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Capturing Uncertainty in Fatigue Life Data

Brent D. Russell

CAPTURING UNCERTAINTY IN FATIGUE LIFE DATA

THESIS

Brent D. Russell

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DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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Brent D. Russell, B.S.

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Approved:

___________//signed//______________________	______26 June 2014 ___
Dr. Raymond R. Hill (Chairman)	date
___________//signed//______________________	______26 June 2014 ___
Maj. Jason K. Freels (Member)	date

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Abstract

Time-to-failure (TTF) data, also referred to as life data, are investigated across a wide range of scientific disciplines and collected mainly through scientific experiments with the main objective of predicting performance in service conditions. Generally, fatigue life data are times measured in cycles until complete fracture of a material in response to a cyclical loading. Fatigue life data have large non-uniform variation, which is often overlooked or not rigorously investigated when developing predictive life models.

This research develops a statistical model to capture dispersion in fatigue life data which are used to extend deterministic life models into probabilistic life models. Additionally, a predictive life model is developed using failure-time regression methods. The predictive life and dispersion models are investigated as dual-response using nonparametric methods. After model adequacy is examined, a Bayesian extension and other applications of this model are discussed.

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To my loving parents and supportive friends and family

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1. Introduction

1.1 Background

Time-to-failure (TTF) data, also referred to as life data, are values measuring the period from some time initiation until a defined end of an component's life. The data can be measured in standard units of time, ranging all the way up to years, or quantities specific to the application of interest, such as miles for automobile parts. The initiation of the time collection is commonly the beginning of testing for a previously untested product or structure, but may not be possible in the cases of observable data, especially with organic subjects. The end of time collection must be a previously and commonly defined "failure", generally when a component or structure is completely unable to perform its intended function.

Life data are investigated across a wide range of disciplines, such as reliability in engineering, survival analysis in biology, and fatigue in mechanical sciences. In reliability, developers can modify designs to increase product or system life, establish inspection and maintenance schedules, and predict performance in variable operating environments. In survival analysis, analysts can quantify the impact of genetic and environmental factors on health, and determine pricing structures for insurance policies. In fatigue, engineers can characterize metal alloys, quantify material responses to testing conditions, and develop inferences for material performance in a larger system.

The majority of life data are collected through scientific experiments with testing environments emulating normal operating conditions. The main test objective is to predict product or system performance in service conditions. Other specific goals of life testing are to identify the most important variables in terms of impact on failure, to accurately monitor structural degradation, and to develop diagnostic tools for estimating product or material integrity (Mancini & Volta, 1980). With these objectives under consideration, life models are developed, often emphasizing design factors influencing the quality of the product or the stress and environmental conditions of the test (Saunders, 2007; Mancini & Volta, 1980).

Prior to testing, experimenters and experts must discuss and agree upon how results can lend information to conclusions or inferences about the product or system in service conditions. This may present a challenge, however, as sample sizes are frequently small, have few statistical replications, and have test and processing conditions which do not always accurately represent service conditions, and consequently, may lead to questionable conclusions about structure or system performance regardless of statistical validity (Little & Jebe, 1975). Hence, there is a prevalent need for analysts who can identify failure causes and construct mathematical life models for these failures (Saunders, 2007).

As detailed by Ebeling (1997), poorly designed testing and unsound service performance inferences have resulted in an alarming number of system failures with safety and economic impacts:

- crash of Lockheed Constellation aircraft killing four crew members in 1946;
- recall of 7.5 million Firestone steel-belted radials in 1972;
- recall of Ford Pinto after numerous reported deaths in 1978;
- collapse of the Hartford Civic Center Coliseum’s roof in 1978;
- partial meltdown of nuclear reactor on Three Mile Island in 1979;
- collapse of Manus River Bridge killing three in 1983; or the

- explosion of the space shuttle Challenger killing the entire crew in 1986.

Failures for these large and complex systems stem from a number of sources, but may not be readily identifiable or commonly accepted among investigative experts. However, sometimes the failure is the result of a fracture in a specific material within the system. Ebeling discusses the collapse of the Tacoma Narrows Bridge in the Puget Sound in 1940; oscillations in metal supports from vibrations of high winds over the five-month period after construction caused the bridge fall (1997). Numerous failures of airfcraft, transporters, and other civil structures have been attributed to fractures in degraded construction materials (Swanson, 1974).

In material sciences and mechanical engineering, fatigue refers to "the special behavior pattern exhibited by materials in response to cyclic loading" (Conway & Sjodahl, 1991, p. 1). As cracks or degradation in these materials cause failures in larger systems, such as aircraft panels, experiments are designed to examine fatigue characteristics. The resulting data from these fatigue experiments are analyzed to gain understanding of how a particular material would behave as part of a system in service conditions. Fatigue life of the material, time until complete fracture, is frequently the focus. Deterministic life equations are developed with the goal of predicting the life of a given material under specific service conditions.

Several parameters can be investigated in fatigue testing. Factors such as loading, specimen geometry, material behavior, and thermal or chemical conditions are frequently of consideration (Little & Jebe, 1975). These factors are quantified and investigated in relation to foundational fatigue concepts such as stress/strain amplitude or cycle, elasticity, plasticity, and cyclic softening and hardening (Conway & Sjodahl, 1991).

Although the scope and objectives vary between fatigue experiments, general testing considerations are applicable, as outlined by Swanson (1974):

- clearly defined testing objectives;
- consideration of processing effects and geometric discontinuities in testing specimens;
- consideration of cost factors;
- consideration of other limiting constraints;
- identification of environmental variables, both control and noise;
- clear measurement of effect of variables on fatigue behavior;
- randomization to reduce bias;
- consistency in testing preparation and execution;
- rigorous analysis and comprehensive reporting of results; or
- identification of sources of error and uncertainty in data.

Error and uncertainty play a role in the development of fatigue and other life models, but their importance is often under-emphasized. Thacker et al. (2001) have highlighted the need for greater consideration of uncertainty and error in probabilistic engineering analysis by first establishing their difference; uncertainty is inherent in statistical analysis, while errors are the result of the analytic process stemming from several sources, including insufficient data, measurement, incorrect distribution or transformation selection, or mathematical approximation. While uncertainty can be incorporated in modeling, errors should be identified and reduced.

Unfortunately, the uncertainty of fatigue models is often not investigated rigorously (Conway & Sjodahl, 1991).

1.2 Research Objectives

The goal of this research is capturing variance in fatigue-life data by developing a dispersion model which can be used to extend deterministic life models into probabilistic life models. This is accomplished through the use of regression modeling with the median absolute deviation of fatigue life as the response and examination of its relationship to predictive variables, specifically temperature and tensile stress. This relationship is established and further investigated using statistical testing methods.

