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Accuracy of Time Phasing Aircraft Development using the Continuous Distribution Function

Gregory E. Brown

ACCURACY OF TIME PHASING AIRCRAFT DEVELOPMENT USING THE CONTINUOUS DISTRIBUTION FUNCTION

THESIS
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Abstract

Early research on time phasing primarily focuses on the theoretical foundation for applying the continuous distribution function, or S-curve, to model the distribution of development expenditures. Minimal methodology is provided for estimating the S-curve's parameter values. Brown, White, and Gallagher (2002) resolve this shortcoming through regression analysis, but their methodology has not been widely adopted by aircraft cost analysts, as it is judged as overly broad and not specific to aircraft. Instead, analysts commonly apply the 60/40 "rule of thumb" to aircraft development, assuming 60 percent expenditures at 50 percent schedule. It is currently unknown if the 60/40 heuristic accurately describes contemporary aircraft development programs. Therefore, using a sample of 26 DoD aircraft programs, we first test the accuracy of 60/40, discovering that, as a heuristic, the 60/40 cannot account for differences between new start and upgrade programs. Next, we improve upon prior research by using program characteristics to construct an aircraft-specific methodology for estimating parameters. Finally, we conclude our research by comparing the accuracy of our Rayleigh, Weibull, and Beta S-curve models. Our Weibull model explains 74.6 percent of total variation in annual budget, improving the estimation of budgets by 6.5 percent, on average, over the baseline 60/40 model.

Acknowledgments

I would like to warmly thank my research advisor, Dr. Edward White, for his guidance and support during my thesis work. His profound knowledge of model building and regression resulted in countless improvements to this research. Additionally, I wish to express the deepest gratitude towards Lt Col Jonathan Ritschel and Mike Seibel; their combined experience proved crucial in ensuring this research is applicable to the cost analysis practitioner, and not just an inane contribution to theory.

Gregory E. Brown

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ACCURACY OF TIME PHASING AIRCRAFT DEVELOPMENT USING THE CONTINUOUS DISTRIBUTION

I. Introduction

Once a point estimate is established for a Department of Defense (DoD) research, development, test and evaluation (RDT&E) program, the program manager must "time phase", or spread, funding across the program's expected fiscal years. One method presented by current DoD cost analysis literature is the application of the continuous probability distribution, commonly referred to as an S-curve (AFCAH, 2007: 15-25). A comprehensive literature review reveals that the single-parameter Rayleigh distribution (a special case of the Weibull distribution) is the most frequently studied and applied S-curve model for time phasing development efforts (Norden, 1970; Putnam, 1978; Watkins, 1982; Abernathy, 1984; Lee et al., 1997). Recently published research shows that increased precision may be achieved through the application of the two-parameter Weibull and Beta distributions (Brown et al., 2002; Burgess, 2006). However, no aircraft development-specific means currently exists to estimate the parameters for the Rayleigh, Weibull, and Beta distributions; as a result, many cost analysts instead favor the application of a 60 percent expenditures at 50 percent schedule "rule of thumb" (Lee et al., 1997; NASA, 2002).

Therefore, this research has three goals. First, this research will attempt to identify the source of the 60 percent expenditures by 50 percent schedule heuristic, and test its applicability against a database of contemporary aircraft development programs. Second, this research will develop a standard methodology for estimating parameters for the Rayleigh, Weibull, and Beta distributions. Finally, using the developed methodology for estimating parameters, this research will compare the predictive ability of the Rayleigh, Weibull, and Beta distribution models, with the intent of defining a best-fit, robust model to be utilized for future time phased estimates of aircraft development expenditures and budgets.

Problem Statement

One common method for allocating dollars over fiscal years is the application of the continuous distribution, often referred to as an S-curve within DoD cost estimating literature. In applying the S-curve, the Air Force Cost Analysis Handbook (AFCAH) cautions that:

> No single S-curve…describes the funding profile of all development programs; rather, estimators adjust the general S-shape of the curve to model a program's particular expenditure pattern of more/less effort earlier/later in the development program (AFCAH, 2007: 15-26).

