3-14-2014

Analysis of Geolocation Approaches Using Satellites

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Recommended Citation

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ANALYSIS OF GEOLOCATION APPROACHES USING SATELLITES

THESIS

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AFIT-ENY-14-M-07

DEPARTMENT OF THE AIR FORCE
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AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty
Department of Aeronautical and Astronautical Engineering
Graduate School of Engineering and Management
Air Force Institute of Technology
Air University
Air Education and Training Command
in Partial Fulfillment of the Requirements for the
Degree of Master of Science in Aeronautical Engineering

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March 2014

AFIT-ENY-14-M-07

ANALYSIS OF GEOLOCATION APPROACHES USING SATELLITES

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AFIT-ENY-14-M-07

Abstract

A space based system capable of geolocating radio frequency signals of interest has wide reaching application to the Air Force. This system would provide increased situational awareness to the warfighter on the battlefield. The Air Force Institute of technology is developing a satellite to conduct research on geolocation using CubeSats. A methodology to evaluate space based geolocation systems by varying orbital altitude and transmitter position for a given geolocation algorithm and satellite configuration was developed. This method allows multiple satellite configurations and geolocation algorithms to be compared during the design process of a space based geolocation system. The method provides a tool to facilitate decision making on the configuration design and geolocation methods chosen for a given system design. This research explains the geolocation methods and provides comparisons for one through four satellite configurations for time difference of arrival and angle of arrival geolocation algorithms.

Table of Contents

List of Figures

List of Tables

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I. Introduction

In this section we will discuss background information, the motivation for this research, the problem statement, research focus, limitations and assumptions and an overview of the thesis format.

1.1 Background Information and Motivation

CubeSats are popular research platforms and learning tools for universities due to their relatively short design cycle and their relative in expense. A CubeSat is defined in terms of units or Us; one U is defined as a 10×10×10 cm^3 cube. The Air Force Institute of Technology (AFIT) is no different than other universities in that it is leveraging the benefits of CubeSats for research and hands on learning of spacecraft system design. AFIT recently launched its first satellite, a 3U CubeSat, the AFIT low Earth orbit integrated miniaturized electrostatic analyzer carbon nanotube experiment, or ALICE for short. In addition to its recent satellite launch, AFIT hosts a CubeSat design course during which students complete a sequence of three courses on CubeSat design. During the courses students translate stakeholder requirements into mission and system requirements, create a paper design of a CubeSat system and finally build a functioning educational model of a CubeSat. This year's class sequence involved the development of two 6U CubeSat systems, the first being a laser communications payload and the second a radio frequency geolocation payload; the geolocation project has been conducted for two years as part of this course sequence and it is an eventual goal to build and launch a geolocation capable CubeSat.

Geolocation of radio frequency (RF) signals is a capability that has broad application, in fact many people use geolocation frequently without even realizing it. Cell phone providers use a geolocation method called time difference of arrival (TDOA) and the global positioning satellites to estimate your cell phone's location when a user requests it or for electronic 911 services. The military uses geolocation techniques to locate signals of interest on the battlefield, providing the warfighter increased situational awareness by locating potential threats. The geolocation CubeSat being developed at AFIT seeks to demonstrate that a geolocation capability can be provided using small satellites; to date, geolocation on a CubeSat has not been demonstrated. In order for the AFIT geolocation mission to successfully demonstrate this capability it is essential that the system design is valid. To the author's knowledge, a tool that can estimate the geolocation accuracy of satellite system does not currently exist, this thesis seeks to provide a method through which geolocation accuracy of a variety of system designs may be estimated and compared against each other.

1.2 Problem Statement

This research seeks to answer the question, how can the performance of different geolocation methods be compared for a wide variety of satellite mission configurations? Designing a space based geolocation system is a complex process with a large number of variables to consider. Having a method to evaluate geolocation method accuracy and comparing the results as these variables are changes is a critical part of designing a space based geolocation system.

1.3 Research Focus and Limiting Assumptions

The focus of this research is to develop a method to evaluate the accuracy of different geolocation methods in a wide range of different mission configurations. The research focuses exclusively on low Earth orbits (LEO) and configurations consisting of one to four satellites. It is important to note that the method developed has broad application to any space based geolocation system, however the analysis involves varying orbital parameters so non-space based systems cannot be evaluated by this method.

System lifecycle considerations were not taken into account for this research. Two body orbital mechanics were assumed and no perturbations were taken into account. In order to evaluate the system lifecycle a higher order model of specific orbital configurations would need to be considered and simulated over the desired system lifetime of the system. This is of importance for the multiple satellite cases where the spacing of the satellites relative to one another is critical.

1.4 Overview

In Chapter two, the literature review on relevant topics to this project will be discussed. Chapter three describes the methodology of how the research was conducted. Chapter four presents the results of the research. Chapter five discusses conclusions and recommended future work on this project.

II. Background

In this chapter we will discuss geolocation methods including time difference of arrival and angle of arrival methods. The time difference of arrival methods discussed are the exact solution and Taylor series method; the angle of arrival solution is computed using the multiple signal classification algorithm. Satellite orbit and orbit propagators will be also be discussed.

2.1 Geolocation Methods

2.1.1 Time Difference of Arrival.

Time difference of arrival (TDOA) is a method used to geolocate a signal by determining a hyperboloid whose surface represents all of the possible signal transmitter locations [1, 4–6, 8–11, 13–15, 20]. By measuring the TDOA of a signal between two spacially separated receivers, a unique hyperboloid can be calculated that represents all of the possible locations of the signal transmitter [4, 5, 9, 14]. A minimum of two TDOA solutions are needed to produce a geolocation solution [9].

Figure 2.1 shows the intersection of three hyperboloids with Earth's surface. The hyperboloids were created using the TDOAs from receivers on four satellites, the label Hyp 1-2 means that the hyperboloid was generated using the time difference between satellite one and satellite two. The four satellite configuration spaces the satellites such that a diamond shape is formed as the constellation crosses the equator; all of the configurations use a subset of the four satellites shown. Satellite configurations will be further discussed in Chapter 3. A geolocation solution is calculated by estimating the point at which the three hyperboloids intersect, yielding an estimate of the location of the signal transmitter [1, 4–6, 8–10, 20]. Several methods can be used to find the intersection of the hyperboloids; some common methods will be discussed further in the following sections.

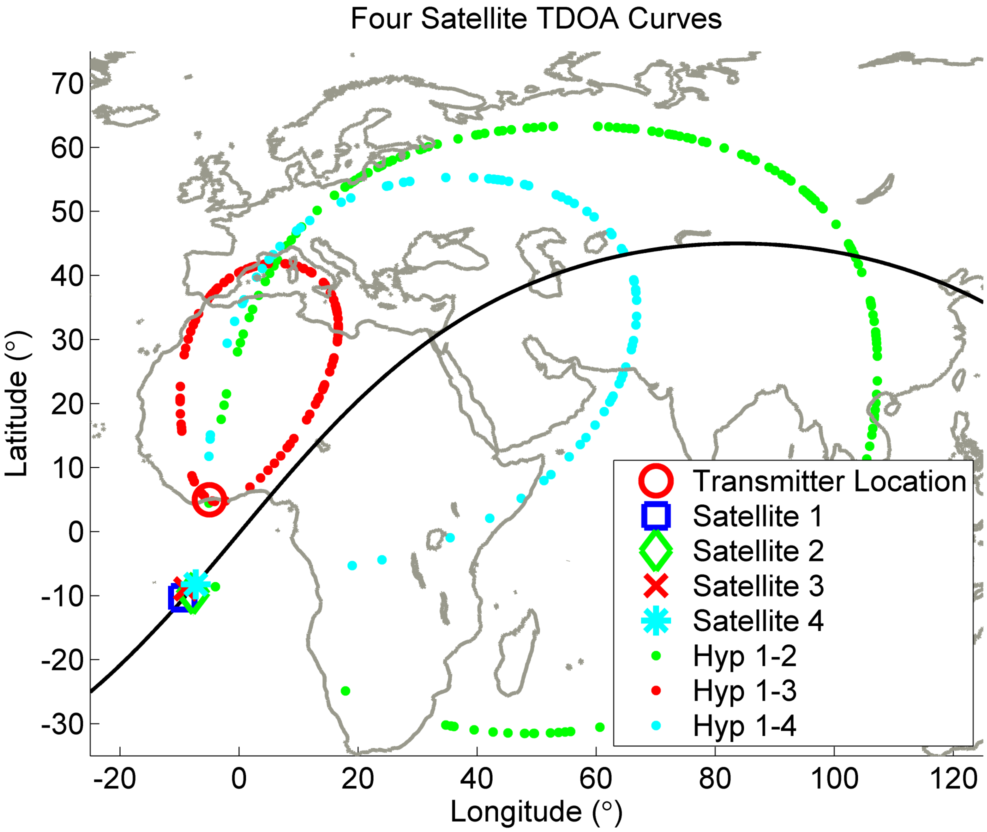


Figure 2.1: TDOA Hyperbolic Curves Produced from Four Satellite Receivers

2.1.1.1 Explicit Solution.

In this section, an exact solution to the TDOA equations will be discussed. Advantages and disadvantages of this method will also be discussed.

An explicit solution can be computed for the intersection of the TDOA hyperboloids [4, 5, 14]. The solution is computed by transforming the nonlinear TDOA estimates into a set of linear equations through use of an intermediate value found using a linear least squares approach [4, 5, 14]. This method is applicable to configurations consisting of three or more receivers. The three receiver configuration produces two TDOA solutions; utilizing these two solutions and making the assumption that the transmitter is located on the Earth's surface allows for the estimate of the transmitter location to be calculated.

Figure 2.2 shows the range and range difference explained in Eqs.(2.3) and(2.4). A signal arrives at the spatially separated receivers at different times.

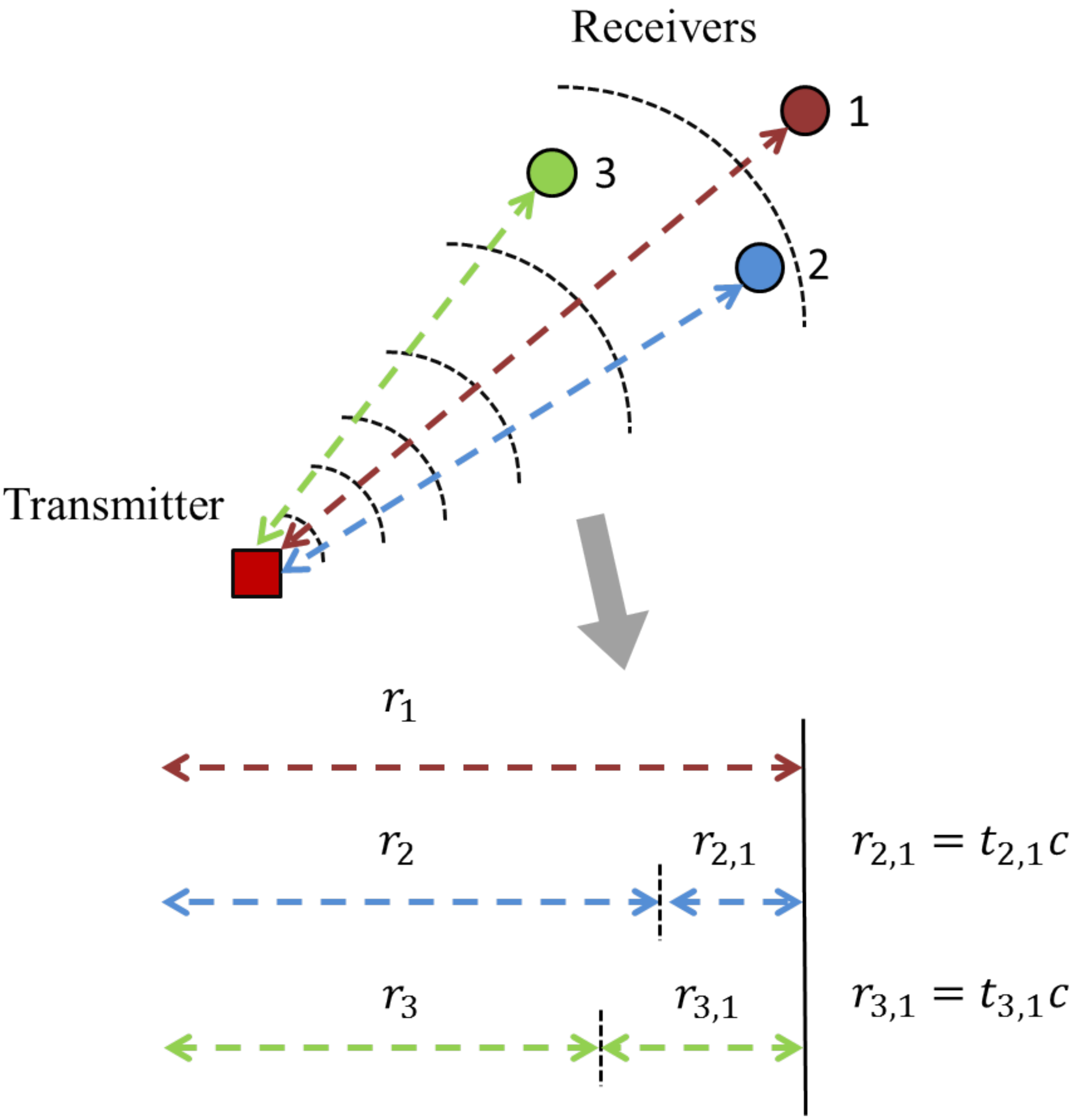


Figure 2.2: TDOA Three Receiver Example

The TDOA $t_{i,1}$ of the signal between receiver one and the ith receiver is given by

$$\begin{aligned} t_{i,1} &= t_1 - t_i \\ & \quad i = 2, 3, ..., N \end{aligned} \tag{2.1}$$

where t_1 is the time the signal arrives at receiver one and t_i is the time the signal arrives at the ith receiver. The measured TDOAs are then converted to range differences $r_{i,1}$ using

$$\begin{aligned} r_{i,1} &= ct_{i,1} \\ & \quad i = 2, 3, ..., N \end{aligned} \tag{2.2}$$

where c is the speed of light [8, 14].

The range r_i between the transmitter and the ith receiver can be written

$$\begin{aligned} r_i &= \sqrt{(x_i - x_T)^2 + (y_i - y_T)^2 + (z_i - z_T)^2} \\ & \quad i = 1, 2, 3, ..., N \end{aligned} \tag{2.3}$$

where x_i, y_i, and z_i are the coordinates of the ith receiver, x_T, y_T, and z_T are the coordinates of the transmitter, and N is the number of receivers [14]. Now, the known range difference $r_{i,1}$ between receiver one and the ith receiver is

$$\begin{aligned} r_{i,1} &= r_i - r_1 \\ & \quad i = 2, 3, ..., N. \end{aligned} \tag{2.4}$$

For clarification of terminology; range, as given by Eq.(2.3), is the distance between a satellite and transmitter; range difference between two receivers is the subtraction of the two receiver ranges, given by Eq.(2.4).

For convenience of computation the term K is defined as [14]

$$\begin{aligned} K_i &= {x_i}^2 + {y_i}^2 + {z_i}^2 \\ & \quad i = 1, 2, 3, ..., N. \end{aligned} \tag{2.5}$$

Now we define the transmitter location

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}^{-1} \frac{1}{2} \begin{bmatrix} K_1 + {r_E}^2 - {r_1}^2 \\ K_2 + {r_E}^2 - {r_1}^2 - 2r_{2,1}r_1 - {r_{2,1}}^2 \\ K_3 + {r_E}^2 - {r_1}^2 - 2r_{3,1}r_1 - {r_{3,1}}^2 \end{bmatrix} \tag{2.6}$$

where the right hand side of the equation is the set of linear equations given by the surface of the Earth and the two time differences. All terms in the equation are known with the exception of x_T, y_T, z_T and r_1, in other words this equation is the estimated transmitter locations in terms of r_1. The transmitter locations from Eq.(2.7) can now be substituted into Eq.(2.3) yielding a fourth order polynomial in terms of in terms of r_1 [4, 5, 14]. The four roots are computed and then checked to see which one gives a transmitter location closest to Earth's surface; this root is selected as the estimated location of the transmitter. For a more complete derivation of these equations see [4, 14].

A four receiver configuration produces three TDOA solutions where the assumption the transmitter is on the Earth's surface is no longer required. The transmitter location is now defined as

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = \begin{bmatrix} x_{2,1} & y_{2,1} & z_{2,1} \\ x_{3,1} & y_{3,1} & z_{3,1} \\ x_{4,1} & y_{4,1} & z_{4,1} \end{bmatrix}^{-1} \frac{1}{2} \begin{bmatrix} {r_{2,1}}^2 + 2r_{2,1}r_1 - K_2 + K_1 \\ {r_{3,1}}^2 + 2r_{3,1}r_1 - K_3 + K_1 \\ {r_{4,1}}^2 + 2r_{4,1}r_1 - K_4 + K_1 \end{bmatrix} \tag{2.7}$$

where the right hand side of the equation is the set of linear equations given by the three time differences. The transmitter locations are again in terms of r_1; as in the two TDOA solution, the transmitter location from Eq.(2.7) are substituted into Eq.(2.3). This substitution yields in a second order polynomial in terms of r_1 resulting in two possible transmitter locations.

2.1.1.2 Taylor Series Estimation.

In this section, we will discuss a Taylor series solution method to the TDOA equations. Unlike the explicit solution, the Taylor series method is iterative which means it is likely to be more computationally intensive; however, the Taylor series method allows for a means for error checking. The advantages and disadvantages of this method will be discussed in further detail.

Taylor series estimation can be used to compute a least sum squared error solution of the linearized TDOA equations [8, 11, 14]. An initial guess for the transmitter position is required for this method [8, 14]. Convergence to a solution is tied to the initial guess of the transmitter position with some initial positions taking more iterations to converge or failing to converge entirely [8, 14]. The need for an initial guess is a disadvantage compared to the explicit method which does not need this input. The two methods may be used in conjunction, an explicit solution can be used to ”seed” the Taylor method providing the required initial transmitter position guess. This method produces an error ellipse allowing for the accuracy of the method to be easily checked which is a fundamental advantage of the Taylor estimation method when compared to other methods [8, 14]; the explicit solution provides no such means to verify accuracy.

