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On Pecuniary Resiliency, Early Warning, and Market Imitation Under Unrestricted Warfare

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DISSERTATION

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DEPARTMENT OF THE AIR FORCE
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IMITATION UNDER UNRESTRICTED WARFARE

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ON PECUNIARY RESILIENCY, EARLY WARNING, AND MARKET

IMITATION UNDER UNRESTRICTED WARFARE

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Abstract

This study extends established financial market approaches to account for key econophysical attributes, low probability/high impact events, and a market's potential use as early warning for threats. Any disparity between established financial practices and true market conditions may provide incentive for exploitation and may harm national security objectives and interests through cascading effects. These national security concerns may include, in particular, the health of a reserve currency for those countries whose currency serves as one. This is a preferred approach with Unrestricted Warfare-type operations as these techniques may not enable repudiation of the antagonist. Since this approach may remain a strong incentive for such tactics for the foreseeable future, it is imperative to develop techniques that hedge against financial miscalculations and subversive efforts. This research relaxes key assumptions of standard finance theory and applies these approaches to currency dynamics and portfolio selection which provides insight on areas of vulnerability. Early warning measures of threats are developed and compared to critical world events. Vulnerabilities to capital markets are studied, and their effects on reserve currencies are also analyzed. Lastly, a mathematical framework is developed that enables imitation of the aforementioned econophysical attributes in a simulated environment thereby bridging the divide between certain aspects of standard finance theory and econophysics for future study.

I dedicate my work to three inspirations that have led me throughout my journey. Firstly, I dedicate this work to mankind's ceaseless pursuit of knowledge fueled by unbounded curiosity to understand the universe and its place within it. This journey is not lonesome but filled with great minds which have paved the way before and others who will continue this journey in the spirit of Enlightenment as Immanuel Kant so famously declared.

"Aufklärung ist der Ausgang des Menschen aus seiner selbstverschuldeten Unmündigkeit. Unmündigkeit ist das Unvermögen, sich seines Verstandes ohne Leitung eines anderen zu bedienen. Selbstverschuldet ist diese Unmündigkeit, wenn die Ursache derselben nicht am Mangel des Verstandes, sondern der Entschließung und des Muthes liegt, sich seiner ohne Leitung eines anderen zu bedienen. Sapere aude! Habe Muth, dich deines eigenen Verstandes zu bedienen! ist also der Wahlspruch der Aufklärung." - Immanuel Kant, 1784

Secondly, no effort regardless of how powerful a person's devotion is can be fruitful without the correct guidance. For this, I am eternally grateful to my advisor and mentor Dr. Richard F. Deckro. His ceaseless dedication and perpetual devotion guaranteed that my motions obtained the necessary vector for my journey. Few individuals are ever so fortunate as I have been. Not only did he sharpen my skills to become a rigorous analyst, he broadened my scope and challenged my perceptions on any and all topics we would ponder in our fruitful discussions which I have very much enjoyed and will sincerely miss.

Most importantly, no person is replete with inspiration and dedication without the perpetual sacrifices and love of that person who makes them whole. I dedicate this to you who has been with me long before the beginning of this adventure, who has provided ceaseless support throughout my journey, and who has been excited to commence with the next chapter of what this life has to offer for us. As with everything I do, I dedicate this work to my wife.

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I. Introduction

1.1 Introduction

The impetus of this dissertation is to ascertain the viability of some unrestricted warfare-like operations in the financial domain. Unrestricted warfare-like operations are those instruments which surpass older generations of warfare [36]. The instruments of unrestricted warfare typically circumvent the strengths of a foe and rather focus on those ill-protected domains which enable a superior foe to achieve its advantage. In today's military, especially for the United States, such domains would include the economic (specifically financial) sector and information technology. These domains are particularly vulnerable as well as vital to ensure a superior military deterrent. They pose lucrative targets for any practitioners of these types of tactics.

The evolution of warfare from one generation to the next generally include an increase in technological sophistication and those corresponding tactics to exploit these technological advances. 4GW is no exception. However, as will be discussed throughout this and subsequent chapters, is the fact that this evolutionary step to 4GW now enables the blurring between citizen and soldier as well as concealing diminishing traditional battle spaces and elevating the location of conflict to domains not traditionally associated with military conflict. This is of elevated importance as globalization increases and traditional military tactics may potentially bear large negative economic consequences for those involved either directly or

indirectly. Therefore the instruments of unrestricted warfare could enable politics by other means in a globalized economy.

Although unrestricted warfare is a useful framework for operations, its adaptation is not sufficient alone to achieve desired outcomes. The continuing effectiveness of these types of operations are possibly linked to the level of secrecy involved when actuating a specific tactic. If a foe becomes aware that they are under attack, then they likely will try to defend against such an attack and possibly attempt a counter attack. A means to achieve continued success for these types of operations is to retain a high degree of ambiguity of whether or not an attack is impending, or currently ongoing. In practice, this may entail obfuscating whether a complex system which already exhibits risk is under attack or is simply behaving within broader expectations. To attain this ambiguity, it is of interest to target assets in environments which exhibit a high level of inter-connectivity and a natural state having higher volatility. By emphasizing these environments over other battlespaces, the likelihood for success of clandestine, or *sub rosa*[1], attacks may be higher. Potentially lucrative environments include the financial and information sectors. These two sectors exhibit a highly integrated and complex environment having many heterogeneous and interactive nodes and agents. Large networks in general exhibit power law distributions [28, 42] which provide leptokurtic[2] behavior in their volatility and which can then increase the probability of Black Swan[3] events.

This dissertation addresses this issue on several fronts in order to provide a broad assessment of the incentive of unrestricted warfare, the specifics of unrestricted financial warfare, possible offensive and defensive strategies, early warning for impending attacks or vulnera-

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[1]Today typical vernacular when describing covert operations. Translated from Latin, the phrase means "under the rose" and is used to imply secrecy.

[2]Distributions which have more probability in their tails than that of the Normal Distribution. More extreme values are probable.

[3]A metaphor for highly improbable which have a large effect on a system [63].

bilities, and the development of a mathematical framework for data replication which mimics those attributes of financial data for further study.

1.2   Chapter Synopsis

In order to provide an easily managed study, this work has been divided into seven additional chapters. Although each chapter is self-contained, it is suggested that each chapter be read in the order presented. Certain concepts such as Unrestricted Warfare (UW) are introduced and expanded early on which provides the impetus for additional chapters. Chapter 2 presents pertinent literature review for Chapters 3 through 7. Chapter 3 provides the basis for this research by attempting to make the case that UW concepts are the instruments toward implementing 4GW-type operations and other irregular warfare. One facet of UW is financial warfare which is the focus of this study. It provides the introduction and background necessary to understand finance as a weapon and how financially-focused strategies may influence the financial markets and thereby the status of a reserve currency[4] as the U.S. dollar (USD). Chapter 4 initiates the study by linking the equity market capitalization of a nation with the status of a reserve currency. It will show that the status of a reserve currency has a strong connection with the equity market of the currency's country of origin. Furthermore, it studies the relationship of a reserve currency relative to equity markets of the country of origin and how these relates to the Composition Of Foreign Exchange Reserves[5] (COFER) of these reserve currencies. It provides the understanding of how unrestricted warfare may be utilized in financial markets and how it may influence the standing of a reserve currency. Chapter 5 expands the assumption of normality for log-adjusted asset returns, providing an

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[4]A currency that is held in significant levels by foreign countries.

[5]Composition of a central bank's currency reserves. The International Monetary Fund tracks the 140 countries who voluntarily provide their compositions. These numbers are strictly confidential and only reported publicly as aggregates. The vast majority of these reserves are held in reserve currencies as the U.S. dollar, the Euro, the Japanese Yen, the British pound sterling, and the Swiss Franc.

alternate formulation modeling paradigm which indicates why assets are likely leptokurtic[6]. Furthermore, it studies how this leptokurtosis exhibits certain attributes which expose these assets to potential Black-Swan types of weaknesses. These attributes are then utilized to provide a method of offensive counter finance by developing target lists of financial assets as well as defense counter finance by developing a mathematical formulation to harden a portfolio against these types of events. Chapter 6 suggests evidence that certain market measures may provide warning signals for Black Swan-type events. Such events could be considered large terrorist attacks as the USS Cole bombing and the 11 September attacks, or financial crises as the Sub-prime Mortgage Crisis and the Global Financial Crisis. This approach assumes that for both of these scenarios the volatility gradually increases until the proposed event occurs. Specifically for attacks the assumption is that those individuals aware of an attack or are funding an attack wish to financially benefit from said attack and exploit this *a priori* information to achieve personal gain. This chapter incrementally develops more sensitive measures which first link those derived measures to attacks and crises to types of measures which provide a lead time before an attack. Chapter 7 develops a process of financial data imitation. Since financial systems are large, heterogeneous, and mostly complex, simulation may not be possible or those simulations developed may not be a sufficiently accurate representation. This may be due to hidden processes in the private sector firms as well as the lack of knowledge about those proprietary systems deployed. Furthermore, the aggregate effects may not be those desired and may require perpetual adjustment to achieve those dynamics desired. This chapter's objective is to develop a process of imitation that would enable an analyst to adjust the aggregate effects that would also incorporate both standard financial assumptions as well as econophysical attributes, as desired while achieving those desired attributes with minimal adjustment. Chapter 8 provides an aggregate conclusion for

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[6]Distributions which have more probability in their tails than that of the Normal Distribution. More extreme values are probable.

the findings in this study as well as which research trajectories should be valuable for further study.

II. Pertinent Literature

The motivation for this study is a response to the evolutionary pressures due to globalization. Current military doctrine addresses these pressures through its understanding of 4th Generation Warfare and what it implies for the war fighter. Although military doctrine such as the ICI COIN Guide, Irregular Warfare JOC (signed 2007), Allied Joint COIN Operations, FM 3-24/MCWP 3-33.5 (signed 2006), AFDD 2-3 (signed 2007), and JP 3-24, Counterinsurgency Operations (signed 2009) began to address these pressures, they retreated from these gains to strategies more accustomed with traditional military operations. The *Joint Warfare of the Armed Forces of the United States: JP 1* began to incorporate Military Operations Other Than War (MOOTW) however by 2006, the *Doctrine for Joint Operations: JP 3-0* (2001) began to retreat from these ideas, at least officially.

Some of the theoretical work and early indications that we are evolving into a new type of warfare was carried out by Lind, Nightengale, Schmitt, Sutton, and Wilson in *The Changing Face of War: Into the Fourth Generation*. They indicate that increased independence of the warfighter from command and logistics of each generation will continue to increase with a further decrease in resource reliance while increasing the exploitation of maneuver warfare. They also posit that these strategies are inherent in terrorist-type operations. These types of tactics further exploit the slow adaptability of laws of their foe and bring the combat to the enemy's population directly. Works that extend these ideas of 4th Generation Operations include *Fourth Generation Operations: Principles for the 'Long War'* by Artelli and Deckro, *The Evolution of War: The Fourth Generation* and *Insurgency: Modern Warfare Evolves into the Fourth Generation* by Hammes, *The Principles of War, with Reference to the Campaigns of 1914-1915* by Fuller, *Counterinsurgency Warfare* by Gagula, *Understanding Fourth Generation War* by Lind, and *Wars of the 21st Century* by Münkler.

These evolving tactics toward 4th Generation Operations have been incorporated into foreign doctrine, specifically Chinese doctrine, as well. The terminology utilized however is "Unrestricted Warfare", or directly translated "Warfare Beyond Bounds." Unrestricted warfare was proposed by Colonels Qiao Liang and Wang Xiangsui of the People's Liberation Army and first surfaced in 1999 with the book that bears the same name. Their focus is on ambiguous and often non-reputable tactics in order to win against a technologically superior foe. Their work specifically addresses the emphasis the United States lays on technology as a potential panacea against military casualties in a conflict and develops strategies to exploit this tactic. Furthermore, they expand the traditional battle space to include those domains which may not necessarily be considered areas where conflict has historically been waged. A few of these domains would be the financial sector, illegitimate use of international laws, and economic warfare, among others. They argue that these approaches can have significant effects on a target of interest without resorting to typical military approaches especially against a strong military foe as the United States.

There are indications that the United States has increased its awareness of these types of attacks. Johns Hopkins' Applied Physics Lab, along with the Department of Defense, and other agencies have conducted war games to understand the vulnerability of the United States to these types of attacks. Furthermore, the question if we are even under attack is difficult to answer. This refers again to the ambiguity and non-repudiation of an unrestricted warfare-type of approach. Jeremy Recurd, in *Bounding the Global War on Terrorism* argues that the conflict can be seen as an example of "war and nonwar." Howard in *'9/11' and After: A British View* considers the name Global War on Terrorism (GWOT) misleading maintaining this is more of a conflict than a war since it cannot be won in the traditional view of armed conflict. These ideas can be expanded to other domains of unrestricted warfare as well.

Hammes asserts in *Modern Warfare Evolves into a Fourth Generation* that 4GW are typically those which are dangerous for large military powers. The United States lost its conflicts in Vietnam, Lebanon, and Somalia; the Soviet Union lost Afghanistan and Chechnya; the French in Vietnam and Algeria; and Israel in Lebanon. Hammes argues that 4GW has been the most successful type of warfare in the last 50 years. Unlike previous generations of warfare, 4GW utilizes "all of societies' networks - political, economic, social, and military-to carry on the fight." He does argue, however, that 4GW is not a panacea. Several examples exist where such tactics were implemented and still failed as Malaya (1950s), Philippines (1950s), Oman (1970s), and El Salvador (1980s). Victory can be achieved but requires "coherent, patient action that encompasses the full range of political, economic, social, and military activities." In addition, traditional military forces are ill-suited for this type of warfare and thus a formidable military is rendered essentially useless. The objective of 4GW is to draw out a war such that it destroys a foe's political will, becomes too costly, and provides no clear structure for declaring victory.

Flynn in *Resilience to Unrestricted Warfare: Threats to the Homeland* begins a strategy on how to deal with this type of warfare [20]. He argues that under the post 9/11 Bush administration there was an emphasis to conduct this type of warfare utilizing military operations abroad. This was due to the view in Washington of the inability to protect the vast number of domestic soft targets which could be of particular interest to adversaries fighting abroad. Flynn argues that it is vital that we "not just be able to throw a punch but [be] able to take a punch." He specifically argues that Al Qaeda and other terrorist organizations have limited resources to plan and carry out attacks and therefore these attacks are likely to be large in order to maximize effect.

It directly follows that the Homeland must protect its soft targets by hardening them to attack. Kinetic terrorist attacks, however, only make up a small part of what may be considered unrestricted warfare. Soft targets beyond those susceptible to kinetic attack

that are likely terrorist targets must also be hardened. This is increasingly vital when the premise of limited resources may not remain valid if a nation state or wealthy interest group is incentivized to fund attacks that reach beyond kinetic terrorism. Today's UW warfighters understand that the main benefit of these types of actions is not necessarily the immediate damage of an attack but the "collateral consequences of eroding the public's trust in services on which it depends." [38]

Since these types of strategies may be used in the future by individuals, interest groups, or nation states, it is of national interest to be able to defend against these strategies and counter them. The United States possesses several key attributes which would make unrestricted warfare a valuable approach to inflict damage. Specifically, the U.S. dollar is the premier reserve currency. This implies that the U.S. dollar is one of only very few currencies which is used for global trade, is used exclusively in other countries, or is the currency to which other national currencies are pegged or anchored. Currencies are no longer backed by a commodity or a basket of commodities; their value is derived by the *trust* individuals and governments alike have in the good faith in the currency to maintain its value for the purpose of exchange of goods and services. Therefore to affect the trust in a currency is to affect the value of that currency and thereby affecting its standing relative to other currencies in global circulation.

Only a handful of currencies have reserve currency statues. This implies that those nations home to these reserve currencies have achieved a particular set of difficult criteria to make their currency lucrative enough to circulate beyond their domestic borders. Gary Shilling, in his contribution *The Dollar will Remain on First* to Eisen's book *Currencies after the Crash* [15] lists those conditions which aid a currency to achieve reserve currency status. They are as follows:

1. Rapid economic growth and GDP per capita, promoted by robust productivity growth[1].

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[1]The increase of "the optimal use of all resources-labor, capital, material, energy, and technology. It includes both "physical capital" and "human capital."[21]

2. A large economy, possibly the world's largest.

3. Deep and broad financial markets.

4. Free and open financial markets and economy.

5. Lack of substitutes.

6. Credibility in the value of the currency.

Shilling argues that due to the high relative productivity growth of the United States, the ongoing struggles with the Eurozone, specifically Greece, and the lack of any credible alternatives, the U.S. dollar will remain, at least for the immediate future, as the premier reserve currency. This is reflected in its use throughout the world and as a store of value by central banks. Although the BRICS[2] nations have insisted on decreasing their dependence on the U.S. dollar, their efforts do not seem consistent. The likeliest of the BRICS nations to achieve reserve currency status is China due to its large relative economy with respect to its other BRICS partners. In 2009-2010 China received pressure from the United States and Europe to readjust its currency peg to the U.S. dollar. Reluctantly, they complied as the U.S. Senate passed a bill which would penalize Beijing for artificially devaluing their currency. This control of a currency, Schilling argues, is not the type of behavior which promotes a currency to reserve status.

However, China faces further challenges when it comes to these criteria. China's economy is tightly controlled and the government runs its economy without respect to international free-trade norms. An example of this can be found with rare earth metals of which China holds a monopoly on their mining and refining [15]. The biggest threat, according to Shilling, is that large foreign holders of dollars and treasuries, such as China, will dump their treasuries and dollar assets for political reasons. He continues by stating "If foreigners refused to recycle

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[2]The largest emerging economires. They are Brazil, Russia, India, China, and South Africa.

their ongoing trade surpluses with the United States into American investments and tried to dump their existing holdings of Treasuries and other American assets as well, the dollar would collapse, and so would global financial markets and the global economy" [15]. However this could be considered the "mutually assured destruction" à la finance and would benefit nobody. A different strategy would include small proxy wars in the financial sector which would achieve an objective without having the entire financial system implode. Minkova provides the incentives for a nation to obtain reserve currency status. These are:

1. Seignorage - "the difference between the face value of the money and its production costs." [46]

2. Lower Transaction Costs

3. Liquidity premium for bond markets

4. 'Exorbitant Privilege' - 'excess returns on assets and the ability to avoid or deflect the burden of adjustment to current account imbalances' [57]

Therefore there is an incentive to not only pursue reserve currency status of one's own currency, as may be the case with the BRICS nations, but also to weaken the position of other currencies and thus encourage the use of a possible alternative. Several alternatives the U.S. dollar exist and their strengthening would be a possible weakening of the dollar as premier reserve currency. To affect this "weakening" it is necessary to understand how to achieve this for a given nation. Furthermore, it is also of importance to know how to counter these efforts by "hardening" a nation in whichever domain an attack may occur. Attacks on a reserve currency may occur as a secondary affect through an attack on financial markets. Therefore it is prudent to harden financial markets for severe yet rare - Black Swan - events. However the financial sector makes assumptions about how markets behave which are rooted in the findings by Louis Bachelier[2?]. For his analysis, Bachelier was able to

determine that one possible solution to the probability formula is the *normal distribution*. Since Bachelier's work, a general mathematical formula for the uncertainty probability curve has been established. Therefore, variability is often modeled to behave in accordance with the normal distribution. One of the major steps into utilizing this concept within financial markets appeared with Markowitz in the 1950s [40]. Markowitz had been interested in determining equations that capture the tradeoffs between risk and reward. Building on the foundations set for by Bachelier and others, Markowitz developed his efficient portfolio selection[3] which is described in (2.1) [8]. In (2.1) the objective function represents the risk of the vector of available security percentages $\mathbf{x}$. This risk is to be minimized with a required minimum expected return, r. The expected return is expressed as $\mathbf{e}^{\mathrm{T}}\mathbf{x}$ where $\mathbf{e}$ is the vector of average returns on the securities [8]. $\mathbf{\Sigma}$ represents the covariance[4] matrix[5] of the returns of the portfolio assets. The constraint, $\mathbf{1}^{\mathrm{T}}\mathbf{x} = 1$ mandates that the weights of the securities balance to unity. Finally, the constraint $\mathbf{x} \geq \mathbf{0}$ mandates that no selection is negative.

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[3]A portfolio is efficient if no higher expected return is possible to obtain without additional variability [45]

[4]The covariance of two random variables is the difference between the expected value of their product and the product of their separate expected values. For random variables X and Y,

$$\mathrm{cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$$

If X and Y are independent then $\mathrm{cov}(X, Y) = 0$. However, if $\mathrm{cov}(X, Y) = 0$ then X and Y may not be independent. [5]

[5]Also known as a dispersion matrix. A square symmetric matrix in which the elements on the main diagonal are variances and the remaining elements are covariances. Suppose $X_1, X_2, ..., X_p$ are random variables and the variance of X_j is σ_j^2 and the covariance of X_j and X_k is $c_{jk} = c_{kj}$. Then the variance-covariance matrix is $\mathbf{\Sigma}$ given by

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1^2 & c_{12} & \cdots & c_{1p} \\ c_{21} & \sigma_2^2 & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p1} & c_{p2} & \cdots & \sigma_p^2 \end{pmatrix}$$

The same term is also used for the corresponding matrix based on sample values.[5]

$$\min \mathbf{x}^{\mathrm{T}} \boldsymbol{\Sigma} \mathbf{x}$$
$$\text{s.t. } \mathbf{e}^{\mathrm{T}} \mathbf{x} \geq r$$
$$\mathbf{1}^{\mathrm{T}} \mathbf{x} = 1 \tag{2.1}$$
$$\mathbf{x} \geq \mathbf{0}$$

The efficient frontier that the portfolio selection to (2.1) provides enables investors to analytically determine which portfolio selection to make in order to limit the uncertainty in their portfolio given a minimum level of return they are seeking which is dependent on an individual's preferences. This approach, however, focuses on individual investors and portfolios [41]. In 1954, Sharpe [56] extended the work by analyzing risk to capital asset prices under market equilibrium. As many individual investors engage in determining efficient portfolios, the likelihood is that they will arrive at a small group of nearly identical or identical portfolios (provided subgroups of investors have the same acceptance level of risk) is higher since they will be utilizing the same tools to develop their portfolios. Furthermore, if we regard the market service users who push large transaction volumes in the foreign exchange (FOREX) markets, namely central banks, with respect to Sharpe's analysis this would then indicate that they too would develop few optimal portfolios of foreign currency *ceteris paribus*. This would imply that although central banks keep their composition confidential we could estimate their currency portfolio compositions.

However, this approach assumes that uncertainty in an asset follows those assumptions set forth by Bachelier. Fama [18] and Mandelbrot [40] provided instances in cotton markets[6] that showed this commodity, like many other commodities, does not, in fact, follow the aforementioned assumptions. The log-adjusted prices follow a leptokurtic distribution which provide a higher probability for extreme values which are not accounted for by the normality assumption [30, 42]. In addition to the disparity between probability density functions is

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[6]The price of cotton has been tracked since 1816 and provides a larger data set than other commodities.

the inconsistency of independent observations. Although this is another assumption for Bachelier's results, evidence by Johnson [30], Mantegna [42], and Gopikrishnan [23, 24] indicate that this assumption does not hold true for financial markets. Their results indicate that although a near zero autocorrelation exists between past and future values of an asset, the autocorrelation of the magnitudes of the past and future values have autocorrelations which may range between 0 and 0.25 for the assets they considered at different time scales. These results suggest that future observations are dependent on previous observations and that this temporal dependency may give rise to episodes of volatility clustering[7].

From the literature search and review conducted, no literature was found that provides a relationship between the magnitude of volatility clustering based on autocorrelation of log-adjusted absolute values and the number of outliers produced with respect to the normality assumption. This would provide the inconsistency of the traditional approaches dating back to Bachelier and indicates that these empirical observations which are borrowed from physics (the term econophysics is used) indicate that financial dynamics may be more sophisticated than Bachelier's approach would warrant. Chapter 5 of this work will begin to address this issue.

Most unrestricted warfare-like tactics involve ambiguous and non-reputable attacks. Some, however, do not fulfill this assumption. Terrorism is an exception to this rule and although its attack may be indirect and cascade which is an intent of an unrestricted warfare-like approach. Although these attacks may affect financial markets, an analysis of certain market measures may provide insight into anomalous trading activity which may provide early indications of an impending attack. The idea of utilizing financial, commodity, and currency markets as a predictor of an impending attack by an adversary is not new. *A priori* information about a planned attack affords a financial market advantage over competitors,

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[7]The posterior probability of high volatility episode is higher with a prior high volatility episode and vice versa.

which in turn may be reflected in the purchase price or selling price of an asset. Hence, we use asset as a generic term to reference stocks, bonds, commodities, and currencies. As market service users desire an advantage over their peers to garner larger profits, agencies committed to national security also desire an informational advantage to identify threats to national interests. Having a warning capability based on market conditions would provide the ability to quantitatively assess threats.

This concept was tested by the Central Inteligence Agency (CIA) via Project Prophecy from April 2002 to 2004. In Project Prophecy, market data was used to predict threats using such metrics as volatility, trade volume, put-call ratios, short interest, and momentum, and to search for trading anomalies akin to insider trading dynamics [53]. The anomalies which were found could then be analyzed and assessed by financial and intelligence analysts. Several studies on this matter have also been conducted by more academic financial institutions. Poteshman's study [51], the Swiss Finance Institute study [9], and the 9/11 Commission [53] concluded that anomalous behavior within American Airlines, United Airlines, Boeing, Delta Air Lines, and Koninklijke Luchtvaart Maatschappij (KLM) were likely subject to informed trading activities prior to terrorist attacks against the United States on 11 September 2001 that analyzed four commercial passenger aircraft as weapons to destroy these targets.

Although the idea is not new, the open literature on this topic is limited. Financial indicators that can suggest such anomalies may be proprietary by private firms or closely guarded by government agencies. This dissertation proposes and demonstrates an early warning system for possible financial exploitation of market conditions with *a priori* information of an attack based on the concepts developed.

The potential issue with any pecuniary resiliency approach and early warning measures proposed is that development may be stifled by the fact that the same data may be used to develop each subsequent approach. Further development may be too restricted to the current financial data set and may not posses the variability necessary if financial dynamics change.

If one would like to prepare for behavioral changes or test different conditions, it would be difficult if only the current data set were available. Therefore it would be advantageous to have a framework which will enable flexibility and generate data set which provide a target distributions and specific attributes. This is the objective of the work on financial market imitation. The data set generated would imitate actual financial data so that the artificially generated data would be indistinguishable from a real empirical data set. The literature search provided several models as the 'El Farol Market Model' [30] which also could imitate econophysical attributes. However the difference with this model is that the end aggregate results were not tailorable to fit the conditions of empirical data sets. Mandelbrot was able to incorporate econophysical attributes in his fractional fractal approach [40]; however, this approach is bottom-up as well and not top-down as would be desired by an analyst.

Since at least the 1950s, there has been a debate as to whether or not these assumptions form a sufficient basis to accurately capture the behavior of financial markets [42]. Some of the major proponents of econophysics[8] (along with some of their areas of emphasis) are Jean-Philippe Bouchard in utilizing statistical physics in risk management [7], Kirill Ilinski in gauge[9] modeling in non-equilibrium pricing [28], Benoit Mandelbrot in leptokurtic[10] price distributions [40], J. Doyne Farmer in developing automated trading systems in financial markets using Chaos Theory[11] [19], Dirk Helbing in self-organizing systems [26, 27], Per Bak in self-organizing criticality[12] [3] and punctuated equilibria[13] [4], Janos Kertesz in long-term

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[8]The interdisciplinary field which utilizes theories and techniques which originated in physics to topics in economics.

[9]A type of field in which the Lagrangian in invariant under specific transformations. [28]

[10]Distributions that exhibit more probability in the tails of its distribution.

[11]The study of phenomena which appear random, but in fact have an element of regularity which can be described mathematically. [6, 12]

[12]A property in dynamical systems with a critical point as an attractor. At the macroscopic level the system displays a combination of spatial and temporal scale-invariance characteristics of the critical point of a phase transition. [3]

[13]The theory that most species remain in stasis, or have little net evolutionary change, for long periods of time. The theory suggests that most significant evolutionary change occurs in relatively rare and rapid events of branching speciation called cladogenesis. This is the process when a species spontaneously bifurcates instead of a gradual transformation. [22]

memory within systems [32], Rosario Mantegna in scaling in finance [43], price changes in financial markets [44], simulation of Lévy stable stochastic processes[14] [44], Robert Engle with the development of the autoregressive conditional heteroskedasicity[15] (ARCH) models (and their variations) which are used to predict financial volatility [16], and many others. They have addressed issues in established theory and pose strong arguments that the standard finance theory may not be a sufficient representation of market dynamics in its current form.

Currently, both approaches have advantages and disadvantages. The advantages of standard finance theory are its relative simplicity, its long use within the financial community, and its ability to be expressed analytically given its assumptions. Its disadvantages lie in that some of the assumptions upon which it is built may not be sufficiently realistic. Inaccuracies might misrepresent the risk associated within a currency, derivative, or option and provide an artificially low or high expectation of risk for a calculated portfolio or pricing of an asset [30, 40]. Unlike standard finance theory, econophysics provides a more flexible strategy when dealing with financial markets [42]. It relaxes the assumptions of standard finance theory and allows for a more complex understanding of possible market conditions. Its tradeoff lies in the fact that most extensions are more complicated, originating in physics. Its complexity and origin might limit acceptance and implementation. Furthermore, closed form solutions are seldom achievable; therefore numerical solutions are commonplace. This provides an extra level of complexity when implementing these models into the computer systems which underpin financial markets.

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[14]A stochastic process similar in setup as the random walk with the normal distribution.
[15]These are random variables with varying levels of variance.

III. Unrestricted Warfare As an Instrument of 4th Generation Operations

3.1 Introduction

The impetus of this research stems from the evolution of warfare to accommodate an increasing level of economic interdependence and integration of nations and markets. As globalization increases, the likelihood of national operations that influence economic sentiment is also likely to rise. Therefore the nature of military operations to influence political objectives must also likely evolve to adapt to this change. The instruments necessary to transition from 4th Generation Warfare (4GW) to 4th Generation Operations (4GO) are different from those instruments which evolved previous evolutions of warfare. These instruments would include operations which blur the line between citizen and soldier as well as attack assets on which traditional kinetic military operations would not focus. Therefore 4GW are those types of conflicts that blur the lines between war and politics, combatants, and civilians. The term was first used by American analysts in 1989 as Lind *et. al.* [37] to describe the return of warfare to a decentralized form. While warfare elicits images of armed conflict, operation spans all phases of the continuum from peace to conflict. Indeed, in this modern interconnected world a nation may be under attack and not even realize it until it is too late, if at all. Technology coupled with the necessary changes drove previous generations evolution while their interconnectedness presents unique difficulties. The transition from 4GW to 4GO blurs the line between the definition of a warfighter as well as what is defined as the battle space. 4GO includes those operations which combine elements "of guerrilla tactics, terrorism, traditional warfare, and the ability to exploit and skip generations of technology to conduct operations, particularly to target the will and morale of the enemy's support structure, in order to achieve political victory" [1]. In this context, UW is

not as much a new type of warfare in its own right as an instrument that facilitates further military evolution. Hammes [25] and Artelli [1] provide the following tactical traits that are present in 4GO, namely:

- Exploitation of dissimulation through complex battle spaces via low-intensity conflict.

- Inclusion of lower generational tactics.

- Dissimulation of tactics utilizing cross-border strategies

- Utilization and expansion beyond traditional battle spaces through political, economic, military networks.

UW focuses on these traits by expanding the traditional concept of "battlespace" to include those domains such as financial, economic, and cyber warfare to name only a few (a more extensive list is forthcoming in the next pages) which may not typically be associated with warfare. Recent restatement of this strategic expansion was developed by Colonels Qaio Liang and Wang Xiangsui of the People's Liberation Army in February 1999. In their book, they expand the notion of military operations to other domains which are not typically associated with warfare. They coined the term unrestricted warfare[1] to designate incorporation of these less traditional battlespaces.

An added premise of UW is to develop strategies which circumvent accepted advantages enjoyed by those countries with their superior military portfolios. There is no incentive to attack a foe on their terms which they are organized, trained, and equipped to fight. Many potential foes saw the folly of going toe to toe with the United States and its coalition partners in the first Gulf War. Rather, the objective is to determine which tactics would make these advantages obsolete. Unrestricted warfare, then, is adopting a strategy which fully embraces the philosophy of Sun Tzu and others. By avoiding a foe where they are strong

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[1]Directly translated from from Chinese, UW means "warfare without bounds."

and exploiting them where they are weak, practicing dissimulation, and hiding beneath "a cloak of disorder" [64], enables asymmetric warfare which neutralizes actual military advantages. These domains include, but are not limited to [35, 38, 54, 62, 68]:

- Cultural Warfare - The tactic of exploiting cultural climate of a target nation or group.

- Financial Warfare - The tactic of exploiting the financial system and monetary policy of a target nation.

- Drug Warfare - Injecting illicit drugs into an enemy country to reap profits and to degrade the enemy nation.

- Economic Aid Warfare - Providing aid with stipulations that the recipient take certain political actions. One example would be the supply of grain and other food stuffs by the United States, South Korea, and Japan to North Korea in exchange for their dismantling of their atomic weapons program [62].