However, the problem arises that no standard baseline exists from which a program manager may begin analysis. The closest to a commonly accepted baseline is the 60/40 S-curve, which is cited as a "rule of thumb" by the NASA Cost Estimating Guide (2002: 168) and Lee et al. (1997), and used as an example curve in both the DoD Basic Cost Estimating (BCE) (2005) and AFCAH (2007) literature. Unfortunately, the origin of the 60/40 S-curve remains unpublished and therefore unclear to many cost analysts, making it difficult to justify to decision makers. To further convolute the rule of thumb, Air Force guidance reports that the ratio 60/40, which describes the S-curve's skew, may be interpreted to have two different definitions (AFCAH, 2007). Under the first, less commonly used definition, 60/40 would describe a program with 60 percent expenditures at 40 percent schedule. Under the second, more commonly accepted definition, 60/40 would describe a program with 60 percent expenditures at 50 percent schedule. For these reasons, the cost analyst may be apprehensive to accept the 60/40 S-curve as a standard baseline for aircraft development. And if the 60/40 S-curve is accepted as a baseline, no published methodology exists for further adjusting the distribution parameters based on program characteristics. For example, it is currently unknown whether the distribution of expenditures for upgrade programs differs from the distribution for new starts. Therefore, it becomes evident that a requirement exists for an aircraft-specific S-curve model capable of being adjusted by the development program's characteristics. This requirement leads us to develop our research approach, which is presented next.

Research Approach

The real world applicability of past S-curve research has been limited, as many researchers attempted a "one size fits all" approach of defining an optimal Rayleigh distribution for fitting all development programs. Limited consideration was given to how a program's characteristics drive the parameters of the selected Rayleigh or Weibull distribution. Brown, White, and Gallagher (2002) resolve this limitation, utilizing a program's length of development, program type, and branch of service to estimate Weibull parameter values. Using their Weibull technique, Brown et al. report an average correlation of 0.607 between the actual budget distribution and the Weibull-predicted budget distribution.

As a follow-on effort to Brown et al.'s research, we will pursue a similar research approach. As summarized in Figure 1, this approach proposes that development program characteristics may be used to estimate distribution parameters, which in turn predict the distribution of expenditures. The distribution of expenditures is later converted to a predicted budget profile before measuring goodness-of-fit (R^2). However, our research is uniquely different from Brown et al., as we will consider aircraft-specific development characteristics, such as time to first flight and aircraft type, within the model. We postulate that the inclusion of these aircraft-specific characteristics will lead to greater predictive ability, in turn leading to more appropriate time-phased projections of future aircraft development budgets.

