3-23-2018

Parameter Study of an Orbital Debris Defender using Two Team, Three Player Differential Game Theory

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PARAMETER STUDY OF AN ORBITAL DEBRIS DEFENDER USING TWO TEAM, THREE PLAYER DIFFERENTIAL GAME THEORY

THESIS

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AFIT-ENY-MS-18-M-295

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Wright-Patterson Air Force Base, Ohio

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AFIT-ENY-MS-18-M-295

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THESIS

Presented to the Faculty

Department of

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

in Partial Fulfillment of the Requirements for the

Degree of Master of Science in Aeronautical Engineering

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March 2018

AFIT-ENY-MS-18-M-295

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Abstract

The United States Air Force and other national agencies rely on numerous space assets to project their doctrine. However, space is becoming an increasingly congested, contested, and competitive environment. A common risk mitigation strategy for the orbit debris problem is either performing evasive maneuvers, or placing additional shielding on the satellite before launch. Current risk mitigation strategies have significant consequences to satellite operators and may not produce sufficient risk mitigation. This research poses that an orbital debris defender, which would defend the primary satellite from orbital debris, may be a more effective risk mitigation strategy. By assuming the worst case scenario, an optimally performing pursuer, this research can show when and how often the defender can intercept debris. The results of this research provide the performance trade space for the orbital debris defender, and additional recommendations to future satellite designers. Additionally, this researched derived a way to generate a pseudo cooperation between defender and evader. This cooperation between evader and defender is a new way to solve differential games, and is not limited to the space domain considered herein.

This research is dedicated to all the friends and family that aided me in both my professional and personal life.

Acknowledgements

Earning a Master's degree is the summation of many individuals' work, much more than my own. The following list, while not exclusive, consists of individuals whose contributions to my AFIT education are pivotal.

I would like to thank my thesis advisor, who served as a first go with my research and results, and was essential in distilling those results into easily communicated ideas.

I would like to extend gratitude to the members of my committee for investing their invaluable time and effort in this research. Their commitment ensured both the quality and quantity of this research.

I would also like to thank the RPO working group who provided incredible insight, and helped solve unsurmountable problems that could not have been achieved without our teamwork. Finally, I would like to thank my friends and family, whose tireless support and tolerance, not just in my thesis but over the course of my life, will be the reason I receive my diploma.

David F. Spendel

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List of Symbols

e, d, p	subscript signifying evader, defender, pursuer, respectively
i	Iteration number
t	Time
t_f	Final time
${}^i t_f^p$	Capture time of Evader by Pursuer during iteration i
t_0	Time at start of differential game
J	Cost
ψ	Terminal constraint
ϕ	Terminal cost
λ	Costates
x	Radial position in LVLH Frame
y	Along track position in LVLH Frame
$\dot{x}$	Radial velocity in LVLH Frame
$\dot{y}$	Along track velocity in LVLH Frame
$\vec{x}$	Position (x, y) and velocity $(\dot{x}, \dot{y})$ in Cartesian coordinates using the LVLH frame
$\vec{r}$	Position (x, y) in Cartesian coordinates using the LVLH frame
$r_{pd}(t_f)$	The distance between pursuer and defender at t_f
$r_{pe}(t_f)$	The distance between pursuer and evader at t_f
T_i	Maximum thrust of player i at t_0
E_i	Exhaust velocity of player i

List of Abbreviations

GEO	Geosynchronous Orbit
GPOPS	General Purpose OPtimal control Software
HCW	Clohessy-Wiltshire Dynamics
LEO	Low Earth Orbit
LTI	Linear Time Invariant
LVLH	Local Horizontal Local Vertical Frame
NCM	Natural Circumnavigational Motion
PSO	Particle Swarm Optimization
USAF	United States Air Force

PARAMETER STUDY OF AN ORBITAL DEBRIS DEFENDER USING TWO TEAM, THREE PLAYER DIFFERENTIAL GAME THEORY

I. Introduction

1.1 Background

The United States Air Force (USAF), as well as numerous other civil and private agencies, rely heavily on national assets located in orbit around Earth. The USAF uses space to project national security policy [1]. However, today's space environment is a congested environment, with an ever-increasing chance of cascading collisions, threatening the USAF's satellites and further degrading USAF's capabilities [6, 7]. For example, in 2009, a Russian Cosmos satellite collided with an Iridium satellite, generating 1,500 new pieces of orbital debris, many which remain in orbit around Earth [1]. Figure 1 shows the growth of space debris, and Figure 2 shows the growth of launches. Figures 1 and 2 suggest that the number of launches will continue to increase, and the number of debris objects is increasing as well. Additionally, the main debris events, the Chinese ASAT and Iridium-COSMOS collisions have occurred recently. The space debris problem is escalating, especially in geosynchronous orbit (GEO), where numerous key USAF platforms are located [7, 1].

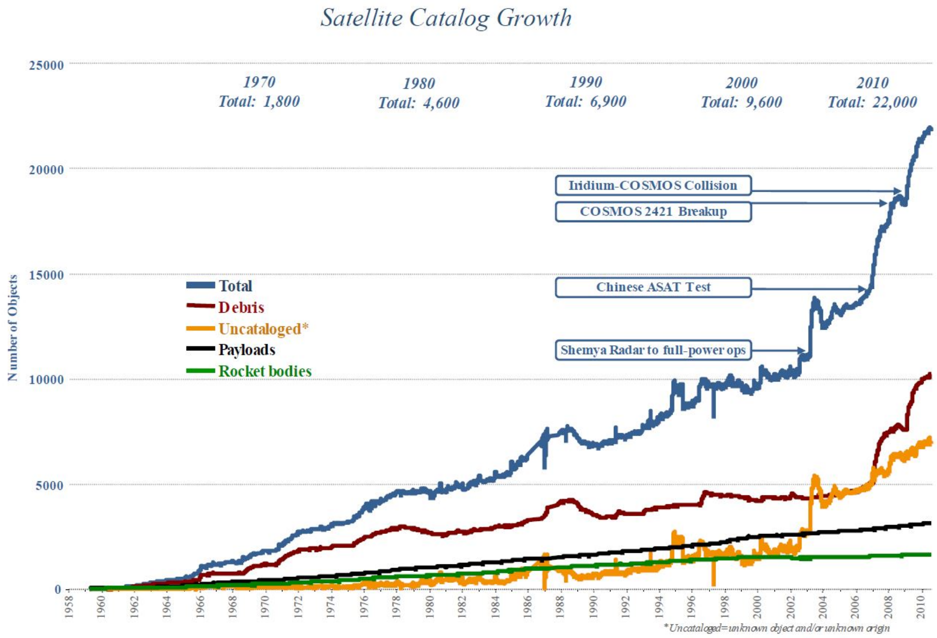

Figure 1. Increasing number of objects in space, especially uncontrollable objects such as debris [1]

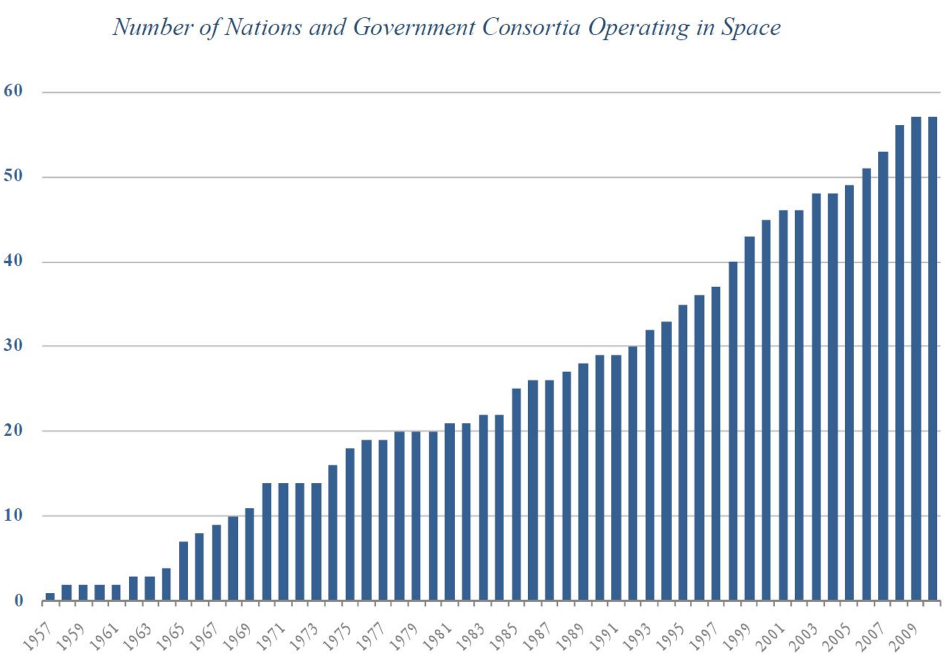

Figure 2. Number of space agencies increasing [1]

The collision risk is currently mitigated to some degree by satellites performing evasive maneuvers, which decrease the operational lifetime of the satellite, as well as potential adverse consequences to operations [8]. An example of collision avoidance occurred when Landsat 7 maneuvered away from the Iridium/Cosmos 2251 debris cloud [9]. This evasive maneuver impacted the operations of Landsat 7 for two months, as Landsat's coverage was reduced due to the maneuver [10]. Maneuvering may be unacceptable for certain satellite operators as a means to mitigate the orbital debris problem, as a coverage gap caused by a maneuver could reduce the satellite's effectiveness. Another mitigation strategy for the orbital debris problem in use is shielding, which may be impractical and is expensive and unavailable to satellites in orbit now [8].

A maneuvering satellite that intercepts and mitigates potential collisions on behalf of other satellites may be a viable risk mitigation strategy, which this research will term as an orbital debris defender. The satellite that the orbital debris defender is protecting will be termed the primary satellite. Some active debris mitigation strategies exist, such as NASA's Lightforce project, which would use photon pressure to nudge debris instead of having the satellites maneuver [11]. Thomson proposed a satellite vehicle for orbital debris protection, similar to this research, however Thomson's research was focused on the external shielding required, as opposed to capabilities and initial state vectors recommended in this research [12]. The parameters investigated in this research includes the thrust ratios of the various players, initial positions, and the optimal behaviors of the defender and the primary satellite.

This research poses the space debris problem as the worst case scenario using differential game theory, in order to ensure the successful defense of the primary satellite. If the orbital debris defender can intercept an optimally maneuvering piece of debris attempting to impact the primary satellite, then the defender can intercept debris with the same initial conditions. The following sections in this chapter will outline the motivation, problem statement, resource and methodology, limitations, similar research and finally an overview of the thesis.

1.2 Motivation

An orbital debris defender, if designed and implemented optimally, may be an effective and potentially cost-reducing risk mitigation strategy for the space debris problem. The defender would intercept the debris, so that it would not collide with the satellite. This research is focused on the performance trade space of the proposed orbital debris defender.

Due to the potentially cascading effects of collisions, the USAF's current risk mitigation strategy may be unsustainable. While a single collision may not significantly degrade the USAF's overall operational capabilities, the debris cloud generated could cause a cascading series of debris, and eventually renders regions of space unsuitable for satellites [13].

1.3 Problem Statement

This research seeks to provide an additional risk mitigation strategy for the orbital debris problem. The hypothesis of this research is that an orbital debris defender is an effective risk mitigation strategy for other satellites to use. This hypothesis can be broken into the following three questions:

- Research Question 1: What are the key parameters of an orbital debris defender enabling the defender to successfully protect the primary satellite?

- Research Question 2: Can the primary satellite implement any strategies to aid the orbital debris defender?

- Research Question 3: Can an active orbital debris defender be an effective risk mitigation strategy for the space debris problem?

This research will answer those research questions by simulating many possible scenarios, and analyzing the outcome of those scenarios in order to draw any conclusions.

1.4 Limitations and Assumptions

Using a differential game theory approach to solving this problem necessities the use of limitations and assumptions, in order to reduce the number of design variables. Some of these limitations and assumptions are caused by the dynamics used in the simulation. These limitations include neglecting perturbations to orbits, such as J2, solar radiation and atmospheric drag. This research also assumes that all information between players is complete, deterministic and instantaneous. However, this research also assumes that the pursuer does not alter its behavior in the presence of the defender. In order to use the Decomposition Method, the evader must eventually lose to the pursuer, in the absence of the defender. This research assumes that none of the maneuvering players are at risk of causing a collision with a non-player satellite. Finally, cost and schedule trade spaces are not investigated in this research.

1.5 Overview

The application of a two team, three person differential game to a satellite debris defender is unique to this research. Three person differential games have been applied to numerous problems, primarily focused on aircraft survivability [14]. However, very little research has been accomplished on the pseudo-cooperating defender-evader pair in orbit. This thesis will evaluate previous work, describe the specific methodology used to generate the trade study of the orbital debris defender, and provide various examples of trade space and design recommendations. Chapter II provides a literature review of differential game theory, optimal controls, and the dynamics used in this research. Afterwards, Chapter III explains the mathematical models and algorithms used, as well as the various parameters used in the study. Following Chapter III, Chapter IV shows the results of this study. Chapter V summarizes the results, and provides recommendations on future work in this field. This research aims to provide the USAF with a novel and viable tool to design an orbital debris defender in order to prevent collisions in space.

II. Literature Review

2.1 Introduction to Problem Background

The orbital debris defender relies on three well-established knowledge areas. The first areas is system of equations of motion to model the propagation of satellites in reference to each other. Another area is leveraging classical optimal control thesis. Optimal control is a subject of applied mathematics that minimizes or maximizes a scalar value functional by either a direct or indirect method [15]. One of the benefits of optimal control is the wide array of existing software that can solve these problems quickly, such as a genetic algorithm [16, 17]. The third area is differential game theory. Differential game theory captures real-world scenarios and poses them as math problems to generate optimal solutions to those problems. Using all three areas, a simulated space debris scenario can be generated and solved using a variety of parameters, such as initial state vectors of the debris and the defender, as well as each player's thrusting capabilities. The following sections provide a literature review on the information used to generate the simulations used in this research.

2.2 Dynamics

Ever since the start of the space race, many mathematicians desired to simplify the satellite rendezvous mission through the use of new coordinate frames [18]. Mathematicians in the 1960s, using the work of George William Hill, developed a new frame that focused on the relative motion between two bodies [19], called the Local Horizontal Local Vertical (LVLH) frame. The LVLH simplifies the computations required to solve for the relative orbit of two satellites with respect to each other [18]. The LVLH frame is a rotating, non-inertial frame centered around a chief satellite. The x-axis is aligned with the negative radial direction, the z-axis is orthogonal to the orbital plane and the y-axis completes the right-handed orthogonal coordinate system. If the satellites are in a circular orbit, then the y-axis is aligned with the velocity vector of the satellite. Figure 3 shows the relationship between the Earth Centered, Earth Inertial (ECI) and LVLH frame [19].

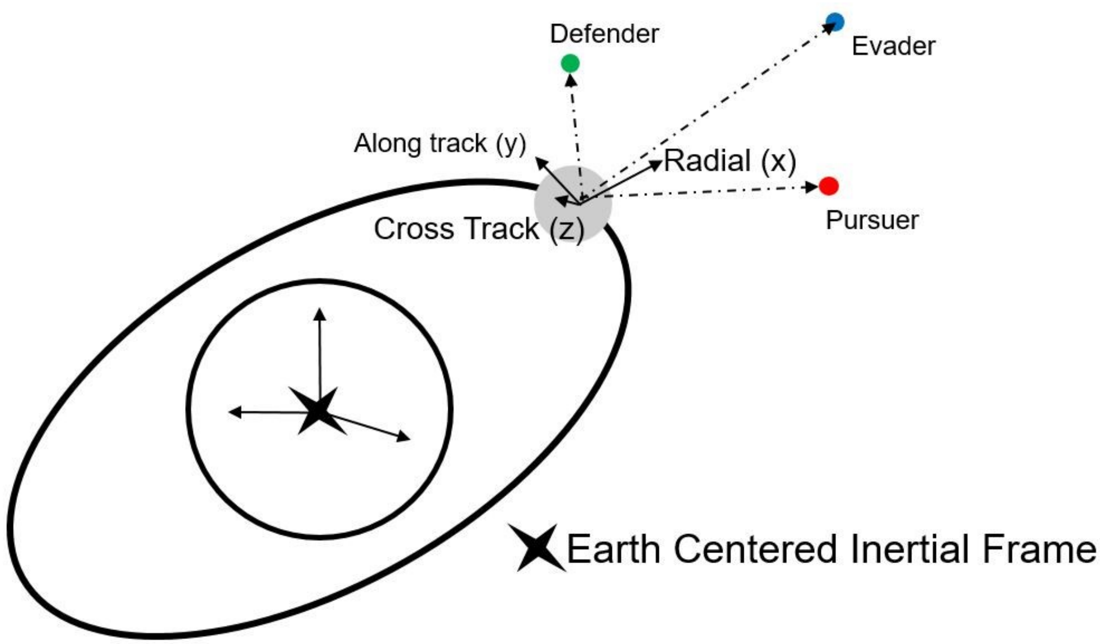

Figure 3. Relationship between ECI and LVLH frame

Using these assumptions, it is possible to derive the equations of motion for each satellite relative to the chief satellite. The following section derives the equations of motion.

2.2.1 Clohessy Derivation.

The nonlinear equations of motion used to model the propagation of one satellite with reference to another satellite were derived in [20]. These equations of motion are

$$
\begin{aligned}
\ddot{x} - 2\dot{f}(\dot{y} - y\frac{\dot{r}_c}{r_c}) - x\dot{f}^2 - \frac{\mu}{r_c^2} &= -\frac{\mu}{r_d^3}r_c + x \\
\ddot{y} + 2\dot{f}(\dot{x} - x\frac{\dot{r}_c}{r_c}) - y\dot{f}^2 &= -\frac{\mu}{r_d^3}y \qquad , \\
\ddot{z} &= -\frac{\mu}{r_d^3 z}
\end{aligned}
\tag{2.1}
$$

where the x, y, and z (and their derivatives and double derivatives) are the Cartesian coordinates in the LVLH frame. r_c is the distance from the satellite at the origin of the coordinate system to the center of the Earth, and $\dot{r}_c$ is its time derivative. r_d is the distance between the satellites, and $\dot{r}_d$ is its time derivative. μ is the Earth standard gravity parameter. f is the true anomaly of the satellite, and $\dot{f}$ is its time derivative.

In order to reduce the dynamics to a linear system of equation, the following simplifications are made. First, the chief is placed in a circular orbit. This implies

$$\dot{f} = n, \tag{2.2}$$

where n is the mean motion of the chief (and is constant). Additionally, this research assumes the distance between the relative satellites and the chief satellite is significantly less than the semi-major axis of the chief satellite, or

$$r_c \approx r_d. \tag{2.3}$$

The equations of motion are therefore further reduced to

$$\begin{aligned} \ddot{x} - 2n\dot{y} - 3n^2x &= 0 \\ \ddot{y} + 2n\dot{x} &= 0\cdot \\ \ddot{z} + zn^2 &= 0 \end{aligned} \tag{2.4}$$

Equations of motion can be expressed as a matrix multiplied by the state vectors, also known as a state space transformation. The following section provides a background on state space transformations.

2.2.2 State Space Transformation.

Holmes derived a method of describing the behavior of mathematical systems using differential equations [21]. A way to capture a series of differential equations is state transition matrix. These relate the states of a system to the systems derivatives. This is outlined in Equation (2.5), assuming a linear time invariant system,

$$\dot{\vec{x}} = \mathbf{A}\vec{x}, \tag{2.5}$$

where $\mathbf{A}$ is a matrix that maps the relationship between the states and the derivatives. However, for a system with inputs and outputs, the system can be modeled as

$$\begin{aligned} \dot{\vec{x}} &= \mathbf{A}\vec{x} + \mathbf{B}\vec{u} \\ \vec{y} &= \mathbf{C}\vec{x} + \mathbf{D}\vec{u} \end{aligned}, \tag{2.6}$$

where $\vec{u}$ is an input vector, and $\mathbf{B}$ and $\mathbf{D}$ maps the input and outputs, respectively, to the dynamics, and the $\mathbf{C}$ matrix maps the states to the output. In a linear system, only the states are varying, whereas in a nonlinear system, the matrices may also vary with respect to time. The state space transformation for the above system of equations of motion, assuming zero input, is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & n^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}. \tag{2.7}$$

In order to further reduce computational complexity, only planar motion is assumed. To ensure only planar motion, the following constraint,

$$z = \dot{z} = 0, \tag{2.8}$$

is required. A virtual chief implies that the satellite at the origin is not a real satellite, but instead an artificial satellite used to simplify the mathematics. Therefore, all satellites are propagated with respect to a virtual chief. The state space used in this research, assuming zero input, is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & -2n & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}. \tag{2.9}$$

2.2.3 Trajectory of Satellite without maneuvers.

The state transition matrix solved for above is a linear, time invariant (LTI) state transition matrix. A LTI state transition matrix is defined as a system of equations that are not dependent on time, and are a combination of linear variables. A benefit of the LTI is that it has an analytical solution, if the satellites do not maneuver. Assuming $t_0 = 0$, and the initial state of the satellite, the state vector at the time t can be found using [22], if the satellite does not maneuver. The states at time t can be calculated using

$$
\begin{aligned}
x(t) &= \frac{\dot{x}_0}{n}\sin(nt) - \left(3x_0 + \frac{2y_0}{n}\right)\cos(nt) + \left(4x_0 + \frac{2y_0}{n}\right) \\
y(t) &= \left(6x_0 + \frac{4y_0}{n}\right)\sin(nt) + \frac{2\dot{x}_0}{n}\cos(nt) - (6nx_0 + 3\dot{y}_0)\,t + \left(y_0 - \frac{2\dot{x}_0}{n}\right), \\
\dot{x}(t) &= \dot{x}_0\cos(nt) + (3nx_0 + 2\dot{y}_0)\sin(nt) \\
\dot{y}(t) &= (6nx_0 + 4\dot{y}_0)\cos(nt) - 2\dot{x}_0\sin(nt) - (6nx_0 + 3\dot{y}_0)
\end{aligned}
\tag{2.10}
$$

where $x_0, y_0, \dot{x}_0, \dot{y}_0$ are the initial states (at time t_0) of a satellite relative to the virtual chief satellite. Inputs are discussed in the next section.

2.2.4 Controls for Players.

The pursuit, evasion defender problem posed in this research requires that all players be able to maneuver. In order to maneuver, the satellites must generate a thrust, which can be incorporated into the state transition matrix. Satellite inputs generally require mass loss in order to maneuver [23, 24]. Furthermore, satellites within this research have no limit on the orientation of the thrust in plane, nor how quickly the satellite can reorient the thrust vector. In order to account for both mass loss, the control input (**B** matrix and $\vec{x}$ input) to the state transition matrix is

$$
\vec{u} = \begin{bmatrix} 0 \\ 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} \frac{T}{1 - t\frac{T}{E}}, \tag{2.11}
$$

where θ is the angle of thrust, as measured from the x axis. T is the thrust at time $t = 0$, and E is the exhaust velocity [24].

2.2.5 Full State Space Representation.

The full state space representation used in this simulation is therefore

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 2n \\ 0 & -2n & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} \frac{T}{1 - t\frac{T}{E}}.$$
$$\vec{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \quad . \tag{2.12}$$

The $\mathbf{D}$ is a vector of zeros for this research. This research uses the in-plane frame and linear dynamics (with nonlinear controls) in order to solve these problems. However, the algorithm developed is not limited to the in plane LVLH frame with linear dynamics. Future work could use the nonlinear dynamics, as well as allowing for cross-track motion.

2.3 Optimal Control

Optimal Control is a field within applied mathematics for finding the best solution to a functional. An example of a problem solved using optimal control is the minimum time to a final state given a system of equations, initial state and a thrust capability [5]. The following sections outline the components of a dynamical optimal control problem, such as the cost functional and the Hamiltonian.

2.3.1 Cost Functionals.

A cost functional is at the core of every optimization problem. The cost functional must be a scalar value, but can be the sum of many scalars. The optimal solution to a problem produces the lowest possible value of the cost. A common form of a cost equation, is

$$J = \frac{1}{2}x^T(t_f)\mathbf{Q}(t_f)x(t_f) + \int_0^{tf} \frac{1}{2}x^T(t)\mathbf{Q}x(t) + u(t)^T\mathbf{R}u(t)dt. \tag{2.13}$$

The matrix $\mathbf{Q}(t_f)$ is the cost imposed on the states at the final time, whereas the constant matrix $\mathbf{Q}$ is the cost imposed on the states before the final time. $\mathbf{R}$ is the constant cost matrix imposed on the control usage. Time varying $\mathbf{R}(t)$ and $\mathbf{Q}(t)$ were not considering in this research.

The first half of the equation, before the integral, is known as the terminal cost. This cost is only dependent on the final state and final time. A common terminal cost is the final time. The second half of the equation, within the integrand, is known as the running cost, or the Lagrange cost. A common running cost is the control of the system. To use classical techniques in order to solve the optimal control problem, $\mathbf{R}$ must be a positive definite matrix, and $\mathbf{Q}$ must be a positive semi-definite matrix [5]. Using Equation (2.13), the optimal control can be solved by first generating the Hamiltonian[15]. The Hamiltonian applies the principle of least action in order to amend the state dynamics to the cost equation [25].

2.3.2 Hamiltonian.

The Hamiltonian is a summation of the Lagrange costs, as defined in Section 2.3.1, as well as the state dynamics multiplied by the costates. Costates, also known as Lagrange multipliers or λ, enforce the dynamics on the optimal control problem.

The standard form of the Hamiltonian is

$$H = L(t) + \lambda(t)^T (f(x(t), u(t), t)), \tag{2.14}$$

where L is any running cost, $\lambda(t)$ are the costates at time t, and $f(x(t), u(t), t)$ are the state dynamics. Using the Hamiltonian, it is possible to solve optimal control problems using the boundary and necessary conditions.