- Economic Warfare - Exploiting trade policy, regulations, utilizing embargoes, black-listing of foreign firms, strategic purchases of critical materials, manipulating import and export quotas, tariffs and subsidies, manipulation of export-import prices, and preemptive buying.

- Ecological Warfare - Introducing invasive species to an ecosystem to decimate native species and therefore would have negative economic consequences for the ecosystem.

- Lawfare - "The use of the law as a weapon of war"[13]. It is the illegitimate utilization of domestic or international law in order to financially cripple, wasting an opponent's resources and time with litigation to delay their objectives as a strategic lawsuit against public participation (SLAPP), and to win a public relations victory.

- Media Warfare - The tactic of manipulating foreign media.

- Network Warfare - Utilizing cyber warfare exploits to disrupt information systems and their dependent infrastructure.

- Psychological Warfare - Deceptive exploitation of people's perceptions of its capabilities and intentions.

- Resource Warfare - Monopolizing scarce resources in order to influence their market value.

- Smuggling Warfare - Infiltration of illegal or counterfeit goods into a market thereby weakening legitimate manufacturers.

- Terrorism - The pursuit of a political aim using violence and intimidation.

Figure 1 illustrates the relationship between a foe utilizing UW operations versus the United States. The foe's intentions are obscured by the complexities of the chosen battlespaces. This approach is indirect and could be considered a soft attack. The target, who does not embrace unrestricted warfare-style tactics, is unsure whether they are under attack, the originator of the attack or effort, and how they are to respond. Ambiguity and non-repudiation are essential elements to neutralize traditional types of retaliation.
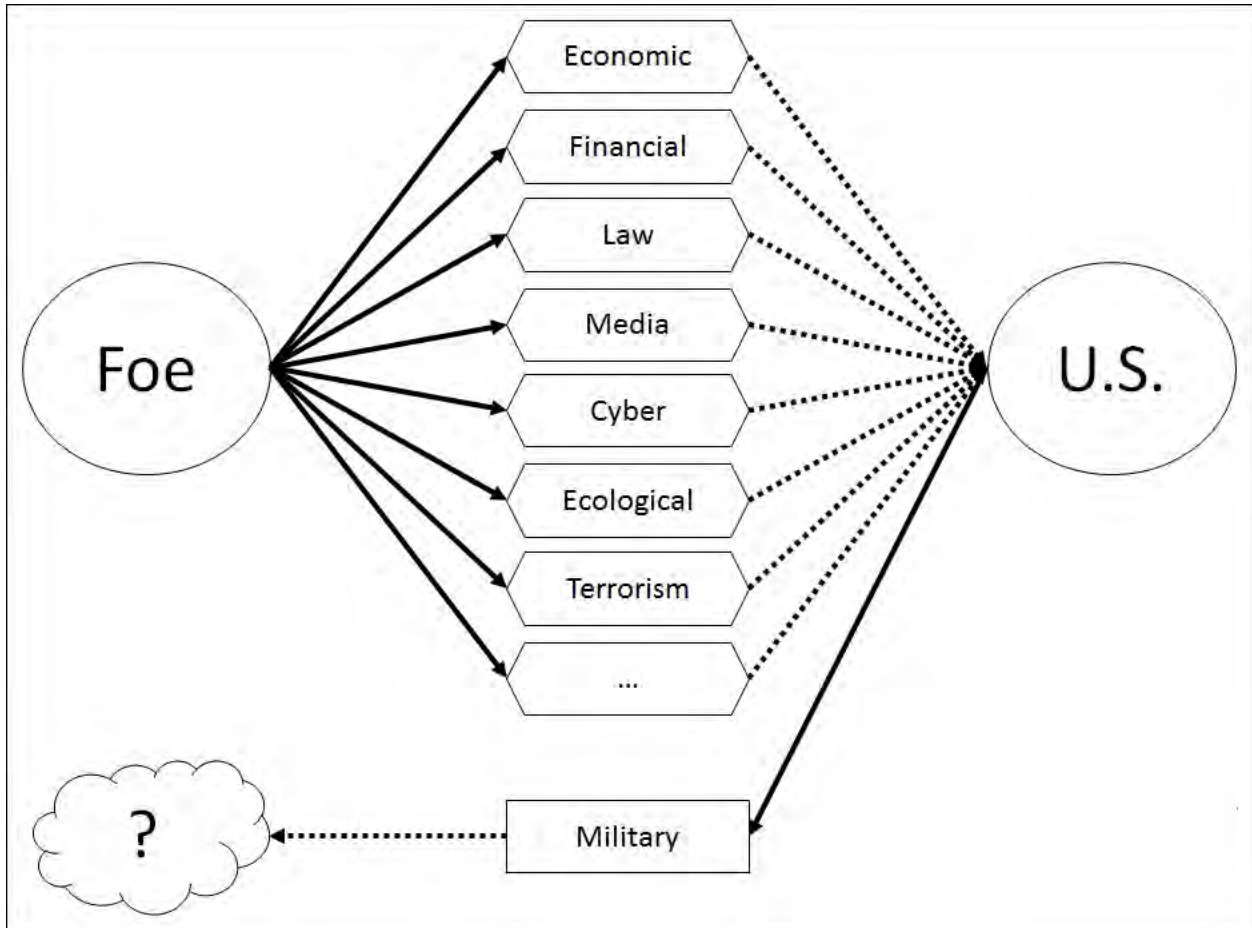
Figure 1. Illustration of a foe and the U.S. as a target of UW. Conventional military tactics may not be sufficient to retaliate.

Although unrestricted warfare would be the vehicle of operation, it is not sufficient alone. The continuing effectiveness of these types of operations are possibly linked to the level of secrecy involved when actuating a specific operation. If a foe becomes aware that they are under attack, then very likely they will try to defend against such an attack and attempt a counterattack. The objective for these types of operations is to retain a high degree of ambiguity of whether or not an attack is ongoing within a complex system which already exhibits variability is simply behaving within broader expectations. To obtain this ambiguity it would be of interest to target those environments which exhibit a high level of inter-connectivity and a natural state of higher volatility. By emphasizing these environments over

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other battlespaces the success of clandestine, or *sub rosa*[2], attacks may be higher. Potentially viable solutions would specifically be the financial and information systems. These two systems exhibit a highly integrated and complex environment with many heterogeneous and interactive nodes or agents. Large networks in general exhibit power law distributions [28, 42] which provide leptokurtic[3] behavior in their volatility and which can then lead to Black Swan[4] events. Figure 1 illustrates this type of operation. A nation state, group, or individual would influence nontraditional battlespaces such as those previously mentioned, which would have indirect and possibly cascading effects on a specific target. These effects might be perceived as typical behavior of those domains utilized, allowing a concerted effort to be misinterpreted as natural system behavior. Essential elements of UW are then ambiguous between a concerted effort vis-à-vis typical system dynamics and non-repudiation to the origin of the attacker.

Some suggested UW-type operations may include:

- The alleged U.S., Afghanistan/USSR counterfeit currency operations in 1985 where $2B in counterfeit Afghan currency was distributed to deflate the value of the real currency in circulation as well as to cast doubt that the currency in circulation was legitimate. This retarded Soviet local logistics replenishment. This forced Soviet logistics to import essentials rather than addition troops and munitions, thereby degrading their military operations [62].

- The U.S./Iran Nuclear non-proliferation through Executive Order 13599 on 6 February 2012 to present. This executive order blocks property and financial institutions from utilizing the Society for Worldwide Interbank Financial Telecommunication (SWIFT)

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[2]Today typical vernacular when describing covert operations. Translated from Latin, the phrase means "under the rose" and is used to imply secrecy.

[3]Distributions which have more probability in their tails than that of the Normal Distribution. More extreme values are probable.

[4]A metaphor for highly improbable which have a large effect on a system [63].

which is a information system that is utilized to send and receive information about financial transactions in a secure, standardized and reliable environment. This effectively disabled Iran's financial conduit to the international financial world [17, 52].

- The alleged counterfeiting of U.S. $100 Federal Reserve notes (supernotes) by the Democratic People's Republic of Korea is another example; North Korea may have printed and circulated $45 million [52].

- Currency manipulation from 1921-1936, 1967-1987, and 2010 to present [52].

- Alleged hacking of the Office of Personnel Management by China which could affect 4 and 18 million past and present government employees [14].

- The bombing of the USS Cole, the 9/11 Attacks, and the Khobar towers attacks.

IV. Influences on Reserve Currency Status

4.1 Introduction

For a nation's currency to achieve reserve currency status, several necessary conditions must be met. Schilling outlines these conditions as follows [15]:

1. Rapid economic growth and GDP per capita, promoted by robust productivity growth[1].

2. A large economy, possibly one of the world's largest.

3. Deep and broad financial markets.

4. Free and open financial markets and economy.

5. Lack of substitutes.

6. Credibility in the value of the currency.

Currently, the U.S. dollar (USD), European euro (EUR), Japanese yen (JPY), British pound (GBP), and Swiss franc (CHF) are the five reserve currencies. The use of these currencies as reserve currencies affords these countries certain benefits over countries having non-reserve currency status. These are [46]:

1. Seignorage - "the difference between the face value of the money and its production costs." [46]

2. Lower transaction costs

3. Liquidity premium for bond markets

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[1]The increase of "the optimal use of all resources-labor, capital, material, energy, and technology. It includes both "physical capital" and "human capital."[21]

4. 'Exorbitant Privilege' - "excess returns on assets and the ability to avoid or deflect the burden of adjustment to current account imbalances" [57].

There exists an incentive for a non-reserve currency country to posture its currency to become a reserve currency, join a currency union to benefit from the reserve currency incentives, and possibly to diminish a monopoly a reserve currency has over its competitors so that no one reserve currency benefits strongly from these incentives. Several countries such as Brazil, Russia, India, China, and South Africa (known as BRICS) have begun to cooperate to diminish the influence the United States has through the U.S. dollar. One attempt to weaken the U.S. dollar was to peg a currency artificially low. From 2009-2010, China received pressure from the United States and Europe to readjust its currency peg to the U.S. dollar. They complied reluctantly as the U.S. Senate passed a bill which would penalize China for artificially devaluing their currency. Another endeavor was to initiate direct currency swaps between BRICS partners and thereby eliminate the use of the U.S. dollar as a conduit in the process. Although these tactics do show limited promise, they do not seem to provide the required level of force to diminish the U.S. dollar as premier reserve currency. What is required is an alternate approach and a possible metric to measure its effect.

In Figure 2, Papaioannou illustrates the international functions of a currency [50]. One metric that could be useful is the size of international reserves governments hold in each currency. Central banks hold these reserves to regulate their own national money supply. Moreover, central banks hold reserve currencies which provide low volatility in their assets. Therefore the composition of total reserve currency portfolio would be an indicator of a reserve currency performance. As economies increase, however, so do their equity markets to accommodate this increase. Therefore, additional reserves are purchased and traded. Understanding this relationship between equity markets and central bank reserves and how these influence a reserve currency standing is the focus of this chapter.

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Function of Money	Government	Private Agents
Store of Value	International Reserves	Investment Currency (Currency substitution and asset substitution)
	Financial Markets	
Medium of Exchange	Foreign Exchange Trading	Trade Invoicing
Unit of Account	Anchor for Currency Peg	Quotation currency for goods and assets

Figure 2. International functions of a currency [50]

4.2 Equity Market Capitalization

An equity market is a market wherein shares are traded through exchanges or over-the-counter markets. Equity market capitalization is the measure of the total value of an equity market. Central banks hold currencies as international reserves as a store of money. These international reserves are mostly comprised of reserve currencies[2]. These reserves are held in a Currency Composition of Official Foreign Exchange Reserves (COFER) portfolio. The health of a reserve currency can be measured by the consumer confidence in its use. This suggests that the increased use of a currency over any substitutes indicates a higher level of confidence over alternatives.

The first analysis conducted in this study is to understand whether a country's capital market is correlated to currency's COFER status. For countries having large capital markets, this would be logical. If their own large capital market and trade are in distress relative to substitutes, then this might imply that market players may search for more secure alternatives. Of all the reserve currency countries[3] the United States has the largest capi-

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[2]The Australian Dollar (AUD), British Pound (GBP), Canadian Dollar (CAD), EMU Euro (EUR), Japanese Yen (JPY), Swiss Franc (CHF), and the United States Dollar (USD) are reserve currencies. The Canadian dollar (CAD) and the Australian dollar (AUD) are now included into COFER but only as recent as Q4 of 2012 and thus very little data exists of their impact and dynamics as reserve currency. Therefore all but these two reserve currencies will be considered sufficient for calculations and the discussion regarding reserve currencies.

[3]We define a reserve currency country as singular sovereign nation which originates a reserve currency. These are Great Britain, Japan, and the United States.

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tal market at approximately \$19T, followed by Japan at \$4T, and Great Britain with \$3T in 2012 (values rounded to nearest \$1T; data from Quandl). This provides the idea that, since the capital market of the United States is a magnitude higher than its reserve currency country alternatives, its COFER holdings should be more tightly linked to its capital market structure. This is indeed the case. The equity markets of the United States, Japan, and Great Britain and their COFER levels have respective correlations of $\rho \approx 0.845$, $\rho \approx 0.648$, and $\rho \approx -0.832$. These results suggest that the size of a reserve country's capital markets are related to the influence of their reserve currency. However, if a nation's own capital markets are not large enough to counter external currency demand pressures then this relationship may no longer be valid. This may be the case with Great Britain. Additional detail will follow to address this particular result. Switzerland's capital market value is approximately \$1T and has a correlation to its market capitalization of $\rho \approx 0.380$. Although the Swiss franc is a currency union[4], it behaves much like a currency from a reserve currency country. This is due to the fact that its members are culturally similar and its members' fiscal plans are aligned. The correlation is low, and this may be attributable to people investing in Swiss Francs as a hedge against domestic fiscal issues. The EMU[5] has a market capitalization of \$10T and a correlation to its market capitalization of $\rho \approx 0.335$. This implies that currency unions may also have low correlations since their individual members have their own capital markets which may be driven by market factors beyond the currency union proper.

The second analysis conducted compares paired correlations. The first paired correlation, ρ_1, is that between a reserve currency country equity capital market and the other capital markets of the 106 listed countries[6]. The second paired correlation, ρ_2, compares the

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[4]A monetary union with two or more states which share a common currency.

[5]The European Monetary Union, a currency union comprised of Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, and Spain.

[6]The countries analyzed in alphabetical order of their respective country codes (CC) are the United Arab Emirates (ARE), Argentina (ARG), Armenia (ARM), Australia (AUS), Austria (AUT), Belgium (BEL), Bangladesh (BGD), Bulgaria (BGR), Bahrain (BHR), Bermuda (BMU), Bolivia (BOL), Brazil (BRA), Bar-

COFER level of the reserve currency country in the first paired correlation with the equity capital markets of the country in the first paired correlation. Figure 3 depicts the results of this relationship for the three reserve currency countries. The detailed results are listed in Appendix A.

We observe that there is evidence to suggest that a near linear relationship exists between these two factors. Unlike Japan and the United States which have a positive correlation between these two factors, Great Britain has a strong negative correlation. This implies that the demand of the pound sterling may be influenced by stronger factors than the market capitalization levels of Great Britain. In addition, many data points for Japan and the United States populate their respective graphs for low values of ρ_1 while the opposite is true for Great Britain. This feature suggests that many of the 106 countries under consideration behave differently than the United States and Japan with respect to their COFER levels and equity markets.

Figure 3 provides this relationship for the United Kingdom (left), Japan (middle) and the United States (right). Each depicts the paired correlation of capital markets of reserve currency countries ρ_1 (x-axis) and other 106 countries with the paired correlation of the COFER level of that reserve currency country's currency with the equity market levels of the

bados (BRB), Botswana (BWA), Canada (CAN), Switzerland (CHE), Chile (CHL), China (CHN), Côte d'Ivoire (CIV), Colombia (COL), Costa Rica (CRI), Cyprus (CYP), Czech Republic (CZE), Germany (DEU), Denmark (DNK), Egypt (EGY), Spain (ESP), Estonia (EST), Finland (FIN), Fiji (FJI), France (FRA), United Kingdom (GBR), Georgia (GEO), Ghana (GHA), Greece (GRC), Guyana (GUY), Hong Kong (HKG), Croatia (HRV), Hungary (HUN), Indonesia (IDN), India (IND), Ireland (IRL), Iran (IRN), Iceland (ISL), Israel (ISL), Italy (ITA), Jamaica (JAM), Jordan (JOR), Japan (JPN), Kazakhstan (KAZ), Kenya (KEN), Kyrgyz Republic (KGZ), St. Kitts and Nevis (KNA), South Korea (KOR), Kuwait (KWT), Lebanon (LBN), Sri Lanka (LKA), Lithuania (LTU), Luxembourg (LUX), Latvia (LVA), Morocco (MAR), Mexico (MEX), Macedonia (MKD), Malta (MLT), Mongolia (MNG), Mauritius (MUS), Malaysia (MYS), Namibia (NAM), Nigeria (NGA), Netherlands (NLD), Norway (NOR), Nepal (NPL), New Zealand (NZL), Oman (OMN), Pakistan (PAK), Panama (PAN), Peru (PER), Philippines (PHL), Papua New Guinea (PNG), Poland (POL), Portugal (PRT), Paraguay (PRY), Qatar (QAT), Romania (ROU), Russian Federation (RUS), Singapore (SGP), El Salvador (SLV), Serbia (SRB), Slovak Republic (SVK), Slovenia (SVN), Sweden (SWE), Swaziland (SWZ), Thailand (THA), Trinidad and Tobago (TTO), Tunisia (TUN), Turkey (TUR), Tanzania (TZA), Uganda (UGA), Ukraine (UKR), Uruguay (URY), the United States (USA), Venezuela (VEN), Vietnam (VNM), South Africa (ZAF), Zambia (ZAM), and Zimbabwe (ZWE).

country before, ρ_2 (y-axis). The relationship of the United Kingdom is $\hat{\rho}_2 = 0.044\rho_1 - 1.067$ with $R^2 = 0.877$. Japan's is $\hat{\rho}_2 = 0.027\rho_1 + 0.731$ with $R^2 = 0.685$. The relationship for the United States is $\hat{\rho}_2 = 0.034\rho_1 + 1.083$ with $R^2 = 0.927$.
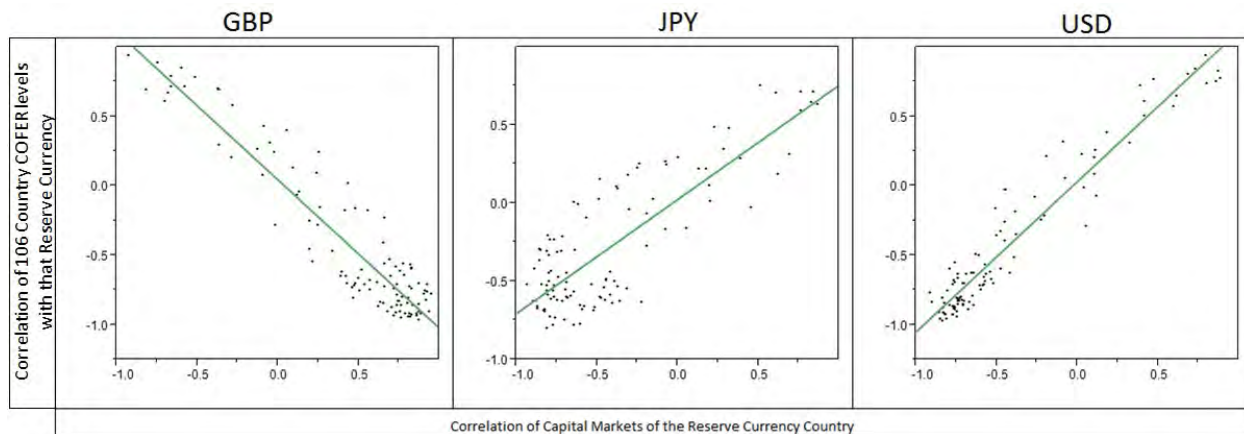
Figure 3. Relationship between the paired correlations capital markets of reserve currency countries (x-axis) and the other 106 counties with the paired correlation of the COFER levels of that country (y-axis).

There is not a singular behavior common to all EMU members. Countries as Spain, Slovakia, and Slovenia have strong positive correlations between their respective market capitalization and the EUR's COFER levels ($\rho \approx 0.831$, $\rho \approx 0.788$, and $\rho \approx 0.645$ respectively) whereas Finland, the Netherlands, and Germany have strong negative correlations ($\rho \approx -0.900$, $\rho \approx -0.878$, and $\rho \approx -0.854$ respectively). The aggregate effect is a low correlation between EMU members with their own reserve currencies. This may be due to heterogeneous international trading partners. Those EMU countries which have high positive correlations with the EUR trade mostly within the EMU[7], whereas EMU countries

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[7]Spain's top 5 import origins and export destinations are respectively Germany (11%), France (11%), China (7.0%), Italy (6.4%) the Netherlands (4.4%), and France (15%), Germany (10%), Italy (7.7%), Portugal (7.1%), and the United Kingdom (6.2%), for Slovakia they are Germany (16%), Czech Republic (13%), Other Europe (12%), Russia (9.0%), South Korea (8.5%), and Germany (22%), Czech Republic (12%), Poland (6.7%), Austria (6.1%), and Hungary (5.8%), and for Slovenia they are Italy (17%), Germany (16%), Austria (7.9%), China (5.5%), France (4.1%), and Germany (21%), Italy (12%), Austria (8.0%), France (5.8%), and Russia (4.7%). Data obtained from the Observatory of Economic Complexity. Data is from 2012.

which have high negative correlations with the EUR trade largely outside of the EMU[8]. This may indicate that, as countries participate in international trade beyond their currency union (if they participate in one), there exists a likelihood they require reserves in other reserve currencies. For larger EMU members, international trade is paramount, and thus these members require the USD over the EUR to actuate these international transactions.

4.3   BRICS Nations & Currency Relationships

The BRICS nations are those emerging countries that wish to increase their world influence. Therefore, it is of interest to understand how their emergence may affect COFER levels of current reserve currencies. As Table 1 indicates, the market capitalization of these countries have a positive correlation with the COFER holdings of the EUR and the GBP while having a negative correlation to the COFER holdings of USD, JPY, and CHF. Although no causal link can be confirmed between these two influences, these countries have indicated on several occasions that they wish to distance themselves from the USD via currency direct swaps, the creation of a BRICS bank, and investment in alternative currencies or diverse assets.

Table 1 provides a comparison of BRICS nation market capitalization to COFER reserve currency levels. The table shows that all BRICS nations have a positive correlation with the EUR and GBP while exhibiting a negative correlation with the USD, JPY, and CHF. The data utilized is from 1999 to 2012 at yearly increments.

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[8]Finland's top 5 import origins and export destinations are respectively Russia (16.0%), Germany (13.0%), Sweden (10.0%), China (8.3%), the Netherlands (5.6%), and Sweden (11.0%), Germany (9.3%), Russia (9.0%), the United States (6.8%), and the Netherlands (6.0%), for the Netherlands Germany (14.0%), Belgium-Luxembourg (9.3%), Russia (8.0%), China (8.0%), the Unites States (6.2%), and Germany (20.0%), Belgium-Luxembourg (17.0%), United Kingdom (9.4%), France (6.1%), and Italy (5.3%) for Germany, the Netherlands (8.9%), China (8.6%), France (7.2%), the United States (5.5%), Italy (5.2%), and France (8.8%), the United States (8.1%), China (6.4%), United Kingdom (6.2%), and the Netherlands (5.8%). Data obtained from the Observatory of Economic Complexity. Data is from 2012.

Table 1. **Comparison of BRICS nation market capitalization to COFER reserve currency levels. Data Source: World Bank.**

	USD	EUR	GBP	JPY	CHF
Brazil	-0.7506	0.714	0.784	-0.705	-0.697
China	-0.7758	0.749	0.826	-0.775	-0.597
India	-0.8396	0.799	0.884	-0.795	-0.671
Russia	-0.833	0.804	0.946	-0.874	-0.635
South Africa	-0.8385	0.819	0.874	-0.854	-0.68

Unlike those direct approaches previously discussed which openly try to diminish the influence of the U.S. dollar, this approach suggests that as BRICS nation equity markets increase, the COFER reserves are increased with Euro and pounds. If a COFER substitutes the U.S. dollar with the Euro, then this would lessen the demand for the dollar and thereby diminish its influence. Such an action would have a slow yet gradual effect.

4.4 Global Relationships Between Equity Markets and COFER Levels

The highest fidelity of analysis is possible when examining all 5,565 unique paired correlations of the 106 country market capitalization as well as the 530 correlations of the market capitalizations and COFER levels of the top five reserve currencies. This is illustrated in Figure 4. The left figure illustrates the sorted pairwise correlations of 5,565 unique country correlations of market capitalization with its quadratic regression (shown in green). This indicates that 3,648 of these pairs are positively correlated with each other while 1,917 are negatively correlated with each other which implies that two clusters of correlations exist with a near 2:1 ratio. The quadratic regression relationship is $\hat{\rho} = 0.891 - 1.11\text{E-}4x - 3.76\text{E-}8x^2$, $R^2 \approx 0.999$. The figure on the right illustrates the sorted values of the 530 pairwise correlations between unique country market capitalization COFER levels of the five reserve currencies with the Logistic 4P regression (shown in blue). The parameters of the 4P logistic regression equation are $a = -0.015$, $b = 229.569$, $c = -0.821$, and $d = 0.928$ with $r^2 \approx 0.997$. This analysis suggests that market capitalization behaves as two clusters. One cluster comprises $\frac{2}{3}$ of all

capital markets while $\frac{1}{3}$ comprises the other. Since there exist two markets and reserve currencies strongly correlate (either positively or negatively) to a reserve currency, this may imply that reserve currencies too form clusters. The first indication of this phenomenon is also depicted in Figure 4.

National Market Capitalization in each Of the five reserve currencies

5,565 Paired Market Capitalizations
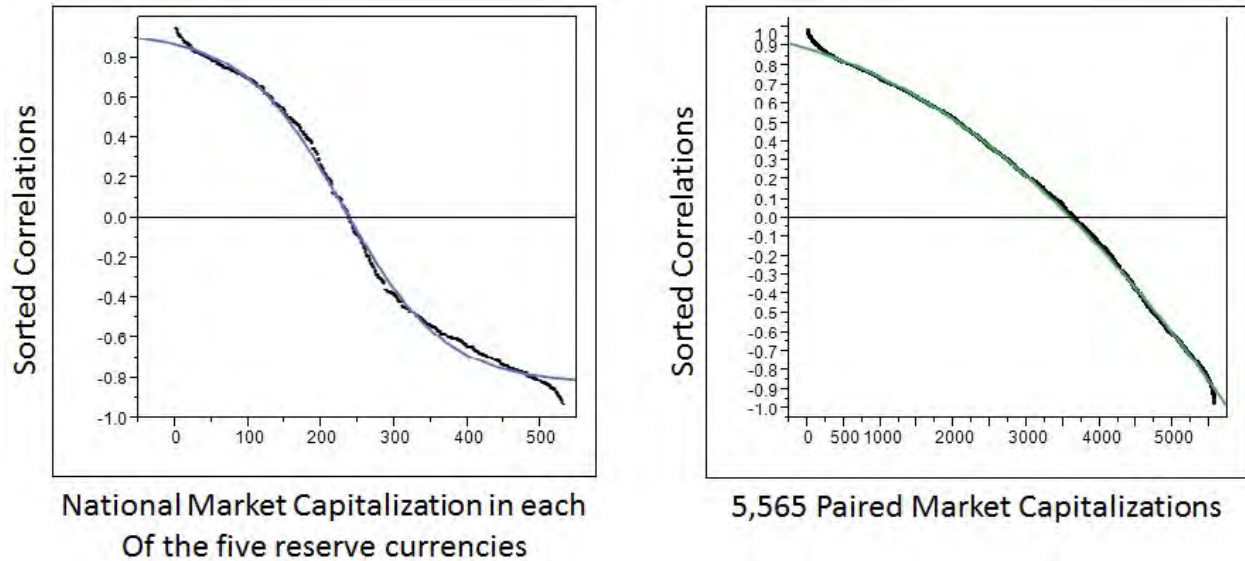
Figure 4. Correlations of national market capitalizations (left) and correlations of market capitalizations and COFER holdings (right). Market equity data: Quandl; COFER holdings data: World Bank.

Since the analysis suggests that two global markets seem to exist and that these prefer different reserve currencies, this would suggest that reserve currencies too may be clustered into two groups. Table 2 indicates the pairwise correlations of world equity capitalizations and COFER levels between different reserve currencies from 1999-2012. As anticipated, the five reserve currencies cluster into two groups. The first group is comprised of the CHF, JPY, and USD while the second is comprised of the EUR and GBP. This suggests that a possible strategy to weaken a reserve currency in one cluster is to strengthen one or more reserve currencies in another cluster.

Table 2. Pairwise correlations of World Equity Capitalization and Reserve Currencies Distribution in COFER Portfolio (1999-2012) in different reserve currencies. Data Source: World Bank.

Currency Pair		Pairwise Correlation
EURO	USD	-0.998
JPY	EURO	-0.984
JPY	GBP	-0.972
GBP	USD	-0.966
CHF	GBP	-0.958
CHF	EURO	-0.927
CHF	JPY	0.921
CHF	USD	0.940
GBP	EURO	0.957
JPY	USD	0.979

4.5   Summary

This chapter demonstrated that there is a preponderance of evidence to suggest that equity markets and COFER levels are linked. The analysis conducted herein indicates that global equity markets may be split into two clusters. Each cluster possesses either the U.S. dollar or the euro as chief reserve currency. Since these clusters are negatively correlated, this implies that as one strengthens by exogenous factors, the other, as a consequence, will weaken in standing. Furthermore, the analysis showed that the about $\frac{1}{3}$ of all countries observed prefer the U.S. dollar cluster while $\frac{2}{3}$ prefer the Euro cluster. Specifically the BRICS nations which pose a new threat to the U.S. dollar prefer the euro cluster.

This suggests that one method to weaken the standing of a reserve currency is to prefer its competitor's reserve currency. Although this concept is not new within normal economic competition, knowing who your competitors are is import when choosing a side. With respect to weakening the U.S. dollar, an adversary should increase their use of the euro and the British pound sterling, which is the case for the BRICS nations already.

V. Extending Pecuniary Resiliency

5.1 Introduction

This chapter focuses on incorporating resiliency in financial models by addressing concerns of extreme events, known as Black Swan events, which may take place. The normality assumption in the uncertainty for asset pricing implies that these extreme events occur much more rarely than they actually do. This would suggest that additional factors play a role in the behavior of financial markets than is accounted for by the normality assumption alone. Failure to address these concerns with an appropriate approach exposes any pricing strategy to further risk, which is then underrepresented by the aforementioned assumption. This under-representation may then cause additional vulnerability based on the pricing models being used as well as providing incentive for the financially savvy foe to exploit these vulnerabilities as a means of unrestricted financial warfare.

To understand the genesis of these Black Swans, this chapter will examine certain traits which have been a focus in econophysics, namely the study of economic effects utilizing approaches borrowed from physics. By studying these traits and examining securities from their perspective, we will attempt to provide conditions for which Black Swans become more prevalent. This will then provide the conditions necessary to hedge against and to be aware of vulnerability assessment.

5.2 Relaxing the Normality Assumption

The assumption of normally distributed volatility is a consequence from assuming that log-adjusted price dynamics of assets follow a Brownian motion [30]. This consequence stems from the assumption of independence for price fluctuations from prior observations. However, for many financial time series, there is a non-zero autocorrelation in the absolute

value of the asset prices for long lags. These attributes of the empirical data will affect the overall distribution. Unlike Brownian motion, for which the discrete distribution tends toward normality in the limit, this will likely not be the case with financial data. Here we modify Brownian motion; by including dependence in transition probabilities, we obtain the required volatility clustering[1] as observed in empirical financial data.

$$p\left(y_{t+1}\middle| \left(\left|\sum_{j=1}^{t} c_j x_j\right| \le B_L(t)\right) \text{ or } \left(\left|\sum_{j=1}^{t} c_j x_j\right| \ge B_U(t)\right)\right) = \frac{1}{2} + \epsilon_1 \qquad (5.1)$$

$$p\left(y_{t+1}\middle| \left(\left|\sum_{j=1}^{t} c_j x_j\right| > B_L(t)\right) \text{ and } \left(\left|\sum_{j=1}^{t} c_j x_j\right| < B_U(t)\right)\right) = \frac{1}{2} - \epsilon_2 \qquad (5.2)$$

$$p(y_t|t = 0) = \frac{1}{2} \qquad (5.3)$$

Let $y_t \in \mathbb{R}^+$ represent the log-adjusted price at time t; $x_j \in \{-1, 1\}$ represents the direction in which the asset price is moving, where 1 indicates an increase in price and -1 indicates a decrease in price; and $c_j \in \mathbb{R}$ represent the relative weight of an observation. Furthermore, $B_L(t)$ and $B_U(t)$ respectively represent the lower and upper bound conditions at a given value t, whereas $\epsilon_1, \epsilon_2 < \frac{1}{2}$ represent the respective probability increase or decrease for a particular condition. Furthermore, (5.1) is utilized when either condition in the expression is true while (5.2) is utilized when both conditions are true.