The Taylor series method begins in a similar way as does the explicit solution. Equations(2.1) and(2.2) convert the TDOAs $t_{i,1}$ into range differences $r_{i,1}$. Next, the initial guess of the transmitter position x_T, y_T, and z_T is converted into range estimates between the transmitter position and the ith receiver using Eq.(2.3) [8, 14]. The estimated range difference between receiver one and receiver i is then calculated using Eq.(2.4) [8, 14]. Now, the matrices for the least squared solution are defined

$$\mathbf{z} = \begin{bmatrix} r_{1,1,measured} - r_{1,1,guess} \\ r_{2,1,measured} - r_{2,1,guess} \\ \vdots \\ r_{N,1,measured} - r_{N,1,guess} \end{bmatrix} \tag{2.8}$$

where $\mathbf{z}$ is the difference between the range difference from the measured TDOA $r_{N,1,measured}$ and the transmitter position guess $r_{N,1,guess}$ and N is the number of receivers [8, 14]. The partial derivatives of the range difference with respect to the transmitter position x_T, y_T, and

z_T is given by

$$\begin{aligned}
a_{i,1} &= \frac{\partial r_{i,1,guess}}{\partial x_T} = \left[\frac{x_T - x_i}{r_i} - \frac{x_T - x_1}{r_1}\right] \\
a_{i,2} &= \frac{\partial r_{i,1,guess}}{\partial y_T} = \left[\frac{y_T - y_i}{r_i} - \frac{y_T - y_1}{r_1}\right] \\
a_{i,3} &= \frac{\partial r_{i,1,guess}}{\partial z_T} = \left[\frac{z_T - z_i}{r_i} - \frac{z_T - z_1}{r_1}\right]
\end{aligned} \tag{2.9}$$

$$i = 2, 3, ..., N.$$

Next, a matrix **A** is defined

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ & \vdots & \\ a_{N,1} & a_{N,2} & a_{N,3} \end{bmatrix} \tag{2.10}$$

where **A** is composed of the partial derivatives of the range difference between receiver one and receiver N [8, 14]. Finally covariance matrix **R** is defined as

$$\mathbf{R} = \sigma \mathbf{I} \tag{2.11}$$

where σ is the Gaussian variance of the measurement and **I** is the identity matrix [8, 14]. From the matrices defined in Eqs.(2.8),(2.10), and(2.11) the least square error δ is computed [8, 14]

$$\delta = \left[\mathbf{A}^T \mathbf{R}^{-1} \mathbf{A}\right]^{-1} \mathbf{A}^T \mathbf{R}^{-1} \mathbf{z}. \tag{2.12}$$

The updated transmitter position is given by

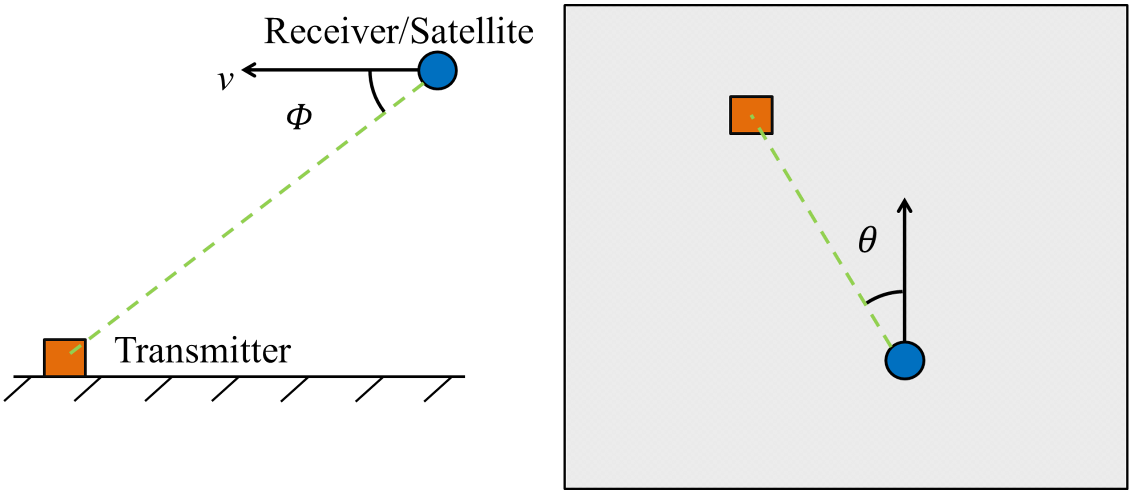
$$\begin{aligned}
x_{T,new} &= x_T + \delta x \\
y_{T,new} &= y_T + \delta y \\
z_{T,new} &= z_T + \delta z
\end{aligned} \tag{2.13}$$

where $x_{T,new}$, $y_{T,new}$, and $z_{T,new}$ define the updated transmitter position [8, 14]. This updated position becomes the new transmitter position guess. The process defined by Eqs.(2.8) through (2.13) is repeated until the determinate of δ is within a specified error limit [8, 14].

2.1.2 Angle of Arrival.

In the section, angle of arrival (AoA) geolocation will be discussed. AoA methods calculate the angle from which a signal is propagating which can be used to generate a line of bearing [7, 16]. AoA is fundamentally different then the TDOA methods; it utilizes an array of closely space antenna to produce a geolocation solution, whereas the TDOA methods require multiple spatially separated receivers. This is a key advantage of the AoA method because all receivers may be located at a single location. Angle of arrival signal processing method are well documented in the literature and a wide variety of methods and variations of methods exist [2, 7, 12, 16, 17, 19]. For the purpose of this research only the multiple signal classification algorithm is consider, this method will be discussed in detail later in this section.

Figure 2.3 shows the definition of the angles ϕ and θ that define a line of bearing [7]. The orange box represents the transmitter on the ground and the blue circle represents a receiver housed in a satellite passing overhead where the arrow is the satellite velocity vector. A unique line of bearing is generated by each receiver during the geolocation process. To convert the lines of bearing into a geolocation solution, a single line is intersected with Earth's surface or if there are multiple lines, a least squares method is used to calculate a point closest to the four lines.



(a) Definition of ϕ - Side View

(b) Definition of θ - Top View

Figure 2.3: Line of Bearing Angle Definitions

The AoA geolocation method requires an antenna array in order to calculate the AoA, for the purposes of the derivation presented in this section uniform linear arrays will be assumed [7].

Figure 2.4 shows an example of a uniform linear antenna array; the squares show the antennas in the array, D is the distance between the antenna, $s(t)$ is the propagated signal, θ is the AoA of the signal, N is the number of antenna in the array, k is the element number of the array, $\triangle t_k$ is the time difference between when the signal arrives at element zero and element k, and $x_k(t)$ is the signal data that is detected at array element k at time t [7].

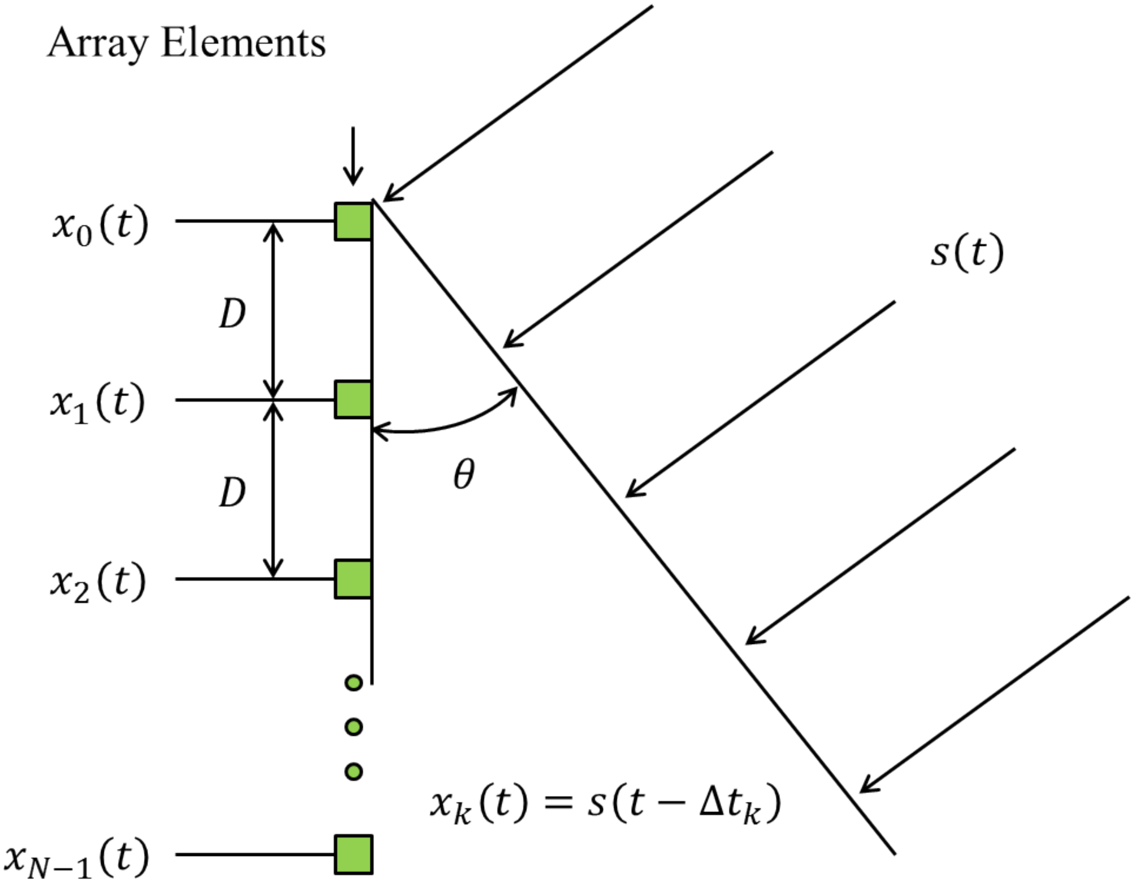


Figure 2.4: Propagating Wave Received at Uniform Linear Array [7]

In wireless digital communication systems, the propagation delay across the array is much less than the period T over which the signal is sampled [7]

$$T \gg \triangle t_k, k = 0, 1, ..., N - 1. \tag{2.14}$$

The signal can be approximated by

$$x_k(nT) \approx s(nT)\, e^{-j2\pi f_c \triangle t_k} \tag{2.15}$$

where x_k is the signal, nT is the discrete time at which the signal reaches the array element, and f_c is the signal frequency [7]. In order to prevent aliasing of the signal, the distance D between the antenna nodes must be less than or equal to $\lambda/2$, where λ is the wavelength.

By relating the speed of light c and f_c through $c = \lambda f_c$ and setting the distance between the array elements to $\lambda/2$, Eq. (2.15) can be written as

$$x_k(nT) \approx s(nT)\, e^{-j\pi k \sin\theta} \tag{2.16}$$

[7]. The steering vector $a(\theta_r)$ of signal of the rth signal is defined as

$$a(\theta_r) = e^{-j\pi k \sin\theta} \tag{2.17}$$

where r is the total number of signals present [7]. Now, the baseband signal sampled at element k in the antenna array can be expressed as

$$x_k(nT) \approx \sum_{i=0}^{r-1} s_k(nT)\, a(\theta_i) \tag{2.18}$$

[7]. Equation(2.18) can be written in matrix form

$$\begin{bmatrix} x_0(n) \\ x_1(n) \\ \vdots \\ x_k(n) \end{bmatrix} = \begin{bmatrix} a_0(\theta_0) & a_0(\theta_1) & \cdots & a_0(\theta_{r-1}) \\ a_1(\theta_0) & a_1(\theta_1) & \cdots & a_1(\theta_{r-1}) \\ \vdots & \vdots & \ddots & \vdots \\ a_k(\theta_0) & a_k(\theta_1) & \cdots & a_k(\theta_{r-1}) \end{bmatrix} \begin{bmatrix} s_0(n) \\ s_1(n) \\ \vdots \\ s_{r-1}(n) \end{bmatrix} + \begin{bmatrix} v_0(n) \\ v_1(n) \\ \vdots \\ v_k(n) \end{bmatrix} \tag{2.19}$$

where $v_k(n)$ is the additive noise at each array element [7]. Simplifying Eq.(2.19) into matrix notation results in

$$\mathbf{x}_n = \mathbf{A}\mathbf{s}_n + \mathbf{v}_n \tag{2.20}$$

or for the case assuming no noise

$$\mathbf{x}_n = \mathbf{A}\mathbf{s}_n \tag{2.21}$$

where A is composed of the steering vectors of the r signals [7].

An eigendecomposition of the spatial covariance matrix R_{xx} can be defined as

$$\mathbf{R}_{xx} = \mathbf{Q}\mathbf{D}\mathbf{Q}^H = [\mathbf{Q}_s\mathbf{Q}_n] \begin{bmatrix} \mathbf{D}_s & 0 \\ 0 & \sigma^2\mathbf{I} \end{bmatrix} [\mathbf{Q}_s\mathbf{Q}_n]^H \tag{2.22}$$

where $\mathbf{Q}$ is partitioned into an $N \times r$ matrix whose columns, $\mathbf{Q}_s$, are the eigenvectors of the signal subspace, and an $N \times (N - r)$ matrix whose columns are the ”noise” eigenvectors [7]. For a more complete derivation of the equations in this section, see reference [7].

2.1.2.1 The Multiple Signal Classification Algorithm.

This section will discuss the derivation of the Multiple Signal Classification (MUSIC) algorithm and its limitations. The MUSIC algorithm is a method to process signal data and provide lines of bearing from which to compute geolocation solutions. The MUSIC algorithm is more computationally expensive than the TDOA methods because the entire range of possible AoAs are considered to find where the algorithm peaks. The derivations in section 2.1.2 form the basis of the MUSIC algorithm.

The MUSIC algorithm is defined as

$$P_{MUSIC}\left(\theta\right) = \frac{1}{\mathbf{A}^{H}\left(\theta\right)\mathbf{Q}_{n}\mathbf{Q}_{n}^{H}\mathbf{A}\left(\theta\right)} \tag{2.23}$$

where θ varies from$-\pi/2$ to $\pi/2$, **A** is the matrix of steering vectors for signals with AoA θ, and $\mathbf{Q}_n$ contains the ”noise” eigenvectors of the signal to be located [7, 16]. To generate the **A** matrix, the steering vectors for all the values of θ are calculated [7, 16]. In theory $\mathbf{A}^{H}\left(\theta\right)\mathbf{Q}_{n} = 0$, **A** spans the signal subspace and $\mathbf{Q}_n$ contains the eigenvectors of the noise subspace; by definition, the vectors are orthogonal [7, 16]. In practice, there are errors estimating the value $\mathbf{Q}_n$, so it will not be precisely orthogonal to **A**. The MUSIC algorithm, Eq.(2.23), produces a very large value when the two vectors are close to orthogonal; these peaks in the values of P_{MUSIC} correspond to the AoA of the signal [7, 16].

Figure 2.5 shows an example of the MUSIC spectrum plot using a 10-element uniform linear array with three signals present. The peaks in the plot are the estimated angles of arrival for the three signals and are the values at which $\mathbf{A}^{H}\left(\theta\right)\mathbf{Q}_{n} = 0$ therefore the value of Eq. 2.23 is large.

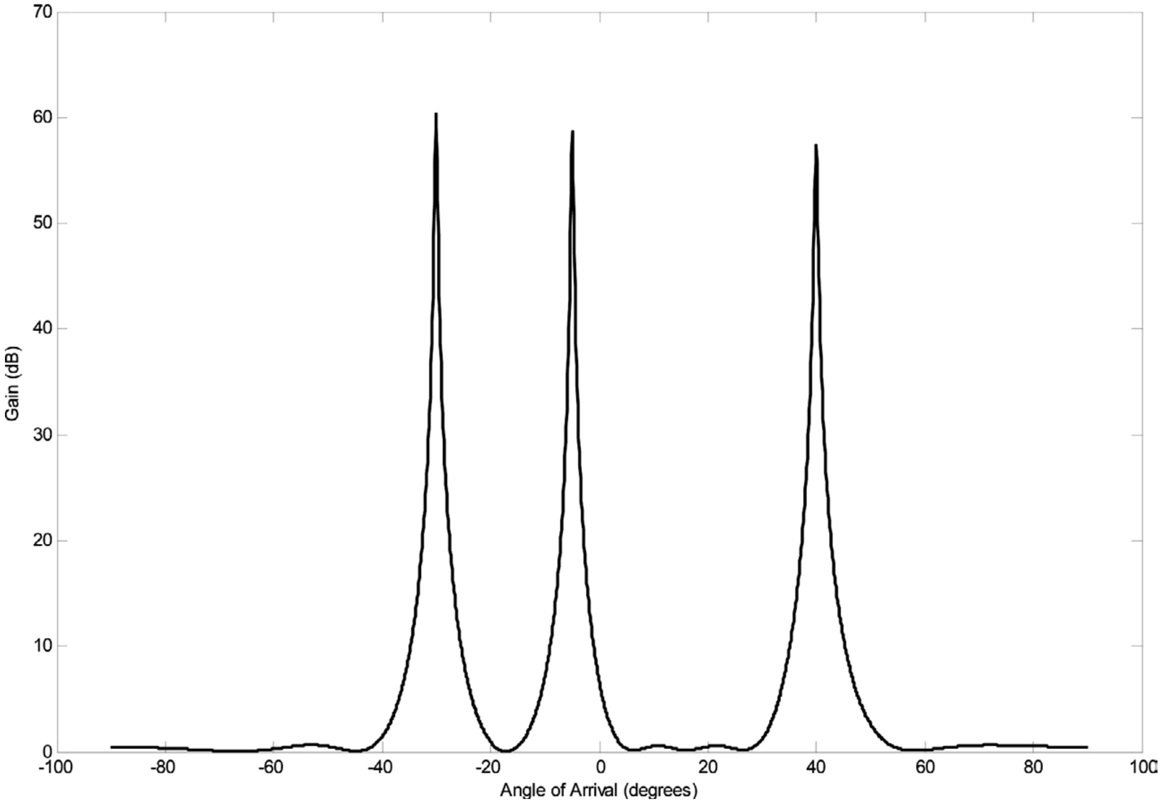


Figure 2.5: MUSIC Spectrum with Three Signals Present [7]

2.1.3 Classical Orbital Elements and Orbit Propagators.

This section discusses information on orbits and orbit propagation. To evaluate space based geolocation, the position of the satellites on which the receivers are located must be known. Knowledge of a satellite's position in its orbit at a given moment in time allows the geolocation solution for that time to be calculated. Propagation of the satellites through their orbits allows for the simulation of a satellite passing over a transmitter which is the basis of the experiments conducted for this research.

One way to express a satellite's position and velocity is to use six classical orbit elements or Keplerian elements [3, 18, 21]. Each Keplerian element provides distinct information about a satellite's orbit; semimajor axis a and eccentricity e the size and shape of the orbit; inclination i and right ascension of the ascending node Ω describe the orientation of the orbital plane; argument of perigee ω gives the orientation of the semimajor axis; true anomaly ν tells the position of the satellite in its orbit [3, 18, 21]. A list of the orbital elements used to fully describe the satellite orbit is found in Table 2.1.

Given the orbital parameters listed in Table 2.1 and the time t, the satellite position and velocity vectors can be calculated [3, 18, 21]. By taking the initial time t and adding a time step $\triangle t$, the satellite is propagated to the point in its orbit $(t + \triangle t)$. This process is repeated in order to propagate the satellite through the entire time period desired.

Table 2.1: Keplerian Elements

Orbit Size and Shape	a	Semimajor Axis
	e	Eccentricity
Orbital Plane Orientation	i	Inclination
	Ω	Right Ascension of the Ascending Node
Semimajor Axis Orientation	ω	Argument of Perigee
Satellite Position	ν	True Anomaly

III. Methodology

In this chapter, we will discuss the methods that will be used to conduct the analytical experiments in this thesis. A total of 3360 experiments were conducted that varied satellite configuration, altitude, transmitter location and geolocation method. The geolocation algorithms, orbital constellations, and the experiments conducted will be discussed. Assumptions that were made in order to scope the problem will be discussed.

3.1 Assumptions

Several assumptions were made in order to bound the experiments to analyze the geolocation algorithm performance under a variety of scoped configurations.

The receiver electronics in the experiments are hosted in satellites in low Earth orbit. All satellites in a given constellations are at the same orbital altitude and are in the same circular orbit; the maximum altitude to be tested is 2000 km. For configurations consisting of more than one satellite, it is assumed that the individual satellites are be capable of maneuvering into and maintaining their respective positions in the constellation.

All experiments are conducted over a single pass of the satellite constellation. This allows for the orientation of the orbital ground trace to remain fixed with respect to the transmitter location so that comparisons in the experiments are valid.

3.1.1 Geolocation Assumptions.

In order for a geolocation solution to be calculated, all satellites in a constellation must be in view of the transmitter. Prior to calculating a geolocation solution, a visibility check must be performed; as shown in Figure 3.1. The visibility check in Fig. 3.1 shows that satellites one and three have a direct line of sight to the transmitter but the Earth obstructs satellite two's view of the transmitter. At this point it is still technically possible to obtain a geolocation solution using the two available satellites. However, to allow for analysis of the effect the number of satellites has on geolocation accuracy, these solutions are not considered. This assumption ensures that the geolocation solution is from the entire satellite constellation and not from a subset of the total satellites.