2.3.3 Boundary and Necessary Conditions.

Applying the calculus of variations to the Hamiltonian, the following boundary conditions are used to find the optimal solution. The necessary conditions to solve the optimal control problem, presented in [5], are

$$\begin{aligned} \frac{\partial H}{\partial u} &= 0 \\ \frac{\partial H}{\partial x} &= -\dot{\lambda}(t). \\ \frac{\partial H}{\partial \lambda} &= \dot{\vec{x}}(t) \end{aligned} \tag{2.15}$$

In addition to necessary conditions shown above, there is also a boundary condition to ensure the optimal solution. The boundary condition, also presented in [5], is

$$\begin{aligned} 0 = &\left(\frac{\partial \phi}{\partial x}(x^*(t_f), t_f) - \lambda^*(t_f) \right)^T \delta x_f \\ &+ \left(H(x^*(t_f), u^*(t_f), \lambda^*(t_f)) + \frac{\partial \phi}{\partial t}(x^*(t_f), t_f) \right) \delta t_f \end{aligned} . \tag{2.16}$$

ϕ is the terminal cost, x_f are the final state and t_f is the final time. ∂ is a partial derivative, and δ is the variational derivative. Using the boundary condition, shown in Table 1, a two point boundary problem is generated.

Table 1. Different types of games [5]

End Condition	Implications	Equation
Fixed Final State	The optimal control problem terminates at a fixed state	$x(t_f) = x(t_f)$
Free Final State	The optimal control problem does not have a fixed final state	$\frac{\partial\psi}{\partial x}(x^*(t_f), t_f) - \lambda^*(t_f) = 0$
Fixed Final Time	The optimal control problem terminates at a fixed time	$t_f = t_f$
Free Final Time	The optimal control problem does not have a fixed final time	$H(x^*(t_f^*(t_f), \lambda * t_f), t_f) + \frac{\partial\phi}{\partial t}(x^*(t_f), t_f) = 0$

The two points in the two-point boundary problem are the initial state (defined by the user) and the end state (generated by the type of game). The two point boundary problem can be solved using a variety of methods, such as forward, backwards or multiple shooting methods [26]. A secondary solution method to solve optimization problems is to employ heuristic optimization software [26].

2.4 Heuristic Optimization Software

Heuristic optimization software attempt to solve optimization problems more quickly than traditional methods, through a variety of approaches. Heuristic optimization software also can solve for a local, as opposed to the global, optimal solution quickly, which may be sufficient, depending on the application [26]. The two heuristic optimization software tools used in this research are the Particle Swarm Optimization (PSO) and the General Purpose Optimization Program (GPOPS).

2.4.1 Partial Swarm Optimization.

Kennedy and Eberhart developed a heuristic optimization program inspired by the migration of birds in the 1990s, called PSO [16]. The governing equation behind PSO is

$$v^{(i,k+1)} = v^{(i,k)} + c_1 r_1 (x_p^{(i,k)} - x^{(i,k)}) + c_2 r_2 (x_g^{(k)} - x^{(i,k)})), \tag{2.17}$$

where v is the particle velocity. r_1 and r_2 are random numbers between 0 and 1. x_p is the optimal solution amongst the current iteration of candidate solutions, and the x_g is the optimal solution of all previous iterations. The user defines c_1 and c_2, which are the cognitive and social parameters respective. PSO generates candidate solutions to the optimization problem, and then those candidate solutions are compared to their peers. These solutions are migrated based on the cognitive and social parameters. Numerous research utilized a particle swarm approach to solve an optimal trajectory [24].

2.4.2 GPOPS.

Another optimization software, which is the main optimization software used in this research, is GPOPS. GPOPS uses a pseudo spectral method, where the controls and states are approximated using polynomials, and differential-algebraic equations are enforced at the roots of those polynomials, or a linear combination of the polynomials and their derivatives. This allows a continuous-time optimal control problem to be posed as a nonlinear program, which is then solved using a nonlinear program (NLP) solver [17]. This research used GPOPS as the optimization software, unless otherwise noted. A potential drawback of using GPOPS is that only a local optimal solution may be found, as opposed to a global solution. A local optimal solution implies that the solution produces the best value of the cost functional within a small neighborhood of that solution, but is not the best possible solution [5]. This research can only claim local optimality. Pseudo spectral methods have been shown to solve optimal trajectories in [27].

2.5 Differential Games

Differential games attempts to find the best outcome when a variety of players have different objectives. Solving a differential game often means solving for the Nash Equilibrium. A Nash Equilibrium (in a two player noncooperation game) is defined as a solution in which, if player one performs optimally, no better solution exists for player two. This research is modeling a two team, three player game. A team is defined as a group of players whom have a mutual objective. This research is a two on one differential game, where the evader and defender are one team against the pursuer. The pursuer and evader are playing a classical differential game, called Pursuit Evasion.

2.5.1 Pursuit Evasion.

One of the first differential games is the *'homicidal chauffeur'*, popularized by Isaacs [2]. The homicidal chauffeur has a slow but maneuverable pedestrian, whose objective is to not be captured by the chauffeur. The chauffeur is a fast but sluggish car that is attempting to capture the pedestrian. The solution to the homicidal chauffeur was derived by Isaacs, to show where the pedestrian escapes, and where the chauffeur captures the pedestrian, shown in Figure 4.

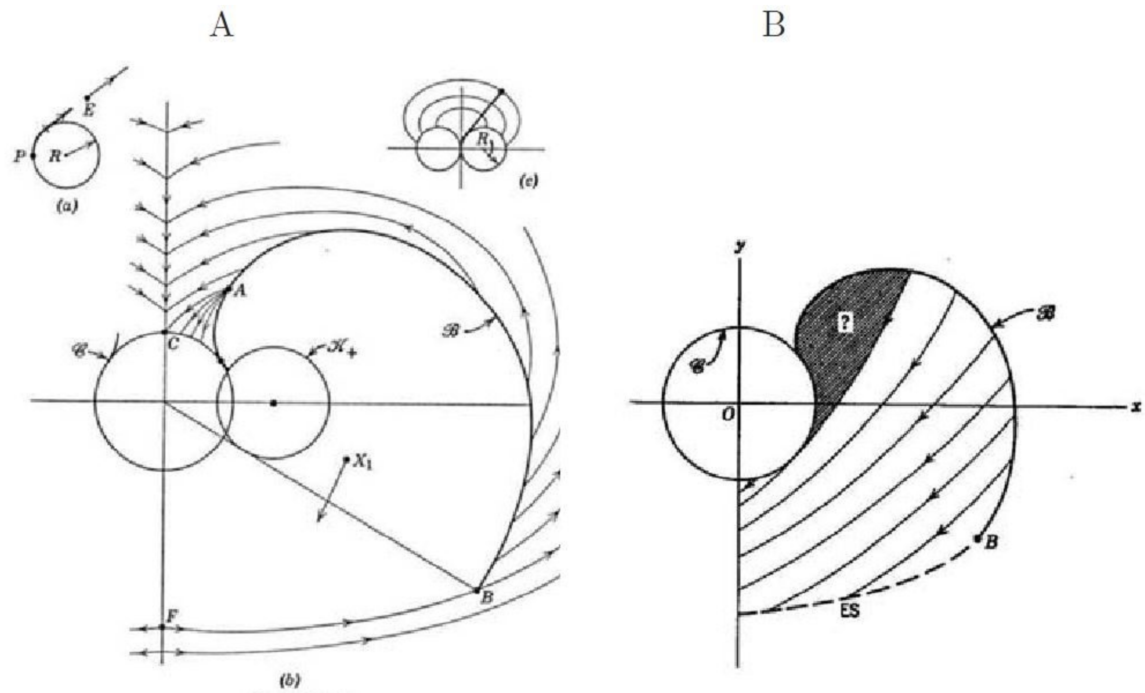

Figure 4. Solution to homicidal chauffeur [2]

Since the initial work on the homicidal chauffeur, other research has applied the principles of that homicidal chauffeur to many different environments, such as using the linearized relative orbit mechanics in the LVLH frame.

2.5.2 Pursuit Evasion in the LVLH Frame.

Pursuit-evasion games in the LVLH frame were solved by both Stupik and Jagat. Stupik solved the two-point boundary problem using a particle swarm optimization software in the LVLH frame [24]. Jagat instead used a nonlinear regulator with heavy control penalty [28]. Stupik was used a comparison for this research, instead of Jagat's, because Jagat does not have bounded control which this research uses and is more realistic. This research also adds a defender satellite, which was not included in either of Stupik's or Jagat's work. For a defender satellite, a comparison to the loyal wingman concept in the fighter community is useful, which can be solved using an Apollonius Circle.

2.5.3 Apollonius Circle.

Three player differential games pose a significant problem to find a Nash Equilibrium. One solution for a three player game is to use an Apollonius Circles approach. An Apollonius Circle is defined as the capture region of the evader by the pursuer. The Apollonius Circle is found by finding the maximum distance the evader can achieve with any heading before the pursuer intercepts the evader. In order to apply Apollonius Circles, a simplified coordinate system and dynamics are required [3]. Figure 5 shows an example of an Apollonius Circle.

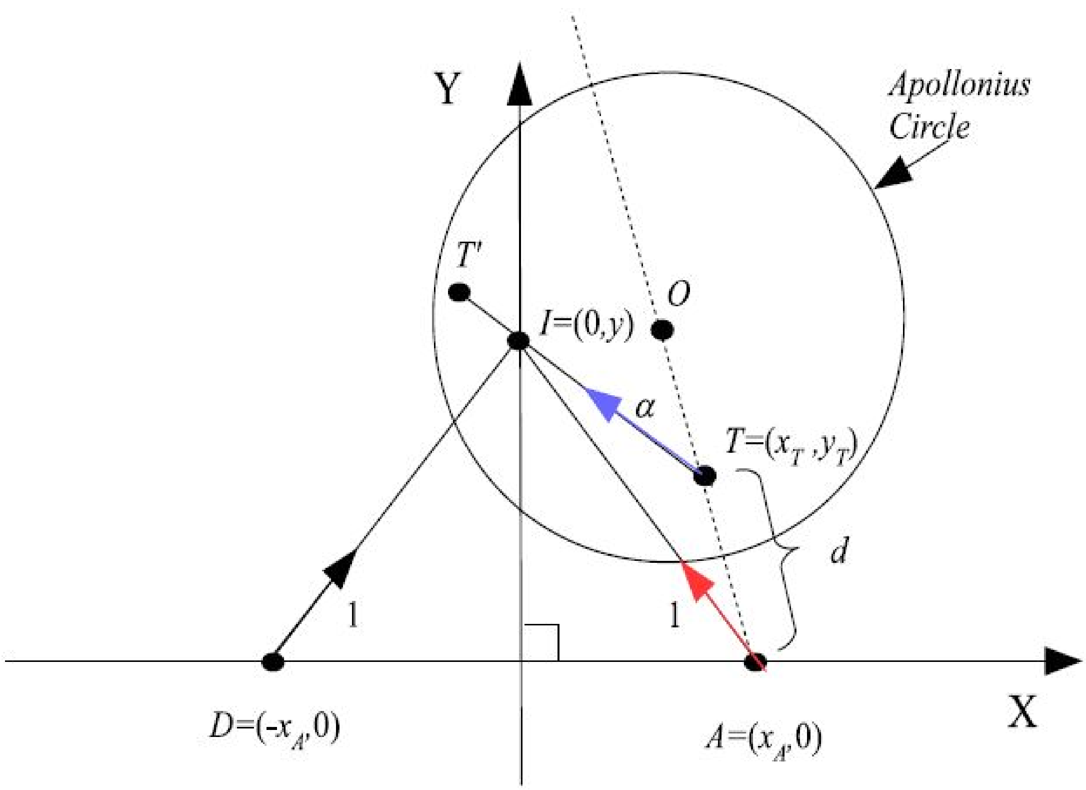

Figure 5. Example of the Apollonius Circle [3]

By generating the Apollonius Circle, it is possible to derive the Nash Equilibrium between the three players. However, in order to solve for the Apollonius Circle, numerous simplifies are required, such as a constant velocity and a simplified coordinate system. This research does not use Apollonius Circles due to those limitations. Another possible way to solve for the Nash Equilibrium is limiting the number of maneuvers a player is allowed, such as having only a finite number of maneuvers.

2.5.4 Finite Maneuvers.

Differential Game problems often attempt to reach a Nash Equilibrium. However, for the three player game, this can pose difficulty in convexifying the problem. A solution method for a differential game is to limit the number of available maneuvers all players are allowed. By limiting the number of available maneuvers, there exists a finite number of possible games. By solving for the outcome in each game, it is possible to solve for the Nash Equilibrium. This approach was solved in space, using a target, attacker and defender differential game [29]. However, this research allows all players to orient the thrust in any planar direction. Therefore, this approach is ill-suited for this research, as there are an infinite number of games possible. This research instead used the Decomposition Method to solve for its differential game.

2.5.5 Decomposition Method.

A fundamental problem with solving the optimization problem is that the problem is not generally stable when solve separately. However, Raivio developed a method to avoid a two-point boundary problem and stabilize the problem, called the Decomposition Method [30]. The Decomposition Method solves for the Nash

Equilibrium by solving classical optimal control problems. An example of the Nash Equilibrium was solved for a one-on-one air combat using medium range air-to-air missiles. Figure 6 shows the saddle point generate for one player.

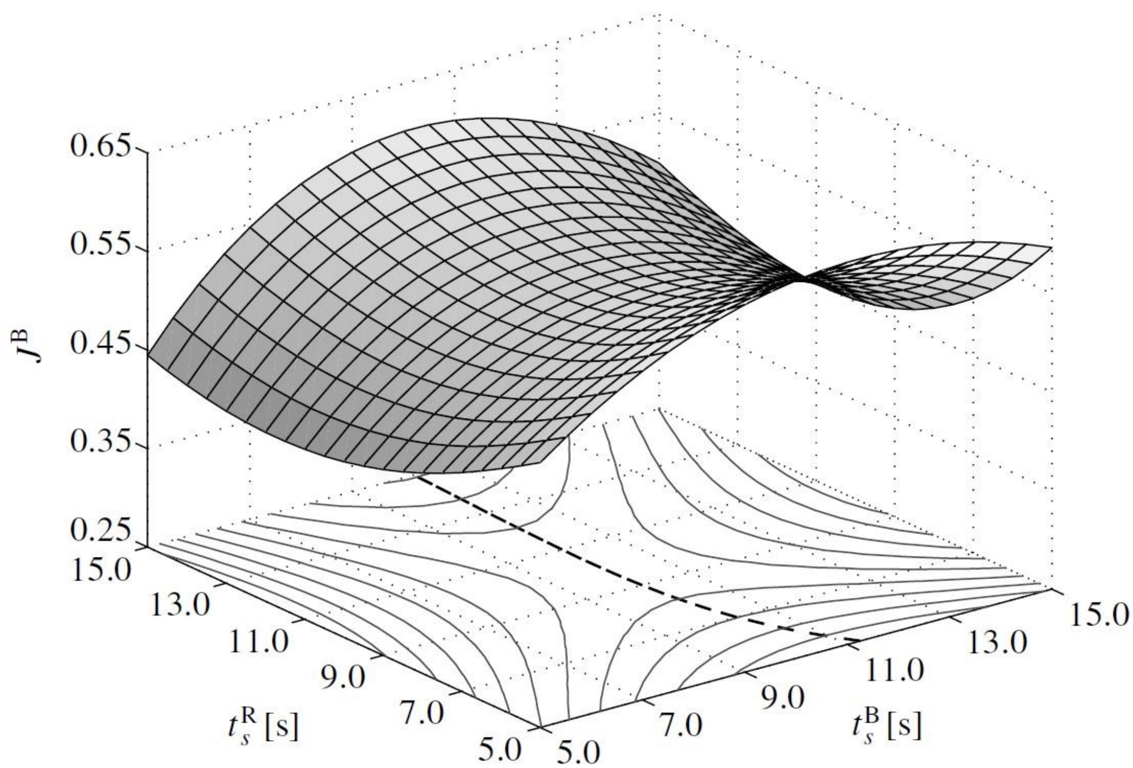

Figure 6. Decomposition Method produces a Nash Equilibrium in air to air combat [4]

This method will be expanded upon in the following chapter.

2.6 Summary

This research uses three well-established knowledge areas to pose the differential game. Those areas are equations of motion that propagate one satellite with reference to another satellite, classical optimal control problems, and differential game theory. Chapter III outlines the methodology used in this research.

III. Methodology

3.1 Introduction

The methodology for this research into the orbital debris defender can be segmented into various key areas. The first area is to generate relevant initial conditions and capabilities. Another area is to convert the differential game into a series of classical optimal control problems. Finally, the third area is to devise a scoring function in order to evaluate the outcome of the game. The full process is shown in Figure 7.

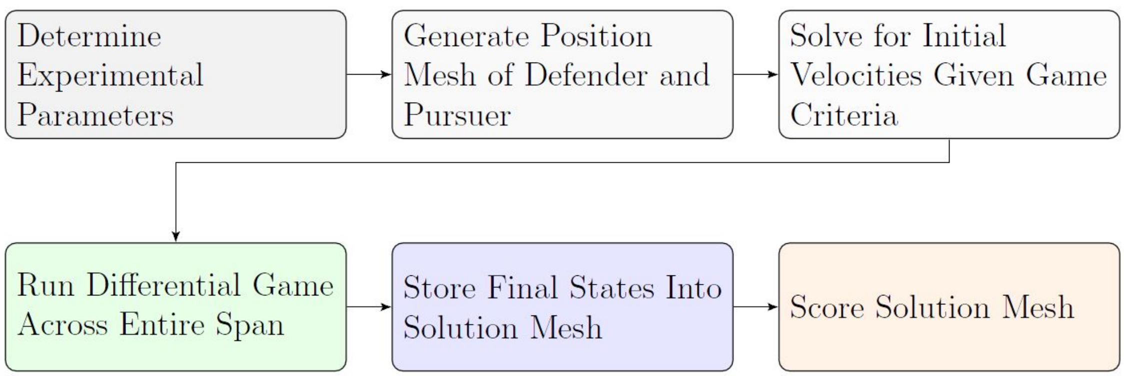

Figure 7. Flow chart of research process

This chapter provides a detailed derivation and application for each of those areas.

3.2 Initial Position and Velocity Vectors of All Players

The goal of this research is an evaluation of various parameters in the two team, three player game. A variety of initial conditions and capabilities are required, to ensure a robust sample size. The following sections outline the methodology to find the initial state vectors of all players. All players require an assumption in order to consistently populate their initial state vector at the start of the game.

3.2.1 Pursuer Initial State Vector.

In order to model the worst case scenario (of the orbital space debris problem), the pursuer will begin on a collision course with the evader. Therefore, a time until impact is required to generate an initial state vector for the pursuer. This research recommends the use of β, where β is defined as the amount of orbits until the debris will impact the evader, if neither the debris nor evader maneuver. β can be found by solving

$$t_c = \frac{2\pi}{n}\beta, \tag{3.1}$$

where t_c is the time until impact. The recommended values of β are between 0.1 and 1.

As outlined in the previous chapters, a variety of pursuer initial positions are used in this research. Therefore, the pursuer's initial position is known at the start of the differential game. Given the pursuer's initial position, as well as β, the pursuer's initial state vector is

$$\vec{x}_p(0) = \begin{bmatrix} x_p(0) \\ y_p(0) \\ \frac{nx_p(0)(4-3\cos(nt))+2(1-\cos(nt))\dot{y}_p(0)}{\sin(nt)} \\ \frac{(6x_p(0)(nt-\sin(nt))-y_p(0))n\sin(nt)-2nx_0(4-3\cos(nt))(1-\cos(nt))}{(4\sin(nt)-3nt)\sin(nt)+4(1-\cos(nt))^2} \end{bmatrix}, \tag{3.2}$$

where $x_p(0), y_p(0)$ are the initial x and y coordinates of the pursuer in the LVLH frame, and $t = t_c$. β therefore is analogous to a warning time the evader and defender have before impact, an operationally important metric.

3.2.2 Defender Initial Position and Velocity Vector.

The assumption of the orbital debris defender was a system dedicated to defend a primary satellite [12], and not a system on the primary satellite itself. In order to reduce fuel consumption protecting the primary satellite, the defenders may operate in a passive stable relative orbit. The solution to this orbit is a Natural Motion Circumnavigation (NMC), which is solved by reducing the secular terms within the initial state vector. The secular term in Equation (2.10) is

$$(6nx_0 + 3\dot{y}_0). \tag{3.3}$$

To nullify the secular term, Equation (3.3) must equal 0. This generate the following initial state vector for the defenders initial y velocity,

$$\dot{y}_0 = -2nx_0. \tag{3.4}$$

Additionally, there is a constant offset term in Equation (2.10), which is the center of the NMC in the y component. This research assumes that the NMC will be centered around the virtual chief. For the NMC to be centered around the origin,

$$y_0 - \frac{2\dot{x}_0}{n} \tag{3.5}$$

must equal zero. The following initial condition for the defender's initial velocity in the x direction is

$$\dot{x}_0 = \frac{ny_0}{2}. \tag{3.6}$$

The initial state vector for the defender is

$$\dot{x}_d(0) = \begin{bmatrix} x_d(0) \\ y_d(0) \\ \frac{ny_d(0)}{2} \\ -2nx_d(0) \end{bmatrix}, \tag{3.7}$$

where $x_d(0)$ and $y_d(0)$ are the initial x and y coordinates of the defender in the LVLH frame. Figure 8 shows the initial velocity vector of the pursuer (with β=0.1) and defender based on their initial position. The initial velocity of the defender is independent of β, and is only a function of the initial x and y coordinates.

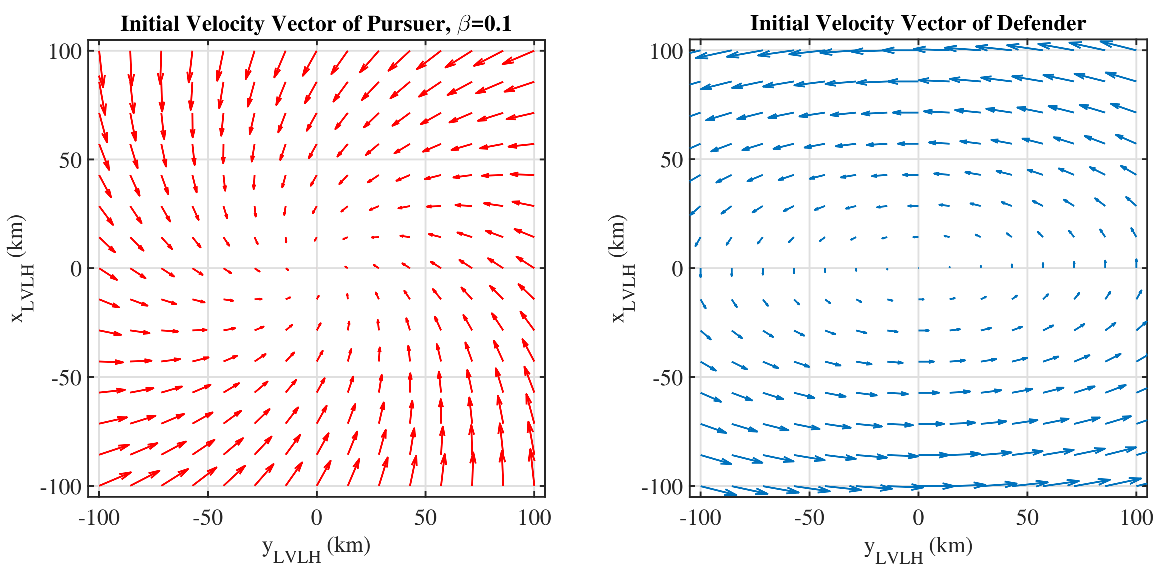

Figure 8. Velocity vectors of players at initial positions

The evader's initial state vector is defined as

$$\vec{x}_e(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{3.8}$$

as the evader and the virtual chief will begin collocated at the start of every game.

3.3 Control for all players

As outlined in Chapter II, the defender's capabilities will remain constant. This reduces the number of independent variables. Therefore, the capabilities of the evader and pursuer are defined compared to the defender, labeled as T_e/T_d and T_p/T_d respectively. The control for the defender is

$$\vec{u}_d = \begin{bmatrix} 0 \\ 0 \\ \cos\theta_d \\ \sin\theta_d \end{bmatrix} \frac{T_d}{1 - t\frac{T_d}{E}}. \tag{3.9}$$

The control for the pursuer is

$$\vec{u}_p = \begin{bmatrix} 0 \\ 0 \\ \cos\theta_p \\ \sin\theta_p \end{bmatrix} \frac{T_d\frac{T_p}{T_d}}{1 - t\frac{T_d\frac{T_p}{T_d}}{E}}. \tag{3.10}$$

The control for the evader is

$$\vec{u}_e = \begin{bmatrix} 0 \\ 0 \\ \cos\theta_e \\ \sin\theta_e \end{bmatrix} \frac{T_d\frac{T_e}{T_d}}{1 - t\frac{T_d\frac{T_e}{T_d}}{E}}. \tag{3.11}$$

3.4 Decomposition Method with pseudo-cooperating defender evader

This research takes the Decomposition Method and then adds two contributions. The Decomposition Method is a way of solving the pursuit evasion problem by separating the pursuer and evader's control problem and reaching a Nash Equilibrium through an iterative approach [27]. The first contributions is a defender, which attempts to intercept the pursuer before the purser captures the evader. Another component added to the Decomposition Method is evader strategies. Traditionally, two team, three player games are solved without the evader and defender cooperating [31], or by using a set amount of strategies [32]. However, by taking advantage of the derivation of the Decomposition Method, a terminal cost which is a function of the evader's final position, and the initial position of the defender and pursuer, can be used to implemented pseudo-cooperation between the evader and defender.

The first phase of this algorithm is a traditional pursuit-evasion problem. Following the first phase, the second phase, labeled defense, determines if the defender can intercept the pursuer at any point. Finally, the third phase, labeled evaluation, scores the outcome of the game. The following sections provide additional information on each phase.

