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An Approximate Dynamic Programming Approach for Comparing Firing Solutions in a Networked Air Defense Environment

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AN APPROXIMATE DYNAMIC PROGRAMMING APPROACH FOR COMPARING FIRING SOLUTIONS IN A NETWORKED AIR DEFENSE ENVIRONMENT

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Abstract

The United States Army currently employs a shoot-shoot-look firing policy for air defense. As the Army moves to a networked defense-in-depth strategy, this policy will not provide optimal results for managing interceptor inventories in a conflict to minimize the damage to defended assets. The objective for air and missile defense is to identify the firing policy for interceptor allocation that minimizes expected total cost of damage to defended assets. This dynamic weapon target assignment problem is formulated first as a Markov decision process (MDP) and then approximate dynamic programming (ADP) is used to solve problem instances based on a representative scenario. Least squares policy evaluation (LSPE) and least squares temporal difference (LSTD) algorithms are employed to determine the best approximate policies possible. An experimental design is conducted to investigate problem features such as conflict duration, attacker and defender weapon sophistication, and defended asset values. The LSPE and LSTD algorithm results are compared to two benchmark policies (e.g., firing one or two interceptors at each incoming tactical ballistic missile (TBM)). Results indicate that ADP policies outperform baseline polices when conflict duration is short and attacker weapons are sophisticated. Results also indicate that firing one interceptor at each TBM (regardless of inventory status) outperforms the tested ADP policies when conflict duration is long and attacker weapons are less sophisticated.

Key words: air and missile defense, dynamic weapon target assignment problem, Markov decision processes, approximate dynamic programming, approximate policy iteration, least squares policy evaluation, least squares temporal difference

This research is dedicated wife for her loving support and to my daughter and son - may you seek to always improve yourselves in all that you do.

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AN APPROXIMATE DYNAMIC PROGRAMMING APPROACH FOR COMPARING FIRING SOLUTIONS IN A NETWORKED AIR DEFENSE ENVIRONMENT

I. INTRODUCTION

Over 35 countries have theater ballistic missile (TBM) capabilities. Some TBMs have ranges of up to 3000 kilometers and the ability to deliver payloads of 1000 kilograms [1]. Some nations (e.g., North Korea and Iran) stockpile less sophisticated versions while other nations (e.g., China) continue to advance their technology to include faster moving missiles, missiles with multiple reentry vehicles, and maneuverable missiles capable of significantly altering their ballistic trajectory.

Throughout the first half of 2016 North Korea launched numerous TBMs to include a Musudan Intermediate Range Ballistic Missile in June that traveled almost 250 miles before crashing into the sea between North Korea and Japan [5]. Secretary of Defense Ash Carter affirmed the United States commitment to TBM defense in response to this launch [5]. North Korea then fired a KN-11 ballistic missile from a submarine on July 9th, further provoking tensions in the region [14]. In response to these launches, the United States and South Korea agreed to move a Terminal High Altitude Air Defense (THAAD) battery to the peninsula on July 13th [13]. This move has been highly criticized by Chinese and Russian officials as a destabilizing action [12]. These events, as well as the continued military presence of the United States in the Middle East, underscore a critical need for an intelligent TBM defense policy.

The United States divides its TBM defense into three segments: the boost defense segment, the mid-course defense segment, and the terminal defense segment [1]. The

Missile Defense Agency (MDA) has expended significant resources on boost segment defense, yet it remains difficult to intercept TBMs with any level of accuracy when they are in the boost phase of their trajectory. Therefore, the MDA limits efforts in this segment mostly to providing early launch detection. This policy is underscored by a 2012 National Research Council report that stated, “Boost-phase missile defense is not practical or cost-effective under real-world conditions for the foreseeable future” [7]. Budget cuts within the MDA have reduced its budget by 23 percent over the past eight years from 11 billion dollars to 8.5 billion dollars. Former MDA director Lt. Gen. Trey Obering (ret.) recently called for more aggressive research into boost phase defense stating,

> “Even if we’re only talking about North Korea and Iran, we have to invest in this R&D to keep up with that limited threat, because those threats are evolving and they’re becoming more mature, and then, of course, if we’re talking about a very aggressive China or a more belligerent Russia, we’ve got a long way to go to address that as well.”[23]

Unfortunately, the boost phase limitations reduce intercept opportunities to the mid-course and terminal phase of the TBM’s trajectory.

The Aegis Ballistic Missile Defense system provides a mid-course defense with the Standard Missile-3 (SM-3) interceptor. The United States Navy currently employs 33 Aegis capable platforms - five cruisers and 28 destroyers - and significant efforts are underway to place the Aegis ashore variant in high risk areas. The Aegis began development under the Reagan administration, performing its first successful flight test intercept in 2002 and becoming fully operational in 2005 [1].

The terminal defense segment offers the highest probability of intercept by current air and missile defense systems, but also portends the highest threat to the defended assets. The United States currently relies on the Patriot air defense system, the THAAD system, and the Aegis with its Standard Missile-2 (SM-2) interceptor for terminal defense [1]. Raytheon designed the original Patriot in 1969 and it made its first successful intercept in 1975 [1]. Although the system has undergone several technological updates to include enhanced computing, new interceptors, and most recently an update to the radar's front end, it is still heavily reliant on 1960's technology [1]. Lockheed Martin designed the THAAD in 1987 and it successfully intercepted a test target in 1999 [1].

These three systems indicate two major concerns with the United States missile defense capabilities. Current systems are all based on thirty-year old or older technology, and there is at least a ten-year lag between system design and initial fielding of the system. In 1999, the United States Army began to seek a replacement for the Patriot with a greater detection range and a 360 degree radar capability. Lockheed Martin won the contract for an international venture called the Medium Extended Air Defense System (MEADS) in 2005. Though this program showed promise in detection range, networkability, and multi-function 360 degree capability, it was canceled in 2015 after the Army expended over two billion dollars on the project [1].

The United States and its allies face not only an ever growing enemy arsenal of TBMs, but also an ever improving TBM technology impelled by countries like China and Iran. With the loss of the MEADS program, the United States must rely on outdated technology to counter both mass attacks by unsophisticated TBMs and pointed attacks by very technologically sophisticated TBMs. Moreover, the United States will likely not experience a vast improvement in TBM defense capabilities for a decade due to the long lag time required for the procurement process. This forces the United States to rely more heavily upon the segmented defense-in-depth strategy advocated by the MDA as it cannot rely solely on the Patriot or the THAAD to defend assets in the terminal phase. Paramount to this strategy is the ability to network the limited air and missile defense assets available together in order to provide a common air picture, provide early detection, and allow for a larger and more tailored coverage area. The United States Army is currently developing, under contract with Northrup Grumman, the Integrated Air and Missile Defense Battle Command System (IBCS) [1]. Once fielded, this networked command and control system will allow individual systems like the Patriot and the THAAD to network together and truly provide an integrated defense-in-depth.

Although a networked system of air defense assets improves the ability to detect, identify, track, and engage an enemy TBM, it also creates the added burden of deciding which air and missile defense system within the network should engage the TBMs and with how many interceptors. A fundamental tension exists between the potential catastrophic damage caused by TBMs and the extremely limited number of air and missile defense system interceptors available. Given an air and missile defense battery versus a single salvo of incoming missiles, formulating and solving a static weapon-target assignment problem could determine the best firing solution to protect the defended asset. Unfortunately, in a high intensity conflict, the air and missile defense battery must expect numerous salvos of incoming TBMs. The defender must now consider how many interceptors to fire at the current wave, while anticipating future attacks. Addressing a multi-salvo missile defense situation changes the problem from a static weapon-target assignment problem to a dynamic weapon-target assignment problem. The problem is further complicated when considering multiple air and missile defense batteries with overlapping target coverage and the ability to engage the same set of incoming TBMs with differing probabilities of kill.

Previous work examines situations concerning the location of integrated air and missile defense systems assets (e.g., [9]) and the control of such assets in a multi-salvo engagement (e.g., [6]). However, such work assumes the air and missile defense systems operate in parallel (i.e., they are capable of engaging the same targets at the same time). This assumption is somewhat unrealistic due to the limited number of air defense batteries available for asset coverage. There are very few assets, if any, that would be defended by multiple air defense systems of the same type. However, it is possible that an asset will be defended by multiple air defense systems at different segments within the MDA's defense strategy. This defense-in-depth strategy assumes the individual air and missile defense systems operate in series when engaging TBMs. For example, an Aegis may have the ability to engage during the mid-course segment at one point in time, and a THAAD or Patriot system may have the ability to engage during the terminal phase at a later point in time.

Due to the extremely high speed of TBMs, the air defense community generally adopts a shoot-shoot-look policy in the terminal phase [1]. This policy allows air defense assets to fire two interceptors at an incoming missile before it penetrates the defended assets "keep out zone." This shoot-shoot-look policy increases the probability of a kill, but it is much more resource intensive than the policy of shoot-look-shoot where the decision maker is able to fire one interceptor, assess the battle damage, and then if need be, fire another interceptor [8]. Knowing that the defender will always fire two interceptors in a shoot-shoot-look policy or only one in a shoot-look-shoot policy can significantly decrease the action space for a dynamic program.

An appropriate set of research questions of interest to the missile defense community is as follows. Does a hybrid of these policies exist that performs closer to the optimal policy and that can be more reasonably implemented in an actual combat environment? Does a networked air and missile defense allow for better management of resources and/or less expected cost to the defended assets? Is it better to have a more effective air and missile defense system at the mid-course or in the terminal phase? How do different types of incoming TBMs affect the firing policy? How does a defended asset's remaining value affect the firing policy?

This thesis provides two ways to address this networked, defense-in-depth, air and missile defense problem and answer the research questions of note. First, a Markov decision process (MDP) model is developed that allows sequential decisions to be made as the defender encounters a salvo of incoming TBMs by the first air and missile defense asset (during the mid-course segment), then at a later decision epoch another air defense asset encounters the salvo (during the terminal segment). This model allows the system to determine the optimal firing solution over an infinite horizon of decision epochs. If the series of air defense assets fail to destroy all TBMs in an incoming salvo, the TBMs will decrease the defended assets health with a specified probability of hit. The system continues to evolve until it reaches an absorbing state wherein all defended assets are destroyed. Although formulating and solving this MDP provides the optimal solution, it may take hours to determine the solution for practically-sized problem instances. Moreover, the solution is often too complicated to be administered by air defense coordinators.

Therefore, we utilize approximate dynamic programming (ADP) to develop strategies based on approximation algorithms. This allows for attaining solutions to larger problem instances while handling dimensionality issues that might otherwise make the problem computationally intractable. To answer our relevant research questions, we employ a least squares policy evaluation (LSPE) and a least squares temporal difference (LSTD) ADP approach. We compare these ADP solutions to three 'closed loop' policies based on current doctrine. We investigate the 'closed loop' policy of shooting one interceptor at each incoming TBM and the 'closed loop' policy of shooting two interceptors at each incoming TBM (as long as the inventory of interceptors allow). We also investigate a hybrid of these two policies wherein the defender fires one interceptor at traditional TBMs and two interceptors at MeRV TBMs at the mid-course phase while shooting two interceptors at traditional TBMs and one interceptor at the MeRV TBMs at the terminal phase. Although this hybrid ‘closed loop’ policy requires some level of radar discrimination of the incoming TBMs, it has performed the best in initial policy evaluation simulations.

The remainder of this thesis is organized as follows. Chapter 2 presents a literature review for the dynamic weapon target assignment problem and ADP. Chapter 3 offers a more extensive description of the networked air and missile defense problem. Chapter 4 presents the MDP model formulation as well as the ADP solution approach. Chapter 5 describes the findings when applying the aforementioned methodology. Chapter 6 provides conclusions and suggested future research efforts.

II. LITERATURE REVIEW

Two areas of literature inform the development and analysis of the networked air defense in depth problem. The first area concerns the weapon-target assignment problem (WTAP). The second area involves approximate dynamic programming (ADP).

2.1 WTAP

The WTAP dates back to the 1950s when Manne [17] developed a linear programming approximation to solve the problem. Even then he noted that for military applications a simultaneous decision is unrealistic and should be modeled in a sequential manner. This distinction led to the development of two primary classes of the WTAP, the static and the dynamic.

Xin et al. [27] describe the classes as follows. In a static WTAP, all targets are known, and all weapons are assigned to the targets in a single stage. In a dynamic WTAP, the decisions occur over many stages, so at one decision point weapons are assigned to the currently known targets and then a new set of targets is presented.