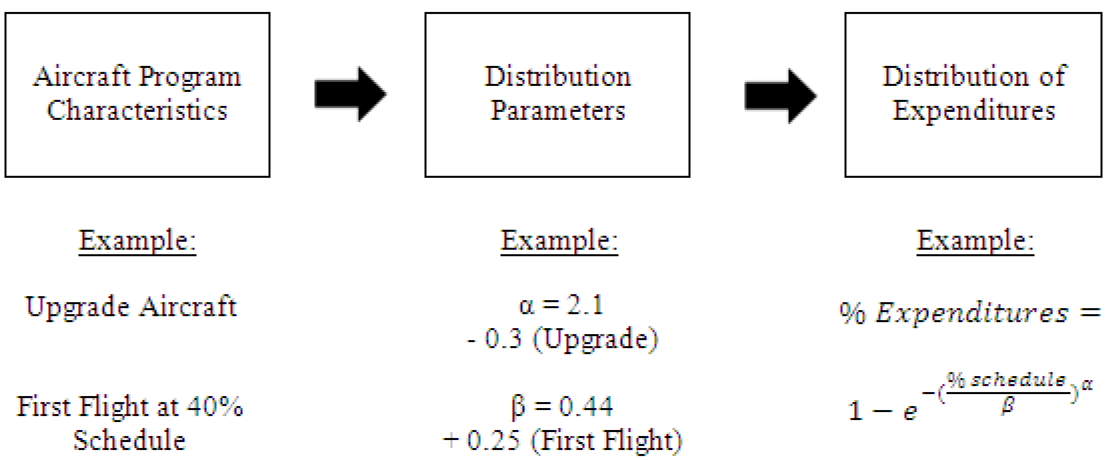


Figure 1: Research Approach for Aircraft Development S-curve Model

Research Implications

Research by Belcher and Dukovich (1999) documents that funding provided in the wrong fiscal years of a development program result in productivity inefficiencies, schedule slips and increased program costs. Unger (2001) confirms these findings, and coins the term "inappropriate funding" to describe programs which initially have adequate total funding, but receive the funding in the wrong fiscal years. Examples of inappropriate funding are given by AFCAH (2007), which warns some analysts resist "putting adequate monies in the front end of a new program to fund technical personnel and analysts when the program most needs them. A program can inadvertently weaken itself by stretching this requirement out over a number of fiscal years." (AFCAH, 2007: 15-9). Similarly, Better Buying Power 1.0 (2010) proposes that underfunding the peak years of a development program results in a "leisurely acquisition timeline" and schedule growth. Inappropriate funding leads to schedule growth, which consequently leads to cost growth.

> As all programs compete for funding, the usual result is that a program settles into a level-of-effort pattern of annual funding that does not deviate much from year to year…thus a one-year extension of a program set to complete in 10 years…result[s] in 10 percent growth in cost as the team working on the project is kept on another year (BBP 1.0, 2010: 4).

Therefore, it is implied by Belcher and Dukovich (1999), Unger (2001), AFCAH (2007) and BBP 1.0 (2010) that an adequate cost estimate may be ruined by inappropriate time phasing and funding shortfalls, which in turn create schedule and cost growth. To finish on-time and on-budget, a program requires both adequate funding and appropriate time phasing. As a result, the objective of this research is to minimize schedule and cost growth by allowing the cost analyst to more appropriately time phase his or her aircraft development program. To meet this objective, we will next present the investigative questions that form the basis of our research.

Research Questions

The objective of this research is best summarized by the following questions:

1 – Is the "rule of thumb" that 60 percent of expenditures occur by 50 percent schedule (60/40 S-curve) accurate for contemporary aircraft development programs?

2 – What program and/or schedule characteristics best predict distribution parameters?

3 – Which distribution (Rayleigh, Weibull, or Beta) provides the best S-curve model for time phasing contemporary aircraft development programs?

Summary

Current acquisition policy emphasizes the importance of developing an accurate point estimate, while undervaluing the importance of applying an appropriate methodology to the time phasing of the estimate. Research has already established that funding provided in the wrong fiscal year of the program may result in both cost and schedule inefficiencies (Belcher et al., 1999) (Unger, 2001). Therefore, we propose the development of an S-curve model capable of more accurately predicting aircraft development expenditures.

In the next chapter, Chapter 2, the common methods for time phasing, historical S-curve research, origins of the 60/40 rule of thumb, and attributes of each distribution-type are discussed. Chapter 3 reviews our data collection procedures and proposed methodology for the estimation of a best-fit S-curve model. Chapter 4 presents the results of our applied methodology; in particular, the R^2 and robustness of the model is reviewed. Finally, Chapter 5 summarizes and discusses our results, outlines research limitations, and suggests topics for future aircraft time phasing research.

II. Literature Review

Chapter Overview

This chapter is an overview of topics related to time phasing and the application of the S-curve. First, the purpose and three common methods for time phasing are provided. Next, a literary review of the evolution of the S-curve time phasing method is presented, in chronological order. Subsequently, a literature review identifying the origin of the 60/40 "rule of thumb" is offered. Finally, this chapter concludes with a brief review of the characteristics of the Rayleigh, Weibull, and Beta distributions. For the cost analysis practitioner unfamiliar with these distributions, the effect of increasing or decreasing the distribution parameters is shown.

Methods of Time Phasing

Contemporary cost analysis literature offers three methods for time phasing: schedule/milestone, analogy, and the S-curve (AFCAH, 2007). The choice of one method over another method is often influenced by the availability of schedule information on the program to be estimated, as well as the availability of historical data on analogous programs.

The schedule/milestone method is considered the most exact means of phasing, but it is also the most complex. When completed correctly, the schedule/milestone method is easily explained to decision makers, and is highly defendable during outside analyses. However, the accurate application of the schedule/milestone method requires a master program schedule and work breakdown structure (WBS) that may be unattainable for the initial time phased estimate of a RDT&E program (AFCAH, 2007: 15-15). When a master schedule is not available, the cost analyst must either estimate the schedule or rely on an alternate time phasing method (BCE, 2005: 16-6).