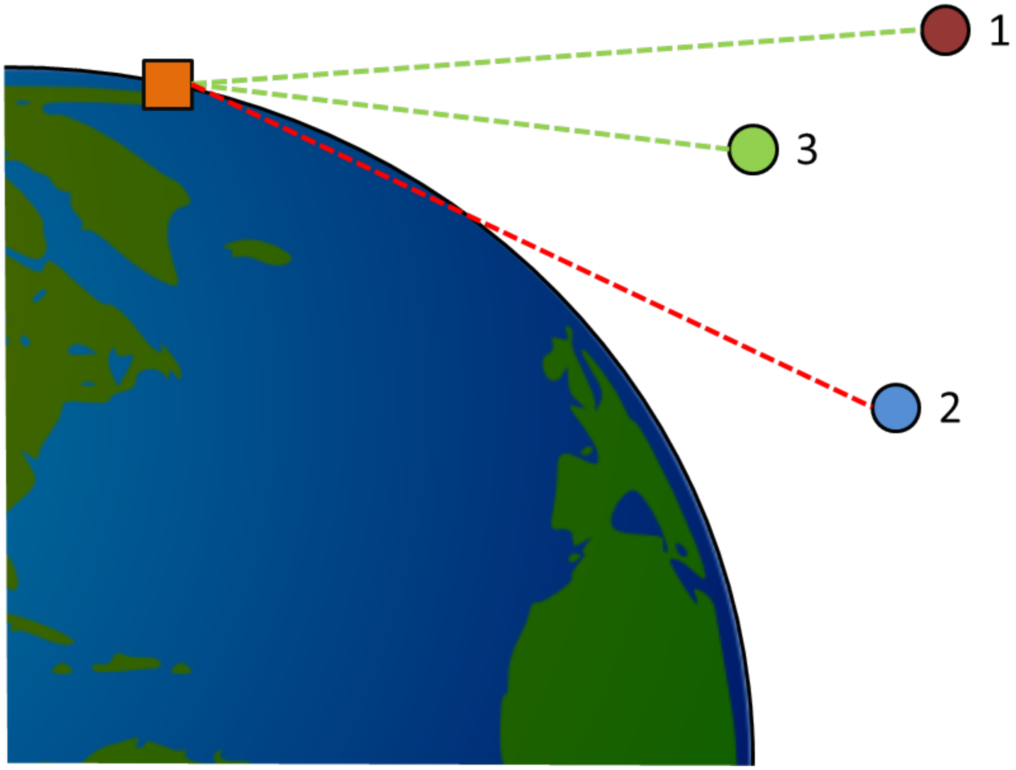


Figure 3.1: Visibility Check for Three Satellite Configuration

Each analysis case is run over a simulated period of 2000 seconds which allows enough time for a configuration to complete an entire pass over the transmitter. The definition of a pass is the time period between the first time at which the transmitter is visible to the satellite configuration until first time at which the transmitter is no longer visible. Geolocation solutions are calculated once per second; this frequency also dictates the time step used to propagate the satellites forward in their orbits. Syncing the time step used for orbit propagation to the frequency of geolocation ensures the satellites are at the correct position to obtain the geolocation solutions desired.

For the TDOA calculations, the signal properties are unimportant. It is assumed that if the visibility check passes, the receivers are capable of detecting the transmitted signal. The TDOA geolocation solutions include Gaussian timing error of $\sigma = 9 \times 10^{-9} s$.

For the AoA calculations, signal properties become more important. The frequency of the transmitted signal dictate the spacing of the antenna in the array used to calculate the AoA as noted in Section 2.1.2. Spacing the antenna array elements half a wave length $\lambda/2$ apart allows for geolocation on a single satellite configuration to be possible. The frequency of the signal is, 1315MHz; this frequency is used for all AoA calculations. The AoA geolocation solutions include Gaussian measurement error in the AoA measurement of $\sigma = 0.1^{\circ}$. Other sources of error such as additive white Gaussian noise were not introduced into the geolocation solutions.

3.2 Satellite Mission Analysis

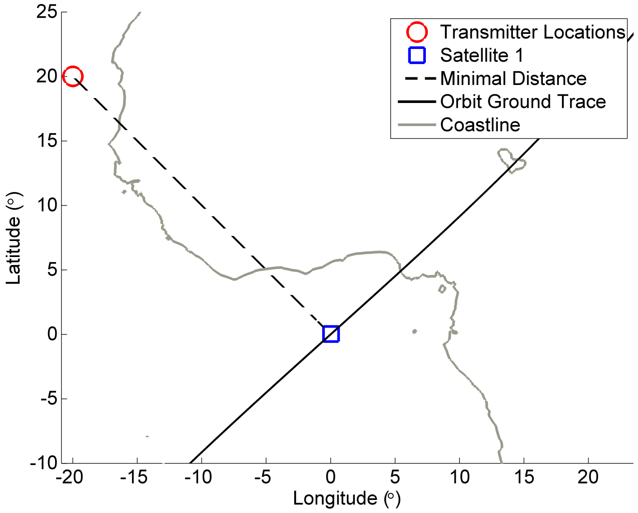
This section will discuss the three variables that were analyzed. The analysis focuses on evaluating the geolocation methods when number of satellites is varied, the minimal ground trace distance of the transmitter from the orbital path is varied, and the orbital altitude is varied. By evaluating all of the transmitter positions at all of the satellite configuration altitudes, a matrix of geolocation solutions is computed. The data computed allows for error mechanisms of geolocation algorithm and satellite combinations to be easily seen and analyzed further. Additionally since all configurations are evaluated at the same conditions, comparisons between configurations and geolocation methods can be made easily.

3.2.1 Orbital Altitude Analysis.

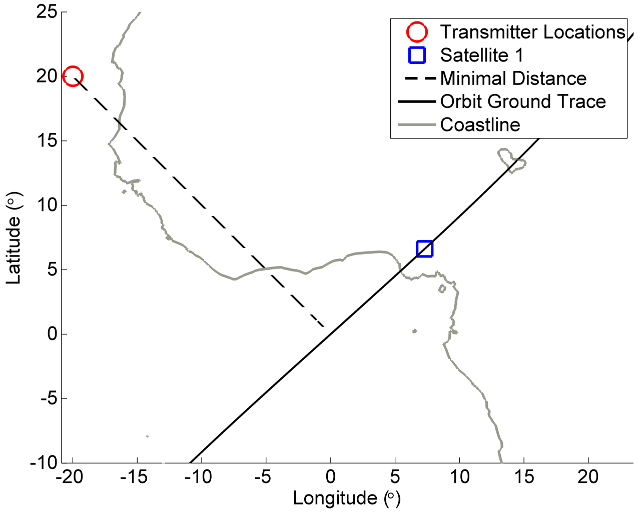
The orbital altitude analysis varies the orbital altitude from 100 to 2000 km which spans what is typically considered low Earth orbit (LEO). This experiment investigates the effect that varying orbital altitude has on the geolocation accuracy of the different geolocation methods and configurations. The satellite configurations will vary in altitude by increments of 100km, this results in twenty altitudes between 100 and 2000 kilometers that are evaluated by this experiment.

3.2.2 Minimal Ground Trace Distance Analysis.

In this experiment the minimal ground trace distance from the satellite orbital path to the transmitter is varied. Figure 3.2a illustrates how minimal ground trace distance (shown as a dashed line) is defined for the purpose of this experiment; this is a curvilinear distance over the Earth's surface. Note that the distance is not a function of the satellite position in the orbit, this can be seen by comparing Figs. 3.2a and 3.2b. In Fig. 3.2a, the satellite is at the point in its orbit that defines the minimal distance. In Fig. 3.2b, the satellite has propagated further in its orbit however the definition of the minimal ground trace distance is unchanged.



(a) Minimal Ground Trace - Satellite at Minimal Distance Point



(b) Minimal Ground Trace - Satellite Past Minimal Distance Point

Figure 3.2: Minimal Ground Trace Definition

In this experiment, a total of 20 transmitter locations will be tested; each location corresponds to a different geolocation experiment. The first transmitter location is located at zero latitude, zero longitude. Subsequent cases will be spaced by in increments of one degree latitude and negative one degrees longitude. Figure 3.3 shows the 20 transmitter locations used in this analysis. The transmitters are located near the western coast of Africa, this location was chosen for ease of experimentation since the transmitter locations can be offset from the zero latitude, zero longitude location.

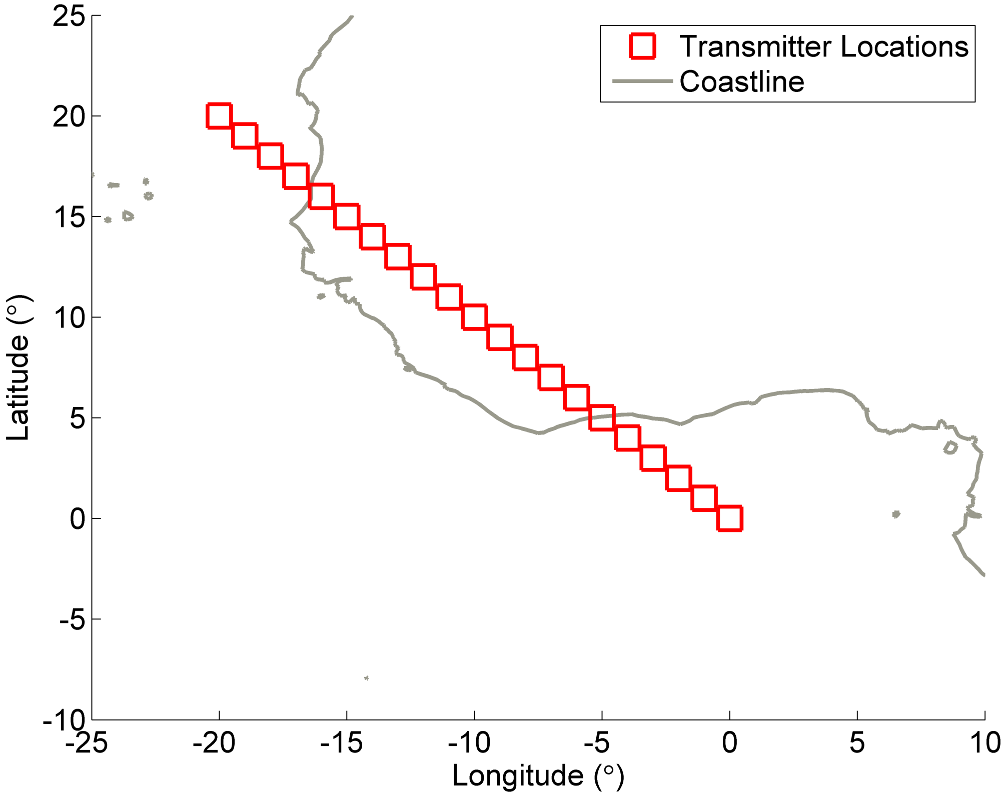


Figure 3.3: Latitude and Longitude Transmitter Positions

3.3 Satellite Orbital Configuration

In this section, we will discuss the orbital configuration of the satellites. The configurations were chosen so that analysis could be performed on the effect on different geolocation techniques of adding satellites to the constellation. This method can be applied to any number of other satellite configurations to evaluate the geolocation accuracy of those configurations.

For all experimental configurations eccentricity e is zero, inclination i is 45°, argument of perigee ω is zero, and the semimajor axis a varies according to the altitude set in the current configuration as discussed in Section 3.2.1. For the multiple satellite configurations, the satellites are phased in RAAN Ω and true anomaly ν.

An example of the one satellite configuration and its ground trace is seen in Fig. 3.2. The one satellite configuration serves as the basis for the the multiple satellite configurations; that is, the Ω and ν phasing is referenced off of the satellite in this configuration. The phasing values for the two satellite configuration are:

Table 3.1: Ω and ν Phasing for Two Satellite Configuration

	Satellite 1	**Satellite2**
ν (°)	0	1
Ω (°)	0	1

Figure 3.4 shows the Ω and ν phasing for the two satellite configuration with the ground traces for the two satellites plotted; the satellites are in prograde orbits so their velocity vectors are pointing from the bottom left to top right of the frame. This configuration offsets the orbital planes of the satellites by $\triangle\Omega$ which causes the space between the two ground traces. Phasing ν moves the second satellite forward in its orbit by one degree causing satellite two to be slightly ahead of satellite one in the orbit.

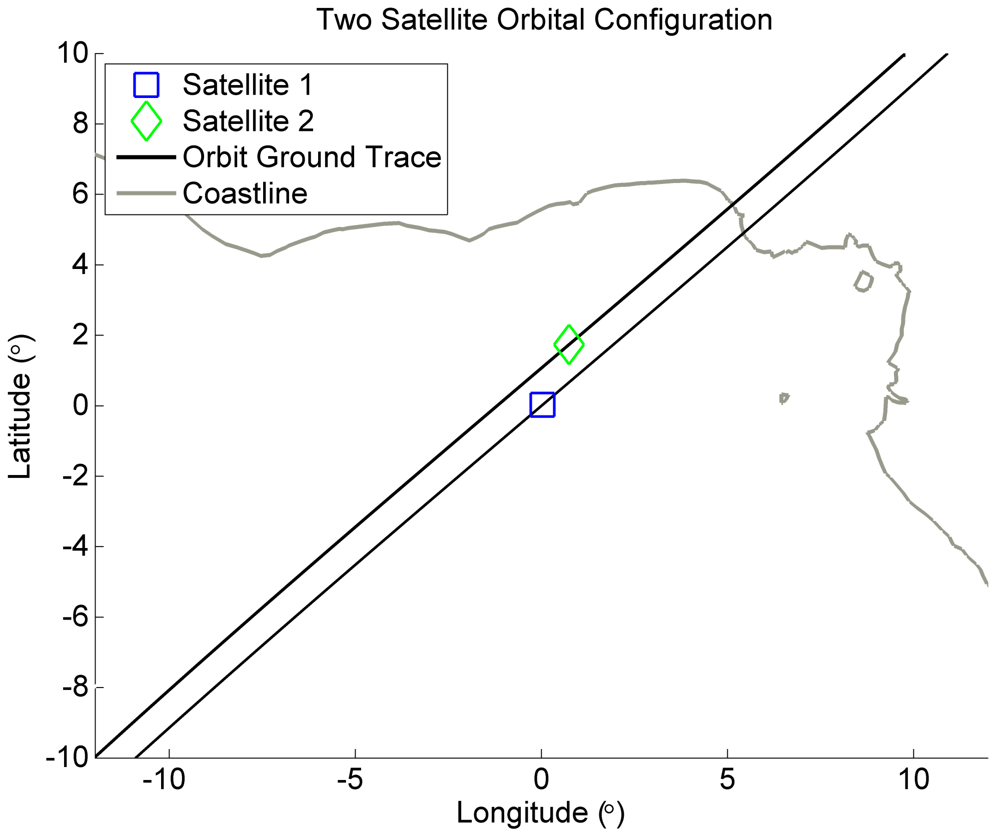


Figure 3.4: Two Satellite Ground Trace and Satellite Constellation

The phasing values for the three satellite configuration are:

Table 3.2: RAAN and ν Phasing for Three Satellite Configuration

	Satellite 1	**Satellite 2**	**Satellite 3**
ν (°)	0	1	2
Ω (°)	0	1	-1

Figure 3.5 shows the Ω and ν phasing for the three satellite configuration with the ground traces for the three satellites plotted; the satellites are moving from bottom left to top right of the frame. This configuration keeps both satellites from the two satellite

configuration and adds a third satellite that is phased by $-\Omega$ and 2ν from satellite one. All satellites in this configuration are in different orbital planes.

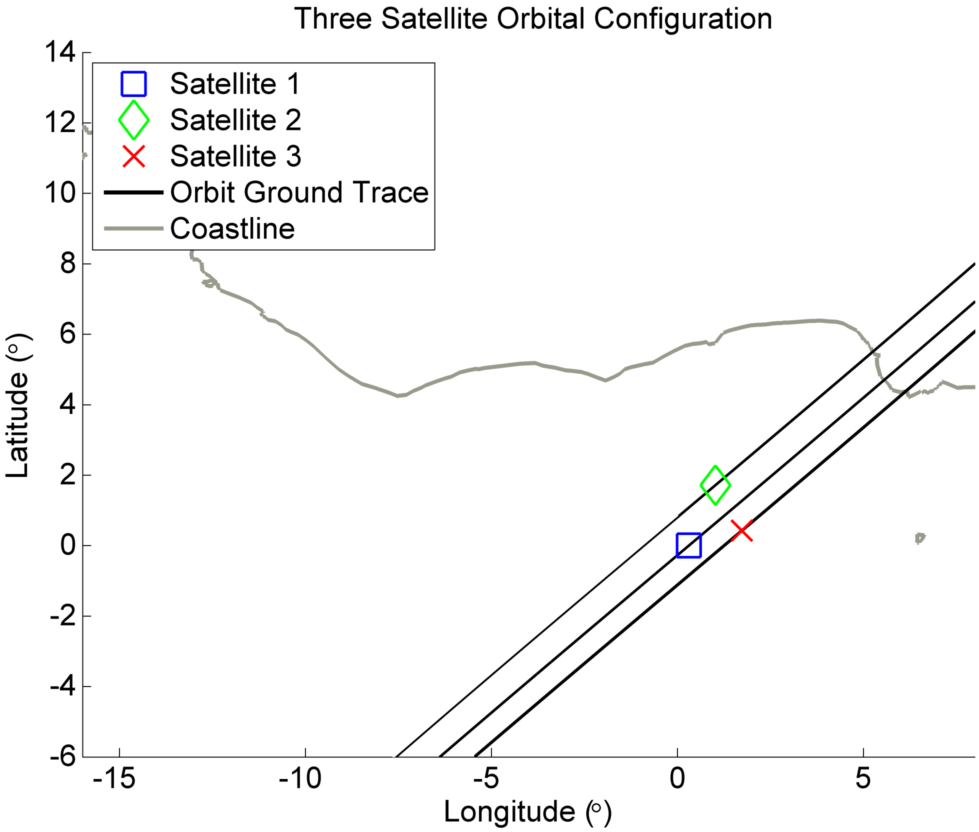


Figure 3.5: Three Satellite Ground Trace and Satellite Constellation

The phasing values for the four satellite configuration are:

Table 3.3: RAAN and ν Phasing for Four Satellite Configuration

	Satellite 1	**Satellite 2**	**Satellite 3**	**Satellite 4**
ν (°)	0	1	2	3
Ω (°)	0	1	-1	0

Figure 3.6 shows the Ω and ν phasing for the four satellite configuration with the ground traces for the four satellites plotted; the satellites are moving from bottom left to top right of the frame. This configuration adds one satellite to the three satellite configuration; the new satellite is not phased in Ω but is phased by 3ν. This means that the fourth satellite is in the same plane as the first satellite, however it is phased three degrees ahead of the first satellite in the orbit. The reason that the orbital ground traces between satellite one and four are slightly offset from each other is due to the rotation of the Earth; this causes the satellites to pass over Earth's surface at different locations even through the satellites are in the same plane. By using this convention to define the satellite constellations, the spacing between the satellites increases as the satellite orbit is increased, to maintain the same spacing between satellites, a different convention would be required.

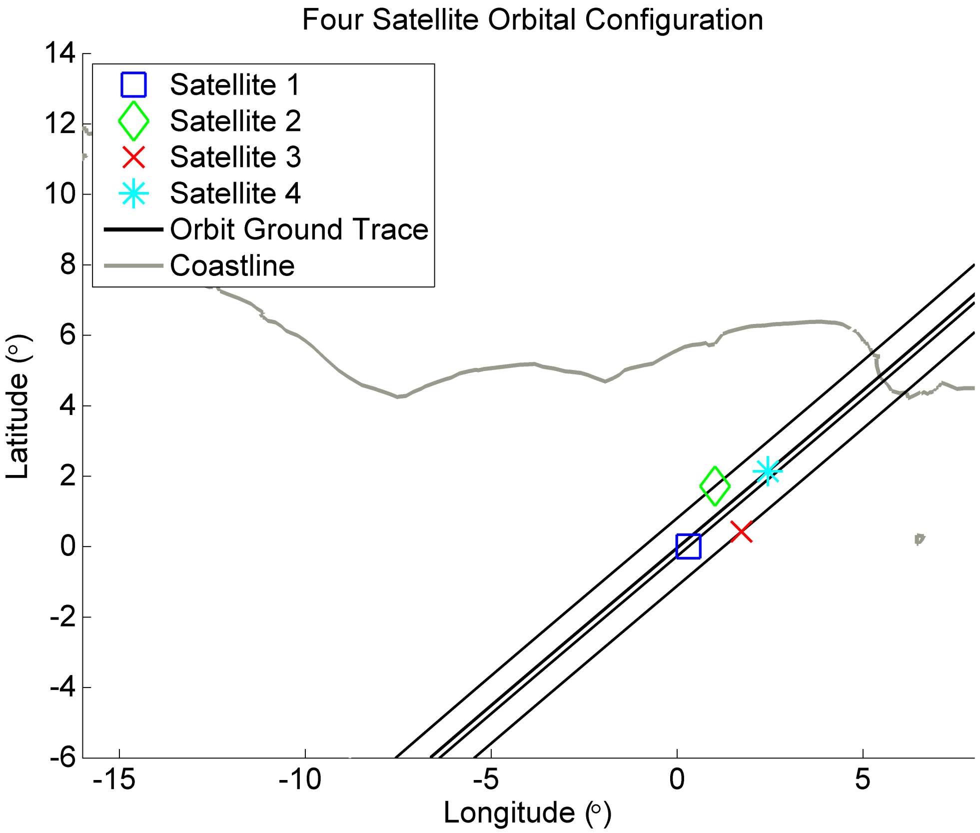


Figure 3.6: Four Satellite Ground Trace and Satellite Constellation

Through comparison of Figures 3.2 - 3.6, it is seen that all of the configurations are subsets of the four satellite configuration. The constellations were designed in this way to allow for comparison of geolocation accuracy as more satellites are added to a constellation.