Additionally, a predictive life model including the same predictive variables is developed using failure-time regression methods. This first approximation serves as an initial deterministic model which can later be replaced by a non-empirical model developed by subject matter experts.

The life and dispersion models are investigated together as dual-response using nonparametric methods. After model adequacy is examined, a Bayesian extension and other applications of this model are discussed.

1.3 Outline

Section 2 presents a literature review for this work including modeling efforts for fatigue life and reliability data, statistical regression, and uncertainty modeling approaches. Section 3 provides the methodology to develop the life and dispersion models for the fatigue data. Section

4 gives analysis results and extensions for our dual-response model, and Section 5 discusses conclusions from this investigation and recommendations for future work.

2. Literature Review

2.1 Modeling of Fatigue Life Data

The most widely used mathematical models in fatigue analysis are fitted equations to the S-N curve, where the independent variable S is an observable stress index and the dependent variable N is the number of cycles until reported failure of the tested specimen (Little & Jebe, 1975). These models allow for inferences in fatigue metrics, including predicted failure proportion and fatigue endurance. Figure 1 provides an example of an S-N curve.

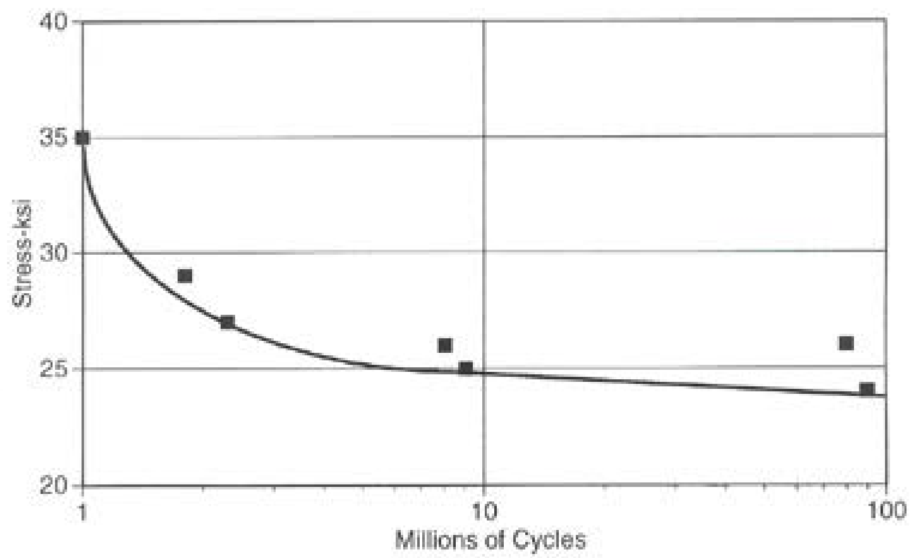

Figure 1. S-N curve example

The most simplified fatigue models are generalized linear models incorporating the pattern on the S-N curve. Examples of these models are

$$\log N = A_0 + A_1 \log S \quad (1)$$

$$N = A_0 + A_1 \log S \quad (2)$$

$$\log N = A_0 + A_1 S \quad (3)$$

where the *A*-parameters are usually determined using a least squares estimation method (Conway & Sjodahl, 1991).

These equations, and other fatigue life models, have been extended to include other explanatory factors, additional response variables, and any relationships between. Rao et al. (1988) modified the cyclical fatigue testing of alloy Inconel 617 to include tensile and compression hold times as predictive variables, in addition to strain rate. The work investigated the influence of these variables on fatigue life and other reported responses in the specimens, including deformation behaviors and crack initiation. Additionally, the effects of hold time variations, such as tensile-only, compression-only, and symmetric, were noted in experiment results (Rao et al., 1988).

Altus and Herzage (1994) expanded the S-N relationship to consider bi-axial testing, as opposed to uni-axial tension and compression. The study allowed for a richer understanding of material behavior without the use of additional test variables and established numerical relationships to the uni-axial testing models. Furthermore, a cumulative damage function was incorporated into prediction methods (Altus & Herszage, 1995).

Popelar (1997) developed a fatigue model including the factors of temperature, elasticity, and creep. The model was used to predict fatigue life of solder joints in flip chips, which was incorporated in the determination of optimal design parameters settings in the larger reliability study. General validation techniques for parametric flip chip testing were also discussed and extended (Popelar, 1997).

Several fatigue models are developed using the data resulting from the investigator's designed tests. However, fatigue life models can be determined using theoretical and approximation methods, often when the model is a piece of a larger and more complex reliability study. Soares and Garbatov (1999) applied reliability-based techniques to welded joints in the shells of tankers by assessing fatigue damage as part of an overall reliability model. Without fatigue testing, approximate fatigue stresses and loadings were calculated from system-wide factors. Established methods, such as Paris-Erdogan equations, were applied to develop the theoretical fatigue life and the system reliability models (Soares & Garbatov, 1999).

The majority of fatigue life models in literature are deterministic equations which do not quantify uncertainty or risk for life predictions. Lodeby et al. (1999) developed a methodology for identifying sources of variation in life models, including errors stemming from random and systematic processes. Random errors were assumed to be normally distributed, while systematic errors were considered constant in repeated measurements or calculations. The correlation between the sources of variation was examined and total variation for any observation was determined using the Gauss Approximation formula (Lodeby et al., 1999).

Fatigue and other life models can be extended into cumulative damage models, which quantify degradation and wear of the material or system of interest. These models are useful for creating inspection schedules, developing repair policies, and monitoring life cycle costs (Bogdanoff & Kozin, 1985). This work developed probabilistic modeling techniques using discrete-time Markov chains with defined states of degradation and determined probabilities of transition from one state to the next; these models were extended to include continuous-time Markov chains, non-stationary state transition probabilities, and identification and incorporation of major sources of variability (Bogdanoff & Kozin, 1985).

These works highlight the increase of statistical rigor in the development of fatigue life model outlined in the literature. However, regression analysis remains the most widely used statistical technique for analyzing fatigue data and developing deterministic fatigue prediction models.

2.2 Modeling with Regression

Regression modeling relates response variable behavior as a function of some set of predictor variable(s) settings. Such models aid in description, control, and prediction (Kutner et al., 2004). Consider the case of one response Y and p-1 predictor variables $X_1, \dots, X_{p-1}$. A first-order regression model is:

$$Y_i = \beta_0 + \sum_{j=1}^{p-1} \beta_j X_{ij} + \varepsilon_i \tag{4}$$

where the β's are parameters, the ε_i are independent and normally distributed with an expected value of 0 and a constant variance σ^2, and assumes the effect of X_i on the expected response does not depend on the level of all other X's, for all i. Regression models can also include interaction terms ($X_{ij}X_{ih}$), second-order terms (X_{ij}^2), or other higher-order functions of X's.