3.4.1 Phase One: Pursuit Evasion using the Decomposition Method.

Phase one of this algorithm uses the Decomposition Method to solve for the Pursuit Evasion differential game. In order to derive the Decomposition Method, the assumption is the evader eventually loses the differential game. Therefore, the final states of the pursuer and evader are defined as

$$\begin{aligned} \vec{r}_e &= (x_e(t_f), y_e(t_f))^T \\ \vec{r}_p &= (x_p(t_f), y_p(t_f))^T \end{aligned}, \tag{3.12}$$

where $\vec{r}_e$ and r_p are the final position at capture time, t_f. The terminal constraint is the pursuer captures the evader, or both the pursuer and evader have the same position but not necessarily the same velocity. This capture condition (ψ_p),

$$\psi_p = (r_e - r_p) = \vec{0}, \tag{3.13}$$

states the evader loses at the final time, which is an assumption within this research. The pursuer's objective is to minimize that capture time, or

$$J_p = t_f. \tag{3.14}$$

The evader wishes to maximize the time to capture. Instead of directly maximizing capture time, the evader instead maximizes the capture condition. This is done by taking a first-order Taylor series around the evader's previous final position. In order to simplify the math, the following equation defines the evader's final position at the (i-1) iteration,

$$\vec{e} =^{i-1} r_e. \tag{3.15}$$

A Taylor series expansion taken around the evader's previous final position results is [27]

$$J_p(^i r_e) \approx J_p(\vec{e}, r_p) + (\frac{\partial J_p}{\partial \vec{e}})(^i r_e - \vec{e}). \tag{3.16}$$

In optimization problems, scalar values in the cost functional do not affect the optimal solutions [5]. The first term in the Taylor Series expansion is a scalar.

Therefore, $J_p(\vec{e}, \vec{r}_p)$ can be ignored, because, as a scalar, $J_p(\vec{e}, \vec{r}_p)$ does not affect the optimal solution, only the optimal value. The second term is calculated by taking the gradient of $J_p(\vec{e})$ with respect $\vec{e}$, or the evader's previous final position, derived in [27], is

$$\frac{\partial J_p}{\partial \vec{e}} = \frac{\partial}{\partial \vec{e}}\phi(\vec{e}, r_p) +^i \lambda_p \frac{\partial}{\partial \vec{e}}\psi(\vec{e}, r_p). \tag{3.17}$$

The evader wants to maximize that value. In the traditional Decomposition Method, the evader has no terminal cost other than maximizing capture time. However, in order to implement strategies for the evader, a terminal cost can be employed as long as it is dependent on the evader's final position, as well as the defender's and pursuer's initial state vectors. The derivative of the other terminal cost is

$$\frac{\partial}{\partial \vec{e}}\phi(\vec{e}, r_p) = \begin{bmatrix} \frac{\partial \phi_e}{\partial e_x} & \frac{\partial \phi_e}{\partial e_y} & 0 & 0 \end{bmatrix}^T. \tag{3.18}$$

Removing the scalar and simplifying, the evader's objective function is

$$J_e = \begin{bmatrix} \lambda_p^x(t_f) + \frac{\partial \phi}{\partial e_x} & \lambda_p^y(t_f) + \frac{\partial \phi}{\partial e_y} \end{bmatrix} \begin{bmatrix} {}^i x_e(t_f) - e_x \\ {}^i y_e(t_f) - e_y \end{bmatrix}. \tag{3.19}$$

This generates the iterative optimal control problem between the pursuer and evader. First, the pursuer solves a fixed final state, free final time problem. The objective of the pursuer is a minimum time to the evader's previous final state. Afterwards, the evader solves a free final state, fixed final time problem to maximize the capture condition. This process is repeated until the evader's final position does not exceed a threshold. This is a local optimal solution, and may not be a global optimum solution. For the algorithm, phase one is captured in a flow chart, shown in Figure 9.

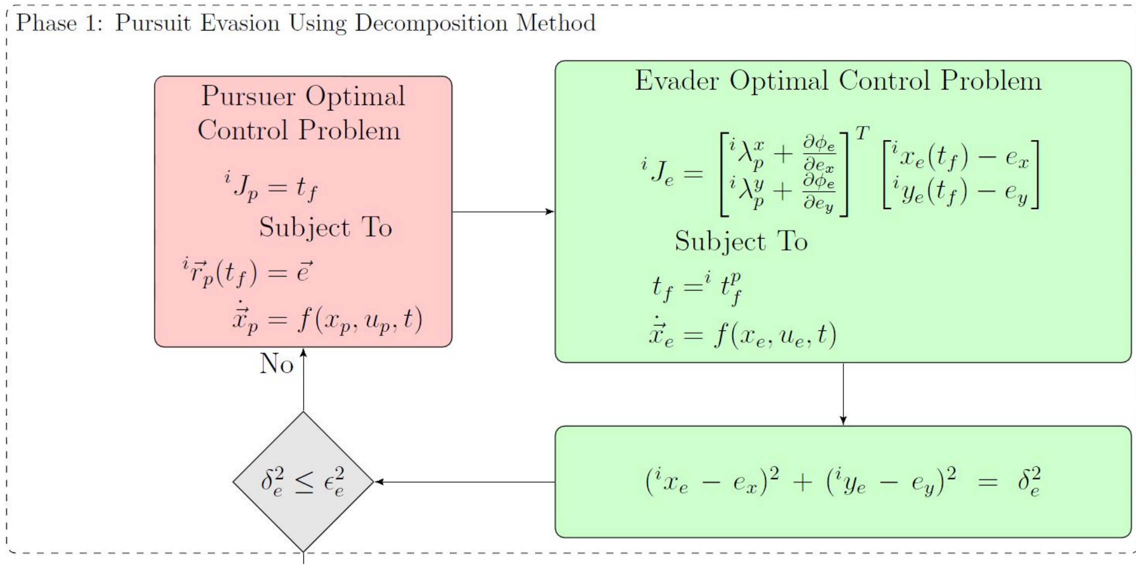

Figure 9. Phase one: Pursuit evasion using Decomposition Method

An additional application of the Decomposition Method, the forced rendezvous problem, is derived and demonstrated in Appendix A.

3.4.2 Phase two: Defense.

A significant benefit of the Decomposition Method is turning differential game theory into classic optimal control problems. Adding a defender only requires two additional optimal control problems. The first optimal control problem is a free final state, free final time problem. The defender is attempting to capture the pursuer before the pursuer captures the evader. Therefore, the objective of the defender is to see the minimum interception distance between pursuer and defender. This portion of the algorithm is shown in Figure 10.

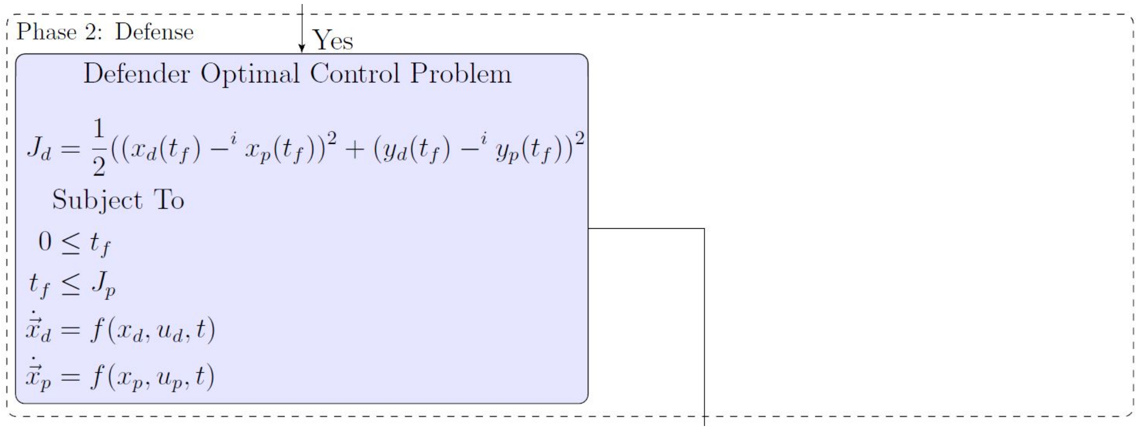

Figure 10. Phase two: Defense

3.4.3 Phase three: Evaluation.

The final optimal control problem depends on the output of phase two. Figure 11 shows the two different optimal control problems in phase three.

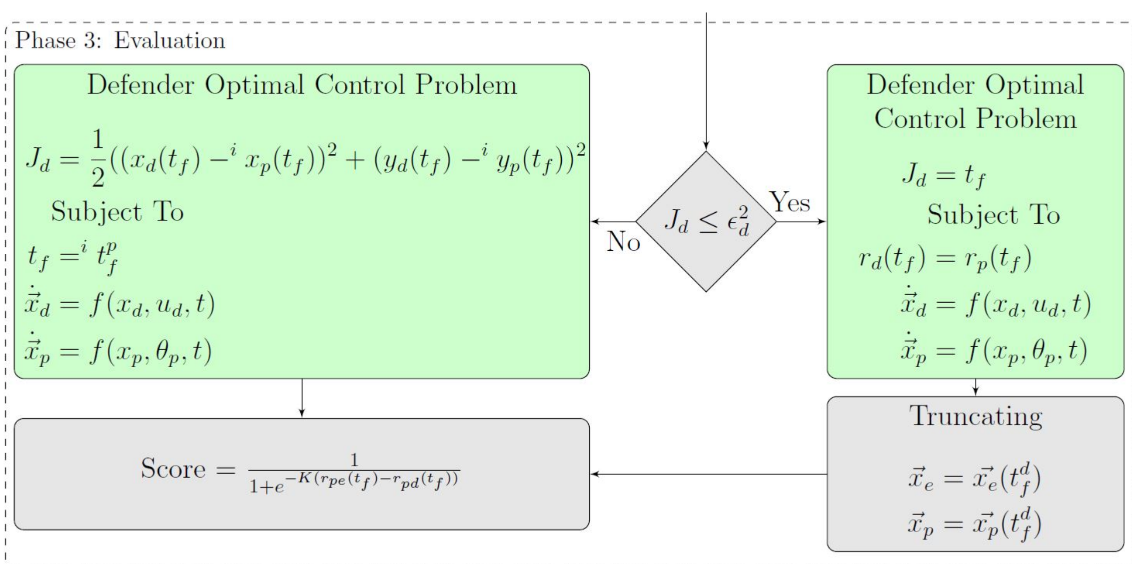

Figure 11. Phase three: Evaluation

If the defender can capture the pursuer, then there is a minimal time to capture. This creates a fixed final state, free final time with an objective of minimizing final time. The states of the evader and pursuer are truncated at the time the defender captures the pursuer. The truncation is done for scoring purposes, to find the margin of victory.

However, if the defender can not capture the pursuer, a fixed final time, free final state problem to see how close the defender was to intercepting the pursuer when the pursuer captures the evader. How close the defender was to intercepting the pursuer is considered the loss margin, and used for scoring purposes. Figure 12 shows the entire algorithm. Appendix B provides a visual aid similar to how it is coded in MATLAB.

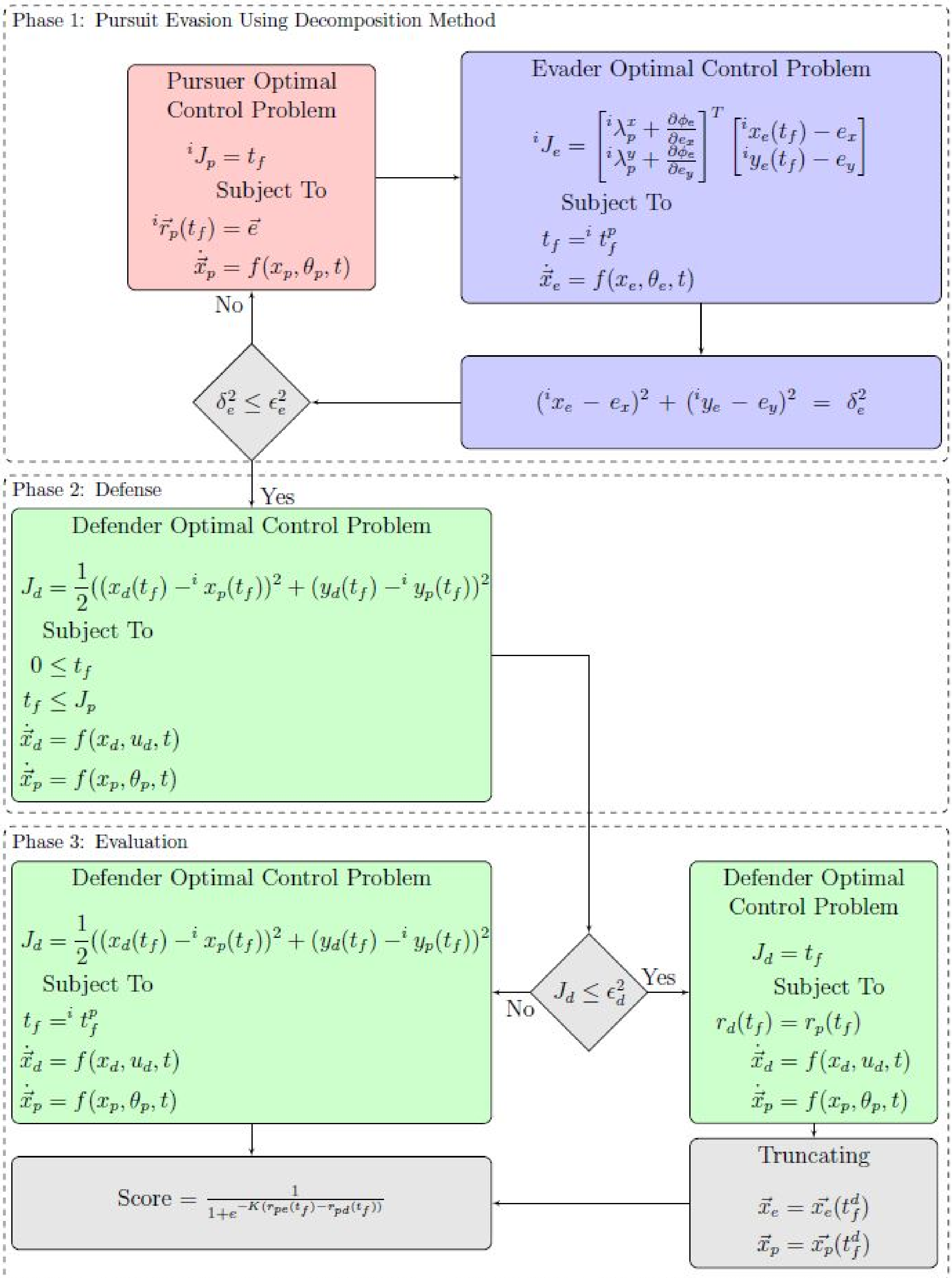

Figure 12. Flow chart of Decomposition Method with defender

3.5 Adding Strategies for the Evader

The Decomposition Method requires taking a Taylor series around the evader's final position. In the original decomposition paper, the first-order term is only a function of the costates of the pursuer. However, if the evader's objective value is not only maximizing capture time, but also a function of position, then the second term in the evader's objective ($\frac{\partial \phi}{\partial e}$) is not zero, as used in [27]. This research purposes using that term to generate strategies for the evader, or pseudo-cooperation between evader and defender. The evader accepts a worse capture time for a potential advantage in its trajectory.

Ideally, the evader would have full knowledge of the defender's states at all times, however a mathematical solution for a full cooperating defender-evader team has not been solved within the assumptions of this differential game [31]. However, in this algorithm, the defender and pursuer's initial state vectors are known at the start of the game. Therefore, as long as the terminal cost of the evader is a function of only the initial state vectors of the pursuer and defender and the final state of the evader, it can be incorporated into the Taylor series. These strategies require the evader to know the defender and pursuer's initial position. This research poses six different strategies, shown in Tables 2. Table 2 provides a narrative descriptor, and the terminal cost used. However, these strategies are not exhaustive. α is a weighting term between capture time and the final position of the evader, and is defined as

$$\alpha = \frac{1}{2}(1 - \cos(\theta_{dep})) = \frac{1}{2}\left(1 - \frac{\vec{r}_d(0) \cdot \vec{r}_p(0)}{r_d(0) r_p(0)}\right). \tag{3.20}$$

Figure 13 shows α as a function the angle between the defender, evader and pursuer. Figure 14 shows an example of α. $\cos(\theta_{dep}))$ is the angle between the pursuer and defender, from the evader's perspective.

Table 2. The evader strategies and their terminal cots

Strategy	Terminal Cost $\phi_e(t_f)$	Description
Strategy one	0	Maximizing capture time
Strategy two	$(x_e(t_f) - x_d(0))^2 + (y_e(t_f) - y_d(0))^{0.5}$	Maximizing capture time and minimizing distance to defender
Strategy three	$\alpha(x_e(t_f) - x_d(0))^2 + (y_e(t_f) - y_d(0))^{0.5}$	Weighting maximizing capture time and minimizing distance to defender
Strategy four	$-(x_e(t_f)(x_d(0) - x_p(0)) + y_e(t_f)(y_d(0) - y_p(0)))$	Maximizing capture time and luring pursuer towards defender
Strategy five	$\frac{1}{2}(x_e(t_f) - x_d(0))^2 + (y_e(t_f) - y_d(0))^2$	Maximizing capture time and minimizing the square distance to defender
Strategy six	$\frac{\alpha}{2}(x_e(t_f) - x_d(0))^2 + (y_e(t_f) - y_d(0))^2$	Weighting maximizing capture time and the square minimizing distance to defender

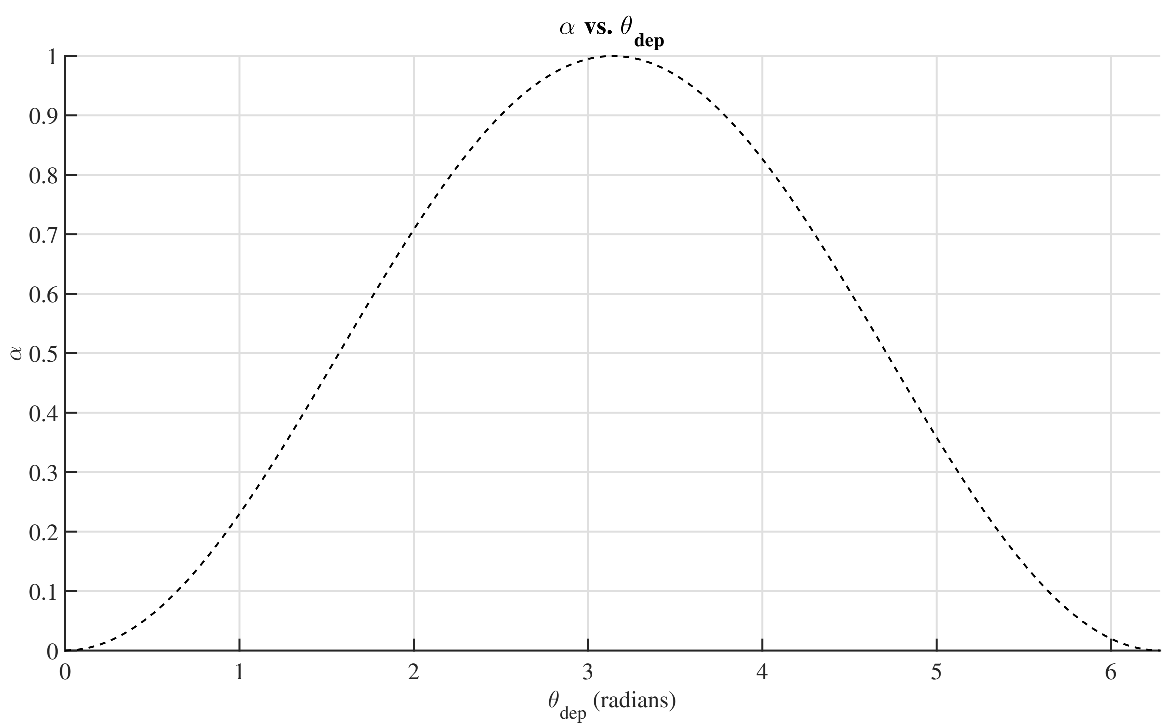

Figure 13. α vs. initial angle between three players

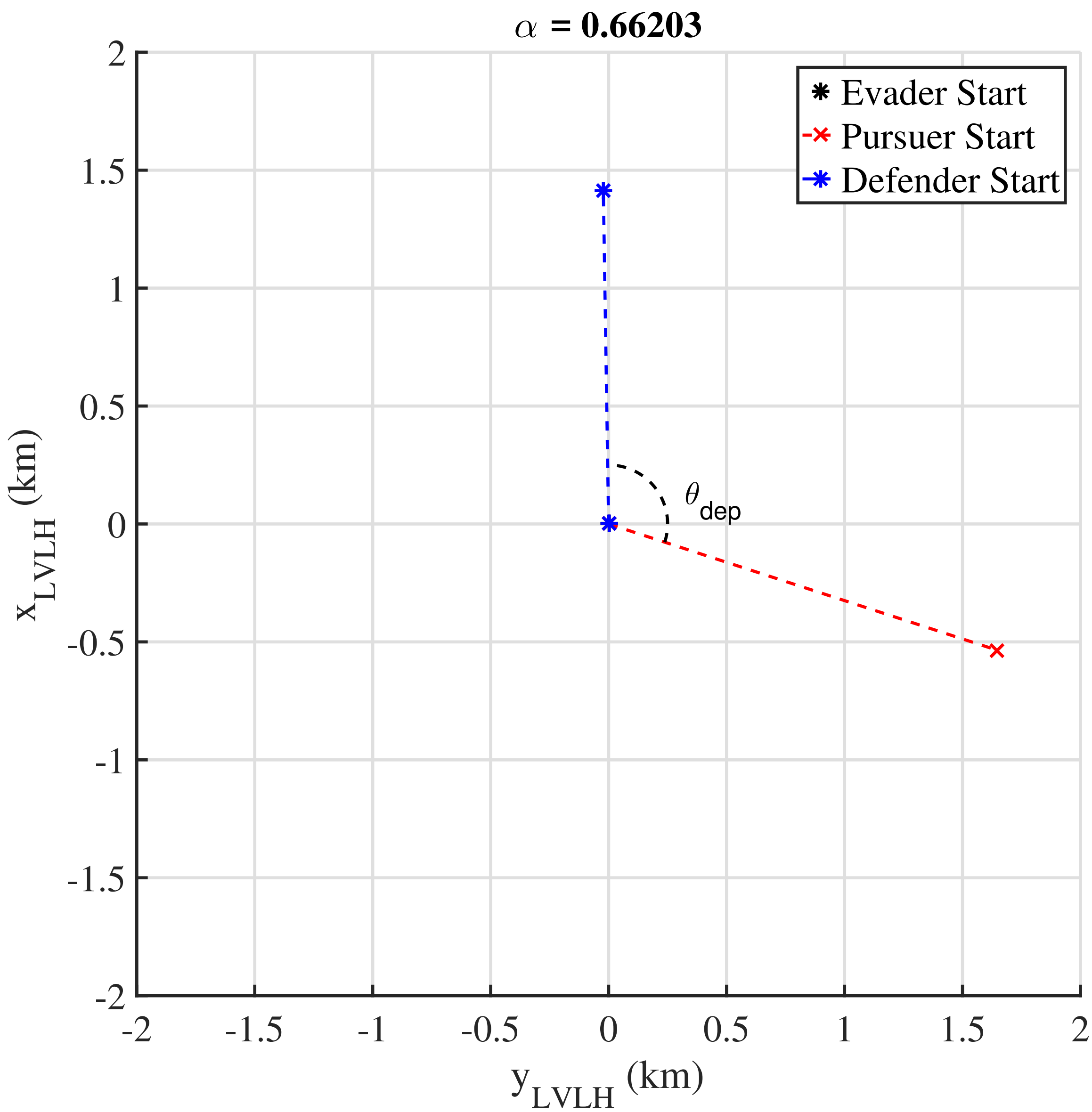

Figure 14. Example of α for strategies three and six

3.6 Scoring

A scoring metric was required to evaluate the outcome of each differential game. Using a strict binary win/loss did not yield unique results. A significant portion of the literature for pursuit-evader-defender differential games concerns aerial dog fighters, which have different scoring requirements. Therefore, a new scoring function was required. The following criteria were used to generate the scoring function.

1. Values between 0 and 1
2. High values are desirable for the Defender/Evader team
3. Sufficient sensitivity to aid design requirements
4. Incorporate the relative distances between pursuer-evader and pursuer-defender

A candidate function was the logistic function. The logistic function is defined as

$$f(x) = \frac{L}{1 + \exp\left(-K(x - x_0)\right)}, \tag{3.21}$$

where, exp is the natural logarithm base, x_0 is the midpoint value, L is the maximum value and K is the steepness of the curve. To meet the criteria, the following values are used in the scoring function,

$$\begin{aligned} x_0 &= 0 \\ L &= 1 \\ x &= r_{pe}(t_f) - r_{dp}(t_f) \end{aligned} \quad . \tag{3.22}$$

Equation (3.22) is also known as the soft Heaviside function. Additional scoring functions were not considered. The scoring function used in this research is

$$\text{Score} = \frac{1}{1 + e^{-K(r_{pe}(t_f) - r_{dp}(t_f))}}, \tag{3.23}$$

where $r_{pe}(t_f)$ is the distance between evader and pursuer at the final time, and $r_{dp}(t_f)$ is the distance between the defender and pursuer at the final time. This leaves K as an unresolved constant. Figure 15 shows how K influences the score.

