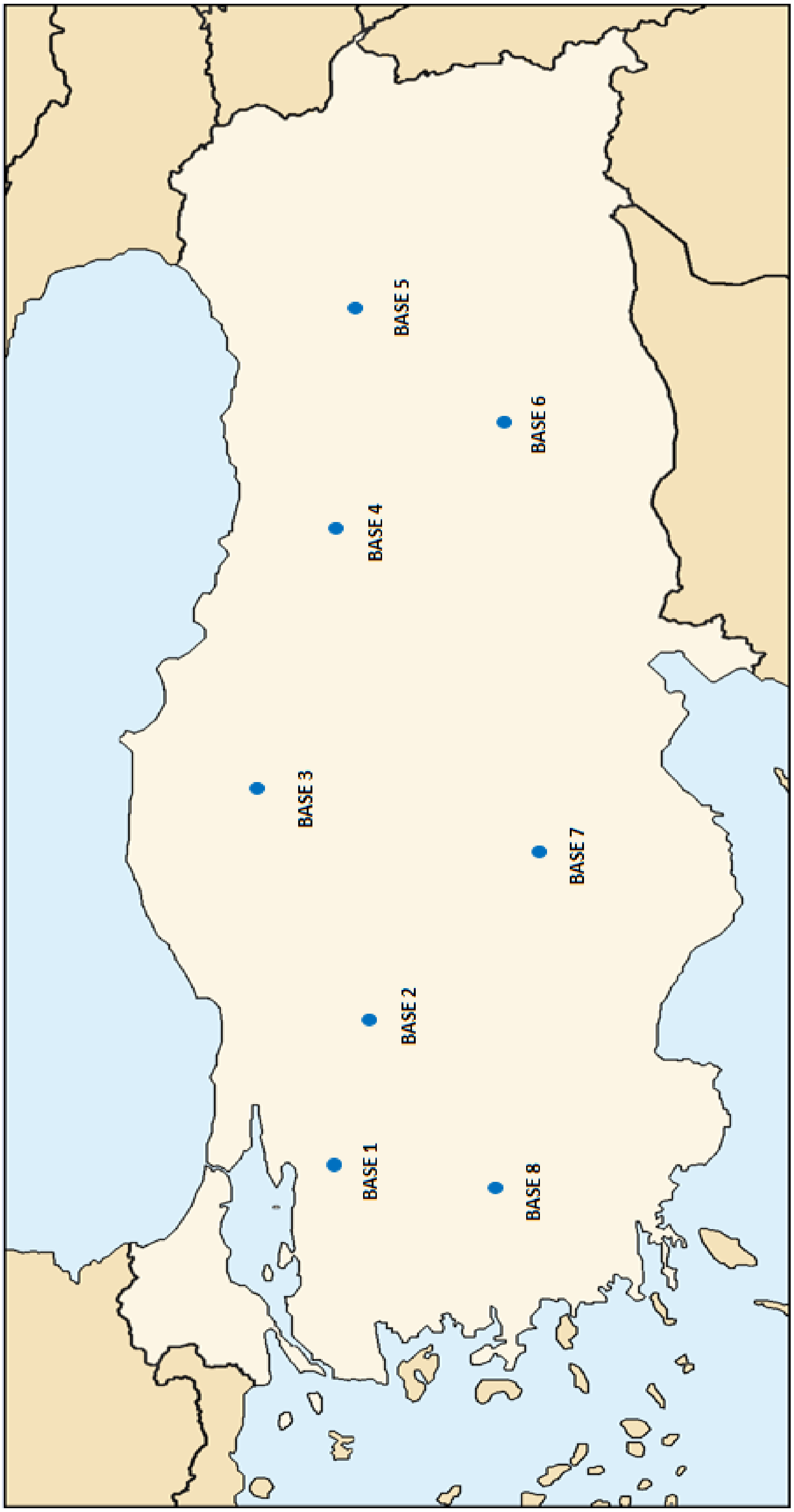


Figure C.1: Notional Base Locations

Appendix D. Usable Base Locations for Resource Packages

A resource package consists of a number of usable bases with particular aircraft and weapon capacities. 5 different resource packages, which are notional but represent real world situations for the TUAF, are constructed and these are used in the cost efficiency analysis in this research. The following figures illustrates the locations of the usable bases for 5 different resource packages in this research.