This configuration enables two conditions which Brownian motion does not. The first condition in (5.1) enables an increase in probability in the center and tails of the distribution. This induces episodes of relative calm (absolute value of summations are smaller than $B_L(t)$) to have a higher probability of remaining calm while it also yields episodes of volatility (absolute value of summations are are larger than $B_U(t)$), which results in volatility clustering.

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[1]The posterior probability of high volatility episode is higher with a prior high volatility episode and vice versa.

The second condition in (5.2) diminishes the intermediate probabilities between these two extremes. The third condition in (5.3) represents the initial condition setup, as is the case with Brownian motion.

The probabilities of the individual trajectories are depicted by Figure 5 while their aggregate effect that these conditions have on the overall distribution is illustrated in Figure 6 wherein the black curve represents the Normal Distribution and the gray curve represents a simulated example of equations (5.1) and (5.2). As expected, the probability distribution in the center and the tails are larger than that of the normal distribution, and the mid sections have a lower probability than that of the Normal Distribution.
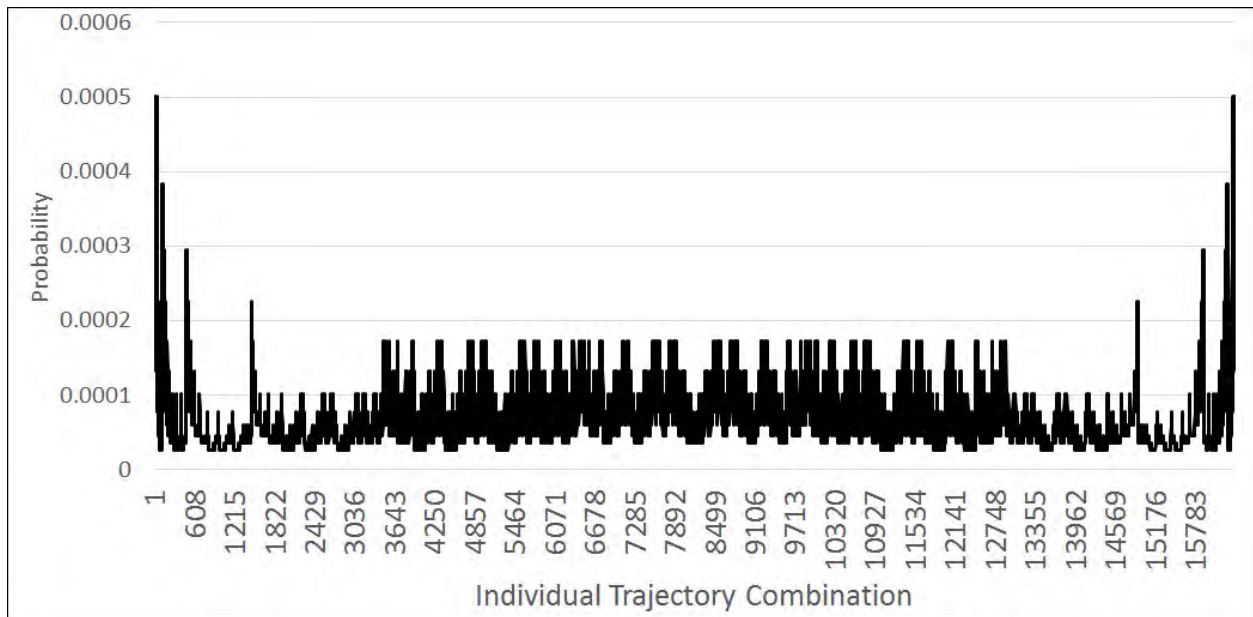
Figure 5. Probabilities of the individual trajectories. We observe that the highest probabilities are focused in the center and in the tails as expected by the formulation.
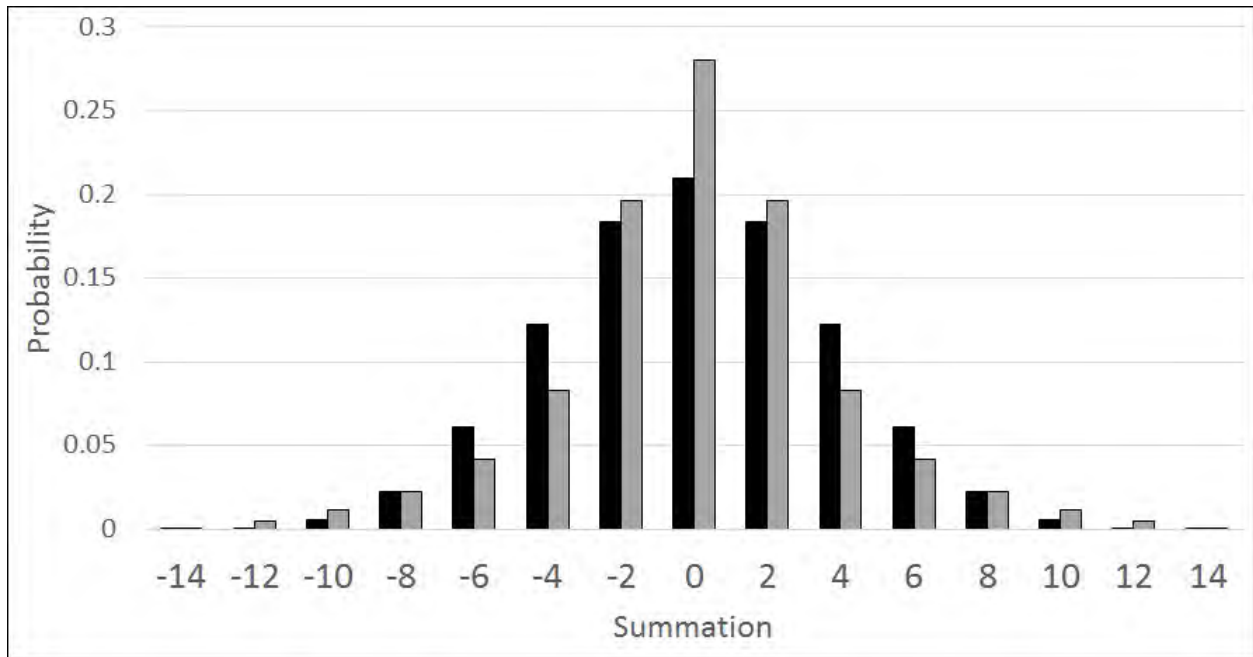
Figure 6. Comparison of the trajectories following Brownian motion (black) which resembles the Normal Distribution and distribution according to the aforementioned formulation (gray).

Furthermore, since observations are no longer independent, the variance calculations for the aforementioned approach must incorporate the covariance term as well. For the simulated example in Figure 6, the variance increases at a quadratic rate as t increases compared to a linear rate for the Normal Distribution. This is illustrated in Figure 7. This suggests that, not only can this model have a higher variance than the Normal Distribution, but it also can diverge over time. This further suggests that if financial conditions do behave as this configuration portends that volatility would increase much faster over time than the Normal Distribution would predict, and that Black Swan-type events would happen much more frequently than accounted for by the normality assumption.
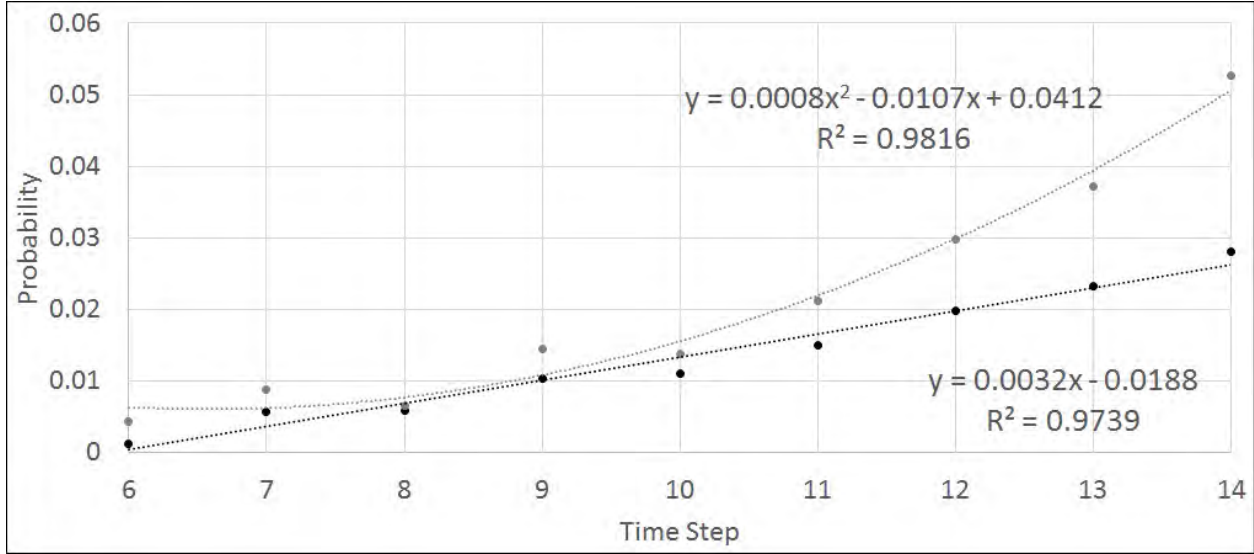
Figure 7. Depiction of the variance of the Brownian motion trajectories (solid black) and the proposed formulation (dashed gray) for the first 12 process iterations.

We do know through empirical studies by Fama [18], Johnson [30], Mandelbrot [40], and Mantegna [42] that volatility clustering and dependence of asset pricing exists. Many financial systems are interconnected and develop pricing strategies based on historical data. By utilizing historical data into future strategies, the data no longer remains independent. This results in the development of a long-term memory in financial dynamics. In addition, many of these assets are interdependent and therefore their prices may also be dependent on each other, providing an added level of complexity and potential volatility. In order to limit the exposure to these factors, we must study how they may affect an asset price.

5.3 Accounting for Extreme Events in Portfolio Selection

In modern portfolio selection, the assumption is that the log-adjusted price values are normally distributed and independent [40, 45]. From the previous section we observed that normality and independence are unlikely. This may present difficulties if the true distribution then is leptokurtic and large levels of autocorrelation are present. If leptokurtosis is present then extreme events are more likely than normality would predict. Furthermore, a non-zero

level of autocorrelation would increase the possibility that subsequent extreme events may occur. The objective here is to determine what the relationship may be between autocorrelation, and extreme event probability, and how to modify portfolio selection to account for this risk. By analyzing the parameters of the Johnson S_U distribution and autocorrelation with the probability of outliers, only autocorrelation seems to have a coherent effect. This suggests that, although leptokurtosis may affect the severity of the outlier, the number of outliers is primarily dependent on autocorrelation. Figure 8 depicts the hypothesized relationship between the square root of autocorrelation and probability of outliers for a particular large capital stock. An outlier is defined here as a value beyond three standard deviations for the Normal distribution. Since normality is assumed in standard finance theory, these values are compared with respect to that assumption. The independent axis represent the square root of the large cap autocorrelation while the dependent axis represents the probability of an extreme event from the empirical data. The green line depicts the best fine linear regression with an $R^2 \approx 0.740$ and RMSE < 0.01.

Those industries with the largest outlier probabilities are Financials (specifically REITs[2], Consumer Finance, Diversified Financial Services, and Multi-line Insurance), Reports (REITs), and Information Technology (Internet & Software Services, Publishing, and Data Processing & Outsourced Services). Those with the smallest outlier probability are Financials (Multi-sector Holdings), Consumer Staples (Distillers & Vintners, Biotechnology, and Household Products), Industrials (Railroads), and Health Care (Health Care Equipment & Services). Therefore, there seems to be a preponderance of evidence to suggest that the field and, specifically in accordance with Global Industry Classification Standard (GICS), subindustry behave similarly in their autocorrelations.

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[2]Real Estate Investment Trust is a security that behaves like a stock and is traded on major exchanges.
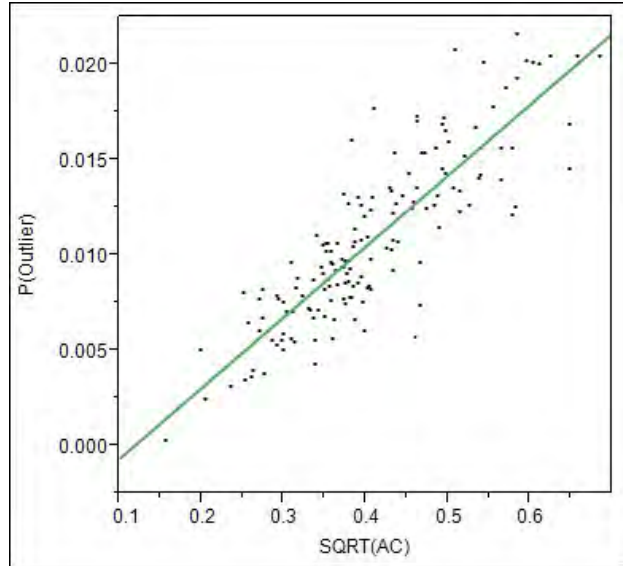
Figure 8. Scatter diagram of autocorrelation of log-adjusted market data for 221 large cap stocks (S&P 500) versus the probability of an outlier from 1 Jan 1990 to 25 Jan 2015. Data: Yahoo Finance.

Another factor to consider is when paired correlations of log-adjusted absolute values (LAAVs) are high. If we assume that a security has a large autocorrelation and its LAAVs are highly correlated with another security, then this security will also have volatility clusters which are synchronized with the volatility clusters of the first security. For portfolios that are heavily invested in such securities, there is a higher probability that when these volatility clusters do happen they will cause severe problems for a larger part of the portfolio. A mitigation strategy is to asynchronize these episodes when developing a viable portfolio. Figure 9 provides an abstraction of autocorrelation for log-adjusted market data to depict episodes of volatility clustering with synchronized episodes versus asynchronous episodes. Modern portfolio theory minimizes risk between different assets by incorporating the correlation of these assets with one another. Similarly the objective here is to minimize correlation, or synchronization, of volatility episodes with each other. This approach then minimizes the extreme shocks to which a portfolio may be subjected.

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This approach would minimize the probability of extreme events from multiple securities happening in parallel. This provides additional incentive to diversify from identical or similar GICS sub industries to increase resiliency against synchronous volatility episodes of a portfolio. Analysis of paired correlations of LAAVs for selected securities[3] indicate that industries in the same or similar GICS subindustries are higher than those which are not.
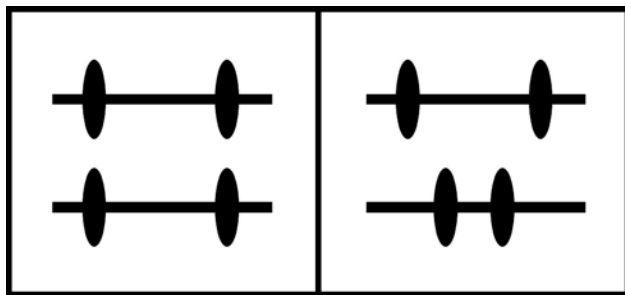
Figure 9. Abstraction of autocorrelation for log-adjusted market data to depict episodes of volatility clustering with synchronized episodes versus asynchronous episodes.

The level of paired correlation of LAAVs can be approximated by the correlation of their log-adjusted returns. Figure 10 illustrates a scatter diagram of paired correlations of the log-adjusted returns (ρ_1) versus the paired correlation of the LAAVs (ρ_2) for the 41 securities under consideration. The data was gathered from 2 Jan 1990 to 24 Jan 2015. Lowest levels of LAAVs would exist around $\rho_1 \approx 0.05$. This suggests that optimal asynchronization is near a zero correlation but not at zero. This analysis implies that an optimal pairing to minimize volatility episodes lies around a value of 0.05 and not exactly zero as one would expect. This suggests that a complete asynchronization (a correlation of zero) is not optimal to mitigate synchronized volatility clustering, and a weak correlation at the previously discussed value may provide a better alternative.

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[3]This data was chosen to provide sufficient data to evaluate extreme event behavior in each security. However not every security of the S&P 500 existed at this time. Therefore evaluation of these traits for younger large cap securities, although important, is the tradeoff until further data can be collected and analyzed. The securities under consideration then for this evaluation are AA, AAPL, ABT, ADBE, ADI, ADM, ADP, ADSK, AEP, AET, AFL, AGN, AIG, ALTR, AMAT, AME, AMGN, AON, APA, APC, APD, ARG, AVP, AVY, AXP, BA, BAC, BAX, BBY, BCR, BDX, BEN, BF.B, BK, BMY, C, CA, CAH, CAT, CB, and CCL.
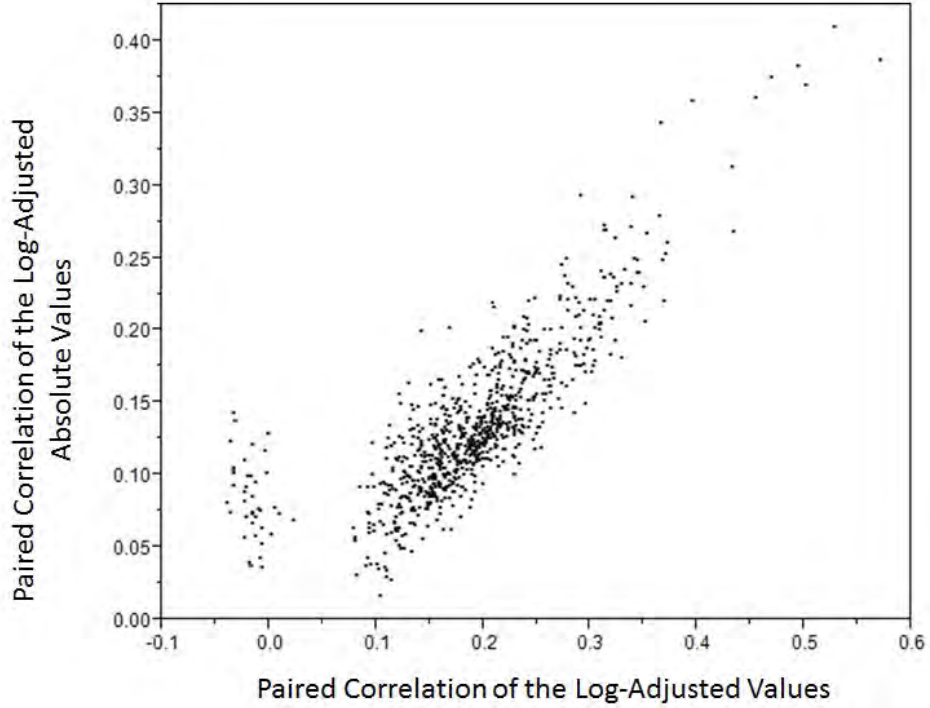
Figure 10. Scatter diagram of paired correlations of the log-adjusted returns (ρ_1) versus the paired correlation of the LAAVs (ρ_2) for the 41 securities under consideration.

To utilize these insights, it is necessary to modify a portfolio strategy. The mathematical program in (5.4) provides the necessary extensions to the classical approach. Let $\boldsymbol{x}$ represent the vector of securities, $\boldsymbol{y}$ the vector of the square root of the autocorrelations, and $\boldsymbol{\rho}$ the vector paired correlations of securities under consideration for a portfolio respectively. Furthermore, let $\boldsymbol{\Sigma}^*$ represents the modified covariance matrix in which values are based on the Johnson S_U variances and not those of the Normal distribution, and $\lambda \in [0, 1]$, $\eta \in [0, 1]$, and $q \in \mathbb{R}_0^+$ represent the tolerance based on extreme events, risk tolerance based on outliers over volatility clustering, risk tolerance due to variance, respectively. In addition, let $\delta \in \mathbb{R}$ represent the shift factor necessary to minimize LAAV synchronization as illustrated in Figure 10, and let $\boldsymbol{R}$ represent the vector of returns for this vector of securities. Finally, $\boldsymbol{1}$ and L represent the unity matrix and the lower triangular matrix operator, respectively.

$$\min \ \mathbf{x}^{\mathrm{T}} \left((1-\lambda) \mathbf{\Sigma}^* \mathbf{x} + \lambda((1-\eta)\boldsymbol{y} + \eta \mathrm{Tr}\left[\boldsymbol{x} L (\boldsymbol{\rho} - \delta \mathbf{1})^T (\boldsymbol{\rho} - \delta \mathbf{1}) \right] \right) - q \mathbf{R}^{\mathrm{T}} \mathbf{x}$$

$$\text{s.t. } \mathbf{1}^{\mathrm{T}} \mathbf{x} = 1, \tag{5.4}$$

$$\mathbf{x} \geq \mathbf{0}.$$

The mathematical program in (5.4) provides the modifications for the concerns previously discussed. The $\mathbf{\Sigma}^* \boldsymbol{x}$ minimizes the variance of the asset price distribution. Since it utilized the Johnson S_U distribution, its characterization of the variance of the portfolio securities provides a tighter statistically significant fit than under the normality assumption. Since the deviation between the variance under the normality assumption and the Johnson S_U distribution is not proportional from the securities considered, no simple correction term is likely possible and thus calculation is required per security. Utilizing the insight from Figure 8, the term $\boldsymbol{y}$ is appended. This term adjusts risk in accordance with the probability of outliers due to autocorrelation. The analysis suggests that a higher autocorrelation is strongly tied to the probability of outliers of a security. Thus to minimize exposure to these outliers, this term needs be minimized as well. The term $\mathrm{Tr}\left[\boldsymbol{x} L (\boldsymbol{\rho} - \delta \mathbf{1})^T (\boldsymbol{\rho} - \delta \mathbf{1}) \right]$ provides a penalty for volatility cluster synchronization. By utilizing the insight from Figure 10 which suggests that this synchronization is related to the correlation of the log-adjusted returns, we may minimize synchronization by minimizing the correlations relative to a specific adjustment, which seems to be around $\delta \approx 0.05$. These terms thus far are weighted relative to one another by utilizing λ and η. The variable λ provides an investor the option to weight standard variance techniques with the suggested new approach while the variable η weights the importance of outlier probability minimization to asynchronization of volatility clusters of a portfolio. Finally, the term $q \mathbf{R}^{\mathrm{T}} \mathbf{x}$ provides the impact of portfolio return with respect to the uncertainty discussed. The variable q provides the weight for its influence on the model.

This approach does not suggest that all amendments provide a higher return over the traditional model. Rather, these amendments may provide a more accurate representation of uncertainty of the asset prices. Each of these amendments is likely to change the composition of a portfolio since each amendment does address a particular observation in the empirical data. Thus, when addressing the advantage of each amendment in this study, it is important to note that the primary objective is to limit uncertainty to extreme events while returns are a secondary objective. To illustrate the incremental advantages of the mathematical program in (5.4), we choose three of the aforementioned securities: Abbot Laboratories (ABT), Analog Devices, Inc. (ADI), and Bank of America Corp (BAC). First we test the portfolio composition utilizing the normality assumption versus the Johnson S_U distribution only. Table 3 provides the necessary values to calculate the composition for each. In addition, we assume for now that $q = 0$. Therefore portfolio composition is currently strictly based on the variability of the securities.

Table 3. Necessary values to calculate the first two amendments to the traditional portfolio mathematical program.

Security	R	$x^T\Sigma x$	$x^T\Sigma^* x$	y
ABT	0.528	0.011	0.008	0.261
ADI	5.010	0.014	0.014	0.462
BAC	0.353	0.014	0.012	0.578

Utilizing the normality assumption, the standard deviation for the portfolio is approximately 0.008, while the standard deviation for the portfolio with Johnson S_U distribution is approximately 0.007. This value does not differ by much, but the compositions do. Figure 11 illustrates each portfolio's composition. The composition of ABT, ADI, and BAC are approximately 0.501, 0.253, and 0.246 respectively under the normality assumption. The composition of ABT, ADI, and BAC are approximately 0.630, 0.158, and 0.212 respectively. The returns are 1.620 and 1.201, respectively under the Johnson S_U assumption. As suggested, the normality assumption misrepresents uncertainty of these securities and suggests

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a higher return than the Johnson S_U distribution would suggest. This miscalculation of uncertainty could supply vulnerability to those portfolios which use the normality assumption.
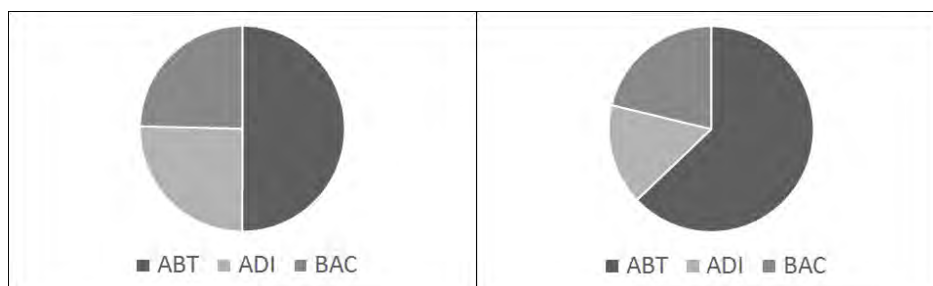
Figure 11. Portfolio composition under different population distributions. Left: Under the normality assumption. Right: Under the Johnson S_U assumption.

By including the second amendment ($\eta = 0$), we notice that the standard deviation and autocorrelation for ABT dominate ADI and BAC. This would suggest that, for most values of λ, the example portfolio ABT has a monopoly ($w_{ABT} = 1$) of the composition. This is indeed the case until $\lambda > 0.96$. As λ increases, so too does the weight of both ADI and BAC while the weight of ABT decreases. This is illustrated in Figure 12. When $\lambda \leq 0.96$ the portfolio is dominated by ABT. This is due to the fact that this security's standard deviation is the smallest as well as its square root of its autocorrelation. Right: As $\lambda > 0.96$ the paired correlation of the securities play an increasing role. This adjusts the portfolio composition. When $\lambda = 1$, the solution is identical as with the first amendment. In either case, the variance of the math program with just the first or with the first and second amendment is still lower than that portfolio assuming normality.
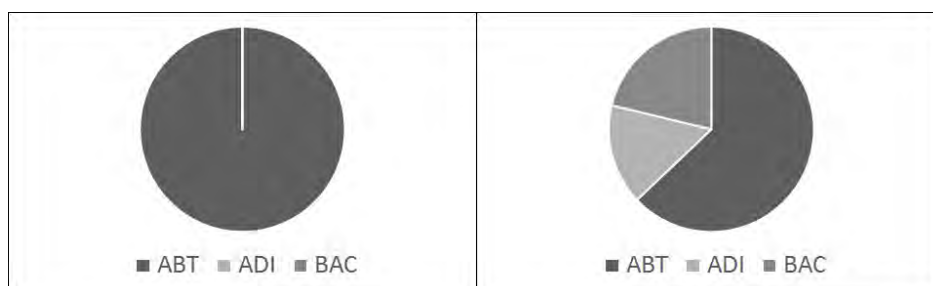
Figure 12. Portfolio composition utilizing the second amendment.

The third amendment focuses on volatility cluster asynchronization. To minimize synchronization, we account for the necessary adjustment in LAAVs discussed in Figure 10. Table 4 provides a visualization of paired correlations of autocorrelations of LAAVs for different securities. Incorporating securities with large correlation values can be particularly dangerous since their volatilities are synchronized. The expectant result may be catastrophic for a portfolio that utilizes these security combinations without properly estimating and considering the risk. Although the example portfolios are relatively small, they do indicate how the weighting in η and λ affect the composition for the securities under consideration.

5.4 Exploiting Pecuniary Vulnerabilities

The previous section discussed how an individual investor, company, hedge fund, or nation could hedge against extreme risk by modifying current practices. It did not address the origin of this variability. Furthermore, it did not address how to exploit this uncertainty nor where particularly critical vulnerabilities may lie. This section explores these questions and provides a meaningful approach toward the overall examination of reserve currency manipulation.

5.4.1 Securities.

As previously discussed, there is evidence to suggest that market capitalization influences the affluence of a reserve currency. Market capitalization is dependent on the individual firms that drive this capitalization. Therefore a vulnerability to market capitalization is a secondary vulnerability to reserve currency status. Thus, seeking and exploiting vulnerabilities in these markets is an alternative to a direct approach. Certain GICS sub industries are more vulnerable than others. By influencing these subindustries directly (through competition), or indirectly (through resource manipulation, trade warfare, price gouging, and so forth) these industries may be adversely affected more readily than others. Tailoring

offensive strategies based on GICS fields and GICS subindustries may provide an effective strategy for currency manipulation.
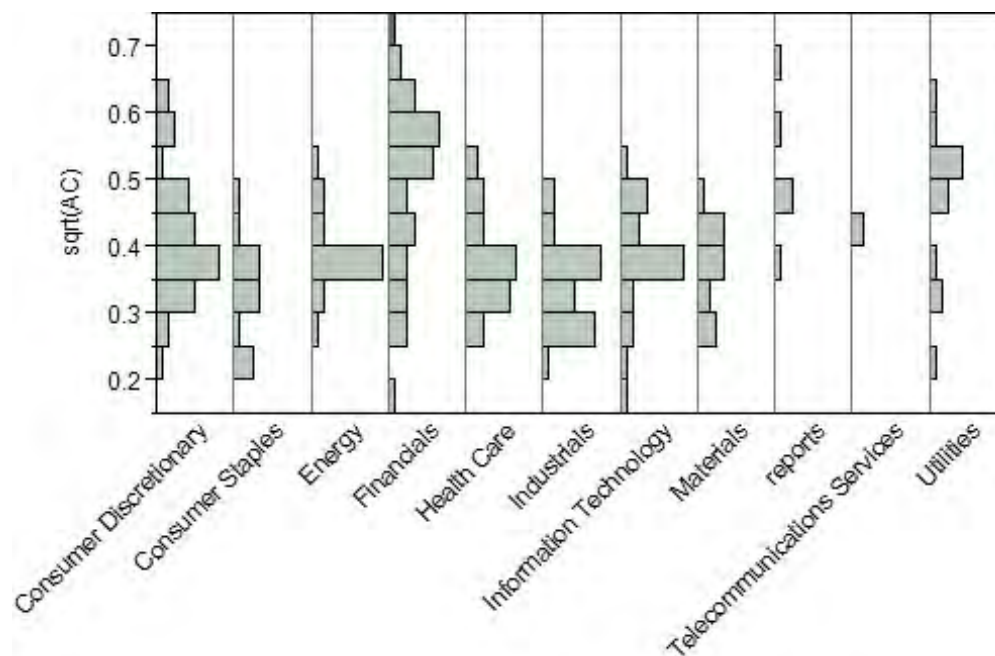
Figure 13. Categorical histograms of LAAVAs for 221 securities of the S&P500 by GICS field.

From Figure 13, we observe that fields as Consumer Discretionary, Financials, Reports, and Utilities have LAAVAs greater than 0.600. This suggests that these fields are more vulnerable to extreme events and volatility clustering than other fields. Table 4 provides the top 30 securities of the 221 examined with largest autocorrelations by Field and GICS subindustry (GICS SI) where F-Financials; R-Reports; CD-Consumer Discretionary; U-Utilities; IT-Information Technology; HC-Health Care; and E-Energy. Furthermore, the legend for GICS SI is: LP-Leisure Products; IC-Industrial Conglomerates; SS-Specialty Stores; EU-Electric Utilities; HCDS-Health Care Distributors & Services; DFS-Diversified Financial Services; BCTV-Broadcasting & Cable TV; B-Banks; MI-Multi-line Insurance; ISS-Internet Software & Services; CF-Consumer Finance; MUUP-Multi-Utility & Unregulated Power; IOG-Integrated Oil & Gas; HCES-Health Care Equipment & Services; AMCB-Asset Management & Custody Banks; and BT-Biotechnology. This suggests that Financials is the

most vulnerable field based on extreme event probability. The largest GICS subindustry represented is Real Estate Investment Trusts (REIT) (six present).

Table 4. Top 30 securities of the 221 examined with largest autocorrelations by Field and GICS subindustry (GICS SI).

Ticker	$\sqrt{AC}$	Field	GICS SI	Ticker	$\sqrt{AC}$	Field	GICS SI
HBAN	0.714	F	LP	FITB	0.582	F	ISS
AIV	0.684	F	REITs	DFS	0.581	F	CF
GGP	0.663	F	IC	BAC	0.578	F	B
AVB	0.657	R	REITs	EQR	0.578	F	REITs
HOT	0.649	CD	IC	CNP	0.575	U	MUUP
GNW	0.648	F	SS	CBS	0.570	CD	BCTV
BXP	0.625	F	REITs	DISCA	0.565	CD	BCTV
AEP	0.611	U	EU	AN	0.564	CD	SS
ESS	0.610	F	HCDS	HIG	0.555	F	IOG
AMP	0.602	F	DFS	AKAM	0.542	IT	ISS
DISCK	0.601	CD	BCTV	AEE	0.540	U	MUUP
HCN	0.599	R	B	A	0.539	HC	HCES
AIZ	0.595	F	MI	CHK	0.537	E	IOG
CCI	0.585	F	REITs	AMG	0.533	F	AMCB
AMT	0.584	F	REITs	BIIB	0.526	HC	BT

Since empirical data from each of these securities is not necessarily available, we revert to the 41 securities previously examined in the set of 221 securities previously discussed in Section 3.3 to test volatility clustering synchronization. There is a preponderance of evidence to suggest that volatility cluster synchronization is higher for those securities in the same or similar field, or GICS subindustry. Table 5 provides the paired correlations of autocorrelations for LAAVs for the 41 securities under consideration. Although not every subindustry is present, these results do suggest that this may be the norm for the general behavior of volatility clusters.