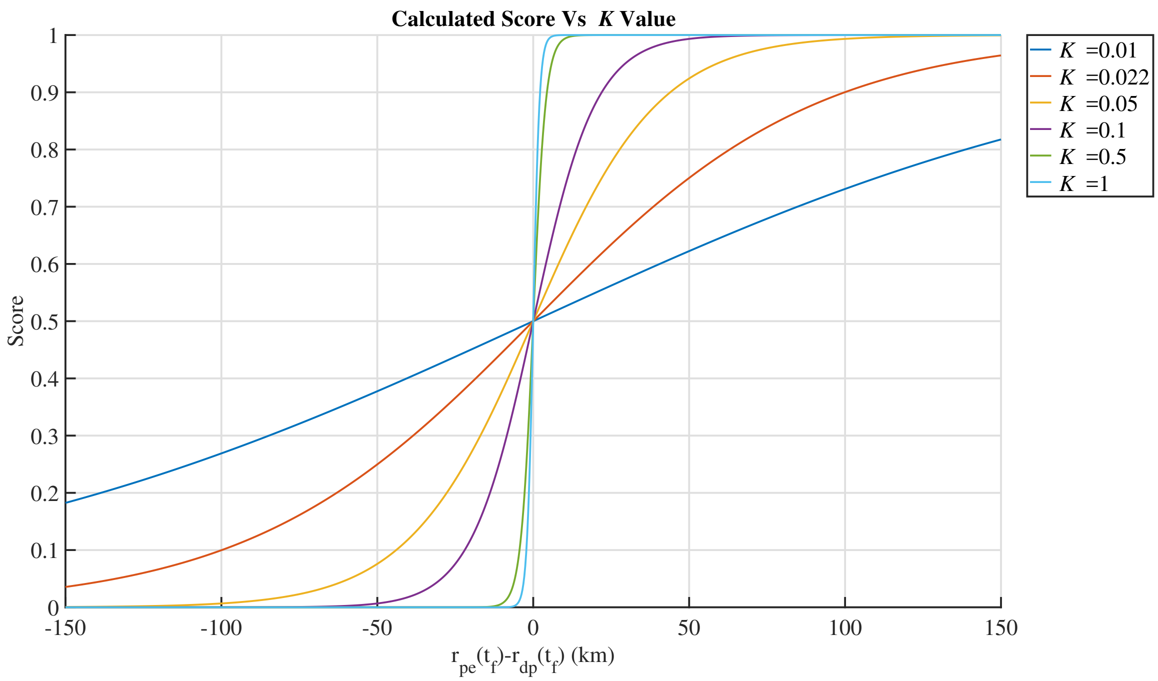

Figure 15. Influence of K on score

A possible method of determining the K value in the scoring function is to determine a distance (r) and a score (S) for that distance value. Solving Equation (3.22) using r and S for K yields

$$K = \frac{1}{r} \ln\left(\frac{S}{1-S}\right). \tag{3.24}$$

For example, assigning a score value of 0.9 at a distance of 100 km yields

$$K = 0.022, \tag{3.25}$$

which is the K valued used in this research unless otherwise indicated. In addition to a scoring function, a scoring metric was required to evaluate the strategies as compared to each other. Scoring metrics were used in this research to further investigate the data. Table 3 outlines the scoring metrics used. The best average score amongst all scenarios may not yield sufficient details, as one strategy might not perform well under some initial conditions, but for very specific scenarios may be the optimal strategy. As this is a completely new use of the Decomposition Method, a more detailed analysis of the data may be warranted.

Table 3. Scoring metrics to compare strategies

Metric	Description of criteria
High Scorer	% of times this strategy produced the best outcome of all strategies
Low Scorer	% of times this strategy produced the worst outcome of all strategies
Unique Winner	% of times this strategy produced the only winning outcome of all strategies
Unique Loser	% of times this strategy produced the only losing outcome of all strategies
Mean Score	Mean Score over all games

3.7 Test Matrices

This research requires a large number of differential games, with a variety of initial state vectors and capabilities of the pursuer, evader, and defender, in order to investigate the orbital debris defender. This research used different test matrices, depending on the parameter investigation. The following sections outline the test matrices used during this research.

3.7.1 Evader Capability Test Matrix.

One of the initial parameters investigated how T_e/T_d and T_p/T_d influenced the mean score of a game. A number of games, with a variety of pursuer and defender initial state vectors and varying values T_e/T_d and, T_p/T_d were solved, and averaged. Table 4 outlines the parameters used in this investigation.

Table 4. Test matrix for Evader strategies study

Variable	Value
Defender initial x and y bounds	-70 km to 70 km
Pursuer initial x and y bounds	-100 km to 100 km
Number of initial positions for each player	16
Spacing between points	equal along x and y axis
ϵ_e	10 cm
ϵ_d	1 mm
Number of differential games per data point	256

3.7.2 Defender Initial Position Test Matrix.

The initial position of the defender was a parameter investigation. However, in order to ensure a robust data set, a large amount of differential games were played. Table 5 outlines the parameters used in this investigation.

Table 5. Test matrix for defender initial position study

Variable	**Value**
Defender initial x and y bounds	-100 km to 100 km
Pursuer initial x and y bounds	-105 km to 105 km
Number of positions for each player	100
Spacing between points	equal along x and y axis
ϵ_e	10 cm
ϵ_d	1 mm
Number of differential games per data set	10,000

The parameter study of the evader maintaining its mission also used the same parameters. The LEO parameter used similar parameters, outlined in Table 6.

Table 6. Test matrix for defender initial position study in at 2000 km altitude

Variable	**Value**
Defender initial x and y bounds	-20 km to 20 km
Pursuer initial x and y bounds	-21 km to 21 km
Number of positions for each player	100
Spacing between points	equal along x and y axis
ϵ_e	10 cm
ϵ_d	1 mm
Number of differential games per data set	10,000

3.7.3 Time In Orbit and Optimally Phased Defenders Test Matrix.

As a result of the defender's initial position study, the next parameter study was determining the variation in the mean score of the same defender while in the NMC around the evader. This was measured as the angle between the defender's initial position and the negative x axis, demonstrated in Figure 16.

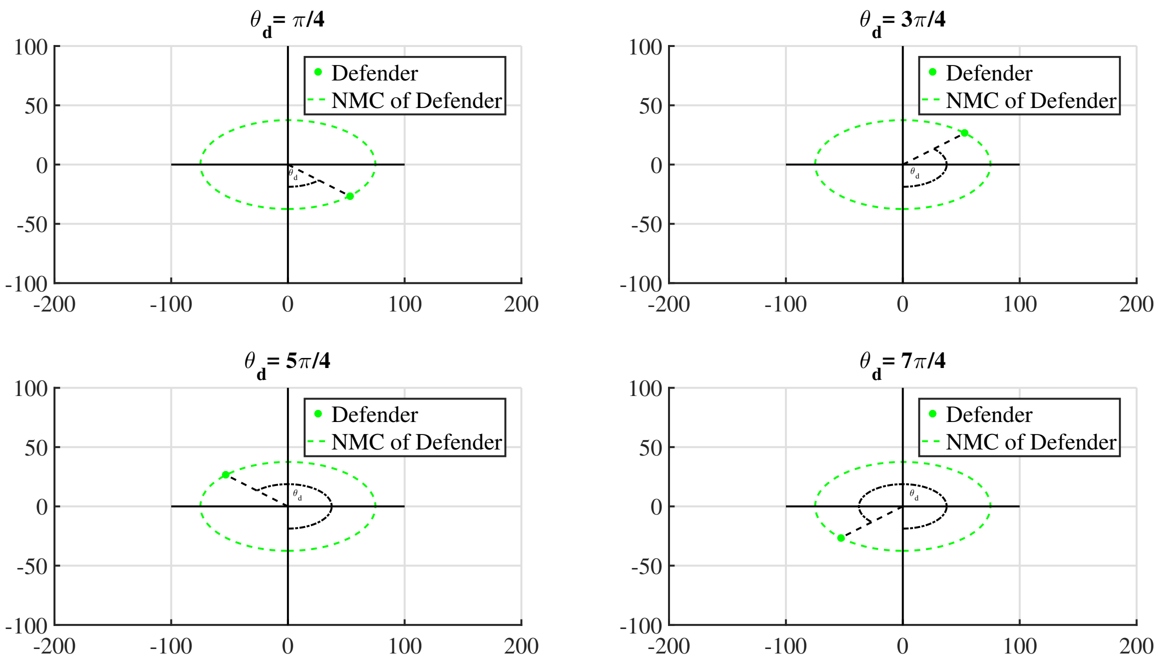

Figure 16. Variety of defender initial position during NMC as measure by θ_{dep}

The following test matrix outlines the parameters used in this research. Table 7 outlines the parameters used in this investigation.

Table 7. Test matrix for defender time in orbit, optimal phasing, and the performance of two optimally phased satellites

Variable	**Value**
Defender relative radius	75 km
Defender number of angles	20
Pursuer initial x and y bounds	-105 km to 105 km
Number of pursuer positions	100
Spacing between points	equal along x and y axis
ϵ_e	10 cm
ϵ_d	1 mm
Number of differential games per data point	2,000

This data set was also used in the optimal phasing of the two defender system, as well as the optimally phased defenders.

3.7.4 Evader Strategies Test Matrix.

In order to determine the optimal strategy, a variety of games were played. All of those games involved the same initial positions (outlined in Table 8), but the initial capabilities were varied. The capabilities used to generate a large data sample are shown in Table 9.

Table 8. Test matrix for player capabilities study

Variable	Value
Defender initial x and y bounds	-100 km to 100 km
Pursuer initial x and y bounds	-105 km to 105 km
Number of initial positions for each player	36
Spacing between points	equal along x and y axis
ϵ_e	10 cm
ϵ_d	1 mm
Number of differential games per scenario	1296

3.8 Limitations

The research presented here has a variety of limitations, due to the initial assumptions and the approach used. One of the biggest limitations is during phase two and phase three if the defender can intercept the pursuer. The pursuer's control are passed into GPOPS, but in order to solve the optimal control problem, GPOPS interpolated the pursuer's control. This potential causes the defender to chase a

Table 9. Capabilities used in strategies studies

Scenario	T_e/T_d	T_p/T_d	β
1	0.20	0.80	0.50
2	0.20	0.80	0.80
3	0.30	1.20	0.25
4	0.30	0.90	0.15
5	0.30	1.20	0.15
6	0.40	0.80	0.20
7	0.40	1.00	0.20
8	0.50	1.20	0.25
9	0.50	0.90	0.15

pursuer who is using a control with error caused by the interpolation. Future work may deal with this limitation by incorporating the true dynamics of the pursuer into the equations of motion, thus avoiding the interpolation. However, incorporating the true thrust into the equations of motion may cause a significant increase in computational time. Another limitation of this research is that there is no guarantee of the global optimal solution. Future work could use an iterative approach by passing the previous solution of Stupik and the Decomposition Method until both agree upon a trajectory and capture time.

3.9 Summary

This research posed the pursuer-evader-defender differential game into a series of classical optimal control problems, using the Decomposition Method. Two additional optimal control problems were appended to the traditional Decomposition Method, to simulate a defender. Additionally, this research argued that it is possible to generate pseudo-cooperation between the evader and defender using additional terminal costs within the Decomposition Method. Chapter IV provides the results of various simulations corresponding to the test matrices defined in this chapter.

IV. Implementation and Analysis

The goal of this research is to provide performance trade space to satellite designers and operators. However, this research did not have an initial data set or recommendations in order to narrow the search parameters. Therefore, a large amount of differential games were required, in order to generate a data set. This data set was then analyzed in order to determine the key parameters that increase the likelihood of success of the orbit debris defender. Those results are presented in the following sections, however the Decomposition Method is a relatively new method to solve the pursuer evader problem. Therefore, the solution generated using the Decomposition Method was compared to the solution solved using Stupik's method in order to validate the Decomposition Method.

4.1 Initial Validation

This portion of the research solves the pursuit-evasion problem in space using the Decomposition Method. In order to validate that approach, a comparison was required. Stupik solved the same pursuit evasion problem using the necessary conditions and solving the two-point boundary problem [24]. Both algorithms attempt to maximize the capture time of the evader, so if the Decomposition Method produces similar results to Stupik, then it is validated. Figure 17 shows an example of the trajectories of all players, if all players do not maneuver. Note, in the absence of the evader maneuvering, the pursuer will collide with the evader.

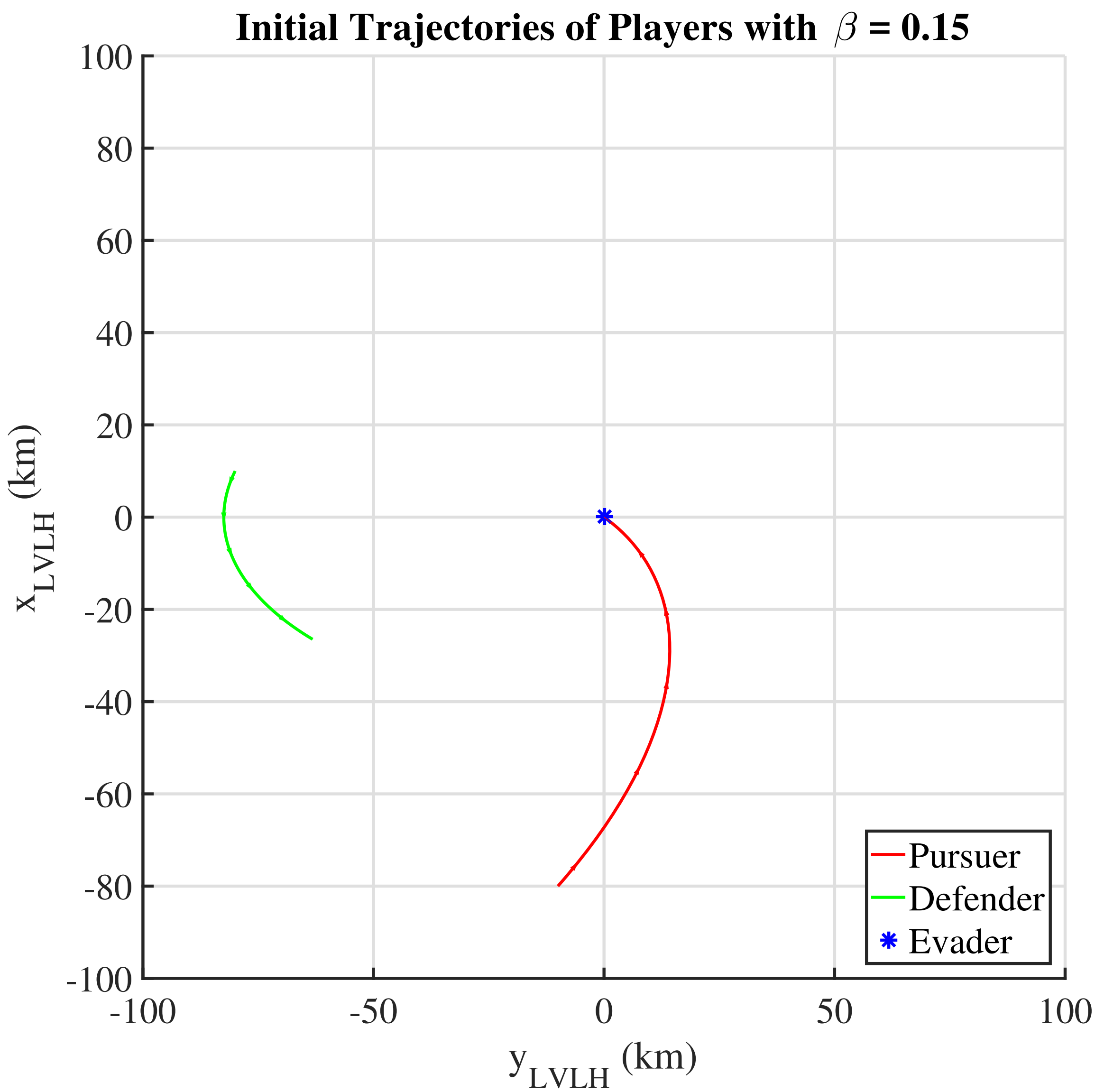

Figure 17. Initial velocity vectors

Using the same initial conditions, the pursuit-evasion portion of the differential game was solved independently of each other using both algorithms. Finally, Stupik's solution was used as an initial guess to the Decomposition Method. The following section outlines the results of the Decomposition Method, and compares the results to Stupik's method.

4.1.1 Decomposition Method.

The Decomposition Method requires iterations, between the evader and pursuer, to generate the Nash Equilibrium. Delaying the capture time produces the optimal solution for the evader. The evader is eventually captured by the pursuer, which is required for both the Decomposition Method and Stupik's method. Figure 18 shows the change in evader's final position as the number of iterations increases.

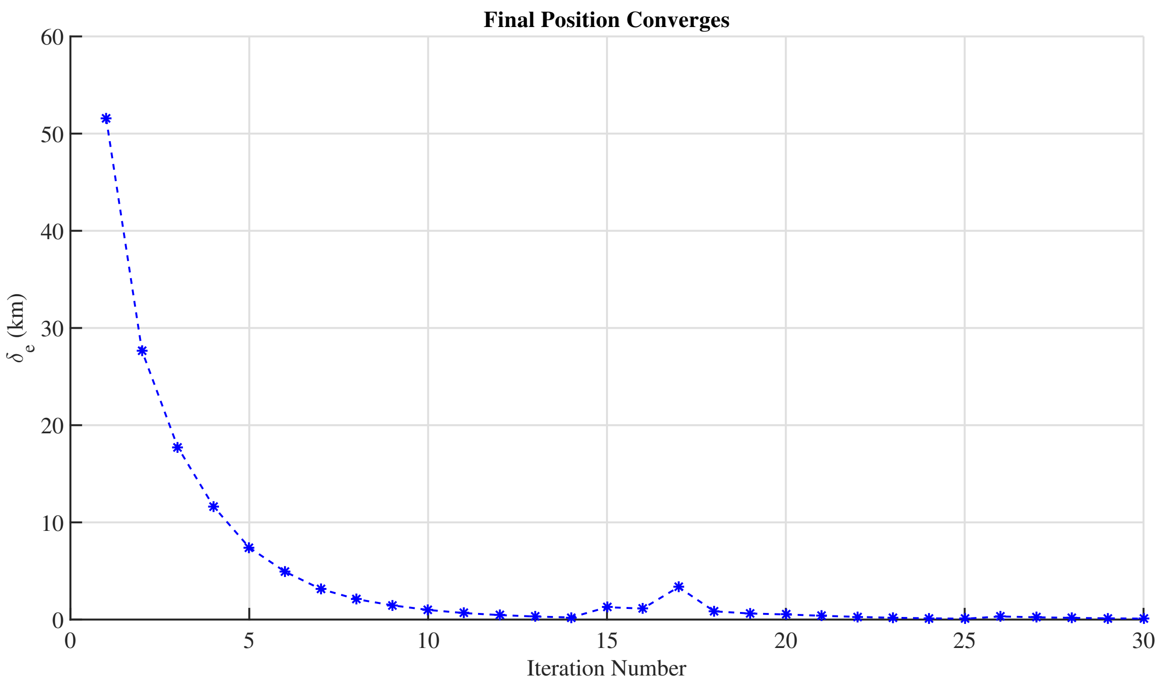

Figure 18. Evader reaches Nash Equilibrium as iteration number increases

The pursuer's objective, capture time, also reaches a final value as the iteration number increases, shown in Figure 19. Figures 18 and 19 show that the Decomposition Method is stable when applied to the HCW equations of motion.

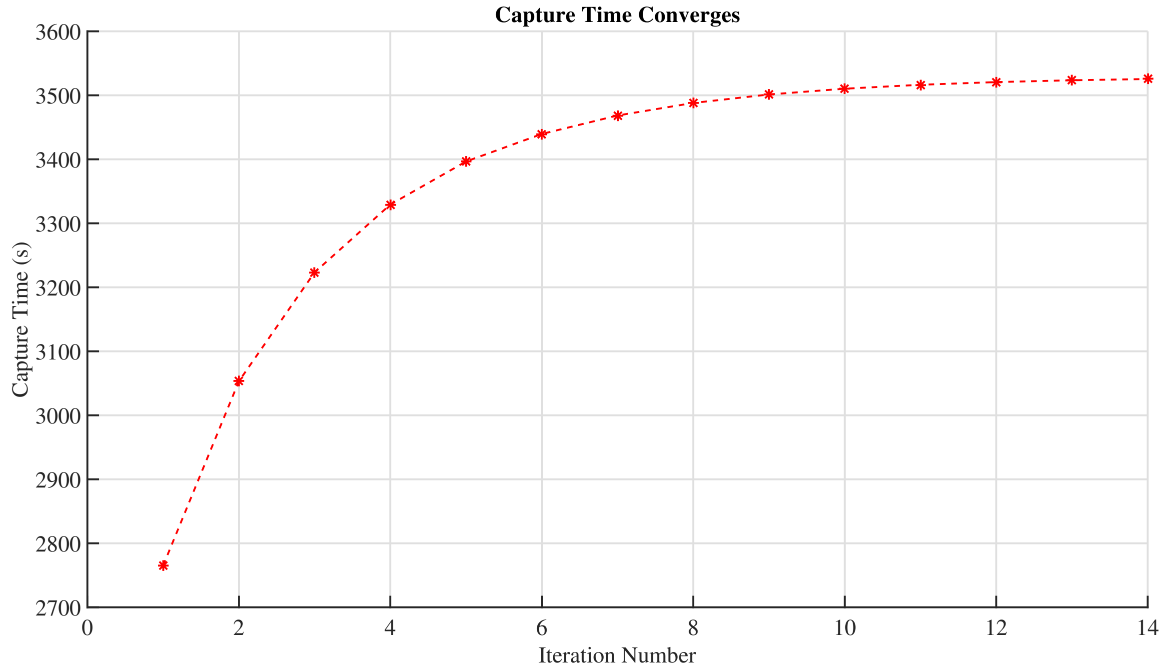

Figure 19. Pursuer reaches Nash Equilibrium as iteration number increases

The trajectories generated between the two players during the Decomposition Method are illustrated in Figure 20, with the final trajectory shown in Figure 21.

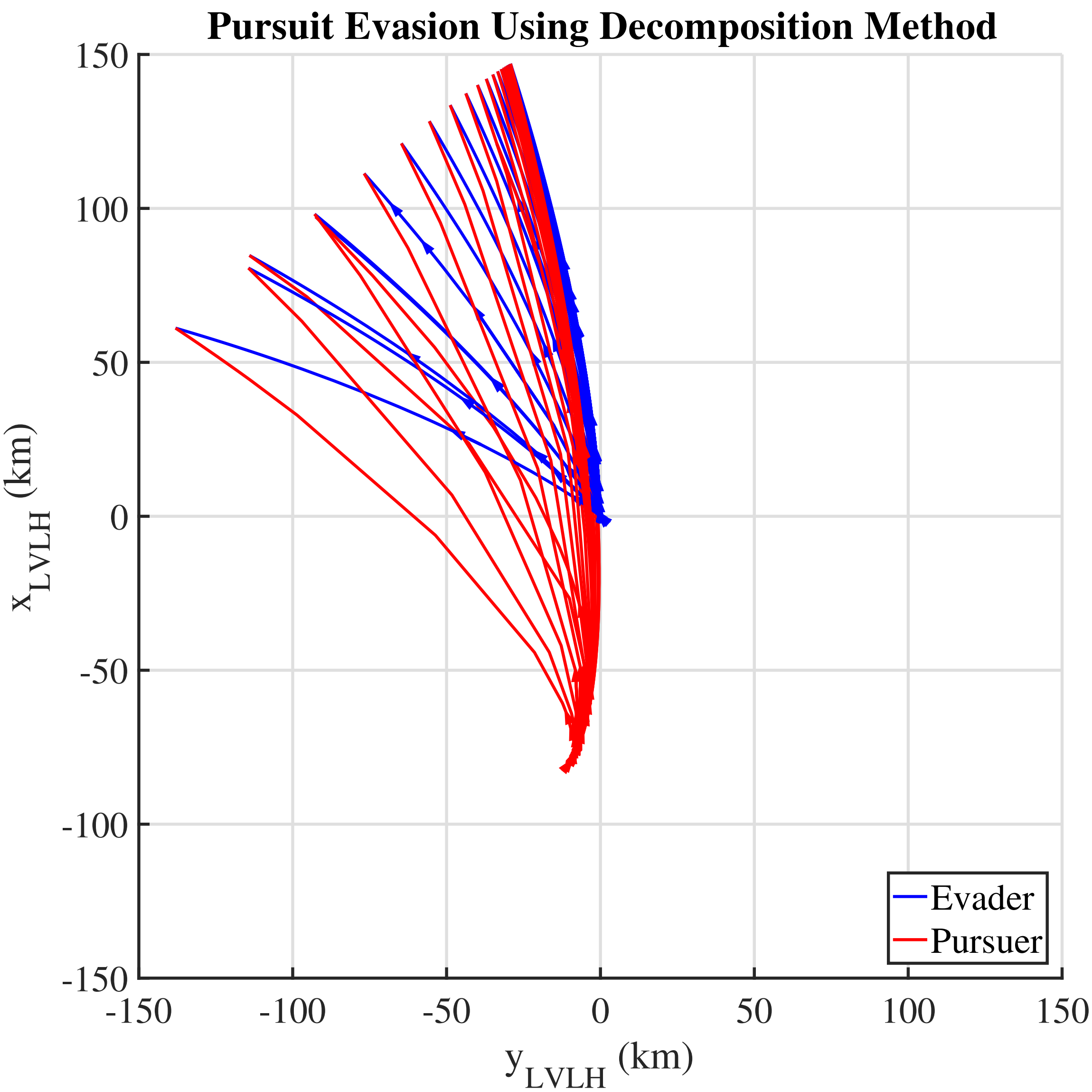

Figure 20. Iterative trajectories using Decomposition Method

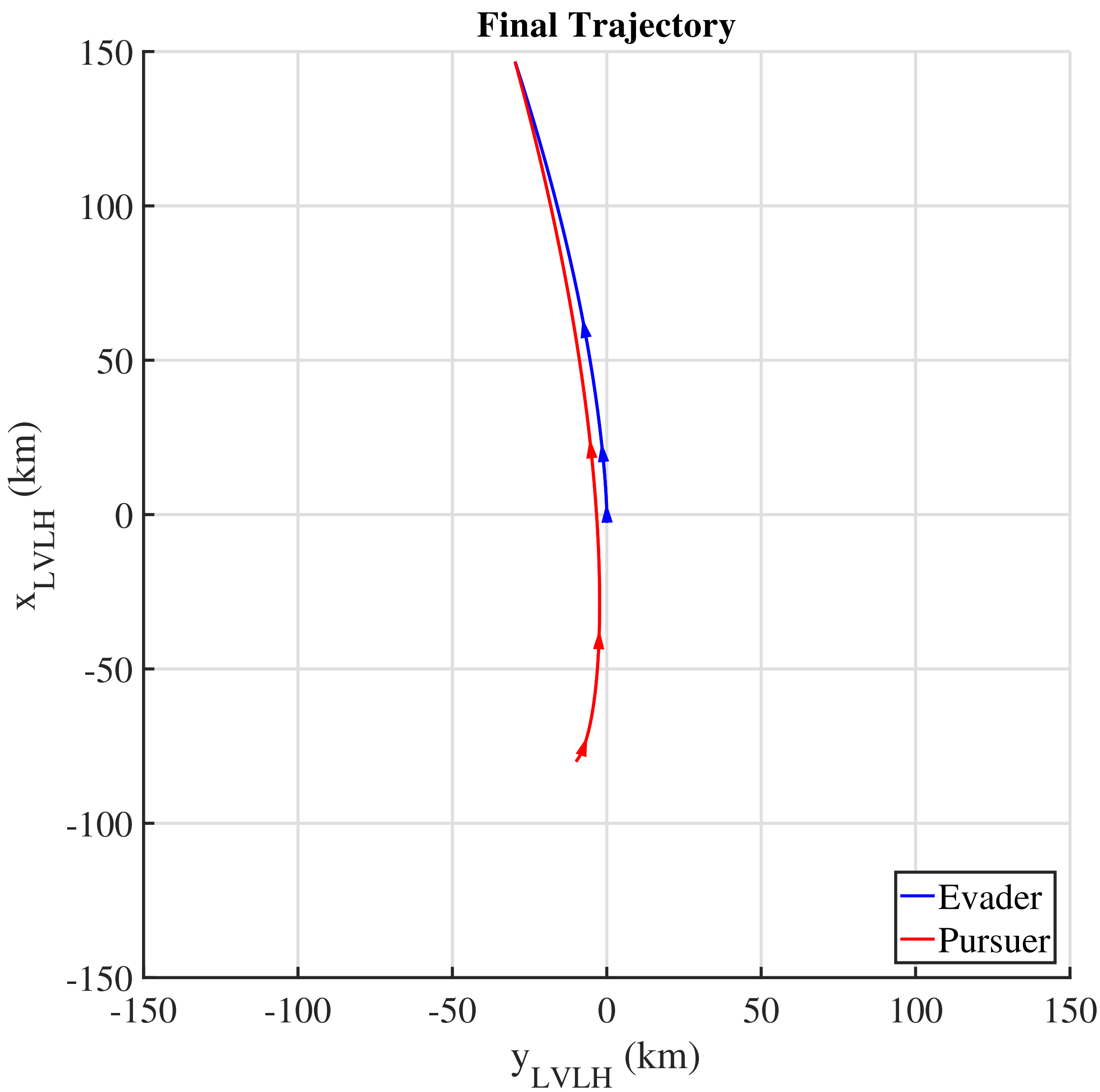

Figure 21. Final Pursuit Evasion trajectories using Decomposition Method

The controls for both players after final iteration are shown in Figure 22. These results agree with the results found in Stupik, that $u_e = u_p$ [24]. This control law is consistent with the necessary condition $\frac{\partial H}{\partial u} = 0$, which can be reduced to

$$\lambda_{\dot{x}}(t)\sin(\theta) = \lambda_{\dot{y}}(t)\cos(\theta). \tag{4.1}$$

Neither of the velocity costates are constant, nor are their slopes the same. This boundary condition produces a varying burn angle, which is shown in Figure 22.