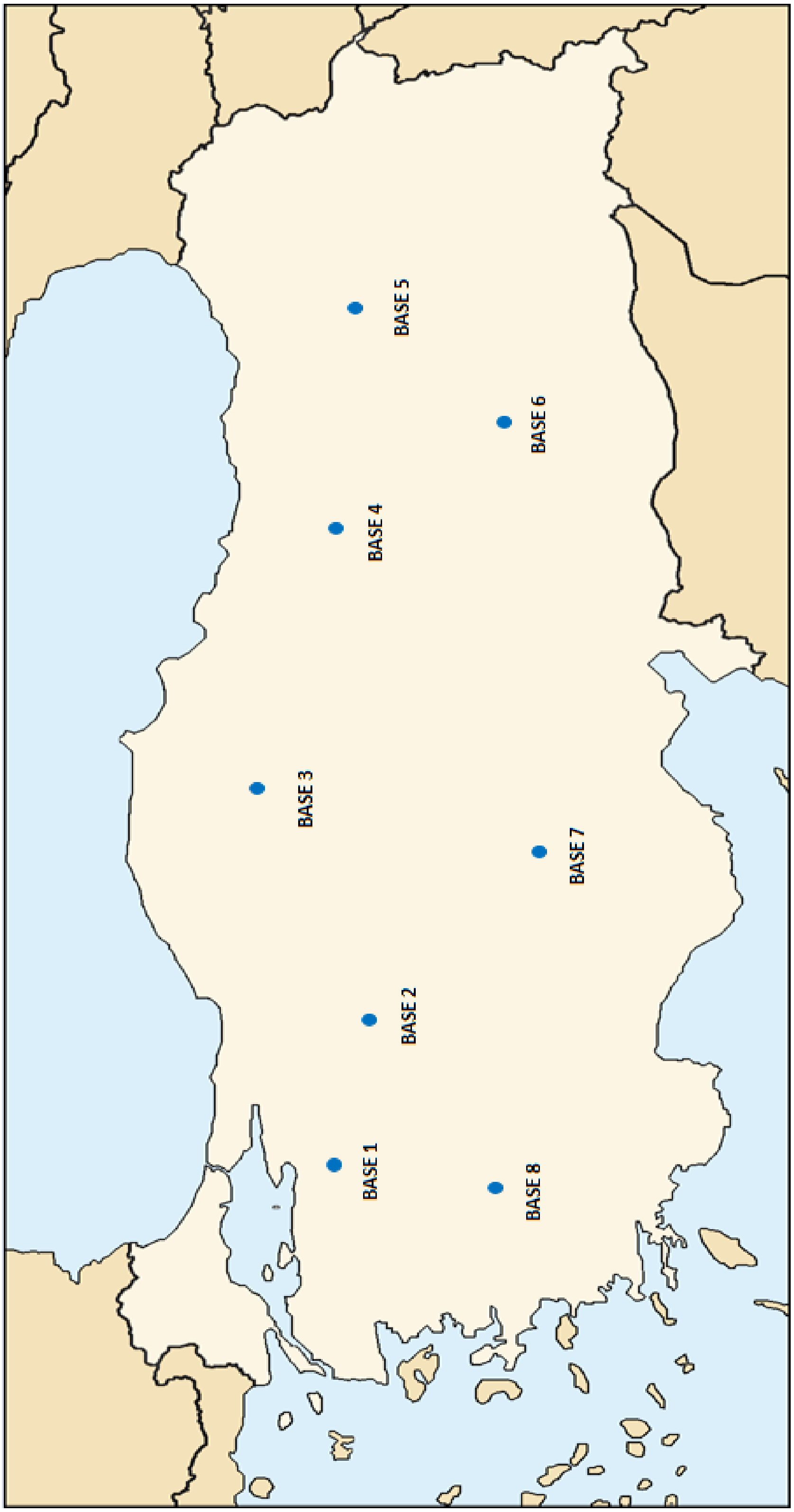


Figure D.1: Usable Base Locations for Resource Package 1

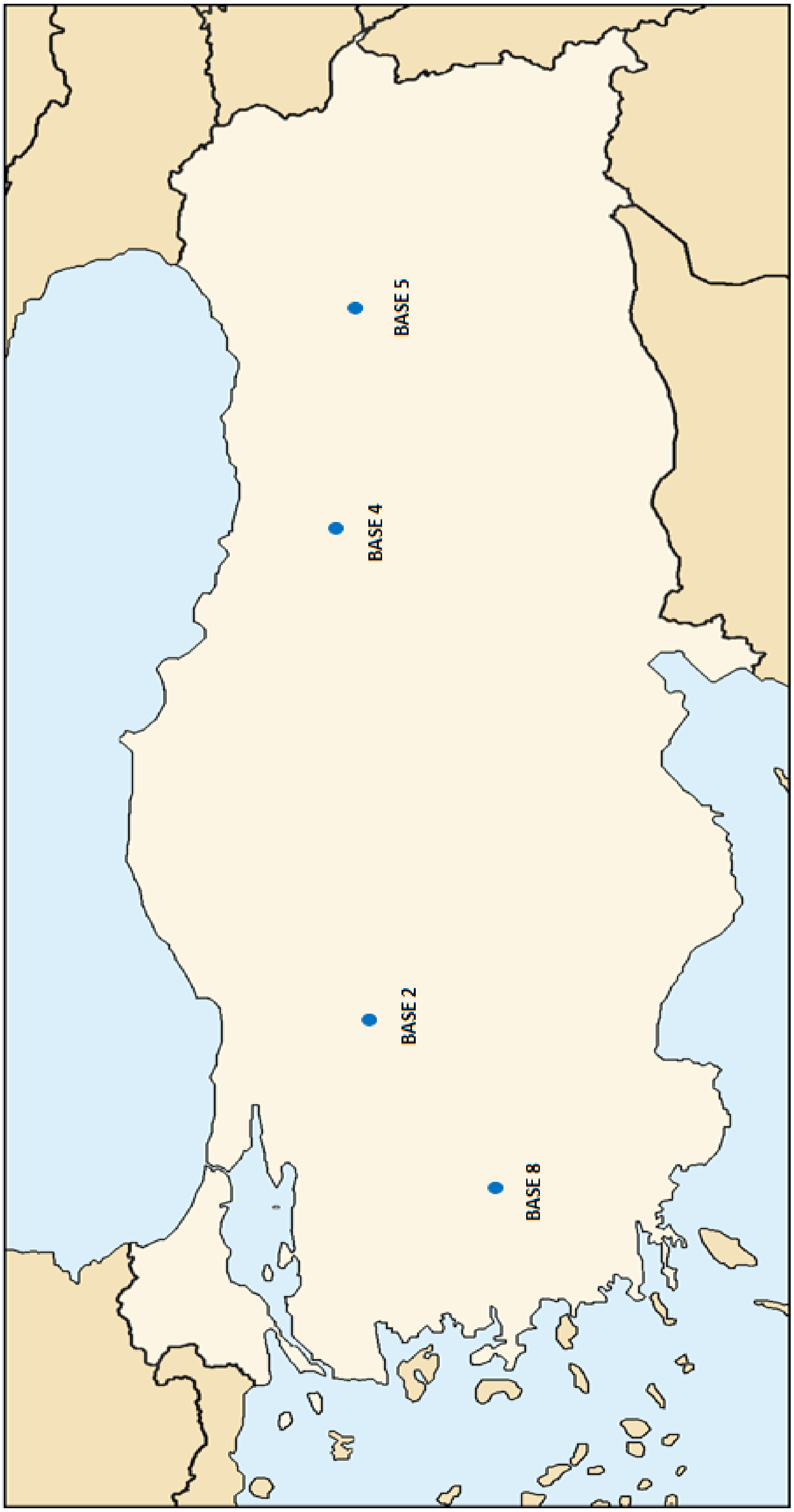


Figure D.2: Usable Base Locations for Resource Package 2

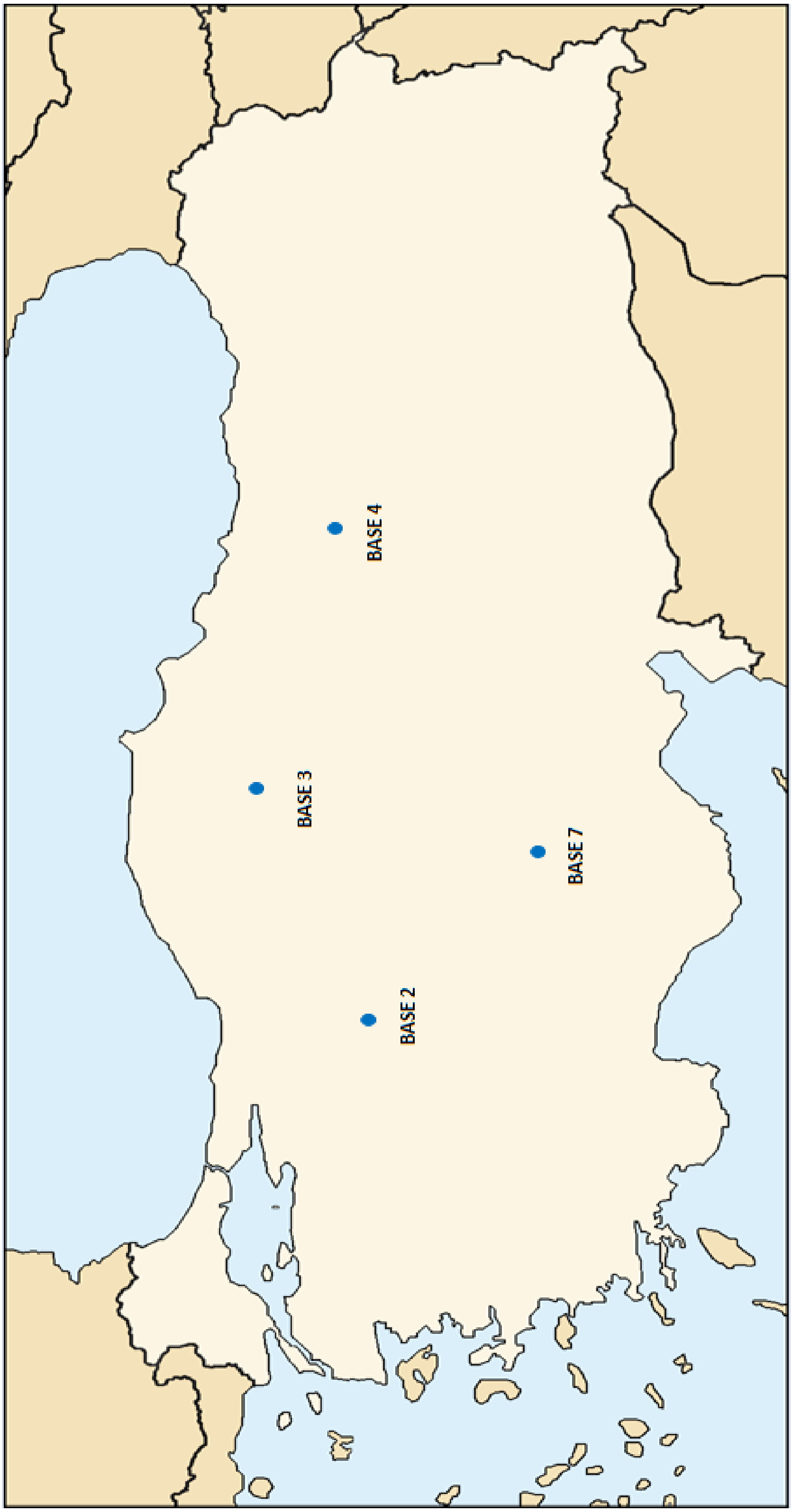


Figure D.3: Usable Base Locations for Resource Package 3

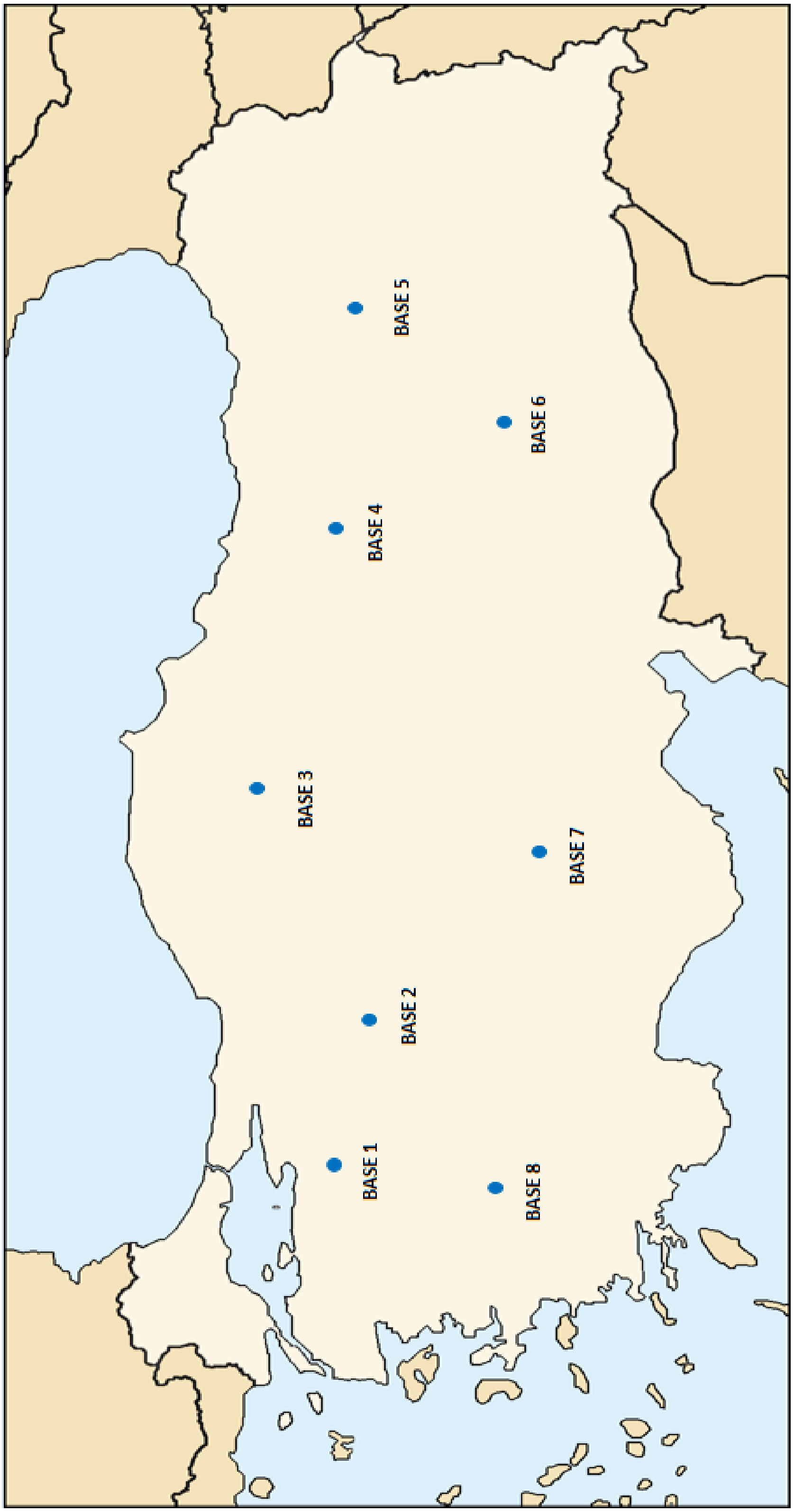


Figure D.4: Usable Base Locations for Resource Package 4

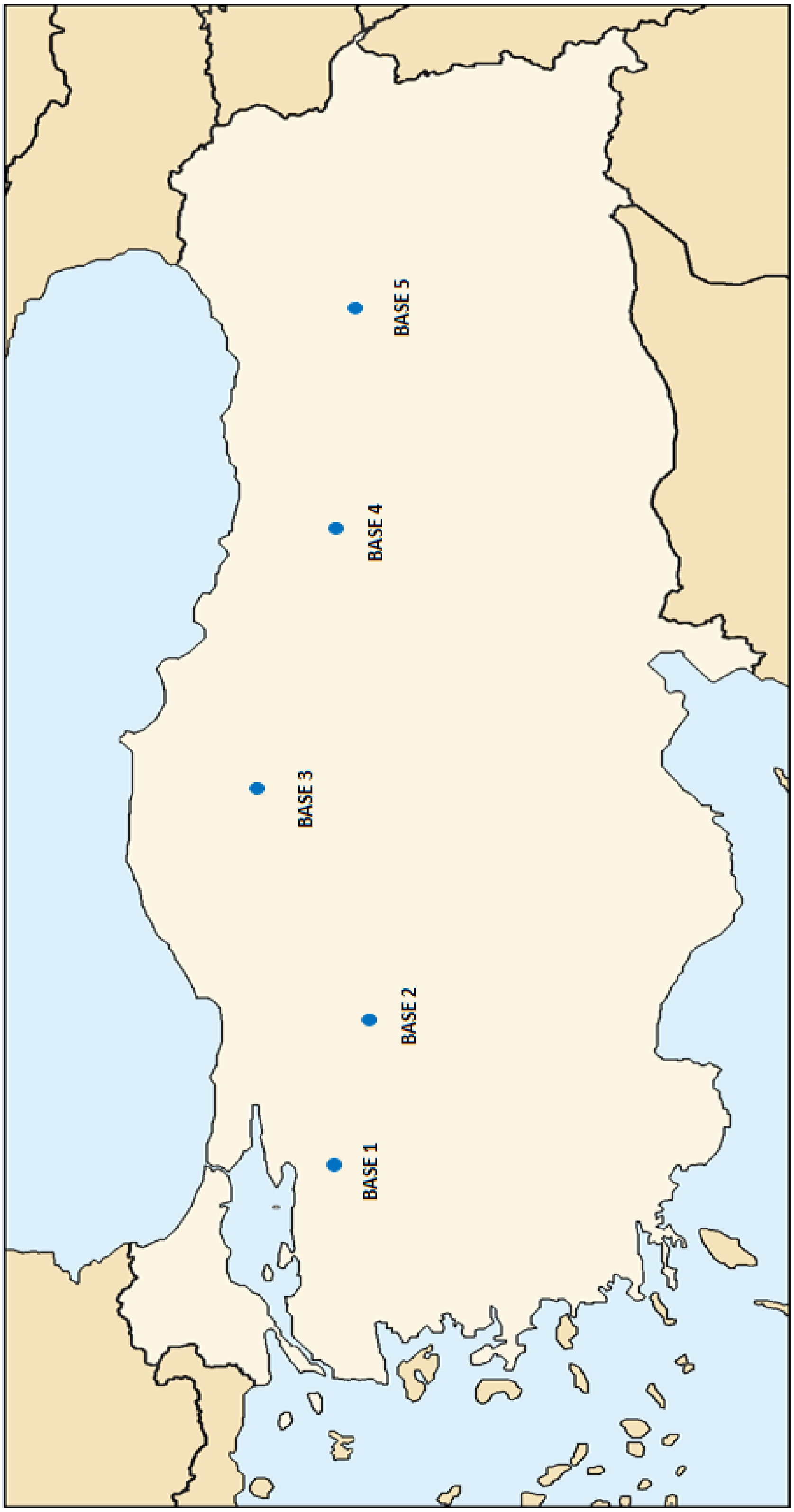


Figure D.5: Usable Base Locations for Resource Package 5

Appendix E. Aircraft Capacities at Usable Bases for Resource Packages