Table 5. Some of the top paired correlations of those securities under consideration.

Ticker	Field	GICS SI	Ticker	Field	GICS SI	ρ
ALTR	IT	SC	ADI	R	SC	0.4098
APC	E	OGEP	APA	E	OGEP	0.3867
BK	F	B	BAC	F	B	0.3827
BAC	F	B	AXP	F	CF	0.3749
AMAT	IT	SC	ADI	R	SC	0.3693
C	F	B	BAC	F	B	0.3602
BAC	F	B	AFL	F	LHI	0.3586
BMY	HC	HCDS	AMGN	HC	BT	0.3436
BK	F	B	AXP	F	CF	0.3127
BCR	HC	HCES	AVP	CS	PLP	0.2931
C	F	B	AIG	R	PCI	0.2916
AXP	F	CF	AFL	F	LHI	0.279
AVY	M	PP	AEP	U	EU	0.2726
ADI	R	SC	ADBE	IT	AS	0.2718
APD	M	IG	AEP	U	EU	0.269

The results from Tables 4 and 5 show that individual security autocorrelation, as well as pairwise correlated autocorrelations, can have a strong influence on the number of outliers a security or securities exhibit. These outliers provide a level of uncertainty for which standard finance theory does not account. It therefore follows that causing fluctuations in these sectors which exhibit a higher level of these attributes may be one avenue where unrestricted financial warfare operations may occur. This approach would not likely cause suspicion since these securities inherently manifest these vulnerabilities. Based on this analysis, an adversary would wish to focus their attention in manipulating securities in the Financials and Consumer Discretionary industries, particularly those with the highest levels of autocorrelation (i.e., those between 0.55 and 0.8). For Financials, these are specifically REITs. Large values of correlated securities are typically in the same subindustry. This implies that negatively influencing one of more of these securities may have cascading effects throughout the entire subindustry and those industries which are closely related. For REITS, this would include

other subindustries in the Financials as banks, diversified financial services, and multi-line insurance which would themselves possibly cause further propagation.

5.5   Summary

This chapter introduced an alternate formulation of financial behavior. This behavior deviated from the normality assumption and indicated that behavior with long-term memory may lead to a leptokurtic distribution of asset pricing. The resultant long-term memory incorporates non-zero levels of autocorrelation of the LAAVs which leads to volatility clustering and extreme events known as Black Swans. An additional concern was the synchronization of these Black Swan-type events. These are more likely to occur in industries and subindustries which produce similar products or provide similar services. These attributes are not accounted for in standard portfolio selection, and therefore this chapter attempted to develop a formulation that hedges against these types of factors. The result was a trade-off between return of a portfolio and resiliency of a portfolio (which were adjusted through the parameters λ and η) against these types of vulnerabilities.

VI. Early Warning Utilizing Currency and Commodity Market Metrics

6.1 Introduction

The idea of utilizing financial, commodity, and currency markets as a predictor of an impending attack by an adversary is not new. *A priori* information about a planned attack affords a financial market advantage over competitors, which in turn may be reflected in the purchase price or selling price of an asset. Hence, we use asset as a generic term to reference stocks, bonds, commodities, and currencies. As market service users[1] desire an advantage over their peers to garner larger profits, agencies committed to national security also desire an informational advantage to identify threats to national interests.

Having a predictive capability based on market conditions would provide the ability to quantitatively assess threats. This concept was tested by the CIA via Project Prophecy from 2002 to 2004. In Project Prophecy, market data was used to predict threats using metrics such as volatility, trade volume, put-call ratios, short interest, and momentum, and also to search for trading anomalies akin to the dynamics exhibited by insider trading [53]. The anomalies which were found could then be further analyzed and assessed by financial and intelligence analysts. Several studies on this matter have also been conducted by academic financial institutions. Poteshman's study [51], the Swiss Finance Institute study [9], and the 9/11 Commission [53] concluded that anomalous behavior within American Airlines, United Airlines, Boeing, Delta Air Lines, and Koninklijke Luchtvaart Maatschappij (KLM) were likely subject to informed trading activities prior to the terrorist attacks against the United States on 11 September 2001 that utilized four commercial passenger aircraft as weapons to destroy these targets.

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[1]Market service users are market users whose motivations typically fall within one of three categories: speculators, hedgers or arbitrageurs. They may vary in size, their time horizon, objective, and perception and handling of risk. [30]

This type of attack fits within the model of unrestricted operations. Unrestricted operations leverage those tactics for which non-repudiation is possible and the true objective is achieved through indirect attack that typically does not involve loss of life and ideally never results in open armed conflict [35]. The attacks on 11 September 2001 had at least three main effects: loss of life, loss of property, and the loss suffered in financial markets. With this single attack, the financial losses were estimated to be 16% of assets traded in the U.S. exchange markets, or nearly $640 billion [53]. Thus, with one series of attacks, 19 hijackers were able to carry out a highly asymmetric type of warfare and attain a substantial return on investment.

Recent restatement of this strategic expansion was developed by Colonels Qaio Liang and Wang Xiangsui of the People's Liberation Army in February 1999. In their book, they expand the notion of military operations to other domains which include financial warfare, cultural warfare, media warfare, network warfare, Lawfare (the use of laws to achieve an objective over a foe), resource warfare, ecological warfare, smuggling warfare, and terrorism [35, 38]. They coined the term unrestricted warfare to designate incorporation of these less traditional battlespaces.

In order to classify the general nature of warfare, it is necessary to expand the concept of combat operations to a broader framework that incorporates strategies and battlespaces not traditionally considered. To develop this, we revert back to Clausewitz's general observation that, within combat, there is always the "fog of war." This observation translates to the presence of uncertainty, when extended to other domains not typically associated with warfare. When the outcome for this uncertainty is strictly negative, then it is considered risk. Therefore we may define a battlespace as any facet of a force or society which may exhibit risk. In a true military sense this may seem trivial as any risk in combat operations, logistical support, communications and so forth may dictate the ultimate fate of a battle and may decide the outcome of a war. Beyond the traditional sense warfare starts

to become nebulous. Risk is ubiquitous in every facet of life, suggesting this definition may be too broad. Yet this generalization does allow for the assimilation of the advent of new operations that are not necessarily associated with classical military warfare.

Unrestricted warfare may be considered an evolutionary pressure of warfare to adapt to an increasingly interconnected world. As economies converge toward globalization, the return on investment of traditional warfare decreases. Traditional military approaches can have immediate economic consequences for participants and others through cascading effects. A further evolutionary pressure is the development of strategies to circumvent accepted advantages enjoyed by those countries having superior conventional military portfolios. There is no incentive to attack an adversary in the battlespace domain for which they are well organized, trained, and equipped to fight. Rather, the objective is to determine which tactics would make these advantages obsolete. Unrestricted warfare, then, adopts a strategy which fully embraces the philosophy of Sun Tzu: avoiding a foe where they are strong and exploiting them where they are weak, practicing dissimulation, and hiding beneath "a cloak of disorder" [64], thereby waging asymmetric warfare which neutralizes actual military advantages.

Although unrestricted warfare does provide some asymmetric advantages, it is no panacea. The implementation of these types of strategies within nontraditional battlespaces does not occur without effect. Every action has a reaction, but it may be obscured by the complexity of the battlespace through which it is actuated. The objective of this research is to develop tools that effectively identify anomalous behavior and differentiate such behavior occurring due to attacks from anomalies caused by environmental variations, with the purpose of identifying attacks and enabling repudiation. Herein, we illustrate one approach to develop such tools for the financial sector.

6.2 Unrestricted Warfare (UW) as an Instrument toward Fourth Generation Warfare

The adoption of UW enables the evolution of military operations. Fourth Generation Warfare (4GW) is a conflict which blurs the line between combatant and civilian by utilizing atypical battlespaces such as financial markets or the cyber domain, for example, increases independence on the individual combatant, diminishes the necessity for logistical support, and continues to increase its strategies based on maneuver warfare by applying ambiguous and highly non-reputable strategies[1, 36]. It includes operations which combine elements "of guerrilla tactics, terrorism, traditional warfare, and the ability to exploit and skip generations of technology to conduct operations, particularly to target the will and morale of the enemy's support structure, in order to achieve political victory" [1]. In this context, UW is not so much a new type of warfare in its own right but an instrument that facilitates further military evolution. The evolutionary pressures incurred by economic interdependence through globalization provide an incentive to move toward UW are exactly some of those mechanisms which enable such a transition. Hammes [25] and Artelli [1] provide the following tactical traits that are present in Fourth Generation Warfare:

- Exploitation of dissimulation through complex battlespaces via low-intensity conflict.

- Inclusion of lower generational tactics.

- Dissimulation of tactics utilizing cross-border strategies.

- Utilization and expansion beyond traditional battlespaces through political, economic, military networks.

Furthermore, the utilization of UW enables a foe to focus on targets which "include such things as the population's support for the war and the enemy's culture" [36]. These

Continuum of Conflict

State Conflict: Employs large-scale military force and sophisticated military technologies across multiple domains to defeat the enemy. May include use of WMD, anti-access/area denial systems, global strike systems, undersea platforms, advanced cyber tools, and counter-space systems, among other capabilities.

Hybrid Conflict: Blends conventional and irregular forces to create ambiguity, seize the initiative, and paralyze the adversary. May include use of both traditional military and asymmetric systems.

Non-State Conflict: Employs small units and networks to undermine governments and gain control over populations. May include use of IEDs, small arms, propaganda, and terror.

Unrestricted Warfare

CONSEQUENCE

PROBABILITY

Figure 14. Illustration state conflict, hybrid conflict, non-state conflict, and UW. Depending on the strategy employed, UW may traverse each of these battlespaces. Source: 2015 National Military Strategy Plan.

methods do engage the enemy directly but circumvent the strength of a foe and focus on those areas for which the foe is ill-equipped to address through military strategy. This link is established through the utilization of terrorism rather than traditional military conflict [1, 37], as well as exploitation of laws which govern traditional military operations (Lawfare) [35]. As this evolution further progresses, it is likely to cause a "demilitarization of warfare" [48] so that no clear target can be determined. Figure 14 illustrates the different types of conflict including UW.

Therefore it is of interest to anticipate the effective utilization of UW as instruments to wage 4GW. By focusing on these instruments and developing effective techniques and systems to monitor and assess their use, we may be able to develop equivalent techniques of ISR for the UW area.

6.3   Motivation

As Unrestricted Warfare becomes increasingly preferred by state and non-state actors due to its relative lack of requirements for resources, its influence on financial market behavior is likely to rise. This may not be merely to degrade market levels but to also exploit them for personal gain. The knowledge of *a priori* information provides an advantage over those with *posterori* information of an event. This may incentivize those who may benefit politically, tactically, or strategically from an attack to also benefit financially. Some research has already indicated that markets may possess predictive qualities for impending attacks [9, 51, 53]. However only selected and specific securities were analyzed for those studies. Herein, we seek to analyze metrics of combinations of currencies and commodities to illustrate that broader measures may exist which might provide an early warning capability for asymmetric attacks within UW operations such as terrorist attacks.

For many, gold is considered to be a traditional hedge against market risk. Therefore, in times of high volatility the price of gold will surge as demand for this commodity increases. The reverse is experienced in times of market stability and periods of lower volatility. Moreover, people are less likely to invest in another precious metal commodity such as silver during periods of high volatility if gold is available. However, gold and silver do behave similarly in pricing strategy in times of low volatility; the correlation of gold (XAU) and silver (XAG) prices in different currencies is very high. Their correlation values for the currency basket[2] under consideration are within $0.910 \leq \rho \leq 0.998$ overall. Thus of these two commodities,

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[2]For this study, the currency basket is limited to CAD, CHF, GBP, JPY, NOK, SEK, USD, and ZAR.

Table 6. Snapshot of the highly correlated nature of gold and silver in different currencies. Data: Yahoo Financial using Invest Excel.

CIC1	CIC2	ρ		CIC1	CIC2	ρ
XAGSEK	XAGNOK	0.998		XAGZAR	XAUZAR	0.977
XAGCAD	XAGUSD	0.997		XAGZAR	XAUUSD	0.975
XAGNOK	XAGCAD	0.997		XAGZAR	XAUCAD	0.973
XAUSEK	XAUNOK	0.997		XAGGBP	XAUUSD	0.973
XAGSEK	XAGCAD	0.996		XAGZAR	XAUNOK	0.972
XAUNOK	XAUCAD	0.996		XAGUSD	XAUUSD	0.970
XAUCAD	XAUUSD	0.995		XAGGBP	XAUGBP	0.970
XAGNOK	XAGGBP	0.995		XAGSEK	XAUUSD	0.970
XAUNOK	XAUGBP	0.995		XAGNOK	XAUUSD	0.969
XAGNOK	XAGUSD	0.995		XAGCAD	XAUUSD	0.968

regardless of currency, move similarly in times of low volatility. Table 6 lists Correlations between gold (XAU) and silver (XAG) daily prices for a currency basket from 1 Jan 1992 to 11 April 2014 for different Commodity In Currencies (CIC). The currency basket contains the U.S. dollar (USD), the Canadian dollar (CAD), the British pound (GBP), the Japanese yen (JPY), the South African rand (ZAR), the Norwegian krone (NOK), the Swiss franc (CHF), and Swedish krona (SEK). The left table indicates the highest paired correlations. These are same commodity with different currencies. The right table indicates the highest correlations for different commodities and different currencies. The lowest of all paired correlations is 0.910 which indicates a very high paired correlation regardless of pairing.

An exchange rate from one currency to another, *ceterus paribus*, allows a commodity to be purchased with the same purchasing power parity[3] (PPP). Therefore an ounce of gold in the U.S. should have the same purchasing power of the Euro, or any other currency, if exchange

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[3] "The theory that exchange rates between currencies are determined in the long run by the amount of goods and services that each can buy. In the absence of transport costs and tariffs, if the price of tradeable goods were lower in one country than another, traders may gain by buying goods in the country where they are cheaper and selling in the other one: relative price levels thus determine the equilibrium exchange rate. Not all goods are tradeable, and even for tradeables transport costs and tariffs mean that prices need not be equal but the same forces of arbitrage limit their differences, and thus limit deviations and exchange rates from PPP. An alternative form of PPP says that changes in equilibrium exchange rate are determined by changes in relative price levels." [5]

rates are efficient *ceterus paribus* as postulated under the Efficient Market Hypothesis[4]. This is not necessarily always the case and prices are exposed to minimal lags due to updating and variability in transaction costs; this variability can be observed when comparing the prices of various goods in different currencies with respect to the stated exchange rate of a commodity in different currencies. Therefore, the official exchange rate may be more akin to an average over a certain time frame rather than the instantaneous or perfectly consistent value. This would be considered random variation and should not exhibit extreme occurrences. If extreme events happen in a well-defined system, then arbitrage mechanisms would exist and markets would adjust to exploit these, which would theoretically counter the effect. If, however, an attack or operation were planned, information of this attack or operation may be known to insiders and others made aware of it through the interpersonal networks of those insiders. Since people respond to incentives [49], this information may then leak in the form of market pricing for different securities, commodities, and currencies. This effect is similar to insider trading; people with insider information may gain an unfair advantage by favorably positioning their assets. If those with insider information were to invest in gold, as many generally do to hedge against volatility, this is likely to cause a spike in its price differential with respect to silver. This differential may propagate differently through different currencies. If that is the case, it follows that this price differential would be reflected in a PPP differential spike between gold and silver. If the PPP differential possesses a lead period, then it may be possible to develop sensory measures and techniques to increase security, heighten alert levels, and increase intelligence assets to seek the impending threat.

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[4] "The theory that, where assets are traded in organized markets, prices take account of all available information, so that it is impossible to predict whether some assets will give better risk-adjusted returns than others. This cannot be predicted because it depends on news, that is, information which is not yet available, and cannot be deduced from information which is available. There are several variants of the Efficient Markets Hypothesis. Weak-form efficiency asserts that it is not possible to use historical share prices to construct a trading strategy that yields excess returns. Semi-strong efficiency states that excess returns cannot be earned on the basis of public information. Strong-form efficiency states that excess returns cannot be earned by trading on the basis of private information. Empirical evidence seems to support semi-strong efficiency as a description of markets." [5]

In general, these techniques fall into the category of prediction markets[5] [11, 51, 55] and utilize market behavior as a threat sensor.

Let c_1, c_2, and γ be the values of the respective individual currencies and the target commodity under consideration at time t. Then $P(\gamma, c_j, t)$ represents the price of commodity c_j in the currency γ at time t. Then (6.1) is the PPP for this parameter triplet.

$$PPP(\gamma, c_1, c_2, t) = \frac{P(\gamma, c_1, t)}{P(\gamma, c_2, t)} \tag{6.1}$$

The PPP differential, or $\varphi(t)$, for the currency pair (CP) is then expressed in (6.2). For the aforementioned example of XAU and XAG, we set γ_1 and γ_2 as XAU and XAG respectively.

$$\varphi(t) = PPP(\gamma_1, c_1, c_2, t) - PPP(\gamma_2, c_1, c_2, t) \tag{6.2}$$

The φ-values were examined using the Anderson-Darling statistic to test for normality for daily data from 1 Jan 1992 to 11 April 2014. Each of these tests considered the hypothesis in (6.3) and was conducted for each of the six CP combinations under consideration.

$$H_0 : \varphi \text{ data is normally distributed}$$
$$H_a : \varphi \text{ data is not normally distributed} \tag{6.3}$$

Their results are listed in Table 7. We observe that, for the CPs selected, all but USDGBP reject H_0. Several tools exist to transform data to normality. The typical approach is the Box-Cox transformation but, for the CP combinations tested in this study, this technique did not provide an adequate transformation since nearly half of all data sets were negative. Chou [10] proposed a solution to this problem by transforming non-normal data to normal

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[5]Artificial speculative markets devised to attempt to predict actual market behavior.

Table 7. Summary of normality tests of the selected CPs using the Anderson-Darling test. All CPs except USDGBP fail the normality assumption at the $\alpha = 0.01$ level of significance.

	AD Crit	AD	Reject?	μ	σ
USDCAD	3.907	10.704	yes	N/A	N/A
USDGBP	3.907	3.453	no	1.282E-5	2.0242E-08
USDJPY	3.907	6.843	yes	N/A	N/A
USDZAR	3.907	97.308	yes	N/A	N/A
USDNOK	3.907	61.284	yes	N/A	N/A
USDCHF	3.907	4.408	yes	N/A	N/A
USDSEK	3.907	89.067	yes	N/A	N/A

data via the Johnson system of distributions transformations[6]. This transformation will serve two purposes in the next pages: since many φ calculations are Johnson S_B or Johnson S_U-distributed, this will enable a process to obtain normality of the data through a transformation and since the data is distributed standard normal, we will be able to test how often outliers should occur for the measures that will be developed.

In an attempt to achieve normality, we will utilize these Johnson transformations. As the data under consideration ranges from $(-\infty, \infty)$, only the Johnson S_B and Johnson S_U transformations are viable; we dismiss the Johnson S_L, or Lognormal Distribution, since it does not fulfill the domain requirements. The necessary transformation equations for the Johnson S_B and Johnson S_U are provided in (6.4) and (6.5), respectively, wherein Φ^{-1} is the inverse cumulative distribution function, U is uniformly distributed random variable, and its four parameter values are γ, $\delta > 0$, θ, and $\sigma > 0$. For the rest of this study, we designate the feasible Johnson transformation to normality for a data set as $T(\varphi)$.

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[6]The Johnson Distributions are modifications of the Standard Normal Distribution. The family is defined by

$$Y = g\left(\frac{Z - \alpha}{\beta}\right)$$

where α and β are constants, Z is the Standard Normal Distribution, and Y is the Johnson random variable. There are three types of Johnson Distributions defined by S_U, S_B, and S_L. Their respective g functions are $\sinh x$, $\frac{1}{2}(1 + \tanh x)$, and e^x. [5]

$$\Phi^{-1}(U) = \gamma + \delta \sinh^{-1}\left(\frac{x - \theta}{\sigma}\right) \tag{6.4}$$

$$\Phi^{-1}(U) = \gamma + \delta \ln\left(\frac{x - \theta}{\theta + \sigma - x}\right) \tag{6.5}$$

Table 8 presents the results of each transformation of the six remaining CPs. Five of the six CPs fail to reject normality following one of the two Johnson transformations for the Anderson-Darling test at the $\alpha = 0.01$ level of significance. The critical values for each data series was 3.907. The CP USDCAD fails normality for both transformations. Although this CP does not pass the normality assumption for statistical control, its kurtosis post the S_B transformation is nearly zero. This would imply that few outliers past three standard deviations should exist. This would imply that a statistical control on this data set then would be more conservative than if it did pass the normality assumption. Each data set, following either transformation results in a standard deviation of 1.

We observe that, for nearly all data sets, a transformation to normality exists that is statistically significant at the $\alpha = 0.01$ level. This need not always be the case. The CP USDCAD fails the normality assumption after both transformations. However, each data set attains a standard deviation of 1 after transformation and, with that transformation which provides the closest data set to normality, yields a negative excess kurtosis[7]. This suggests that, although some data sets may fail the normality assumption, we may still be able to use them by excluding the normality criteria in another context. The excess kurtosis, given the proper transformation is negative, suggests that outliers beyond three standard deviations should occur with a lesser probability than with a Normal Distribution. Therefore, utilizing

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[7]A readjusted calculation of kurtosis to provide a value of 0 for the Normal Distribution. Kurtosis is a measure of the level of "fatness" of the tails of a distribution. The higher the kurtosis (or excess kurtosis) the larger the probability in a distribution's tails which is a higher likelihood of extreme values or outliers.

Table 8. Normality testing of resultant data sets after Johnson S_B and S_U transformations for the remaining six CPs. Data: Yahoo Financial through Invest Excel. Analysis: EasyFit, Minitab, JMP, Excel.

	T	Johnson Distribution Parameters				AD	Reject?	Kurt
		γ	δ	θ	σ			
USDCAD	SB	-0.202707	2.3876929	-0.00001589	0.0000306	5.4623	yes	-0.016
USDCAD	SU	438.0062	54.313107	0.000167	1.0501E-07	10.625	yes	-0.362
USDJPY	SB	1.0408	3.8442	-0.000000292	6.8251E-07	3.486	no	-0.025
USDJPY	SU	-276.3	23.605	-0.000001	1.67E-11	4.4989	yes	-0.229
USDZAR	SB	-516.846	66.345008	-0.077325	0.0773574	97.157	yes	3.261
USDZAR	SU	-0.00734	1.1429741	8.8926E-09	3.7538E-07	0.99363	no	-0.143
USDNOK	SB	-677.9236	104.43462	-0.021404	0.0214369	61.105	yes	2.392
USDNOK	SU	0.0130667	1.3584577	5.326E-09	3.1892E-07	1.1312	no	-0.059
USDCHF	SB	0.064888	2.871144	-0.000018	0.000036363	1.853	no	-0.022
USDCHF	SU	-632.7917	85.816322	-0.000264	3.3146E-07	4.1887	yes	-0.292
USDSEK	SB	1367.4224	96.681601	-0.00002863	39.738246	1.0877	yes	2.556
USDSEK	SU	-0.045441	1.1957795	-1.291E-08	2.4927E-07	88.627	no	-0.127

results of this form would provided a more conservative estimate than if the transformation had passed the normality assumption.

Statistical process controls can be leveraged to detect outliers from the standardized population. The intent here is to determine singular outliers. When these singular outliers are then mapped to their corresponding dates, the ensuing objective would be to find contributing events for these outliers in PPP disparity. The Lower Control Limit (LCL) and Upper Control Limit (UCL) provide boundaries for which the process should deliver without fundamental changes [65]. Furthermore, normality is not a necessary assumption for these control charts, which is an added benefit for their use [66, 67]. The reason for applying the Johnson transformation is therefore not to utilize statistical control but to test the behavior of these CPs and future measures in this study on a standard scale. Since we are interested in individual observations (i.e. the sample size is 1) and these values are continuous data, the I-Chart is the appropriate choice [47].

From Figure 15 and Table 9 we observe the aggregated data sets for those CPs which passed the normality test alone (e.g., USDGBP) and those which pass the normality test

after the appropriate Johnson transformation. If we continue to assume that outliers are those points which extend beyond $\pm$ 3 standard deviations, then those points of interest are those above or below the respective UCL and LCL. Several outliers are visible. Those of interest are those for which values are below -3 standard deviations. These occurrences indicate that the prices of gold are much tighter than those of silver, which happens in times of high volatility. Several examples are listed: A) Srinagar car bomb, B) USS Cole Bombing, and C) 11 September Attacks. Not every φ-outlier can be clearly mapped to a large event.

The intent of the φ-measure, we are specifically interested in those outliers which are negative. As volatility increases, there will be a higher increase in demand for gold than for silver. As this commodity is traded at increasing volume, the disparity in the PPP between currencies decreases. Therefore we expect that φ-values would turn negative. In extreme situations, the value of φ should decrease considerably and do so to statistically significant levels. Of interest are those dates which these levels drop below -3 standard deviations, the LCL. One further observation is that many outliers tend to be very near each other in pockets of volatility, leading to volatility clustering[8]. This may suggest that these outliers are due to some event and not simply due to chance. Some of the largest outliers occurred on 12 October 2000 (USS Cole Bombing) and 7 September 2001 (days before the 9/11 Attacks). Although these dates surpass the lower control limit (LCL), we observe an increase in volatility leading up to these dates. Investigating this volatility leading up to outliers is the next focus.

The next measure investigates the hypothesis that volatility clusters begin before these outliers occur. As some of these outliers may be mapped to days on which terrorist attacks occurred, conflicts escalated, and other world events took place, such behavior may indicate market service users acting on *a priori* information before the rest of the market reacts on

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[8]Episodes of high volatility are followed by similar episodes of high volatility while episodes of low volatility are followed by similar episodes of low volatility [41]
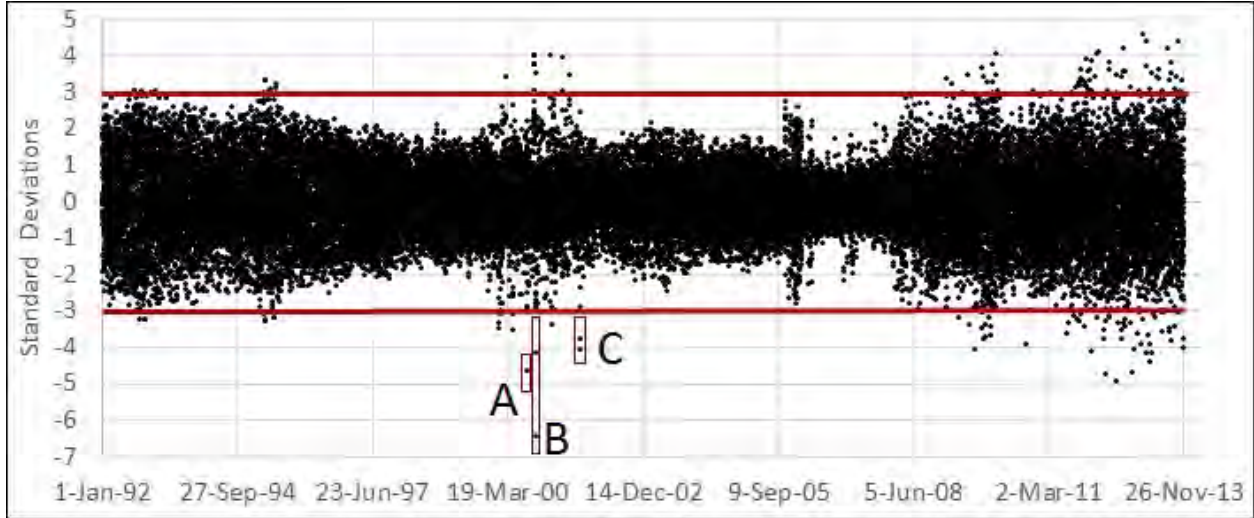
Figure 15. Statistical control chart for the six CPs for which normalization is possible.

posterori information. If this were not the case, then values before an outlier should not necessarily increase in volatility until an event has occurred. After an event has occurred, volatility is commonplace as market service users increase trading to hedge against possible losses caused by either the attack or other market service users' trading. Therefore we wish to develop a measure which would indicate a rise in volatility before an event. A rise in volatility would be reflected in increased values of the excess kurtosis of $T(\varphi)$. To handle non-stationarity and thus enable the use of control charts, we seek a measure that examines time series data. An obvious candidate would be to measure the rate of change of the running excess kurtosis for a sample data set of $T(\varphi)$.

The rate of change for the running excess kurtosis of $T(\varphi)$ is calculated according to (6.6) where w represents the warning period availability and κ is the excess kurtosis for the φ data set from $(t-1-\Delta)$ time units to $(t-1)$ time units compared to the excess kurtosis values from $(t-1-w-\Delta)$ time units to $(t-1-w)$ time units. For this study, we consider $\Delta = 150$ and $w = 5$ trading days. The Δ-parameter value was chosen to provide Δ a sufficiently large sample to detect outlier values while w was chosen to reflect one trading week. These values may change as they were selected for demonstration purposes.

Table 9. Outliers below the LCL for the six CPs categorized by intervals.

Value of φ-Measure			
$-7 \leq \varphi < -6$	$-6 \leq \varphi < -5$	$-5 \leq \varphi < -4$	$-4 \leq \varphi < -3$
12-Oct-00	7-Sep-01	10-Aug-00	1-Oct-92
	7-Mar-14	12-Oct-00	14-Nov-92
		2-Sep-09	15-Nov-92
		6-Jan-12	2-Nov-92
		23-Apr-12	22-Feb-93
		19-Jul-12	30-Mar-95
		10-Nov-12	13-Apr-95
		19-Nov-12	18-Apr-95
		19-Feb-13	23-Jun-95
		18-Mar-13	24-Jun-95
		26-Mar-13	26-Jun-95
		1-Jan-14	6-Jan-00
		24-Mar-14	24-Apr-00
			7-Sep-01
			16-Jul-09
			17-Oct-09
			20-Oct-09
			23-Jul-12
			19-Jun-13
			20-Jun-13
			17-Oct-13
			7-Jan-14
			15-Jan-14
			24-Jan-14
			26-Mar-14

$$K(T(\varphi), t) = \frac{1}{w} \left(\kappa(T(\varphi), t - 1 - \Delta, t - 1) - \kappa(T(\varphi), t - 1 - w - \Delta, t - 1 - w) \right) \quad (6.6)$$

Figure 16 provides an illustration of these values from 3 June 1992 through 11 April 2014. There are several episodes of relatively extreme increases in excess kurtosis, with many exhibiting similar behavior at certain clusters of dates. This would suggest that volatility
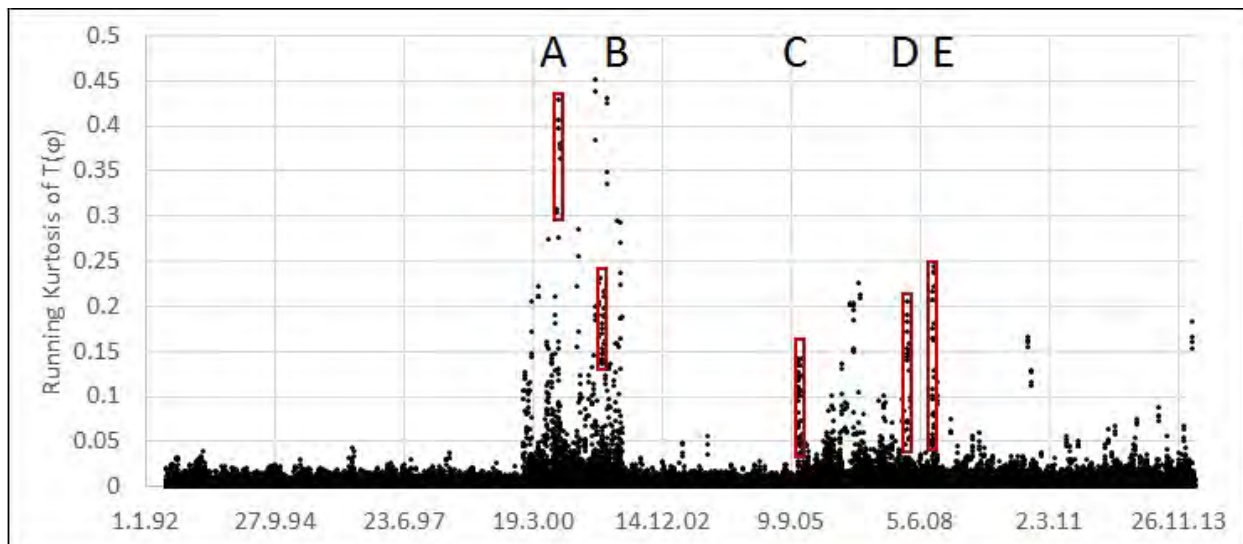
Figure 16. Hypothesized relationship between a subset of world events and relatively extreme successive rates of excess running kurtosis of $T(\varphi)$ utilizing all CP results. A: USS Cole Bombing (12 October 2000), B: September 11 Attacks, Amman Bombings (5 November 2005), D: Subprime Mortgage Collapse (February 2008) and E: Global Financial Crisis (September 2008).

around these dates increased very rapidly, leading to the outliers previously observed in $T(\varphi)$.