2.2 Static WTAP

The static WTAP investigates the assignment of weapons to targets without regarding the impact of time. Consider the following situation as a motivating example. Suppose there are 10 tanks and 15 anti-tank teams that represent the weapons. In this class of WTAP, the battle manager selects the weapon-target assignment decision that maximizes the expected value of the destroyed tanks based on each anti-tank weapon's associated probability of kill. Before the advent of modern computers, problems like this proved difficult and time consuming to solve. Today large-scale instances of static WTAP can be solved rather easily with linear programming and heuristic algorithms. While interesting in some cases, this class of problem does not achieve the level of detail required for realistic air defense related problems. Indeed, very few situations exist in which an air defense battle manager would have the ability to consider all incoming TBMs at one point in time and assign interceptors to maximize a selected optimality criterion. Instead, the battle manager will likely observe a single incoming salvo of TBMs at a time and be forced to make the decision on how many interceptors to fire at the incoming salvo while knowing that future salvos are likely. The number and size of incoming salvos can be informed by knowing what phase of a conflict the battle manager is in and by having intelligence on how many threat TBMs the enemy has placed within range of the defended asset. Knowing this information and seeking to formulate a more realistic problem class takes us to the dynamic WTAP.

2.3 Dynamic WTAP

Similar to the static WTAP, the dynamic class seeks to assign weapons to targets in the most effective manner to ensure the highest probability of a kill, the greatest decremented value of the target, or the least decremented value of defended assets. Different in this problem class, as compared to the static case, is that the decisions are made in a sequential manner as more information presents itself. Consider the tank example described in Section 2.2. In a dynamic WTAP, the battle manager does not consider the simultaneous engagement of all 10 tanks. Instead, the battle manager might observe a grouping of five tanks and be able to assign some number of the anti-tank weapons to those five tanks, knowing that future tank sightings are likely. Once assigned the battle manager then moves to the next decision epoch wherein another grouping of tanks is presented, and the assignment decision must be made again. The dynamic WTAP allows a much more realistic representation of combat decision making under uncertainty. However, it also makes the problem far more complex and with each level of complexity the problem becomes more computationally intractable.

Uncertainty in an air defense related problem comes from several sources. The battle manager may not know the number and types of the TBMs the enemy will fire during any given salvo. The duration of the engagement, represented by the number of incoming salvos, may be uncertain. The probability of detect for the battle manager's radar systems and the probability of kill for any fired interceptors model inherent uncertainties present in the problem. The accuracy of any networked capabilities may also be uncertain. Exploring just a few of these uncertainties creates a very large problem instance. Due to the nature of air defense, the decisions of how many interceptors to fire must be made in a matter of minutes, if not seconds, which requires any solution method to be implemented quickly. Optimally solving large-scale dynamic WTAPs instances can take computers several hours if not days. This challenge suggests the appropriateness of using approximate dynamic programming to implement algorithms that will provide high-quality solutions in very short periods of time.

Leboucher et al.[16] ignore the general assumption in a dynamic WTAP that the defender knows exactly what asset the TBM is targeting and instead only reveal a particular region that the TBM is targeting. This feature adds realism to the air defense problem in that even though radars can accurately predict a TBM's general path, they cannot truly assess what target the TBM will hit during the boost- or mid-course phase. In this thesis this uncertainty is addressed by assigning a probability that an incoming TBM hits each asset. Modeling this problem feature allows a TBM to be ignored by all defending assets and still not destroy its target.

The dynamic WTAP can be solved through heuristic methods such as the work done by Xin et al. [26] and Hoisen et al. [10]. Although these authors do not give the actual optimal solutions, this is common due to the complexity of the problem. These problems can be solved using genetic algorithms such as the anytime algorithm created by Wu et al. [25]. They developed an algorithm that evolved over time as more information became available. This algorithm improved gradually but always had a reasonable and feasible decision ready for implementation. The problem can also be solved by formulating an integer linear program like the one designed by Karasakal [11], and though this paper considered both point and area defense, it did so by making the assumption of a shoot-look-shoot policy that severely limits the action space and therefore the true optimal solution.

Bertsekas et al. [3] discuss a much more complex WTAP than that of the single weapon static case. In this case the defender must decide how many weapons to assign to each target in the current wave of attack and how many to hold for later waves. Due to the curse of dimensionality, which denies the ability to find an exact solution in medium- or large-scale problems, the authors use neuro-dynamic programming approach to help handle the increased number of dimensions. This approach determines a sub-optimal yet high-quality solution to the problem, and the authors develop four policies to approximate the solution to the dynamic WTAP.

Davis et al. [6] discussed the dynamic WTAP from the defender's perspective, considering a smart attacker that knew the outcome of each salvo and fired appropriately at surviving targets. They allowed an overlapping of each air and missile defense site's coverage area so one asset could be defended by two SAM sites. This allowed for the investigation of optimal firing policies when one SAM site was low on interceptors and another was not, or when some defended assets had lower values and others had higher values. Their problem instance was small enough to find an exact solution; they also investigated the quality of ADP approaches.

2.4 ADP

Assigning interceptors to missiles in a dynamic WTAP is a stochastic process that must be performed under uncertainty. Formulation of an MDP model allows us to determine the optimal decision now (i.e., for the current salvo) while accounting for the uncertain future salvos. Unfortunately, because of the curse of dimensionality, we are unable to quickly find an optimal solution to large-sized problems of this class. Therefore, for the dynamic WTAP we employ ADP techniques. Powell [18] provides a thorough starting point for ADP procedures. Earlier works include Bertsekas and Tsitsiklis [2] and Sutton et al. [22].

We can achieve solutions through two different algorithmic approaches: approximate value iteration (AVI) and approximate policy iteration (API). For the particular dynamic WTAP variant examined in this thesis, we utilize an API algorithmic strategy to map the system state (i.e., incoming salvo make up, asset health, and interceptor inventory) to the action (i.e., how many interceptors to fire at each missile) in order to maximize the expected value of the defender's surviving assets.

Powell [20] describes four different policies for solving an ADP. The first policy he addressed was a myopic cost function wherein the defender attempts to minimize damage for just one decision epoch. Next, he described a look-ahead policy wherein the defender would start to plan over a set number of decision epochs, but only takes the action for the current period. Some problems benefit from using policy function approximations such as look-up-tables, neural networks, or linear regression. For the DWTAP a defender might have the policy that when it is above a prescribed threshold interceptor inventory it utilizes a shoot-shoot-look policy and when it is below that inventory level it uses a shoot-look-shoot policy. The final policy discussed by Powell [20] is based on value function approximations. For our problem, we utilize a value function approximation scheme, adopting a basis function approach to determine the value of the post-decision state. Van Roy et al. [24] used a modified Bellman's equation with the post-decision decision state to reduce the outcome space making large-scale problems more easily solved. As we conduct the policy evaluation part of our API algorithm, we update the value function approximation using least squares temporal difference (LSTD) learning. Bradtke and Barto [4] showed that LSTD was an efficient algorithm to find an approximate solution to a fixed policy. Lagoudakis and Parr [15] advanced this method as they investigated the interactions of state and action pairs.

III. Methodology

3.1 Problem Description

Theater ballistic missiles (TBMs) and cruise missiles (CMs) present an extremely dangerous threat to United States forces in the early stages of combat. Although efforts are made to destroy enemy TBM and CM stockpiles before moving friendly forces into a protected area of interest, the United States cannot expect to completely negate the enemy's use of these weapons. While the United States has several options for defending assets from TBMs and CMs, such protective systems are available in relatively small quantities. This forces the United States to leave some assets unprotected and nearly guarantees that assets will only be protected by one air defense asset.

Over the past several years the Army has worked on developing a networked air defense capability that will allow available air defense assets to work in concert, providing defense in depth as a TBM or CM moves through a protected area. This capability gives the defender several decisions to make when developing an air defense plan. This includes determining which assets will be defended and by what type of air defense system, how many interceptors to provide each air defense site, how many interceptors to fire at a given salvo, and what firing policy to utilize.

In our problem instance of interest, the defender has two assets to protect, each with a co-located air defense system (i.e., surface to air missile (SAM) site) providing terminal phase protection. An air defense system is also located closer to the enemy launch site, providing mid-course protection. Each friendly asset has an associated value and health state. As asset's health state is decremented if the asset is hit by an incoming missile. Each SAM site has a predetermined number of interceptors that is not replenished during the engagement. An asset, but not the co-located air defense asset, is destroyed if its health state decreases to zero. The attacker has predetermined numbers of two types of TBMs that are fired in salvos. The number of salvos is uncertain from the perspective of the defender. The attacker does not know if its previous salvos successfully destroyed the defended asset so it could continue to fire missiles at a completely destroyed asset. The two types of TBMs fired by the attacker include a traditional TBM and a TBM with multiple reentry vehicles (MeRV). Once the attack commences, the defender decides how many interceptors to fire from the mid-course air defense system and, if it declines to fire or if the interceptors miss, it must decide how many interceptors to fire from the terminal phase defense systems. If the salvo contains a MeRV TBM and is not destroyed by the mid-course defense system this TBM will split into three missiles (targets). The defender seeks a policy that minimizes the expected value of the assets remaining after all incoming salvos.

3.2 Methodology

This section describes the MDP model formulation of the DWTAP and provides the mathematical underpinning for the ADP algorithm discussed later in this chapter.

MDP Formulation

The MDP model is formulated in the following manner.

1. Let $\mathcal{T} = \{1, 2, ..., T\}, T \leq \infty$ be the set of decision epochs.

2. The state space consists of three components: the status of each asset, the inventory of each SAM site, and the number of TBMs in each SAM's area of responsibility.

   (a) The asset status component is defined as

$$A_t = (A_{ti})_{i \in \mathcal{A}} \equiv (A_{t1}, A_{t2}, ..., A_{t|\mathcal{A}|}),$$

where $\mathcal{A} = \{1, 2, ..., |\mathcal{A}|\}$ is the set of all assets, and $A_{ti} \in \{0, 0.25, 0.5, 0.75, 1\}$. A_{ti} is the health status of asset $i \in \mathcal{A}$ at decision epoch t and shows what percentage of the asset remains.

(b) The SAM inventory status is defined as

$$R_t = (R_{ti})_{i \in \mathcal{A}} \equiv (R_{t1}, R_{t2}, ..., R_{t|\mathcal{A}|}),$$

where $R_{ti} \in \{0, 1, ..., r_i\}$, and $r_i =$ initial inventory of interceptors at SAM site $i \in \mathcal{A}$. R_{ti} is the number of interceptors at SAM site $i \in \mathcal{A}$ at decision epoch t.

(c) Let $\hat{\mathcal{M}}_{tj} = \{1, 2, ..., |\hat{\mathcal{M}}_{tj}|\}$ be the set of all fired attacker missiles of type $j \in \mathcal{J}$ at decision epoch t, where $\mathcal{J}$ is the set of all TBM types that can be fired by the attacker. For example, $j \in \mathcal{J}$ indicates whether the missile is a traditional TBM or a MeRV and its location. $\hat{\mathcal{M}}_{tj}$ is the collection of observed incoming TBMs of type $j \in \mathcal{J}$ that must be targeted by the defense at time t. The attack salvo is expressed as

$$\hat{M}_t = (\hat{\mathcal{M}}_{tji})_{j \in \mathcal{J}, i \in A},$$

where $\hat{\mathcal{M}}_{tji} \subseteq \hat{\mathcal{M}}_{tj}$ is the set of missiles of type $j \in \mathcal{J}'$ targeting asset $i \in \mathcal{A}$ at decision epoch t. The information provided by $\hat{M}_t$ is available to the defender at time t.

Using these components, we define $S_t = (A_t, R_t, \hat{M}_t) \in \mathcal{S}$ as the state of the system at decision epoch t, where $\mathcal{S}$ is the set of all possible states.

3. At each epoch t, the defender must decide how many to assign to each TBM targeting an asset. The defender must make this choice from among the SAM sites that have the given asset within their respective protection radii. We can deduce a coverage matrix for the entire defended area from the *a priori*

placement of SAM sites relative to the cities. From this coverage matrix, we can determine which SAM sites can intercept each incoming missile. Let $x_{tijk} \in \mathbb{N}^0$ be the number of interceptors fired by SAM site $i \in \mathcal{A}$ against missile $k \in \hat{\mathcal{M}}^A_{tij}$ at decision epoch t, where $\hat{\mathcal{M}}^A_{tij}$ is defined as the set of missiles of type $j \in \mathcal{J}$ that can be intercepted by SAM site i at decision epoch t. Let $x_t = (x_{tijk})_{i\in\mathcal{A}, j\in\mathcal{J}, k\in\hat{\mathcal{M}}^A_{tij}}$ denote our decision vector. We define the set of all feasible defender actions (i.e., assignment of interceptors to missiles) as

$$\mathcal{X}_{S_t} = \{x_t : \sum_{j\in\mathcal{J}} \sum_{k\in\hat{\mathcal{M}}^A_{tij}} x_{tijk} \leq R_{ti},\ \forall\ i \in \mathcal{A}\},$$

where the constraint $\sum_{j\in\mathcal{J}} \sum_{k\in\hat{\mathcal{M}}^A_{tij}} x_{tijk} \leq R_{ti}$ ensures that each SAM site $i \in \mathcal{A}$ cannot fire more interceptors than it has in inventory.