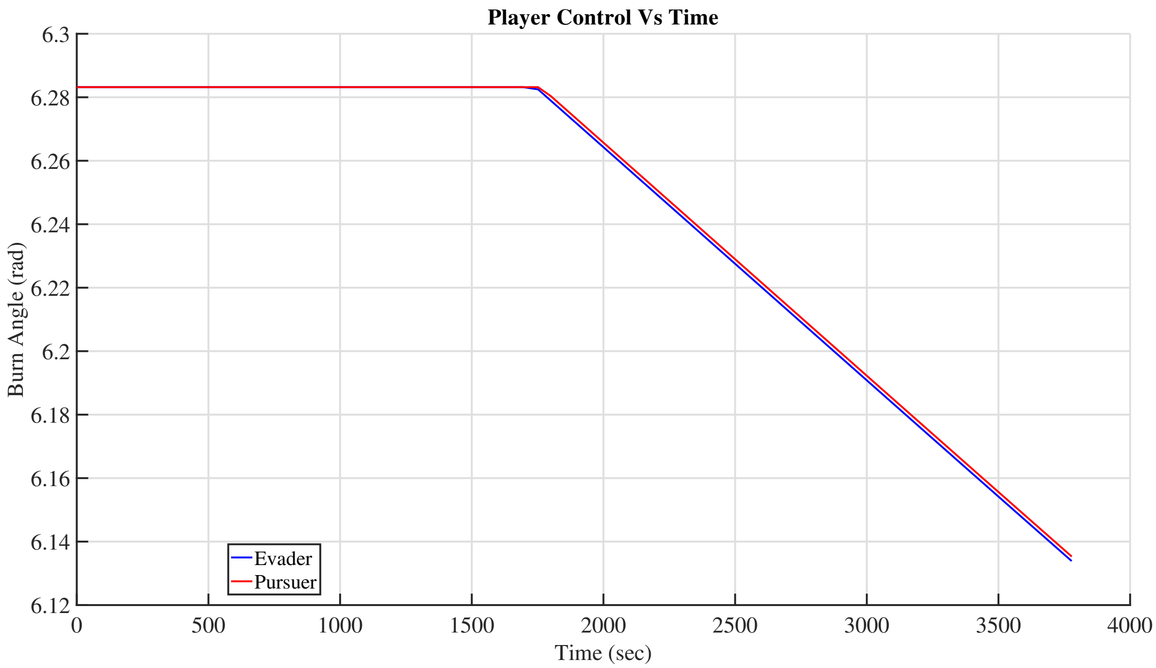

Figure 22. Control of players at end of phase one, meets necessary conditions of the same burn angle between Evader and Pursuer

The Hamiltonians for both players after the final iteration are shown in Figure 23. The Hamiltonian meets the boundary condition that

$$H_p(t_f, u^*, \vec{x}_p^*, \lambda_p^*) = -1. \tag{4.2}$$

Equation (4.2) was solved from the boundary condition equation

$$H_p(t_f, u^*, \vec{x}_p^*, \lambda_p^*) + \frac{\partial \phi}{\partial t} = 0, \tag{4.3}$$

where

$$\frac{\partial \phi}{\partial t} = 1. \tag{4.4}$$

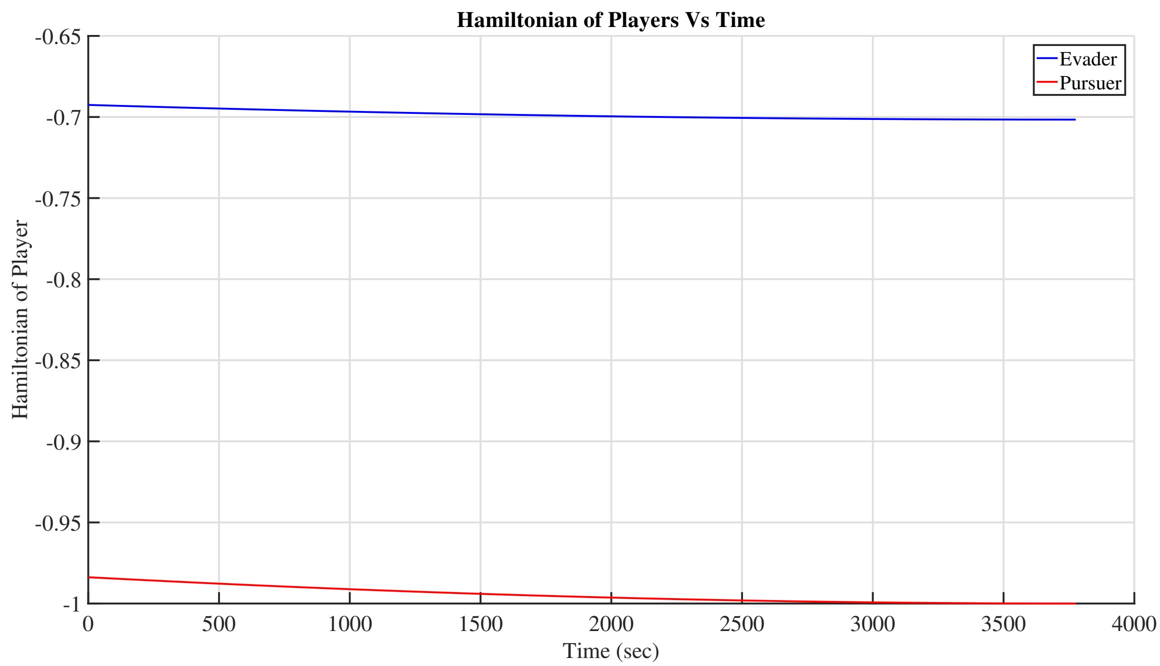

Figure 23. Hamiltonian of players at end phase one

The costates for both players after the final iteration are shown in Figure 24. The Decomposition Method meets boundary conditions solved in Stupik [24], where

$$\lambda_p^*(t) = \lambda_e^*(t), \tag{4.5}$$

and

$$\lambda_p^*(t_f) = \begin{bmatrix} v_1 \\ v_2 \\ 0 \\ 0 \end{bmatrix}. \tag{4.6}$$

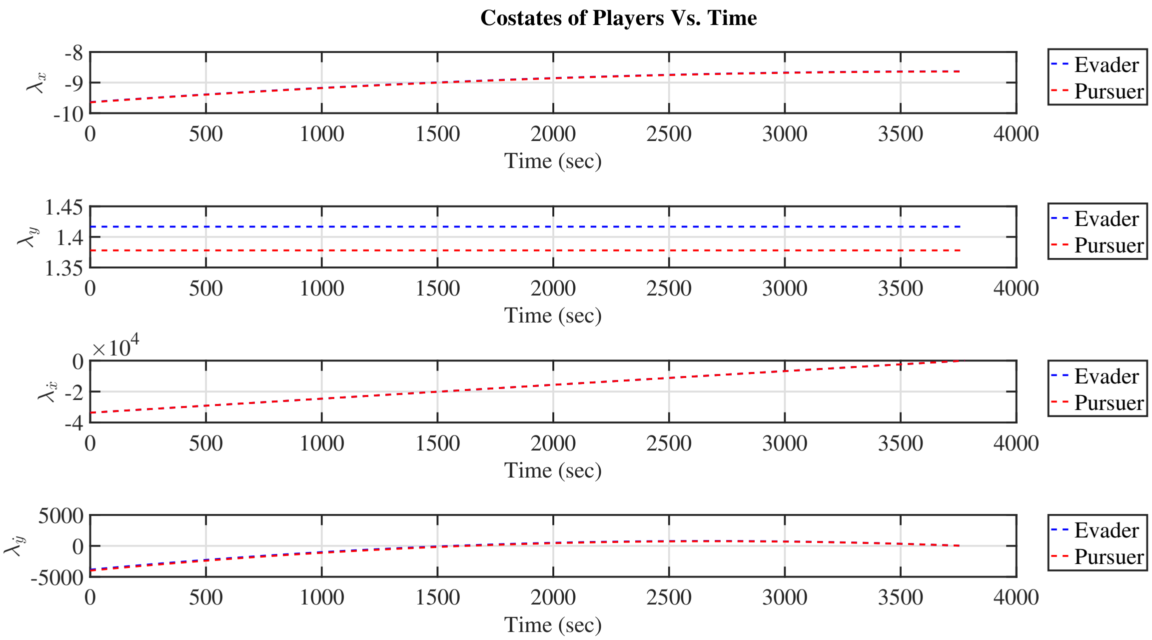

Figure 24. Decomposition Method produces solution that meets the necessary and boundary condition for Pursuit-Evasion problem

Figure 25 shows the resulting thrust of each player after phase one. There is not as much symmetry between the actual thrust magnitudes due to the difference in the denominator of both the evader and pursuer's control equation.

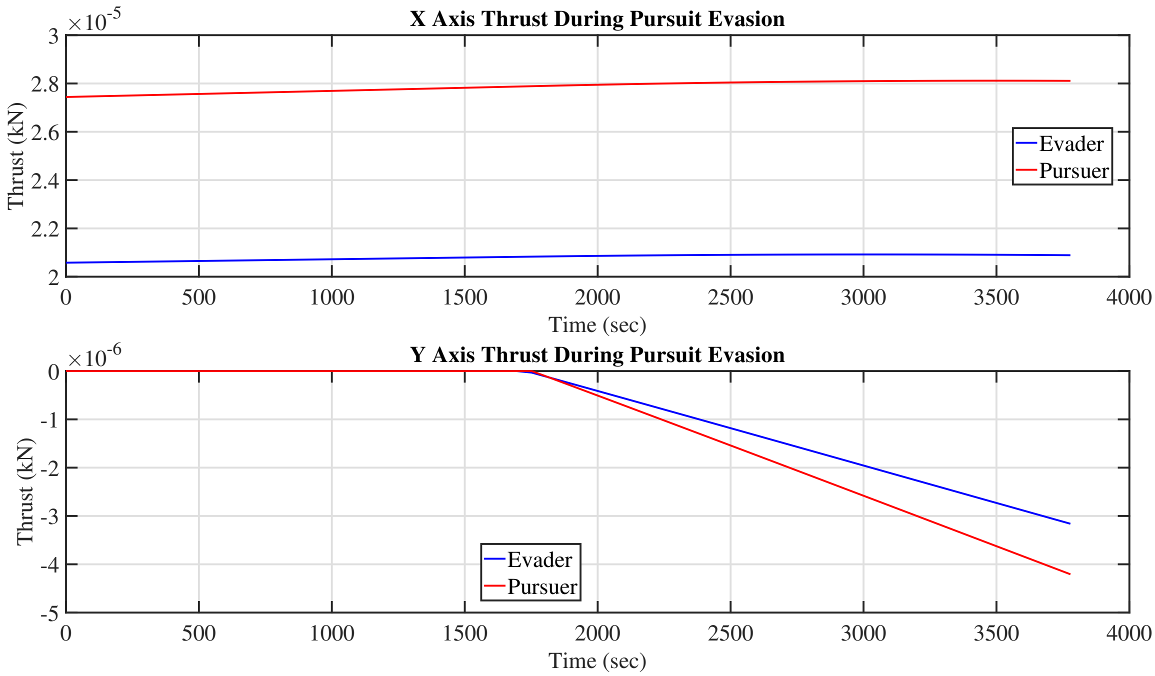

Figure 25. Thrust of players during Pursuit Evasion, meets necessary conditions

4.1.2 Comparison to Existing Literature.

Under the same initial conditions, the pursuit evasion problem was solved using Stupik's approach and the Decomposition Method. Figure 26 shows the resulting trajectories, which are significantly different. This means that there are potentially multiple saddle points along that Nash Equilibrium, and therefore there may exist multiple local optimal solutions. Neither method guarantees the global optimal solution.

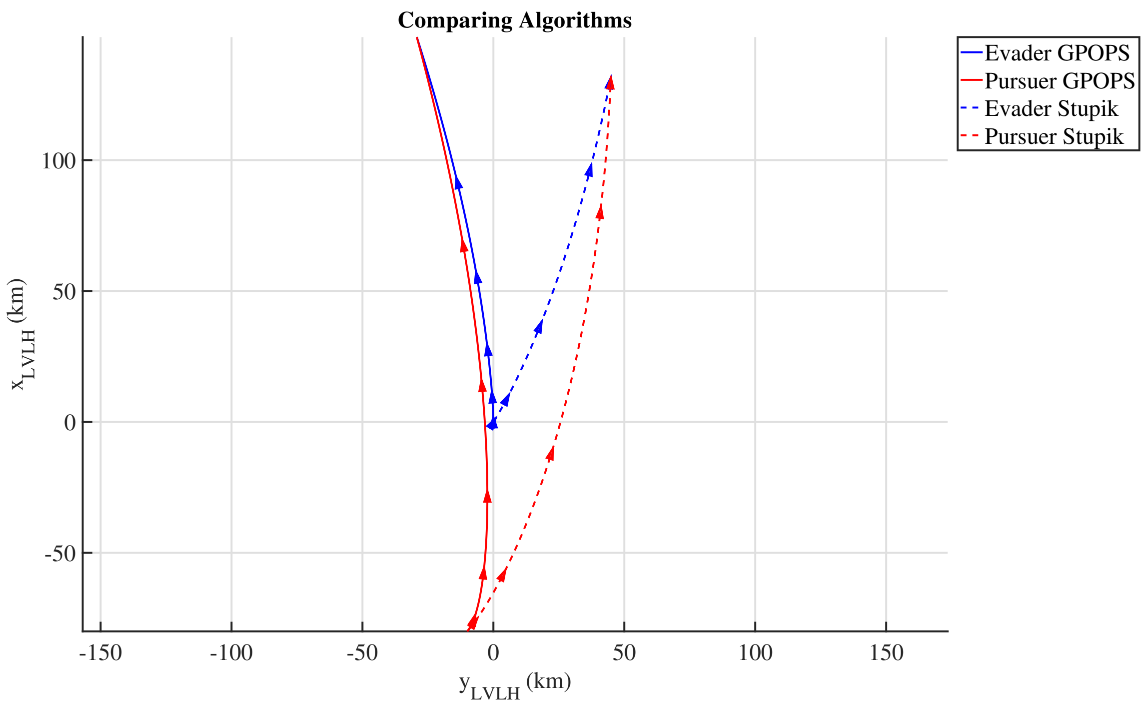

Figure 26. Decomposition Method and Stupik's method produce different trajectories, but both solution satisfy the necessary and boundary condition

As there are at least two saddle points to a Nash Equilibrium between the evader and pursuer, a third comparison was required to see if any additional solutions could be found. This third approach was providing GPOPS with Stupik's solution as an initial guess in the Decomposition Method. Figure 27 shows the resulting trajectories of all three approaches. The Decomposition method, even when primed with Stupik's solution as an initial guess, produced the same trajectory as without the Stupik's solution.

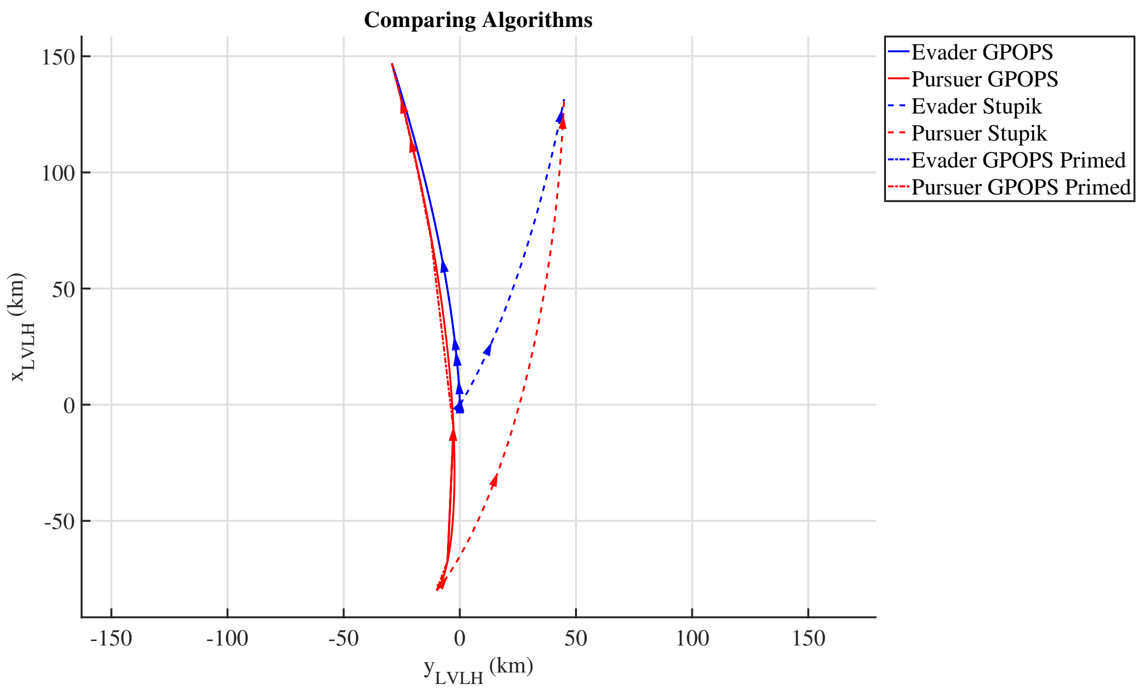

Figure 27. Decomposition Method with an initial guess of Stupik produces the original Decomposition Method solution using these initial conditions and software

However, as all three algorithms attempt to maximize capture time of the evader, a better comparison is capture time. Table 10 outlines the capture time between all three approaches. All three approaches produced similar capture times. This validates using the Decomposition Method to solve phase one of the algorithm. The next validation is to see if phase two produces a result without a large amount of error. It should be noted that the Decomposition Method did not always solve for a better final capture time than Stupik's method, and the Decomposition Method primed with Stupik's solution did not always produce the same trajectory as the

Table 10. The Decomposition Method produces a better final capture time than Stupik using these initial conditions and software

Solution Method	Capture Time (s)
GPOPS	3778.32
Stupik	3644.63
GPOPS primed with Stupik	3777.32

Decomposition Method that is not primed with Stupik's solution as an initial guess. This indicates that there may be many local optimal solutions. A method to ensure the global, as oppose to local, optimal solution is left for future work.

4.1.3 Example: Phase two.

The next optimal control problem is to see if the defender is able to intercept the pursuer, shown in Figures 28 and 29.

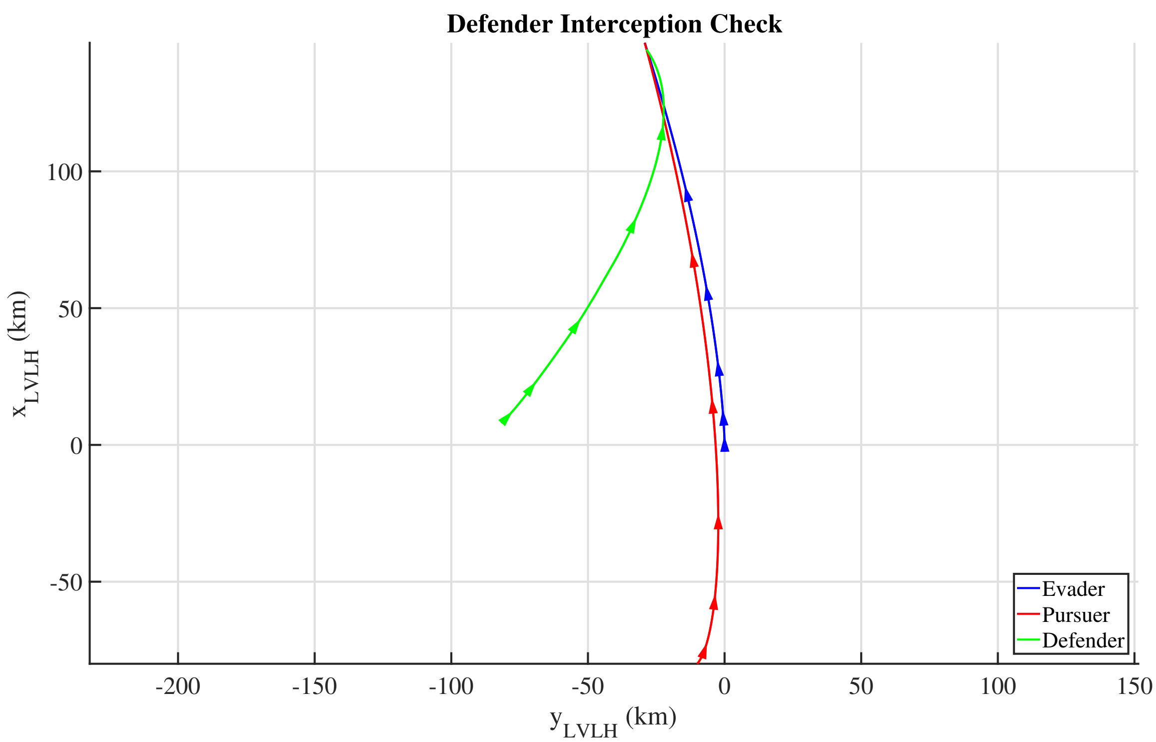

Figure 28. Defender intercepts pursuer

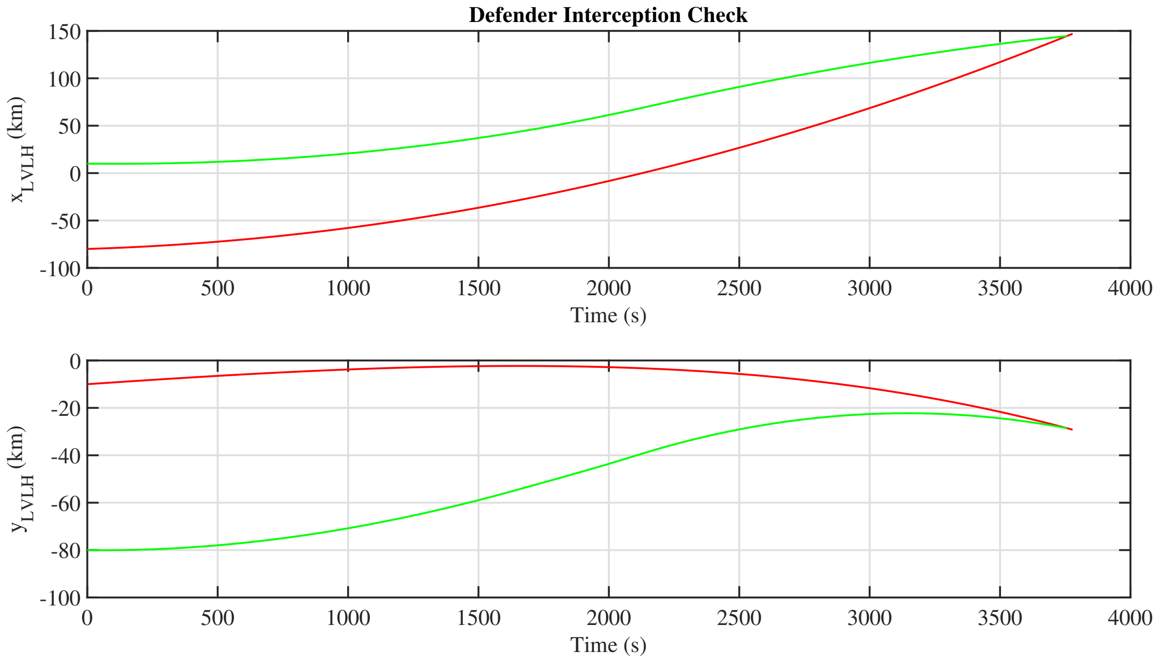

Figure 29. Minimal errors caused by interpolation of pursuer's control

While the control of the pursuer requires interpolation, it does not appear to induce sufficient error to invalidate this approach. Therefore, phase two is a reasonable approach. The final validation is for phase three, to see if a minimum time to intercept is possible.