To ensure that these results are not coincidental based on the procedure developed, we conduct a test utilizing a simulation wherein a standard normal variates were generated. If the probability of extreme values for the K-measure adheres to the same distribution as the K-measures from market data, then there is a likelihood that the K-Measure is the result of random processes. If, however, the distribution of the simulated K-measure differs significantly from that of actual market data, then it can be assumed that these results cannot be attributed to coincidence. A simulation of this was run and is depicted in Figure 17. It generated 60,000 positive K-measure values from a 100,000 standard normally distributed data set utilizing the same values of Δ and w. We notice that the majority of values are below 0.1 and progressively becomes more and more rare at larger values. Furthermore, clustering only includes very few values. Additionally there are no distinct episodes with the simulation as there are with φ.

These values, as well as those of the six combined CP data sets, were compared to a pool of 47 probability distributions[9]. Each of the two data sets were then tested for their goodness-of-fit utilizing the Anderson-Darling test statistic. Based on their fitness, each received a rank for the 47 probability distributions. Each data set was assigned the distribution for which it received an ordinal rank of 1 (the best of those in the pool) if the distribution at rank 1 failed to reject the hypothesis test for that data set at the $\alpha = 0.01$ level of significance. Furthermore, both data sets were compared to the rank 1 distribution of the other using the same hypothesis test and α level. This procedure was applied to determine whether they may fit each other's distribution. Table 10 provides a summary for distribution fitting for CP data and simulation data. We observe that neither distribution chosen for one passes the hypothesis test for the other. Therefore, we may assume that each data set is different at the α-level of significance. Again, the Anderson-Darling test statistic was utilized to test for goodness-of-fit. The "D" and "W" correspond to those instances where Dagum (4P) Distribution and Wakeby Distribution are utilized respectively. The critical value for the Anderson-Darling Test for each scenario was 3.907 at the $\alpha = 0.01$ level of significance. Furthermore, the analysis shows that the 24,110 market values of the six CPs are Wakeby

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[9]The distribution pool is comprised of: Beta, Johnson S_B and S_U, Kumaraswamy, Pert, Power Function, Reciprocal, Triangular, Uniform, Cauchy, Error, Error Function, Gumbel Max, Gumbel Min, Hypersecant, Laplace, Normal, Student's t, Burr, χ^2, Dagum, Erlang, Exponential, Fatigue Life, Frechet, Gamma, General Gamma, Inverse Gaussian, Levy, Log-Gamma, Log-Logistic, Lognormal, Nakagami, Pareto, Pareto 2, Pearson 5, Pearson 6, Rayleigh, Rice, Weibull, General Extreme Value, General Logistic, General Pareto, Log-Pearson 3, Phased Bi-Exponential, Phased Bi-Weibull, and Wakeby distributions.

Table 10. Summary for distribution fitting for CP data and simulation data.

	Rank	k	α	β	γ	δ	ξ	AD	Reject?
CP (D)	3	5.30E-01	1.95E+00	8.35E-03	3.34E-07	N/A	N/A	11.2	yes
CP (W)	1	N/A	6.91E-03	1.21E+00	2.17E-03	6.87E-01	2.71E-05	1.97	no
Sim (D)	1	3.65E-01	2.70E+00	1.61E-02	2.83E-07	N/A	N/A	3.90	no
Sim (W)	19	N/A	1.15E-02	1.03E+00	3.45E-03	4.38E-01	1.91E-05	268	yes

distributed[10] whereas the simulation values are Dagum (4P) distributed[11]. Neither data set fits the other's distribution at the α-level of significance. Furthermore, no combination of acceptable distributions for one were acceptable for the other when observing values from the entire pool of distributions. This would also suggest that these two data sets have different properties. This would suggest that the K-measure values calculated from financial data are not coincidental and may be attributable to causal effects such as attacks and other significant world events.

---

[10]The Wakeby Distribution is defined by the quantile function

$$x(F) = \xi + \frac{\alpha}{\beta}\left(1 - (1-F)^\beta\right) - \frac{\gamma}{\delta}\left(1 - (1-F)^{-\delta}\right)$$

where

- $\alpha, \gamma \neq 0$
- $\beta + \delta > 0$ or $\beta = \gamma = \delta = 0$
- if $\alpha = 0$, then $\beta = 0$
- if $\gamma = 0$ then $\delta = 0$
- $\gamma \geq 0$
- $\alpha + \gamma \geq 0$

. The domain is

- $\xi \leq x < \infty$ if $\delta \geq 0$ and $\gamma > 0$
- $\xi \leq x \leq \xi + \frac{\alpha}{\beta} - \frac{\gamma}{\delta}$ if $\delta < 0$ or $\gamma = 0$

[11]The Dagum (4P) Distribution is defined as

$$f(x) = \frac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha k - 1}}{\beta \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha\right)^{k+1}}$$

where $k > 0$ and $\alpha > 0$ are continuous shape parameters, $\beta > 0$ is a continuous scale parameter, and γ is a continuous location parameter with domain $\gamma \leq x < \infty$ [39].
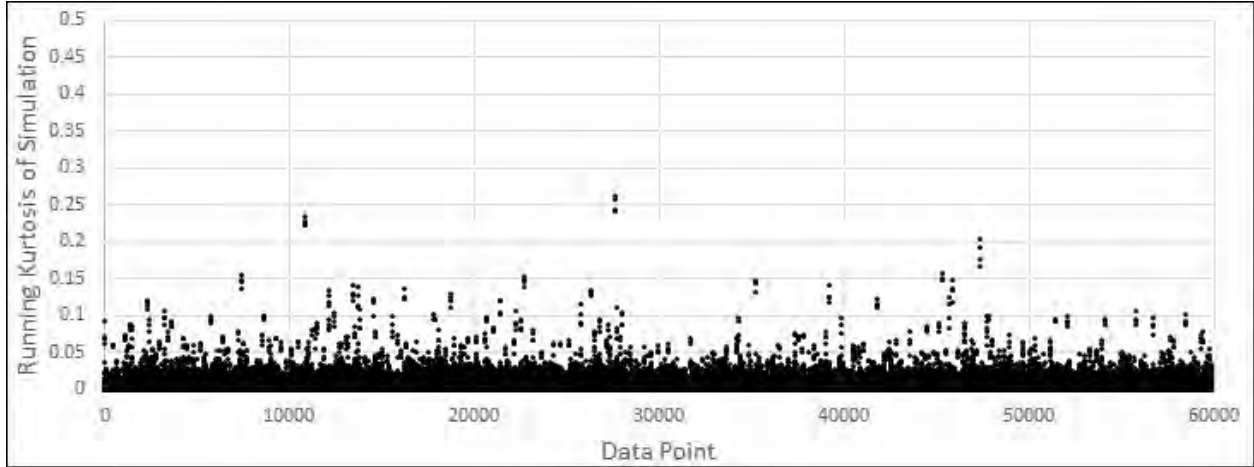
Figure 17. Example simulation output for positive rates of excess running kurtosis.

6.4   Statistical Control of Volatility Clustering - The S-measure

Thus far φ provided a metric to identify a possible relationship between market behavior and impending significant world events, $T(\varphi)$ provided a transformation so that this effect could be compared utilizing control charts and later to indicate that the K-measure could signal an anomalous market variation in volatility preceding an event. The final measure, the S-Measure, utilizes the Johnson S_U and S_B transformation once more, specifically on the K-measure. Since some CPs are more sensitive in extreme K-measure values, it is unlikely that all will fit normality after transformation. This was the case after testing each of the six CPs under consideration. Each CP failed to accept normality at the $\alpha = 0.01$ level of significance. However this transformation is performed to ensure all CPs are on the same scale. The concept that the transformation would provide a conservative estimate is not the case when examining this idea to the S-measure. In this case, due to the higher excess kurtosis of those more sensitive CPs in the K-measure, there is a likelihood that these CPs would create false positives. Therefore, it is necessary to observe the aggregate effects of those CPs which fail the normality assumption and those which pass the normality assumption. Applying the Johnson S_B and Johnson S_U transformations to the CP K-measures, we obtain the results

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Table 11. Johnson transformation and subsequent normality testing on each of the K-measures for each of the six CPs. The base currency for each remains the USD.

		Johnson Parameters				AD	Reject?	Ex. Kurt
	T	γ	δ	θ	σ			
JPY	SB	-544.483	62.151	-8545.024	8546.364	930.230	yes	249.335
JPY	SU	0.001	1.019	0.000	0.007	5.124	yes	0.907
ZAR	SB	-254.764	25.345	-9896.460	9896.887	956.470	yes	298.423
ZAR	SU	-0.063	0.846	0.000	0.004	3.097	no	0.478
NOK	SB	-339.696	24.735	-438017.400	438017.900	752.040	yes	111.397
NOK	SU	-0.044	0.881	0.000	0.006	3.212	no	0.482
CHF	SB	1679.815	128.746	-2.918	1353716.800	862.020	yes	87.321
CHF	SU	-0.013	0.853	0.000	0.006	6.997	yes	0.626
SEK	SB	-357.385	25.723	-2499477.000	2499479.100	1863.000	yes	360.494
SEK	SU	-0.032	0.654	0.000	0.006	10.661	yes	0.559
GBP	SB	-109.800	11.405	-1495.758	1495.857	66.645	yes	8.583
GBP	SU	0.076	1.435	0.001	0.009	1.520	no	0.248

in Table 11. As expected those CPs with higher K-measures fail to accept normality at the $\alpha = 0.01$ level. The CPs USDGBP, USDZAR, and USDNOK, however, fail to reject the normality hypothesis.

The final step is to aggregate those $T(K)$ which fail to reject the normality assumption. These are the first generation S-measures; those $T(K)$ which failed to reject the normality assumption but have the lowest excess kurtosis will be used but are considered as second generation S-measures. Therefore, the CPs USDZAR, USDNOK, and USDGBP belong to the first generation, and USDJPY, USDCHF, and USDSEK below to the second generation. The results of these two generation S-measures are depicted in Figure 18. We observe that outliers occur eleven days before the USS Cole Bombing (value 6.122), three days before the September 11 Attacks (3.384), ten days before the Amman Bombings (3.261), as well as the onset of the Subprime Mortgage Collapse and the Global Financial Crisis. Furthermore, many of the first and second generation S-measures coincide which may imply non-normality, provided near zero excess kurtosis is adequate to qualify as a viable S-measure. Although causal analysis will still be required to determine what may induce the higher volatility, these
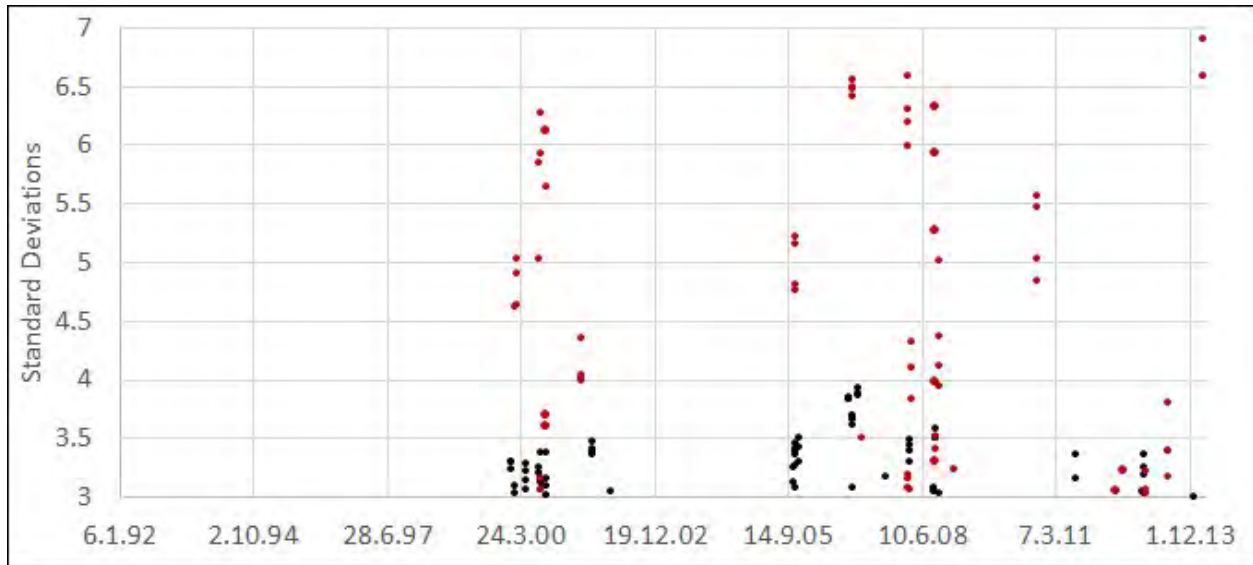
Figure 18. Outliers for first generation (black) and second generation (red) transformations.

results clearly demonstrate the approach may be used as a warning mechanism that signals when further analysis is warranted.

6.5 Summary

Global markets act as a comprehensive information processor. Market service users process supply and demand of goods and services, and this is reflected through asset prices. The act of processing supply and demand is fueled by the current state and the information it is being supplied. If an attack is being planned, then this information can be advantageous to those who act on it before the market can react. This incentive may cause certain asset prices to move which can then be detected which may provide early warning.

This study addressed three different measures based on the relationship of gold and silver, namely the φ-, K-, and S-measure. Although $T(\varphi)$ did not provide predictive qualities its solution illustrated how gold decouples from silver in episodes of extreme volatility due to an attack or financial crisis. The largest of these decouplings surfaced during the USS Cole Bombing while others preceded the 9/11 Attacks, the 2008 Subprime Mortage Collapse,

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and Global Financial Crisis. The S-measure, namely the rate of increase of running excess kurtosis tested the assumption that *a priori* information may be present in the gold and silver relative to different currency pairs. The analysis conducted suggested that this is not only the case but that reserve currencies, especially the USD, JPY, and CHF serve as the preferred warnings of potential black swan-type events. For the terrorist events, this measure with respect to those CPs utilized were able to signal the potential of the USS Cole Bombing and 9/11 attacks 10 and 3 days respectively. As a signaling mechanism for intelligence analysis, the approach shows promise.

VII. Imitation of Modern Financial and Econophysical Effects

Some of the earliest work on uncertainty in markets was carried by Louis Bachelier [2, 40]. In his dissertation work in 1900, Bachelier investigated the overarching probability function governing market behavior in the French bond market. In his work, Bachelier made several significant and reasonable assumptions: 1) *The market does not believe, at any given instant, in a rise nor a fall in the true price* and 2) *the mathematical expectation*[12] *of a speculator is zero*, and 3) *prices may vary between* $(-\infty, \infty)$ [2]. Here the first assumption suggests that markets are governed by what came to be known from later work as the Efficient Markets Hypothesis[3]. If a market is isolated from outside information then the market would stay at rest. This is due to the fact that all current information has been processed and no further adjustments need to be performed as the optimal state has been reached. As information begins to flow into the market once again, it will be immediately available to all market service users. Furthermore, the second assumption assumes that there is no preference of direction of the price of an option based on previous performance. The

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[1]The expected value of a random variable X, is denoted by $\mathbb{E}(X)$ and may be interpreted as the long-term average value of X. In the case of a discrete random variable, taking values $x_1, x_2, ...$

$$\mathbb{E}(X) = \sum_j x_j P(X = x_j)$$

For a continuous random variable with probability density function f,

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x) \mathrm{d}x$$

The expected value of X is often referred to as the **expectation** of X or as the mean of X. [5]

[2]Bachelier defines mathematical expectation as "the potential profit [which is] defined as the product of that profit by the corresponding probability.[2]

[3]"The theory that where assets are traded in organized markets, prices take account of all available information, so that it is impossible to predict whether some assets will give better risk-adjusted values than others. This cannot be predicted because it depends on news, that is, information which is not yet available, and cannot be deduced from information which is available. There are several variants of the efficient markets hypothesis. Weak-form efficiency asserts that it is not possible to use historical share prices to construct a trading strategy that yields excess returns. Semi-strong efficiency states that excess returns cannot be earned on the basis of public information. Strong-form efficiency states that excess returns cannot be earned by trading on the basis of private information. Empirical evidence seems to support semi-strong efficiency as a description of markets." [5]

mathematical expectation is always zero. This implies that future prices are independent of previous levels and follow the same probabilistic process at each time step. Although prices may vary only between $[x_0, \infty)$ (since prices of an underlying asset cannot be negative) with x_0 being the current absolute price, Bachelier, for the sake of simplicity, expands the interval to include all possible values $(-\infty, \infty)$. This allows, he argues, for mathematical simplicity while indicating that this expansion has a negligible effect on the outcome [2]. Furthermore, a margin call could generate losses beyond what is invested.

For his analysis, Bachelier determined that an acceptable probability formula with respect to his assumptions is the Normal Distribution. Since Bachelier's work, a generally accepted mathematical formula for the uncertainty probability curve had been established. Variability is often modeled in accordance with the Normal Distribution. However there is a preponderance of evidence to suggest that normality may be a special case of a more complicated nature of financial systems [7, 18, 30, 40]. Today a specialized branch of economics, namely econophysics, attempts to address these anomalies through complex mathematical representations and analysis to provide a more acceptable representation of financial dynamics. Therefore in order to imitate financial behavior to placate both standard and econophysical experts it will be necessary to develop a framework that fulfills the requirements of both.

7.0.1 Imitation versus Simulation.

The intent of this study is to develop a framework which enables the imitation of financial dynamics. Here we understand imitation is unlike simulation. Simulation which utilizes a simplified model of reality to produce data while imitation mimics reality through whatever mechanisms exist. The objective then is to develop a product (in this case data sets) which are virtually indistinguishable from real market data. Subjected to a Turing test then, the data generated from the imitation should then exhibit the same characteristics of actual financial data. An imitation that would possess this ability then would capture the essentials

of actual financial dynamics and would supply an analyst with the ability to test strategies beyond actual financial data in a replicable and controlled environment.

Externalities[4] influence market service users[5] which further influence market behavior to reflect the impact of these externalities. Due to the lack of perfect information of these influences, it is unlikely to precisely model market performance. This imprecision complicates data analysis and skews potential conclusions. Since financial systems are large and have many market service users, it is potentially unknowable to evaluate all factors influencing these dynamics. However, to test a hypothesis, it is imperative to conduct a replicable experiment. With respect to financial markets this is unlikely. Therefore to achieve an acceptable alternative of market data replication, it is necessary to provide an approximation of market behavior which adheres to required realism while providing data that is tailorable to particular scenarios. This approximation is achieved in this study by imitating market behavior. It is not a simulation as it is unknowable if the dynamics which are involved follow the actual market behavior or if the behavior is only mimicked. Thus we use the term imitation since the attributes of the proposed model in this study fulfills the characteristics that financial markets exhibit. This approach is similar to the Turing test but for financial modeling.

7.1   Motivation

7.1.1   The Evolution toward Unrestricted Warfare.

Due to evolutionary pressures which exist with an increasingly integrated global economy, the utility of traditional warfare becomes increasingly impractical and progressively

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[4]An influence which does not occur to the person, group, or organization actuating the activity under consideration.

[5]Market service users are market users whose motivations typically fall within one of three categories: speculators, hedgers or arbitrageurs. They may vary in size, their time horizon, objective, and perception and handling of risk. [30]

economically inefficient. Economic responses to traditional military approaches are likely to be swift and severe and are therefore ill-suited for deployment as economic interdependence strengthens. The objective then is to develop approaches which will achieve the desired political objectives via atypical strategies without the use of a traditional military portfolio. One possible alternative is in the adoption of what is known as unrestricted warfare. Unlike traditional military strategy, unrestricted warfare embraces the philosophy of avoiding a foe where they are strong and exploiting them where they are weak, practicing dissimulation, and hiding beneath "a cloak of disorder"[64], and enables asymmetric warfare which neutralizes actual military advantages. This "cloak of disorder" can be interpreted as exploiting the complexity and chaos in a system to one's own benefit. These atypical battlespaces include but are not limited to the economic, financial, law, ecological, and cyber landscapes [35, 38]. Of interest in this chapter is the financial landscape. Financial data is the aggregate effect of many supplies and demands working through market service users to obtain their highest level of utility. This suggests that financial markets may pose an ideal environment to cloak this type of warfare beneath the inherent complexity of this financial environment. Understanding how to imitate financial market behavior is the objective of this chapter.

7.1.2 Econophysical Attributes.

There is evidence to suggest that financial data for securities, currencies, commodities, indexes, bonds, derivatives, and so forth, are influenced by "feedback, non-stationarity, many interacting agents, adaptation, and evolution" [30]. The effects of these influences are visible in a leptokurtic[6] distribution of price changes, non-trivial scaling properties[7], fast decay of

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[6]Distributions which have more probability in their tails than that of the Normal Distribution. More extreme values are probable.

[7]This refers to data sets which do not appear to be from the same distribution when viewed at different scales. Brownian Motion is a counter example since regardless of scale the data is viewed at, it follows normality.

log-adjusted values[8], slow decay of log-adjusted absolute values[9] (LAAV) which leads to volatility clustering[10] [30].

Furthermore, the distribution of the LAVs is unlikely to remain the same between any two assets under consideration. Therefore utilizing a single distribution with few parameters may be too limiting. A distribution, however, with the ability to modify itself component-wise may provide a more suitable alternative. Although any distribution could be utilized in a component fashion as suggested, several distributions pose advantages. These particular distributions are discussed next.

7.2 The Distribution of Root Causes

In order to develop a model which may mimic complex behavior it was advantageous to understand possible origins of motivation itself. If we assume that there is an origin of a motivation then we may consider this a root cause. The actions that result from this fundamental motivation are then considered effects. Thus, the causal reason of an effect is one of its root causes. A root cause cannot be further dissected into root causes otherwise the initial root cause would be an effect and not a root cause. If an effect is stochastic then at least one of its root causes is likely stochastic as well. Assuming the action's distribution and that a root cause, or causes, are the parameters of the effect's distribution, we propose that this may be regarded as a hierarchical model of root causes and effects as in (7.1), where X_k is a random variable, f_k is the probability density function (PDF), and $\boldsymbol{\theta}_k$ is the parameter vector for the f distribution k.

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[8]Although several types of data manipulation exist such as linear price change, discounted or de-trended price change, and simple value to name a few, this chapter will utilize the log-adjusted values [30] (LAV). The LAV technique enables a proportionate rescaling of the data. Thus a percent change of one year would be similar to the same percentage another year [41]. Since data under longer time periods (typically decades) are under consideration for this study, this approach will be utilized.

[9]This implies a non-zero autocorrelation.

[10]Episodes of high volatility are followed by similar episodes of high volatility while episodes of low volatility are followed by similar episodes of low volatility [41]

$$X_1 \sim f_1(x_1|\boldsymbol{\theta}_1)$$

$$\Theta_1 \sim f_2(\theta_2|\boldsymbol{\theta}_2)$$

$$\vdots$$

$$\Theta_n \sim f_n(\theta_{n-1}|\boldsymbol{\theta}_{n-1})$$

(7.1)

Once an effect reaches a certain set of distributions, their parameters remain at this distribution as well. These distributions may be regarded as hierarchically invariant as further hierarchical analysis does not deviate from its current distribution. Mathematically, PDFs that fulfill this characteristic satisfy (7.2). Here we provide a short derivation of the condition to retain hierarchical invariance. We wish to develop a functional expression such that a hierarchical distribution retains the same functional form as the parameter distribution for any parameters of the PDF. Therefore we wish to solve a one-stage hierarchical model as in (7.2).

$$X|\Theta_1 \sim f(\theta_1)$$

$$\Theta_1 \sim f(\theta_2)$$

(7.2)

We obtain (7.3) by rewriting the marginal density as an expression of conditional density and marginal density in the variables θ_1 and θ_2. For a distribution to be invariant means that the functional form of the hierarchical model must match that of the original model.

$$P(X = x) = \int_{-\infty}^{\infty} f(x, \theta_1)\mathrm{d}\theta_1$$

$$= \int_{-\infty}^{\infty} f(x|\theta_1)f(\theta_2, \theta_1)\mathrm{d}\theta_1$$

$$= f(x|\boldsymbol{\theta})$$

(7.3)

Therefore, a PDF retains hierarchical invariance if and only if (7.4) is true.

$$f(x|\boldsymbol{\theta}) = \int_{-\infty}^{\infty} f(x|\theta_1)f(\theta_2, \theta_1)\mathrm{d}\theta_1 \tag{7.4}$$

Several distributions which possess this characteristic are the Normal Distribution (when Θ_1 is the mean), the Cauchy Distribution (when Θ_1 is the median), the Laplace Distribution, and the Pareto/Power Law Distribution pair[11]. However, (7.4) shows how one root cause may act. In complex systems, there are likely multiple root causes and each may have one of the aforementioned distributions. Furthermore, a root cause may be conditional based on circumstances. This suggests that in certain situations a varying number of root causes may affect a process and that a system with more than one root cause may exhibit non-stationarity if certain root causes are not ubiquitous over others. If conditions arise wherein only one root cause affects the process, then it dominates the process outcome. If, however, multiple root causes influence a process, then their interactive influence determine the outcome. This study does not imply that a factor that is normally distributed is a root cause to a process but rather that the factors become indistinguishable from characteristics of those of root causes. These are likely criteria a root cause must fulfill, but it does not necessarily make a factor a root cause of a process. Root causes may also interact at different depths within a process.

This chapter will assume root causes at the same depth k and develop a framework around a particular instance of the aforementioned distributions and interactions without loss of generality. This framework provides one example of this general concept and illustrates the versatility of this approach. Due to its relative ease of use, finite moments, and general familiarity, we will utilize the Normal Distribution as a general root cause distribution and convolution as the interactive process between root causes.

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[11]The Pareto/Power Law Distribution pair holds true for the 2-stage hierarchical model. A hierarchical Pareto Distribution becomes a Power Law Distribution and vice verse for the Power Law Distribution.

This chapter develops the Synchronous Interactive Gaussian Mixture Aggregate (SIGMA or Σ) Distribution for static and dynamic root causes as well as a possible Interaction Aggregate Framework (IAF) to approximate financial data with variable accuracy and to imitate financial dynamics per the aforementioned attributes. Unlike many approaches which hypothesize the use of a static distribution and test its validity, the approach in this chapter will show how the Σ distribution may be tailored to the distribution under consideration.

7.3 Development of the Σ Distribution (Static Number of Influences n)

As previously discussed, root causes may monopolize an effect at a specific point in time or may interact with aggregated effects or other root causes at a point in time. Depending on which of these is taking place, the stationarity of a process may change. If we assume X_1 and X_2 exist, that $X_1 \overset{\text{iid}}{\sim} N(\mu_1, \sigma_1^2)$, and that $X_2 \overset{\text{iid}}{\sim} N(\mu_2, \sigma_2^2)$, then the interactive effects of both X_1 and X_2 is represented in (7.5) while individual interactions remain the same as their original distribution.

$$X_{1,2} \sim N(\mu_1, \sigma_1^2) \otimes N(\mu_2, \sigma_2^2) = N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2) \tag{7.5}$$

Since root causes may be conditional and may not always affect the outcome of an event, it follows that (7.5) makes up only a possible portion of potential scenarios. Since X_1, X_2, and $X_{1,2}$ comprise all possible scenarios, the overall distribution of the effects due to root causes and their interactions given by (7.6), where c_1 and c_2 are mixture coefficients such that $c_1 \geq 0, c_2 \geq 0$ and $c_1 + c_2 \leq 1$ necessarily.

$$c_1 N(\mu_1, \sigma_1^2) + c_2 N(\mu_2, \sigma_2^2) + (1 - c_1 - c_2) N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2) \tag{7.6}$$

Since $\sigma_1^2 + \sigma_2^2 > \sigma_1^2$ and $\sigma_1^2 + \sigma_2^2 > \sigma_2^2$, the interaction of multiple root causes provides higher variability and kurtosis than individual root causes alone. The exact values, however,

depend on its mixture values. As n increases, so do the possible combinations of root cause effects, as the number of combinations is $2^n - 1$. If we assume that n root causes are present, then (7.7) through (7.14) provide the necessary setup to represent their aggregate distribution. This distribution will be referred to the Synchronous Interactive Gaussian Mixture Aggregate (SIGMA or Σ) Distribution (see Appendix B for derivation).

$$\Sigma_0(x|n, \mathbf{c}, \boldsymbol{\mu}, \boldsymbol{\sigma}) = \frac{1}{2} \sum_{d=1}^{2} \sum_{j=1}^{2^n-1} c_j N \left[(-1)^d \sum_{k=0}^{n-1} \mu_{n-k} b_k, \sum_{k=0}^{n-1} \sigma_{n-k}^2 b_k \right] \tag{7.7}$$

$$\sum_{j=1}^{2^n-1} c_j = 1 \tag{7.8}$$

$$c_{2^{w-1}} \geq c_{2^w} \tag{7.9}$$

$$1 \leq w \leq \log_2 n \tag{7.10}$$

$$a_{j,0} = j \tag{7.11}$$

$$b_{j,0} = 0 \tag{7.12}$$

$$a_{j,k+1} = \frac{1}{2}(a_k - b_k) \tag{7.13}$$

$$b_{j,k} = 2\left(\frac{a_k}{2} - \left\lfloor \frac{a_k}{2} \right\rfloor\right) \tag{7.14}$$

Equation (7.7) represents the overall PDF of n root causes. The vectors $\boldsymbol{c}$, $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ represent the mixture, mean, and standard deviation vectors of the components, respectively. The parameter a provides a recursive process to provide n as a binary expression which is represented in the binary parameter b. This process is expressed in (7.13) and (7.14) and is also utilized in (7.7). Equation (2.9) ensures a mixture distribution while Equation (2.8) sorts all distribution components based on the typical binary counting scheme. This is advantageous since n can be increased and factors can then still be compared from one n value to another. This ensures consistency of components when analyzing multiple data sets for identical n. Equation (2.7) provides the necessary number of values in order to sort the main factors from Equation (2.8) correctly. Equations (7.11) and (7.12) initialize a and b variables accordingly for the binary counting scheme and Equations (7.13) and (7.14) provide the iterative values to complete a breakdown of j. Essentially, the variables a and b provide a binary representation of the j value such that its main components remain at specific positions throughout the representation such that a change in n can still be compared to previous results without having to recompute the parameters for the target distribution under consideration.

A special case of the Σ Distribution is when the target distribution is symmetric. In this case, we refer to the appropriate Σ Distribution as Σ_0. For the rest of this study, we will refer to the generic case Σ, but it will be implied to utilize the Σ and Σ_0 Distributions for asymmetric and symmetric distributions, respectively. For the Σ Distribution, we eliminate the $\frac{1}{2}$ factor as well as the outer summation of (7.7). Furthermore, it is of interest to note that at most two solutions exist for any symmetric data set. Each solution is either skewed to the left or to the right (each with parameter vectors $\boldsymbol{c}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$ and $\boldsymbol{c}$, $-\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, respectively). By combining both possible solutions, we must divide by two in order to maintain a PDF. This is the reason for the $\frac{1}{2}$ coefficient before the outer summation in (7.7). Financial data is not symmetric as symmetry would imply log-adjusted price changes orbit near a specific

Table 12. Illustration of 24 securities and their respective skewness and excess kurtosis.

Ticker	Skewness	Ex. Kurtosis	Ticker	Skewness	Ex. Kurtosis
AA	-7.244	177.513	AIG	23.864	1382.611
AAPL	-4.219	91.720	ALTR	-3.522	61.811
ABT	-15.290	427.854	AMAT	-5.912	110.291
ADBE	-4.169	75.830	AME	-6.970	165.216
ADI	-1.197	20.510	AMGN	-11.899	321.132
ADM	-1.220	25.510	AON	-5.039	97.442
ADP	-14.795	445.174	APA	-3.570	101.380
ADSK	-4.028	75.731	APC	-5.721	142.442
AEP	-0.478	25.789	APD	-9.744	298.485
AET	-8.457	212.140	ARG	-5.802	135.117
AFL	-6.204	149.769	AVP	-9.039	228.062
AGN	-8.694	253.368	AVY	-6.893	222.486

price value. This is not the case, and prices change due to market conditions, not specific valuations. Unless market dynamics remain static, prices will fluctuate too. This implies that the distributions should be skewed. This is visible in many financial data sets. Table 12 provides a list of 24 securities and their respective skewness and excess kurtosis. Note that each of the securities is skewed and with a very high level of kurtosis. This suggests that the Σ Distribution over the Σ_0 Distribution must be used for these financial data sets. Furthermore, the extremely high excess kurtosis indicates further that the Normal Distribution is not a good fit since its excess kurtosis is zero.

7.4 Approximating Continuous PDFs Using Σ Distribution

The difficulty with utilizing a specific distribution is that not every data set can be approximated with a specific statistical significance. Furthermore, it may be difficult to generate random variables of a specific PDF chosen if that PDF is analytically intractable. The Σ Distribution provides an alternative approach to mitigating these issues. Since it allows arbitrary approximation of analytical distributions or empirical data by adjusting n, it is not limited to a fixed number of parameters and thus would enable a tailored approach.