4. The transition functions explain how the the system evolves as new information becomes known [19]. We define the asset status transition function as

$$A_{t+1,i} = \begin{cases} 0 & \text{if } A_{ti} = 0, \\ \hat{A}_{t+1,i}(x_t) & \text{otherwise,} \end{cases} \quad \forall i \in \mathcal{A},$$

where $\hat{A}_{t+1,i}(x_t)$ is a random variable representing the status of each asset $i \in \mathcal{A}$ after salvo $\hat{M}_t$ and the interceptor allocation decision x_t. This information depends on x_t since the number of interceptors fired at the inbound TBMs affects an asset's health status. We define the inventory status transition function as

$$R_{t+1,i} = R_{ti} - \sum_{j\in\mathcal{J}} \sum_{k\in\hat{\mathcal{M}}^A_{tij}} x_{tijk},\ \forall\ i \in \mathcal{A},$$

and note that the asset status transition function is stochastic whereas the inventory status transition function is deterministic since there is no probability associated with firing the interceptor —once the decision to fire the interceptor is made we reduce the inventory. Concerning the transition of the attacker missiles status, let $\hat{M}_{t+1}, j(x_t)$ denote a random variable representing the status of incoming TBMs of type $j \in \mathcal{J}' \subset \mathcal{J}$, where $\mathcal{J}'$ is the set of types with terminal locations.

The state transition function is defined as $S_{t+1} = S^M(S_t, x_t, W_{t+1})$, where $W_{t+1} = \hat{A}_{t+1}, \hat{M}_{t+1}$. W_{t+1} represents all the information (i.e., asset status and attacker salvo) that becomes known at decision epoch $t+1$.

5. At each decision epoch t, the defender incurs an uncertain, immediate cost as a result of its decision. We define this cost as $\hat{C}(S_t, x_t, \hat{A}_{t+1,i}) = \sum_{i \in \mathcal{A}} v_i(A_{ti} - \hat{A}_{t+1,i})$, where v_i is the value of asset $i \in \mathcal{A}$. We rewrite the cost function in terms of only the current state and decision by taking its expected value

$$C(S_t, x_t) = \mathbb{E}\left\{\sum_{i \in \mathcal{A}} v_i(A_{ti} - \hat{A}_{t+1,i}) | S_t, x_t\right\}.$$

We seek the policy that minimizes our expected total cost savings. That is, we are trying to maintain as much value as possible in the assets. This optimal policy is denoted as π^*, and our objective is denoted as

$$\min_{\pi \in \Pi} \mathbb{E}^{\pi}\left\{\sum_{t=0}^{T} \gamma^t C(S_t, X_t^{\pi}(S_t))\right\}.$$

The notation $\mathbb{E}^{\pi}$ shows that the expectation is dependent upon the defender's actions.

The parameter $\gamma \in (0,1)$ is a discount factor that implicitly models the number of salvos or decision epochs T. We note the following relationship

$$\mathbb{E}[T] = \frac{1}{1-\gamma}. \tag{1}$$

The defender does not know how many salvos they need to defend against which makes this case more difficult than a simple optimization problem. The infinite time horizon requires the defender to make optimal decisions in the face of an uncertain number of incoming salvos of TBMs. To determine the optimal policy, we must find a solution to the Bellman equation

$$J(S_t) = \min_{x_t \in \mathcal{X}_{S_t}} (C(S_t, x_t) + \gamma \mathbb{E}\{J(S_{t+1})|S_t, x_t\}). \tag{2}$$

6. We define the decision function (i.e., policy) as

$$X^{\pi}(S_t) = \arg\min_{x_t \in \mathcal{X}_{S_t}} (C(S_t, x_t) + \gamma \mathbb{E}\{J(S_{t+1})|S_t, x_t\}),$$

where π represents a policy.

ADP Formulation

Although this MDP model enables the determination of an exact solution to the DWTAP, it is only computationally tractable for very small problems. In any instance of interest to the air defense community, the problem quickly becomes too large to solve optimally. For example, if we look at the size of the state space $\mathcal{S}$, where $S_t = (A_t, R_t, \hat{M}_t) \in \mathcal{S}$ is an arbitrary state. The tuples A_t, R_t, and $\hat{M}_t$ represent the status of each asset, the status of each SAM battery's inventory, and the attacker TBMs at decision epoch t, respectively. Since asset status can be from $0, .25, \ldots, 1$ there are $5^{|\mathcal{A}|}$ possibilities for A_t. The different SAM sites have different max inventories, but if they each had a max of 12 interceptors then there are $13^{|\mathcal{R}|}$ possibilities for R_t. If M is the maximum number of attacker missiles that can be located in any SAM's area of responsibility at any epoch t, then there are $\binom{|A|+M}{M}$ possibilities for $\hat{M}_t$. This means that an instance of this problem with three SAM sites, a max of 12 interceptors per site, and 12 points where missiles can be located creates a state space of nearly one billion different states. Exhaustive enumeration of a state space this size is computationally intractable to find the exact solution. Additionally, the DWTAP air defense problem suffers from the curse of diminsionality in reference to the action space as well as the state space. Since the defender can choose to fire zero or up to two interceptors at every TBM in the corresponding SAM site's area of responsibility, the feasible actions can increase into the millions with only 14 available firing points. Such a large action space makes solving this problem to the optimal solution computationally intractable even if the state space did not.

ADP offers solution strategies to handle both of the issues described in the previous paragraph. The approximate policy iteration (API) algorithmic strategy approximates solutions utilizing Equation (2). Therefore we rewrite the Bellman equation and use the post-decision state variable convention. Letting $J^x(S_t^x)$ be the value of being in post-decision state S_t^x, we can show the relationship between $J(S_t)$ and $J^x(S_t^x)$ with the following equations

$$J^x(S_{t-1}^x) = \mathbb{E}\{J(S_t)|S_{t-1}^x\}, \tag{3}$$

$$J(S_t) = \min_{x_t \in \mathcal{X}_{S_t}} (C(S_t, x_t) + \gamma J^x(S_t^x)), \tag{4}$$

$$J^x(S_t^x) = \mathbb{E}\{J(S_{t+1})|S_t^x\}$$

By substituting Equation (4) into Equation (3), we obtain the Bellman equation around the post-decision state variable

$$J^x(S_{t-1}^x) = \mathbb{E}\left\{ \min_{x_t \in \mathcal{X}_{S_t}} (C(S_t, x_t) + \gamma J^x(S_t^x)) \middle| S_{t-1}^x \right\}.$$

Using the post-decision state form instead of the standard form of the Bellman equation requires the swapping of the expectation and minimum operators and allows us to avoid approximating the expectation inside of the optimization problem. This allows us to control the structure and take advantage of approximation techniques.

With ADP we will step forward in time and solve the problem stochastically instead of enumerating the entire state space and using techniques like backward induction to solve the problem exactly. We are able to randomly choose a pre-decision state S_t and make a decision x_t to move to the post-decision state S_t^x.

We can now handle large state spaces, but we still must contend with approximating the expectation. We can do this by constructing a *post-decision state* variable which allows us to avoid this approximation. Van Roy *et al.* [24] first used this term, and Powell and Van Roy [21] define the post-decision state variable as the state at time t which is right after a decision x_t is made, but before any new information $\hat{W}_{t+1}$ arrives. Now the state transition function $S_{t+1} = S^M(S_t, x_t, W_{t+1})$ can be broken into two steps

$$S_t^x = S^{M,x}(S_t, x_t),$$

and

$$S_{t+1} = S^{M,W}(S_t^x, W_{t+1}),$$

where S_t^x is the post-decision state variable. For this DWTAP air defense problem, the post-decision state is given by $S_t^x = (A_t^x, R_t^x)$, where $A_t^x = (A_{ti}^x)_{i \in \mathcal{A}}$ is the component concerning asset status and $R_t^x = (R_{ti}^x)_{i \in \mathcal{A}}$ is the component concerning interceptor inventory status.

Value Function Approximation

The value function is approximated using regression methods. Similar to linear regression where we seek to find a vector using observations to fit a model that will predict a new unknown observation using a set of variables, for value function approximation we seek to find a parameter vector θ using observations that are created from a set of basis functions $(\phi_f(S_t))_{f\in\mathcal{F}}$. The set $\mathcal{F}$ of basis functions reduces the size of the state variable to those factors that we are most concerned with. For example, a basis function $f \in \mathcal{F}$ for our problem might be the remaining value of a defended asset. Using the post-decision state, we write our value function in a similar way from linear regression

$$\bar{J}^x(S_t^x) = \sum_{f\in\mathcal{F}} \theta_f \phi_f(S_t^x). \tag{5}$$

Our Bellman equation is then expressed as follows

$$\bar{J}^x(S_{t-1}^x) = \mathbb{E}\left\{ \min_{x_t\in\mathcal{X}_{S_t}} (C(S_t, x_t) + \gamma \sum_{f\in\mathcal{F}} \theta_f \phi_f(S_t^x)) \middle| S_{t-1}^x \right\}.$$

Algorithmic Strategy

API uses a series of inner loops to evaluate a set policy. It then uses an outer loop to improve the policy. For least squares temporal difference (LSTD), this is done by updating the θ vector after each inner loop completes and using the updated θ vector to better approximate the value function in the next outer loop iteration. Each time the algorithm finishes an inner loop it updates the θ vector and performs another iteration of the outer loop to seek further improvement. Algorithm 1 shows API-LSTD modified to solve the air defense problem.

The algorithm consists of K policy evaluation loops and N policy improvement loops. After initializing a θ vector as the representation of a base policy, the policy evaluation loop begins by generating a random post-decision state. Once the value $\phi(S_{t-1,k}^x)$ is recorded, we simulate forward to the next pre-decision state and select the best decision using exhaustive enumeration. We could also use a genetic algorithm to increase time savings, but exhaustive enumeration allows us to investigate a wider range of basis functions. We record the cost $C(S_{t,k}, x_t)$ and basis function evaluations of the post-decision state, $\phi(S^x_{t,k})$. We obtain K temporal difference sample realizations where the kth temporal difference given the parameter vector θ^n is $(C(S_{t,k}, x_t) + \gamma\phi(S^x_{t,k})^T\theta^n) - \phi(S^x_{t,k-1})^T\theta^n$.

The policy improvement loop occurs once the Kth temporal difference sample realizations is collected. We can describe the basis function vectors and the cost vector in the following manner. Let

$$\Phi_{t-1} \triangleq \begin{bmatrix} \phi(S^x_{t-1,1})^\top \\ \vdots \\ \phi(S^x_{t-1,K})^\top \end{bmatrix}, \quad \Phi_t \triangleq \begin{bmatrix} \phi(S^x_{t,1})^\top \\ \vdots \\ \phi(S^x_{t,K})^\top \end{bmatrix}, \quad C_t \triangleq \begin{bmatrix} C(S_{t,1}) \\ \vdots \\ C(S_{t,K}) \end{bmatrix},$$

where matrices Φ_{t-1} and Φ_t contain rows of basis function evaluations of the sampled post-decision states, and C_t is the cost vector. We perform a least squares regression of Φ_{t-1} and Φ_t against C_t to ensure the sum of the K temporal differences equals zero and calculate $\hat{\theta}$. We update our estimate of θ using $\alpha_n = \frac{a}{a+n-1}, a \in (0, \infty)$ as our smoothing function. The smoothing function manages the rate at which the function converges. Higher values of a slow the rate that α_n drops to zero, which allows later N loop iterations to have more impact on the θ vector. Smoothing θ completes one policy improvement step.

For least squares policy evaluation (LSPE), we obtain a collection of M value and post-decision state pairs and use least squares regression to fit a linear model to approximate our a value function. This is done by updating the θ vector after each inner loop completes and using the updated θ vector to better approximate the value function in the next outer loop iteration. Each time the algorithm finishes an inner loop it updates the θ vector and performs another iteration of the outer loop to seek further improvement. Algorithm 2 shows API-LSPE modified to solve the air defense problem.