By comparison, the analogy method is the simplest method as it utilizes the time phased expenditures of a single, analogous development program as the basis for spreading funding. However, AFCAH instructions warn “the process of finding a truly analogous program…may be difficult and time consuming” (AFCAH, 2007: 15-21). Additionally, the analogy method’s reliance on a single program may introduce additional uncertainty and risk which could be mitigated through the use of the S-curve, which accrues data from multiple programs.

Therefore, the use of the S-curve, or cumulative distribution function (CDF), is often the preferred method for RDT&E programs initiated with considerable schedule uncertainty. When contrasted with the analogy method, the S-curve is theoretically superior, as it may utilize historical data from multiple analogous programs to form an estimated distribution of expenditures. The resulting expenditure distribution may then be adjusted based on unique program characteristics expected to shift effort earlier or later into the program’s development (AFCAH, 2007: 15-25). As shown in Figure 2, the S-curve is given its name from the shape formed when cumulative program expenditures are plotted over time.

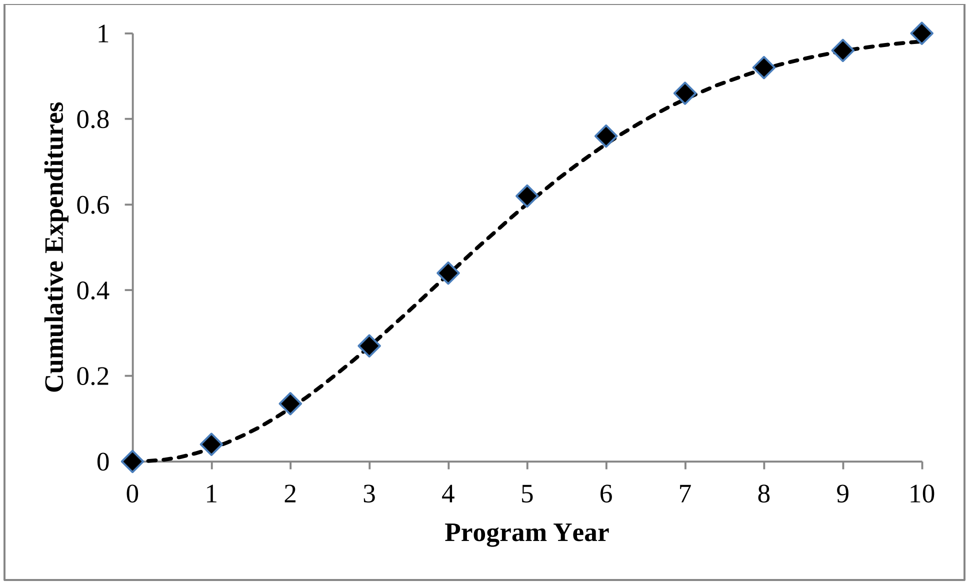


Figure 2: S-Curve of Cumulative Costs over Time

Evolution of the S-curve in Cost Analysis Literature

No single continuous distribution type describes the S-curve. As shown in Table 1, a review of literature reveals that a variety of continuous distributions may be used to model the relationship between a program's cost and time. Beginning with Norden's publication of "Useful Tools for Project Management" in 1970, the single parameter Rayleigh distribution is the most frequently applied and researched application of the S-curve. Conversely, Weida (1977) offers the solution of applying two quadratic equations. Brown et al. and Unger et al. subsequently improve upon previous research, showing that the two parameter Weibull distribution offers a more robust fit for RDT&E programs (Brown et al., 2002; Unger et al., 2004). Finally, Burgess (2006) simultaneously tests the Rayleigh, Weibull, and Beta distributions; his results show that the Weibull distribution outperforms both the Rayleigh and Beta distributions in fitting expenditure patterns for

26 DoD RDT&E space contracts (Burgess, 2006: 24-25). For the remainder of this subchapter, we will review the findings of these publications in greater detail.