The most widely used method for estimating the model coefficients is the method of ordinary least squares, which determines the β-parameters by finding the values which minimize the sum of squared errors (e_i^2) expressed by:

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (\widehat{Y_i} - Y_i)^2 \tag{5}$$

where $\widehat{Y_i}$ is the fitted value of the model for a specific set of β-parameters (Kutner et al., 2004).

Linear first-order regression models can also be expressed in matrix form:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{6}$$

where **Y** is an n x 1 vector of responses, $\boldsymbol{\beta}$ is an p x 1 vector of parameters, **X** is an n x p vector of constants, and $\boldsymbol{\varepsilon}$ is an n x 1 vector of independent normal random variables with an expected value of zero and a diagonal covariance matrix with a constant variance as every element on the main diagonal. Then the ordinary least squares estimator of the parameters is given by

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-\mathbf{1}}\mathbf{X}'\mathbf{Y}. \tag{7}$$

The assumptions on the error terms are important to regression analysis; statistical tests on regression coefficients and inferences applied to any predicted value are based upon the error term assumptions. Residual analysis is focused on the assessment of error term assumptions and modifications used to ensure assumptions are sufficiently valid; diagnostic tests and remedial measures have been developed for regression to detect when one or more of the assumptions above are violated (Kutner et al., 2004).

Initial residual analysis is often done visually, by creating plots of the residuals against the fitted values of the model (Montgomery et al., 2012). Other, sometimes more useful plots in residual analysis include using other forms of the residual including studentized and PRESS, or plotting these residuals against predictive variables (Kutner et al., 2004). If the assumption of constant variance is violated, there will be observable trends on these plots, as opposed to random scatter. However, these graphical methods are mostly subjective.

The most widely used formal test for non-constant variance is the Breusch-Pagan test. This test requires the regression of squared residuals on the same predictive variables used in the initial model with a test statistic ratio following a chi-square distribution (Breusch & Pagan, 1979). This approach was extended to include a test with a relaxation on the normality assumption of the error terms in regression, in addition to estimation of error terms when the constant variance assumption has been violated (White, 1980).

Several model modifications and remedial measures have been presented in the field of statistical regression when the assumption of constant variance has been violated. Most involve a transformation of the response variable and some include a simultaneous transformation on predictive variables (Kutner et al., 2004). Often this transformation on the response falls into the family of power transformations, and can be automatically identified computationally (Box & Cox, 1964).

Although transformations of the response variable are used to remediate problems due to non-constant variance, subsequent formal testing must occur to gauge the effectiveness of the transformation. Specific tests have been developed for log-transformed data (Bartlett & Kendall, 1946). This work also develops and explores statistical techniques regarding other model assumptions after data transformation.

Regression has been used for model building across a wide range of disciplines (see Montgomery et al., 2012; Kutner et al., 2004), and has been incorporated into the field of reliability engineering for developing life models of products or systems with increasing complexity.

2.3 Modeling with Failure-Time Regression

Frequently with fatigue or reliability testing, analysts wish to use a parametric distribution to describe failure processes in order to generate more flexibility in the inferences and predictions about their modeled product. These location-scale (or log-location-scale) distributions are derived mathematically and must have sufficient ability to describe the failure or reliability of the tested component or system (Ebeling, 1997). Weibull, lognormal, and normal are examples of such distributions containing two or more parameters.

The majority of data sets widely used in reliability academia contain response failure times from constant testing conditions. For these models, the location and scale parameters are constant values determined by maximum likelihood methods. However, when predictor variables such as thermal and physical conditions have multiple levels, the distributional parameters can be expressed as functions of these explanatory variables. As a result, model inferences are conditional on fixed, observed values of the predictive variables (Meeker & Escobar, 1998).

Often in life and reliability experiments, it is necessary, due to time and cost considerations, to terminate a test before the component or system has reached its failure state. This reported data is designated as right censored and requires special considerations during analysis. Maximum likelihood methods can account for this data in linear regression (Miller & Halpern, 1982) and in failure-time regression (Meeker & Escobar, 1998).

Similar to normal regression, failure-time regression models have assumptions and diagnostics, which vary with the model chosen. For example, the errors terms may not have constant variance as is normal regression. This can be incorporated into the model with a scale parameter expressed as a function of the explanatory variables (Meeker & Escobar, 1998).

However, these efforts to model dispersion in reliability or life data may not be sufficient, and more rigorous methods have been explored.

2.4 Modeling of Dispersion

In addition to statistical efforts for measuring central tendency of data, such as in predictive life equations, data dispersion may require separate modeling efforts. The estimation of these functions are a form of statistical regression, but have not had the wealth of literature and research dedicated to it as with mean response functions. As with regression on means, the intent is to capture variance as a function of predictors. Several techniques have been established for variance function estimation after the development of a mean function, including maximum and pseudo likelihood, weighted residual, logarithmic method, Rodbard, and Sadler-Smith (Carroll & Ruppert, 1988).

Often, mean and variance functions are developed simultaneously. With the presence of replicates at design points spanning the design space, the variance of the response can be modeled using a log-linear equation, which does not produce serious violations of regression assumptions (Myers & Montgomery, 1995; Bartlett & Kendall, 1946). The log-linear parameters are estimated using maximum likelihood methods and improved algorithms for this estimation are present in literature (Aitken, 1987).

The log-linear variance model uses a separate criterion for the detection of and consequent need to capture non-constant variance across the design space. This criterion is a specific application of the likelihood ratio test, where the regression model with assumed constant variance serves as the null model, and the model with a separate equation for the variance of the response serves as the alternative model (Myers & Montgomery, 1995).

The idea of dual response mean and variance model estimation is not limited to additive variance functions. Multiplicative variance models have been developed using maximum likelihood methods, with a simpler form of the likelihood ratio test, and have been shown to produce improved parameter estimations for both functions (Harvey, 1976). Furthermore, it is has been shown both functions can be generalized linear models with simplified likelihood equations and estimation algorithms having good convergence properties (Smyth, 1989).

If the data do not contain replicates at design points, alternate techniques for variance modeling have been presented in literature. “Near-neighbor” observations in the design space can be binned together to estimate variance at a mean predictive level, which can be used in a regression model for the estimated variance separately (Montgomery et al., 2012). Box and Meyer (1986) established data pooling techniques in a two-level experiment without replicates. This technique was further expanded using maximum likelihood estimation methods for parameter identification in the selected model and applied to an experimental design for truck leaf springs (Nair & Pregibon, 1988).