Algorithm 1 LSTD-API Algorithm [19]

```
Step 0: Initialize θ^0.
Step 1:
for n=1 to N (Policy Improvement Loop)
    Step 2:
    for m=1 to M (Policy Evaluation Loop)
        Generate a random post-decision state, S^x_{t-1,m}.
        Record φ(S^x_{t-1,m}).
        Simulate transition to next pre-decision state, S_{t,m}.
        Determine decision x = X^π(S_{t,m}|θ^{n-1}) through exhaustive enumeration of feasible actions.
        Record cost C(S_{t,m}, x).
        Record basis function evaluation φ(S^x_{t,m})
    end for
    End
    Update θ^n and the policy:
    θ̂ = [(Φ_{t-1} − γΦ_t)^T (Φ_{t-1} − γΦ_t)]^{-1} (Φ_{t-1} − γΦ_t)^T C_t
    θ^n = α_n θ̂ + (1 − α_n) θ^{n-1}
end for
Return X^π(S_t|θ^N) and θ^N.
End
```

The algorithm consists of K policy evaluation loops and N policy improvement loops. After initializing a θ vector as the representation of a base policy, the policy evaluation loop begins by generating a random post-decision state. Once the value $\phi(S^x_{t-1,k})$ is recorded, we simulate forward to the next pre-decision state and select the best decision using exhaustive enumeration. We record the value $V(S_{t,k}, x_t)$ and basis function evaluations of the post-decision state, $\phi(S^x_{t,k})$.

The policy improvement loop occurs once the Kth policy evaluation sample realizations is collected. We can describe the basis function vectors and the cost vector in the following manner. Let

$$\Phi_{t-1} \triangleq \begin{bmatrix} \phi(S^x_{t-1,1})^\top \\ \vdots \\ \phi(S^x_{t-1,K})^\top \end{bmatrix}, \quad V_t \triangleq \begin{bmatrix} V(S_{t,1}) \\ \vdots \\ V(S_{t,K}) \end{bmatrix},$$

where matrix Φ_{t-1} contains rows of basis function evaluations of the sampled post-decision states, and V_t is the value vector. We perform a least squares regression of Φ_{t-1} against V_t. We update our estimate of θ using $\alpha_n = \frac{a}{a+n-1}, a \in (0, \infty)$ as our smoothing function. Smoothing θ completes one policy improvement step.

Algorithm 2 LSPE-API Algorithm [19]

Step 0: Initialize θ^0.
Step 1:
for n=1 **to** N **(Policy Improvement Loop)**
    Step 2:
    for m=1 **to** M **(Policy Evaluation Loop)**
        Generate a random post-decision state, $S^x_{t-1,m}$.
        Record $\phi(S^x_{t-1,m})$.
        Simulate transition to next pre-decision state, $S_{t,m}$.
        Determine decision $x = X^\pi(S_{t,m}|\theta^{n-1})$ through exhaustive enumeration of feasible actions.
        Record cost $V(S_{t,m}, x)$.
    end for
    End
    Update θ^n and the policy:
    $\hat{\theta} = [(\Phi_{t-1})^T(\Phi_{t-1})]^{-1}(\Phi_{t-1})^T V_t$
    $\theta^n = \alpha_n\hat{\theta} + (1-\alpha_n)\theta^{n-1}$
end for
Return $X^\pi(S_t|\theta^N)$ and θ^N.
End

IV. Results

4.1 Computational Results

In this chapter, we examine a problem of interest to the military and utilize the approximate dynamic programming (APD) techniques described in Chapter 3 to seek policy solutions to this problem. We compare policies found by our ADP algorithms to current baseline policies used by the air defense community. We construct a theater ballistic missile (TBM) defense scenario as the tactical underpinning for our analysis. From this scenario, we create 32 test instances and, for each instance, determine approximate solutions for each baseline policy using simulation techniques. We solve each instance approximately by employing the ADP solution methodologies. A set of designed experiments is conducted to identify which ADP algorithmic parameter-values result in the best solution. We conduct computational experiments for both least squares policy evaluation (LSPE) and least squares temporal difference (LSTD), and compare the two ADP algorithms to each other to determine the best overall ADP algorithm (and policy) for each of the 32 instances. We also compare the current air defense policies and the acquired ADP policies using vignettes that are of interest from earlier simulation-based experiments.

Representative Scenario.

We present a networked TBM defense utilizing the Missile Defense Agency's (MDA) defense-in-depth plan for a mid-course and terminal phase defense. For this scenario the defender seeks to protect two assets. This scenario places an Aegis air defense system at the mid-course point, a THAAD with the first defended asset, and a Patriot with the second defended asset. See Figure 1 for a detailed diagram of this scenario. All TBMs pass through the Aegis' area of responsibility, and the Aegis has an opportunity to fire up to two interceptors at each TBM. We allocate 12 interceptors to the Aegis, which constitutes half of the payload of the Aegis equipped ship [1]. The THAAD and the Patriot can only fire at TBMs targeting their defended asset, but they also have the opportunity to fire up to two interceptors at each TBM. The attacker fires a combination of traditional TBMs and multiple reentry vehicle (MeRV) TBMs. If the MeRV is missed (or not targeted) by the Aegis, it splits into three missiles (targets) before the THAAD or Patriot have an opportunity to fire at it. The TBMs, if missed or not fired at in the terminal phase, have a given probability of hitting its intended target. This probability of hit models the technical sophistication of the attacker's weaponry (e.g., flight control, guidance, and warhead technology). If the TBM hits its targeted defended asset, it decrements the asset by a preassigned amount of one quarter of the asset's total value. The defended asset can sustain up to four hits before being completely destroyed. A discount factor is used to model how many expected salvos the defender will encounter. See Davis et al. (2016) for a description of this modeling approach.

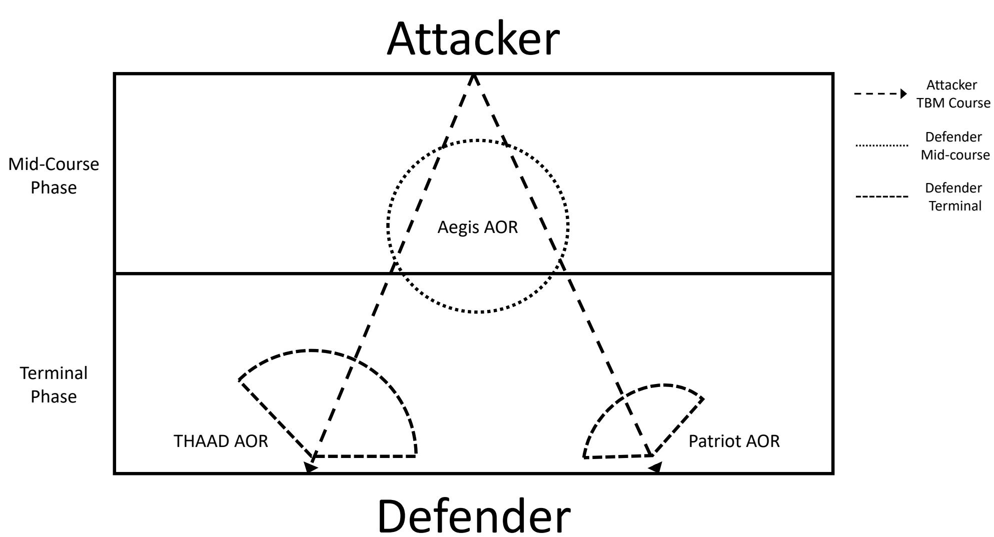


Figure 1. Scenario Diagram

From this basic scenario, we develop 32 test instances by varying four of the problem features. We first varied the number of salvos the defender can expect to engage, or the duration of the attack, as indicated by γ. Exploratory simulations, based on the number of available interceptors in each phase, allowed us to choose two γ-values, 0.8 and 0.9, to investigate the impact the expected number of salvos had on the ADP policy.

The second problem feature we varied was the enemy's level of technological sophistication – that is, the enemy may have fairly accurate TBMs or inaccurate TBMs. We chose a probability of hit of 0.8 for the technologically superior attacker and a probability of hit of 0.5 for the technologically inferior attacker.

The third problem feature we varied was the defender's level of technological sophistication, seeking to capture the accuracy of the defender's interceptors to successfully engage the TBMs. We chose a probability of kill for the technologically superior defender of 0.8, 0.9, and 0.85 for the Aegis, THAAD and Patriot, respectively. We chose a probability of kill for the technologically inferior defender of 0.7, 0.8, and 0.75, for the Aegis, THAAD, and Patriot respectively.

The fourth problem feature we varied was the defended asset value. We wanted to investigate how a higher, lower, or equal value of the defended assets protected by the THAAD and Patriot would affect the ADP policy. We assigned equal values of 24 for both Asset 1 and Asset 2 for the Low/Low case, values of 48 for Asset 1 and 24 for Asset 2 for the High/Low case, values of 24 for Asset 1 and 48 for Asset 2 in the Low/High case, and equal values of 48 for both Asset 1 and Asset 2 in the High/High case. Table 1 shows the problem feature settings for each test instance.

Table 1. Test Instances

	Problem Features			
	Expected Conflict Duration	Attacker's Technological Sophistication	Defender's Technological Sophistication	Defended Asset Values
1	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	Medium (pK = 0.7, 0.8, 0.75)	Low/Low
2	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	Medium (pK = 0.7, 0.8, 0.75)	High/Low
3	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	Medium (pK = 0.7, 0.8, 0.75)	Low/High
4	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	Medium (pK = 0.7, 0.8, 0.75)	High/High
5	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	Low/Low
6	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	High/Low
7	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	Low/High
8	Short ($\mathbb{E}[T] = 5$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	High/High
9	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	Low/Low
10	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	High/Low
11	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	Low/High
12	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	High/High
13	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	Low/Low
14	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	High/Low
15	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	Low/High
16	Short ($\mathbb{E}[T] = 5$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	High/High
17	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	Medium (pK =0.7, 0.8, 0.75)	Low/Low
18	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	Medium (pK =0.7, 0.8, 0.75)	High/Low
19	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	Medium (pK =0.7, 0.8, 0.75)	Low/High
20	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	Medium (pK = 0.7, 0.8, 0.75)	High/High
21	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	Low/Low
22	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	High/Low
23	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	Low/High
24	Long ($\mathbb{E}[T] = 10$)	Low (pH = 0.5)	High (pK = 0.8, 0.9, 0.85)	High/High
25	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	Low/Low
26	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	High/Low
27	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	Low/High
28	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	Medium (pK = 0.7, 0.8, 0.75)	High/High
29	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	Low/Low
30	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	High/Low
31	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	Low/High
32	Long ($\mathbb{E}[T] = 10$)	High (pH = 0.8)	High (pK = 0.8, 0.9, 0.85)	High/High

Experimental Design.

For each of the 32 test instances, we wish to determine the best parameter settings for Algorithms 1 and 2. We focus on parameters N, K, ϕ, a, and η. Table 2 shows the 2-level, 5-factor experimental design, and Table 3 shows the set of features for each design level of the ϕ factor. The levels for each factor were chosen based on initial experimental runs of the model. These experimental runs also suggested that the instrumental variables (IV) method for LSTD would not perform well for these instances, and so the IV method was not utilized.

Experimental Results.

For each test instance, we ran a full factorial experiment for three random number seeds (i.e., three replications) for a total of 96 runs. For each run, we recorded the mean and standard deviation, and calculated the difference between the ADP policy means and the means garnered from our two baseline policies. For each scenario, we chose the ADP policy (and noted the attendant parameter settings) that provided the largest difference between the baseline policies and the ADP policy.

Table 2. Experimental Design for Algorithmic Features

N	K	ϕ	a	η
25	1000	1	10	10
50	1000	1	10	10
25	2000	1	10	10
50	2000	1	10	10
25	1000	2	10	10
50	1000	2	10	10
25	2000	2	10	10
50	2000	2	10	10
25	1000	1	100	10
50	1000	1	100	10
25	2000	1	100	10
50	2000	1	100	10
25	1000	2	100	10
50	1000	2	100	10
25	2000	2	100	10
50	2000	2	100	10
25	1000	1	10	100
50	1000	1	10	100
25	2000	1	10	100
50	2000	1	10	100
25	1000	2	10	100
50	1000	2	10	100
25	2000	2	10	100
50	2000	2	10	100
25	1000	1	100	100
50	1000	1	100	100
25	2000	1	100	100
50	2000	1	100	100
25	1000	2	100	100
50	1000	2	100	100
25	2000	2	100	100
50	2000	2	100	100

Table 3. Basis Function Features

ϕ	$\phi_0(S)$	$\phi_1(S)$	$\phi_2(S)$	$\phi_3(S)$	$\phi_4(S)$	$\phi_5(S)$
1	1	A_t	R_t^x	A_t^x		
2	1	R_t^x	A_t^x	$(R_t^x)^2$	$(A_t^x)^2$	$R_t^x A_t^x$

4.2 Least Squares Policy Evaluation

Using the problem features and experimental design described in Section 4.1, we implemented the LSPE algorithm annotated in Algorithm 2. This required 3072 runs to perform the full factorial experiment for all problem and algorithmic features with three replications. The LSPE ADP algorithm provided a θ-vector for each of these 3072 runs. We then utilized a simulation to determine the mean performance and standard deviation for each of those 3072 θ-vectors. We executed 2000 simulation runs for each θ-vector in order to gain confidence that we found an accurate mean.