Table 1: Notable S-curve Research

Year	Author	Distribution Applied
1970	Norden	Rayleigh
1977	Weida	Quadratic Equation
1978	Putnam	Rayleigh
1982	Watkins	Rayleigh
1984	Abernethy	Rayleigh
1997	Lee et al.	Rayleigh
2001	Unger	Rayleigh and Weibull
2002	Brown et al.	Rayleigh and Weibull
2006	Burgess	Rayleigh, Weibull, and Beta

Applications of the Rayleigh Distribution (Special Case of the Weibull)

Norden (1970) first applies the continuous distribution to development expenditures by utilizing the Rayleigh distribution, which is a special case of the Weibull Distribution, to relate monthly manpower usage and elapsed project time. Norden observes that "there are regular patterns of manpower buildup and phase-out in complex project. The cycles…seem to be a function of the way groups of engineers and scientists tackle complex technological development problems" (Norden,1970: 80). By using the assumption that the rate of problem solving increases as a linear function of time, a relationship may be derived between manpower and project time. Nordon represents this relationship using the Rayleigh cumulative distribution function

$$y = K(1 - e^{-at^2}) \qquad (1)$$

where y = cumulative manpower utilized at time t, K = total cumulative manpower utilized by end of the project, a = Rayleigh scale parameter, and t = elapsed time since the start of the project. Norden states that on average, projects of high importance will reach peak manpower sooner, while projects of lower importance will take longer to reach peak effort. Therefore, highly important projects are numerically represented by a larger scale parameter a; projects of lower importance are represented with a smaller scale parameter a.

Putnam (1978) builds on Norden's approach of applying the Rayleigh distribution to RDT&E projects. Putnam relates Norden's Rayleigh model to software development by analyzing the recorded manpower-hours of 50 U.S. Army Computer Systems Command contracts. Putnam calculates that the peak (inflection point) of manpower effort occurs at 39.45 percent of software's development lifecycle (Putnam, 1978: 349). These findings validate Norden's theory that, on average, manpower requirements for development projects have a shorter rise and longer exponential tail. However, Putnam also notes that a small subset of contracts within his dataset do not follow the expected Rayleigh distribution, and instead exhibit a stepwise increase to peak effort, followed by steady effort requirements until completion. Putnam postulates that projects which do not conform to the Rayleigh distribution are the result of inefficient management acting contrary to system requirements. Putnam summarizes his assessment by saying that "usually management adapts to the system signals, but generally responds late because the signal is not clear instantaneous with the need" (Putnam, 1978: 348). In such a case, the project is overfunded or overstaffed during the later years or months of the development program, resulting in management continuing to support peak manpower levels after the manpower is no longer required.

Following Putnam, cost researchers Watkins and Abernethy further confirm the viability of the Rayleigh distribution for time phasing. Watkins (1982) uses the Rayleigh distribution to forecast cost growth in contracts. By applying the shape of the Rayleigh distribution against the difference between actual cost of work performed, budgeted cost of work performed, and budget cost of work scheduled, Watkins hypothesizes that total cost growth (as defined by budget at completion) may be estimated early within a development contract. Abernethy (1984) models the Rayleigh distribution against cost data from 21 Navy contracts, to include Navy aircraft and missiles. Although Abernethy successfully fits the Rayleigh distribution to individual contracts, he cannot produce a Rayleigh parameter that fits all contract data to within five percent of its final values. Abernethy theorizes that an increased sample size will result in additional precision for future studies.

Finally, cost researchers Lee et al. (1997) offer two theory-based contributions to the Rayleigh S-curve. First, Lee et al. write that it has been previously observed that the peak expenditure rate (t_p) for "aircraft development programs often comes at, or slightly before, the time of first flight" (Lee et al, 1997:32). Therefore, the Rayleigh's scale parameter a may be calculated by setting the derivative of the Rayleigh's probability distribution function equal to 0 at the time of first flight. When simplified, this equates to $a = 0.5{t_p}^{-2}$ for the Rayleigh equation given in (1). Second, Lee et al. offer a solution for those applying the 60/40 S-curve. By setting $a = 3.5$, the Rayleigh distribution will always produce a S-curve for which 60 percent expenditures occur at 50 percent schedule, when truncated to control for the Rayleigh's infinite tail. Lee et al. truncate their S-curve model by assuming that schedule completion occurs at 97 percent of expenditures. It should be noted that we will again reference Lee et al.'s 60/40 Rayleigh S-curve in Chapter 4 of this thesis, as the model will serve as our standard "baseline" model for comparison.

Application of the Quadratic Function

Rather than rely on an established distribution, Weida (1977) offers the approach of using quadratic functions to estimate the S-curve. Weida's method estimates two independent quadratic equations; one quadratic equation represents all expenditures before the inflection point, with the other equation representing expenditures following inflection. After aggregating the cost performance reports from 17 DoD weapon systems, Weida finds that the mean inflection point for aircraft development contracts occurs at 56.2 percent of expenditures and 45.2 percent of time (Weida 1977: 8).