4.1.4 Example: Phase Three.

Since the defender is able to intercept the pursuer, a minimum time to intercept is required. Figure 30 shows the final states. Note that even though the defender is using an interpolated pursuer control, it still intercepts the pursuer's path, as desired.

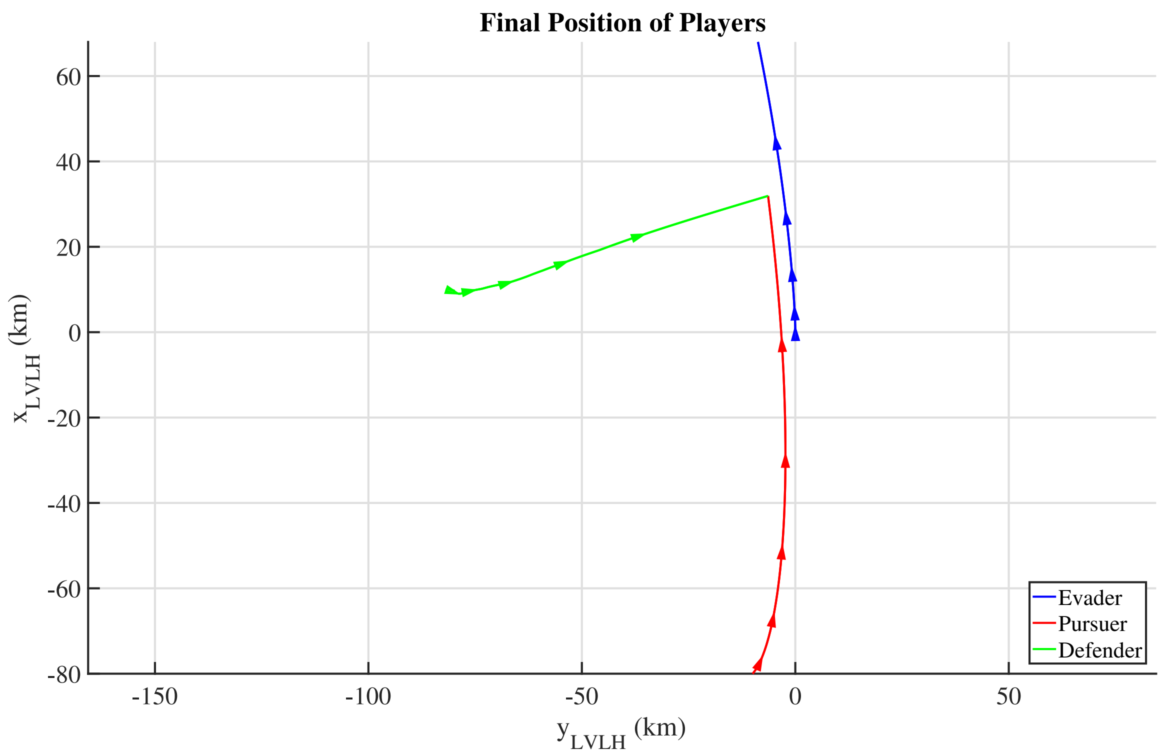

Figure 30. Trajectories of all players

Figure 31 shows the costates of all players at the end of the algorithm. The defender meets the necessary condition that the velocity costates are zero at the final time. The pursuer's and evader's velocity costates are not zero due to the truncation.

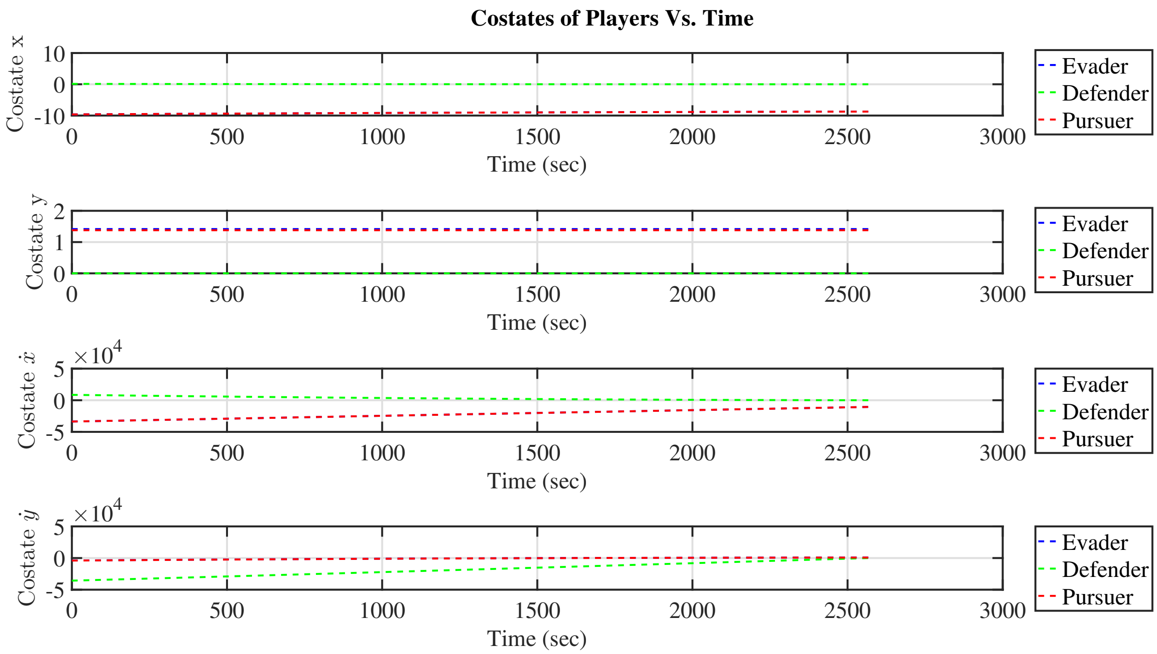

Figure 31. Costates of all players

Figure 32 shows the Hamiltonian at the end of the algorithm. The Hamiltonian for the defender is expected to be smooth, and this is likely due to numerical errors within GPOPS.

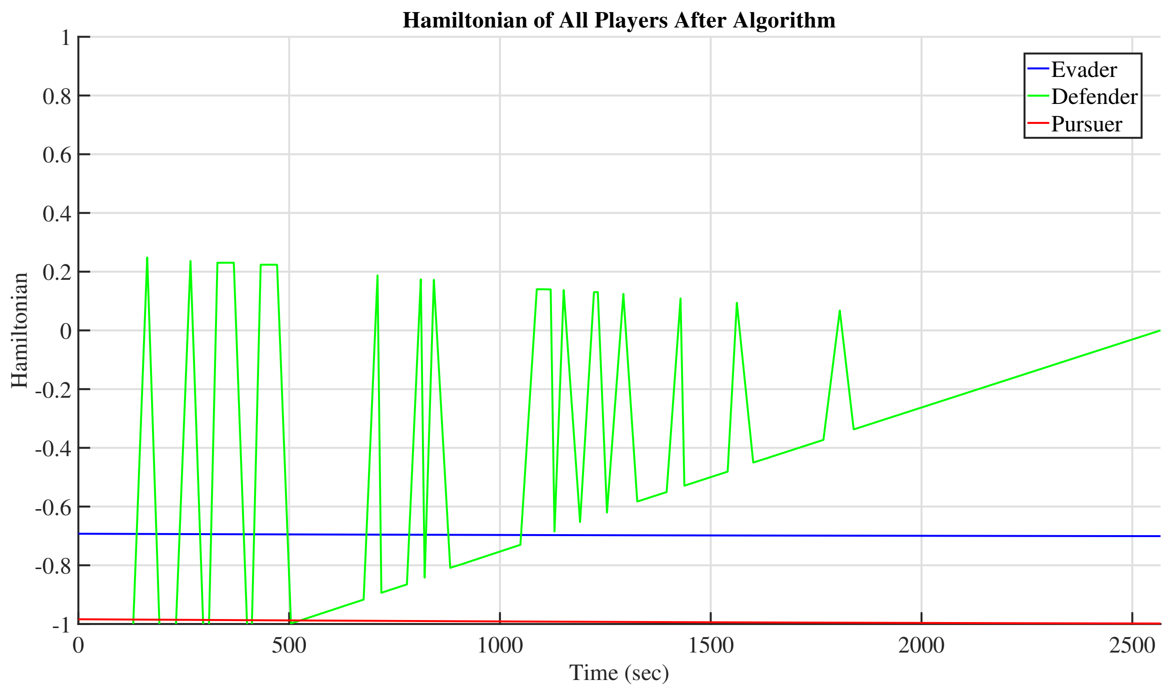

Figure 32. Hamiltonian of players at end phase three

Figure 33 shows the control for all players at the end of phase three. A nonconstant defender's control angle is expected, and likely the optimal control is a smooth continuous function. Further refinement of the GPOPS solution is likely to produce such a result. The defender's control appears similar to that of the pursuer and evader, with an offset to the angle but the same shape. This result was not analyzed further in this research.

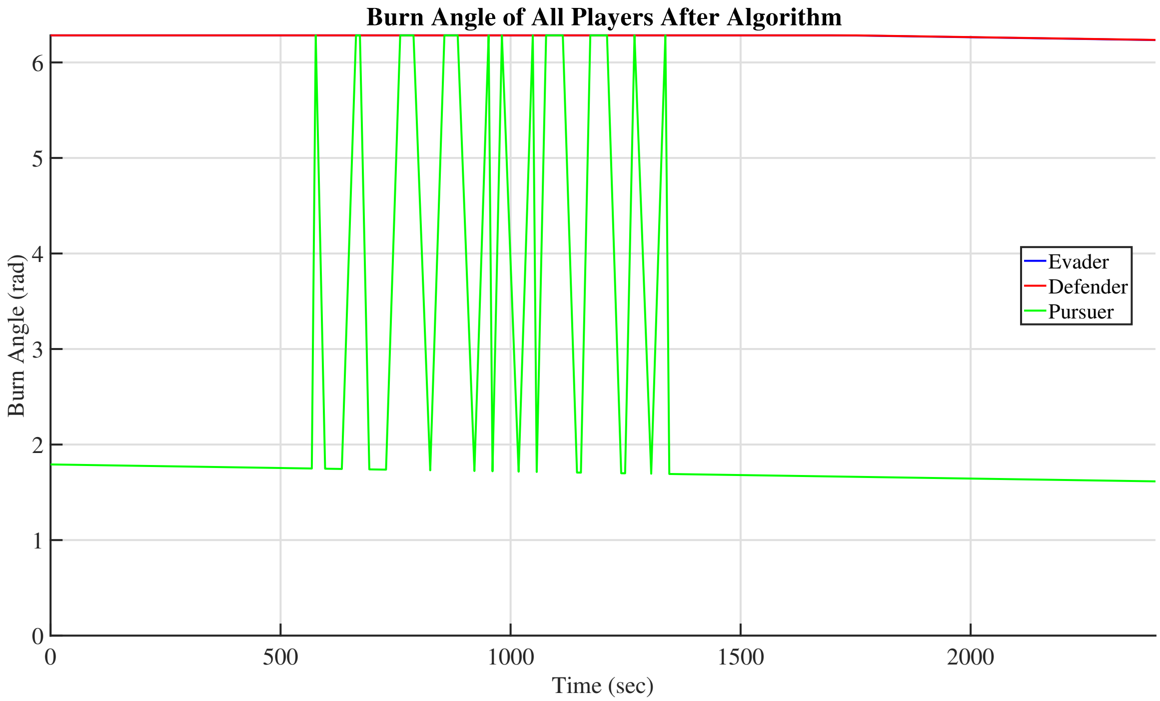

Figure 33. Control of players at end of phase three

Figure 34 shows the thrust for all players at the end of phase three. There does not appear to be any correlation between the defender's thrust and the pursuer's thrust.

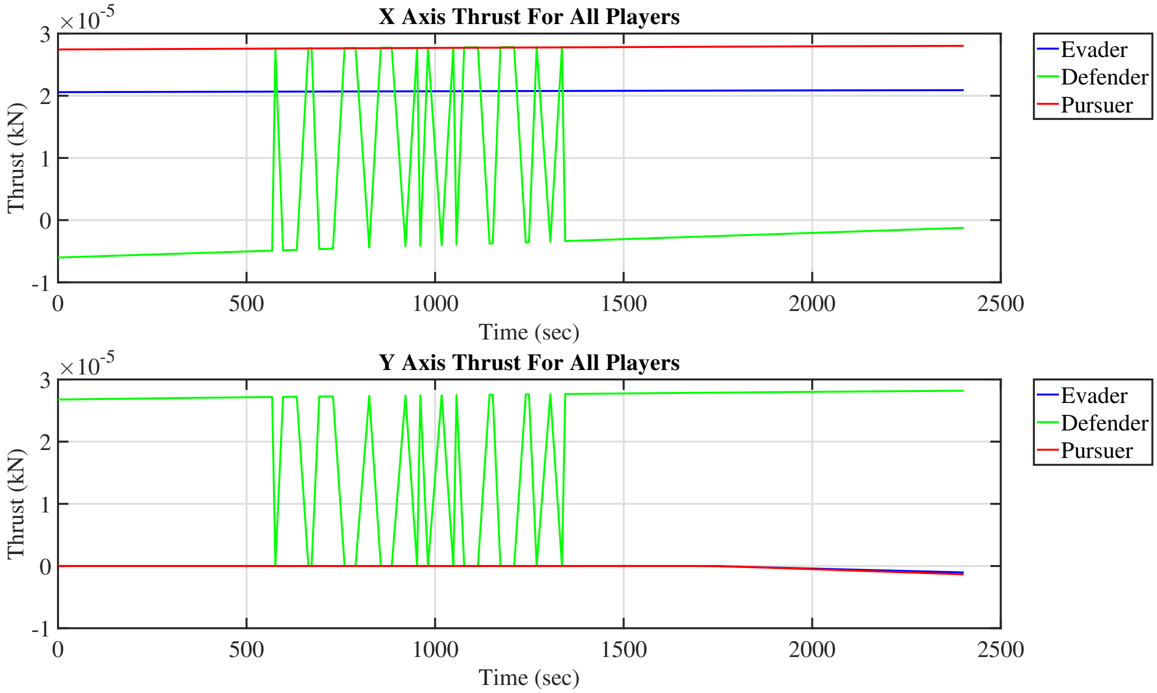

Figure 34. Thrust of players after end of simulation

The above results validate the three different phases in this algorithm. A potential improvement to the algorithm solve the first iteration using Stupik's method, and then solve the following iterations using the Decomposition Method. This may produce a better solution. The following sections outline the different parameter investigation performed using this research's algorithm.

4.2 Parameter Investigation: Players Capability

This research investigated the ability of player's capabilities to influence the mean score of the game. For this, a game of variety of defender and pursuer positions were played, but different capabilities were assigned to the pursuer and evader. Figure 35 shows the results of those.

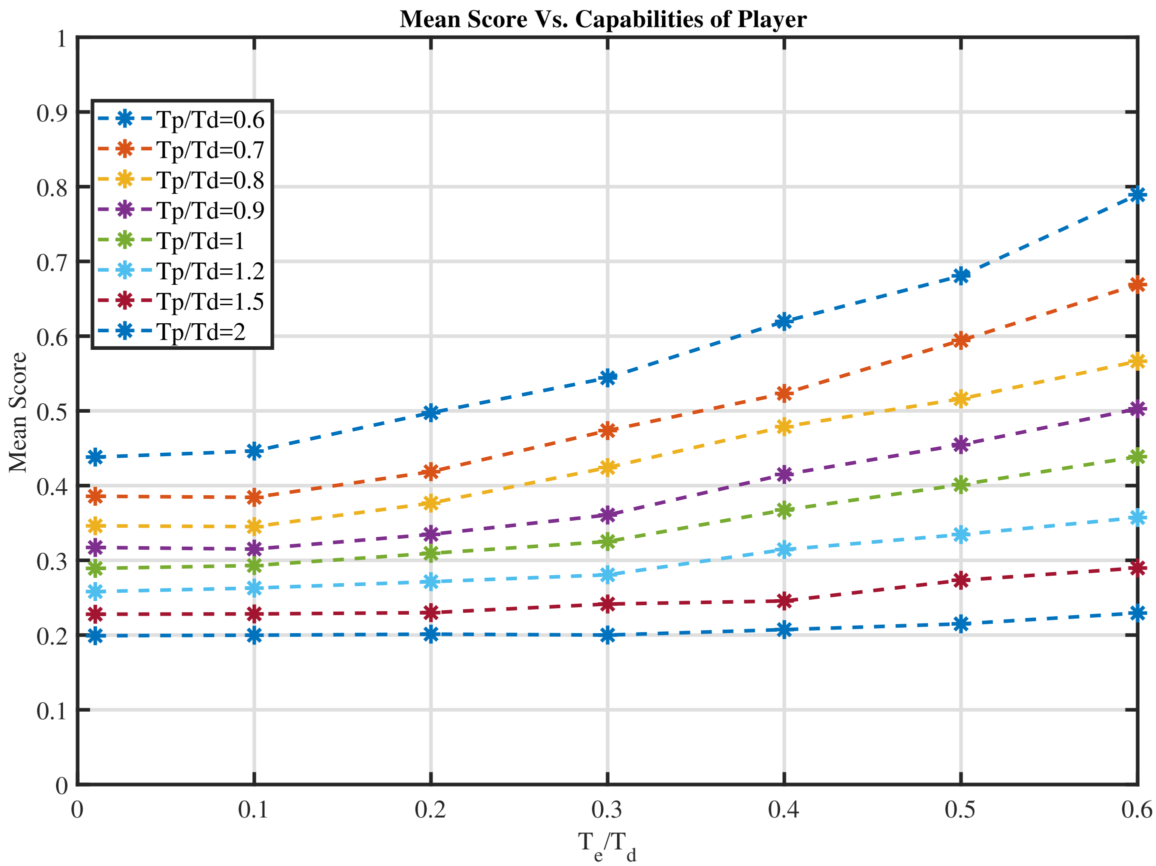

Figure 35. Mean score vs. player capabilities (2D)

Figure 36 is a plot of the same results in three dimensions.

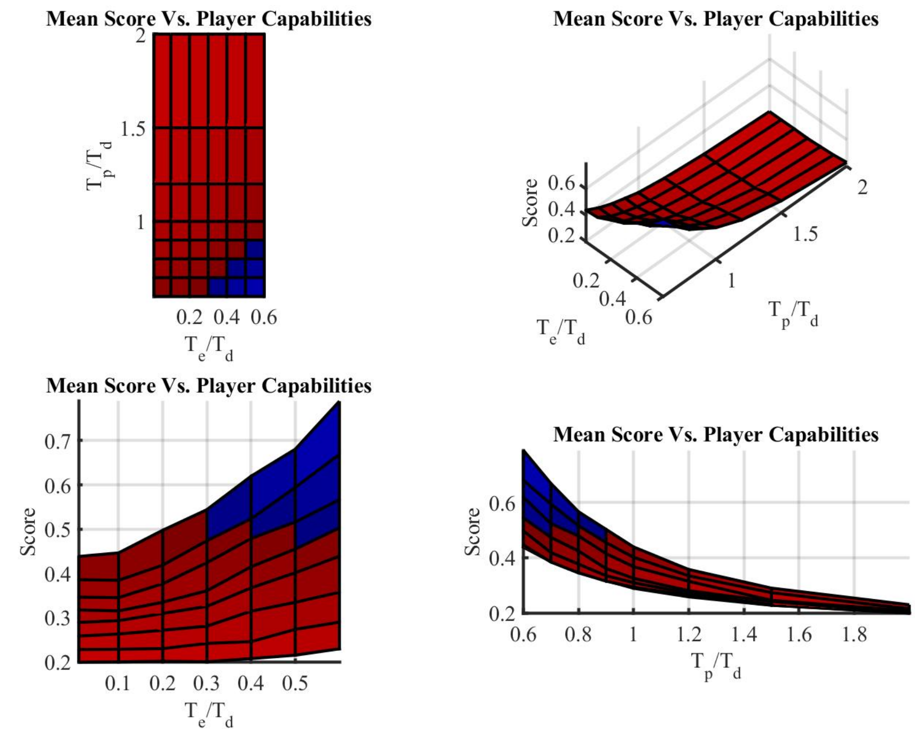

Figure 36. Mean score vs. player capabilities (3D)

In these scenarios, the defender appears to have an advantage over the pursuer, likely due to lack of interaction in the pursuer-defender pairing. However, in order to exploit this hypothesized advantage, the defender requires more capability than the pursuer. For comparison, the same mean score was achieved when $\frac{T_e}{T_d} = 0.1$ and $\frac{T_p}{T_d} = 0.8$ when compared to $\frac{T_e}{T_d} = 0.6$ and $\frac{T_p}{T_d} = 1.2$ The evader needs to add 0.5 defender ability, if the pursuer gains 0.4 defender ability. There is also an evader's capabilities in which it does not impact the outcome of the game.

4.3 Parameter Investigation: Initial Position of Defender

This research posited that the orbital debris defender is in NMC around the evader. Therefore, a design parameter for the orbit debris defender is its initial position in a variety of NMCs. Figure 37 shows the mean scores as a function of the defender's initial position.

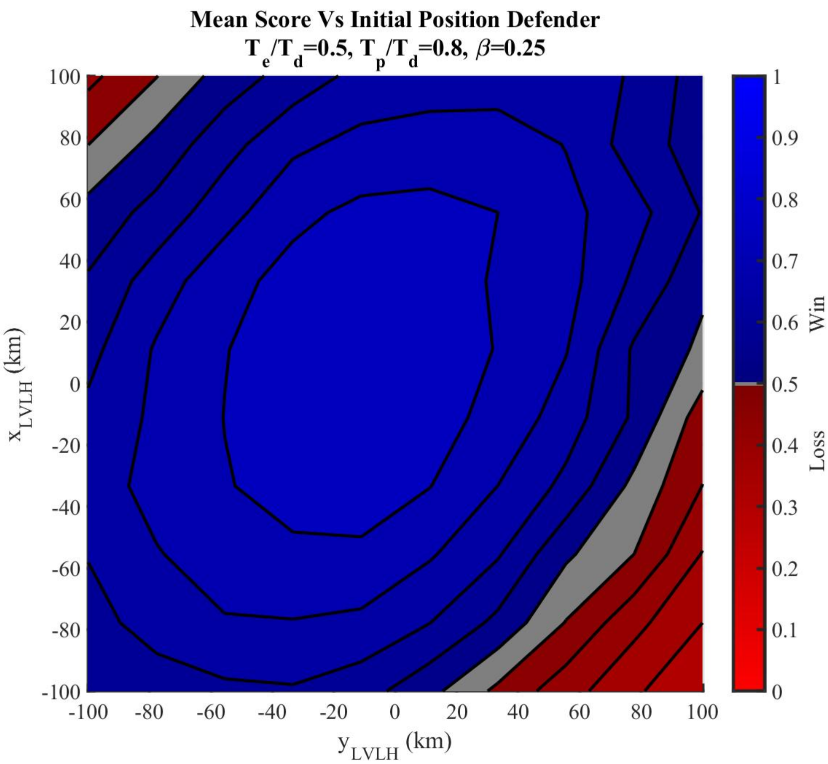

Figure 37. Mean score of defender with T_e/T_d=0.5, T_p/T_d=0.8, and β=0.25

Figure 37 is the mean score of the defender, at that initial position, when attempting to intercept 100 different pursuer's initial state vectors. Figure 37 is an interpolated plot, in order to better compare the solution structure. A consistent rotated ellipsoid appears to exist. The shape of the ellipsoid may be dependent on the initial capabilities of all players, and β when operating in a conservative gravity field. However, a conclusive relationship between those variables and the ellipsoid shape remains unresolved, and is left for future work. This shape was found consistently when varying all three parameters (T_e/T_d, T_p/T_d, and β). The diagonal nature of the solution suggests that the defender may experience a decrease in performance as its orbits the evader. Therefore, the next parameter investigation was the variation in mean score as the defender orbits the evader.