We compared the ADP results to the two baseline policies. The first baseline policy (Baseline Policy 1) was to fire one interceptor at each incoming TBM (as long as the SAM site inventory allowed), and the second baseline policy (Baseline Policy 2)was to fire two interceptors at each TBM (if the SAM site inventory did not allow for firing two, only then would the SAM fire one). We executed 2000 simulation runs for the two baseline policies. In exploratory runs of the simulation, we found that firing one interceptor at each incoming TBM generally outperformed firing two interceptors for the problem features being explored. We compared the means of the policies found by our LSPE algorithms to the two baseline policies. The results for the best ADP policy versus the baseline polices for each scenario are shown in Table 4.

Table 4. LSPE Results - Quality of Solution Using Best θ-vector

Instance (γ, pH, pK, Asset Value)	Best Algorithm Parameters (N,k,a,η,ϕ)	ADP Policy 95% CI	Baseline Policy 1 95% CI	Baseline Policy 2 95% CI
1 (0.8, 0.5, 1, Low/Low)	25, 2000, 100, 1, 2	**† † 5.75 ± 0.71**	7.55 ± 0.64	7.59 ± 0.82
2 (0.8, 0.5, 1, High/Low)	25, 1000, 10, 10, 2	**† † 5.68 ± 0.71**	8.01 ± 0.68	7.97 ± 0.85
3 (0.8, 0.5, 1, Low/High)	25, 2000, 100, 10, 1	**6.09 ± 0.73**	7.42 ± 0.63	6.77 ± 0.78
4 (0.8, 0.5, 1, High/High)	25, 2000, 10, 1, 2	**6.05 ± 0.72**	7.55 ± 0.63	7.2 ± 0.8
5 (0.8, 0.5, 1, Low/Low)	25, 2000, 100, 1, 2	4.78 ± 0.67	**4.29 ± 0.52**	7.25 ± 0.84
6 (0.8, 0.5, 2, High/Low)	50, 2000, 100, 10, 2	4.71 ± 0.68	**4.43 ± 0.51**	7.41 ± 0.84
7 (0.8, 0.5, 2, Low/High)	25, 1000, 100, 1, 2	4.99 ± 0.7	**4.13 ± 0.46**	6.48 ± 0.78
8 (0.8, 0.5, 2, High/High)	50, 1000, 10, 10, 2	5.15 ± 0.71	**4.18 ± 0.5**	6.42 ± 0.78
9 (0.8, 0.8, 2, Low/Low)	50, 2000, 10, 1, 1	**†7.11 ± 0.8**	11.68 ± 0.84	8.53 ± 0.88
10 (0.8, 0.8, 1, High/Low)	25, 1000, 10, 1, 2	**†7.68 ± 0.83**	10.39 ± 0.77	9.15 ± 0.91
11 (0.8, 0.8, 1, Low/High)	50, 2000, 100, 10, 1	**7.49 ± 0.81**	10.8 ± 0.78	8.33 ± 0.85
12 (0.8, 0.8, 1, High/High)	25, 2000, 100, 1, 1	**† † 7.37 ± 0.81**	10.82 ± 0.79	9.36 ± 0.92
13 (0.8, 0.8, 2, Low/Low)	25, 2000, 10, 10, 2	6.07 ± 0.77	**5.79 ± 0.58**	7.98 ± 0.89
14 (0.8, 0.8, 2, High/Low)	50, 2000, 10, 10, 2	6.12 ± 0.79	**5.69 ± 0.58**	6.68 ± 0.81
15 (0.8, 0.8, 2, Low/High)	25, 1000, 10, 10, 1	5.97 ± 0.78	**5.94 ± 0.6**	8.65 ± 0.91
16 (0.8, 0.8, 2, High/High)	50, 2000, 100, 10, 2	**6.02 ± 0.76**	6.25 ± 0.62	7.31 ± 0.84
17 (0.9, 0.5, 1, Low/Low)	25, 1000, 100, 10, 1	21.11 ± 1.3	**20.88 ± 1.13**	24.46 ± 1.35
18 (0.9, 0.5, 1, High/Low)	25, 1000, 100, 10, 1	21.02 ± 1.29	**19.9 ± 1.12**	22.58 ± 1.32
19 (0.9, 0.5, 1, Low/High)	50, 1000, 100, 10, 2	20.27 ± 1.29	**20.17 ± 1.12**	23.6 ± 1.34
20 (0.9, 0.5, 1, High/High)	25, 2000, 100, 10, 1	**21.16 ± 1.3**	21.49 ± 1.15	23.4 ± 1.32
21 (0.9, 0.5, 1, Low/Low)	25, 1000, 100, 1, 2	19.46 ± 1.29	**⋆ ⋆ 15.22 ± 1.06**	22.46 ± 1.34
22 (0.9, 0.5, 2, High/Low)	50, 2000, 10, 1, 1	19.55 ± 1.29	**⋆ ⋆ 14.33 ± 1.02**	22.46 ± 1.35
23 (0.9, 0.5, 2, Low/High)	25, 2000, 10, 10, 2	19.6 ± 1.29	**⋆ ⋆ 14.44 ± 1.04**	21.47 ± 1.31
24 (0.9, 0.5, 2, High/High)	25, 1000, 10, 1, 1	19.53 ± 1.28	**⋆ ⋆ 14.5 ± 1.03**	21.6 ± 1.33
25 (0.9, 0.8, 2, Low/Low)	50, 1000, 10, 1, 2	**†22.97 ± 1.34**	25.83 ± 1.2	25.66 ± 1.38
26 (0.9, 0.8, 1, High/Low)	50, 1000, 100, 1, 1	**†23.34 ± 1.35**	26.98 ± 1.24	25.94 ± 1.38
27 (0.9, 0.8, 1, Low/High)	25, 2000, 100, 1, 1	**†22.85 ± 1.34**	25.16 ± 1.21	25.91 ± 1.38
28 (0.9, 0.8, 1, High/High)	50, 1000, 10, 1, 1	**23.15 ± 1.33**	25.76 ± 1.21	25 ± 1.37
29 (0.9, 0.8, 2, Low/Low)	25, 1000, 100, 10, 1	21.38 ± 1.34	**⋆ ⋆ 17.95 ± 1.11**	24.19 ± 1.39
30 (0.9, 0.8, 2, High/Low)	50, 1000, 10, 10, 2	21.12 ± 1.32	**19.64 ± 1.15**	23.83 ± 1.38
31 (0.9, 0.8, 2, Low/High)	25, 1000, 10, 1, 2	20.88 ± 1.34	**⋆ ⋆ 18.01 ± 1.12**	24.53 ± 1.4
32 (0.9, 0.8, 2, High/High)	50, 2000, 10, 1, 1	21.12 ± 1.34	**19.42 ± 1.17**	22.74 ± 1.36

†† denotes statistical significance (as compared to the next best policy) with 95% confidence
† denotes statistical significance (as compared to the next best policy) with 90% confidence
⋆⋆ denotes statistical significance (as compared to the ADP policy) with 95% confidence
⋆ denotes statistical significance (as compared to the ADP policy) with 90% confidence

The LSPE policy achieves statistically significant improvement over the two baseline policies in 8 of the 32 test instances. The instances that show LSPE policy superiority at the 95% confidence level are 1, 2, and 12. Instances 9, 10, 25, 26, and 27 show statistical significance at the 90% confidence level. LSPE attains the best mean result in 14 of the 32 instances. We see that LSPE outperforms the baseline policies when the duration of the conflict is short or when the enemy has weapons with a high probability of hit. It is not surprising that in circumstances where if missed the incoming TBM has a high likelihood of damaging its targeted asset that the LSPE policy outperforms the baseline policies, but it is interesting that in shorter duration conflicts when the two baseline policies show very similar means that the ADP is able to outperform at a statistically significant level.

Baseline Policy 1 outperforms LSPE in Instances 21, 22, 23, 24, 29, and 31 at the 95% confidence level. Examining these instances, we find common characteristics: long duration conflict where the attacker had lower quality weapons and the defender had higher quality weapons.

It is of further interest that Baseline Policy 2, which is currently the Army's implemented policy, is never significantly better than the LSPE policy or Baseline Policy 1. This suggests that as the military moves to an integrated, defense-in-depth strategy it needs to consider a different firing policy for networked air defense systems with both mid-course and terminal systems.

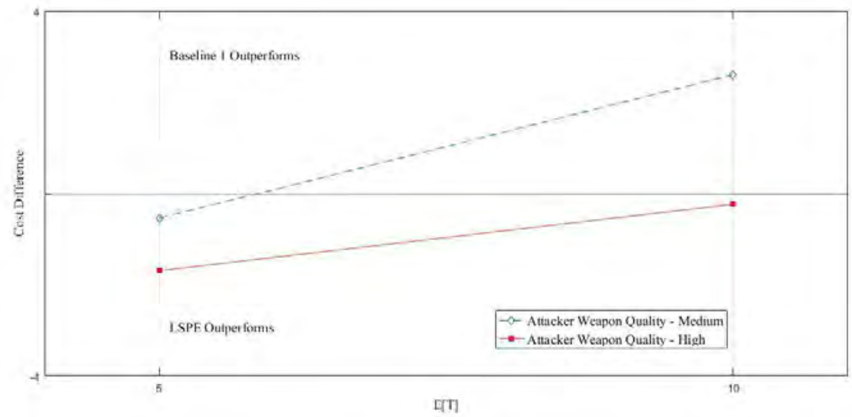


Figure 2. Cost Difference between High and Low Attacker Weapon Quality

Figure 2 highlights the cost difference between LSPE and Baseline Policy 1 when we look at the two different levels of attacker weapon quality for short and long duration conflicts. In this we observe that for high quality attacker weapons LSPE performs better than the baseline policy regardless of conflict duration, but for low quality attacker weapons Baseline Policy 1 is superior for long duration conflicts. Note that this graphic implies a linear relationship that might not exist.

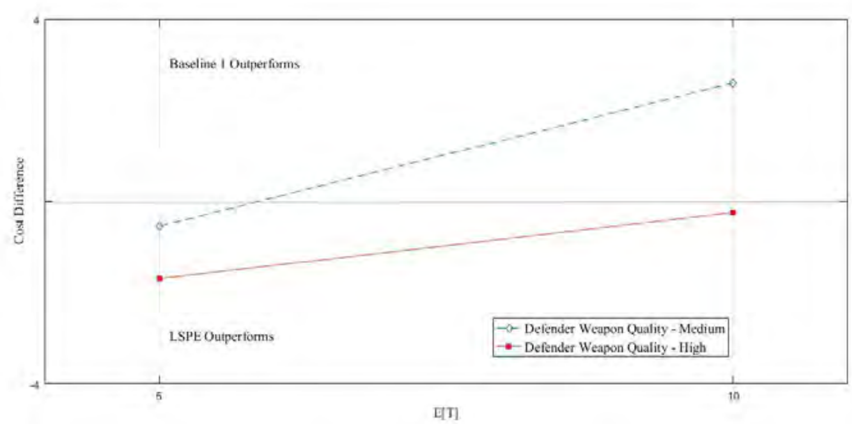


Figure 3. Cost Difference between High and Medium Defender Weapon Quality

Figure 3 highlights the cost difference between LSPE and Baseline Policy 1 when we look at the two different levels of defender weapon quality for short and long duration conflicts. In this we observe that for medium quality defender weapons

LSPE performs better than the baseline policy regardless of conflict duration, but for high quality defender weapons Baseline Policy 1 is always superior. Note that this graphic implies a linear relationship that might not exist.