Applications of the Weibull & Beta Distributions

Unger (2001) first recommends that the Weibull distribution is a better predictor of RDT&E expenditure profiles than the Rayleigh distribution. Unger tests the ability of both the Rayleigh and Weibull to predict variation and cost and schedule growth, finding that the Weibull outperforms the Rayleigh when fit to individual programs. However, in his findings, Unger annotates a significant limitation of his model: no method currently exists to estimate the Rayleigh and Weibull parameters for future programs.

Brown, White, and Gallagher (2002) resolve Unger's stated limitation, using multi-stage regression techniques to estimate Rayleigh and Weibull parameters. Brown et al. define their Weibull cumulative distribution function as

$$F(t) = 1 - e^{-(\frac{t-\gamma}{\delta})^{\beta}} \qquad (2)$$

where t = time, γ = location parameter, δ = scale parameter, and β = shape parameter. Using 128 completed DoD RDT&E programs, Brown et al. estimate that the Weibull scale parameter δ is a linear function of program duration, defined as $\delta = 0.726(Duration)$. Similarly, the shape parameter β is a function of duration, branch of service, and program type, and defined as $= 1.299 + 0.972\ln(Duration) - 0.461(NonAF) - 0.543(Electronic) - 1.099\,(Space)$. Brown et al. report their scale parameter model coefficient of determination as 0.922; the shape parameter model coefficient of determination is 0.309.

In addition to duration, branch of service, and program type, Brown et al. test the predictive ability of total program cost, but find that it does not have a statistically significant influence on the Weibull's scale and shape parameters. Furthermore, they find that the Weibull parameter estimates for missiles, munitions, ships, and vehicles are statistically equivalent to aircraft. When contrasting their Weibull model with their Rayleigh model, Brown et al. determine the Weibull distribution improves budget profile projection by 60 percent, on average. Brown et al. conclude their research by observing that because the Rayleigh distribution is limited to one scale parameter, the Rayleigh distribution often overestimates expenditures during the early phase of programs and underestimates during the later phases (Brown et al., 2002: 51).

Following Brown et al., Burgess (2006) further advances S-curve estimation techniques by simultaneously testing the Rayleigh, Weibull, and Beta distributions. Using contract expenditure data from 26 National Reconnaissance Office and DoD space satellite systems, Burgess concludes that although the Beta distribution is more accurate than the Rayleigh, it is less accurate than the Weibull. Burgess reports that it is not surprising that the Weibull and Beta distribution show greater accuracy than the Rayleigh, "because the Weibull and Beta have more parameters and are therefore better able to accommodate variations among spending profiles" (Burgess, 2006: 17). However, Burgess is not satisfied with the ability of the Weibull distribution to fit the aggregated data set used within his study.

In particular, Burgess observes that the Weibull model fails to capture late program spending. This shortfall in predictive capability is the result of a theoretical assumption of the distribution; when applying a continuous distribution, we must assume that the rate of work (and thus rate of spending) is completely variable and always begins and ends at zero. This assumption is not consistent with reality, where a fixed cost exists for each year of a space program. Therefore, Burgess recommends the inclusion of a constant rate term in the Weibull equation, based on schedule length, to better account for the fixed costs and overhead present within satellite RDT&E programs (Burgess 2006: 25). Burgess publishes his modified Weibull equation as

$$E(t) = d[Rt + 1 - e^{-\alpha t^{\beta}}] \tag{3}$$

where $d = \frac{total\ cost}{R+1-e^{-\alpha}}$, $\alpha = -0.414 + 0.729(units) + 0.0488(months\ duration) + 0.145(\%\ nonrecurring)$, $\beta = 1.71$, and $R = 0.00148(months\ duration)$.

Origin and Applicability of the 60/40 "Rule of Thumb"

Introduced in Chapter 1, the 60/40 "rule of thumb" is a commonly cited heuristic assumption used to approximate the continuous distribution. It is still in use by cost analysts to time phase aviation programs, as confirmed by AFLCMC-FZC staff. Notably, the use of the 60/40 S-curve is not specific to aircraft cost analysis, as a contemporary review of cost analysis literature reveals that the NASA Cost Estimating Guide (2002) references the 60/40 S-curve as a “rule of thumb” for spreading space expenditures (but does not provide an origin or reference). Additionally, the DoD’s BCE (2005) and Air Force’s AFCAH (2007) repeatedly use the 60/40 S-curve as an example within time phasing chapters; however, both documents fail to provide a source. Therefore, we search for the origin of the 60/40 S-curve to better judge its applicability for time phasing contemporary aircraft development.