4.4 Parameter Investigation: Time in Orbit of Defender

In order to determine the variation in mean score of the defender in the same NMC, another series of differential games were played. Figure 38 shows the mean score as a function of the defender's time within that NMC, as defined in Chapter III. A defender experiences large variations of the mean score, with the same

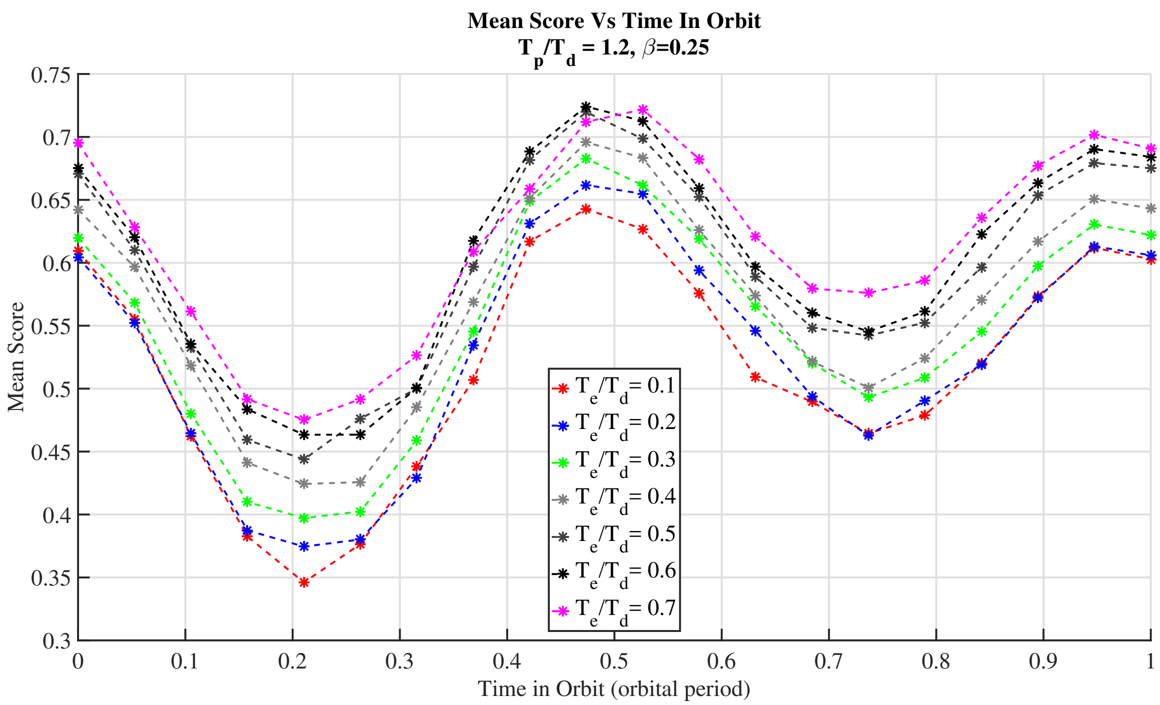

Figure 38. Same defender's interception ability is dependent on its time in orbit

capabilities, during the defender's relative orbit of the evader. These results are consistent with the ellipsoid shown in Figure 37. The periods of better performance are during the perigee and apogee of the debris defender. The variation in performance posses significant problems from a performance trade space. A significantly effective defender may perform poorly depending on when the orbital debris defender is detected. A potential solution, although there are significant cost and schedule concerns, would be two defenders. If two defenders are used, they will be likely have the same relative orbit, but a time phase between the two satellites. The next parameter study was the optimal phasing between a two defender formation.

4.5 Parameter Investigation: Optimal Phasing for Two Defender Formation

If a satellite designer wants to decrease the variation in defender mean score, a two defender formation in NMC may be an adequate mitigation strategy. Figure 39 shows the mean score across the entire orbit as a function of the phasing between the two defenders. The result suggests that the defenders should be separated by

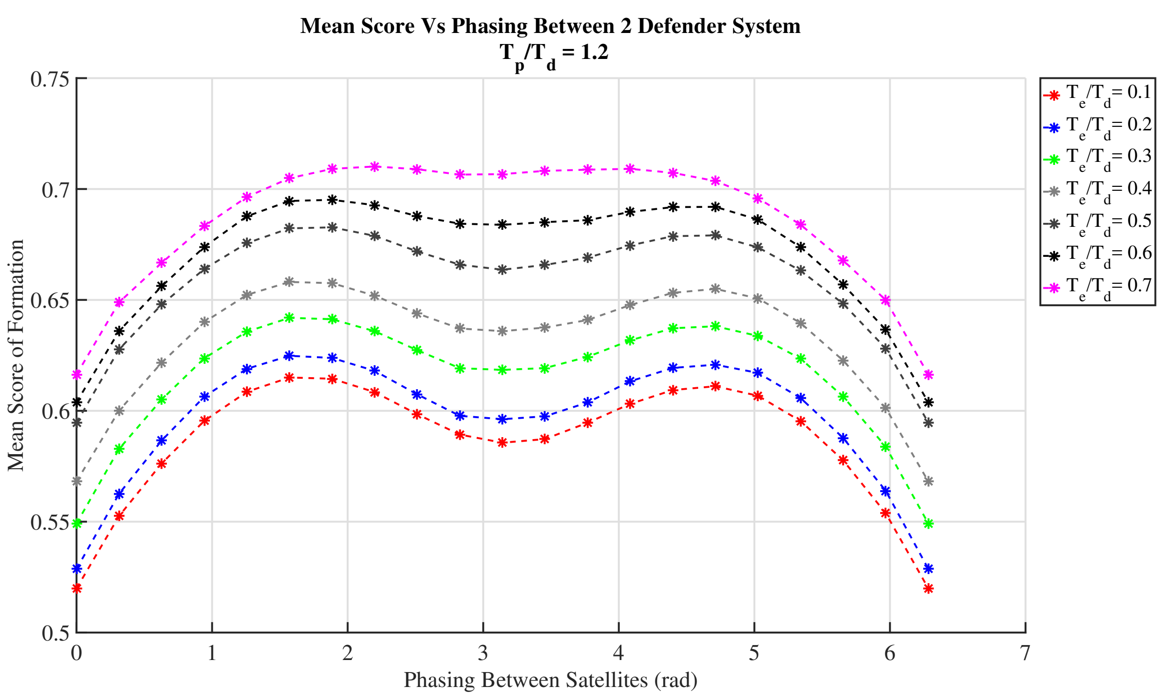

Figure 39. Optimal phasing around $\frac{\pi}{2}$ or $\frac{3\pi}{2}$

1/4 or 3/4 of an orbit, as measured by a reduction of variation of the mean score across the defender's main orbit. Figure 40 shows the resulting performance of the two defender system when optimally phased. A two defender formation both

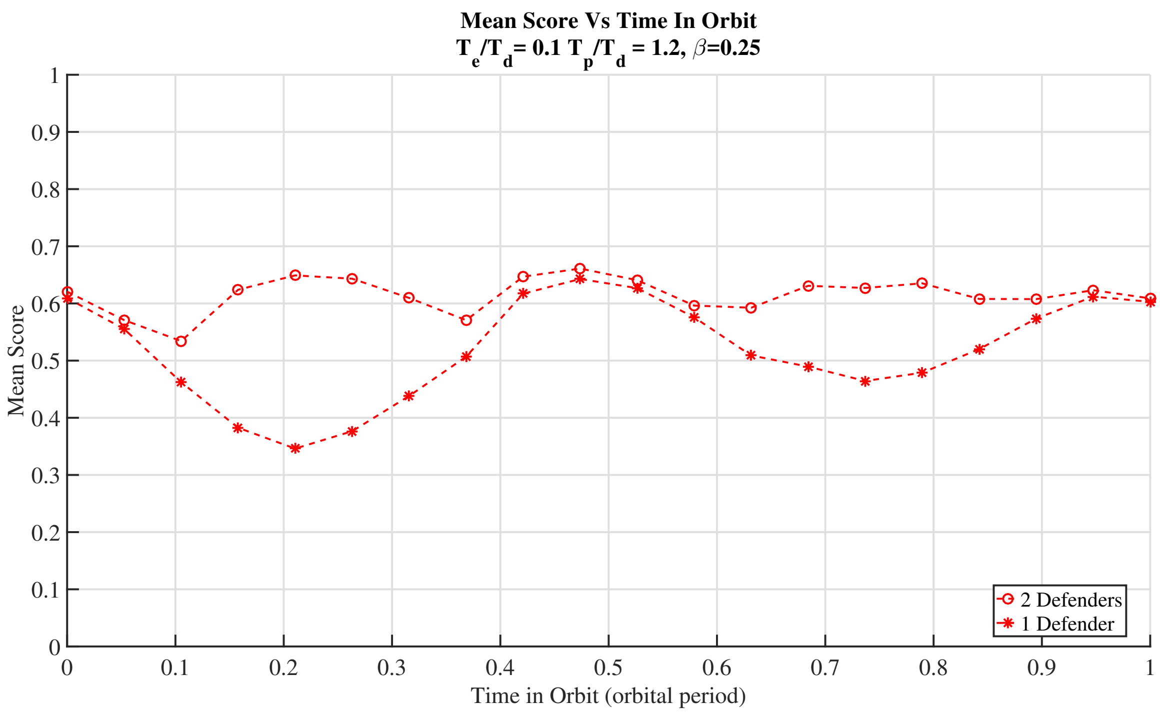

Figure 40. Mean score of optimally phased two defender system with $\frac{T_e}{T_d}$= 0.1 $\frac{T_p}{T_d}$ = 1.2, β=0.25

increases by the mean score and decreases the variation in performance. This is due to one of the two defenders being close to apogee or perigee, which are the most successful initial positions, throughout the orbit.

A highly capable defender may be less effective than two inferior defenders that are phased optimally. This is because a single defender experiences significant variation in the mean score within its orbits, whereas two defenders may have lower peak performance, but better consistent performance. If a two defender system is implemented, a phasing of either 1/4 or 3/4 of an orbit produces a decrease in the variation of the mean score and potentially a better mean score. The determining factor between either an inferior two defender system or a superior single defender may require cost and schedule considerations to determine the best system.

4.6 Parameter Investigation: Evader Strategies

As shown in Chapter III, using terminal constraints can have the evader exchange capture time for a better trajectory. Figure 41 shows the difference in possible trajectories based on the strategy used by the evader using the same initial conditions.

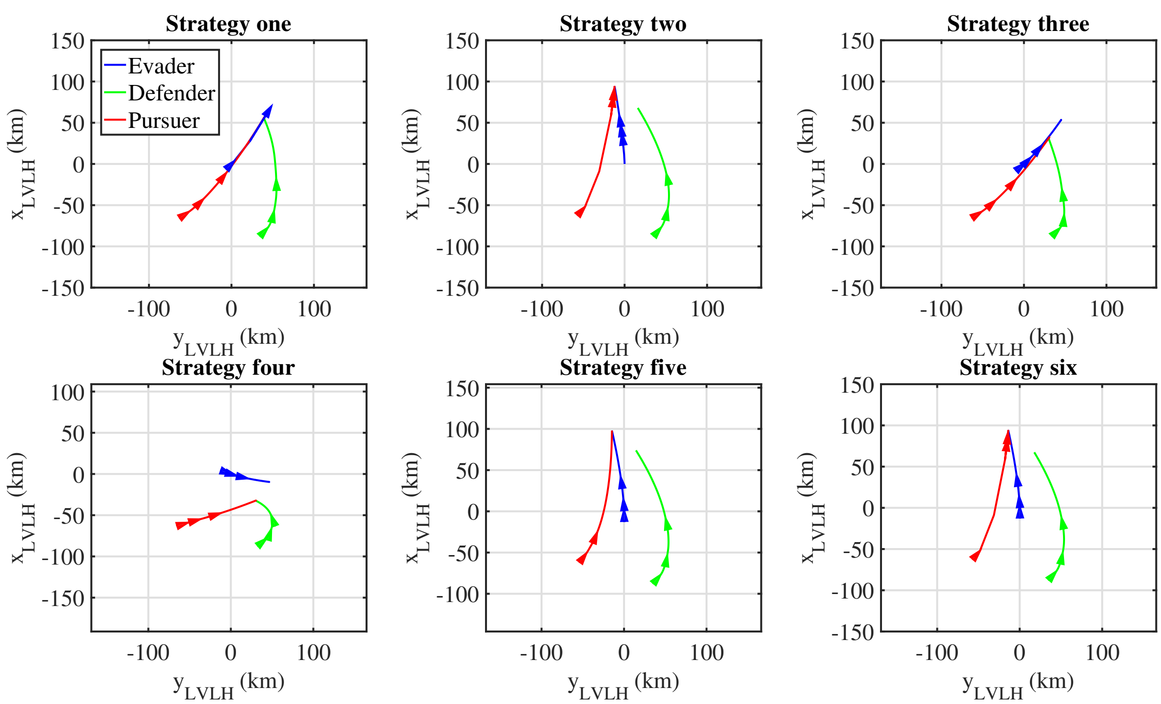

Figure 41. Different strategies produces different trajectories

These different trajectories are generated by solving the optimal control problem with a different terminal cost. Figure 42 shows the pursuer's objective function (capture time) for all strategies as a function of the iteration number.

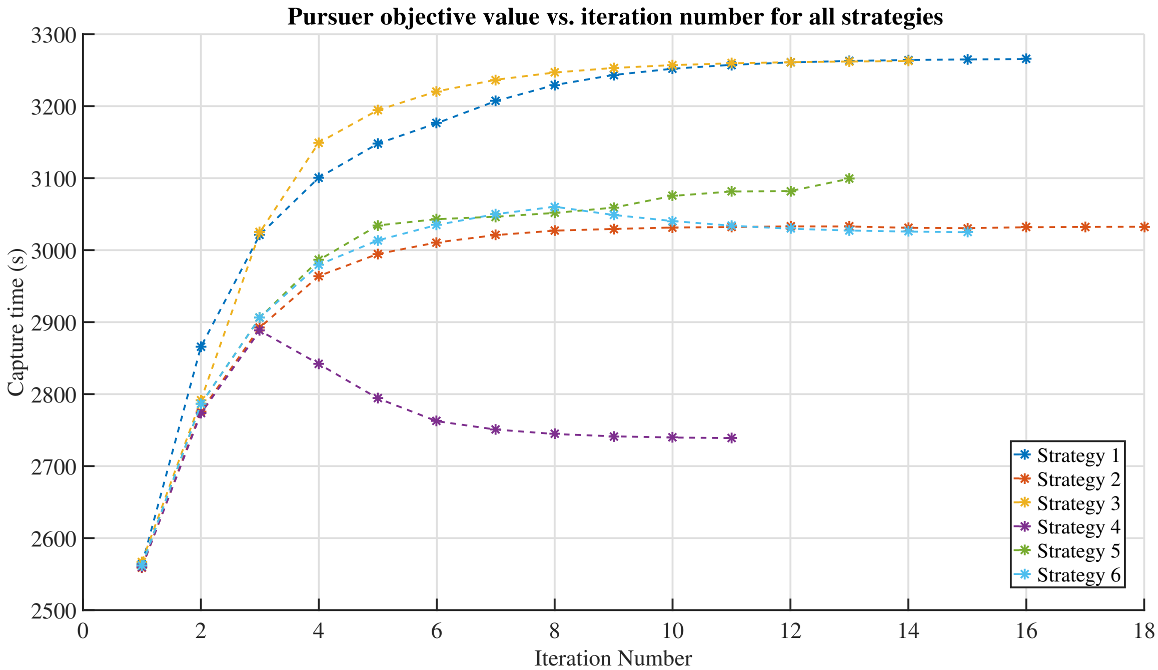

Figure 42. All strategies do not produce a better capture time than strategy one, but a potentially better trajectory for the defender to intercept the pursuer

As demonstrated in Figure 42, all strategies produce an equal or inferior to strategy one, but the other strategies might produce better trajectories, as measured by the final score of the game. Table 11 shows the scores each strategy produced, with the same initial condition. This use of the terminal cost in the Decomposition Method is new to this research. Therefore, in order to determine the best strategy, a large amount of differential games are required to ensure a robust analysis. Table 12 represents the outcome of the six strategies under a number of differential games, using the parameters in Table 8. Based on the positions of the pursuer and defender, each strategy may be the optimal strategy to implement, as shown by each strategy generating a high score and a unique win. Therefore, more

Table 11. Using different strategies produces different scores

Strategy	one	two	three	four	five	six
Score	0.58	0.32	0.63	0.64	0.32	0.30

Table 12. Scoring outcomes when T_e/T_d=0.3, T_p/T_d=1.2 and β=0.25

Strategy	One	Two	Three	Four	Five	Six
High scorer (%)	10.03	7.02	4.48	18.67	13.89	15.66
Low scorer (%)	8.26	13.12	19.68	7.02	12.96	8.72
Unique winner (%)	0.46	0.23	0.31	1.16	0.69	1.77
Unique loser (%)	0.23	0.39	2.16	0.39	0.54	0.39
Mean scores	0.38	0.38	0.35	0.39	0.39	0.41

differential games were played, and then the best strategy was determined using the scoring metrics. Table 13 shows the results of that analysis. Tables 12 and 13 indicate there might not be an optimal strategy for all scenarios. Strategies four and six are the best overall strategies, but they do not always perform the best. These strategies did not always converge on a single final position. A brief analysis of scenarios when the algorithm did not converge indicated that the algorithm was osculating between two solutions. This research did not devise a robust method of determining which outcome to implement between the two solutions, and therefore if any of the strategies did not produce a converging solution, then all data points with those initial conditions were not analyzed.

4.7 Miscellaneous Studies

This research also explored two additional studies. The first study was the influence on the outcome of the evader maintaining its mission. The second was

Table 13. Strategy four and six are the best overall strategy, but are not always the optimal strategy

Strategy	One	Two	Three	Four	Five	Six
Most High Scores	1	0	0	8	0	0
Least Low Scores	0	2	0	4	0	3
Most Unique Wins	1	0	1	3	0	4
Least Unique Loses	3	0	3	1	1	1
Best Mean Score	0	1	0	1	0	7
Total Wins	5	3	4	17	1	15

whether or not the solutions structures generated in this research were generated in other orbital regimes. The following sections outlines the results of that analysis.

4.7.1 Parameter Investigation: Maintaining Mission.

Landsat 7 experienced a significant operational impact after performing a collision avoidance maneuver [9]. Therefore, the satellite's operators may choose to accept more risk in order to reduce the operational impact. This can be modeled by having the evader not performing any maneuvers or $T_e/T_d = 0$. Figure 43 shows a similar structure as if the evader does not maneuver, however the area of successful defense is decreased.

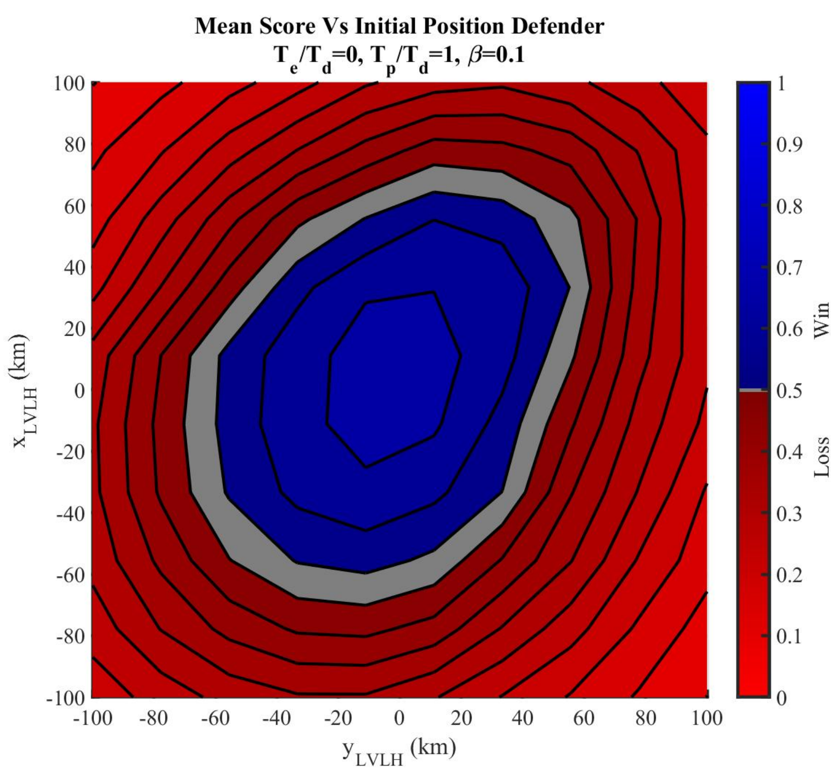

Figure 43. Mean score of defender with T_e/T_d=0, T_p/T_d=1.2, and β=0.1

The results of maintaining mission produces the innovative result that the mean score of the defender decreases if the evader does not aid in escaping.

4.7.2 Parameter Investigation: Defender in LEO.

This research focused on a satellite in GEO. However, many satellites are also in Low Earth Orbit (LEO). Using the same algorithms, but changing the K value ($K = 0.1099$) in order to adjust the initial relative distances, Figure 44 shows a similar structure as in GEO.

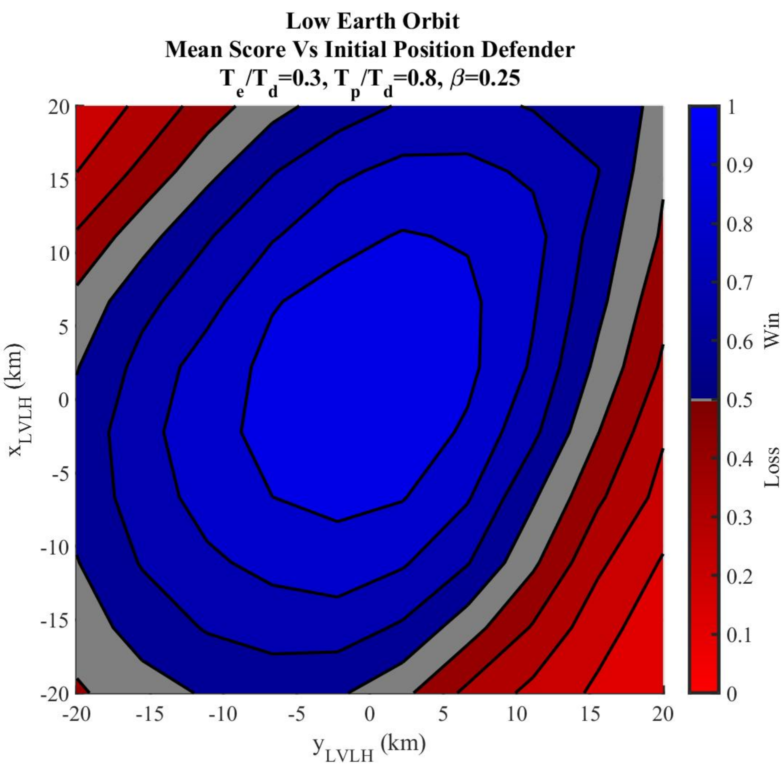

Figure 44. LEO mean score of defender with T_e/T_d=0.3, T_p/T_d=0.8, β=0.25

The existence of the same structure in the LEO region suggests that all the results of this study may be applied to a LEO satellite defender. However, the LEO environment poses additional perturbations, such as atmospheric drag and J2 [19]. Future work could add those perturbations into the dynamics and validate the results presented in this research.

4.8 Summary

The motivation behind this research is whether or not an orbital debris defender is an effective risk mitigation strategy. In order to validate whether or not the orbital debris defender is effective, numerous parameters were investigated. The following provides a summary of those results.

First, Section 4.1 validated the Decomposition Method as a solution to the pursuit-evasion problem in the LVLH frame. Second, Section 4.2 showed that in order to take advantage of the lack of interaction between the pursuer and defender, the defender needs to be more capable than the pursuer. Additionally, there is an evader capability in which for any lower value, the evader does not impact the outcome of the differential game. Third, Section 4.3 showed the mean score as a function of the defender's initial position. This result showed a consistent structure, a rotated ellipsoid. This structure suggests the same defender in NMC will experience large variations in its ability to the defend based on the start of the game. Sections 4.4 and 4.5 expanded on this research, recommending that a two defender system be phased either 1/4 or 3/4 of an orbit. Additionally, two defenders reduce the variation of mean score. This solution structure was also examined if the evader does not maneuver, and furthermore explored the results of the algorithm in the LEO orbit. Finally, Section 4.6 shows the results of the evader studies. No optimal strategies were apparent, as the best outcome was dependent on the initial conditions. However, strategies four and six were generally the most successful strategies overall. Chapter V will provide the final conclusions and recommendations of this research.

V. Conclusions and Recommendations

Space is a congested, contested and competitive environment, with an increasing amount of space objects that pose significant risk to satellites [1]. Current risk mitigation strategies can have significant consequences to satellite operators. This research showed that an orbital debris defender may be an effective risk mitigation strategy. Utilizing differential game theory, the orbital debris defender can intercept debris prior to the debris impacting the evader. This researched modified the Decomposition Method to solve the pursuer-evader-defender problem. This research showed how to design for an effective orbital debris defender with respect to the performance trade space, and posited metrics scoring functions and strategies for the pursuer-evader-defender differential game. Additionally, research also showed a method to generate pseudo-cooperation between the evader and defender, which is not limited to the dynamics used in this research. The remainder of this chapter will summarize the methodology and results, and conclude with recommendations for future work. However, this chapter will first determine whether the research questions have been answered.

5.1 Research Questions

The following sections will outline the answers to the research questions proposed in Chapter I.

5.1.1 Research Question 1.

The first research question was: *"What are the key parameters of an orbital debris defender, enabling the defender to successfully protect the primary satellite"*? One of the first results demonstrated in this research was that there are significant performance gains in the defender being more capable than the pursuer, even if it requires reducing the evader's capabilities. Therefore, there may be an optimal distribution between performance of the evader defender pair, when including other acquisition trade space such as schedule and cost, or such as trading total launch mass between the evader and defender. This research demonstrated that a defender in NMC, a passively stable relative orbit, will experience significant variations in its ability to protect the primary satellite. Therefore, two inferior defenders may produce a better overall outcome than one superior defender, subject to other acquisition trade space.