Table 5. LSPE Results - Robustness

	Algorithm Parameters	Run 1	Run 2	Run 3	Mean	Best	Difference
1	25, 2000, 100, 1, 2	5.75	6.62	6.23	6.20	5.75	0.45
2	25, 1000, 10, 10, 2	7.00	5.68	6.81	6.50	5.68	0.82
3	25, 2000, 100, 10, 1	6.09	6.52	6.27	6.29	6.09	0.20
4	25, 2000, 10, 1, 2	6.05	6.52	6.93	6.50	6.05	0.45
5	25, 2000, 100, 1, 2	5.69	4.78	6.03	5.50	4.78	0.72
6	50, 2000, 100, 10, 2	4.71	5.60	6.14	5.49	4.71	0.77
7	25, 1000, 100, 1, 2	6.04	4.99	5.85	5.63	4.99	0.64
8	50, 1000, 10, 10, 2	5.15	5.94	5.72	5.60	5.15	0.45
9	50, 2000, 10, 1, 1	8.27	9.43	7.11	8.27	7.11	1.16
10	25, 1000, 10, 1, 2	7.68	8.43	8.37	8.16	7.68	0.48
11	50, 2000, 100, 10, 1	8.59	7.49	8.33	8.13	7.49	0.65
12	25, 2000, 100, 1, 1	7.37	8.21	8.50	8.03	7.37	0.66
13	25, 2000, 10, 10, 2	6.07	7.07	7.47	6.87	6.07	0.80
14	50, 2000, 10, 10, 2	6.95	7.15	6.12	6.74	6.12	0.62
15	25, 1000, 10, 10, 1	7.00	5.97	7.35	6.77	5.97	0.80
16	50, 2000, 100, 10, 2	6.02	7.05	6.86	6.65	6.02	0.62
17	25, 1000, 100, 10, 1	22.46	21.11	22.85	22.14	21.11	1.03
18	25, 1000, 100, 10, 1	21.02	22.30	21.47	21.60	21.02	0.58
19	50, 1000, 100, 10, 2	21.77	20.27	22.24	21.42	20.27	1.15
20	25, 2000, 100, 10, 1	22.06	22.92	21.16	22.05	21.16	0.89
21	25, 1000, 100, 1, 2	20.90	21.27	19.46	20.54	19.46	1.09
22	50, 2000, 10, 1, 1	21.00	19.55	21.10	20.55	19.55	1.00
23	25, 2000, 10, 10, 2	19.60	21.19	20.17	20.32	19.6	0.72
24	25, 1000, 10, 1, 1	21.74	21.44	19.53	20.90	19.53	1.37
25	50, 1000, 10, 1, 2	23.56	25.45	22.97	23.99	22.97	1.02
26	50, 1000, 100, 1, 1	23.34	24.44	25.11	24.30	23.34	0.96
27	25, 2000, 100, 1, 1	25.92	22.85	25.10	24.62	22.85	1.78
28	50, 1000, 10, 1, 1	24.80	24.97	23.15	24.31	23.15	1.16
29	25, 1000, 100, 10, 1	21.38	22.06	23.68	22.37	21.38	0.99
30	50, 1000, 10, 10, 2	22.16	21.12	24.05	22.45	21.12	1.33
31	25, 1000, 10, 1, 2	23.06	20.88	22.07	22.00	20.88	1.12
32	50, 2000, 10, 1, 1	23.62	24.48	21.12	23.07	21.12	1.95

Table 5 shows the LSPE-determined three-run averages for the θ-vectors for each replication of the 32 instances. These three-run averages show a general robustness across the θ-vectors garnered from the given parameter settings. For most of the instances, we have a less than 1 point difference between the best mean and the average mean. Though this would impact the statistical significance of those parameter settings versus the baseline policies, it does not indicate that any of the chosen best θ-vectors were simply outliers. This result suggests an overall robustness with respect to the consistency of performance of the LSPE algorithm.

Meta Analysis

Table 6. Parameter Estimates - LSPE

	Estimate	Standard Error	t Ratio	Probability $< \|t\|$
Intercept	-120.26	0.22	-555.56	< 0.0001
N (outer loops)	0.00	0.00	-0.79	0.43
K (inner loops)	0.00	0.00	1.47	0.14
a (smoothing)	0.00	0.00	-2.05	0.04
η (regularization)	0.00	0.00	0.61	0.54
ϕ (basis function set)	-0.01	0.02	-0.34	0.73
Conflict Duration	157.27	0.23	694.73	< 0.0001
Attacker Weapon Quality	5.74	0.08	76.05	< 0.0001
Defender Weapon Quality	-1.44	0.02	-63.43	< 0.0001
Asset 1 Value	0.00	0.00	-1.61	0.11
Asset 2 Value	0.00	0.00	-0.37	0.71
			R-Square Adj	0.99

When examining the parameter estimates in Table 6, we see that conflict duration, attacker weapon quality, and defender weapon quality have the largest impact on the change in the mean of the damage caused by the incoming TBMs. Although not statistically significant at a 95% confidence level, the Asset 1 value factor appears to explain more of the variation than the N, η, ϕ and the Asset 2 value factors. Recall that Asset 1 is protected by the THAAD air defense system and the THAAD had the highest pK across all scenarios. This suggests that having the more effective air defense system co-located with the higher value asset could lead to more impact in minimizing the mean damage incurred, an intuitive result.

Examining the parameter settings for the ADP algorithm, we observe that the smoothing component explains a significant portion of the variation (with a 0.04 p-value). Although the number of inner loops (K) is not statistically significant, it does explain more variation than the other parameter settings. It is likely that the number of outer loops (N) did not have more impact on the mean because the smoothing coefficient did have an impact, and new information garnered from a higher number of outer loops received very little weight. We likely did not have a large enough difference in the number of inner loops, and had we had time to perform 4000 inner loops, we might have seen this coefficient become statistically significant. Since the ϕ did not have an impact we might benefit from searching for other sets of basis functions that perform better than the baseline policies.

4.3 Least Squares Temporal Difference

Using the problem features and experimental design described in Section 4.1 we implemented the LSTD algorithm annotated in Algorithm 1. This required 3072 runs to perform the full factorial experiment of all problem and algorithmic features with three replications. The LSTD ADP algorithm provided a θ-vector for each of these 3072 runs. We then utilized a simulation to determine the performance in terms of the mean and standard deviation for each of those 3072 θ-vectors. We executed 2000 simulation runs to ensure we had confidence in our garnered mean.

We compared these means to the means for the two baseline policies. We compared the means of our LSTD algorithms to the two baseline policies. These results for the best θ-vector versus the baseline for each scenario is shown on Table 7.

Table 7. LSTD Results - Quality of Solution Using Best θ-vector

Instance (γ, pH, pK, Asset Value)	Best Algorithm Parameters (N,k,a,η,ϕ)	ADP Policy 95% CI	Baseline Policy 1 95% CI	Baseline Policy 2 95% CI
1 (0.8, 0.5, 1, Low/Low)	25, 2000, 10, 10, 2	**†6.08 ± 0.74**	7.55 ± 0.64	7.59 ± 0.82
2 (0.8, 0.5, 1, High/Low)	25, 2000, 10, 1, 2	**† † 6.15 ± 0.73**	8.01 ± 0.68	7.97 ± 0.85
3 (0.8, 0.5, 1, Low/High)	25, 2000, 100, 1, 2	**6.17 ± 0.74**	7.42 ± 0.63	6.77 ± 0.78
4 (0.8, 0.5, 1, High/High)	25, 2000, 100, 10, 2	**6.05 ± 0.73**	7.55 ± 0.63	7.2 ± 0.8
5 (0.8, 0.5, 1, Low/Low)	25, 1000, 10, 10, 2	4.83 ± 0.68	**4.29 ± 0.52**	7.25 ± 0.84
6 (0.8, 0.5, 2, High/Low)	50, 2000, 10, 10, 2	4.55 ± 0.66	**4.43 ± 0.51**	7.41 ± 0.84
7 (0.8, 0.5, 2, Low/High)	50, 1000, 100, 10, 1	4.9 ± 0.68	**4.13 ± 0.46**	6.48 ± 0.78
8 (0.8, 0.5, 2, High/High)	50, 2000, 100, 10, 2	4.88 ± 0.7	**4.18 ± 0.5**	6.42 ± 0.78
9 (0.8, 0.8, 2, Low/Low)	25, 1000, 100, 10, 2	**7.29 ± 0.81**	11.68 ± 0.84	8.53 ± 0.88
10 (0.8, 0.8, 1, High/Low)	50, 1000, 10, 1, 2	**† † 7.16 ± 0.79**	10.39 ± 0.77	9.15 ± 0.91
11 (0.8, 0.8, 1, Low/High)	50, 2000, 100, 10, 1	**7.58 ± 0.83**	10.8 ± 0.78	8.33 ± 0.85
12 (0.8, 0.8, 1, High/High)	25, 2000, 10, 10, 1	**† † 7.31 ± 0.8**	10.82 ± 0.79	9.36 ± 0.92
13 (0.8, 0.8, 2, Low/Low)	25, 1000, 100, 10, 1	6.23 ± 0.79	**5.79 ± 0.58**	7.98 ± 0.89
14 (0.8, 0.8, 2, High/Low)	25, 1000, 10, 1, 1	6.3 ± 0.79	**5.69 ± 0.58**	6.68 ± 0.81
15 (0.8, 0.8, 2, Low/High)	25, 2000, 10, 10, 2	5.98 ± 0.77	**5.94 ± 0.6**	8.65 ± 0.91
16 (0.8, 0.8, 2, High/High)	25, 1000, 10, 10, 1	**6.14 ± 0.79**	6.25 ± 0.62	7.31 ± 0.84
17 (0.9, 0.5, 1, Low/Low)	25, 1000, 100, 1, 2	**20.69 ± 1.29**	20.88 ± 1.13	24.46 ± 1.35
18 (0.9, 0.5, 1, High/Low)	25, 1000, 100, 10, 2	21.02 ± 1.29	**19.9 ± 1.12**	22.58 ± 1.32
19 (0.9, 0.5, 1, Low/High)	25, 2000, 10, 1, 1	21.11 ± 1.29	**20.17 ± 1.12**	23.6 ± 1.34
20 (0.9, 0.5, 1, High/High)	50, 2000, 10, 10, 2	**20.78 ± 1.3**	21.49 ± 1.15	23.4 ± 1.32
21 (0.9, 0.5, 1, Low/Low)	25, 2000, 10, 1, 1	19.75 ± 1.29	**⋆ ⋆ 15.22 ± 1.06**	22.46 ± 1.34
22 (0.9, 0.5, 2, High/Low)	25, 1000, 10, 10, 1	19.4 ± 1.28	**⋆ ⋆ 14.33 ± 1.02**	22.46 ± 1.35
23 (0.9, 0.5, 2, Low/High)	25, 1000, 100, 1, 1	19.62 ± 1.3	**⋆ ⋆ 14.44 ± 1.04**	21.47 ± 1.31
24 (0.9, 0.5, 2, High/High)	50, 1000, 100, 10, 2	19.67 ± 1.29	**⋆ ⋆ 14.5 ± 1.03**	21.6 ± 1.33
25 (0.9, 0.8, 2, Low/Low)	50, 1000, 10, 10, 1	**†23 ± 1.34**	25.83 ± 1.2	25.66 ± 1.38
26 (0.9, 0.8, 1, High/Low)	50, 2000, 10, 10, 2	**†23.53 ± 1.36**	26.98 ± 1.24	25.94 ± 1.38
27 (0.9, 0.8, 1, Low/High)	25, 1000, 10, 1, 2	**23.27 ± 1.36**	25.16 ± 1.21	25.91 ± 1.38
28 (0.9, 0.8, 1, High/High)	25, 1000, 100, 10, 2	**23.25 ± 1.35**	25.76 ± 1.21	25 ± 1.37
29 (0.9, 0.8, 2, Low/Low)	50, 1000, 10, 1, 2	21.22 ± 1.34	**⋆ ⋆ 17.95 ± 1.11**	24.19 ± 1.39
30 (0.9, 0.8, 2, High/Low)	25, 1000, 10, 1, 2	21.01 ± 1.34	**19.64 ± 1.15**	23.83 ± 1.38
31 (0.9, 0.8, 2, Low/High)	25, 1000, 100, 10, 2	21.19 ± 1.35	**⋆ ⋆ 18.01 ± 1.12**	24.53 ± 1.4
32 (0.9, 0.8, 2, High/High)	50, 1000, 100, 1, 1	21 ± 1.34	**19.42 ± 1.17**	22.74 ± 1.36

†† denotes statistical significance (as compared to the next best policy) with 95% confidence
† denotes statistical significance (as compared to the next best policy) with 90% confidence
⋆⋆ denotes statistical significance (as compared to the ADP policy) with 95% confidence
⋆ denotes statistical significance (as compared to the ADP policy) with 90% confidence

The LSTD policy achieves statistically significant improvement over the two baseline policies in 6 of the 32 test instances. The instances that show LSTD policy superiority at the 95% confidence level are 2, 10, and 12. Instances 1, 25, and 26 show statistical significance at the 90% confidence level. LSTD attains the best mean result in 15 of the 32 instances. We see that LSTD outperforms the baseline policies when the duration of the conflict is short or when the enemy has weapons with a high probability of hit. It is not surprising that in circumstances where if missed the incoming TBM has a high likelihood of damaging its targeted asset that the LSPE policy outperforms the baseline policies, but it is interesting that in shorter duration conflicts when the two baseline policies show very similar means that the ADP is able to outperform at a statistically significant level.