3.4 Geolocation Solutions and Solution Processing

In this section we will discuss how the geolocation solutions for an entire analysis case are processed to produce a single geolocation solution. Figure 3.7 shows an example of the results for an entire pass of a four satellite constellation for a single geolocation method. A total of 925 individual geolocation estimates were computed during the pass in this example. In order to arrive at a single geolocation estimate, the mean of all the geolocation estimates for each geolocation method is computed. This process is repeated for each geolocation method producing a single geolocation solution for each method used. This process occurs concurrently for all of the geolocation methods, however no data is shared between the different geolocation algorithms.

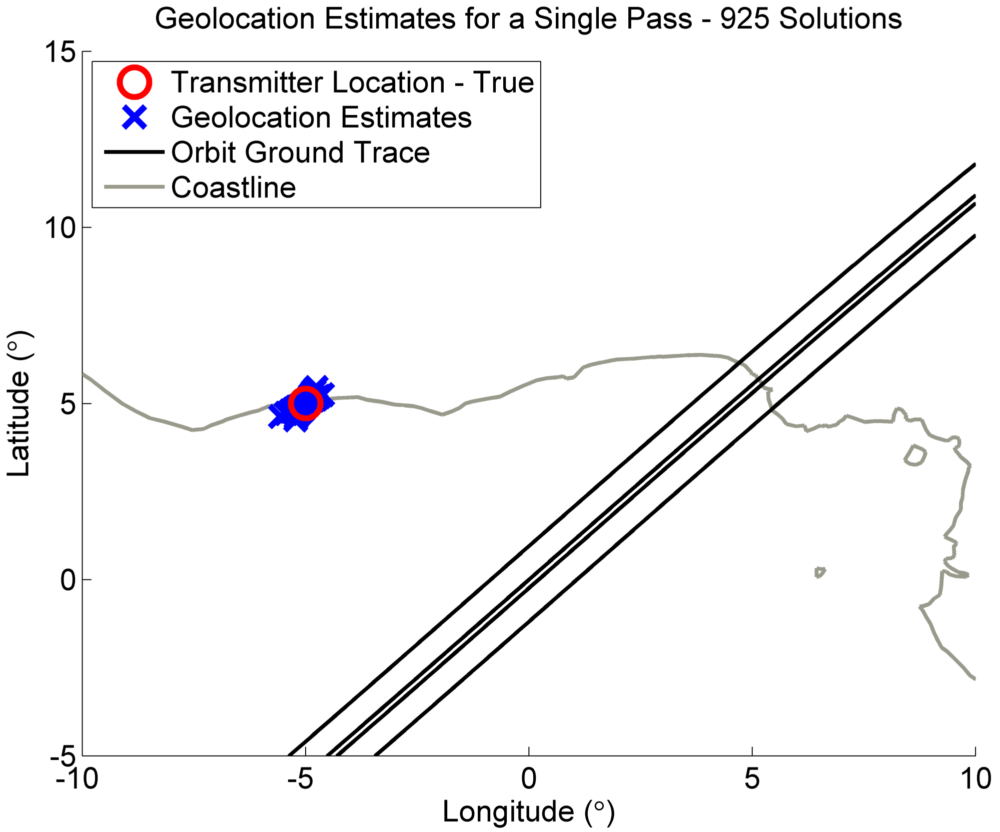


Figure 3.7: Geolocation Solutions Computed for a Single Pass

3.4.1 One and Two Satellite Configurations.

The one satellite and two satellite experiments use the angle of arrival geolocation method. The TDOA methods cannot be used in the one satellite configuration because there is not a second satellite from which to take a time difference. In the two satellite configuration is technically possible to produce TDOA geolocation solutions by gathering single TDOA solutions at different times. These solutions could be combined to produce a geolocation solution once two or more TDOAs were calculated. A methodology to combine TDOA solutions gathered at different times was not considered so TDOA geolocation will not be evaluated for the two satellite configurations.

The angle of arrival solution will utilize the MUSIC algorithm discussed in Section 2.1.2.1, this will will be the only solution method used for the one and two satellite configurations.

3.4.2 Three and Four Satellite Configuration.

The three and four satellite experiments will use the AoA geolocation method utilizing the MUSIC algorithm as well as the two TDOA solution techniques; the explicit solution and the Taylor series method. These methods are fully discussed in Sections 2.1.1 and 2.1.2.

IV. Results

In this chapter we will discuss the results from the altitude and minimal ground trace distance analysis cases. The results from the analysis cases will be presented separately for the different satellite configurations, one through four satellites. Results from the different geolocation algorithms will be discussed in the satellite configuration sections. Comparisons between the satellite configurations will be discussed at the end of the section. The section on the one satellite solution discusses conventions used on the figures for all satellite configurations; these conventions will not be repeated in subsequent sections therefore it is recommended the reader familiarize themselves with this section before reading further in the chapter.

4.1 One Satellite

In this section, the results from the one satellite configuration and an explanation of the format in which the data is presented will be discussed. Figure 4.1 shows the absolute error AoA geolocation data gathered for the one satellite experiments. The average error for all of the geolocation solutions shown in 4.1 is 15589 m. The maximum error is 33353 m which is for the 2000 km altitude, 3100 km minimal ground trace distance analysis case. The minimum error is 41 m which is for the 1000 km altitude, 0 km minimal ground trace distance case. The conventions used in presenting the data will now be discussed further.

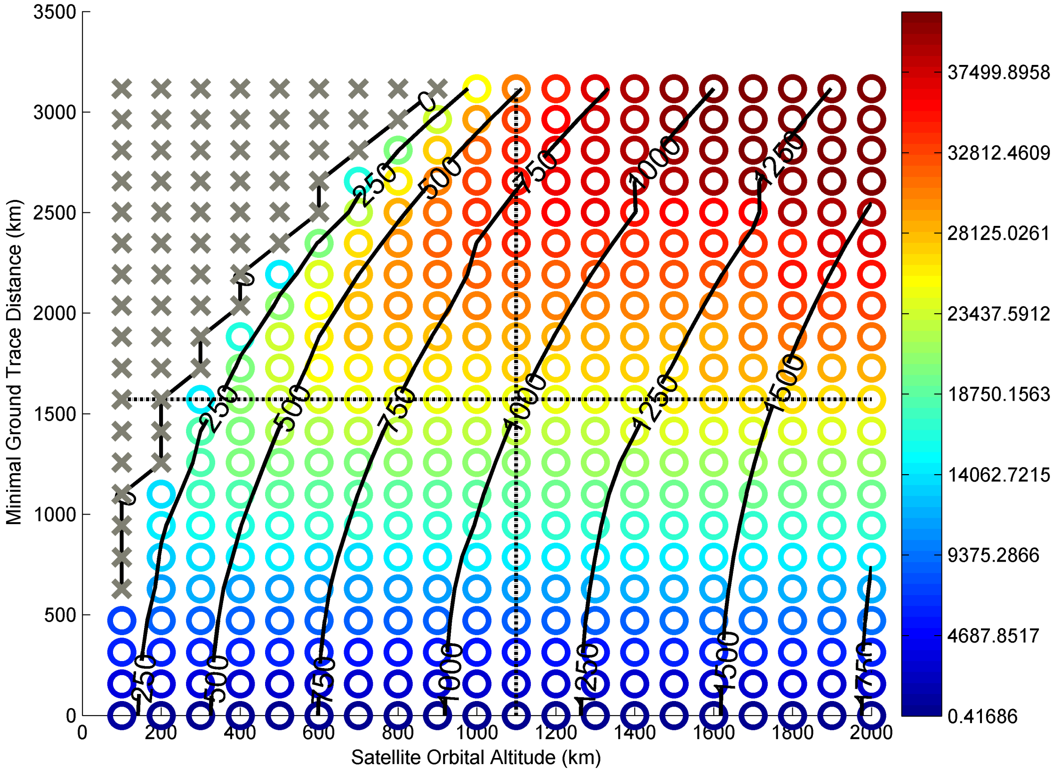


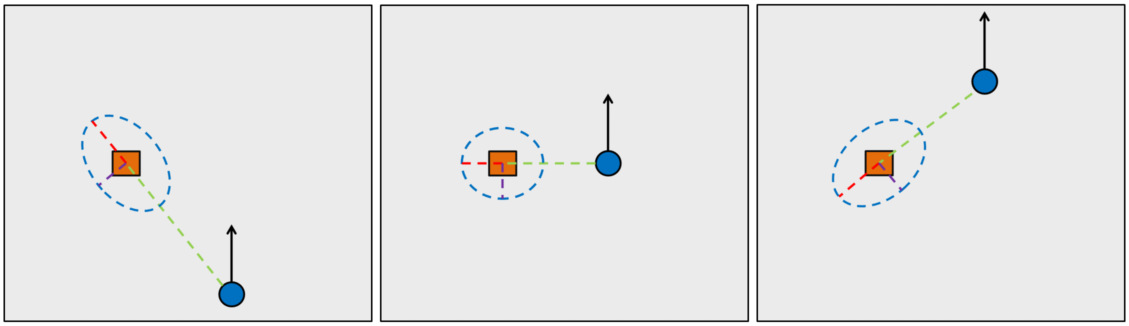
Figure 4.1: One Satellite AoA - Absolute Error in Meters

4.1.1 Data Analysis Explanation.

Each point on the plot represents the mean AoA geolocation solution for a given satellite orbital altitude and minimal ground trace distance. The color of the points corresponds to the absolute error of the geolocation solution in meters; the color bar on the right side defines the upper and lower bounds of the error for the given experiment. The contour lines show the number of individual geolocation solutions included in a solution point. Locations marked by X's are conditions under which no geolocation solution was able to be calculated; discussion of the reasons this may occur will be discussed later in this chapter. The *x*-axis is the satellite orbital altitude; the altitudes plotted are all of the altitudes for the orbital altitude experiment; the *y*-axis is the minimal ground trace distance defined in Section 3.2.2. The horizontal dotted line represents a line of constant minimal ground distance; reading Fig. 4.1 horizontally from left to right shows the effect on geolocation error of varying altitude at a given minimal ground trace distance. The vertical dotted line represents a line of constant altitude, following a vertical line on Fig. 4.1 shows the effect on geolocation accuracy of varying the transmitter minimal ground distance at a given orbital altitude. Lines of constant minimal ground trace distance will be used during discussion of the orbital altitude analysis and lines of constant altitude will be used during the discussion of the minimal ground distance analysis.

Noise was not introduced into the measured signals so the sole source of error in the AoA geolocation solutions is the Gaussian angle measurement error introduced. The effect of angular errors on the AoA geolocation solution are somewhat non-intuitive, a brief discussion of the error mechanisms involved will seek to shed some light on the complexity of these errors.

Figure 4.2 shows the effect of the angle measurement error on the transmitter position error as the satellite propagates through its orbit. The blue dotted line shows the region of error probable caused as a result of the Gaussian measurement error in ϕ and θ. The green dotted line is the line of bearing from the receiver to the transmitter, the red dotted line is the error in the transmitter position estimate due to the error in ϕ and the purple dotted line is the error in the transmitter position estimate due to the error in θ. Figure 4.2 shows that the magnitude of the transmitter position error caused by errors in ϕ and θ changes as the receiver travels past the transmitter. Given the same angular error, the transmitter position error changes when the satellite configuration is changed; varying altitude changes the transmitter position error and as the transmitter minimal distance is changed, the error in transmitter position caused by the same angular error also changes. An exhaustive analysis of this mechanism will not be discussed, however this error mechanism is responsible for the changing errors seen in the AoA anlysis cases.



(a) Approaching (b) Adjacent (c) Departing

Figure 4.2: Effect of Angular Error During a Pass

4.1.2 Orbital Altitude Analysis.

The upper left corner of Fig. 4.1 shows a region where no geolocation solution was obtained. The reason that no solution was obtained for these analysis cases is because the satellite never passed the visibility test shown in Fig. 3.1, in these cases the transmitter was never in view of the satellite. As the satellite altitude is increased, more of the transmitter positions were visible and for altitudes above 1000 km all of the transmitter locations were visible.

Figure 4.3 shows the percent difference in the error between the lowest altitude at which the transmitter could be seen and the highest altitude. This figure obscures the data in the middle altitudes however it is useful for showing the trend in the error as altitude increases. As altitude increases, the error in the geolocation solution increases for all but three of the transmitter locations. The transmitter locations with a minimal ground trace distance less than 300 kilometers had less geolocation error when the satellite altitude was increased. From the contours in Fig. 4.1 it is seen that at low altitudes the minimum number of AoA solutions that go into the mean geolocation solution is less than 250 measurements. As the altitude increases, the number of measurements increases to a maximum at 2000 km altitude where over 1750 measurements go into the solution.

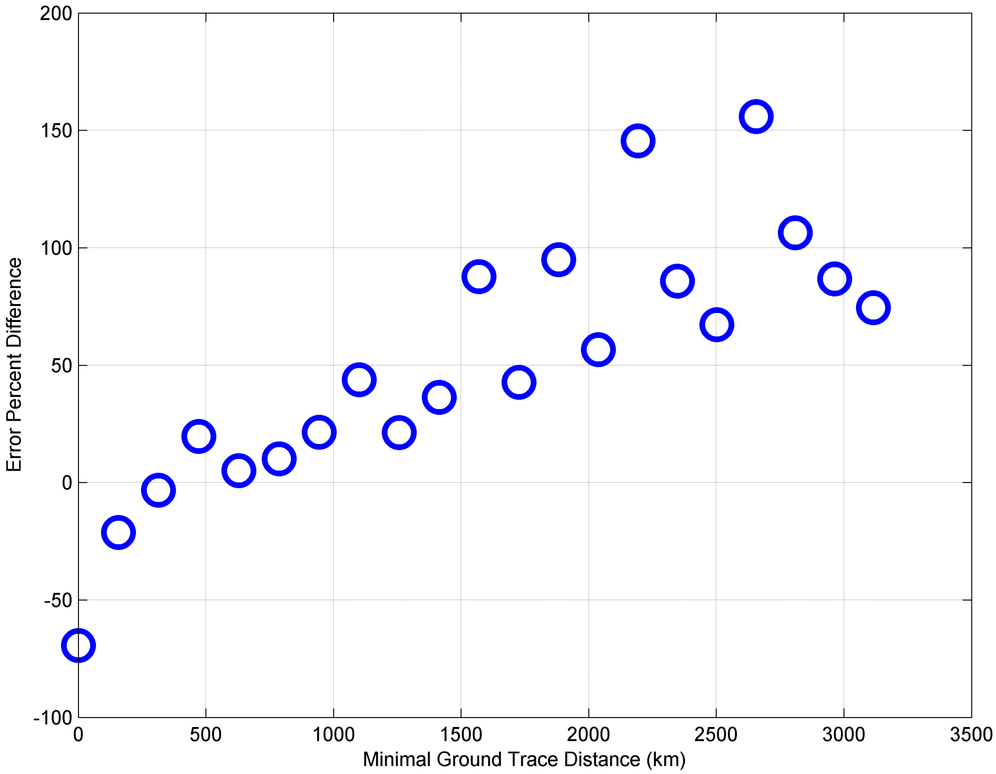
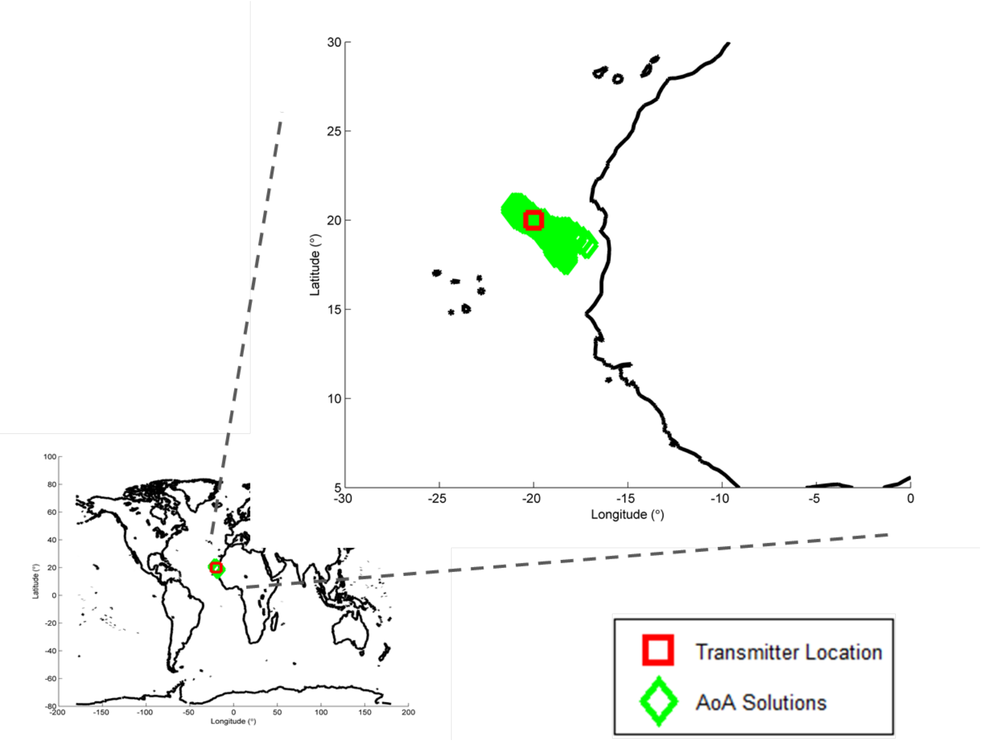


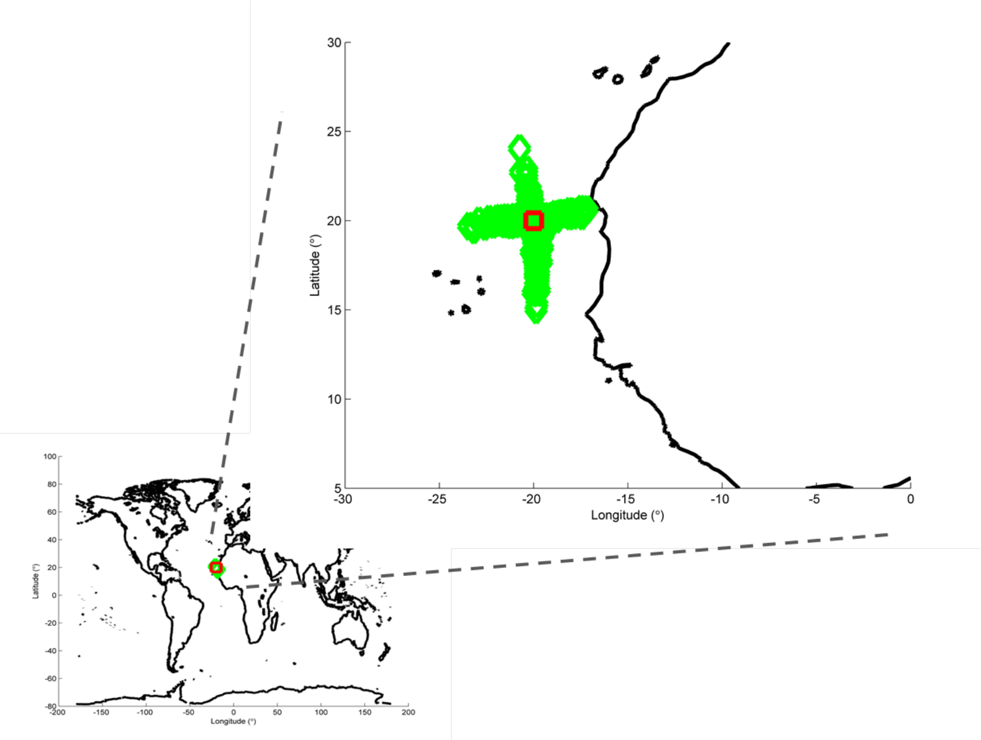
Figure 4.3: Percent Difference in Error Between Lowest and Highest Altitude for Transmitter Locations

Figure 4.4a shows that the geolocation solutions for the 1000 km altitude case, shown as green diamonds, were relatively concentrated near the true transmitter location; there are approximately 300 solution plotted in this case. In this case, the satellite sees the transmitter for only a small amount of time, this means that the angle the receiver makes to the satellite changes very little over the course of the 300 measurements; this is why the data is clustered in a relatively straight line in the direction of the satellite ground path.