Although the true analytical distribution would fit itself perfectly, it does not allow for the added attributes (non-trivial scaling properties, slow decay of LAAVs, and so forth) that the Σ Distribution enables, which is shown throughout the rest of this chapter. This flexibility then allows PDFs with different properties (such as mean, variance, skewness, and kurtosis) to be approximated using a consistent approach, regardless of distribution supplied. To illustrate its versatility, we approximate different distributions using two different techniques. The first technique will minimize the squared error of the difference between the two distributions. Let ψ be the target distribution, then this accomplished by (7.15).

$$\min \int_{-\infty}^{\infty} \left(\psi(x|\boldsymbol{\theta}) - \Sigma_0(x|n, \mathbf{c}, \boldsymbol{\mu}, \boldsymbol{\sigma}) \right)^2 \mathrm{d}x$$
$$\ni \sum_{j=1}^{2^n-1} c_j = 1$$
$$c_j \geq 0 \tag{7.15}$$
$$-c_j - 1 \geq 0$$
$$\sigma_j > 0$$

Figure 19 illustrates this approach graphically while Table 13 provides details for each of the illustrated examples. These examples vary in kurtosis and skewness and provide some insight into the versatility of the approach. We notice that the Σ Distribution provides very low error[12] regressive approximations for both symmetric and asymmetric PDFs respectively for small n. Those distributions tested are the Johnson S_U Distribution[13], the Student's t

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[12]For the distributions tested at $n = 2$ the error did not exceed 0.009 for the interval tested.

[13]The Johnson S_U Distribution is versatile since it supports every combination of skewness and kurtosis. Its PDF is according to

$$p(x|\gamma, \delta, \theta, \sigma) = \frac{\delta}{\sigma\sqrt{2\pi}} \frac{1}{\sqrt{1 + \left(\frac{x-\theta}{\sigma}\right)^2}} \exp\left(-\frac{1}{2}\left(\gamma + \delta \sinh^{-1}\left(\frac{x-\theta}{\sigma}\right) \right)^2 \right)$$

where $\gamma, \theta \in \mathbb{R}$ and $\delta, \sigma > 0$. It was developed by N.L. Johnson in 1949 as a transformation to the Normal Distribution. [31]

Distribution[14] whose mean variance, skewness, and kurtosis are only defined for specific parameter values, the Logistic Distribution[15], the Voigt Distribution[16], the Laplace Distribution[17], the Generalized Extreme Value (GEV) Distribution[18], and the Stable Distribution Family[19].

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[14]The Student's t Distribution with parameter ν is according to

$$p(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

[15]The Logistic Distribution with mean μ and parameter s is given by

$$p(x|\mu, s) = \frac{1}{4s}\text{sech}^2\left(\frac{x-\mu}{2s}\right)$$

[16]The Voigt Distribution is the convolution of Normal Distribution and Cauchy Distribution. It is described by

$$p(x|\sigma, \gamma) = \frac{\mathscr{R}[w(z)]}{\sigma\sqrt{2\pi}}$$

where $\mathscr{R}[w(z)]$ is the real part of the Fadeeva function evaluated for $z = \frac{x+i\gamma}{\sigma\sqrt{2}}$

[17]The Laplace Distribution with mean μ and scale σ is

$$p(x|\mu, \sigma) = \frac{1}{2\sigma}\exp\left(-\frac{|x-\mu|}{\sigma}\right)$$

[18]The Generalized Extreme Value Distribution is also known as the Fisher-Tippett Distribution with location parameter μ, scale parameter σ, and shape parameter ξ as follows

$$p(x|\mu, \sigma, \xi) = \frac{1}{\sigma}t(x)^{\xi+1}e^{-t(x)}$$

where

$$t(x) = \begin{cases} \left(1 + \frac{x-\mu}{\sigma}\xi\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ e^{-\left(\frac{x-\mu}{\sigma}\right)} & \text{if } \xi = 0 \end{cases}$$

[19]The α, β, γ, and δ are parameters for the general form of the stable distribution with $\alpha \in (0, 2]$, $\beta \in [-1, 1]$, $\gamma \in \mathbb{R}^+$, and $\delta \in \mathbb{R}$. A stable distribution is one that autoconvolves. Khintchine [33] and Lévy [34] solved the general problem [42]. The general solution is

$$\ln \varphi(x) = \begin{cases} i\delta x - \gamma|x|^\alpha\left(1 - i\beta\frac{x}{|x|}\tan\left(\frac{\pi}{2}\alpha\right)\right) & \alpha \neq 1 \\ i\delta x - \gamma|x|\left(1 + i\beta\frac{2x}{\pi|x|}\ln|x|\right) & \alpha = 1 \end{cases} \tag{7.16}$$

Table 13. The twelve cases tested utilizing the Σ Distribution approximation approach. They provide a snapshot of the versatility of the Σ Distribution as a possible approximation technique.

Case	Distribution	Parameters			
		1	2	3	4
1		0	1	0	1
2	Johnson S_U	-1	1	0	1
3		1	0.5	0	0.5
4	Student's t	0	1	1	
5		0	1	0.5	
6	Logistic	0	0.7		
7	Voigt	0.3	0.8		
8	Laplace	0	1.5		
9	Extreme Value	0	1		
10		0.8	0	0	1
11	Stable	0.7	0.5	0	1
12		0.6	0.8	0	1

Table 14. Approximation utilizing (7.15) for different distributions for $n = 2, 3$. The, * indicate where values are below the 3 digits level of significance and may be treated as zero.

		Case											
		1	2	3	4	5	6	7	8	9	10	11	12
n=2	c_1	0.460	0.480	0.340	0.439	0.385	0.534	0.801	0.418	0.522	0.372	0.346	0.329
	c_2	0.435	0.287	*	0.077	*	0.457	0.072	0.418	0.448	*	0.346	*
	c_3	0.105	0.233	0.660	0.484	0.615	0.009	0.127	0.164	0.030	0.628	0.308	0.671
	μ_1	*	0.588	-0.391	-0.036	*	0.013	-0.052	-0.140	-0.135	*	1.641	0.744
	μ_2	*	1.991	-2.290	-0.151	2.117	-0.025	-0.227	-1.650	1.139	-1.008	0.770	1.641
	σ_1	0.671	0.794	0.670	0.740	0.736	1.489	0.909	0.758	0.739	0.563	2.680	0.424
	σ_2	1.700	1.760	3.107	2.462	3.623	0.860	2.490	1.418	1.163	2.323	0.488	2.105
n=3	c_1	0.458	0.353	0.137	0.299	0.210	0.771	0.780	0.316	0.377	0.341	0.168	0.167
	c_2	0.458	0.298	0.137	0.295	*	0.114	0.088	0.300	0.375	0.341	0.165	0.167
	c_3	*	*	*	0.020	0.043	*	0.033	0.061	0.030	0.057	0.135	0.149
	c_4	0.026	0.231	0.137	0.141	*	0.114	0.088	0.026	0.219	0.136	*	*
	c_5	*	0.093	0.150	0.000	0.326	*	0.012	0.204	*	0.001	*	*
	c_6	0.001	*	*	*	0.009	*	0.014	0.028	*	0.123	*	*
	c_7	0.057	0.026	0.439	0.246	0.413	*	0.015	0.064	*	0.001	0.532	0.516
	μ_1	-0.198	1.332	-0.770	0.198	-0.190	-0.184	0.136	0.982	1.218	1.204	0.946	1.016
	μ_2	-0.206	0.411	-1.981	0.193	-1.927	0.354	0.631	0.196	0.331	0.063	0.607	0.561
	μ_3	-1.350	2.473	-0.235	0.902	-0.235	-0.616	-1.212	-0.255	-0.513	-0.072	0.494	1.653
	σ_1	1.687	1.103	0.886	0.562	0.502	1.147	0.886	1.278	1.304	2.770	0.633	0.530
	σ_2	0.606	0.643	3.290	0.910	2.376	0.493	3.410	1.303	0.731	0.884	0.316	0.249
	σ_3	0.356	2.452	0.391	3.088	1.107	2.187	1.222	0.357	0.539	0.345	2.688	2.709

Therefore, as an approximation tool for complex distributions, this approach serves to provide a fit for empirical data that lie between these different distributions. The utilization of the Σ Distribution approach would provide for more versatility over a very specific distribution. For distributions for which it is difficult to generate random variables, the Σ Distribution and IAF framework process may be of additional use.

The results in Tables 14 and 15 as well as Figure 22 suggest that solving (7.15) for the Σ Distribution parameters provides an accurate representation of a wide range of target distributions, especially for symmetric ones. Accuracy can be increased with higher n. For asymmetric distributions however, the error is two to three magnitudes higher than for symmetric distributions. This indicates that higher n-values may be necessary for these distributions to achieve the same level of accuracy. Regardless of case, the Σ Distribution provides a viable and adjustable alternative to traditional distribution fitting techniques. Thus far, this technique assists only in distribution approximation of a target distribution. It does not address any econophysical attributes financial data exhibits. The next sections address how the Σ Distribution with the IAF may accommodate for these attributes as well.

To illustrate the ability of the Σ Distribution to fit the distributions in Figure 19 utilizing the parameters in Table 14 we conduct a series of hypothesis tests to decide whether the random variables which the Σ Distribution generates have a statistically significant goodness-of-fit with the target distribution. For these examples, the squared error was approximated for $x \in [-10, 10]$ and was displayed from $x \in [-5, 5]$. Their parameters were chosen to provide consistent x and y values for each example. The results suggest that the data sets generated utilizing the Σ Distribution are indistinguishable from the target distributions under consideration. The first case tests whether the random values generated fit the target PDF. Since the parameters of the PDF are explicitly defined, we may use the Kolmorgorov-Smirnov-Lilliefors test. The second test generates random variables from the Σ and target distributions and tests whether they are from the same distribution. This test utilizes the Cramér-von Mises two sample test. Since the test case parameters are known tests and compare their p-values to the $\alpha = 0.010$ level of significance. If $p < \alpha$ then we reject H_0 for H_a, otherwise we fail to reject H_a.
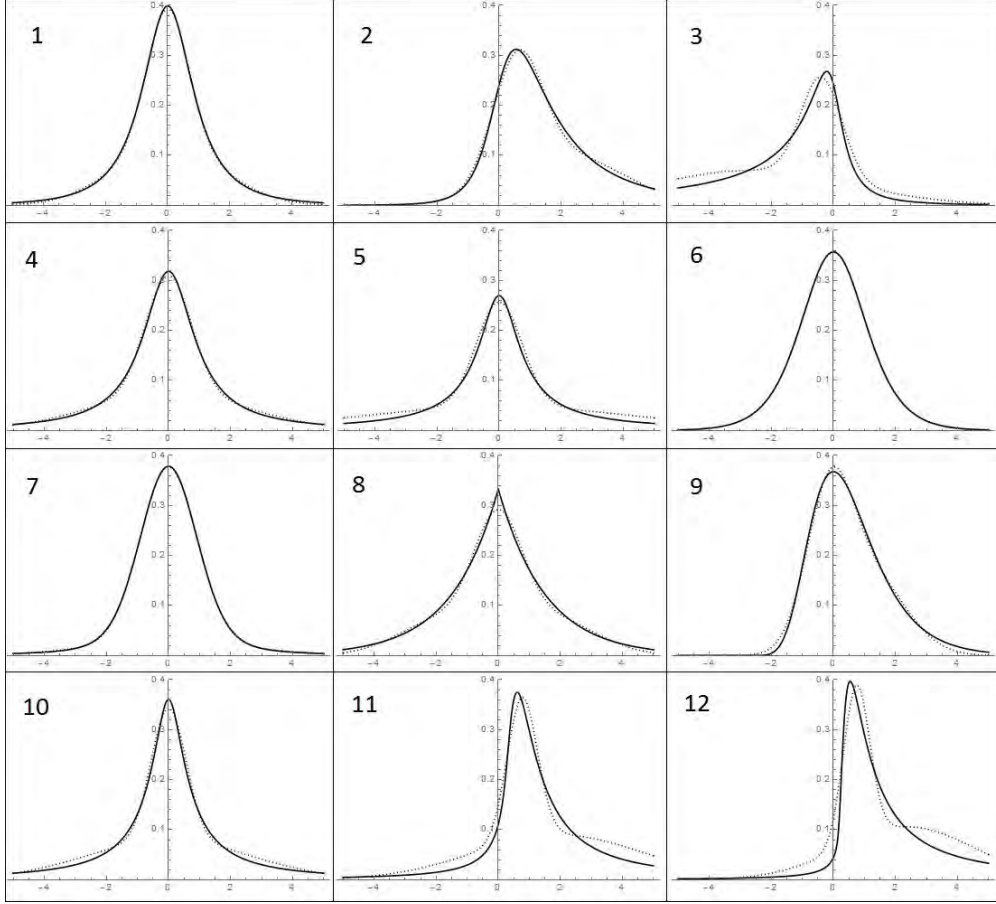
Figure 19. Σ_0 and Σ approximations (dotted) for symmetric and asymmetric PDFs (continuous line) for $n = 2$. Approximations were calculated in Mathematica 10.

$$H_0 : \Sigma\text{-distributed data is } \varphi\text{-distributed.}$$
$$(7.17)$$
$$H_a : \Sigma\text{-distributed data is not } \varphi\text{-distributed.}$$

The reasoning for both tests is as follows. The KSL criteria tests the Σ-distributed random variables with the exact PDF. This would imply that the analyst has perfect information about the distribution of the process. Therefore this PDF would represent perfect information. This is never truly the case and should be considered the lower bound on performance. The Cramér-von Mises criteria tests whether a sample from φ and Σ could be

Table 15. Probability of generated Σ Distribution data to pass for φ Distribution (100 replications each). Increasing n increases the probability of the data set generated will be indistinguishable from a data stream generated by the target distribution.

| | n=2 | | | | n=3 | | | |
| | KSL Test | | Cramer von-Mises | | KSL Test | | Cramer von-Mises | |
Case	100	1000	100	1000	100	1000	100	1000
1	1.00	1.00	0.99	1.00	0.99	0.98	0.99	1.00
2	0.98	0.46	0.96	0.43	1.00	1.00	0.99	0.99
3	0.20	0.00	0.03	0.00	0.24	0.16	0.17	0.15
4	0.99	0.92	0.99	0.97	0.96	0.97	0.98	0.97
5	0.76	0.69	0.99	0.86	0.86	0.87	0.96	0.89
6	0.99	1.00	0.97	1.00	0.99	0.99	1.00	0.99
7	0.98	0.85	0.97	0.75	0.99	1.00	0.99	0.99
8	0.97	0.98	0.97	0.98	1.00	0.99	0.98	1.00
9	0.99	0.94	0.99	0.88	0.99	0.99	0.99	0.99
10	0.98	0.60	0.98	0.56	0.98	0.96	0.99	0.96
11	0.72	0.80	0.86	0.83	0.86	0.84	0.86	0.84
12	0.10	0.00	0.16	0.00	0.29	0.24	0.35	0.37

the same distribution. This relaxes the assumption of perfect information about the target distribution.

Table 15 provides the results of the 100 replications for each scenario. Except for Cases 3, 5, 11, and 12, the hypothesis test failed to reject the data generated by the Σ Distribution coming from the target distribution. Of these four cases, Cases 3 and 12 scored low which is likely due to their high skewness relative to the other cases considered. By increasing $n = 2$ to $n = 3$, the hypothesis test failed to reject a higher percentage of those data sets generated by the Σ Distribution. This suggests that, in order to approximate a distribution, additional Σ Distribution components may be necessary and is likely the case for distributions which have large skewness values.

7.5 Non-Trivial Scaling Properties

Mantagna showed that stable distributions scale and are self-similar [42]. Self-similarity implies that regardless of scale, when rescaled, the stationary properties of the empirical

data remain the same. Unlike stable distributions, the Σ_0 Distribution allows for phase transitions between micro and macro scales. Since the Normal Distribution is part of the stable distribution family, it is self-similar and retains trivial scaling properties, which are scaling properties which retain the identical functional form, or distribution at any scale. The Σ distribution, however, allows components to interact with different mixture properties. Thus as a time scale is changed, the distribution at that time scale may vary. A single root cause is normally distributed (per the definition of the Σ Distribution). For the time interval where it dominates, the effect is self-similar and possesses trivial scaling properties. If the population distribution is not normally distributed, then as the time scale increases, the data that is observed will increasingly distort the Normal Distribution until the population distribution is achieved. This process is illustrated in Figure 20 by utilizing an example from Table 14, specifically the first example of the Student's t Distribution (which is also the Cauchy(0,1) Distribution). Since we know the mixture of the different Normal Distribution components, we may concatenate data sets of these onto one another to form a contiguous data stream. However, the number of data values generated by a component distribution is according to the mixture coefficient of that component distribution. Assuming that we have a population of 100 artificially generated values and wish to approximate the Cauchy(0,1) Distribution. We would require 22, 4, and 24 of those random values to originate from $N(-0.036, 0.740)$, $N(-0.151, 2.462)$, and $N(-0.187, 3.202)$ respectively. Furthermore, since the distribution is symmetric, half of the data points (the 50 remaining) must come from those distributions which are $-\boldsymbol{\mu}$, namely $N(0.036, 0.740)$, $N(0.151, 2.462)$, and $N(0.187, 3.202)$, respectively and in the same quantity is before. Each of these components has a non-zero mean, implying that they set a trend for the data generated. For a positive mean, the trend will be positive and, for a negative mean, the trend will be negative. Thus as more values are generated from a particular component, more of that trend will become pronounced over time. These values can be generated via standard random number generation for the Normal
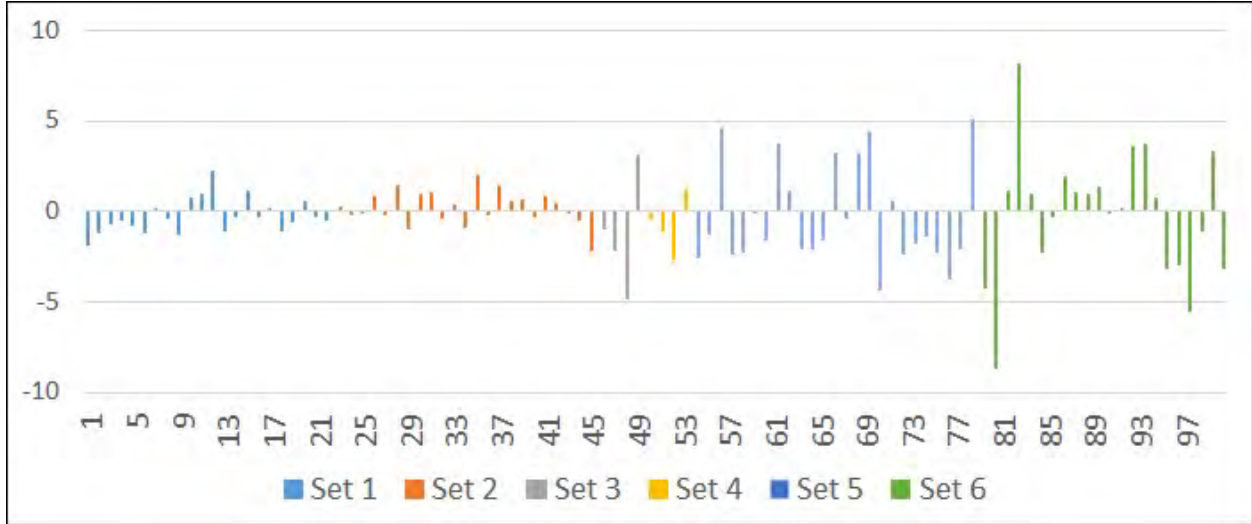
Figure 20. Illustration of sample output to generate an approximation to the Cauchy(0,1) Distribution using the Σ_0 Distribution with $n = 2$. Each data set is colored based on its originating distribution. The aggregate effect of the component distributions is then the Cauchy(0,1) Distribution.

Distribution. This suggests that if the random number generator produces independent random values, this process would also produce independent random values. This does not imply that the magnitudes of these values do not provide other behaviors as will be observed in future sections.

Although the Cauchy Distribution is self-similar [42], if we analyze the data stream with sufficient fidelity we would be able to determine that the underlying distribution is itself not a Cauchy Distribution but a Normal Distribution. This suggests that we are now able to manipulate the scaling property of a data set arbitrarily and tailor this depending on the scale we wish to manipulate to best represent the case under study. In this particular scenario then scaling properties at the micro-scale[20] follow that of a Normal Distribution while those scaling properties at the macro-scale[21] follow that of the Cauchy Distribution. By providing a large enough n we could further tailor the scaling properties to provide

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[20]By "micro-scale" we refer to a time line which consists of only a few time steps. In this example we assume that a micro-scale is less than four time steps which is in line with the number for a transition from one Σ Distribution component to another.

[21]By "macro-scale" we refer to a time line which consists over larger spans of time which are typically months or years.

different distributions at different scales. This approach, however, is only an approximation of the Cauchy Distribution. This approach supplies a mechanism to approximate other distributions with few terms with a predefined level of statistical significance.

7.6 Incorporating Volatility Clustering into the Σ Distribution

If certain component interactions have a much higher variance than other component interactions and their episodes last for several time steps then this may be regarded as volatility clustering. Volatility clustering implies that the magnitudes of data are dependent. Thus a high prior probability for a low volatility results in a high probability for a low volatility in the future. Likewise, a high prior probability for a high volatility results in a high probability for a high volatility in the future. In the context of the Σ Distribution, the high volatility episodes could be attributed to a different combination of root causes than in episodes with low volatility. Thus these volatility clusters have different stationary properties from that of episodes having lower volatility or even for the entire empirical data set. Volatility clustering also implies that the autocorrelation of the magnitudes of the values is greater than zero and thus the transition probability between low (or high) volatility interactions is low (or high) with high volatility interactions.

To accommodate this observation, we require a transition matrix, $T^{(0)}$ that allows different interactions of Σ_0 components to traverse other components. Since there are $2^n - 1$ possible states, we require a $(2^n - 1) \times (2^n - 1)$ transition matrix. Furthermore, we require that the population distribution adheres to the specific values in $\boldsymbol{c}$. If $\boldsymbol{e_j}$ represents the $(2^n - 1) \times 1$ vector with a 1 at the jth entry, zero at all other entries, and $w \in \mathbb{N}$, then (7.18) represents the necessary condition that $T^{(0)}$ must fulfill.

$$\lim_{w \to \infty} \|\mathbf{e}_j^{\mathrm{T}} \mathbf{T}^{(w)} \mathbf{e}_j\| = c_j \tag{7.18}$$

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The limiting probabilities for a specific Σ_0 Distribution mixture are not unique and thus infinitely many different transition probability configurations are possible. To show this, we provide a formal proof.

Theorem 1: *If two different Markov chains of equal dimension have identical limiting probabilities, then any arbitrary product of both matrices with each other will also have the same limiting probabilities as each Markov chain when multiplied only with itself.*

PROOF

Let $\mathbf{T}_1$ and $\mathbf{T}_2$ be two transition probability matrices of size m with $\mathbf{T}_1 \neq \mathbf{T}_2$. We wish to show that, if $\lim_{k\to\infty} \mathbf{T}_1^{(k)} = \lim_{k\to\infty} \mathbf{T}_2^{(k)} = \mathbf{S}$, then any infinite and arbitrary sequence of matrix multiplications of $\mathbf{T}_1$ and $\mathbf{T}_2$ also equals $\mathbf{S}$. Let $n_k, k \in \mathbb{N}$. Since $\mathbf{T}_1^{(\infty)} = \mathbf{T}_1^{(n_1)}\mathbf{T}_1^{(\infty)}$, we could rewrite it as $\mathbf{T}_1^{(\infty)} = \mathbf{T}_1^{(n_1)}\mathbf{T}_1^{(\infty)} = \mathbf{T}_1^{(n_1)}\mathbf{T}_2^{(\infty)}$. Since $\mathbf{T}_1^{(\infty)} = \mathbf{T}_1^{(n_1)}\mathbf{T}_2^{(\infty)}$, we can further write $\mathbf{T}_2^{(\infty)}$ as $\mathbf{T}_2^{(\infty)} = \mathbf{T}_2^{(n_2)}\mathbf{T}_2^{(\infty)} = \mathbf{T}_2^{(n_2)}\mathbf{T}_1^{(\infty)}$. We therefore obtain $\mathbf{T}_1^{(\infty)} = \mathbf{T}_1^{(n_1)}\mathbf{T}_2^{(n_2)}\mathbf{T}_1^{(\infty)}$. We may now extend this as in (7.19).

$$\mathbf{T}_1^{(\infty)} = \prod_{j=1}^{\infty} \mathbf{T}_1^{(n_{2j-1})}\mathbf{T}_2^{(n_{2j})} \tag{7.19}$$

Therefore any arbitrary and infinite sequence of two equal dimension matrices will have the same steady-state as both matrices in isolation. □

This provides proof that the transition probability matrix is tailorable and, depending on this specific tailoring, may provide different characteristics of volatility clustering. From financial data and studies by Mantegna and Stanley [43] and Gopikrishnan [23, 24], we know that there is nearly zero autocorrelation in their LAVs but relatively large autocorrelation in their LAAVs. Figure 21 provides a snapshot of the New York Stock Exchange (NYSE) from 31 December 1965 to 16 July 2012 (9871 data points). Its graphs suggest that although the direction of the market values might be random, the magnitudes of these values have long memory. The regression function used was the linear (shown in green) and the exponential

2P (shown in blue) respectively. We observe that there does not seem to be any discernible pattern in the autocorrelation of its LAVs with a mean essentially zero while the autocorrelation of its LAAVs remains above zero for over 100 days with a clean pattern of tapering [30]. For an imitation it would be important that the model be able to recreate these characteristics. However, for an imitator to be able to emulate market data of a specific index or asset, it is necessary that it can approximate the specifics of the target asset or index. In this case it would be essential that the simulation produce zero autocorrelation in the LAVs while producing an equivalent rate of autocorrelation decay for its LAAVs. Therefore a tailorable mechanism for autocorrelation with respect to $\mathbf{T}^{(0)}$ is required.

The autocorrelation exhibited in the NYSE index values, as well as in specific assets, can be estimated using exponential decay (e.g., via the N-Parameter (P) exponential regression function). The lowest R^2 we obtain from regressing with this approach is with 2P

$$\hat{y} = \kappa e^{\lambda x}$$

and we obtain a value of $R^2 \approx 0.834$ ($R^2 \approx 0.840$ for 3P, $R^2 \approx 0.857$ for 4P, and $R^2 \approx 0.861$ for 5P) which indicates that this model fits the data relatively well. This suggests that the decay may be modeled as an exponential decay. For the NYSE index we obtain $\kappa \approx 0.152$ and $\lambda \approx -0.013$ as parameter values using the exponential regression feature in JMP 10.

Although higher order exponential functions may be used to approximate the LAAV behavior we use the 2P case for simplicity. Let $z(\Delta t)$ represent the exponential 2P regression function for LAAVs, where Δt is the value of lag, then (7.20) represents the relationship.

$$z(\Delta t) = \kappa e^{\lambda \Delta t} \tag{7.20}$$
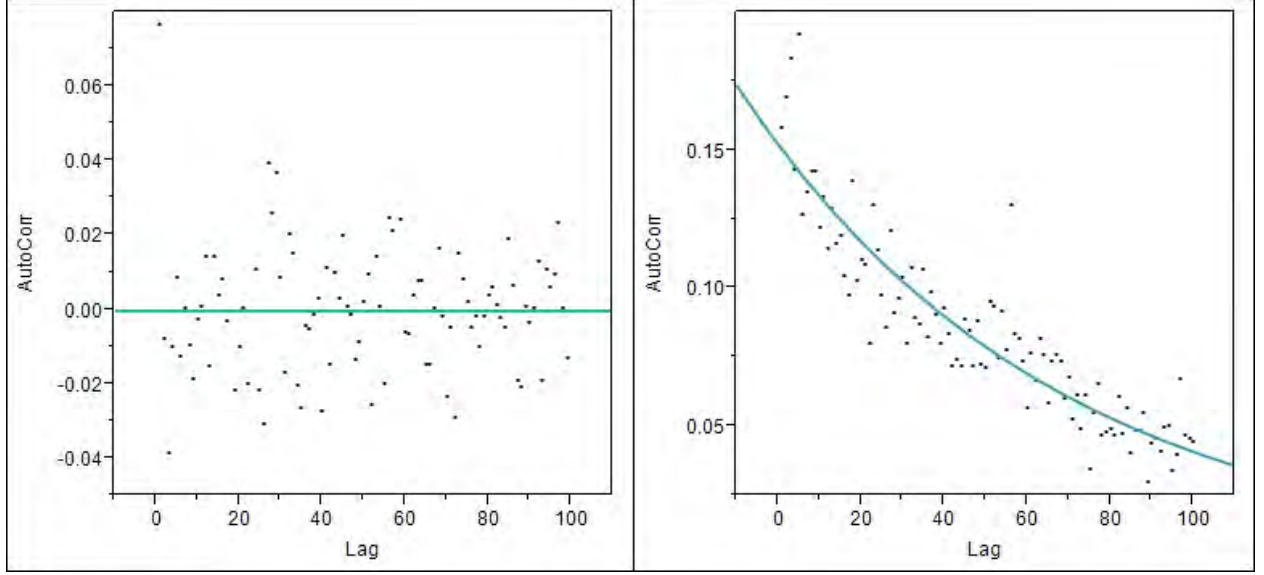
Figure 21. Plot of the first 100 lagged autocorrelations of the LAVs (left) and LAAVs (right) for the NYSE index. Data: NYSE.

$$\min \epsilon^2 = \sum_{s=1}^{\max(\Delta t)} \left(\kappa e^{\lambda s} - \kappa^* e^{\lambda^* s} \right)^2 \tag{7.21a}$$

$$\ni \lim_{m \to \infty} \| \mathbf{e}_j^{\mathrm{T}} \mathbf{T}^{(m)} \mathbf{e}_j \| = c_j \tag{7.21b}$$

$$v^- - t_{j,k}^{(0)} \leq 0 \tag{7.21c}$$

$$t_{j,k}^{(0)} - v^+ \leq 0 \tag{7.21d}$$

$$j - 2^n + 1 \leq 0 \tag{7.21e}$$

$$k - 2^n + 1 \leq 0 \tag{7.21f}$$

The mathematical program in (7.21) provides an approach to approximate the volatility clustering of the Σ_0 Distribution through the transition probability matrix $\mathbf{T}^{(0)}$ for a given set of empirical data for which the regression function parameters are known. The objective function minimizes the squared error between the empirical regression function and the simulated regression function of the autocorrrelations. Constraint (7.21b) ensures

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that the transition probability matrix $\mathbf{T}^{(0)}$ reaches the required mixture of the Σ distribution components at the population size as previously discussed. Constraints (7.21c) and (7.21d) handle minimum and maximum transition probabilities respectively. These may be set by interested parties based on specified limits. Constraints (7.21e) and (7.21f) ensure that the indexes are within the dimensions of $\mathbf{T}^{(0)}$. The inputs for this mathematical program involve the empirical autocorrelation parameters κ and λ; the mixture vector $\boldsymbol{c}$; minimum and maximum thresholds of the transition probabilities, v^- and v^+, respectively; and the number of interacting factors, or size of the Σ Distribution, n. The mathematical program then varies $t_{j,k}^{(0)}$ with respect to these constraints to minimize the squared error between the empirical regression function and those calculated from the Σ Distribution simulation, namely κ^*, and λ^* for the same empirical size.

To show that the mathematical program (7.21) will converge to the objective function for some value of n, we provide a proof. For this, we must show that the autocorrelation of its random values follow an exponential decay relative to lag which is consistent with financial data. For this we need to show that the limiting probabilities of the LAAVs are reached exponentially.

Theorem 2: *The LAAV autocorrelation of artificially generated values of the Σ Distribution with IAF follows an exponential decay over time.*

PROOF

At steady state, further transition has no effect and therefore we have (7.22).

$$\boldsymbol{T}^{(\infty)} = \boldsymbol{T}^{(\infty)}\boldsymbol{T}^{(0)} \tag{7.22}$$

If we assume for now that $\boldsymbol{T}^{(0)}$ is diagonalizable, then $\boldsymbol{T}^{(0)}$ may be decomposed into (7.22) where $\boldsymbol{S}$ is the eigenvector matrix and $\boldsymbol{\Lambda}$ the eigenvalue matrix [61]. Since we previously showed in the proof of Theorem 1 that infinitely many $\boldsymbol{T}^{(0)}$ are constructable that provide

the same limiting probability, if we would develop a $\boldsymbol{T}^{(0)}$ what would not be diagonalizable, we would search for one that would be. Furthermore, we assume that the eigenvalues of $\boldsymbol{\Lambda}$ are λ_1 to λ_{2^n-1} and are necessarily distinct and sorted from smallest to largest, or $\lambda_1 = \lambda_{(0)}$ to $\lambda_{2^n-1} = \lambda_{(2^n-2)}$. This is a further criteria for $\boldsymbol{T}^{(0)}$ selection.

$$\boldsymbol{T}^{(0)} = \boldsymbol{S\Lambda S^{-1}} \tag{7.23}$$

If we apply (7.23) to (7.22), then we obtain (7.24).

$$\lim_{w \to \infty} \boldsymbol{T}^{(w)} = \lim_{w \to \infty} \prod_{j=1}^{w} \boldsymbol{S\Lambda S^{-1}} \tag{7.24}$$

Since $\boldsymbol{S^{-1}S = I}$, (7.24) may be simplified [61] to (7.25).

$$\lim_{w \to \infty} \boldsymbol{T}^{(w)} = \lim_{w \to \infty} \boldsymbol{S\Lambda}^{(w)}\boldsymbol{S^{-1}} \tag{7.25}$$

If u_w then represents the solution at iteration w, then

$$u_w = \boldsymbol{S\Lambda S^{-1}} = \sum_{j=1}^{2^n-1} c_j \lambda_j^w x_j. \tag{7.26}$$

By factoring the largest eigenvalue term, namely λ_{2^n-1}, we obtain

$$u_w = \boldsymbol{S\Lambda S^{-1}} = \frac{1}{\lambda_{2^n-1}^w} \sum_{j=1}^{2^n-1} c_j \left(\frac{\lambda_j}{\lambda_{2^n-1}} \right)^w x_j. \tag{7.27}$$

Since λ_{2^n-1} dominates the decay and this decay is exponential, this implies that the decay of this process is also exponential. This indicates that the limiting probabilities are reached exponentially and thus may be approximated by (7.21). $\qquad\square$

7.7 Adjusting Volatility Clustering

Although most interest in volatility clusters would be from empirical data, it may be of tactical interest to understand which interactions would yield a more volatile situation. Therefore, it is of interest to understand how the dynamics of the system change when certain transitions are emphasized over others. For this we use a generic examples to illustrate this concept.