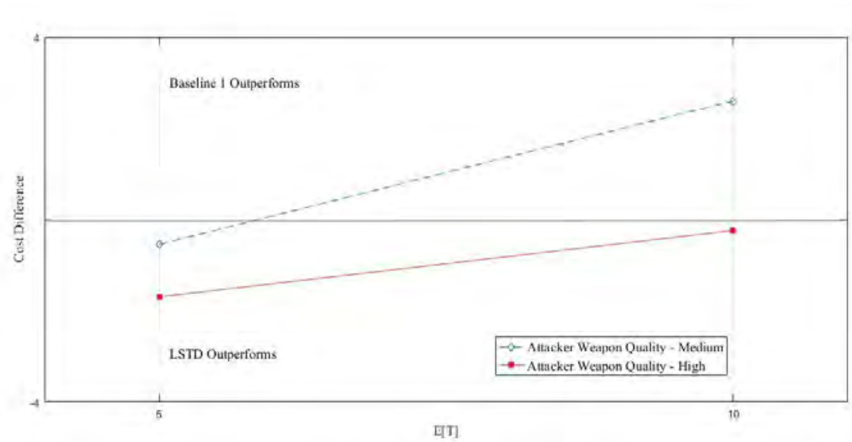


Figure 4. Cost Difference between High and Low Attacker Weapon Quality

Figure 4 highlights the cost difference between LSTD and Baseline Policy 1 when we look at the two different levels of attacker weapon quality for short and long duration conflicts. In this we observe that for high quality attacker weapons LSTD performs better than the baseline policy regardless of conflict duration, but for low quality attacker weapons Baseline Policy 1 is superior for long duration conflicts. Note that this graphic implies a linear relationship that might not exist.

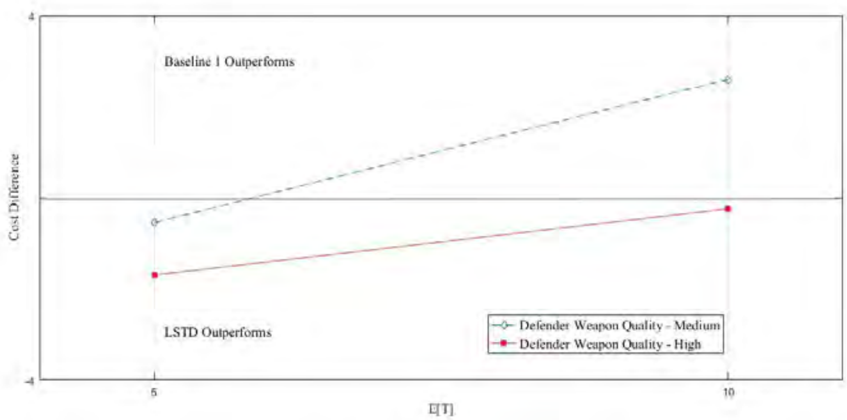


Figure 5. Cost Difference between High and Medium Defender Weapon Quality

Figure 5 highlights the cost difference between LSTD and Baseline Policy 1 when we look at the two different levels of defender weapon quality for short and long duration conflicts. In this we observe that for medium quality defender weapons LSTD performs better than the baseline policy regardless of conflict duration, but for high quality defender weapons Baseline Policy 1 is always superior. Note that this graphic implies a linear relationship that might not exist.

Table 8 shows the three-run average for the best θ-vector for each scenario. Similar to LSPE we see robustness in our best θ-vectors with the difference between the best and the mean damage value being around one point. Additionally, those instances with greater than a one point difference were not the same instances that performed significantly better than the baseline policies, suggesting that even with a different θ-vector LSTD would still perform better for those test instances.

Table 8. LSTD Results - Robustness

	Algorithm Parameters	Run 1	Run 2	Run 3	Mean	Best	Difference
1	25, 2000, 10, 10, 2	7.28	6.08	7.16	6.84	6.08	0.76
2	25, 2000, 10, 1, 2	6.56	6.92	6.15	6.55	6.15	0.40
3	25, 2000, 100, 1, 2	7.50	6.17	7.22	6.96	6.17	0.80
4	25, 2000, 100, 10, 2	6.71	6.05	7.01	6.59	6.05	0.54
5	25, 1000, 10, 10, 2	4.83	6.45	5.88	5.72	4.83	0.89
6	50, 2000, 10, 10, 2	5.99	5.33	4.55	5.29	4.55	0.74
7	50, 1000, 100, 10, 1	5.87	4.90	5.57	5.45	4.90	0.55
8	50, 2000, 100, 10, 2	4.88	5.19	5.41	5.16	4.88	0.28
9	25, 1000, 100, 10, 2	8.46	7.29	8.54	8.10	7.29	0.81
10	50, 1000, 10, 1, 2	8.34	9.50	7.16	8.33	7.16	1.17
11	50, 2000, 100, 10, 1	7.58	8.44	7.64	7.89	7.58	0.30
12	25, 2000, 10, 10, 1	8.51	7.31	8.50	8.10	7.31	0.80
13	25, 1000, 100, 10, 1	6.66	7.30	6.23	6.73	6.23	0.50
14	25, 1000, 10, 1, 1	6.71	7.61	6.30	6.88	6.30	0.57
15	25, 2000, 10, 10, 2	7.08	5.98	6.90	6.65	5.98	0.67
16	25, 1000, 10, 10, 1	6.14	7.02	7.13	6.76	6.14	0.63
17	25, 1000, 100, 1, 2	20.69	21.59	21.53	21.27	20.69	0.58
18	25, 1000, 100, 10, 2	21.02	22.69	22.35	22.02	21.02	1.00
19	25, 2000, 10, 1, 1	22.70	22.49	21.11	22.10	21.11	0.99
20	50, 2000, 10, 10, 2	20.78	21.86	22.28	21.64	20.78	0.86
21	25, 2000, 10, 1, 1	20.51	19.75	20.62	20.29	19.75	0.55
22	25, 1000, 10, 10, 1	20.58	19.40	20.73	20.24	19.40	0.83
23	25, 1000, 100, 1, 1	21.20	19.62	21.29	20.70	19.62	1.08
24	50, 1000, 100, 10, 2	20.91	19.67	20.85	20.48	19.67	0.81
25	50, 1000, 10, 10, 1	24.69	24.85	23.00	24.18	23.00	1.18
26	50, 2000, 10, 10, 2	26.31	23.53	24.24	24.69	23.53	1.17
27	25, 1000, 10, 1, 2	25.32	23.90	23.27	24.16	23.27	0.90
28	25, 1000, 100, 10, 2	23.43	23.89	23.25	23.52	23.25	0.27
29	50, 1000, 10, 1, 2	22.94	23.36	21.22	22.51	21.22	1.29
30	25, 1000, 10, 1, 2	21.01	22.24	23.05	22.10	21.01	1.09
31	25, 1000, 100, 10, 2	22.68	22.88	21.19	22.25	21.19	1.06
32	50, 1000, 100, 1, 1	21.00	24.84	22.60	22.81	21.00	1.81

Meta Analysis

Table 9. Parameter Estimates - LSTD

	Estimate	Standard Error	t Ratio	Probability $< \|t\|$
Intercept	-122.76	0.21	-584.40	< 0.0001
N (outer loops)	0.00	0.00	0.57	0.57
k (inner loops)	0.00	0.00	0.1	0.92
a (smoothing)	0.00	0.00	-0.34	0.73
η (regularization)	0.00	0.00	-0.53	0.60
ϕ (basis function set)	0.00	0.01	-0.32	0.75
Conflict Duration	157.50	0.23	699.06	< 0.0001
Attacker Weapon Quality	5.84	0.08	77.8	< 0.0001
Defender Weapon Quality	0.73	0.01	64.44	< 0.0001
Asset 1 Value	0.00	0.00	-0.27	0.78
Asset 2 Value	0.00	0.00	0.56	0.58
			R-Square Adj	0.99

Unlike the meta analysis conducted for LSPE, when we look at the parameter estimates in Table 9 for the LSTD policy performance, we find that only conflict duration, attacker weapon quality, and defender weapon quality have a significant impact on the mean damage incurred. In fact, with the LSTD algorithm, none of the other terms had values anywhere close to statistical significance. The LSTD algorithm performed nearly as well as LSPE against the two baseline policies, so this might show that the LSTD algorithm performs well, regardless of parameter settings. However, it also suggests that better parameter settings might exist that would allow LSTD to perform better. Future research should include an expanded region of experimentation with respect to the LSTD algorithmic feature space.

4.4 ADP Algorithm Comparison

When comparing LSPE to LSTD we see in Table 10 that LSPE proves superior in 19 of the 32 scenarios. LSTD appears to perform better when duration is short, attacker weapon quality is low, and defender weapon quality is high. Alternately, when duration is long, attacker weapon quality is low, and defender weapon quality is high LSTD also performs better. In most of the other problem instances LSPE is either superior or equal to LSTD.

Table 10. Algorithm Comparison

Best ADP Algorithm	
Best	# of Scenarios
LSPE	19
LSTD	11
Tie	2

4.5 Focused Analysis for Selected Instances

To explore why LSTD performed statistically better than the baseline policies in Instance 12, why LSPE performed statistically better in Instance 10, and why Baseline Policy 1 performed statistically better in Instance 24, we conducted a series of sensitivity analyses where we varied the surface to air missile (SAM) site inventories. We investigated when the SAM site was low (25% of its starting inventory) and when it was high (75% of its starting inventory). We looked at what actions the ADP algorithm took versus the baseline for different inventory combinations of the Aegis, THAAD, and Patriot systems having a low or high starting condition.

Instance 12, LSPE Focused Analysis

Instance 12 from Section 4.2 is a shorter duration conflict with high quality attacker weapons, medium quality defender weapons, a high-valued Asset 1, and a low-valued Asset 2. Table 11 shows the results of running 2000 simulations starting at the different inventory levels. Looking at when the Aegis, THAAD, and Patriot all have low inventories we see that LSPE no longer performs statistically better than the Baseline Policy 1. In fact LSPE only performs better when at least two of the

SAM sites have high inventory statuses. Recall from Table 4 that Baseline Policy 2 (firing two interceptors each) outperformed Baseline Policy 1 (firing one interceptor) in Instance 12, albeit at a non-significant level. Now in this Low/Low/Low inventory vignette we see that Baseline Policy 2 performs significantly worse than LSPE and Baseline Policy 1. In fact as we look across all the investigated inventories for Instance 12, we observe that Baseline Policy 2 performs significantly worse, even with a High/High/High inventory status. This observation suggests that only when the SAM sites are fully stocked with interceptors does Baseline Policy 2 perform well. LSPE performs better at a statistically significant level when all SAM site inventory statuses are at 75%, suggesting little sensitivity in the starting inventory for this problem instance.

Table 11. Instance 12 Policy Performance at Different SAM Inventories (Aegis/THAAD/Patriot)

Instance 12			
Low / Low / Low			
Policy	95% CI		
LSPE	30.95	±	1.35
Baseline 1	27.14	±	1.20
Baseline 2	43.44	±	1.78
High / Low / Low			
Policy	95% CI		
LSPE	21.99	±	1.27
Baseline 1	21.53	±	1.08
Baseline 2	40.82	±	1.78
Low / High / High			
Policy	95% CI		
LSPE	19.64	±	1.20
Baseline 1	21.95	±	0.97
Baseline 2	29.37	±	1.66
High / Low / High			
Policy	95% CI		
LSPE	19.14	±	1.19
Baseline 1	21.32	±	1.03
Baseline 2	34.03	±	1.73
High / High / High			
Policy	95% CI		
LSPE	14.84	±	1.08
Baseline 1	18.23	±	0.89
Baseline 2	29.74	±	1.65

Instance 10, LSTD Focused Analysis

Instance 12 from Section 4.3 is a shorter duration conflict with high quality attacker weapons, medium quality defender weapons, a high-valued Asset 1, and a high-valued Asset 2. Table 12 shows the results of running 2000 simulations starting at the different inventory levels. Unlike in Instance 10 with LSPE, LSTD only performed statistically better than Baseline Policy 1 when all SAM site inventories were at 75%. This suggests more sensitivity to starting inventory conditions. We also see Baseline Policy 1 outperform LSPE when at least two of the SAM sites inventory statuses are low. With this starting condition, Baseline Policy 2 performed better than in Instance 10 since the value of Asset 2 was high and a multiple reentry vehicle (MeRV) TBM had split into three targets for the SAM protecting Asset 2.