In the 2000 km case the geolocation solutions are more spread out in latitude and longitude which is seen in Fig. 4.4b; there are approximately 1300 solutions plotted in this case. The data is more spread out in this case because the receiver is in sight of the transmitter for around five times longer.



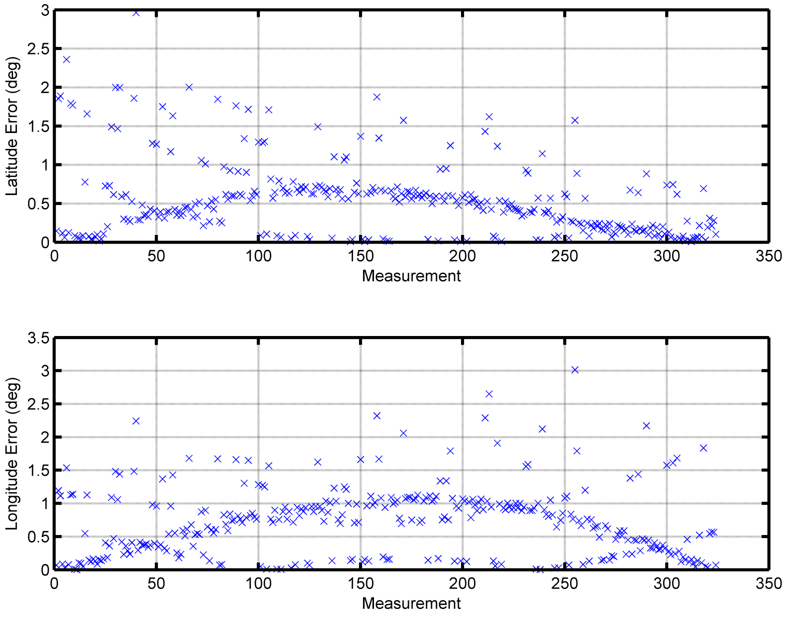
(a) Geolocation Solutions - 1000 km Altitude



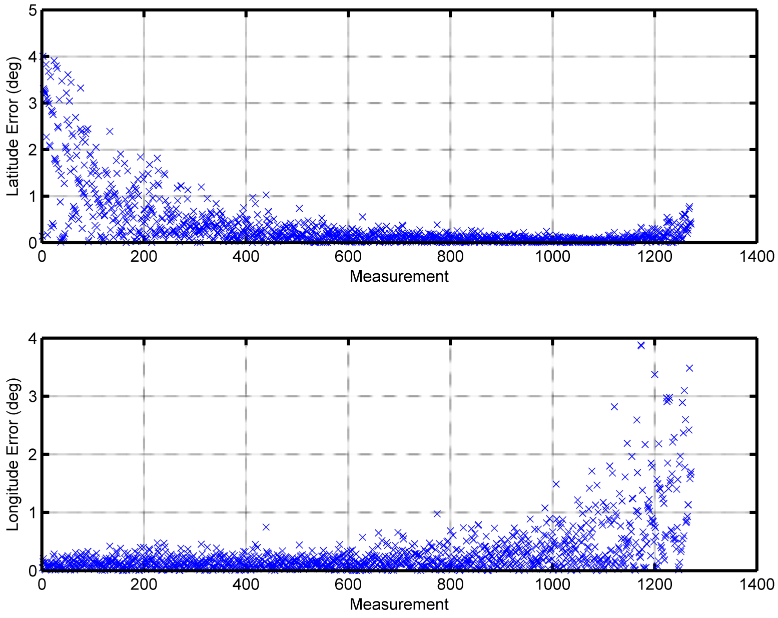
(b) Geolocation Solutions - 2000 km Altitude

Figure 4.4: AoA Geolocation Solutions for Maximum Transmitter Distance

Figure 4.5 provides additional insight into the source of the error. The latitude and longitude errors for the 1000 km altitude case at the maximum transmitter distance are shown in Fig. 4.5a. The majority of the latitude and longitude errors for this case are less than two degrees. Figure 4.5b shows that the latitude errors are dominant at the beginning of the collection period with a substantial quantity above two degrees; the longitude errors show a similar trend at the end of the collection period. These peaks in error are the cause of the increased absolute error observed in Fig. 4.1.



(a) 1000 km Altitude



(b) 2000 km Altitude

Figure 4.5: Latitude and Longitude AoA Geolocation Error for Maximum Transmitter Distance

4.1.3 Minimal Ground Trace Distance Experiment Discussion.

Figure 4.1 shows that in general the geolocation error increases as the minimal transmitter distance is increased. This can be seen by looking at vertical lines of constant altitude, represented by the vertical dotted line in Fig. 4.1. The exception to this is when the transmitter is barely in view of the receiver and only a few hundred geolocation solutions are obtained. The reason the absolute error initially increases and then decreases again is due the fact that the satellite only sees the transmitter for a short period of time, this means the angles between the transmitter and satellite change very little and all solutions are gathered from nearly the same angle.

4.2 Two Satellites

In this section, results from the two satellite experiments will be discussed. The two satellite analysis cases used the angle of arrival geolocation exclusively. Many of the observations discussed in the one satellite configuration discussion are applicable to the two satellite configuration as well.

Figure 4.6 shows the absolute error of the AoA geolocation data gathered for the two satellite experiments. The absolute error for the two satellite cases closely resembles the plot for the one satellite cases seen in Fig. 4.1. The main difference between the two plots is that the magnitude of the error is less for the two satellite cases, a comparison of this difference will be presented at the end of the Chapter. The average error of the two satellite AoA cases in Fig. 4.6 is 14541 m. The maximum error is 30019 m which is for the 2000 km altitude, 3100 km minimal ground trace distance analysis case. The minimum error is 9.5 m which is for the 1800 km altitude, 0 km minimal ground trace distance case.

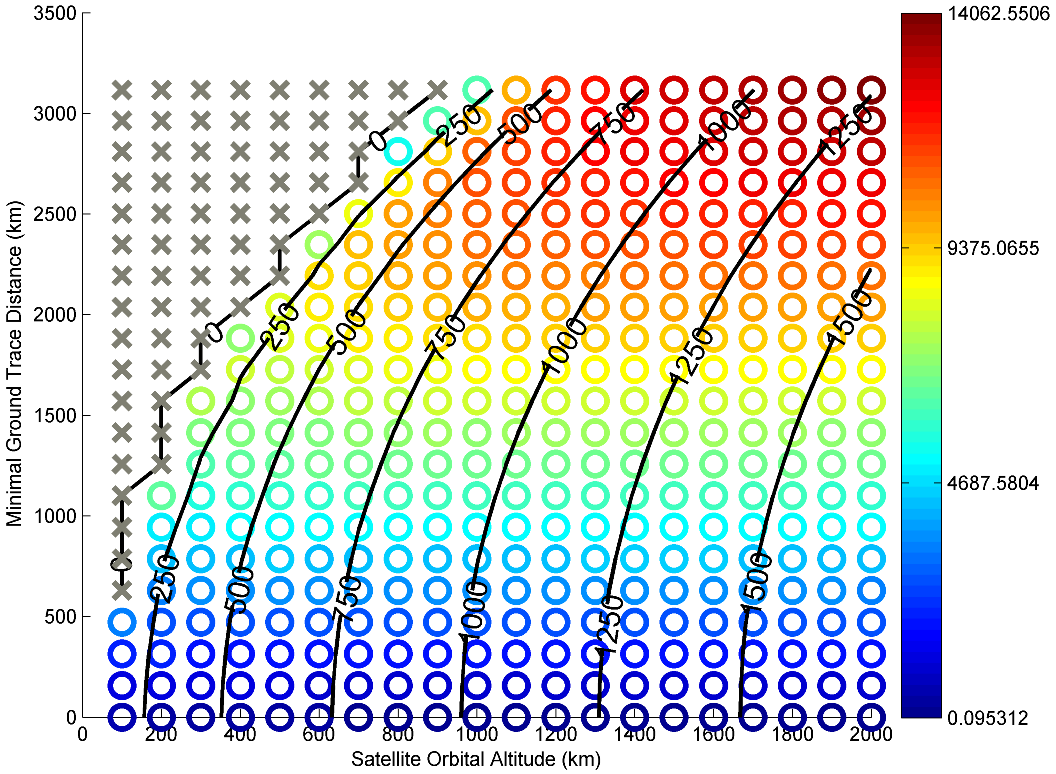


Figure 4.6: Two Satellite AoA - Absolute Error in Meters

4.2.1 Orbital Altitude Analysis Discussion.

For the lower altitude experiments, the satellite constellation is not visible to all of the transmitter locations. As a result, a geolocation solution is not obtained for these experiments; which is what causes the region of no solution at the top left of the chart. For altitudes greater than 1000 kilometers, all twenty of the transmitter locations are visible. The contour lines show that as altitude is increased, the number of geolocation solutions obtained increases for all of the transmitter locations.

Taking a similar approach to analysis as in the one satellite case, the error between the lowest altitude at which the transmitter could be seen and the highest altitude will be discussed. Figure 4.7 shows the percent error between the lowest altitude at which the transmitter can be seen and the 2000 km altitude case. For all but the three largest minimal ground trace distances, there is less than a 50 percent increase in the error between the lowest and highest altitude. For the three furthest minimal ground trace distances, the error is between 100 and 150 percent greater at the highest altitude. For the minimal ground trace distances less than 1750 km, the percent difference in error between the lowest and highest altitude is less than 10 percent and in six of the cases the error decreases as altitude is increased.

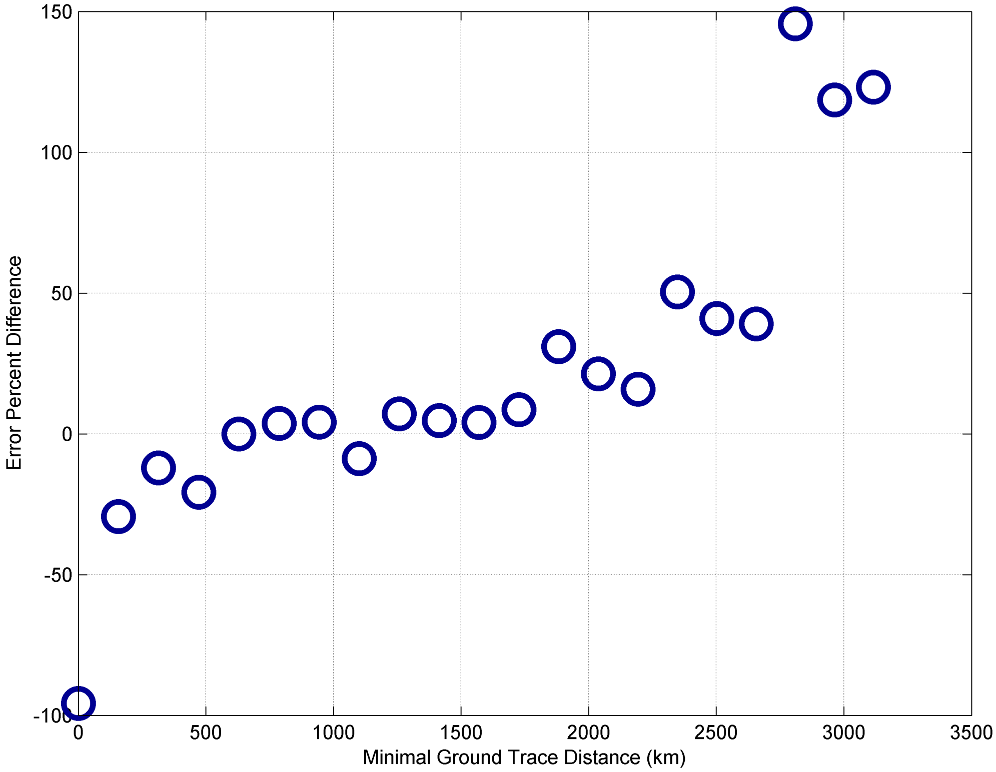


Figure 4.7: Percent Difference in Error Between Lowest and Highest Altitude for Transmitter Locations

4.2.2 Minimal Ground Trace Distance Analysis Discussion.

The minimal ground trace distance shows the same trends in the two satellite case as was seen in the one satellite case. The error increases as transmitter minimal ground distance increases. There are a few cases in the mid altitude ranges where the error increases and then decreases again as the transmitter distance increases to the point that the transmitter is nearly out of view of the satellite.

4.3 Three Satellite

This section will discuss results from the three satellite analysis cases. Unlike the one and two satellite cases, the three satellite cases include time difference of arrival (TDOA) results in addition to the AoA results seen in the previous sections. The two TDOA methods analyzed are the explicit solution and Taylor series method.

4.3.1 Angle of Arrival Solution.

Figure 4.8 shows the absolute error in geolocation solution for the three satellite angle of arrival geolocation cases. These results look similar to what was seen in the one and two satellite geolocation cases however there appears to be more variation in the results than was seen in the one and two satellite cases. The average error for all of the three satellite AoA cases seen in Fig. 4.8 is 8331 m. The maximum error is 69113 m which is for the 800 km altitude, 2800 km minimal ground trace distance analysis case. The minimum error is 88 m which is for the 200 km altitude, 0 km minimal ground trace distance case.

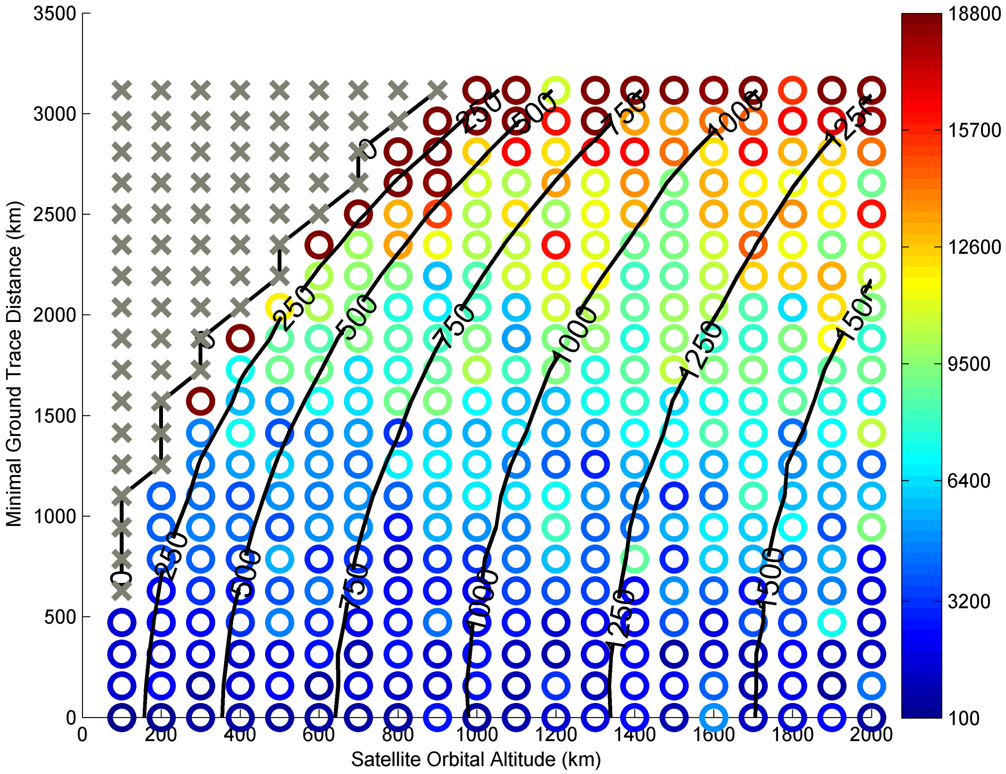


Figure 4.8: Three Satellite AoA - Absolute Error in Meters

4.3.1.1 Orbital Altitude Analysis Discussion.

It is hard to see an immediate trend in the error as altitude is increased in Fig. 4.8. Upon further inspection, there is no noticeable trend in the geolocation error as altitude is varied. For transmitter minimal distance cases that are greater than 2400 km, the error is much greater at the lowest one or two altitudes however as altitude is further increased, no trend is seen in the mean solution error. Further examination of the low altitude cases shows that very few geolocation measurements contribute to these solutions.

Fig.4.9 shows the error for the 2800 km minimal ground trace distance case at 800 km altitude. This plot shows that only 50 measurements went into this solution since the transmitter was only in view for a very short period of time. Latitude and longitude errors peaking at two degrees coupled with the small number of measurement causes the mean solution error in these cases to be large.

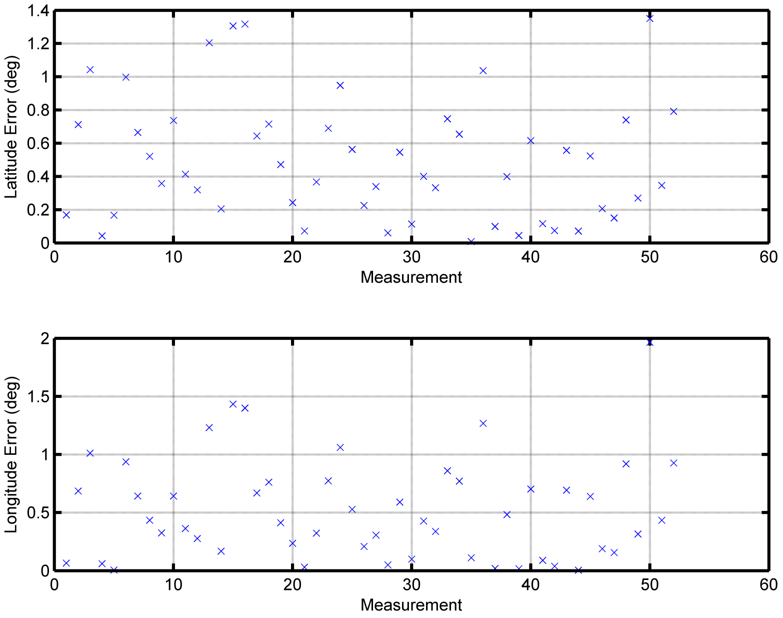


Figure 4.9: Latitude and Longitude AoA Geolocation Error for 2800 km Ground Trace Distance

4.3.1.2 Minimal Ground Trace Distance Analysis Discussion.

It is clear from Fig. 4.8 that the AoA geolocation error increases as the minimal ground trace distance is increased for the three satellite cases. The addition of a third satellite into the constellation caused more variation in the AoA solution and deviated from the similar pattern seen between the one and two geolocation cases.

4.3.2 Explicit Solution.

Figure 4.10 shows the mean absolute error for the three satellite explicit TDOA method cases. The plot shows a few distinct regions; again as in the previously discussed cases, a region exists where the transmitter is never visible to the satellite constellation.

This solution method shows two large regions where the geolocation solution error is much greater than for the other cases. In addition there are regions with much lower geolocation errors. To understand the source of this error, the latitude and longitude errors of the geolocation solutions for the 1400 km altitude, 2600 km minimal ground trace distance case will be analyzed.