Let ψ be the Cauchy(0,1) Distribution for this example. The motivation to choose this distribution is due to its difficulty to use relative to other possible distributions (e.g., undefined mean and variance). Furthermore, we retain the value of $n = 2$. Again, we utilize (7.15) but set the integrand limits to $[-10, 10]$. We obtain $\boldsymbol{\sigma} = \{0.740, 2.462, 3.202\}$. From Theorem 1, we know that there are infinitely many transition states which will lead to the same end state. However, these different transition states may have different properties. Figure 22 depicts three possible configurations where $v^- = 0.001$ and $v^+ = 0.999$ and the objective function maximizes the transition probabilities depicted in black. The transitions depicted in black were those transition probabilities maximized. Not all states produce recognizable volatility clustering. The state at the bottom has the only recognizable pattern of volatility clustering. This is due to higher autocorrelation of the LAAVs of this transition matrix over the others. Their approximate autocorrelations are respectively 0.123, 0.121, and 0.230.

From the first case, we observe that a low transition probability between different states does not indicate high volatility clustering. The autocorrelation of its LAAVs is 0.123 which is relatively low and does not produce well-defined volatility clusters. The second case maximizes the transition probability between those states which have highest variance but does not maximize all states returning to themselves. Its volatility clustering is similar to the first case with a value of 0.121. The third case maximizes the transition from each state

to itself while maximizing the flow of State 2 to State 3. This produces an aggregation in State 3 which only transitions to State 1 with a 0.016 probability. Furthermore, State 2 only transitions to State 1 with a 0.025 probability. This indicates that States 2 and 3 transition the majority for the time between each other with an eventual, and relatively improbable, transition to State 1. Since State 1 have a much lower variance than States 2 or 3 there will be a transition from relatively high variance to one with a relatively low variance *ceteris paribus*. This will cause volatility clustering and is is indicated by an autocorrelation in the LAAVs with a value of 0.230.

The objective then is to cluster the σ vector values to achieve similar conditions for other target distributions as well as different n-values. To achieve high volatility clustering, it is necessary to achieve a very low (near zero) transition probability between clusters of low volatility and clusters of high volatility. For Case 3, we achieve just that. If we define State 1 as Cluster 1 and States 2 and 3 as Cluster 2, then we obtain nearly a twofold increase in autocorrelation over the previous two examples. Until now this process was applied manually in that several computer programs were employed to carry out this process. The next step would be to automate this approach in a single application. As previously indicated, clustering the σ values of the Σ Distribution based on size is the initial step to determine the range of volatility clustering available.

Once these clusters have been formed, the next step it to maximize the transition probabilities within each cluster and to minimize the transition probabilities outside the clusters. To obtain the maximum volatility (or the upperbound), we solve the mathematical program in (7.28). Equation (7.28a) is the objective function where we wish to maximize in order to obtain an upperbound on volatility clustering available; the constraint in (7.28b) provides the necessary Σ distribution component mixture necessary; the constraint in (7.28c) and (7.28d) set lower and upperbounds on the transition probabilities, respectively; the constraints in (7.28e) and (7.28f) are the constraints which permit only those index values which are within

the dimension of the Markov transition matrix; the constraint in (7.28g) ensures that the index values are integer.

$$\max Z = \sum_{j=1}^{2^n+1} \sum_{k=1}^{2^n+1} b_{j,k} t_{j,k} \tag{7.28a}$$

$$\text{s.t. } \lim_{k \to \infty} \|\mathbf{e}_j^{\mathrm{T}} \mathbf{T}^{(k)} \mathbf{e}_j\| = c_j, \forall j \tag{7.28b}$$

$$v^- - t_{j,k}^{(0)} \leq 0 \tag{7.28c}$$

$$t_{j,k}^{(0)} - v^+ \leq 0 \tag{7.28d}$$

$$j - 2^n + 1 \leq 0 \tag{7.28e}$$

$$k - 2^n + 1 \leq 0 \tag{7.28f}$$

$$j, k : \text{integer} \tag{7.28g}$$

The b-values are zero if $t_{j,k}$ is not a transition probability within a cluster and the b-values are one if $t_{j,k}$ is a transition probability within a cluster. Setting up (7.28a) through (7.28g) for our example, we obtain (7.29a) through (7.29f) which provides a maximum volatility clustering of 0.230. The individual objective terms of $t_{j,k}$ were chosen to maximize the transition of high volatility states to other high volatility states, while ensuring that low volatility states remain with a high probability in low volatility states.

$$\max Z = t_{1,1} + t_{2,2} + t_{3,3} + t_{2,3} + t_{3,2} \tag{7.29a}$$

$$\text{s.t. } \lim_{k \to \infty} \|\mathbf{e}_j^{\mathrm{T}} \mathbf{T}^{(k)} \mathbf{e}_j\| = c_j, \forall j \tag{7.29b}$$

$$v^- - t_{j,k}^{(0)} \leq 0 \tag{7.29c}$$

$$t_{j,k}^{(0)} - v^+ \leq 0 \tag{7.29d}$$

$$j - 2^n + 1 \leq 0 \tag{7.29e}$$

$$j, k : \text{integer} \tag{7.29f}$$
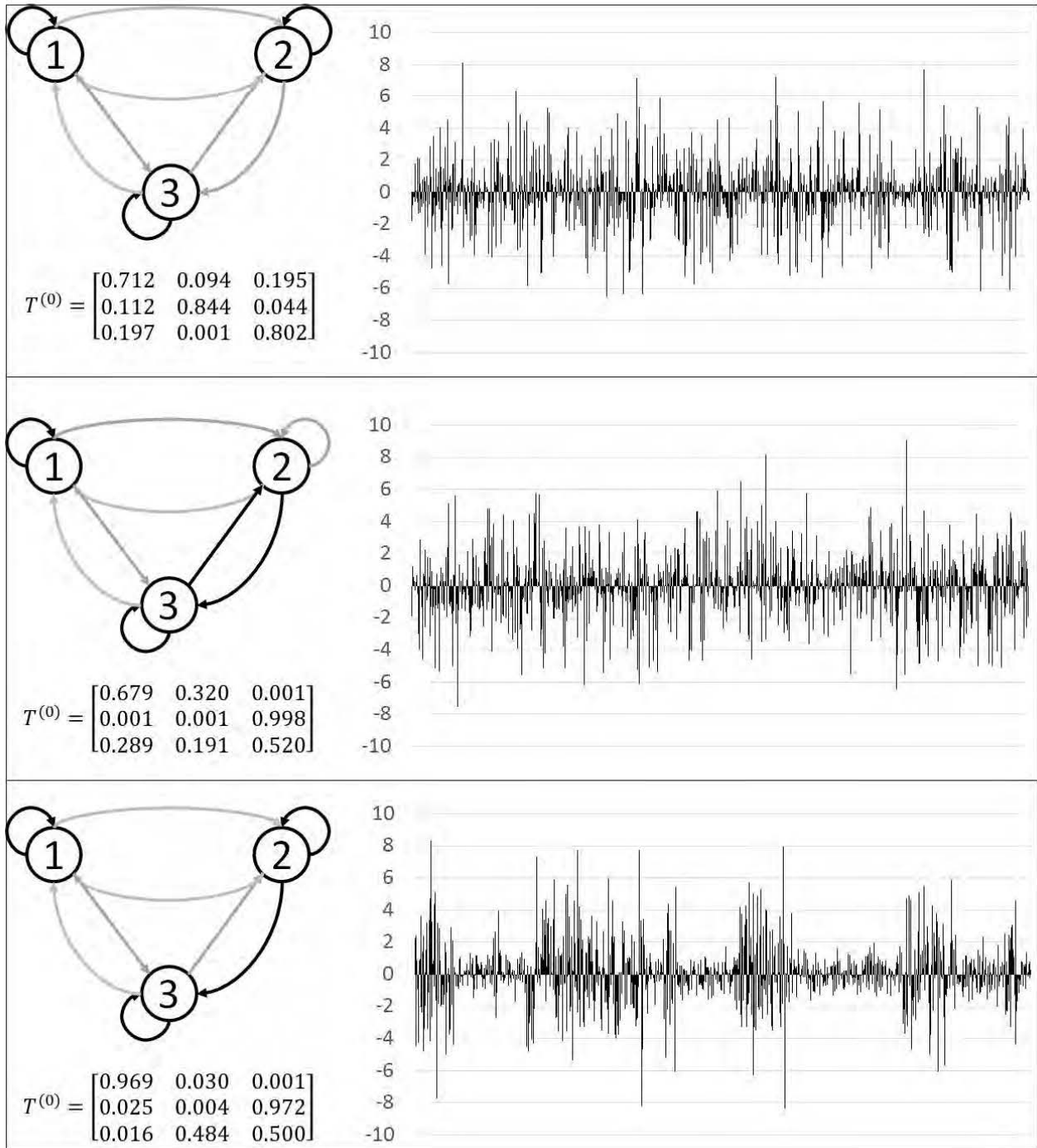
Figure 22. Approximate state transitions with simulated output.

7.8 Non-Normal Scaling Behavior

There is a preponderance of evidence to suggest that the scaling behavior of financial data cannot be accurately modeled by a Normal Distribution. Johnson showed that for the NYSE and Shanghai markets indexes their LAVs are not distributed normally for varying time scales [30]. Non-normal scaling behavior is imitated through component mixtures of the Σ Distribution as well as the transition probabilities of the IAF. To test this, we conduct a series of hypotheses tests for varying lags Δt. Figure 23 illustrates how the Anderson-Darling test statistic of the NYSE index compares to Anderson-Darling statistics of 20 instances of the Σ Distribution with IAF model utilizing a Cauchy(0,1) Distribution. The scaling properties for the data generated in is the proposed manner is non-trivial and exhibits the same properties as actual financial data [30]. Previous analysis entertained $1 \leq \Delta \leq 32$ (horizontal dashed line). For most replications the proposed method provides data which meets these values of Δ as well as much higher values. On average, the Anderson-Darling test statistic values are a magnitude higher than the critical value implying that in many instances the values generated are far removed from trivial non-stationarity properties.

Although the model still utilizes the Cauchy (0,1) Distribution it would be important to note that regardless of distribution chosen this behavior of non-normal scaling remains intact. The results indicate that neither the NYSE index nor the model pass the hypothesis test for normality at different lag scales and thus are unlikely to be from a Normal Distribution. Furthermore, the imitation provides similar test statistic values for all runs conducted at the $\alpha = 0.010$ level.
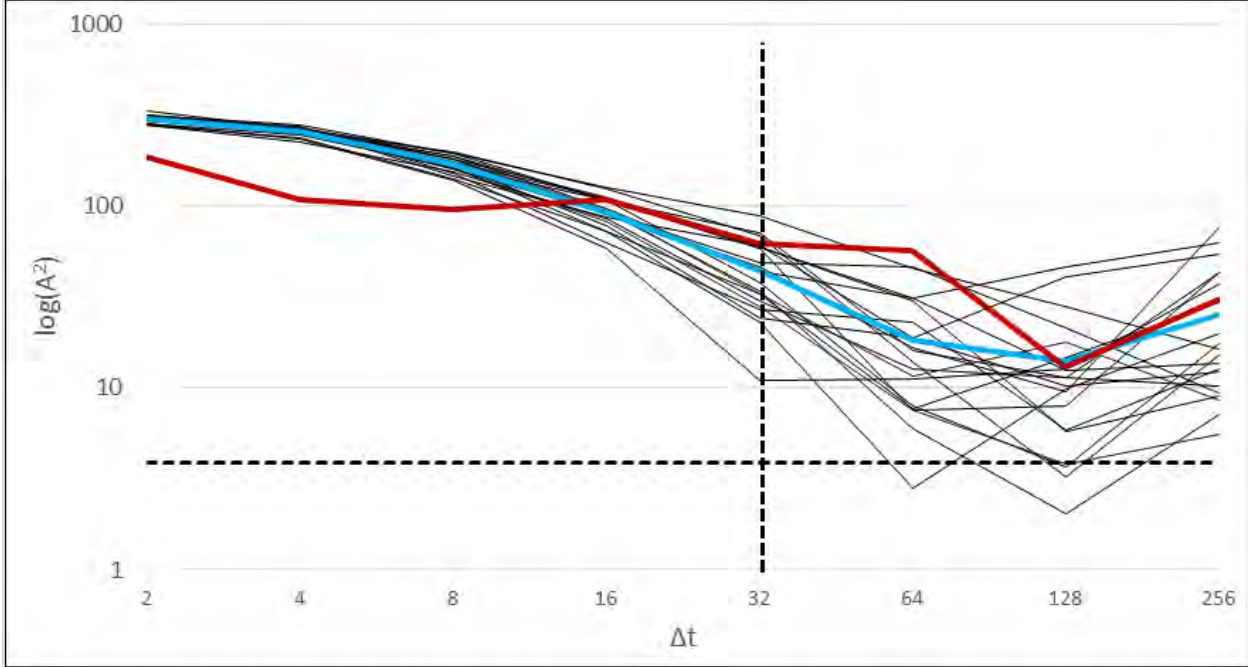
Figure 23. Log-scaled Anderson-Darling test statistic values for the 20 replications of 1,000 artificial data points each using the Cauchy(0,1) as φ and $n = 2$ for different delay values Δ. The critical value for all replications 3.907.

7.9 Current Imitation Limits

7.9.1 IAF Limitations.

Although the Σ Distribution coupled with the IAF may assist in modeling econophysical and other data sets, it is no panacea. As the number of factors n increases, the accuracy between the theoretical distribution and the Σ Distribution tends to improve. However the IAF, at least in its current state, may provide diminishing returns. As we have suggested, volatility clustering may become more pronounced as n increases. This however is based on how individual Σ components may be clustered in order to increase the LAAV autocorrelation. Figure 24 provides a summary of σ values for $1 \leq n \leq 6$. In the case of $n = 1$ the solution is trivial since only one component is present and thus no clustering is possi-

Table 16. Twenty replications of Anderson-Darling test statistics for various delays Δt for Σ Distribution approximation with the Interactive Assistance Framework (IAF) for $n = 2$ for the Cauchy(0,1) Distribution.

Replication	Δt							
	2	4	8	16	32	64	128	256
1	295.160	251.770	176.210	93.507	34.271	5.984	2.029	7.146
2	285.990	245.830	162.980	71.888	32.936	7.649	14.542	29.769
3	313.930	267.320	194.160	107.460	37.745	7.911	3.658	17.590
4	305.780	255.170	159.580	81.529	28.095	7.590	3.840	5.571
5	316.310	279.290	195.730	126.750	88.807	45.807	28.208	16.277
6	301.770	254.730	164.730	107.100	71.083	16.144	11.438	42.899
7	311.980	271.920	152.700	62.481	27.183	23.018	5.823	12.586
8	278.550	236.240	140.110	72.565	23.074	2.781	9.948	19.792
9	284.310	233.890	139.070	58.808	10.901	11.267	12.611	37.218
10	302.950	263.290	176.330	109.490	59.479	30.047	5.771	9.012
11	331.300	263.290	186.890	107.700	58.874	7.590	7.902	42.820
12	278.670	225.480	154.320	85.688	29.961	12.677	11.375	10.179
13	301.080	264.970	187.160	95.944	43.910	31.367	46.301	63.092
14	302.020	268.890	146.150	86.967	60.940	18.850	40.649	54.677
15	306.650	263.690	182.190	90.642	45.957	16.615	9.526	76.752
16	301.880	261.490	197.890	123.880	68.448	13.827	3.249	15.264
17	307.270	262.440	158.710	91.379	48.409	46.127	21.502	9.378
18	308.960	269.630	180.630	95.062	32.943	11.501	17.739	8.609
19	316.620	254.020	168.870	77.771	24.050	18.980	10.284	12.219
20	310.580	266.150	182.450	112.450	59.027	30.985	12.411	13.738
Crit A^2	3.907	3.907	3.907	3.907	3.907	3.907	3.907	3.907
Average	303.088	257.975	170.343	92.953	44.305	18.336	13.940	25.229

ble. Clustering is pronounced for values of $n = 2, 3$ and 4, but it begins to spread out near uniformly for $n \geq 5$.

In order to determine approximate bounds on the autocorrelation that the Σ Distribution and IAF can generate for different n, we tested over 100 replications for $n = 2, 3$. The distribution for all remained the Cauchy(0,1) with Σ Distribution approximation using the regression approximation technique described previously. Since the dynamics of each cluster changes based on the value of n, this would result in the dynamics of the transition probabilities and would have an affect on volatility clustering. For $n = 2$, the minimum and maximum autocorrelation of their LAAVs was 0 and 0.344 respectively while for $n = 3$ they were 0
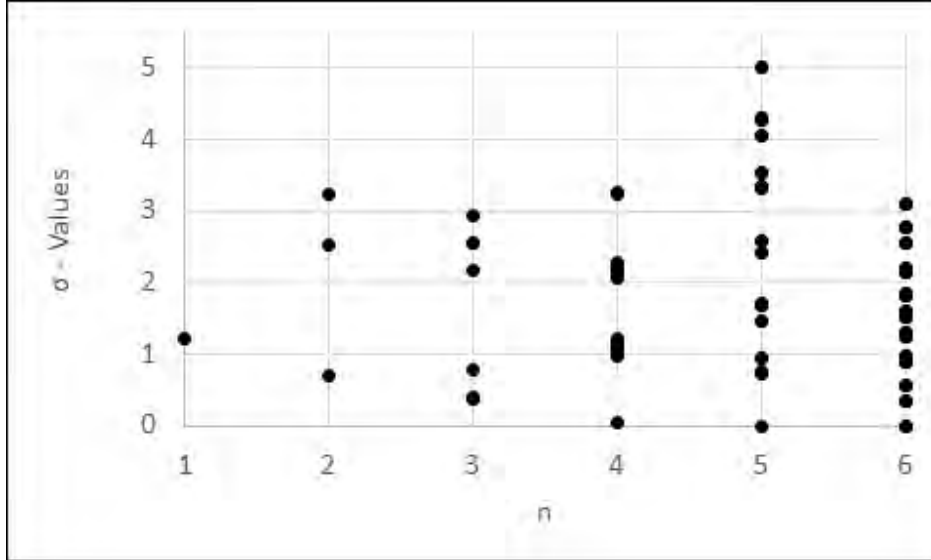
Figure 24. Depiction of σ values for $1 \leq n \leq 6$. These values are mostly heterogeneously distributed for $n \leq 4$ while values become more homogeneously distributed at higher n. This indicates that as n increases, clustering may be of diminishing returns.

and 0.456 respectively. This implies that as clustering becomes more pronounced, higher autocorrelation of its magnitudes become more feasible. This would suggest that volatility clustering would only increase until $n = 4$ since higher values of n start to spread homogeneously between one another. From the investigation in Chapter 4, securities LAAVs from the S&P 500 have are log-normally distributed and have values in the range $[0.024, 0.509]$. Thus for most securities a $n \leq 3$ would suffice to provide a viable model for distribution approximation and volatility clustering.

7.9.2 Exponential Decay of Imitation Remains too Exact.

Although the current approach does provide a slow and exponential decay of the LAAVs, the regression values of their exponential decay remain too exact when compared to those of actual financial data. Figure 25 illustrates that R^2 for the imitated LAAV tends to be much higher ≈ 0.977 when compared to actual financial data. Since autocorrelation of LAAVs originates from the IAF setup and the limiting probabilities of the Markov transition matrix

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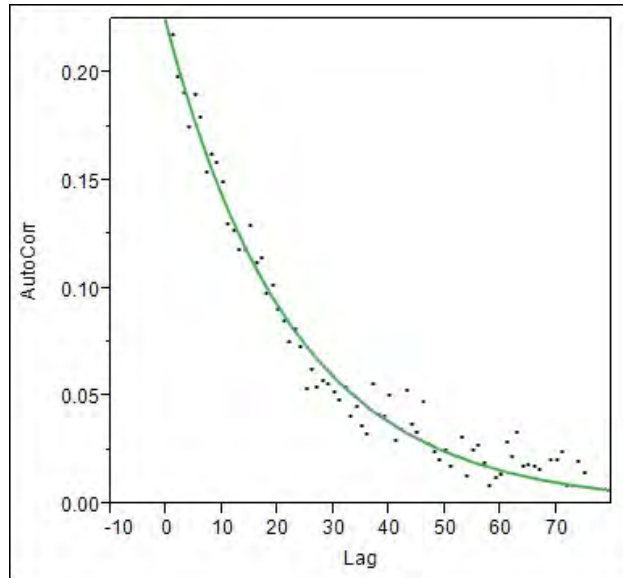
Figure 25. Example of exponential fitting of the Σ Distribution with IAF for the Cauchy(0,1) Distribution.

provide the correct mixture of Σ Distribution components, this suggests that in order to degrade the exponential fitness, a certain level of tolerance should be built into the mixture coefficients. This may lead to a less precise fit of the financial data but yield the required degradation in exponential fitting. This behavior is a topic for future study.

7.10 Summary

The Σ Distribution with IAF provides a valuable mechanism to imitate certain econophysical market behaviors which are replicable. The Σ Distribution allows for flexibility in choosing parameters which can be tailored for a specific asset while maintaining those attributes most enjoyed by the Normal Distribution. This ability was demonstrated with several PDFs to provide evidence that suggests this approach is viable with negligible information loss. This chapter also showed that the IAF complements the Σ Distribution by enabling econophysical extensions as near zero autocorrelation of the LAVs, large autocorrelation of the LAAVs which persisted over a relatively long time, non-normal scaling behavior,

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and different distributional behaviors as the sampling period moves from a micro-scale toward a macro-scale. Since Bachelier's work is based on the Normal Distribution and the Normal Distribution is a subset of the Σ Distribution with IAF, we may expand Bachelier's work with this approach. This approach would then provide one approach to bridge the gap between Standard Finance Theory and econophysics. It suggests that Bachelier's results are not incorrect and may serve as a specialized scenario of a larger framework. Thus, assuming that each Σ Distribution component is normally distributed, each micro-scale component behaves exactly as Bachelier assumed, yet when aggregated to the macro-scale behaves more along the lines as expressed through econophysics.

VIII. Conclusions & Future Research

8.1 Conclusions

Unrestricted warfare is an instrument for nation states, interest groups, or individuals to conduct operations against a technologically superior foe. The battlespaces utilized to carry out these operations need not be considered combat operations in the traditional sense, since many of these operations could be considered military operations other than war. Large countries with formidable militaries such as the United States, the Russia, France, and Israel have had difficulty adjusting their traditional tactics to those adversaries employing unrestricted-warfare type tactics. Therefore, against foes such as these, operations of this sort offer much incentive for their use.

Therefore, the United States must protect itself against these types of threats. Unlike other types of warfare, unrestricted warfare enables ambiguity and non-repudiation which makes retaliation difficult at best. To ensure a high likelihood of ambiguity and non-repudiation, a foe should search for those domains which accommodate these attributes. The financial and cyber domains fulfill this prerequisite. Both of these domains possess many heterogeneous interacting agents. The distribution of their variability is leptokurtic in nature. This allows for a higher probability of more extreme events. By exploiting these effects it is far more difficult to determine whether an extreme event occurred due to the inherent dynamics of the system or due to malicious intent.

One area of concern for such operations is how they may influence the U.S. dollar as a reserve currency. Schilling pointed out the attributes which are likely contributors to a reserve currency. This study illustrated that there is a link between the levels of equity capital markets of a reserve currency country and the COFER levels of that currency as a store of value. The analysis provided evidence that, for the three largest non-currency union reserve currency countries, (i.e., the United States, Japan, and the United Kingdom),

a linear relationship exists. The relationship was positively correlated and was strongest for the United States ($R^2 = 0.927$) and weakest for Japan ($R^2 = 0.685$). For the United Kingdom, the relationship is negatively correlated but still retained a high level of linearity ($R^2 = 0.877$). This suggests that, at least for the U.S. dollar, a strong relationship between equity capital and COFER reserves exists.

Further analysis showed that capital markets' levels relative to reserve currencies' COFER levels divide into two clusters for which the currencies of each cluster behave similarly. The first cluster's makeup is the USD, JPY, and CHF, and the second cluster's makeup is EUR and GBP. This suggests that the behavior of these two clusters is different for the same equity capital market conditions. Thus, a rise in those equity capital markets which aid the USD, JPY, and CHF may cause a negative influence on those of the EUR and GBP. Thus an increase in those equity capital markets which emphasize the EUR or GBP and vice versa.

Many of the strongest EMU members such as Finland, the Netherlands, and Germany seem to aid the resiliency of the USD cluster despite being a part of the Eurozone currency union. Many of the Eurozone members are contributors to the USD, JPY, and CHF cluster, and the BRICS, and non-reserve currency, and non-EMU members are contributors to the euro. Therefore, by not having a reserve currency of their own, the BRICS nations can intend to weaken the USD by investing in the euro. Therefore, the vulnerabilities which are persistent in equity capital markets pose risks to the status of a reserve currency.

Although unrestricted warfare does pose risks to the financial system, under-representing uncertainty that is inherent in financial systems could pose a structural risk. This idea was studied by postulating a formulation that leads to volatility clustering and showed that the aggregate distribution would be more leptokurtic than the Normal Distribution would afford. This suggests that a higher probability would exist for more extreme events, or Black Swans. Analysis indicated that the autocorrelation of the log-adjusted absolute values are positively correlated with the number of outliers for the 221 large cap stocks analyzed. This suggests

that the autocorrelation, which leads to volatility clustering, may assist in achieving extreme fluctuations in asset pricing and may contribute directly to a higher probability of extreme events. Furthermore, any combination of assets which have synchronized volatility clusters add volatility as well. Therefore, any strategy that incorporates assets with these attributes are more exposed to uncertainty than is accounted for by utilizing the standard finance tools.

The analysis indicated that Financials (specifically REITs), Reports, and Consumer Discretionaries such as Industrial Conglomerates provide the highest levels of autocorrelation while Financials in general provided the highest levels of synchronized volatility clustering. This implies that these industries, relative to the spectrum of industries under consideration, possess the highest probability of risk due to extreme events such as Black Swans. Therefore underrepresented risk in these industries or the malicious exploitation of these industries have the highest likelihood of being vulnerable to tactics which emphasize unrestricted warfare-type tactics.

However, knowing where the vulnerabilities lie is only one part of an overall strategy. Of importance is detecting warning signs that an attack may be imminent. The early warning techniques in this study showed that financial indicators may assist in this effort. It also demonstrated that certain commodities with respect to reserve currencies may provide an advantage over non-reserve currency options. This suggests an additional yet incidental benefit of a reserve currency as an anomaly detector. beyond the typical advantages of reserve currencies. Although the subset of currencies tested would suggest this, it remains to be determined whether this is a tautology to the full set of currencies that exist.

This hypothesis was tested by first developing the φ-measure and mapping its outliers to dates. The analysis indicated that many of the outliers map directly to dates of large world events such as terrorist attacks or large economic turmoil. A further hypothesis was posed that volatility clustering, which is ubiquitous in asset pricing, may begin before such an event occurs. This may indicate that *a priori* information is present for this event and individuals,

interest group, and nation states may be exploiting it for economic gain. Therefore the K- and S-measures were developed to test for an increase in volatility clustering and those outliers that form from the K-measure, respectively. These results showed that for many events volatility clustering predates Black-Swan-type events as the USS Cole Bombing, the 9/11 Attacks, the Sub-prime Mortgage Crisis, and the Global Financial Crisis; the range of lead times is between two and ten days. Furthermore, certain currency pairs provide a better indication of warning than others. As previously indicated, reserve currencies (specifically the USD, JPY, and CHF) seem to provide the best results. Although non-reserve currencies did provide warning signals as well, their results were not as pronounced as those with reserve currencies.

These results suggest that reserve currencies coupled with certain commodity pairs are more effective than non-reserve currencies at warning of a vulnerability or attack. Therefore retaining a reserve currency such as the USD may be advantageous not only from a monetary policy perspective but also in terms of threat warning for homeland security.

However these results utilized a finite data set of currency and commodity data. Furthermore, the measures developed were only tested on empirical data. These results may differ if the dynamics of the market may change. There does not seem to be an acceptable way to test these approaches other than for past data. Therefore the final contribution focused on developing an approach to generate data sets that exhibit tailorable characteristics which fit the aforementioned financial attributes.

The Σ Distribution with Interactive Assistance Framework (IAF) illustrated an approach regarding how data streams may be generated to imitate financial data between the spectrum of standard finance theory and econophysics. The Σ Distribution provides a method to approximate the probability distribution for the data set under consideration. For each of the components of the Σ Distribution, the Normal Distribution was the distribution of choice. Since it is a hierarchically invariant and it is the distribution derived by Bachelier for

uncertainty in financial data, this was the distribution chosen, but is not the sole option. This further ensures that imitating financial data according to the standard finance assumptions is a subset of capabilities of the proposed approach. For many distributions, several components may be necessary to reach the desired accuracy. This may be achieved by increasing n, the number of influencing mixture component PDFs in the aggregate PDF.

The IAF provides the transition mechanism between these component PDFs. By utilizing a Markov chain, the IAF enables the transition between different Σ Distribution components. By ensuring that the limiting probabilities reach the necessary mixture values for the Σ Distribution, the distribution of the artificially generated data will imitate the distribution of the financial data to include those attributes that expand standard finance theory to econophysics. From Theorem 1, we know that infinitely many initial conditions for the Markov chain exist, we can tailor these to fit many different combinations of initial conditions. This study illustrated how this proposed approach can integrate certain econophysical attributes observed in empirical data. A wide range of autocorrelations of the log-adjusted absolute values were achieved (between 0 and 0.465) while maintaining very low autocorrelation of the log-adjusted values themselves[1]. The decay of the autocorrelation of the log-adjusted absolute values was exponential which is in line with empirical observations. Those artificially generated data sets with relatively high autocorrelations of their log-adjusted absolute values exhibited volatility clustering. This is a direct consequence of large autocorrelations of this type.

Another attribute achieved was non-stationarity of the distribution under consideration. By allowing a component-wise approach, the aggregate distribution will not achieve stationarity except in the limit. The distribution will continue to change over time and is dependent on the Markov chain configuration. In addition, non-normal scaling properties which are present in empirical data were also observed in the artificially generated data stream. This

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[1]Average autocorrelation was 0.023.

suggests that, as empirical data does not have trivial scaling properties, neither does the proposed approach.

In summary, this research demonstrates that financial warfare may be waged against a foe and that financial assets such as currency may have more uses to national security than meets the eye. This research showed that a currency may not only be utilized as a weapon of choice but also may serve as a warning signal if viewed through proper techniques. Finally, to test different approaches in these areas, it is necessary to develop a framework for scientific testing. The imitation framework was one approach to achieve this outcome. This dissertation developed and illustrated different military applications to a seemingly inert domain of warfare.

8.2 Future Research

8.2.1 Incorporating the Σ Distribution with IAF into Standard Finance Theory.

The Σ Distribution with IAF indicated an alternative approach in representation of financial concepts. By utilizing this alternative approach, it may be possible to expand traditional methods without the difficulty of non-standard distributions. Expanding Standard Finance Theory by utilizing the proposed method might enable a compromise between traditionally accepted approaches and those deemed atypical. Since the Σ Distribution allows for an approximation of a distribution through a mixture of Normal Distributions, we may expand traditional techniques that are based on a single Normal Distribution. This would allow a viable expansion to the standard finance approach with minimal additional complexity in mathematical formulation.

8.2.2 Extending & Automation of the K- and S-Measures.

For this study the K- and S-measures were only used for a very small subset of commodities and currencies. Preliminary results beyond those in this study also hold promise. There may be added utility by expanding the commodity selection beyond gold and silver, which was the focus in this research, to other precious metals and commodities. Furthermore, the techniques explored required several computer applications to retrieve the necessary empirical data, perform statistical testing, and calculate the developed measures. A logical progression would be to develop an application which would automate these steps.

In addition, the expansion of these measures to detect the onset of volatility clustering, signal amplification, and anomaly detection may provide a far more versatile and effective early warning system to benefit the intelligence community's efforts to identify and counter those aforementioned unrestricted warfare-type operations.