Table 12. Instance 10 Policy Performance at Different SAM Inventories (Aegis/THAAD/Patriot)

Instance 10			
Low / Low / Low			
Policy		95% CI	
LSTD	29.87	±	1.34
Baseline 1	27.62	±	1.18
Baseline 2	31.56	±	1.37
High / Low / Low			
Policy	Decision	95% CI	
LSTD	25.67	±	1.33
Baseline 1	23.2	±	1.12
Baseline 2	28.36	±	1.34
Low / High / High			
Policy	Decision	95% CI	
LSTD	20.39	±	1.21
Baseline 1	22.32	±	0.98
Baseline 2	20.4	±	1.21
High / Low / High			
Policy	Decision	95% CI	
LSTD	20.58	±	1.22
Baseline 1	22.29	±	1.09
Baseline 2	25.53	±	1.30
High / High / High			
Policy	Decision	95% CI	
LSTD	14.27	±	1.07
Baseline 1	17.69	±	0.90
Baseline 2	21.61	±	1.24

Instance 24, Baseline Policy 1 Focused Analysis

Instance 24 from Section 4.2 and Section 4.3 is a long duration conflict with low quality attacker weapons, high quality defender weapons, a high valued Asset 1, and a high valued Asset 2. Table 12 shows the results of running 2000 simulations starting at the different inventory levels. We observe no statistical difference between the performance of LSPE and LSTD in this vignette suggesting that these two algorithms perform the same for Instance 24. In all of the investigated initial SAM site inventories, we find that Baseline Policy 1 significantly outperforms the ADP algorithms. We find through these vignettes that even when the two terminal SAM sites have low interceptor inventories that the APD algorithms continue to fire two interceptors at each incoming TBM. This shows a lack of value placed on interceptor inventory. The basis function could be modified to a traditional inventory control problem by adding an indicator variable to find the right interceptor inventory to switch from firing two interceptors at each TBM to firing just one interceptor.

Table 13. Instance 24 Policy Performance at Different SAM Inventories (Aegis/THAAD/Patriot)

Instance 24			
High / Low / Low			
Policy		95% CI	
Baseline 1	30.31	±	1.36
LSPE	36.94	±	1.42
LSTD	36.23	±	1.43
Low / High / High			
Policy	Decision	95% CI	
Baseline 1	25.02	±	1.24
LSPE	32.63	±	1.40
LSTD	32.02	±	1.40
High / Low / High			
Policy	Decision	95% CI	
Baseline 1	24.13	±	1.19
LSPE	31.25	±	1.37
LSTD	30.21	±	1.36
High / High / Low			
Policy	Decision	95% CI	
Baseline 1	26.52	±	1.30
LSPE	33.07	±	1.41
LSTD	32.6	±	1.42

V. Conclusions, Recommendations, and Future Research

As tactical ballistic missiles (TBMs) become more readily accessible to threat nations around the world and as near-peer threats continue to develop more technologically advanced TBMs, the United States must maintain superiority with advanced air defense systems. However, many of the current systems the United States and its allies employ are decades old, and there are not significant improvement on the immediate horizon. This situation requires the United States to employ a networked defense-in-depth to best utilize the air defense systems in the current inventory. As the integrated air and missile defense (IAMD) system becomes operational, the air defense community must reconsider what the best firing strategy is for the limited interceptor inventory.

The Markov decision process (MDP) allows us to look at the dynamic weapon target assignment problem (WTAP) in an elegant manner and obtain optimal firing decisions given small instances. This allows a starting point for comparing the adequacy of other heuristics that can then be used in larger models that are of more interest to the air defense community.

One option for moving to those larger problem instances is the use of approximate dynamic programming (ADP). We utilized both the Least Squares Policy Evaluation (LSPE) and Least Squares Temporal Difference (LSTD) algorithms. We looked at the current policy of firing two interceptors at each incoming TBM and an additional policy of only firing one interceptor. We conducted 2000 runs of each of the 32 problem instances for the two baseline policies and all of the LSPE and LSTD parameter settings.

The large number of simulations gave us the best chance of finding statistical significance in an acceptable amount of time. Had we run closer to 10,000 runs, we likely would have found statistical significance for most of the problem instances.

LSPE outperformed both baseline policies in 3 of the 32 problem instances at the 95% confidence level and an additional 5 instances at the 90% confidence level. Though not at a level of statistical significance, LSPE outperformed the baseline policies in an additional 6 problem instances and was only outperformed by Baseline Policy 1 in 6 problem instances.

With little change between the 3 replications of each algorithmic parameter setting investigated, we found that the best parameter setting for each problem instance showed robustness, and we found through focused analysis that the algorithms were not sensitive to starting inventory levels, which is not desirable.

Similarly, LSTD outperformed both baseline policies in 3 of the 32 problem instances at the 95% confidence level and an additional 3 instances at the 90% confidence level. Though not to a level of statistical significance, LSTD outperformed the baseline policies in an additional 10 problem instances and was only outperformed by Baseline Policy 1 in 6 problem instances.

With little change between the 3 replications of each algorithmic parameter setting investigated, we found that the best parameter setting for each problem instance showed robustness, and we found through focused analysis that the algorithms were not sensitive to starting inventory levels, which is not desirable.

Baseline Policy 1 outperformed both ADP algorithms in 6 of the 32 instances and performed statistically the same as them in 16 of the investigated instances. We found that when conflict duration was short or when defender weapon quality was lower that Baseline Policy 1 did not perform well, but when the duration was long and the defender weapon quality was high, this policy performed as well if not better than the ADP algorithms.

As the IAMD network is employed in the field and the full host of air defense assets are integrated, the air defense community must consider a movement to either an ADP policy, Baseline Policy 1, or some mix of its current policy, Baseline Policy 2, and Baseline Policy 1. Our analysis indicates that the current policy does not outperform the ADP policies to a statistically significant level in any of the problem instances. Though it does outperform Baseline Policy 1 in short-duration, low-defender-quality weapon instances, this could be easily corrected with a static policy directing when to change from firing one interceptor to firing two interceptors based on the expected number of salvos, interceptor inventory, and the interceptors probability of kill

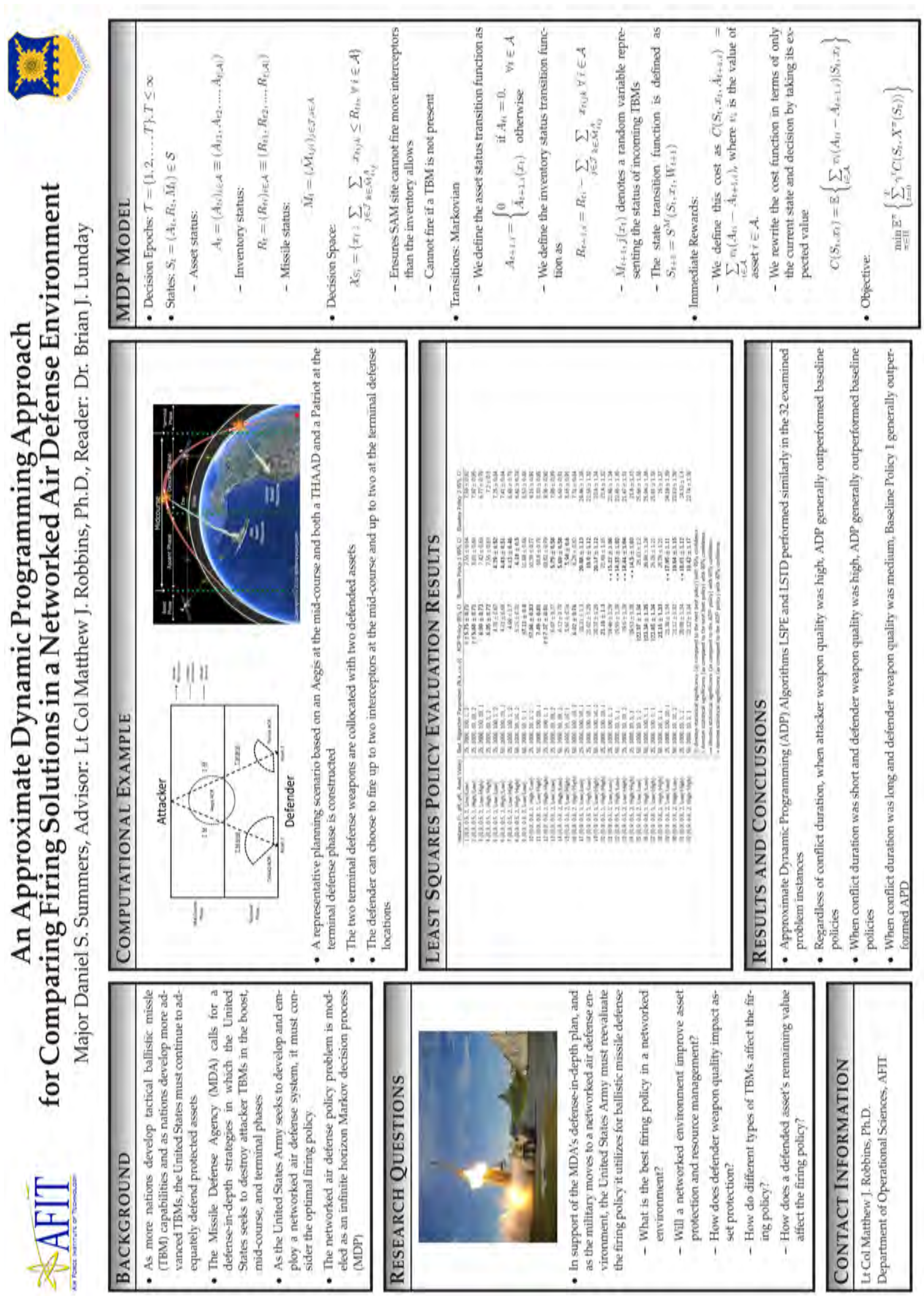
This work assumed the attacker would not know what battle damage (BDA) occurred from the TBMs they fired and would therefore continue to fire interceptors at destroyed targets or targets with lower remaining values than other available targets. This could be made more realistic if we assume the attacker would have visibility of their BDA by having the attacker fire based on the remaining asset value.

Based on the problem instances where the ADP algorithms performed poorly against Baseline Policy 1, we might consider a new basis function set that includes an indicator function that allows the firing decision to change from firing two interceptors to one based on interceptor inventory. This would allow the ADP policy to continue to outperform Baseline Policy 1 in the instances it already does, but also perform at least as well in the instances where Baseline Policy 1 currently outperforms.

This work assumed that the traditional TBM and the TBM with multiple reentry vehicles (MeRV) caused the same amount of damage. It is unlikely that the smaller MeRV warheads would cause as much damage as a traditional warhead. Therefore, another change to enhance the realism would be to investigate how different damage levels from the warhead impacts the decisions. It is likely that with a lower damage level for the MeRV, the ADP policy might ignore MeRVs in the terminal phase when interceptor inventory levels are low.

Due to computational constraints for this work, we only allowed the opportunity to fire one wave of interceptors at the mid-course and one wave at the terminal phase. To truly investigate the firing solution of shoot-shoot-look or shoot-look-shoot problem, we might allow two waves of interceptors at the mid-course and two at the terminal phase. This works off the assumption that there is time during those phases to fire one set of interceptors at incoming TBMs, assess which were destroyed, and then if any TBMs remain, fire another set of interceptors. This, however, increases the size of the state space substantially since instead of the 12 current points in space where a TBM could exist it would be 24.

Appendix A. Quad Chart



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Vita

Major Daniel S. Summers enlisted in the Army as an Intelligence Analyst in 1999. He earned his Bachelor of Science degree from the United States Military Academy at West Point in 2005. Dan was commissioned into the United States Army as a Chemical Officer in 2005.

Major Summers' served in Korea as an Intelligence Analyst and in the 82nd Airborne Division as a Squadron Chemical Officer. While in the 82nd he deployed to Iraq in support of the surge efforts. After this deployment and completion of the Chemical Officer Captain's Career Course, Dan served in the 171st Infantry Brigade as the Assistant Operation Officer and then commanded the Fitness Training Company within that brigade. Upon completion of this command, Dan commanded the Dearborn Army Recruiting Company in Michigan. He then transitioned to Operations Research (FA49) and served at the TRADOC Analysis Center at Fort Leavenworth, KS.

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14. ABSTRACT

The United States Army currently employs a shoot-shoot-look firing policy for air defense. As the Army moves to a networked defense-in-depth strategy, this policy will not provide optimal results for managing interceptor inventories in a conflict to minimize the damage to defended assets. The objective for air and missile defense is to identify the firing policy for interceptor allocation that minimizes expected total cost of damage to defended assets. This dynamic weapon target assignment problem is formulated first as a Markov decision process (MDP) and then approximate dynamic programming (ADP) is used to solve problem instances based on a representative scenario. Least squares policy evaluation (LSPE) and least squares temporal difference (LSTD) algorithms are employed to determine the best approximate policies possible. An experimental design is conducted to investigate problem features such as conflict duration, attacker and defender weapon sophistication, and defended asset values. The LSPE and LSTD algorithm results are compared to two benchmark policies (e.g., firing one or two interceptors at each incoming tactical ballistic missile (TBM)). Results indicate that ADP policies outperform baseline polices when conflict duration is short and attacker weapons are sophisticated. Results also indicate that firing one interceptor at each TBM (regardless of inventory status) outperforms the tested ADP policies when conflict duration is long and attacker weapons are less sophisticated.

15. SUBJECT TERMS

Air and missile defense, dynamic weapon target assignment problem, Markov decision processes, approximate dynamic programming, approximate policy iteration, least squared policy evaluation, least squares temporal difference

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