The presented methods require the development of a model for the mean response prior to any variance modeling. However, it has been shown the development of a variance model is possible without a known mean function in both parametric regression (Hall & Carroll, 1988) and non-parametric (Wang et al., 2008) with further calculations and assumptions.

A specific quality engineering application regarding variance modeling which has received significant attention in academic literature is the Taguchi analysis. This approach seeks to reduce process variance in the presence of noise variables, while ensuring robust product quality by setting target values for the product and maximizing a signal-to-noise ratio (SNR). The Taguchi SNR response function aims at incorporating both mean and variance response measures. This methodology contends experimental product design is more impactful in robustness than on-line control. The Taguchi analysis is a form of product optimization (Taguchi, 1987).

Although many statisticians have criticized the Taguchi approach, others have sought to incorporate the methods into more traditional dual-response modeling approaches (Vining & Myers, 1990). This approach achieves the goals outlined by Taguchi while incorporating a more statistically rigorous methodology, which was further extended to a more efficient and simpler optimization procedure (Lin & Tu, 1995).

2.5 Nonparametric Statistical Methods

Statistical modeling and testing methods common in applied statistics literature are based on parametric assumptions, such as the linear regression and failure-time regression approaches previously discussed. Other common parametric techniques include t-tests, F-tests, and ANOVA, among several others.

Nonparametric methods are a discipline within the statistical field which has developed techniques not purely based upon parametric assumptions. These include modeling methods such as localized regression and splines, and sampling methods such jackknifing and bootstrapping (Krishnaiah & Sen, 1984).

Hypothesis testing remains the major focal point of nonparametric statistics. For example, the nonparametric counterparts for the paired t-test are the Wilcoxen Signed Rank Test, whose test statistic is calculated using the ranks of the observations, and Fisher's Sign Test, whose test statistic is calculated using the signs of the observations. Rejection regions for these tests are based upon combinatorics, and the tests' theoretical underpinnings are based upon random samples being equal in distribution to consequent ranked samples (Krishnaiah & Sen, 1984).

Other nonparametric techniques have parametric counterparts including two-sample location tests Wilcoxen Rank Sum/Mann-Whitney U (t-test for difference of means), Kruskal-Wallis/Friedman (ANOVA), and Spearman rank correlation (Pearson correlation) (Krishnaiah & Sen, 1984).

2.6 Bayesian Statistical Methods

The most common statistical methods, both parametric and nonparametric, are rooted in the frequentist approach, which has developed methods for approximating population parameters that cannot be truly known by only using observed data in parameter calculations. Frequentist statistics employ hypothesis testing controlling for Type I errors to give a concrete conclusion to answer scientific questions.

Bayesian statistical inference uses the laws of probability to model all uncertainty in any statistical method. In contrast to the frequentist approach, Bayesian methods incorporate prior information from a subject matter expert, in addition to the observed data. Additionally, significant tests and intervals about parameter values are based upon posterior probability distributions, often developed using Markov Chain Monte Carlo (MCMC) simulation (Christensen et al., 2011).

Bayesian methods focus primarily on predictive capability of statistical models, as opposed to inferences about the parameters themselves. Despite differing fundamentally from frequentist methods, Bayesian statistics include the majority of methods of frequentist statistics, including parametric and nonparametric testing, linear regression, and failure-time regression (Christensen et al., 2011).

3. Methodology

3.1 Notation

The specific notation used in the subsequent discussion is summarized in Table 1.

Table 1. Notation used.

T_i	Cycles-to-failure of the i-th observation
T_{nk}	Cycles-to-failure of the n-th replicate at the k-the design point
Y_k	Median absolute deviation of replicated observations at the k-the design point
X_{k1}	Temperature (Fahrenheit) level at the k-the design point in the dispersion model
X_{i1}	Temperature (Fahrenheit) level at the i-th observation in the life model
X_{k2}	Tensile stress (ksi) level at the k-the design point in the dispersion model
X_{i2}	Tensile stress (ksi) level at the i-th observation in the life model
β	Population parameter in life model
λ	Population parameter in dispersion model
μ	Distributional location parameter
σ	Distributional scale parameter

3.2 Data

The data analyzed are fatigue test data of titanium alloy Ti 6-4, where the life (T_j) is measured in cycles until complete fracture. One-hundred eighty-six observations were collected using -1 stress ratio, no compressive or tensile hold times, and two predictive variables, temperature (X_1, measured in degrees Fahrenheit) and tensile stress (X_2, measured in thousand-pounds-per-square-inch, or ksi). Forty-three observations are right censored at 10,000,000 cycles. Testing is conducted at 46 design points, 29 of which have replicates varying from 2 runs to 13 runs.

Table 2 below provides all design settings included in the data. Shaded rows represent design settings used in the dispersion model.

Table 2. Replications by design setting.

Temperature (F)	Stress (ksi)	Uncensored	Censored
69.9993	50	2	5
69.9993	55	1	1
69.9993	60	7	0
69.9993	65	6	0
69.9993	70	6	0
69.9993	80	3	0
69.9993	120	1	0
69.9993	130	1	0
69.9993	135	2	0
69.9993	140	2	0
69.9993	145	1	0
69.9993	150	1	0
69.9993	160	1	0
399.998	30	2	6
399.998	35	3	4
399.998	38	1	0
399.998	40	6	4
399.998	45	3	3
399.998	50	11	0
399.998	55	4	0
399.998	60	11	0
399.998	70	3	0
399.998	80	1	0
399.998	85	1	0
399.998	90	2	0
399.998	100	4	0
399.998	105	1	0
399.998	110	2	0
800	25	0	7
800	26	0	2
800	30	7	6
800	35	8	3
800	36	1	0
800	37	1	0
800	40	9	2
800	45	3	0
800	46	1	0
800	50	13	0
800	60	1	0
800	70	1	0
800	75	1	0
800	77	1	0
800	80	2	0
800	85	1	0
800	90	2	0
800	100	2	0

3.3 Dispersion Model

All design points in the experiment which have replicate runs and an uncensored median life are incorporated in the dispersion model. The median absolute deviation (MAD) of the multiple observations at each of the applicable 22 design points, totaling 130 observations, is calculated using Equation 8, if all replications are uncensored, or estimated using mean order numbers (MON) as discussed below.

$$Y_k = MAD\{(X_{k1}, X_{k2})\} = median|T_{nk} - median(T_{nk})| \tag{8}$$

where T_{nk} is the life of each of the replicates at the k-th point in the design space.