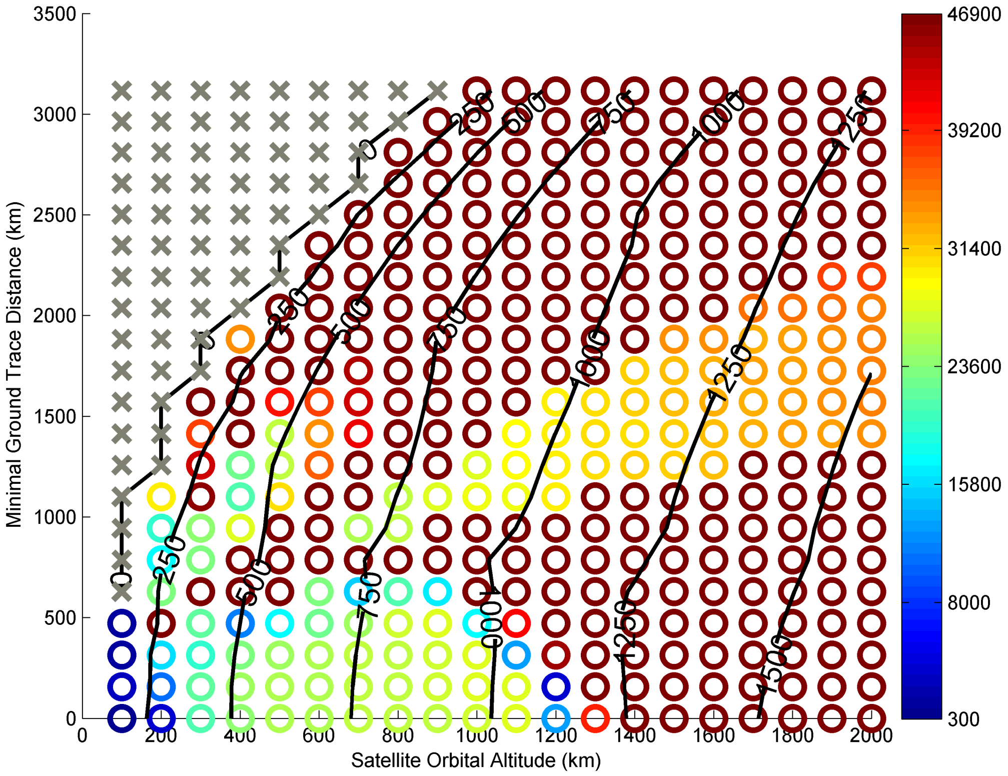


Figure 4.10: Three Satellite TDOA - Explicit Solution - Absolute Error in Meters

Figure 4.11 shows that between the 600 and 700th geolocation measurement, the error in latitude suddenly spikes to 20 degrees and the error in longitude spikes to more than 40 degrees. This sudden spike in error at these locations provides additional insight into the cause of the error. This geolocation method computes the roots of a fourth order polynomial and which in turn gives four possible transmitter locations. Range differences are calculated for the positions given by the roots and then compared to the range difference measured by the receivers; the root that gives the range difference closest to the measured range difference is chosen as the correct root.

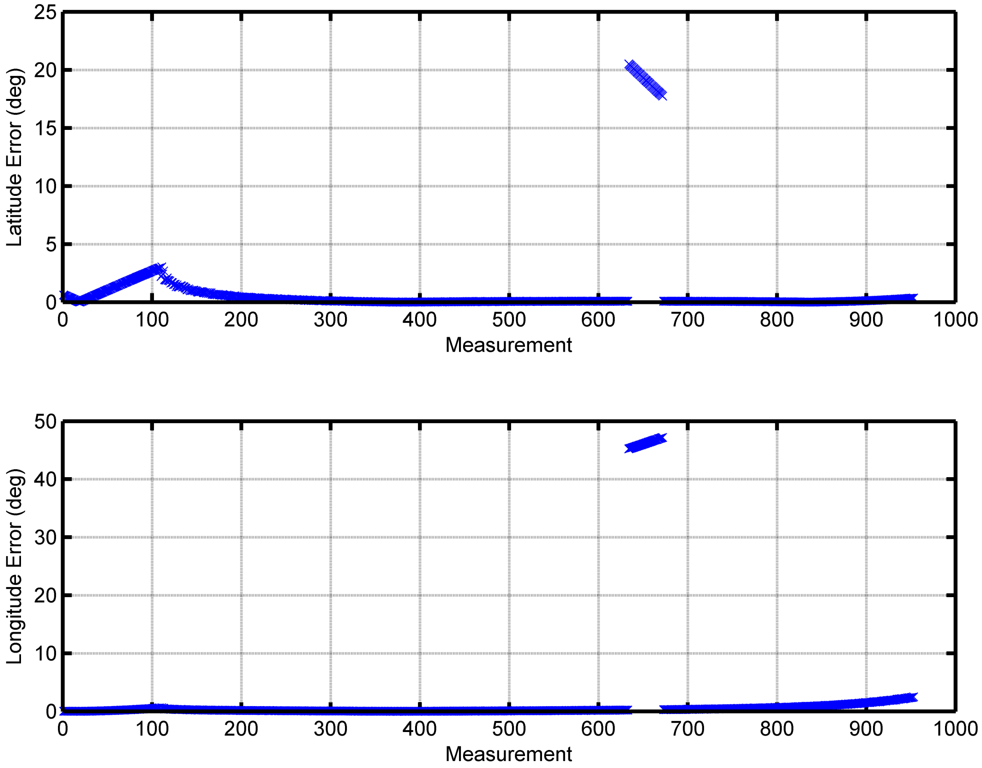


Figure 4.11: Latitude and Longitude Explicit Solution TDOA Geolocation Error for 1400 km Altitude and 2600 km Transmitter Distance

Table 4.1 shows the four possible transmitter locations given by the calculated roots for measurement 660 which is in the region where the error in geolocation solution spikes. The true transmitter location is, 17° latitude, -17° longitude. In this case, the root closest in range difference to the measured values was root two shown in Tab. 4.1. The error this yields is consistent with the errors seen in Fig. 4.11. In order to improve the error of the explicit solution for these cases, an alternative method of root selection is required.

Table 4.1: Four TDOA Roots for 400 km Altitude and 24 km Transmitter Distance - Measurement 660

Root Number	**Latitude**(°)	**Longitude**(°)
1	-10.316	50.349
2	-1.530	29.632
3	22.112	-39.067
4	16.9208	-16.735

4.3.2.1 Orbital Altitude Analysis Discussion.

No clear relationships between orbital altitude and geolocation accuracy can be seen in Fig. 4.10. Upon further investigation, the root selection method contribution to error was wide spread in all of the three satellite explicit solution cases. Even regions shown in Fig. 4.10 with relatively low error still had some measurements that had the incorrect root of the polynomial chosen. This source of error effectively masks trends that may be seen as altitude is varied.

4.3.2.2 Minimal Ground Trace Distance Analysis Discussion.

In the same way that trends were masked in the orbital altitude analysis, trends were also masked for the transmitter minimal distance analysis. For this specific satellite constellation there is likely a relationship between the number of incorrect roots selected throughout a pass and the specific transmitter minimal ground trace distance and orbital altitude. This can be seen in Fig. 4.10 by the distinct regions where the absolute error spikes. Finding an alternative method of root selection would likely eliminate the regions seen and cause the absolute error in these solution to decrease for most of the cases plotted.

In order to properly analyze orbital altitude and transmitter minimal ground trace distance, the analysis cases would need to be rerun with a new root selection method implemented.

4.3.3 Taylor Series Solution.

In this section the Taylor series method for the three satellite configurations will be discussed.

The absolute error for the Taylor series method for the three satellite analysis is shown in Fig. 4.12. It is immediately noticeable by comparing the contour lines to the exact solution contours that fewer solutions are calculated in some regions, shown by the contours extend suddenly to the right and then back to left as on the 750 solution contour line. This pattern in the contours is markedly different than what has been seen in the AoA and exact solutions. It does not appear the error is necessarily increased in these regions however more analysis will be conducted to try to explain a reason for this pattern.

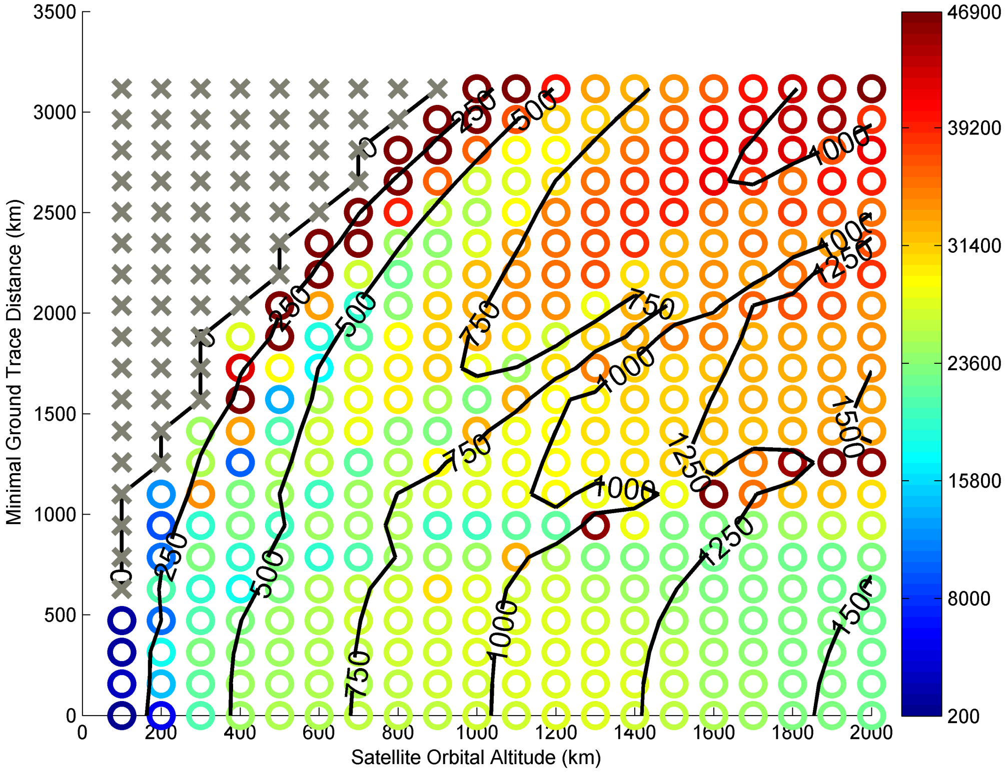
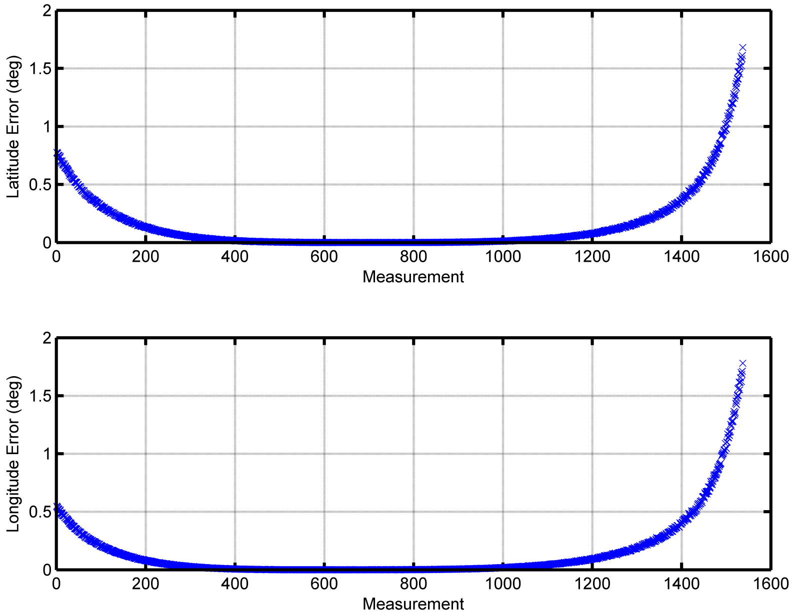


Figure 4.12: Three Satellite TDOA - Taylor Series Solution - Absolute Error in Meters

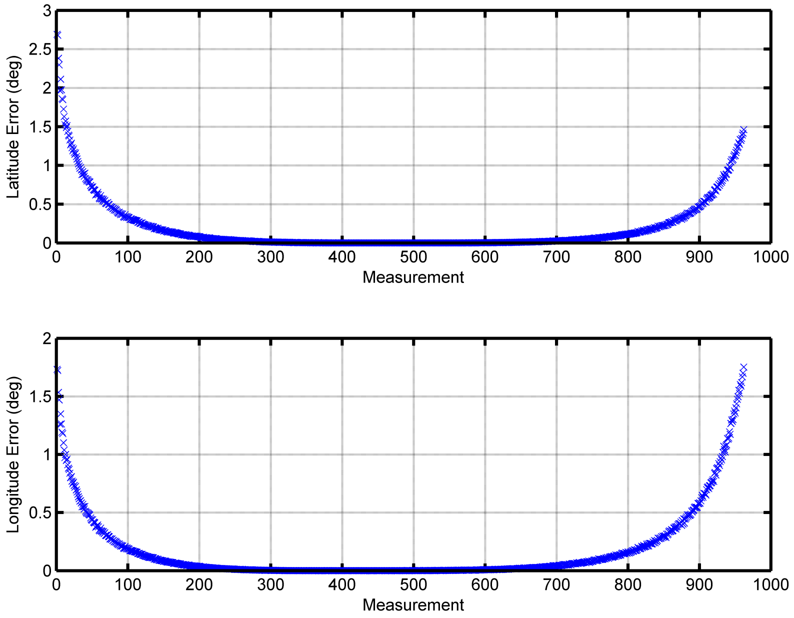
4.3.3.1 Orbital Altitude Analysis Discussion.

For transmitter minimal distances less than 750 km, it is seen that as altitude increases, the geolocation error first increases and then decreases. In order to help understand this, two cases at the 300 km transmitter distance will be looked at more closely, the 1000 and 2000 km altitude cases.

Comparing the error between the 1000 km and 2000 km altitude cases for the 300 kilometer minimal ground distance case, shown in Fig. 4.13, it is seen that the two error profiles look similar. The 1000 km case has fewer total geolocation solution than the 2000 km case however in both cases the error peaks to around 1.5° in latitude and longitude at the end of the collection period. The main difference between the two error profiles is that the lower altitude case has a greater initial error for the first 50 to 100 collects. This greater initial error coupled with the fewer number of collects causes the increased error profile seen in the transmitter minimal ground trace distances less than seven kilometers that is seen in Fig. 4.12.



(a) 1000 km Altitude



(b) 2000 km Altitude

Figure 4.13: Latitude and Longitude Taylor Series Geolocation Error for 300 km Transmitter Distance

4.3.3.2 Minimal Ground Trace Distance Analysis Discussion.

The geolocation error generally increases as the minimal ground trace distance is increased for the three satellite Taylor series solution. There are some exceptions to this rule, such as for 1000 km altitude; Fig. 4.12 shows the error increases and decreases several times as the transmitter minimal ground distance is increased. The reason for this changing error will not be fully investigated in this discussion; the purpose of Fig. 4.12 is to provide insight into error mechanisms affecting the system when Taylor series geolocation is used. A possible reason for the errors seen in the Taylor series is the dependence of the initial guess on the error in the solution. For the Taylor series solution method, the explicit solution position estimate was used as the initial guess; the error seen in Fig. 4.12 appears to have some relation to the errors seen in Fig. 4.10 for this reason.

4.4 Four Satellite

This section will discuss results from the four satellite analysis cases. The four satellite cases include the AoA geolocation method, the explicit solution TDOA and the Taylor series TDOA method. The three and four satellite cases will be compared in the last section of this Chapter to evaluate the TDOA geolocation error as number of satellites are increased.

4.4.1 Angle of Arrival Solution.

The four satellite AoA cases have the smallest overall error out of the satellite cases, the average error for the analysis cases seen in Fig. 4.14 is 6578 m. The maximum error is 36692 m which is for the 800 km altitude, 2800 km minimal ground trace distance analysis case. The minimum error is 81 m which is for the 1400 km altitude, 0 km minimal ground trace distance case.

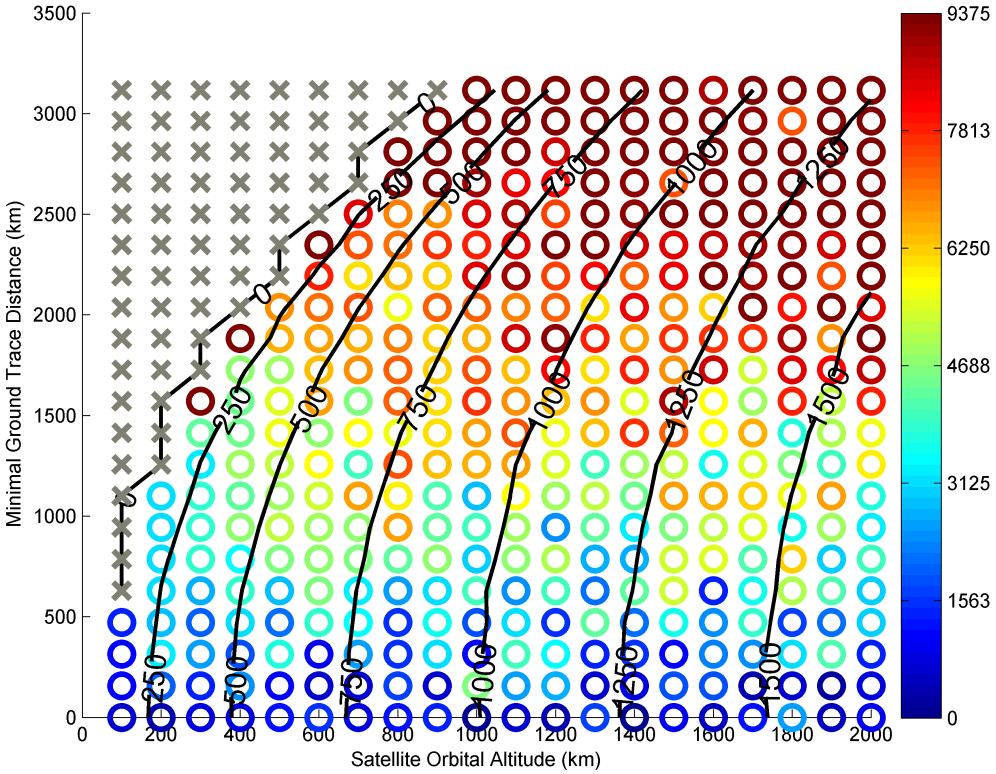


Figure 4.14: Four Satellite AoA - Absolute Error in Meters

4.4.1.1 Orbital Altitude Analysis Discussion.

There appears to be no trend in geolocation error as orbital altitude is increased. No clear trend was seen in the data at any of the transmitter positions; this is similar to what was seen in the three satellite configurations. Causes of error in these solutions has been discussed in previous sections, error mechanisms for the four satellite AoA solutions are due to angle measurement errors and identical to those discussed in previous cases. The decrease in overall error is because four LOBs are used to compute the geolocation solution in this case.

4.4.1.2 Minimal Ground Trace Distance Analysis Discussion.

Like the other satellite configurations, the four satellite analysis cases show that as the transmitter minimal ground trace distance is increased, the error in geolocation solution also increases. The increase in error as minimal ground trace distance is increased is a common trend between the angle of arrival geolocation solutions for all numbers of satellites analyzed.

4.4.2 Explicit Solution.

Figure 4.15 shows two distinct regions of absolute error for the four satellite explicit solution cases. In the upper region, all geolocation solutions have an absolute error less than 900 m; the lower region shows a sudden spike in error to greater than 900 m. The stark division into two regions immediately brings to mind a root selection issue as was seen in the three satellite configurations that exhibited similar regions of increased error, shown in Fig. 4.10. The average error for the analysis cases seen in Fig. 4.15 is 6578 m. The maximum error is 36692 m which is for the 800 km altitude, 2800 km minimal ground trace distance analysis case. The minimum error is 81 m which is for the 1400 km altitude, 0 km minimal ground trace distance case.