There are 5 different resource packages in this research to perform the cost efficiency analysis. The resource packages differ with respect to the usable bases, aircraft capacities and weapon capacities for particular types at each base. Table E.1 shows the aircraft capacities at the usable bases for 5 different resource packages in this research.

Table E.1: Aircraft Capacities at Bases for Resource Packages

	BASE 1	BASE 2	BASE 3	BASE 4	BASE 5	BASE 6	BASE 7	BASE 8
RP 1	20	18	22	16	24	20	22	20
RP 2	N/A	18	N/A	16	24	N/A	N/A	20
RP 3	N/A	18	22	16	N/A	N/A	22	N/A
RP 4	20	18	22	16	24	20	22	20
RP 5	20	18	22	16	24	N/A	N/A	N/A

Appendix F. Weapon Capacities at Usable Bases for Resource Packages

There are 5 different resource packages in this research to perform the cost efficiency analysis. The resource packages differ with respect to the usable bases, aircraft capacities and weapon capacities for particular types at each base. The following tables show the weapon capacities for particular types at the usable bases for 5 different resource packages in this research.

Table F.1: Weapon Capacities at Bases in Resource Package 1

	MK-82	MK-84	GBU-10	GBU-12	AGM 65A	AGM 65G	MK-20	MK-83
BASE 1	80	80	60	40	40	50	100	80
BASE 2	90	80	70	80	60	60	N/A	N/A
BASE 3	90	70	50	70	50	40	120	60
BASE 4	70	60	50	60	70	70	N/A	N/A
BASE 5	80	90	90	50	40	90	N/A	N/A
BASE 6	60	70	40	90	80	80	80	70
BASE 7	90	70	70	60	50	40	90	90
BASE 8	80	90	60	80	60	60	N/A	N/A

Table F.2: Weapon Capacities at Bases in Resource Package 2

	MK-82	MK-84	GBU-10	GBU-12	AGM 65A	AGM 65G
BASE 2	90	80	70	80	60	60
BASE 4	70	60	50	60	70	70
BASE 5	80	90	90	50	40	90
BASE 8	80	90	60	80	60	60

Table F.3: Weapon Capacities at Bases in Resource Package 3

	MK-82	MK-84	GBU-10	GBU-12	AGM 65A	AGM 65G	MK-20	MK-83
BASE 2	90	80	70	80	60	60	N/A	N/A
BASE 3	90	70	50	70	50	40	120	60
BASE 4	70	60	50	60	70	70	N/A	N/A
BASE 7	90	70	70	60	50	40	90	90