If any of the observations obtained at a given design point are censored, the median absolute deviation is estimated using mean order numbers (Kececioglu, 1993). Table 3 provides a specific example from the data at the design point (399.998, 40). A censor code of 0 indicates a true absolute deviation from the median life and a censor code of 1 indicates a censored absolute deviation. The mean order numbers of all uncensored observations calculated using combinatorics and their associated mean rank positions are provided. The deviation estimation with the associated mean rank position of 0.5 is estimated using lognormal probability plotting and fitting.

Table 3. Mean order numbers example.

Censor	Temperature (F)	Stress (ksi)	Cycles	Absolute Deviation	MON	Mean Rank Position
0	399.998	40	179000	6796000	6.5	0.596153846
0	399.998	40	188000	6787000	5.375	0.487980769
0	399.998	40	339000	6636000	4.25	0.379807692
0	399.998	40	1030000	5945000	3.125	0.271634615
0	399.998	40	5470000	1505000	1.5	0.115384615
0	399.998	40	8480000	1505000	1.5	0.115384615
1	399.998	40	10000000	3025000		
1	399.998	40	10000000	3025000		
1	399.998	40	10000000	3025000		
1	399.998	40	10000000	3025000		

A regression equation of the general form

$$\log Y_k = \lambda_0 + \sum_{j=1}^{2} \lambda_{1j} X_{kj} + \sum_{j=1}^{2} \lambda_{2j} (X_{kj} - \overline{X_j})^2 + \lambda_3 (X_{k1} - \overline{X_1})(X_{k2} - \overline{X_2}) + \varepsilon_i \quad (9)$$

where the λ's are parameters, X's are known constants and centered in the non-linear terms , and the ε_i are assumed independent and normally distributed with an expected value of 0 and a constant variance, is fit to this calculated data using ordinary least squares. All λ-parameters are tested for significance and all terms with λ-parameters not significantly different than zero are removed.

The resulting estimated model is

$$\log Y_k = 21.401688 - 0.005126 X_{k1} - 0.141388 X_{k2} + 0.0012652 (X_{k2} - 72.5)^2 \quad (10)$$

Table 4 gives the summary of fit of the model in Equation 10, and Table 5 provides the results of the tests of significance for the model's λ-parameters. Figure 2 gives a surface plot of the model, in log cycles.

Table 4. Summary of fit of dispersion model.

Summary of Fit	
RSquare	0.91653
RSquare Adj	0.902619
Root Mean Square Error	1.119742
Mean of Response	9.96026
Observations (or Sum Wgts)	22

Table 5. Tests of significance for dispersion model.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	21.401688	0.942051	22.72	<.0001 *
Temperature	-0.005126	0.000907	-5.65	<.0001 *
Stress	-0.141388	0.010166	-13.91	<.0001 *
(Stress-72.5)*(Stress-72.5)	0.0012652	0.00024	5.26	<.0001 *

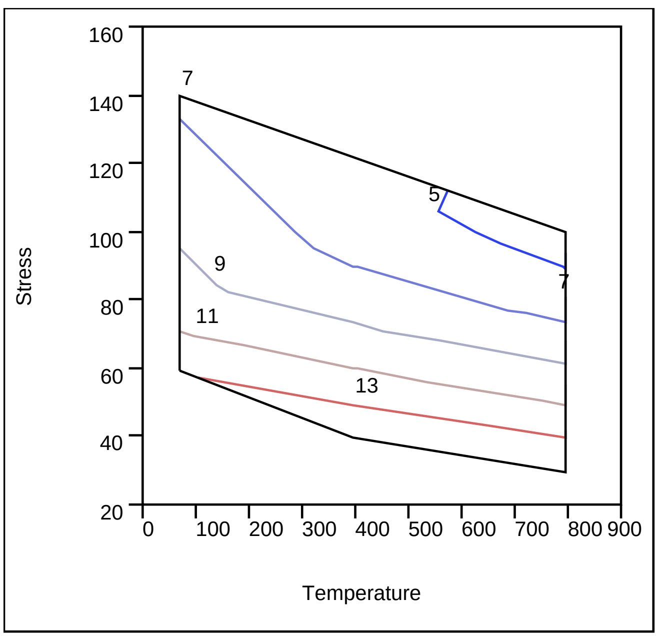

Figure 2. Surface plot of dispersion model in Equation 10.

Prior to drawing inferences with the model, diagnostics are used to ensure there is no violation of the assumption of the errors being normally distributed with an expected value of 0 and a constant variance. Figure 3 shows a histogram and normal quantile plot of the residuals of our model. The Shapiro-Wilk test calculates a p-value of 0.9634 under the null hypothesis of the residuals being from the normal distribution. Therefore, the null is not rejected and the conclusion is residuals are sufficiently normally distributed.

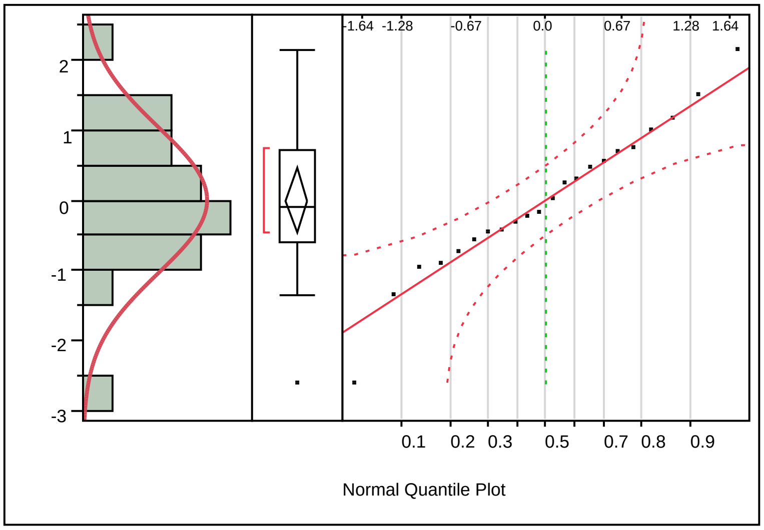

Figure 3. Normality plots of residuals of dispersion model.

Figure 2 shows a plot of the model raw residuals against the predicted values. Visually, there does not appear to be any violation of the assumption of the constancy of error variance. The Breusch-Pagan test calculates a p-value of 0.2092 under the null hypothesis of constant error variance. Therefore, we do not reject the null and conclude the residuals of this model have a sufficiently constant error variance.