5.1.2 Research Question 2.

The second research question was: *"Can the primary satellite implement any strategies to aid the orbital debris defender"*? This research presented six strategies to generate pseudo-cooperation between the evader and defender, using the derivation of the Decomposition Method. Each of those strategies produced the best outcome, depending on the initial conditions and capabilities. Additionally, while the previous results may be limited to the space domain, this method of providing evader strategies is not. This may allow previous problems that had a fixed target to allow the target to maneuver and still reach a Nash Equilibrium.

5.1.3 Research Question 3.

The third research question was: *"Can an active orbital debris defender be an effective risk mitigation strategy for the space debris problem"*? This research showed definite proof that the orbital debris defender can be an effective risk mitigation strategy, with respect to performance trade space. In order to measure that effectiveness, this research used a mean score approach. A mean score greater than 0.5 indicates the defender-evader team wins more often than it loses, or is effective. This research showed that even if the debris behaved as an optimally performing piece of debris, the defender can intercept the debris if the defender has enough capability and warning time. A two defender formation is highly effective against debris if the defenders are warned before the debris is within the defender's NMC. This research does not consider either cost or schedule, however an orbital debris defender does not need to be unique for each satellite. Therefore, there is a potential for significant cost savings. Each individual satellite designer team needs to determine which risk mitigation strategy is the best decision for their teams specific project. However, for satellites where maneuvers and shielding are impractical, an orbital debris defender may be the best mitigation strategy.

5.2 Potential Future Research

While numerous insights were found in this research, there is still significant amount of future work. This research only analyzed games that begin very close, to stay within the Chlossey-Wiltshire linear dynamics [18]. However, nothing in this algorithm is limited to linear dynamics. By changing the continuous function in the optimization software, this algorithm can solve the pursuer, evader and defender problem using the more accurate nonlinear dynamics. Additionally, these strategies are not limited to the space domain. A second assumption used in this research was that all players have complete information, which is not often a realistic scenario. Carr shows a method to add uncertainty to the pursuer's motion and states, in the context of a Receding Horizon control and real-time optimal control problem [27]. This research recommends adding uncertainty to the evader, as this continues the assumption of the worst-case scenario, and limits the information. This research posed six strategies, and showed that there are conditions in which each strategy produces the optimal result. There may exist certain indicators as to when to use which strategy. This research was unable to determine what those indicators are. An example indicator might be if the defender is more capable than the pursuer by a large margin but the pursuer is significantly closer to the evader, the best option for the evader is to implement strategy one, or maximize capture time. This provides the defender the most time to leverage the inherent advantage the defender has over the pursuer. Finally, providing GPOPS with a better initial guess can produce a better final result than used in this research. The existence of the ellipsoid structures should be examined further, as only the initial conditions were investigated. Validation using a better initial guess, or producing the same plots using Stupik's approach is recommended, or finding another coordinate frame, other than the LVLH frame, may cause the ellipsoids to disappear.

5.3 Conclusion

The space environment has evolved significantly over the last half century, however satellites in use right now are not designed for the realities of the domain in which they now are forced to operate. Current risk mitigation strategies for the orbital debris problem can be impractical for some satellite operators. An orbital debris defender may be a preferred risk mitigation strategy, as it may be more effective than existing strategies. Additionally, the use of strategies in the evader defender team provides for an exciting new opportunity to solve new types of differential games.

Appendix A: Forced Rendezvous

The Decomposition Method was derived for a pursuer attempted to capture the evader. However, it is also possible for the pursuer to rendezvous with the evader instead. A forced rendezvous is defined as a pursuer desires to to match both the position and velocity of the evader, where as the evader desires to delay that outcome for as long as possible. The following requires an additional assumption that the evader knows the pursuer is trying to rendezvous, and is attempting to maximize the time to capture. The terminal constraint is

$$\vec{x}_p(t_f) = \vec{x}_e(t_f), \tag{A.1}$$

which, when taking the Taylor series around the evader's final position, changes the gradient to

$$^{i}\lambda_p \frac{\partial}{\partial e}\psi(e, r_p) = \begin{bmatrix} ^{i}\lambda_p^x & ^{i}\lambda_p^y & ^{i}\lambda_p^{\dot{x}} & ^{i}\lambda_p^{\dot{y}} \end{bmatrix}^T \tag{A.2}$$

This modification changes the final evader cost function to

$$J_e = \begin{bmatrix} \lambda_p^x + \frac{\partial\phi}{\partial^{i-1}x_e} \\ \lambda_p^y + \frac{\partial\phi}{\partial^{i-1}y_e} \\ \lambda_p^{\dot{x}} + \frac{\partial\phi}{\partial e_{\dot{x}}} \\ \lambda_p^{\dot{y}} + \frac{\partial\phi}{\partial e_{\dot{y}}} \end{bmatrix}^T \begin{bmatrix} x_e(t_f) - e_x \\ y_e(t_f) - e_y \\ \dot{x}_e(t_f) - e_{\dot{x}} \\ \dot{y}_e(t_f) - e_{\dot{y}} \end{bmatrix}. \tag{A.3}$$

This could be a mitigation strategy for the orbit debris defender, in case the debris is a satellite requiring servicing. As shown in Chapter III, the Decomposition Method can calculate the optimal forced rendezvous. The Decomposition Method still requires the evader to lose. For this example, a significantly more capable pursuer was used, as outlined in Table 14 to avoid a zero value of the denominator in the control law.

Table 14. Test matrix for forced rendezvous

Variable	Value
Pursuer initial state vector	$\begin{bmatrix} 50 \\ -35 \\ 0 \\ 0 \end{bmatrix}$
Evader initial state vector	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
ϵ_p	10 seconds
T_e/T_d	0.8
T_p/T_d	1.5

An additional change to the Decomposition Method is suggested. Due to potential scaling problems, instead of comparing the change in evader's final position to determine whether or not to continue the iterative approach, compare the previous pursuer's objective function. Figure A.1 shows the change in the final time of the rendezvous vs iteration number.

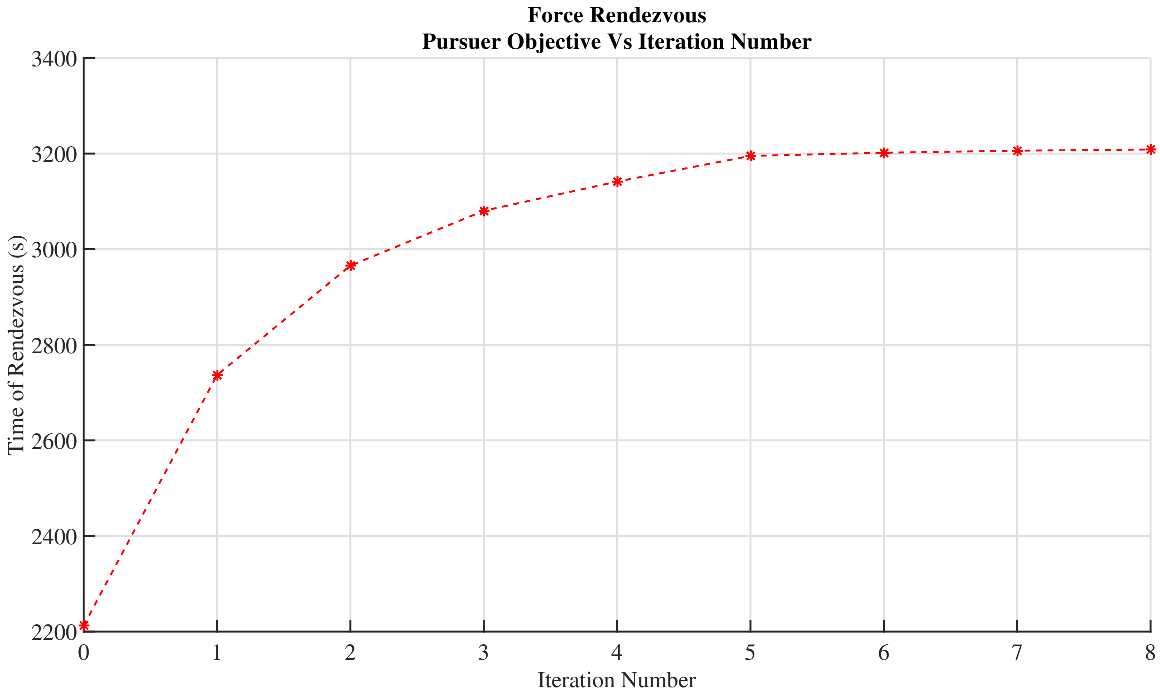

Figure A.1. Pursuer reaches Nash Equilibrium for forced rendezvous

Figure A.2 shows that the decomposition method produces a forced rendezvous trajectory by the pursuer.

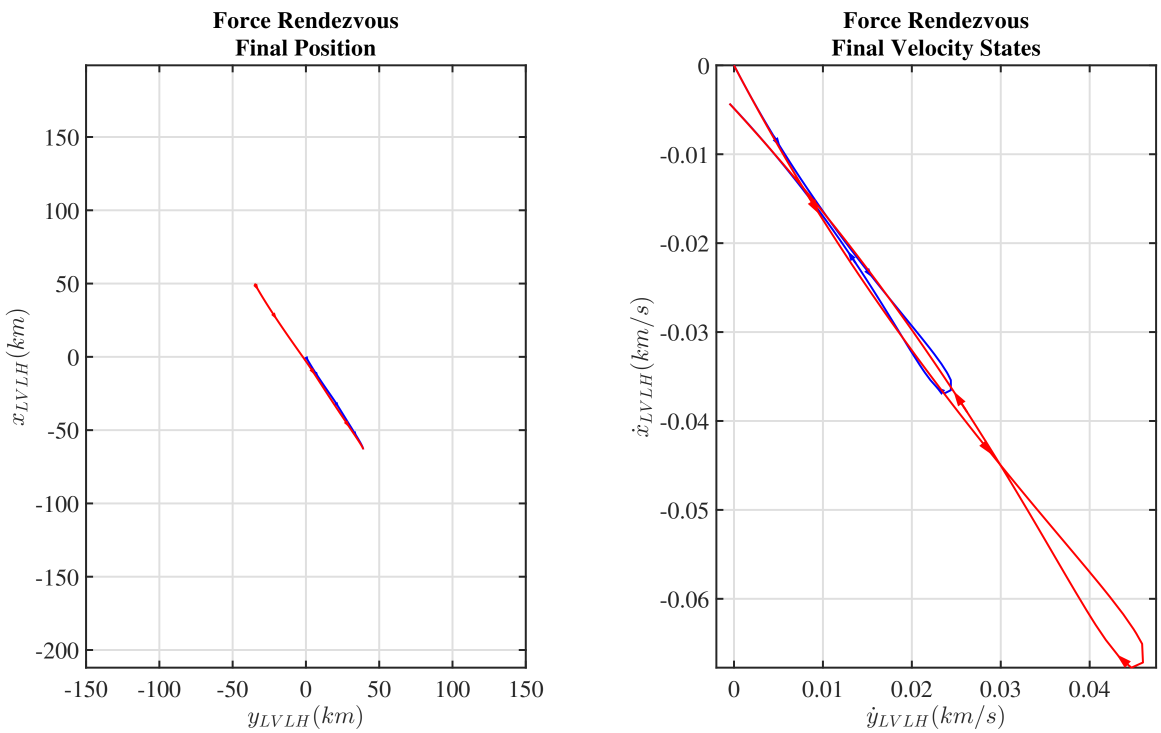

Figure A.2. Final position and velocity states for forced rendezvous

Figure A.3 shows the Hamiltonian of the players within the forced rendezvous.

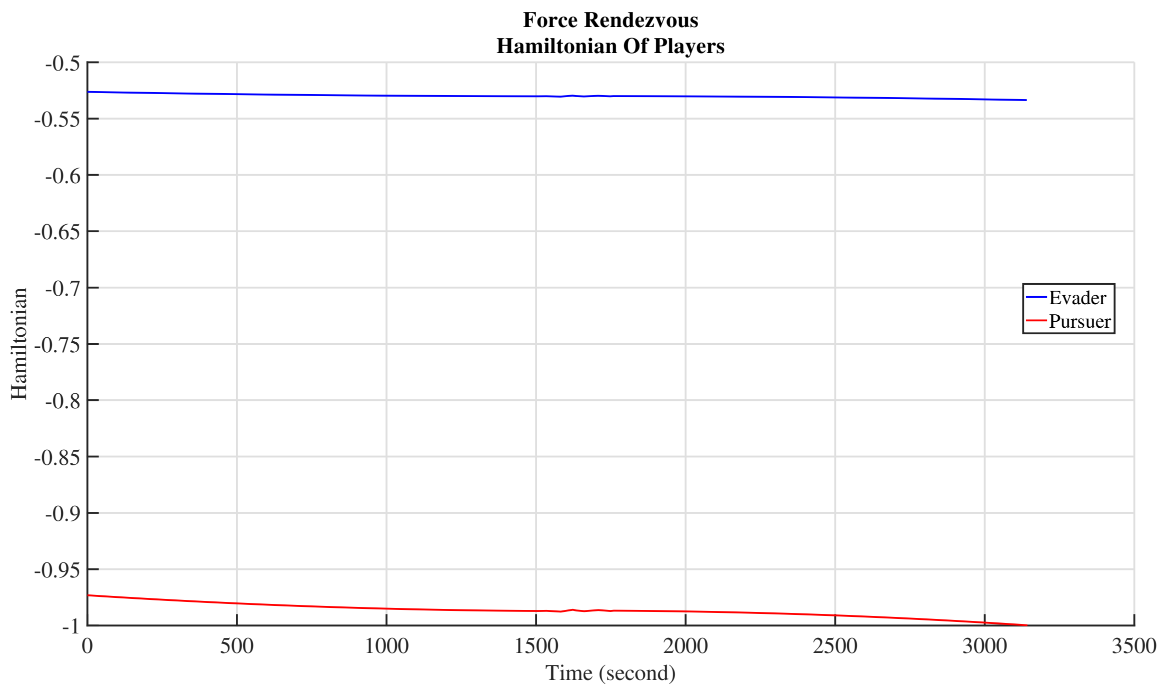

Figure A.3. Hamiltonian for forced rendezvous meets boundary condition

This matches the boundary condition

$$H(x^*(t_f).u^*(t_f), \lambda^*(t_f), t_f) + \frac{\partial \phi}{\partial t}(x^*(t_f), t_f) = 0. \tag{A.4}$$

The final Hamiltonian is negative one, because

$$\frac{\partial \phi}{\partial t}(x^*(t_f), t_f) = 1 \tag{A.5}$$

Additionally, another necessary conditions are

$$\begin{matrix} \lambda_p(t_f) = \lambda_e(t_f) \\ \vec{x}_p = \vec{x}_e \end{matrix}, \tag{A.6}$$

and therefore the only difference between the two Hamiltonians is the magnitude of the thrust, which is significantly smaller than the costates at time t_f. Figures A.4

and A.5 shows the resulting controls of the forced rendezvous problem, with the thrust and burn angle respectively.

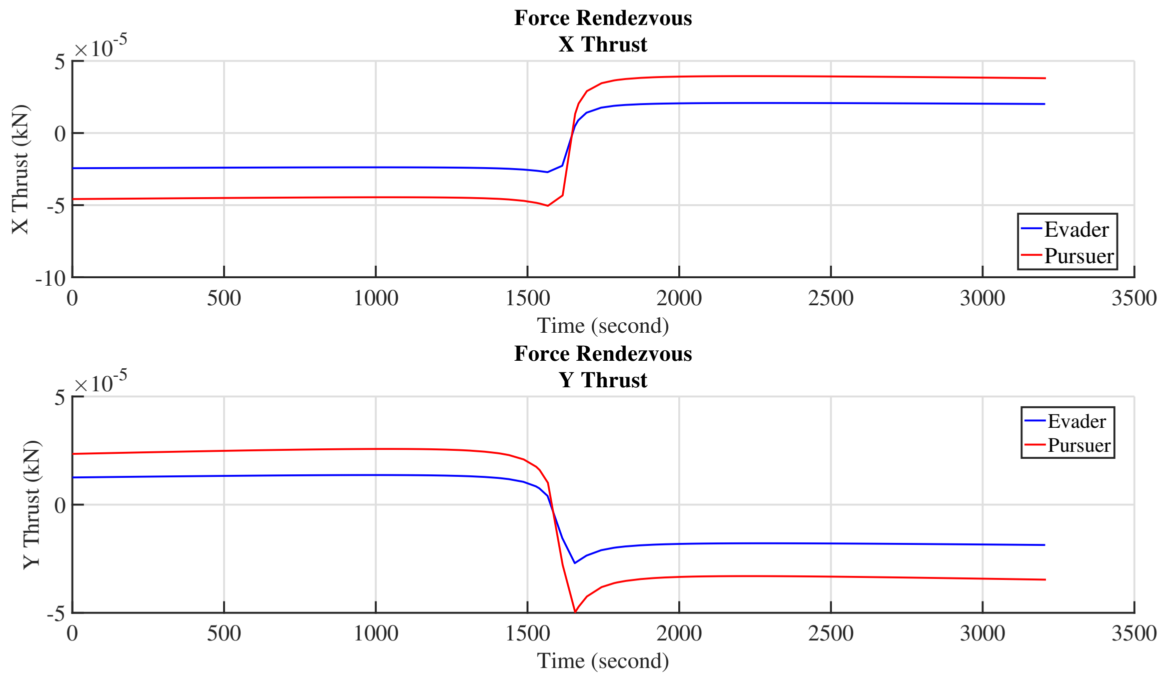

Figure A.4. Thrusts for forced rendezvous

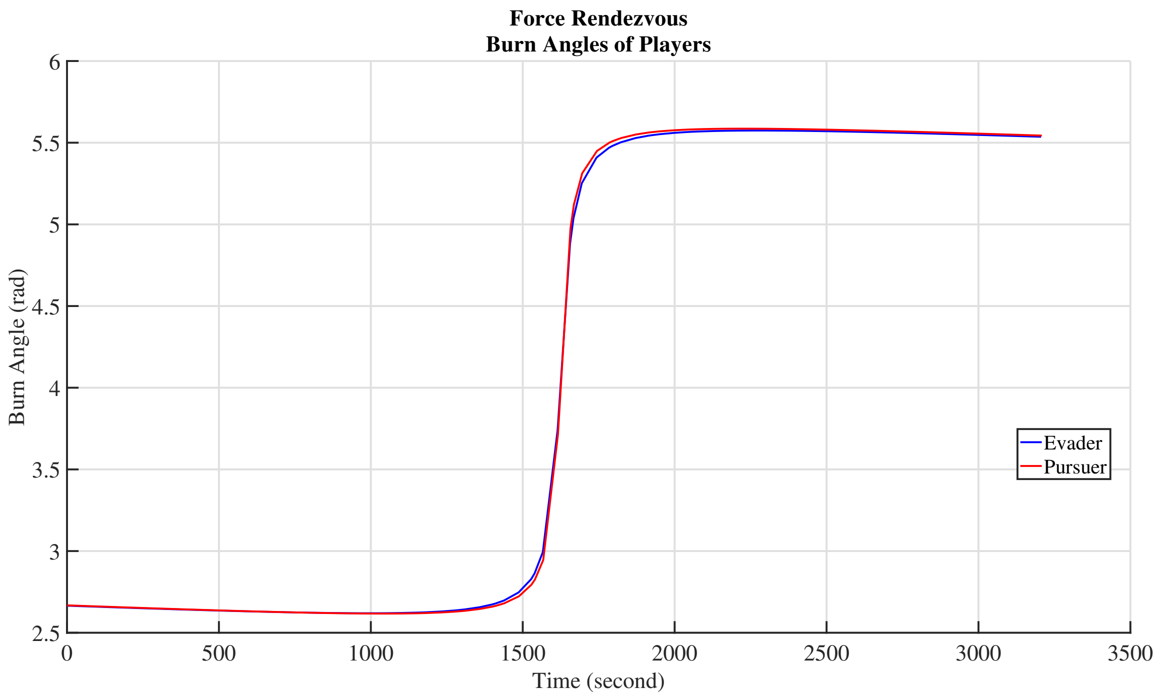

Figure A.5. Burn angle for forced rendezvous meets necessary conditions

The applications of this was not used in this research as the assumption that the orbital debris defender did not need to rendezvous with the orbital debris, just needed to intercept the debris before the debris impacts the evader.

Appendix B:Algorithm

The following is a code outline for the algorithm used in this research.

Algorithm 1 Two Team Three Player Optimal Control Problem

procedure PURSUIT EVASION($\vec{x}_e(t_0), \vec{x}_p(t_0), T_e, T_p, E_e, E_p$) ▷ Initial Vectors and Capabilities
  while $\epsilon_e > \delta_e$ **do** ▷ While Evader's final position is not steady
    Solve Pursuer's Fixed Final State, Free Final Time Problem

$$\begin{aligned} {}^{i}J_p &= t_f \\ &\text{Subject To} \\ {}^{i}\vec{r}_p(t_f) &= \vec{e} \\ \dot{\vec{x}}_p &= f(x_p, u_p, t) \end{aligned}$$

    Solve Evader's Free Final State, Fixed Final Time problem

$$\begin{aligned} {}^{i}J_e &= \begin{bmatrix} {}^{i}\lambda_p^x + \frac{\partial \phi_e}{\partial e_x} \\ {}^{i}\lambda_p^y + \frac{\partial \phi_e}{\partial y_e} \end{bmatrix}^T \begin{bmatrix} {}^{i}x_e(t_f) - e_x \\ {}^{i}y_e(t_f) - e_y \end{bmatrix} \\ &\text{Subject To} \\ t_f &= J_d \\ \dot{\vec{x}}_e &= f(x_e, u_e, t) \end{aligned}$$

    $\epsilon_e = ({}^{i}x_e - e_x)^2 + ({}^{i}y_e - e_y)^2$
  end while
  return $\vec{x}_e, \vec{x}_p$ ▷ Returns States and Control
end procedure
procedure DEFENDER INTERCEPT($\vec{x}_p(t), \vec{x}_d(t_0), T_d, E_d$)
  Solve Defender's Free Final State, Free Final Time Problem

$$\begin{aligned} J_d &= \frac{1}{2}((x_d(t_f) -^{i} x_p(t_f))^2 + (y_d(t_f) -^{i} y_p(t_f))^2 \\ &\text{Subject To} \\ 0 &\le t_f \\ t_f &\le J_p \\ \dot{\vec{x}}_d &= f(x_d, u_d, t) \\ \dot{\vec{x}}_p &= f(x_p, u_p, t) \end{aligned}$$

  return J_d ▷ Returns States and Control
end procedure

if $J_d \leq \epsilon_d$ **then**

procedure MINIMAL TIME TO INTERCEPT($\vec{x}_d, \vec{x}_p$)

Solve Defender's Fixed Final State, Free Final Time Problem

$$
\begin{aligned}
J_d &= t_f \\
&\text{Subject To} \\
r_d(t_f) &= r_p(t_f) \\
\dot{\vec{x}}_d &= f(x_d, u_d, t) \\
\dot{\vec{x}}_p &= f(x_p, u_p, t)
\end{aligned}
$$

end procedure

return t_f^d

$\vec{x}_e(t_f) = \vec{x}_e(t_f^d)$

$\vec{x}_p(t_f) = \vec{x}_p(t_f^d)$

$r_{pd}(t_f) = 0$

$r_{pe}(t_f) = \sqrt{(x_e(t_f) - x_p(t_f))^2 + (y_e(t_f) - y_p(t_f))^2}$

else

procedure MINIMIZE DISTANCE AT IMPACT($\vec{x}_d, \vec{x}_p$)

Solve Defender's Free Final State, Fixed Final Time Problem

$$
\begin{aligned}
J_d &= \frac{1}{2}((x_d(t_f) -^i x_p(t_f))^2 + (y_d(t_f) -^i y_p(t_f))^2 \\
&\text{Subject To} \\
t_f &= J_p \\
\dot{\vec{x}}_d &= f(x_d, u_d, t) \\
\dot{\vec{x}}_p &= f(x_p, u_p, t)
\end{aligned}
$$

end procedure

return $\vec{x}_d$ ▷ Returns States and Control

$r_{pd}(t_f) = \sqrt{(x_d(t_f) - x_p(t_f))^2 + (y_d(t_f) - y_p(t_f))^2}$

$r_{pe}(t_f) = 0$

end if

return $r_{pd}(t_f), r_{pe}(t_f)$

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REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
22-05-2018	Master's Thesis	Aug 2016 - Mar 2018

4. TITLE AND SUBTITLE
Parameter Study of an Orbital Debris Defender Using Two Team, Three Player Differential Game Theory

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Spendel, David F, Capt, USAF

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Institute of Technology
Graduate School of Engineering and Management (AFIT/EN)
2950 Hobson Way
Wright-Patterson AFB, OH 45433-7765

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT-ENY-MS-18-M-295

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Intentionally Left Blank

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
DISTRIBUTION STATEMENT A: Approved For Public Release; Distribution Unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The United States Air Force and other national agencies rely on numerous space assets to project their doctrine. However, space is becoming an increasingly congested, contested, and competitive environment. A common risk mitigation strategy for the orbit debris problem is either performing evasive maneuvers, or placing additional shielding on the satellite before launch. Current risk mitigation strategies have significant consequences to satellite operators and may not produce sufficient risk mitigation. This research poses that an orbital debris defender, which would defend the primary satellite from orbital debris, may be a more effective risk mitigation strategy. By assuming the worst case scenario, an optimally performing pursuer, this research can show when and how often the defender can intercept debris. The results of this research provide the performance trade space for the orbital debris defender, and additional recommendations to future satellite designers. Additionally, this researched derived a way to generate a pseudo cooperation between defender and evader. This cooperation between evader and defender is a new way to solve differential games, and is not limited to the space domain considered herein.

15. SUBJECT TERMS
Rendezvous and Proximity Operations; Differential Game Theory; Orbit Debris; Orbital Debris; Three Player; Pursuer-Evader-Defender;

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
U	U	U	UU	110	Capt Joshuah A. Hess, AFIT/ENY
					19b. TELEPHONE NUMBER *(Include area code)* (937) 255-3636 x4713 joshuah.hess@afit.edu

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