Table F.4: Weapon Capacities at Bases in Resource Package 4

	GBU-10	GBU-12	AGM 65A	AGM 65G
BASE 1	60	40	40	50
BASE 2	70	80	60	60
BASE 3	50	70	50	40
BASE 4	50	60	70	70
BASE 5	90	50	40	90
BASE 6	40	90	80	80
BASE 7	70	60	50	40
BASE 8	60	80	60	60

Table F.5: Weapon Capacities at Bases in Resource Package 5

	MK-82	MK-84	GBU-10	GBU-12	AGM 65A	AGM 65G	MK-20	MK-83
BASE 1	80	80	60	40	40	50	100	80
BASE 2	90	80	70	80	60	60	N/A	N/A
BASE 3	90	70	50	70	50	40	120	60
BASE 4	70	60	50	60	70	70	N/A	N/A
BASE 5	80	90	90	50	40	90	N/A	N/A

Appendix G. 30 Test Cases for the Instance with 20 Targets

For a deeper analysis of the cost efficiency, the scenario consisting of Resource Package 2 and Target Set 4 was selected and 30 different test cases with different target locations but the same desired levels of damages were examined. Table G.1 shows the cost saving percentages for these 30 different test cases.

In addition, the CD associated with this thesis includes LINGO with Excel Spreadsheet Models for these 30 different test cases.

	Cost Saving (%)
Test Case 1	55.71
Test Case 2	58.19
Test Case 3	54.34
Test Case 4	57.62
Test Case 5	55.58
Test Case 6	57.22
Test Case 7	55.59
Test Case 8	55.82
Test Case 9	55.71
Test Case 10	54.07
Test Case 11	56.91
Test Case 12	55.15
Test Case 13	55.58
Test Case 14	56.47
Test Case 15	57.08
Test Case 16	55.90
Test Case 17	58.65
Test Case 18	57.39
Test Case 19	55.30
Test Case 20	55.27
Test Case 21	54.82
Test Case 22	55.81
Test Case 23	55.29
Test Case 24	56.90
Test Case 25	55.10
Test Case 26	58.42
Test Case 27	58.65
Test Case 28	55.70
Test Case 29	55.30
Test Case 30	54.58

Table G.1: The Cost Saving Percentages for 30 Different Test Cases for the Scenario of Resource Package 2 and Target Set 4

Appendix H. Blue Dart

SPENDING LESS MONEY ON ATTACKING TARGETS

One of the main responsibilities of many Air Forces in the world is to protect the national territory against terrorist activities attacking targets, which pose threats to the national territory. It is important to attack targets on time since the enemy may have a prompt intelligence capability about the possible attack. Therefore, attacking targets at a different time than the required time may result in a useless impact on the target. This brings about an unnecessary decrease in the resource capacities.

Air Force resources such as aircraft and weapons are limited and missions must be carefully planned. Any decrease in the resources affects the decision making process negatively since making a decision with a limited number of resources is harder than making a decision with plenty of resources. The decision maker has few options when there are limited resources available. Therefore, limited resources affect the effectiveness and efficiency of the mission. The attacks should be effective. In other words, the decision maker should meet the desired levels of damage on the targets. The desired levels of damage should be met to neutralize the terrorist activities because a desired level of damage on a target is determined based on the operational characteristics of a target. The attacks should also be efficient to prevent unnecessary resource use.

An efficient mission plan also saves a great deal of money since resources are comprised of aircraft and weapons and they both cost a lot. So, how is an effective and efficient mission plan maintained? There are several ways to attack targets using Air Force resources since aircraft can be assigned from several bases and different types and numbers of weapons can be carried by these aircraft when attacking targets. Attacking a target by aircraft taking-off from a base that is closest to the target most probably provides the most efficient attack. However, weapon costs should also be taken into account since they differ greatly. So, a target should be attacked with the weapons that have the least costs, as well.

Resources can also be saved by avoiding having a higher level of damage on a target than the desired level of damage since having a higher level of damage generally requires more resources. There is no need to have a higher level of damage on a target because the desired level of damage on a target is determined by the decision maker based on the operational characteristics of the target. If the desired level of damage on a target is achieved, that means the target cannot continue performing its operational activities. Therefore, having a higher level of damage than the desired level of damage decreases the efficiency of a mission plan and this is an undesired situation from the Air Force standpoint.

A decision maker also needs an automatic tool to quickly determine an effective and efficient plan since making a quick decision is crucial in the battlefield because the enemy may change the current status of a target (i.e., moving a headquarter to another place) using intelligence reports.

This research develops a mathematical model to determine an effective and efficient mission plan. The model in this research attacks as many targets as possible first. Next, it minimizes the total cost of the final mission plan. In addition, an Excel spreadsheet tool is developed that planners may use to make a quick decision in the battlefield. Finally, the model developed in this research provides about 50 % cost saving in the mission planning.

Appendix I. Story Board

The CD associated with this thesis includes the Powerpoint slide for the story board.