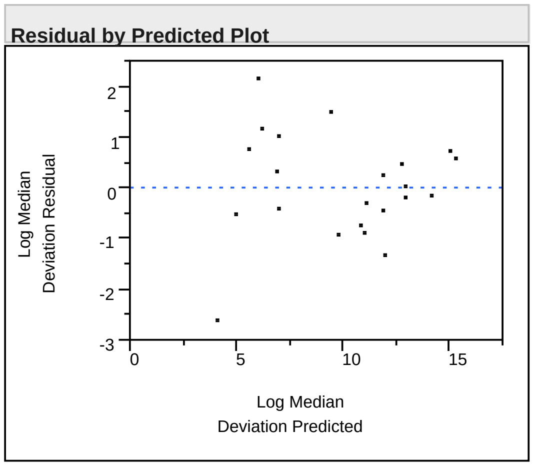

Figure 4. Plot of residuals of dispersion model.

3.4 Life Model

In order to build a dual-response model to output an expected life and expected deviation at any design setting, a predictive life model is developed using failure-time regression. This model is used to determine the adequacy of the dispersion model in Equation 10. Ultimately, the life model can be replaced by a preferred model built from non-empirical methods.

One-hundred forty-nine observations in the design space of the dispersion model, censored and uncensored, are used in the development of the failure-time regression model. A location-scale (or log-location-scale) probability distribution is developed as

$$\Pr(T_i \leq t) = F(t: \mu, \sigma) = \Phi(\frac{g(t)-\mu}{\sigma}) \qquad (11)$$

where T_i is the cycles to failure of a test sample, Φ determines the distribution at a specific design point (normal or smallest-extreme-value, for example), *g(t)* is a transformation of the failure time (logarithmic, for example), and μ, σ are parametric equations given below.

$$\mu_i = \beta_0 + \sum_{j=1}^{2} \beta_{1j} X_{ij} + \sum_{j=1}^{2} \beta_{2j} (X_{ij} - \overline{X_j})^2 + \sum_{j=1}^{2} \beta_{3j} (X_{ij} - \overline{X_j})^3 + \beta_4 (X_{i1} - \overline{X_1})(X_{i2} - \overline{X_2}) + \varepsilon_i \quad (12)$$

$$\sigma_i = \lambda_0 + \sum_{j=1}^{2} \lambda_{1j} X_{ij} + \sum_{j=1}^{2} \lambda_{2j} (X_{ij} - \overline{X_j})^2 + \lambda_3 (X_{i1} - \overline{X_1})(X_{i2} - \overline{X_2}) + \varepsilon_i \quad (13)$$

The parameters in these equations are determined by Maximum Likelihood Estimation (MLE) methods and tested for significance. The specific distribution used is selected from lognormal, Weibull, or loglogistic (all of which require a log-transformation of the failure time), or exponential, and are determined using Akaike's Information Criterion (AIC).

According to the AIC measure, the lognormal distribution is a better fit than the Weibull, loglogistic, and exponential. The fitted failure time model is

$$\Pr(T_i \le t) = F(t: \mu, \sigma) = \Phi_{nor}\left(\frac{\log(t) - \mu}{\sigma}\right) \quad (14)$$

where

$$\mu_i = 21.3916067 - 0.0036379 X_{i1} - 0.1362854 X_{i2} + 0.00151119 (X_{i2} - 58.2282)^2 - 0.0000072563 (X_{i2} - 58.2282)^3 \quad (15)$$

$$\sigma_i = 3.63707445 - 0.0015742 X_{i1} - 0.0237341 X_{i2} \quad (16)$$

is determined using MLE. The results of the likelihood ratio tests for the coefficients in Equations 15 and 16 are given in Table 6. Figure 5 provides a surface plot of the life predictions, in log cyclies, given by Equation 15.

Table 6. Tests of significance for life model.

Effect Likelihood Ratio Tests

Source	Nparm	DF	L-R ChiSquare	Prob>ChiSq	
location: Temperature	1	1	91.5975048	<.0001	*
location: Stress	1	1	180.917234	<.0001	*
location: Stress*Stress	1	1	31.7351959	<.0001	*
location: Stress*Stress*Stress	1	1	7.17230158	0.0074	*
scale: Stress	1	1	43.0737396	<.0001	*
scale: Temperature	1	1	14.8544436	0.0001	*

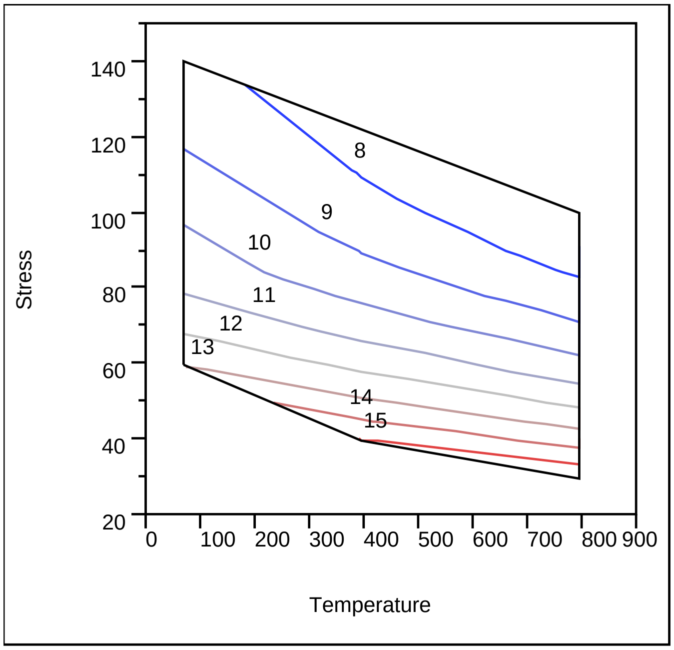

Figure 5. Surface plot of life model in Equation 15.

Prior to drawing any inferences on this model, diagnostics are used to ensure there is no violation of the assumption of lognormally distributed errors. Figure 6 gives the lognormal quantile plot of the standardized residuals of the model. The Kolmogorov' D goodness-of-fit test calculates a p-value greater than 0.15 under the null hypothesis of the Cox –Snell residuals being from the lognormal distribution. Therefore, the null is not rejected and the conclusion is the residuals are sufficiently lognormally distributed.