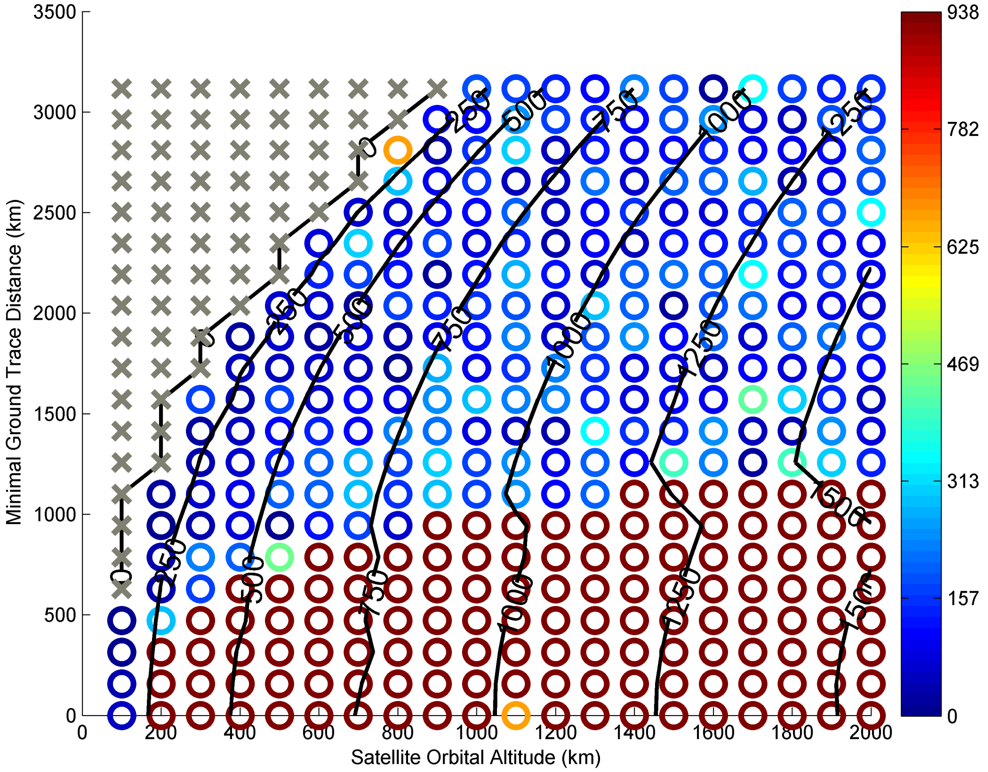


Figure 4.15: Four Satellite TDOA - Explicit Solution - Absolute Error in Meters

Plotting the latitude and longitude error for the 1400 km altitude, 500 km transmitter distance shows that the errors spike during two intervals of the pass which is seen in Fig. 4.16. The latitude error in the first error spike jumps to more than 40 degrees latitude and 30 degrees longitude, this spike occurs near measurement 200. The second spike in error shows a latitude error of approximately 25 degrees and a longitude error peaking at more than 50 degrees. The remainder of the latitude and longitude errors shown in Fig. 4.16 appear to be near zero. These large spikes point towards a root selection problem, to investigate this the roots at measurement 200 will be examined.

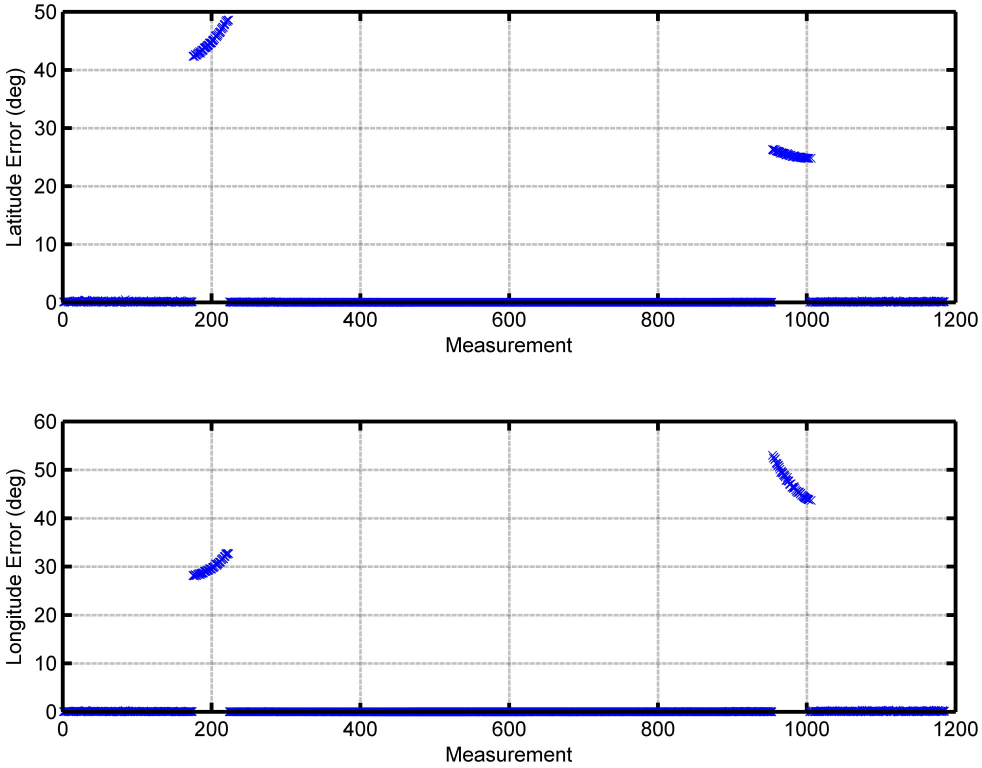


Figure 4.16: Latitude and Longitude Explicit Solution TDOA Geolocation Error for 1400 km Altitude and 500 km Transmitter Distance

Unlike the three satellite configuration solution which solves a fourth order polynomial, the four satellite configuration solves a second order polynomial. The root selection method for the two configurations is the same, the range differences are calculated for the positions given by the roots and then compared to the range difference measured by the receivers; the root that gives the range difference closest to the measured range difference is chosen as the correct root. For measurement 200, the root selected by this method was root one in Tab. 4.2. In this analysis case, the true transmitter position is 4° latitude, -4° longitude. The error caused by the incorrect root selection is consistent with the error seen in Fig. 4.16. Removal of the geolocation solutions that use an incorrect root would likely eliminate the region of increased error seen in Fig. 4.15. As in the three satellite analysis, it is recommended that an alternative root selection method be devised and the analysis cases be conducted again.

Table 4.2: Two TDOA Roots for 400 km Altitude and 500 km Transmitter Distance - Measurement 200

Root Number	**Latitude**(°)	**Longitude**(°)
1	-40.695	-33.587
2	4.022	-3.987

4.4.2.1 Orbital Altitude Analysis Discussion.

For the discussion in of the orbital altitude analysis, only the region with small error in the upper region of Fig. 4.15 will be considered. Transmitter positions greater than 1100 km will be discussed in this section, the positions less than 1100 km will not be considered since the data is obscured by root selection error previously discussed. For all transmitter positions in the upper region of Fig. 4.15, the geolocation error is less than 900 m with the majority of errors being less than 400 m. There is no clear trend as the altitude is increased however there are isolated cases that show a small spike of 100 or 200 m in absolute error. A likely cause for these spikes is Gaussian timing error introduced into the TDOA measurements. With the exception of these small error spikes, the error in this region is very close in magnitude.

4.4.2.2 Minimal Ground Trace Distance Analysis Discussion.

When the minimal ground trace distance is less than 1100 km, the root selection issue begins to manifest in Fig. 4.15. The geometry of the transmitter position with respect to the satellite constellation likely causes the root selection error since the entire region shows the same spike in error; development of a new root selection method would likely fix these errors and the entire plot would resemble the magnitude of errors seen for cases with greater than 1100 km minimal ground trace distance. The contour lines showing number of geolocation solutions obtained moves to the right across the division of the high and low error regions; this occurs because after a geolocation solution is obtained, a visibility check is performed to see if that solution is in sight of the satellite constellation. If the visibility test for the solution fails, the geolocation solution is thrown out. The jump in the contour line indicates that the geolocation solutions given by some of the incorrect roots are not in view of the satellite constellation.

4.4.3 Taylor Series Solution.

Figure 4.17 shows the absolute error in meters for the four satellite Taylor series geolocation analysis cases. The average error for this method is 390 m, a large improvement over all of the other configurations analyzed. The maximum error on this plot is 13741 m and occurs for the 700 km altitude, 300 km minimal ground trace distance case. The minimum error is 180 m and occurs for the 100 km altitude 0 km transmitter minimal ground trace distance case. Again since the Taylor series solution used the explicit solution as the initial guess, a region of higher error can be seen at the bottom of Fig. 4.17; this region is caused by greater error in the initial guess seen in Fig. 4.15.

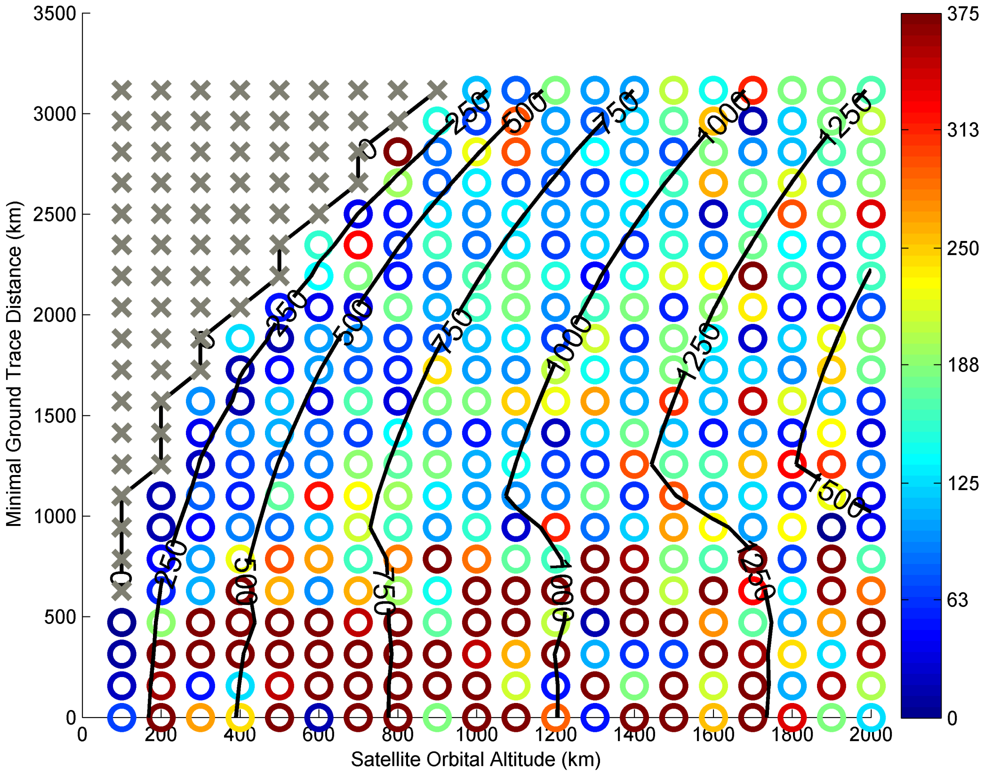


Figure 4.17: Four Satellite TDOA - Taylor Series Solution - Absolute Error in Meters

4.4.3.1 Orbital Altitude Analysis Discussion.

No clear trend in the error is apparent as the altitude is increased. The errors appear to fluctuate randomly across the altitudes with no pattern as altitude is increased.

4.4.3.2 Minimal Ground Trace Distance Analysis Discussion.

There is a noticeable decrease of the solution error as the minimal ground trace distance is increased from the 800 to 900 km analysis case. A clear trend in how the error changes is not seen as the transmitter distance is increased to values greater than 900 km. Additionally at values below 800 km, it appears there is no trend in error as the transmitter distance is decreased to zero.

4.5 Satellite Configuration Comparison

This section will compare the geolocation methods as satellites were added to the constellation. The ability to compare different constellation configurations is the crux of this research, this is the method used to compare the different configurations analyzed. Since data was taken in cases for each orbital altitude and transmitter position analysis case, the data can be easily compared by taking a percent difference in the error between cases.

The average error for all analysis cases for each method is shown in Table 4.3. Comparing the three and four satellite cases for the TDOA methods, it is immediately obvious that the four satellite configurations provided more accurate geolocation solutions than the three satellite configuration. For all of the TDOA methods, the four satellite configuration average error was an order of magnitude more accurate than the three satellite configurations. The angle of arrival solutions shown in the first column of Table 4.3 show that the average error steadily decreased as more satellites were added to the constellation.

Table 4.3: Mean Error of All Analysis Cases for each Geolocation Method

	AoA (m)	TDOA - Explicit (m)	TDOA - Taylor (m)
One Satellite	15589.5	-	-
Two Satellite	14541.5	-	-
Three Satellite	8330.7	337949.5	85686.5
Four Satellite	6578.1	24907.4	390.6

4.5.1 Angle of Arrival.

This section uses the four satellite AoA configuration as a baseline for comparison. The plots shown in this section are in terms of percentage of the four satellite analysis case errors. This allows the configurations with different numbers of satellite to be compared against one another.

The figures seen in this section are similar to those seen in the previous analysis cases however these plots are shaded according to the percent difference in error between two cases. Red denotes that the analysis case being compared has error four or more times greater than the case against which it is being compared. Figure 4.18 shows the percent difference in the errors between the one satellite and four satellite AoA analysis cases. It is seen that for the smaller values of transmitter minimal ground distance, the one satellite angle of arrival performs as well or better than the four satellite configurations. As the transmitter distance is increased, the geolocation error for the one satellite AoA increases as a percentage of the four satellite error at the same analysis case. In nine of the analysis cases, the one satellite error is more than four times greater than the four satellite geolocation error, these are marked by the red circles. On average for the cases compared the one satellite AoA solution error is 1.43 times greater than the four satellite configuration.

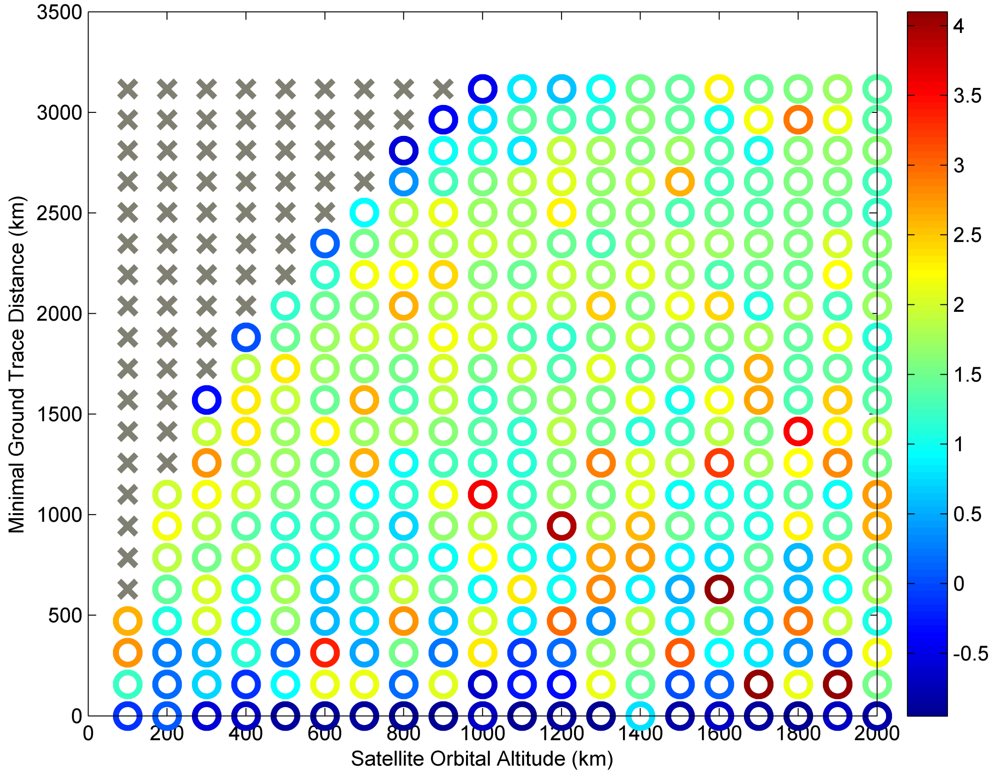


Figure 4.18: One Satellite AoA to Four Satellite AoA Comparison

Figure 4.19 shows the percent difference in the errors between the two satellite and four satellite AoA analysis cases. Again at the smaller values of transmitter minimal ground distance the two satellite case performs as well or better than the four satellite AoA geolocation; there are 26 analysis cases where the two satellite AoA geolocation has less error than the four satellite AoA geolocation. In eight of the cases, the two satellite configuration has error more than four times greater than the four satellite geolocation solution error. Taking the mean for this comparison, the two satellite AoA geolocation error is on average 1.28 times greater than the four satellite geolocation error, an improvement over the error seen in the one satellite configurations.

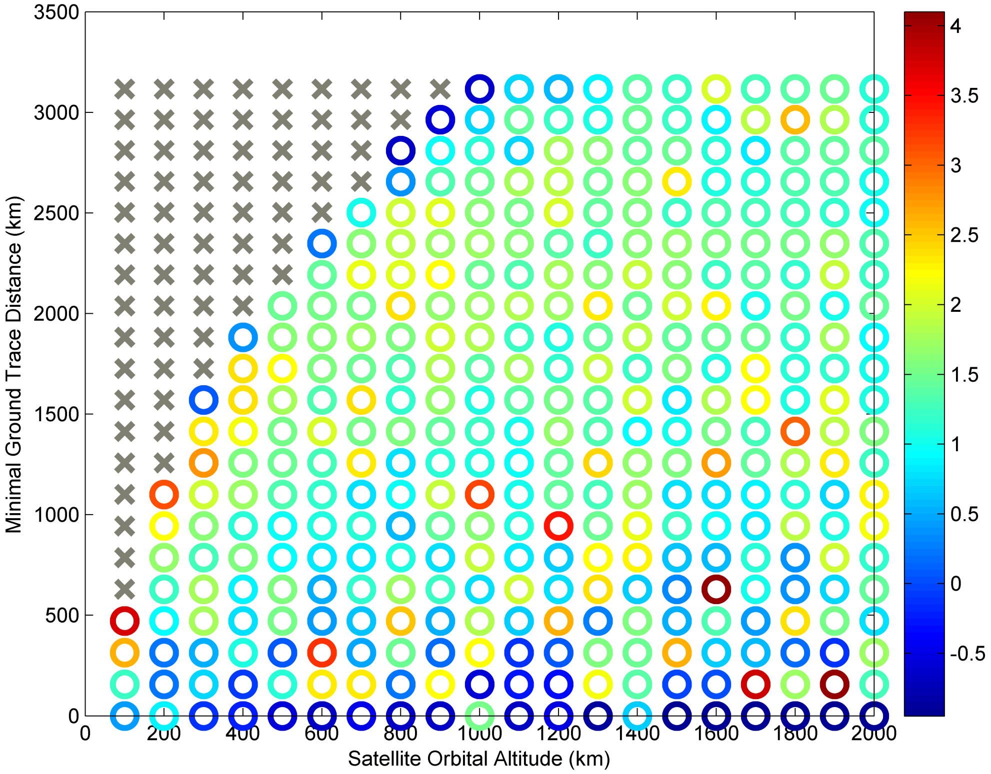


Figure 4.19: Two Satellite AoA to Four Satellite AoA Comparison

The final AoA comparison is the percent difference in the errors between the three and four satellite AoA analysis cases, shown in Fig. 4.20. It is immediately obvious that the three satellite AoA solution error is close in magnitude to the error observed in the four satellite AoA solutions, this is evidenced by the large number of cases that are shaded blue, below the 1.5 mark on the colorbar. Four of the three satellite analysis cases have four or more times greater error than the four satellite AoA cases, shown by the red circles. A total of 121 of the three satellite analysis cases have less error than the four satellite solution. On average, the error for the three satellite AoA analysis cases is 0.35 times greater than the four satellite cases.