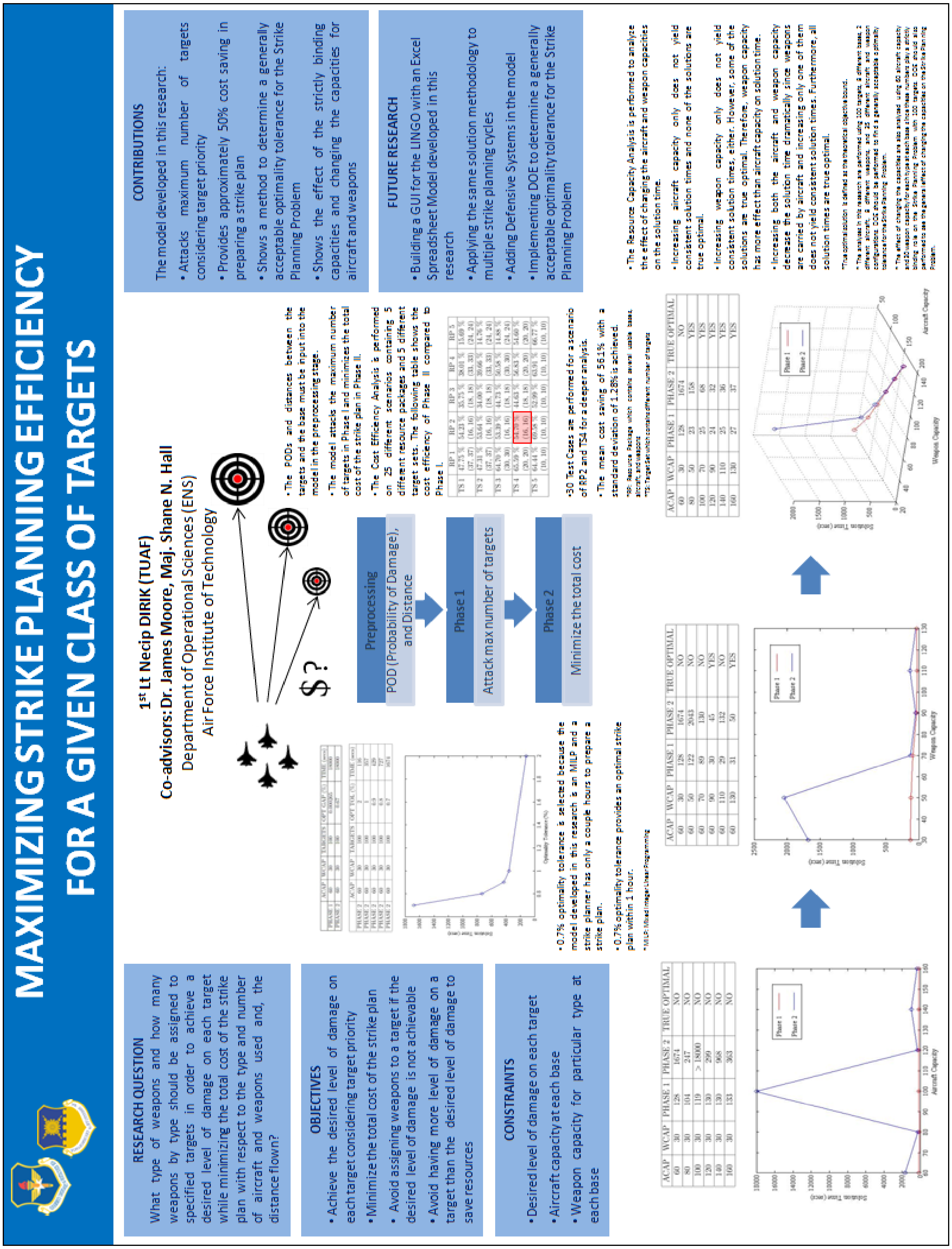


Figure I.1: Story Board

Bibliography

1. A. Yücel, J. M. Rosenberger. "The Generalized Weapon Traget Assignment Problem". *10th International Command and Control Research and Technology Symposium*. McLean, VA, 2005.
2. Arslan, O. *Developing a Tool for the Location Optimization of the Alert Aircraft with Changing Threat Anticipation*. Master's thesis, Air Force Institute of Technology, 2009.
3. Bardak F. S. *Automated SEAD Planning for A Feasible Air Taking Order*. Master's thesis, The Middle East Technical University, 2004.
4. Barth C. D. *Composite Mission Variable Formulation for Real-Time Mission Planning*. Master's thesis, Massachusetts Institute of Technology, 2001.
5. Calhoun K. M. *A Tabu Search for Scheduling and Rescheduling Combat Aircraft*. Master's thesis, Air Force Institute of Technology, 2000.
6. Crawford K. R. *Enhanced Air Tasking Order Optimization Model*. Master's thesis, Air Force Institute of Technology, 1994.
7. Da Silva Castro D. R. *Optimization Models for Allocation of Air Strike Assets with Persistence*. Master's thesis, Naval Postgraduate School, 2002.
8. Dolan M. H. *Air Tasking Order (ATO) Optimization Model*. Master's thesis, Naval Postgraduate School, 1993.
9. F. Johansson, G. Falkman. "An Empirical Investigation of the Static Weapon-Target Allocation Problem". *Proceedings of the 3rd Skövde Workshop on Information Fusion Topics (SWIFT2009)*. Sweden, 2009.
10. Green, D. J. *An Integer Solution Heuristic for the Arsenal Exchange Model (AEM)*. Master's thesis, Air Force Institute of Technology, 1994.
11. Griggs B. J. *An Air Mission Planning Algorithm for a Theater Level Combat Model*. Master's thesis, Air Force Institute of Technology, 1994.
12. Koewler D. A. *An Approach for Tasking Allocated Combat Resources to Targets*. Master's thesis, Air Force Institute of Technology, 1999.
13. Li V. C., Curry G. L., Boyd E. A. "Towards the Real Time Solution for Strike Force Asset Allocation Problems ".
14. LINDO Systems Inc. "LINGO User's Guide", 2008.
15. Linus Schrage. "Optimization Modeling with LINGO", 2006.