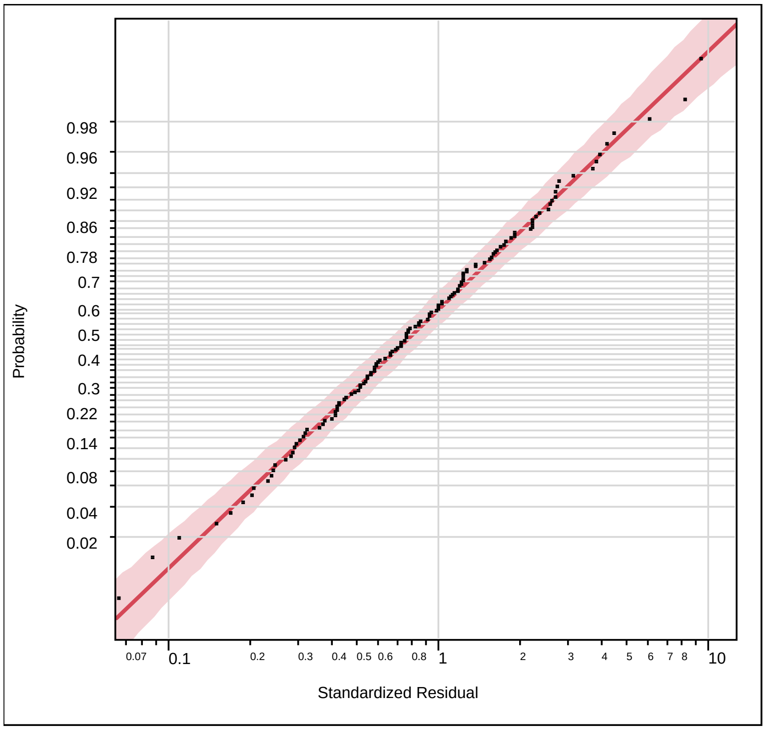

Figure 6. Lognormal quantile plot of residuals of life model.

4. Results and Analysis

4.1 Dual-Response Model

To determine the adequacy of our dispersion model, the expected deviation estimated by the dispersion model Equation 10 is examined in relationship to the median residuals produced by the life model in Equation 15. This is accomplished by ensuring the dispersion model is not consistently overestimating or underestimating deviation, determining measures of association between the two values at each design point, and examining the ability of the confidence intervals produced by our dispersion model to capture the median deviation at each design point. These tasks are done both for data at design settings with replicates and without replicates. Table 7 below provides the values to accomplish this testing for the data at design settings with replicates, where life and deviation values are given in cycles.

Table 7. Actual and expected deviations observations in both models.

Temperature (F)	Stress (ksi)	Expected Life	Median Residual Life	Expected Deviation	Lower 95% Expected	Upper 95% Expected
69.9993	60	427000.272	308000.272	347084.948	126046.575	955741.646
69.9993	65	229912.800	90500.000	150823.615	55850.434	407297.873
69.9993	70	132529.408	53029.408	69819.474	26120.300	186627.220
69.9993	80	52868.711	71131.289	18088.894	6865.222	47661.691
69.9993	135	4280.893	1335.000	990.088	225.877	4339.859
69.9993	140	3620.027	726.000	1111.252	201.869	6117.225
399.998	40	3372363.281	4150500.000	3376208.869	1332964.266	8551456.791
399.998	50	556018.635	525018.635	409431.069	205633.614	815206.218
399.998	55	256983.705	159983.705	156771.472	82825.178	296736.995
399.998	60	128547.301	91547.301	63947.996	34180.193	119640.816
399.998	70	39897.627	78102.373	12863.754	6599.236	25075.051
399.998	90	7812.935	3027.500	1112.073	521.853	2369.837
399.998	100	4518.362	2295.638	477.923	219.233	1041.863
399.998	110	2947.310	1397.310	264.532	114.631	610.456
800	30	6991053.980	3008946.021	4614552.041	1290557.955	16499910.338
800	35	2230278.896	2060278.896	1371895.828	468023.169	4021378.185
800	40	786993.053	414993.053	434496.603	172189.886	1096390.169
800	45	305502.178	186502.178	146597.073	63921.390	336205.171
800	50	129755.535	89755.535	52691.174	23943.576	115954.266
800	80	3714.241	3632.000	428.904	151.799	1211.861
800	90	1823.269	82.000	143.117	46.855	437.140
800	100	1054.430	4.500	61.506	18.918	199.967

To ensure the dispersion model is not consistently overestimating or underestimating the deviation, the differences between the median residual life and expected deviation are examined. If the model is not consistently overestimating or underestimating the deviation, these differences will have an expected value of zero. Table 8 provides the test statistics and associated p-values for the Fisher Sign Test and the Wilcoxen Signed Rank Test for the paired data in Table 7. In addition, Spearman rank correlation and Pearson correlation values are given as measures of association for this paired data. The given p-values under the null hypothesis of a median difference of zero indicate the null is not rejected and the conclusion is there is sufficient evidence the median difference is zero. Hence, the dispersion model is shown to neither consistently overestimate nor underestimate the deviation.

Table 8. Verification tests for dispersion model.

Tests	Fisher	Wilcoxen	Spearman	Pearson
Statistic	14	172	0.9548278	0.935299697
P-Value	0.118594	0.1396		

In addition the strong measures of association between our actual and expected deviation, it is observed from Table 7 the dispersion model is capturing the actual deviation in the 95% confidence intervals at 15 of the 22 design points.

To further examine the adequacy of the dispersion model, the same testing is done on observations at design settings without replicates. Table 9 below provides the values to accomplish this testing on the validation data.

Table 9. Actual and expected deviations observations in life model only.

Temperature (F)	Stress (ksi)	Expected Life	Residual Life	Expected Deviation	Lower 95% Expected	Upper 95% Expected
69.9993	120	6896.132	965.868	1023.434	78.561	13334.971
69.9993	130	5022.060	3335.940	939.642	64.316	13730.902
399.998	80	15915.986	484.014	3332.326	284.455	39047.513
399.998	85	10895.417	2297.583	1864.980	158.260	21983.235
399.998	105	3604.545	962.545	344.442	28.715	4132.857
800	36	1796608.855	1018608.855	1084340.689	82786.923	14208585.432
800	37	1453050.796	4796949.204	859409.066	66460.225	11117803.317
800	46	255615.277	207615.277	118839.212	9849.940	1434386.674
800	60	29998.498	16998.498	8227.977	679.603	99658.154
800	70	9310.727	14689.273	1655.131	131.654	20816.951
800	75	5714.567	565.433	816.229	63.603	10479.371
800	77	4778.416	2281.416	626.177	48.407	8103.471
800	85	2542.614	841.386	239.990	18.028	3196.096

Table 10 provides the test statistics and associated p-values for the Fisher Sign Test and the Wilcoxen Signed Rank Test, and measures of association for the paired data in Table 9. The given p-values under the null hypothesis of a median difference of zero indicate the null is not rejected and the conclusion is there is sufficient evidence the median difference is zero. Hence, the dispersion model is shown to neither consistently overestimate nor underestimate the deviation for this validation data and maintains a strong measure of association.

Table 10. Validation tests for dispersion model.

Tests	Fisher	Wilcoxen	Spearman	Pearson
Statistic	9	69	0.71978	0.734998
P-Value	0.1571	0.0636		

In addition the strong measures of association between our actual and expected deviation, it is observed from Table 9 the dispersion model is capturing the actual deviation in the 95% confidence intervals at all 13 of the 13 design points.