8.2.3 Incorporating Log-Periodic Power Laws into the Σ Distribution with IAF.

Thus far, the Σ Distribution with IAF addresses specific econophysical attributes which the overall data set should exhibit. It does not address any specific dynamics which would lead to extreme events. One such attribute that would be of further interest is the incorporation of those mechanisms which lead to the log-periodic power law (LPPL) as described by Sornette [58, 59, 60] and Jacobsson [29], among others. This log-periodic oscillation describes the behavior of a speculative bubble and its following crash. Its functional form may be described as in (8.1), where p is the price of an instrument at time t, t_c is the time where a crash is most probable, z is the exponential growth parameter, ω is the oscillation amplitude, and the terms A, B, C, and Φ are parameters with no particular structural interpretation.

$$p(t) = A + B(t_c - t)^z + C(t_c - t)^z \cos(\omega \log(t_c - t) + \Phi) \tag{8.1}$$

Equation (8.1) is the typical setup of the LPPL behavior. Currently, the IAF has not been evaluated to determine whether this mechanism can be incorporated into the current IAF and, if so, which modifications would need to considered. The incorporation of this behavior into the current IAF would provide added benefit to the imitation of realistic model financial market performance and risk.

This study focused on utilizing the Normal Distribution as the distribution utilized for the Σ Distribution components. The Normal Distribution is only one of many distributions which is a solution to the hierarchically invariant equation. Due to its ease of use, it was utilized to provide evidence for the utility of Chapter 7's approach. The distribution need not be Normal. Utilizing alternative distributions such as the Cauchy, Laplace, or selected others may provide a higher utility depending on the target distribution under consideration. The evaluation of when to use any of these distribution when utilizing the Σ Distribution would be of interest for further investigation. For those distributions considered to approximate, we assumed that the data perfectly fit those distributions. This will likely never be the case with real-world data. Thus, as we require a certain level of tolerance with the parameters when approximating real-world data via a distribution, we should also provide a certain level of tolerance when calculating the mixture coefficients for the IAF. Within Chapter 7, these mixture coefficients were approximated by minimizing the error. A viable extension would be to provide a range for the mixture coefficients rather than a single fixed value. This will enable the data set to account for, at least in part, non-stationarity. Lastly, for dynamic n-values, the example in Chapter 7 assumes a trivial $\mathbf{W}$ between different Σ Distributions. In future research we may relax this assumption such that the aggregate distribution generated from those n considered produce a distribution which models that empirical data under consideration. This would not require that each component fit the data and then the dynamics are switched through $\mathbf{W}$ but that the dynamics for each n may vary considerably when observed in isolation yet retain the behavior of the empirical

data when all are aggregated. This approach may provide even more versatility than that exhibited by the Σ Distribution with IAF in Chapter 7 and may also provide a way toward incorporating this LPPL behavior.

This dissertation yields a first incremental advance towards bridging finance theory and econophysics. Although a macroscopic unifying theory may never evolve, this research demonstrates useful interrelations between these fields and, in doing so, provides promising methods and tools to detect and analyze financial warfare, an element of unrestricted warfare.

Bibliography

[1] Artelli, Michael J. and Richard F. Deckro. *Fourth Generation Operations: Principles for the 'Long War'* Small Wars & Insurgencies, 19:2, pp. 221-237. Date Accessed: 2 June 2015.

[2] Bachelier, Louis *Théorie de la Spéculation.* Annales Scientifiques de l'É.N.S. 3e série, tome 17 (1900), pp. 21-86.

[3] Bak, Per, Chao Tang, and Kurt Wiesenfeld. "Self-organized Criticality: An Explanation of the 1/ F Noise." *Phys. Rev. Lett. Physical Review Letters* 59.4 (1987): 381-84. Web. 30 Oct. 2014.

[4] Bak, Per, and Stefan Boettcher. "Self-organized Criticality and Punctuated Equilibria." *Physica D: Nonlinear Phenomena* 107.2-4 (1997): 143-50. Web. 30 Oct. 2014.

[5] Black, John. *A Dictionary of Economics.* Oxford: Oxford UP, 1997. Print.

[6] Bothamley, Jennifer. *Dictionary of Theories.* London: Gale Research International, 1993. Print.

[7] Bouchaud, Jean-Philippe, and Marc Potters. *Theory of Financial Risks: From Statistical Physics to Risk Management.* Cambridge: Cambridge UP, 2000. Print.

[8] Calvo, Clara, Carlos Ivorra, and Vicente Liern. "On the Computation of the Efficient Frontier of the Portfolio Selection Problem." *Journal of Applied Mathematics* 2012 (2012): 1-25. Web. 8 Aug. 2014.

[9] Chesney, Marc, Mustafa Karaman, and Ganna Reshetar. "The Impact of Terrorism on Financial Markets: An Empirical Study." *Journal of Banking & Finance* 35.2 (2011): 253-67. Web. 20 Apr. 2015.

[10] Chou, Youn-Min, Alan Polansky, and Robert Mason. "Transforming non-Normal Data to Normality in Statistical Process Control" *Journal of Quality Technology* 30.2 (1998): 133-141. Web. 3 Sep 2014.

[11] Cowgill, Bo, Justin Wolfers, and Eric Zitzewitz. "Using Prediction Markets to Track Information Flows: Evidence from Google." *Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering Auctions, Market Mechanisms and Their Applications* (2009): 3. Web. 12 Apr. 2014.

[12] Devaney, Robert L. *An Introduction to Chaotic Dynamical Systems*. Redwood City, CA: Addison-Wesley, 1989. Print.

[13] Dunlap, Maj. Gen. Charles. "Law and Military Interventions: Preserving Humanitarian Values in 21st Century Conflicts" (2001). Web. 3 Aug. 2014.

[14] "Put up the Firewalls." The Economist. The Economist Newspaper, 13 June 2015. Web. 20 June 2015. `http://www.economist.com/news/united-states/21654078-bureaucratic-inertia-makes-life-easy-foreign-hackers-put-up-firewalls`.

[15] Eisen, Sara. *Currencies after the Crash: The Uncertain Future of the Global Paper-based Currency System*. New York: McGraw-Hill, 2013. Print.

[16] Engle, Robert F. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica* 50.4 (1982): 987. Web. 2 Feb. 2014.

[17] "Executive Order – Blocking Property of the Government of Iran and Iranian Financial Institutions." *The White House*. The White House, 6 Feb. 2012. Web. 2 Jan. 2015.

[18] Fama, Eugene F. "Mandelbrot and the Stable Paretian Hypothesis." *The Journal of Business J BUS* 36.4 (1963): 420-29. Web. 5 Sept. 2014.

[19] Farmer, J., and John Sidorowich. "Predicting Chaotic Time Series." *Physical Review Letters* 59.8 (1987): 845-48. Web. 2 Feb. 2014.

[20] Flynn, Stephen. "Resilience to Unrestricted Warfare: Threats to the Homeland". Johns Hopkins University. 20 Oct 2014.

[21] Grayson, C. Jackson, and Carla S. O'Dell. *American Business, a Two-minute Warning: Ten Changes Managers Must Make to Survive into the 21st Century.* New York: Free, 1988. Print.

[22] Gould, Stephen, and Eldredge Niles. "Punctuated Equilibria: The Tempo and Mode of Evolution Reconsidered". *Paleobiology* 3.2 (1977): 115-151. Web. 4 Jun. 2014.

[23] Gopikrishnan, P., M. Meyer, L.a.n. Amaral, and H.e. Stanley. "Inverse Cubic Law for the Distribution of Stock Price Variations." *The European Physical Journal B Eur. Phys. J. B* 3.2 (1998): 139-40. Web. 2 Oct. 2014.

[24] Plerou, Vasiliki, Parameswaran Gopikrishnan, Luis A. Nunes Amaral, Martin Meyer, and H. Eugene Stanley. "Scaling of the Distribution of Price Fluctuations of Individual Companies." *Physical Review E Phys. Rev. E* 60.6 (1999): 6519-529. Web. 3 Oct. 2014.

[25] Hammes, Thomas X. *Insurgency: Modern Warfare Evolves into a Fourth Generation.* Washington, D.C.: Institute for National Strategic Studies, National Defense U, 2004. Print.

[26] Helbing, Dirk. "A Mathematical Model for the Behavior of Pedestrians." *Syst. Res. Behavioral Science* 36.4 (1991): 298-310. Web. 1 Dec. 2014.

[27] Simonsen, Ingve, Lubos Buzna, Karsten Peters, Stefan Bornholdt, and Dirk Helbing. "Transient Dynamics Increasing Network Vulnerability to Cascading Failures." *Phys. Rev. Lett. Physical Review Letters* 100.21 (2008): n. pag. Web. 1 Dec. 2014.

[28] Ilinski, Kirill. *Physics of Finance: Gauge Modeling in Non-equilibrium Pricing.* Chichester, John Wiley, 2001. Print.

[29] Jacobsson, Emilie. "How to Predict Crashes in Financial Markets with the Log-Periodic Power Law". Statistics, Mathematical, and Stockholm University. (Examenarbete) (2009): n. pag. Web.

[30] Johnson, Neil F., Paul Jefferies, and Pak Ming. Hui. *Financial Market Complexity.* Oxford: Oxford UP, 2003. Print.

[31] Johnson, N. L. "Systems of Frequency Curves Generated by Methods of Translation." *Biometrika* 36.1/2 (1949): 149-76. Web. 15 June 2014.

[32] Vajna, Szabolcs, Balint Toth, and Janos Kertesz. "Modelling Bursty Time Series." *New Journal of Physics New J. Phys.* 15.10 (2013): 103023. Web. 1 Nov. 2014.

[33] Khintchine, Aleksandr and Paul Lévy. "Sur les loi stables". *C.R. Acad. Paris* 202 (1936): 374-376. Web. 2 Oct. 2014.

[34] Lévy, Paul. *Calcul Des Probabilités.* Paris: Gauthier-Villars, 1925. Print.

[35] Liang, Qiao, and Wang Xiangsui. *Unrestricted Warfare.* Beijing: PLA Literature and Arts House Arts, 1999. Print.

[36] Lind, William S., "Understanding Fourth Generation Warfare". *Military Review* 84.5 (2004): 12-16. Web. 5 Apr. 2014.

[37] Lind, William, Keith Nightengale, John Schmitt, Joseph Sutton and Gary Wilson. "The Changing Face of War: Into the Fourth Generation". *Military Review* 69.10 (1989): 2-11. Web. 5 Apr. 2014.

[38] Luman, Ronald R. *Unrestricted Warfare Symposium: Proceedings on Strategy, Analysis, and Technology, 14-15 March 2006.* Laurel, MD: Johns Hopkins U, Applied Physics Laboratory, 2006. Print.

[39] Mathwave Software Documentation. `http://www.mathwave.com/articles/wakeby_distribution.html`. Web. 2 May 2015.

[40] Mandelbrot, Benoit. "The Variation of Certain Speculative Prices." *The Journal of Business J BUS* 36.4 (1963): 394-419. Web. 3 Sept. 2014.

[41] Mandelbrot, Benoit B., and Richard L. Hudson. The (mis)behavior of Markets: A Fractal View of Risk, Ruin, and Reward. New York: Published by Basic, 2004. Print.

[42] Mantegna, Rosario N., and H. Eugene Stanley. *An Introduction to Econophysics: Correlations and Complexity in Finance.* Cambridge, UK: Cambridge UP, 2000. Print.

[43] Rosario N. Mantegna, and H. Eugene Stanley. "Limit Theorems and Price Changes in Financial Markets." *Philosophical Magazine B* 77.5 (1998): 1353-356. Web. 4 Oct. 2014.

[44] Mantegna, Rosario Nunzio. "Fast, Accurate Algorithm for Numerical Simulation of Lévy Stable Stochastic Processes." *Physical Review E Phys. Rev. E* 49.5 (1994): 4677-683. Web. 4 Oct. 2014.

[45] Markowitz, H. *Portfolio Selection: Efficient Diversification of Investments.* New York: Wiley, 1959. Print.

[46] Minkova, Veronika. *Can the Euro Ever Be a Global Reserve Currency?* S.l.: Grin Verlag, 2011. Print.

[47] Montgomery, Douglas C. *Introduction to Statistical Quality Control.* New York: Wiley, 1985. Print.

[48] Münkler, Herfied. "Wars of the 21st Century" *International Review of the Red Cross* 85.849 (2003): 7-22. Web. 5 May 2015.

[49] North, Douglass C., and Roger LeRoy Miller. *The Economics of Public Issues*. New York: Harper & Row, 1971. Print.

[50] Papaioannou, Elias and Portes, Richard. *The euro as an international currency vis-á-vis the dollar*. (2007). Web. 15 Oct. 2014.

[51] Poteshman, Allen M. "Unusual Option Market Activity and the Terrorist Attacks of September 11, 2001*." *The Journal of Business J BUS* 79.4 (2006): 1703-726. Web. 3 Dec. 2014.

[52] Rickards, James. *Currency Wars: The Making of the next Global Crisis*. New York: Portfolio/Penguin, 2011. Print.

[53] Rickards, James. *The Death of Money: The Coming Collapse of the International Monetary System*. NYC: Portfolio/Penguin, 2014. Print.

[54] Scharf, Michael and Elizabeth Anderson "Is Lawfare Worth Defining?". *Case Western Reserve Journal of International Law* 43.1 (2010). Web. 11 Sep. 2014.

[55] Servan-Schreiber, Emile, Justin Wolfers, David Pennock, and Brian Galebach. "Prediction Markets: Does Money Matter?". *Electronic Markets* 14.3 (2004): 243-51. Web. 22 Dec. 2014.

[56] Sharpe, William "Capital Asset Prices: A Theory Of Market Equilibrium Under Conditions Of Risk*." *The Journal of Finance* 19.3 (1964): 425-42. Web. 5 July 2014.

[57] Pisani-Ferry, Jean, and Adam Simon. *The Euro at Ten: The next Global Currency?* Washington, DC: Peterson Institute for International Economics, 2009. Print.

[58] Sornette, Didier. "Critical Market Crashes." *Physics Reports* 378.1 (2003): 1-98. Web. 16 Jan. 2015.

[59] Sornette, Didier *Why Stock Markets Crash: Critical Events in Complex Financial Systems*. Princeton, NJ: Princeton UP, 2003. Print.

[60] Sornette, Didier, Woodward, R. Fedorovsky, M. Reimann, S. Woodard, H., Wei-Xing, Z. *The Financial Bubble Experiment: Advanced Diagnostics and Forecasts of Bubble Terminations Volume II-Master Document*. Cornell university Library. 2010.

[61] Strang, Gilbert. *Linear Algebra and Its Applications*. San Diego: Harcourt, Brace, Jovanovich, 1988. Print.

[62] Taillard, Michael. *Economics and Modern Warfare - The Invisible Fist of the Market*, Palgrave Macmillian, New York, NY. 2012. Print.

[63] Taleb, Nassim Nicholas. *The Black Swan: The Impact of the Highly Improbable*. Random House, New York, NY. 2007. Print.

[64] Tzu, Sun. *The Art of War*. Collins Classics, 2013. Print.

[65] Wheeler, Donald J. *Understanding Variation: The Key to Managing Chaos*. Knoxville, TN: SPC, 1993. Print.

[66] Wheeler, Donald. "Good Limits from Bad Data." *Quality Digest*. N.p., 6 July 2009. Web. 5 Mar. 2015. `http://www.qualitydigest.com/inside/quality-insider-column/good-limits-bad-data.html`.

[67] Wheeler, Donald. "Do You Have Leptokurtophobia?" *Quality Digest*. N.p., 5 Aug. 2009. Web. 5 Mar. 2015. `http://www.qualitydigest.com/inside/quality-insider-column/do-you-have-leptokurtophobia.html`.

[68] Wu, Yuan-li. *Economic Warfare*. New York: Prentice-Hall, 1952. Print.

Appendix A. Correlation of Equity Capitalization & COFER

Percentages by Country

Correlation of Equity Capitalization to COFER Percentages (1999 - 2012) (alpha = 0.01)

Legend: ▮ BRICS ▮ EMU

USD		EUR		GBP		JPY		CHF		USD		EUR		GBP		JPY		CHF	
CI	Mid	CI	Mid	CI	Mid	CI	Mid	CI	Mid	CI	Mid	CI	Mid	CI	Mid	CI	Mid	CI	Mid
FIN	0.882	BWA	0.916	CHN	0.946	NLD	0.867	CRI	0.846	TUR	-0.644	NZL	0.603	COL	0.622	MEX	-0.594	VNM	-0.505
NLD	0.868	AUS	0.900	POL	0.937	FIN	0.838	USA	0.741	IRN	-0.648	TUR	0.575	GHA	0.611	CHL	-0.607	NAM	-0.511
DEU	0.858	ROU	0.835	ROU	0.924	USA	0.824	BMU	0.703	COL	-0.650	LKA	0.574	IDN	0.569	VNM	-0.609	PSE	-0.530
USA	0.845	ESP	0.831	NOR	0.921	GBR	0.764	MLI	0.685	THA	-0.661	EGY	0.571	KNA	0.568	LTU	-0.621	IDN	-0.532
ARG	0.797	KEN	0.827	SRB	0.913	GEO	0.760	GBR	0.654	MEX	-0.671	TUN	0.560	LTU	0.556	LAC	-0.640	BGD	-0.534
GBR	0.795	PNG	0.824	BGR	0.907	CRI	0.691	ARG	0.653	LAC	-0.680	ZMB	0.560	LKA	0.535	GHA	-0.641	COL	-0.536
CRI	0.730	IND	0.819	HRV	0.902	JPN	0.648	ZWE	0.577	BHR	-0.690	UKR	0.553	ZMB	0.513	GUY	-0.646	BHR	-0.537
ITA	0.686	NOR	0.812	CHN	0.884	ARG	0.621	ITA	0.576	LCN	-0.697	TTO	0.551	PSE	0.511	LVA	-0.646	CIV	-0.541
JPN	0.611	POL	0.809	GEO	0.874	ITA	0.607	FIN	0.560	MKD	-0.702	IDN	0.548	MNG	0.503	LCN	-0.647	ZMB	-0.547
GRC	0.591	CZE	0.808	ZAF	0.874	GBR	0.511	SWZ	0.555	BGR	-0.703	MYS	0.540	THA	0.497	KGZ	-0.665	PAK	-0.547
BMU	0.473	RUS	0.804	OMN	0.874	VEN	0.451	DEU	0.464	NPL	-0.708	LTU	0.518	IRN	0.478	CIV	-0.681	CZE	-0.549
IRL	0.411	IND	0.799	PER	0.870	BMU	0.388	VEN	0.398	VNM	-0.708	BGD	0.503	LVA	0.477	UKR	-0.690	LKA	-0.549
FRA	0.411	GEO	0.796	KEN	0.860	SWZ	0.316	IRL	0.373	JOR	-0.708	LVA	0.501	BGD	0.473	ARM	-0.690	MEX	-0.550
SWZ	0.388	SVK	0.788	NGA	0.858	MLI	0.285	FRA	0.346	KNA	-0.709	PAN	0.501	TUN	0.467	NPL	-0.690	DNK	-0.551
VEN	0.321	OMN	0.788	ISR	0.835	FRA	0.228	GRC	0.346	MUS	-0.718	SLV	0.496	PAN	0.457	KOR	-0.698	NGA	-0.556
SWE	0.118	KWT	0.782	MKD	0.827	PRY	0.201	JPN	0.291	KGZ	-0.722	FJI	0.493	HUN	0.434	BRA	-0.705	GHA	-0.563
BEL	0.107	ISR	0.777	UKR	0.827	SWE	0.194	EST	0.218	ARM	-0.732	BRB	0.490	UGA	0.429	CAN	-0.715	HRV	-0.568
PRT	0.106	PER	0.767	RUS	0.826	URY	0.177	PRT	0.166	HRV	-0.733	MNG	0.482	NAM	0.417	EGY	-0.716	LUX	-0.570
PRY	0.105	QAT	0.767	LUX	0.824	ZWE	0.049	CYP	0.109	CHL	-0.733	SGP	0.464	MLT	0.414	SVN	-0.722	MUS	-0.589
URY	0.052	DNK	0.758	LBN	0.819	PRT	-0.002	FJI	0.100	LUX	-0.739	UGA	0.450	PHL	0.391	QAT	-0.741	GEO	-0.593
CYP	0.041	BRA	0.749	PAK	0.818	BEL	-0.069	TZA	0.099	CAN	-0.740	AUT	0.433	MYS	0.388	NGA	-0.749	ZAF	-0.597
ZWE	0.023	ARE	0.748	MAR	0.816	EST	-0.075	BRB	0.059	ARE	-0.740	HUN	0.397	NZL	0.336	KNA	-0.750	ARM	-0.599
ISL	-0.076	LBN	0.747	ZAF	0.814	TZA	-0.080	BEL	0.042	CIV	-0.740	BOL	0.394	TTO	0.261	PER	-0.760	ARE	-0.603
EST	-0.089	PAK	0.747	KOR	0.806	ZWE	-0.156	JAM	0.036	GHA	-0.743	PSE	0.385	ISL	0.249	BHR	-0.766	KWT	-0.604
JAM	-0.192	SRB	0.745	PNG	0.805	HKG	-0.195	PRY	0.011	GUY	-0.750	NAM	0.344	SGP	0.248	LBN	-0.769	PNG	-0.605
MLT	-0.207	KNA	0.740	KAZ	0.803	NAM	-0.196	ISL	-0.025	KAZ	-0.750	PHL	0.335	CYP	0.241	MAR	-0.770	NPL	-0.607
HKG	-0.229	MAR	0.740	ZAF	0.784	PHL	-0.229	HKG	-0.104	ZAF	-0.751	TZA	0.267	BOL	0.215	BRA	-0.775	KGZ	-0.609
TZA	-0.266	KAZ	0.735	KWT	0.779	ISL	-0.240	BOL	-0.153	NGA	-0.760	JAM	0.257	HKG	0.194	LUX	-0.776	BGR	-0.610
PSE	-0.381	KOR	0.729	BRA	0.777	JAM	-0.249	PHL	-0.172	BWA	-0.762	HKG	0.222	URY	0.194	KAZ	-0.786	LAC	-0.612
BOL	-0.383	GHA	0.726	BWA	0.769	SGP	-0.303	HUN	-0.213	PAK	-0.763	MLT	0.189	FJI	0.129	IND	-0.786	NOR	-0.614
HUN	-0.385	CAN	0.723	QAT	0.759	MLT	-0.311	SWE	-0.233	MAR	-0.764	EST	0.124	BRB	0.119	BRA	-0.795	PER	-0.616
PHL	-0.394	NGA	0.719	CIV	0.759	BOL	-0.319	MLT	-0.250	SRB	-0.769	ISL	0.087	SWE	0.090	JOR	-0.795	KOR	-0.617
NAM	-0.422	LUX	0.717	EGY	0.752	MYS	-0.359	TTO	-0.273	IND	-0.769	ZWE	-0.008	BEL	0.052	PAK	-0.797	LCN	-0.620
BRB	-0.445	ARM	0.717	VNM	0.752	UGA	-0.376	MYS	-0.287	ISR	-0.770	CYP	-0.042	TZA	-0.021	BGR	-0.798	CHL	-0.625
AUT	-0.451	JOR	0.717	KGZ	0.750	FJI	-0.377	SVN	-0.298	KWT	-0.772	PRT	-0.082	JAM	-0.029	MKD	-0.802	GUY	-0.628
FJI	-0.452	BRA	0.714	TUR	0.744	BRB	-0.380	SLV	-0.304	LBN	-0.774	BEL	-0.085	PRT	-0.050	ARE	-0.804	SRB	-0.631
LVA	-0.454	GUY	0.713	MEX	0.737	PAN	-0.392	LVA	-0.309	KOR	-0.775	PRY	-0.135	EST	-0.093	KEN	-0.808	ZAF	-0.635
SGP	-0.476	HRV	0.706	AUS	0.733	MNG	-0.406	AUT	-0.322	BRA	-0.776	URY	-0.147	PRY	-0.100	GEO	-0.813	JOR	-0.644
UGA	-0.480	CIV	0.703	JOR	0.733	NZL	-0.418	PAN	-0.360	QAT	-0.791	SWE	-0.187	ZWE	-0.126	OMN	-0.813	TUR	-0.645
LTU	-0.502	BHR	0.699	ARE	0.730	BGD	-0.422	URY	-0.383	IND	-0.792	SWZ	-0.328	FRA	-0.287	SVK	-0.814	KEN	-0.645
TTO	-0.510	CHL	0.698	LAC	0.730	SLV	-0.443	NZL	-0.397	CZE	-0.808	VEN	-0.363	VEN	-0.290	HRV	-0.815	AUS	-0.646
MNG	-0.524	MKD	0.691	LCN	0.729	IRN	-0.444	BWA	-0.416	PER	-0.813	IRL	-0.373	GBR	-0.368	AUS	-0.824	POL	-0.663
BGD	-0.535	NPL	0.686	NPL	0.718	IDN	-0.450	THA	-0.418	OMN	-0.818	FRA	-0.374	IRL	-0.371	POL	-0.827	RUS	-0.671
PAN	-0.539	KGZ	0.683	CAN	0.714	THA	-0.456	LTU	-0.447	GEO	-0.828	BMU	-0.389	SWZ	-0.517	PNG	-0.834	LBN	-0.677
SLV	-0.567	MUS	0.680	GUY	0.705	TUN	-0.476	UGA	-0.448	NOR	-0.829	GRC	-0.605	JPN	-0.580	SRB	-0.846	ZAF	-0.680
ZMB	-0.577	BGR	0.673	BHR	0.696	HUN	-0.483	SGP	-0.464	BRA	-0.833	JPN	-0.618	ITA	-0.598	CHN	-0.854	ROU	-0.681
MYS	-0.577	LCN	0.661	ESP	0.691	LKA	-0.487	MKD	-0.466	PNG	-0.834	ITA	-0.668	DEU	-0.665	KWT	-0.855	OMN	-0.689
EGY	-0.578	VNM	0.653	ARM	0.676	TTO	-0.491	EGY	-0.472	ZAF	-0.839	CRI	-0.683	FIN	-0.667	CZE	-0.860	RUS	-0.697
TUN	-0.579	SVN	0.645	SLV	0.660	PSE	-0.511	IRN	-0.472	IND	-0.840	ARG	-0.748	BMU	-0.685	BWA	-0.872	MAR	-0.701
UKR	-0.586	LAC	0.643	AUT	0.655	ZMB	-0.513	SVK	-0.473	ROU	-0.841	GBR	-0.762	ARG	-0.702	ZAF	-0.874	CAN	-0.710
IDN	-0.600	THA	0.620	SVK	0.655	COL	-0.524	UKR	-0.491	POL	-0.852	USA	-0.803	NLD	-0.751	ISR	-0.886	KAZ	-0.711
LKA	-0.607	MEX	0.619	SVN	0.654	TUR	-0.538	TUN	-0.496	KEN	-0.861	DEU	-0.854	CRI	-0.821	ESP	-0.891	ISR	-0.738
NZL	-0.614	IRN	0.610	CHL	0.650	AUT	-0.566	ESP	-0.499	AUS	-0.904	NLD	-0.878	GBR	-0.832	NOR	-0.895	KNA	-0.761
SVN	-0.634	COL	0.604	MUS	0.634	MUS	-0.581	MNG	-0.499	BWA	-0.914	FIN	-0.900	USA	-0.929	ROU	-0.934	QAT	-0.791

Figure 26. Correlation of the top 108 economies and their COFER levels relative to the top five reserve currencies.

Appendix B. Derivation of the Σ Distribution

The version of the Σ Distribution employed in this research utilizes the Normal Distribution as the component distribution of choice. This is not the only distribution that may be considered, as the research in this work has showed. We assume that we wish to develop an aggregate distribution which employs n main factors, or root causes as discussed. The number of k combinations for all k for a set of n elements is according to (2.1).

$$\sum_{0 \leq k \leq n} \binom{n}{k} = 2^n \tag{2.1}$$

However, we require at least one factor present, and therefore we must subtract one from (2.1) to eliminate the null set. Therefore the number of possible combinations possible is equal to $2^n - 1$. Each one of these $2^n - 1$ combinations has a unique utilization of the n main factors. Therefore, by choosing an ordinal number between 1 and $2^n - 1$ we can uniquely assign an ordinal number to each combination. Several methods to do this are available. The method employed here converts the decimal number to binary by iteratively dividing by 2 and checking to see if a non-zero remainder is available. If the kth remainder is zero then $b_k = 0$ else $b_k = 1$. This ensures that those binary contributors to the ordinal number under consideration are a part of the Σ distribution as well. The $a_{j,0}$ is the ordinal number we begin with and successive values as $a_{j,1}$, $a_{j,2}$, etc. retain the number after iterative conversion from decimal to binary. Furthermore, this approach can be used since the Normal distribution autoconvolves, otherwise this would not function. This process alone provides the following solution for the Σ distribution so far.

$$N \left[\sum_{k=0}^{n-1} \mu_{n-k} b_k, \sum_{k=0}^{n-1} \sigma_{n-k}^2 b_k \right] \tag{2.2}$$

$$a_{j,0} = j \tag{2.3}$$

$$b_{j,0} = 0 \tag{2.4}$$

$$a_{j,0} = j \tag{2.5}$$

$$b_{j,0} = 0 \tag{2.6}$$

When solving for a given distribution utilizing numerical techniques there is yet no constraint which determines which main factor takes on the largest mixture value. Therefore for the same distribution, different iterations of the same process may provide vastly different results for mixture values for a specific component. In order to ensure that there is consistency in the process, we mandate that the lower number main factors have the higher mixture level. Therefore when we change the value of n we can then explore how this change in n also changes the mixture values. Since the main factors have only one '1' in their binary representation and their ordinal numbers are a power of two, we may express this as follows:

$$1 \leq w \leq \log_2 n \tag{2.7}$$

$$c_{2^{w-1}} \geq c_{2^w} \tag{2.8}$$

Since the Σ distribution is a mixture distribution, we must ensure that the mixture sums to unity. We require then:

$$\sum_{j=1}^{2^n - 1} c_j = 1 \tag{2.9}$$

For asymmetric distributions, this setup suffices. The Σ distribution will have one solution. If, however, the target distribution is symmetric then there are possibly two Σ distributions which approximate the target distributions equally well. Each will be skewed opposite of the other. When this occurs, we average both Σ distribution solutions. This results in a final distribution which has zero skew and therefore improves on the individual Σ distribution solutions. The only difference between these two Σ distributions is the vector $\boldsymbol{\mu}$. One solution has a vector $\boldsymbol{\mu}$ while the other has the vector $-\boldsymbol{\mu}$. This is the reason we implement the second summation operation in the Σ distribution and multiply it by the factor $\frac{1}{2}$. The term $(-1)^d$ accounts for the Σ distribution with $-\boldsymbol{\mu}$ when $d = 1$ and accounts for the Σ distribution with $\boldsymbol{\mu}$ when $d = 2$.

Vita

Captain David M. Smalenberger received most of his early education at the Adam Kraft Gymnasium in Schwabach, Germany. His family returned to the United States where he completed his senior year of high school at the Rolling Meadows High School in Rolling Meadows, Illinois and graduated in 1999. He enlisted in 2000 as a Russian cryptologic linguist and was in training In Monterey, California until 2001 when he was accepted to the United States Air Force Academy Preparatory School and then the United States Air Force Academy, where he graduated with a Bachelor of Science degree in Operations Research and received his commission in 2006.

Captain Smalenberger's first assignment was at the 37th Helicopter Squadron, F.E. Warren Air Force Base, Wyoming. David assisted on several missile inspections and assisted in helicopter missions to guard the nation's nuclear deterrent.

In May 2007, he was assigned to Spangdahlem Air Base, Germany where he served as a Communications officer leading up to 80 individuals and in charge of electronic equipment worth $305M at 16 geographically separated units. Furthermore, he deployed to Kandahar Afghanistan to aid the International Security Assistance Forces with our NATO allies. In 2011, he completed his Master's of Science in Operations Research from the Florida Institute of Technology.

In August 2010, David was assigned to the Simulation and Analysis Facility at Wright-Patterson Air Force Base, Ohio. He worked on several advanced weapon systems such as the Miniature Aerial Launched Decoy - Jammer and integrated this and other weapon systems into wargame-type stand-alone simulations as well as interactive human-in-the-loop environments.

In 2012, David entered the Air Force Institute of Technology's Graduate School of Engineering and Management at Wright-Patterson AFB, Ohio. At AFIT, he focused his studies on unrestricted financial warfare. Upon graduation, he will be assigned to Strategic Plans at the Air Staff, in the Department of Defense in Washington, D.C.

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14. ABSTRACT

This study extends established financial market approaches to account for key econophysical attributes, low probability/high impact events, and a market's potential use as early warning for threats. Any disparity between established financial practices and true market conditions may provide incentive for exploitation and may harm national security objectives and interests through cascading effects. These national security concerns may include, in particular, the health of a reserve currency for those countries whose currency serves as one. This is a preferred approach with Unrestricted Warfare-type operations as these techniques may not enable repudiation of the antagonist. Since this approach may remain a strong incentive for such tactics for the foreseeable future, it is imperative to develop techniques that hedge against financial miscalculations and subversive efforts. This research relaxes key assumptions of standard finance theory and applies these approaches to currency dynamics and portfolio selection which provides insight on areas of vulnerability. Early warning measures of threats are developed and compared to critical world events. Vulnerabilities to capital markets are studied, and their effects on reserve currencies are also analyzed. Lastly, a mathematical framework is developed that enables imitation of the aforementioned econophysical attributes in a simulated environment thereby bridging the divide between certain aspects of standard finance theory and econophysics for future study.

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