16. P. A. Hosein, M. Athans. "Some Analytical Results for the Dynamic Weapon-Target Allocation Problem". Massachusetts Institute of Technology, Cambridge, MA, 1990.

17. P. Hosein, J. Walton, M. Athans. "The Dynamic Weapon-Target Assignment Problems with Vulnerable C^2 Nodes ". *Proceedings of the 1988 Command and Control Symposium*. Monterey, CA, 1988.

18. P. Hosein, M. Athans. "The Dynamic Weapon-Target Assignment Problem". *Proceedings of Symposium on C^2 Research*. Washington, D.C., 1989.

19. P. M. Morse, G. E. Kimball. "Methods of Operations Research", 2003.

20. P. M. Thompson, H. N. Psaraftis. "Cyclic Transfer Algorithms for Multi Vehicle Routing and Scheduling Problems". *Operations Research*, 41:935–946, 1993.

21. P. M. Thompson, J. B. Orlin. "The Theory of Cyclic Transfers". *Operations Research Center Report*. Massachusetts Institute of Technology, Cambridge, MA, 1989.

22. R. K. Ahuja, A. Kumar, K. C. Jha, J. B. Orlin. "Exact and Heuristic Algorithms for the Weapon-Target Assignment Problem". *Operations Research*, 1136–1146, November-December 2007.

23. S. Lloyd, H. Witsenhausen. "Weapon Allocation is NP-complete". *Proceedings of the 1986 Summer Conference on Simulation*. 1986.

24. S. Matlin. "A Review of the Literature on the Missile-Allocation Problem". *General Electric Company*, 1968.

25. Tikveṣ Ṣ. *Design and Implementation of an Asset-Target Allocation System for Air Tasking Orders*. Master's thesis, Hacettepe University, 2007.

26. Van Hove J. C. *An Integer Program Decomposition Approach to Combat Planning*. Ph.D. thesis, Air Force Institute of Technology, 1998.

27. W. L. Winston. "Operations Research, Applications and Algorithms", 2004.

28. Wackerly, Mendenhall, Scheaffer. "Mathematical Statistics with Applications", 2008.

29. Weaver P. R. *Development and Evaluation of an Automated Decision Aid for Rapid Re-Tasking of Air Strike Assets in Response to Time-Sensitive Targets*. Master's thesis, Naval Postgraduate School, 2004.

30. Weir, J. D. *An Improved Solution Methodology for the Arsenal Exchange Model (AEM)*. Master's thesis, Air Force Institute of Technology, 1995.

31. X. Zeng, Y. Zhu, L. Nan, K. Hu, B. Niu, X. He. "Solving Weapon-Target Assignment Problem Using Discrete Particle Swarm Optimization". *Proceedings of the 6th World Congress on Intelligent Control and Automation*. Dalian, China, 2006.

32. Z. J. Lee, S. F. Su, C. Y. Lee. "An Immunity-based Ant Colony Optimization Algorithm for Solving Weapon-Target Assignment Problem". *Applied Soft Computing*, 39–47, 2002.

33. Z. J. Lee, C. Y. Lee. "A Hybrid Search Algorithm with Heuristics for Resource Allocation Problem". *Information Sciences*, 155–167, 2005.

34. Z. J. Lee, S. F. Su, C. Y. Lee. "A Genetic Algorithm with Domain Knowledge for Weapon-Target Assignment Problems". *Journal of the Chinese Institute of Engineers*, 25(3):287–295, 2002.

35. Z. J. Lee, S. F. Su, C. Y. Lee. "Efficiently Solving General Weapon-Target Assignment Problem by Genetic Algorithms with Greedy Eugenics". *IEEE Transactions on Systems, Man and Cybernetics*, 33(1), February 2003.

36. Zacherl B. *Weapon-Target Pairing; Revising an Air Taking Order in Real Time.* Master's thesis, Naval Postgraduate School, 2006.

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14. ABSTRACT

Strike planning is one of the fundamental tasks of the Turkish Air Force and involves assignment of strike aircraft to targets with a maximum level of efficiency. Therefore, planning an optimal strike plan based on the preferences of the decision maker is crucial. The efficiency of the strike plan in this research implies attacking the maximum number of targets while considering target priority and the desired level of damage on each target. Another objective is to minimize the cost of the plan. This research develops an exact model that maximizes the efficiency of the strike plan using LINGO with Excel Spreadsheets. Given this efficiency, the aircraft and weapon costs plus the distance flown is minimized while maintaining efficiency. The model also takes into account the aircraft and weapon capacities for particular types at each base to avoid assigning aircraft to targets from a base where there is an insufficient resource in terms of the aircraft and weapon capacity. The results show that the model developed in this research provides a great deal of cost saving (i.e., approximately 50 %) for a strike plan compared to a strike plan which does not consider the total cost.

15. SUBJECT TERMS

Weapon and Target Assignment (WTA), Strike Planning, Air Tasking Order (ATO)

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