4.2 Bayesian Extension of Dual-Response Model

Although the data are assumed lognormal, the deviation estimate provided by Equations 15 and 16 consistently over-estimate residual values across the design space, reinforcing the need for a separate model to investigate dispersion in the data as in Equation 10. The Bayesian extension of this research modifies Equations 15 and 16 as described below to develop the dual response model simultaneously

- A fourth term is added to the scale parameter equation, similar to the dispersion model in Equation 10 (X_2^2 centered)
- Predictive variables are centered in all terms to allow for smoother MCMC approximation

The failure-time regression model for this Bayesian analysis is

$$\Pr(T_i \le t) = F(t{:}\, \mu, \sigma) = \Phi_{nor}\left(\frac{\log(t) - \mu}{\sigma}\right)$$

where

$$\mu = \beta_1 + \beta_2(X_1 - 512.482) + \beta_3(X_2 - 58.2282) + \beta_4(X_2 - 58.2282)^2 + \beta_5(X_2 - 58.2282)^3 \quad (17)$$

$$\sigma = \lambda_1 - \lambda_2(X_1 - 512.482) - \lambda_3(X_2 - 58.2282) + \lambda_4(X_2 - 58.2282)^2 \quad (18)$$

All parameters are assigned a prior distribution of Normal (0, 10,000), and all are assigned an initial value of 0, with the exception of λ_1 which has an initial value of 1, so we have a positive variance value for our distribution to begin.

The MCMC uses 1,025,000 iterations, with a burn-in of 25,000 and a thinning of 1,000 to reduce autocorrelation.

Table 11 below provides median values and 95% probability intervals for the posterior distributions of the model parameters, in addition the value determined by the frequentist approach in Equations 15 and 10. The intercepts are not included, as their values will inherently differ due to the centering of data prior to model building.

Table 11. Comparison of Bayesian and frequentist parameters

Parameter	Frequentist Value	Median of Posterior	Lower 95% Posterior	Upper 95% Posterior
$\beta 2$	-0.0036379	-0.0042895	-0.0054391	-0.0030979
$\beta 3$	-0.1362854	-0.1581	-0.1756	-0.1421975
$\beta 4$	0.00151119	0.0016035	0.0009184	0.0024491
$\beta 5$	-0.00000726	-0.00000663	-0.0000655	-0.000000682
$\lambda 2$	-0.005126	-0.0042895	-0.005439	-0.003098
$\lambda 3$	-0.141388	-0.1581	-0.1756	-0.1422
$\lambda 4$	0.0012652	0.0016035	0.000918	0.00245

The Bayesian extension can be used to build a predictive distribution for T_k at a given design point *k* of interest. From this distribution, the MAD calculation from Equation 8 could be used determine the predicted deviation at *k*, in addition to the median predicted life. This approach is also able to include prior information from scientific experts or previous testing and to serve as a model for future testing using Bayesian updating.

4.3 Simplified Extension of Dual-Response Model

Due to the similarity in parameter estimates of our dispersion and life models, as indicated in Table 11 and the surface plots in Figures 2 and 5, a linear regression model is developed, where expected log life values from Equation 15 are the response, and expected log deviations values from Equation 10 are the single predictor variable. Table 12 provides estimates and tests of significance for the equation's parameters, and Table 13 provides the summary of fit for the equation. Figure 7 provides a plot of the expected log deviation versus expected log life.

Table 12. Equation parameter estimates.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-4.371919	0.345401	-12.66	<.0001 *
Log Life	1.2921139	0.030328	42.60	<.0001 *

Table 13. Summary of fit for simplified extension equation.

Summary of Fit

RSquare	0.989102
RSquare Adj	0.988557
Root Mean Square Error	0.367473
Mean of Response	9.96026
Observations (or Sum Wgts)	22

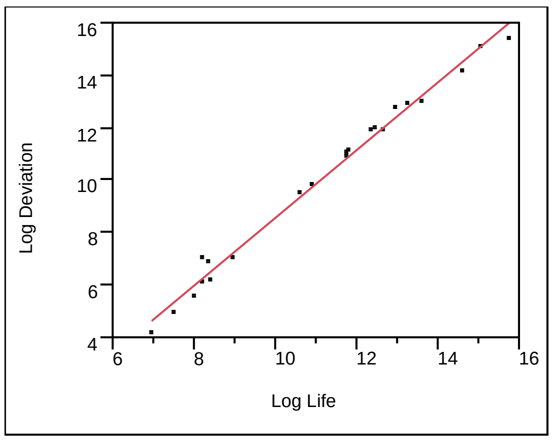

Figure 7. Plot for simplified extension.

5. Discussion

Uncertainty is inherent in any form of statistical analysis and is frequently under-emphasized or not sufficiently examined (Thacker, et al. 2001). Investigating and quantifying uncertainty is complex, and capturing dispersion in data is regularly a secondary objective after determining mean response behavior, such as time-to-failure estimation. However, any predictive mean or life function must consider variance in the data to discuss adequacy of the predictive ability. Although model assumptions and associated diagnostics exist to address this need, remedial measures and entirely separate modeling efforts have been established and are often necessary.

This work develops a model to capture dispersion in fatigue life data separate from a mean life model. The model uses a robust measure for calculating deviation at individual design points, allowing for the incorporation of censored data, and builds a regression model for capturing trends across the design space. Nonparametric testing addresses the adequacy of the dispersion model by comparing results with residuals of a predictive life model. Diagnostics, both quantitative and qualitative, are utilized for the dispersion model and predictive life model. A Bayesian extension of the model allows for predictive life and deviation estimations through sampling predictive distributions.

This work is limited by the lack of elaboration and balance in the test design, the inability to incorporate nominal predictive variables, and the significant amount of highly influential points in the data. Future work should address these limitations and develop the Bayesian and simplified extensions of the models.

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14. ABSTRACT
Time-to-failure (TTF) data, also referred to as life data, are investigated across a wide range of scientific disciplines and collected mainly through scientific experiments with the main objective of predicting performance in service conditions. Fatigue life data are times, measured in cycles, until complete fracture of a material in response to a cyclical loading. Fatigue life data have large variation, which is often overlooked or not rigorously investigated when developing predictive life models.
This research develops a statistical model to capture dispersion in fatigue life data which can be used to extend deterministic life models into probabilistic life models. Additionally, a predictive life model is developed using failure-time regression methods. The predictive life and dispersion models are investigated as dual-response using nonparametric methods. After model adequacy is examined, a Bayesian extension and other applications of this model are discussed.

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