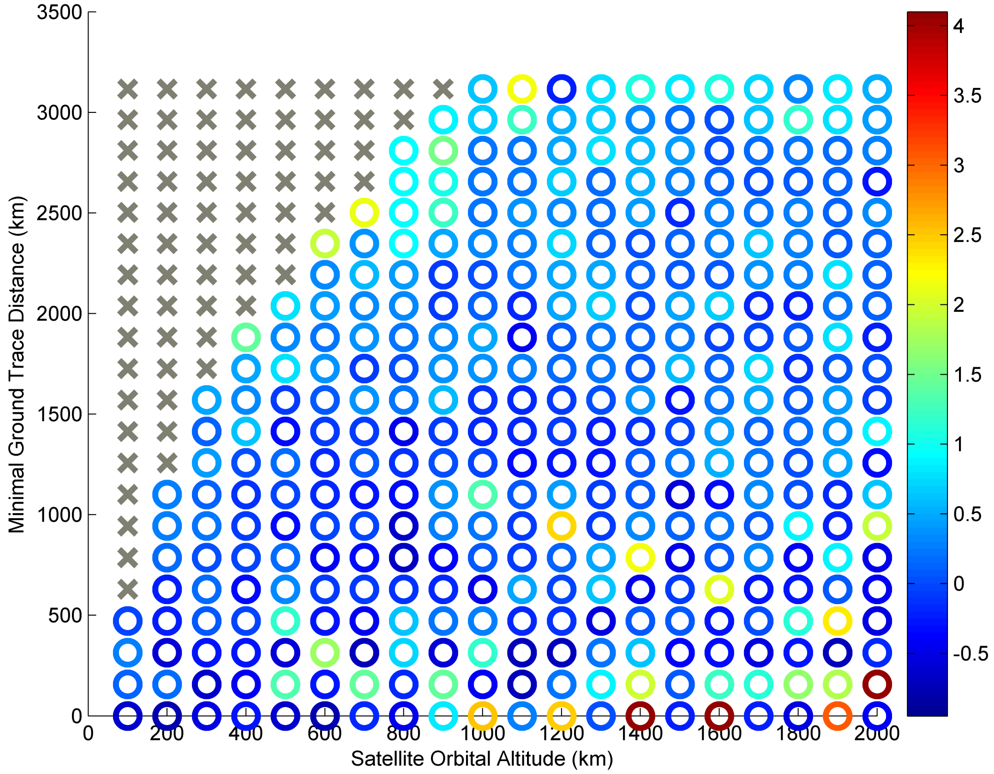


Figure 4.20: AoA - Three Satellite to Four Satellite Error Comparison

As satellites are added to the constellation, the AoA solution shows a steady decrease in average geolocation error, with the four satellite configuration having the least error on average of all of the AoA geolocation cases tested. Additionally the three satellite analysis cases show large reduction in error over the one and two satellite cases.

4.5.2 Explicit Solution TDOA.

The comparison in this section shows the percent difference in error between the three and four satellite explicit solution TDOA method.

The percent difference of the three satellite explicit solution geolocation cases as a percentage of the four satellite explicit solution geolocation cases are shown in Fig. 4.21. Looking back to Figs. 4.10 and 4.15, the form of this plot makes sense. The region at the bottom of the Fig. 4.21 shows the region where the four satellite geolocation solution selected the incorrect root to the geolocation equations. The large region of high error at the top of the plot is the region where the four satellite analysis cases chose the correct root in the geolocation process, coupled with the incorrect root selection of the three receiver explicit solution method causes a region in which the three satellite configuration solution error is much greater than the four satellite solution. The three satellite geolocation is on average more than 3000 times greater than the error for the four satellite explicit solution in this region. In the region of lower error at the bottom of Fig. 4.21 the three satellite error is only 3.44 time greater than the four satellite cases. It is important to note that in this lower region of the plot, both the three and four satellite method choose the incorrect polynomial root for the majority of the cases. In 27 cases, the three satellite explicit method had less error than the four satellite solution; these cases represent the region in which the four satellite geolocation method selects the incorrect root while the three satellite method choses the correct root. In cases where both methods chose the incorrect root, the four satellite explicit TDOA solution always had less error.

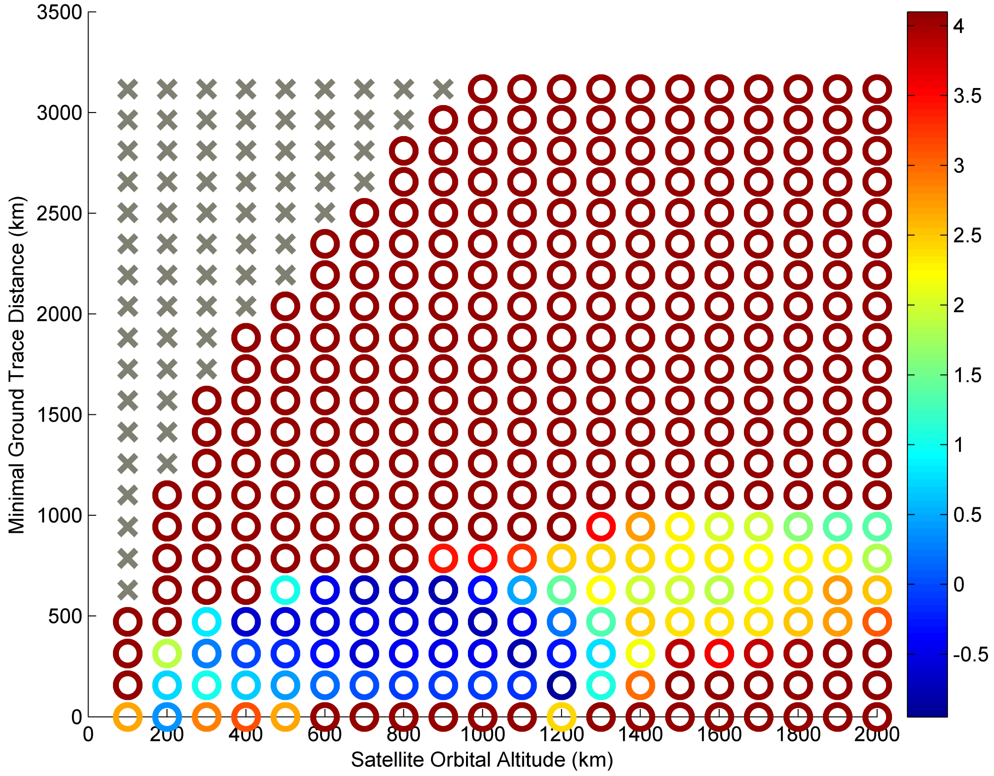


Figure 4.21: Explicit TDOA - Three Satellite to Four Satellite Error Comparison

Fixing the root selection problem with the explicit solution TDOA method would remedy the issues manifest in this plot.

4.5.3 Taylor Series.

The comparison in this section shows the percent difference in error between the three and four satellite Taylor series TDOA solution method.

Figure 4.22 shows the percent difference in error between the three and four satellite Taylor series TDOA solutions. The colorbar shows that the error in the three satellite configuration is much greater than in the four satellite configuration. In ever analysis case, the four satellite Taylor series geolocation has less error than the three satellite case. On average, the three satellite case Taylor series TDOA error is 695 times greater than the four satellite case.

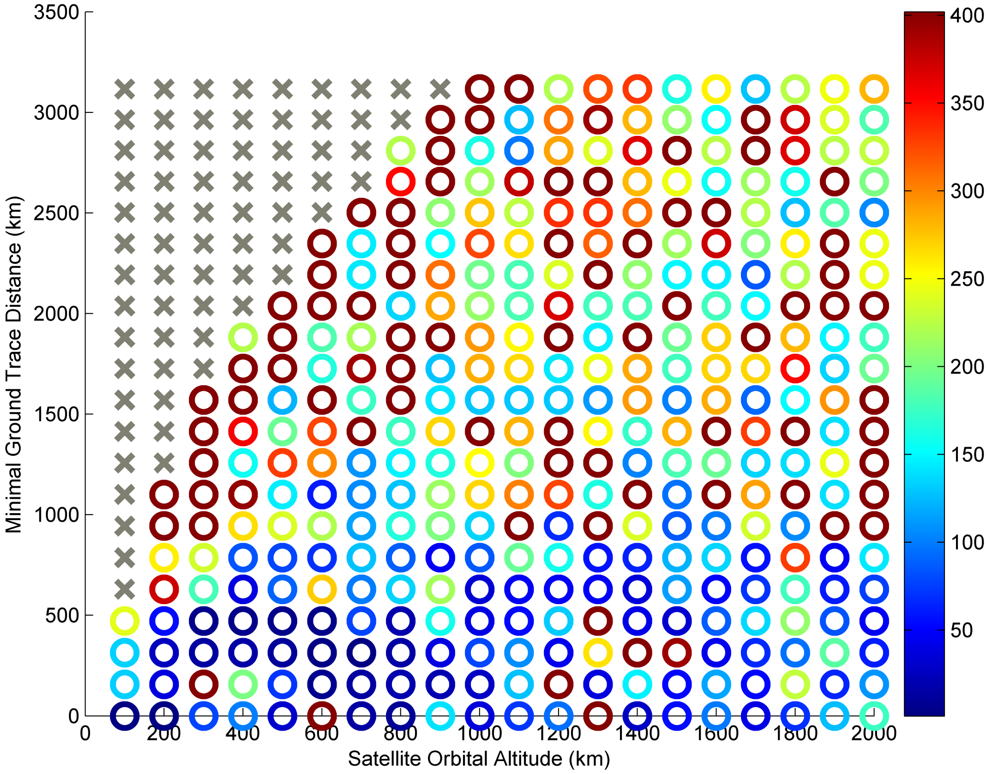


Figure 4.22: Taylor TDOA - Three Satellite to Four Satellite Error Comparison

4.5.4 TDOA to AoA Comparison.

This section show cases how dissimilar configurations and methods may be evaluated against one another using this method.

Fig. 4.23 shows the one satellite AoA solution error as a percentage of the four satellite explicit TDOA solution error. It is clear that for the lower region of the plot where the four satellite explicit TDOA solution selected the wrong root, the one satellite AoA performed better. The error in this region was on average 0.89 times less than the TDOA error in the same region. The large red region at the top of the plot is where the four satellite TDOA solution chose the correct polynomial root during the geolocation solution calculation. In this region, the one satellite AoA had 238 times more error than the four satellite explicit TDOA method.

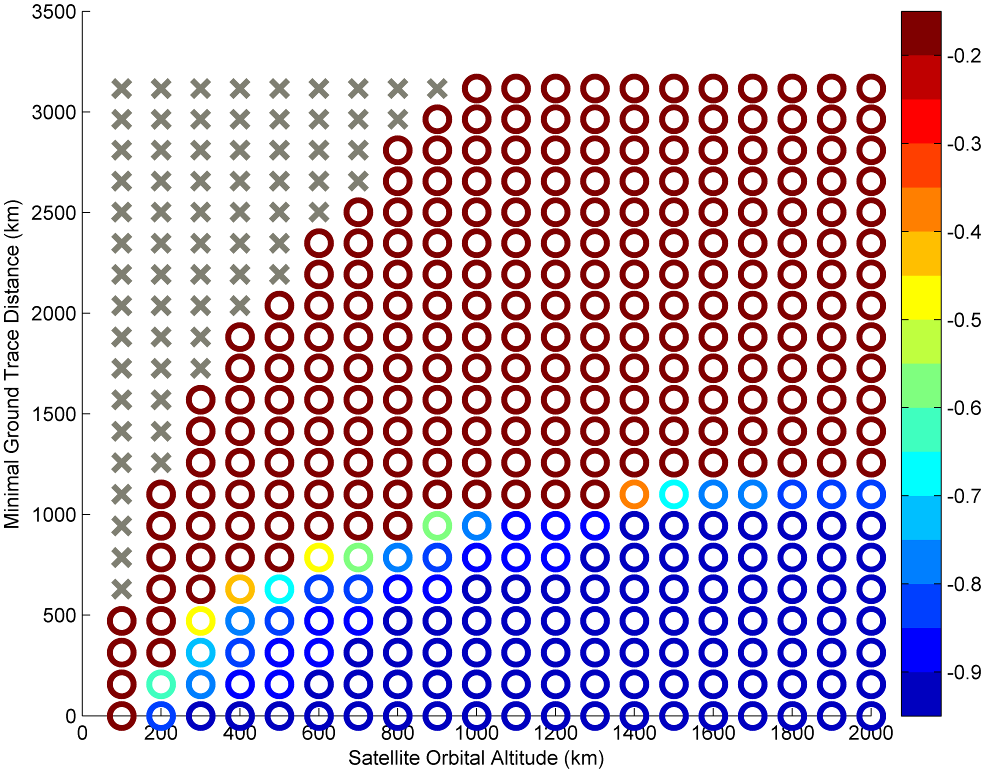


Figure 4.23: One Satellite AoA to Four Satellite Explicit TDOA Error Comparison

Now the four satellite AoA analysis case will be compared to the four satellite explicit TDOA method. Figure 4.24 shows the four satellite AoA solution error as a percentage of the four satellite explicit TDOA solution error. The plot looks almost identical to the plot for the one satellite AoA solution. In the region that the TDOA method selected the wrong root, the four satellite AoA had 0.92 times less error that the TDOA method in the same region. In the top portion of the plot, the AoA method had 92.5 times more error on average than the four satellite explicit TDOA method.

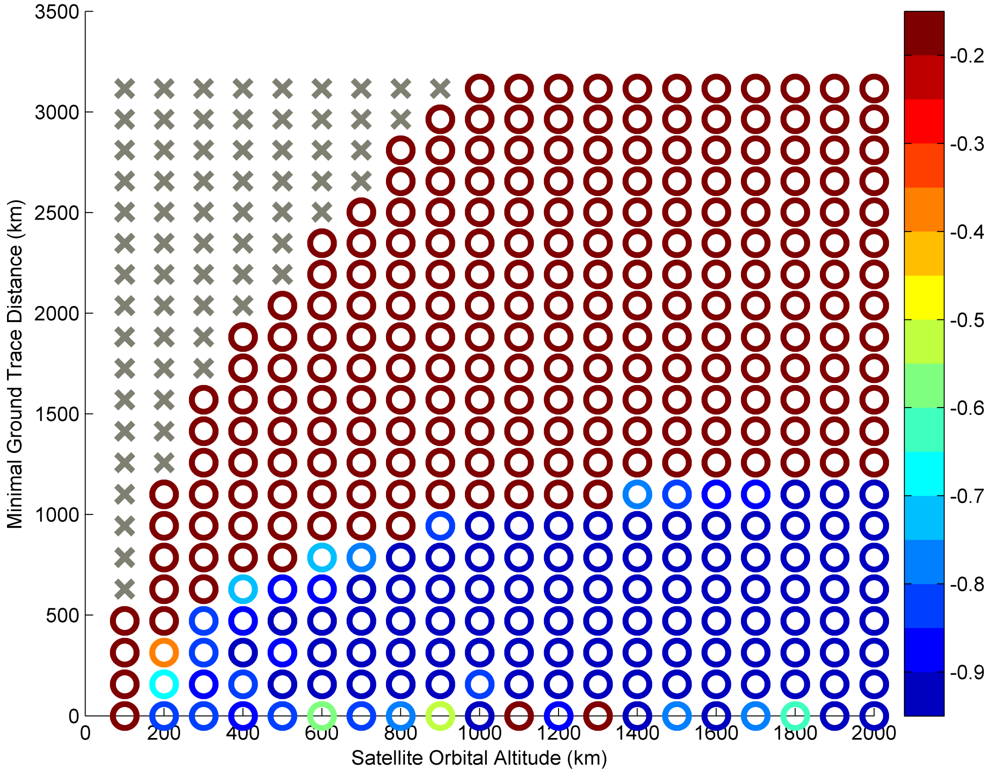


Figure 4.24: Four Satellite AoA to Four Satellite Explicit TDOA Error Comparison

The ability to compare two completely different geolocation methods in two completely different configurations allows tradeoffs between two dissimilar configurations to be quickly seen and analyzed. This method of comparing geolocation methods and satellite configurations may be applied to any geolocation methods that exist currently or that are developed in future work. Additionally this may be used to compare different satellite constellations even if they do not share common satellites as was the case in this research.

V. Conclusion

In this section an overview and conclusion of the work completed will be presented. Motivation for the research, background information, the methodology and the conclusions on analytical results will be discussed.

Evaluation of how different geolocation methods perform under a variety of mission configurations and evaluation of different methods against each other is of prime interest to the Air Force Institute of Technology (AFIT). A tool is needed to evaluate different configurations for a planned geolocation CubeSat being developed at AFIT; this research involved developing a tool to fill the geolocation analysis need.

Several geolocation methods were analyzed including the angle of arrival (AoA) method utilizing the multiple signal classification (MUSIC) algorithm, an explicit solution time difference of arrival (TDOA) method and a Taylor series TDOA method. Additional methods could be devised and analyzed in the same way presented in this paper.

Satellite configurations were selected in order to study the effect of adding additional satellites into a satellite constellation. One, two, three and four satellite configurations are defined in Chapter 3. Each of these constellations was evaluated at twenty low Earth orbits (LEO) ranging from 100 km to 2000 km in altitude; at each altitude, twenty transmitter location were evaluated ranging between zero and 3200 km minimal ground trace distance between the satellite ground path and the receiver, the definition of minimal ground trace distance is found in Fig. 3.2.

5.1 Analysis Conclusions

This section will include a brief summary of the conclusions drawn from the results section of the thesis.

The AoA geolocation method decreased in error as more satellites were added to the constellation, this is clearly seen in figures Section and demonstrated by the decrease in average error for the configurations with more satellite, shown in Table 4.3. The explicit and Taylor series TDOA solution methods both showed decreased solution error between the three and four satellite configuration.

By generating plots such as those seen in Figs. 4.6 and 4.1, two important things may be done. First, error mechanisms of a given configuration and geolocation method can be discovered by showing the areas in which the configuration has high error or where no solution is able to be obtained. This allows for a system to be redesigned or for the geolocation method to be change in order to achieve prescribed geolocation accuracy. Second, this method allows for satellite configurations to be evaluated against each other. For example, the comparison shown in Fig. 4.19 quickly shows that the four satellite angle of arrival geolocation solution has less error than the two satellite configuration in most cases. This same type of comparison could be used to compare any two satellite configurations even if different geolocation methods were used.

The most important thing shown in this work is that dissimilar configurations and geolocation methods can be directly compared. Figures 4.23 and 4.24 compares the AoA method to the four satellite explicit TDOA solution and shows the conditions in which the AoA method performs better than the explicit TDOA solution method. The ability to compare a different configurations and geolocation methods in this manner is a useful when designing a space based geolocation capable system

Geolocation methods and satellite orbit constellations different from those presented in Chapter 4 could just as easily be analyzed through this method. The point to take away from this thesis is that this method can compare any satellite configuration and geolocation method against any other configurations even those utilizing different geolocation methods.

5.2 Future Work

There are a few important areas where work can be done in the future. Updates to the explicit method geolocation solution would fix an issue where an incorrect geolocation solution is selected; this is related to the geolocation method and does not have an overly large impact.

This method can be applied to a real system design in the future. For this work, proper characterization of errors in the system is essential. This method allows for the addition of error to the system so that the real world situation can be matched as closely as possible. Without proper characterization of the system errors, comparisons made would be meaningless.

The angle of arrival geolocation can be tested to evaluate different antenna configurations and the effect of noise on the geolocation solution. A variety of antenna arrays could be compared to each other using this method; this method would provide useful insight for the design antenna design.

A study of the angle of arrival geolocation method accuracy for signal properties would be an interesting study. This thesis did not consider the effect of signal to noise ratio on the error of the angle of arrival solution.

The MUSIC algorithm is capable of detecting and locating multiple signals. Adding multiple signals into a single cases and developing a methodology to distinguish between each signal would more closely represent the situation seen in the real world where many signals are present.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704–0188

1. REPORT DATE *(DD–MM–YYYY)* 27–03–2014	**2. REPORT TYPE** Master's Thesis	**3. DATES COVERED** *(From — To)* Oct 2012–Mar 2014

4. TITLE AND SUBTITLE

Analysis of Geolocation Approaches Using Satellites

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Basel, James P., Second Lieutenant, USAF

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Air Force Institute of Technology
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8. PERFORMING ORGANIZATION REPORT NUMBER

AFIT-ENY-14-M-07

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

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10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT

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13. SUPPLEMENTARY NOTES

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14. ABSTRACT

A space based system capable of geolocating radio frequency signals of interest has wide reaching application to the Air Force. This system would provide increased situational awareness to the warfighter on the battlefield. The Air Force Institute of technology is developing a satellite to conduct research on geolocation using CubeSats. A methodology to evaluate space based geolocation systems by varying orbital altitude and transmitter position for a given geolocation algorithm and satellite configuration was developed. This method allows multiple satellite configurations and geolocation algorithms to be compared during the design process of a space based geolocation system. The method provides a tool to facilitate decision making on the configuration design and geolocation methods chosen for a given system design. This research explains the geolocation methods and provides comparisons for one through four satellite configurations for time difference of arrival and angle of arrival geolocation algorithms.

15. SUBJECT TERMS

Geolocation, Satellite, TDOA, AOA, Space

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			Dr. Eric Swenson (ENY)
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Standard Form 298 (Rev. 8–98)
Prescribed by ANSI Std. Z39.18