We discover that Lee et al. (1997) trace the most likely origin of the rule of thumb to a circa-1980 aircraft study completed by the OSD Cost Analysis Improvement Group (CAIG). The CAIG study remains undocumented, but the results are recalled by Mr. Gary Christle, former Deputy Director of Acquisition Management at OSD (Lee et al., 1997:33). Since the CAIG study remains unpublished and is no longer recoverable, it is not possible to determine the applicability of the study’s methodology and data source. Therefore, we instead evaluate the accuracy of the 60/40 rule of thumb by providing a historical synopsis of findings from all available aircraft development time phasing studies:

1 **–** Weida (1977) reviews seventeen aircraft development contracts and determines that contracts obligate 57 percent of expenditures by 50 percent schedule, on average. Additionally, Weida provides a table of “inflection points”, which identify the time of peak expenditures within a development program. Weida’s inflection point findings are reproduced in Appendix 1.

2 **–** In 1979, a study completed by the General Research Corporation finds that aircraft contracts typically obligate 57 percent of expenditures by 50 percent schedule. An extensive literary search could not locate the original General Research Corporation study; instead, we rely on a summation of the General Research Corporation study provided by Dibbly (1988: 14).

3 **–** Dibbly (1988) reviews 22 aircraft avionics development contracts and discovers that contract type, contract value and engineering complexity have a statistically significant effect on the S-curve distribution. Based on the combination of these characteristics, aircraft avionics development contracts will fall into one of three categories. As summarized in Table 2, contracts will typically obligate either 57 percent expenditure at 50 percent schedule, 68 percent expenditures at 50 percent schedule, or 78 percent expenditures at 50 percent schedule.

Table 2: Dibbly (1988) Avionics Funding Profile Study

Contract Type	Contract Value	Engineering Complexity	S-curve
Cost Plus	Small	High	57/43
Cost Plus	Large	High	
Fixed Price	Large	High	
Cost Plus	Large	Low	
Fixed Price	Small	High	68/32
Fixed Price	Large	Low	
Cost Plus	Small	Low	
Fixed Price	Small	Low	78/22

4 **–** Brown et al. (2002) include 19 aircraft development programs as part of their larger, multi-platform study of the Weibull-based S-curve. Although Brown et al. do not

provide a summary of percent expenditures by 50 percent schedule, we may approximate this statistic using the estimated Weibull parameters provided within Brown's thesis appendix (Brown, 2001: 65-69). Using this data, we estimate that 57.1 percent of expenditures occur at 50 percent schedule, on average.

Characteristics of Probability Distributions

As denoted earlier in this chapter, the S-curve is given its name from the shape formed when a cumulative density function (CDF) is fitted to the plot of cumulative effort versus development time. The AFCAH (2006, 15-25) reports that the normal distribution is typically used. However, a significant limitation of the normal distribution is the assumption that values are symmetric about the mean, or schedule midpoint. Therefore, when applied to development efforts, the normal distribution could only model programs which are expected to expend equal funding during the earlier and later half of the development schedule, as shown in Figure 3. The normal distribution's assumption of symmetry is incongruent with published S-curve research, which find that, on average, aircraft development programs expend a greater percentage of funds during the first 50 percent of schedule (Weida, 1977; Dibbly, 1988; Lee et al., 1997; Brown et al., 2002).

Therefore, to provide a better fit for the positively skewed expenditure pattern, multi-parameter distributions with one or more flexible shape parameters are considered. Based on precedence established by Brown et al. (2002) and Burgess (2006), the Weibull distribution (for which the Rayleigh distribution is a special case) and the Beta distribution are selected for testing. A summary of the Weibull and Beta distribution characteristics follow. To allow for easier comprehension and application of these distributions by the cost analyst practitioner, we will utilize Microsoft Excel's naming convention for distribution parameters for the remainder of this thesis, rather than apply conventions more commonly utilized within mathematical texts.

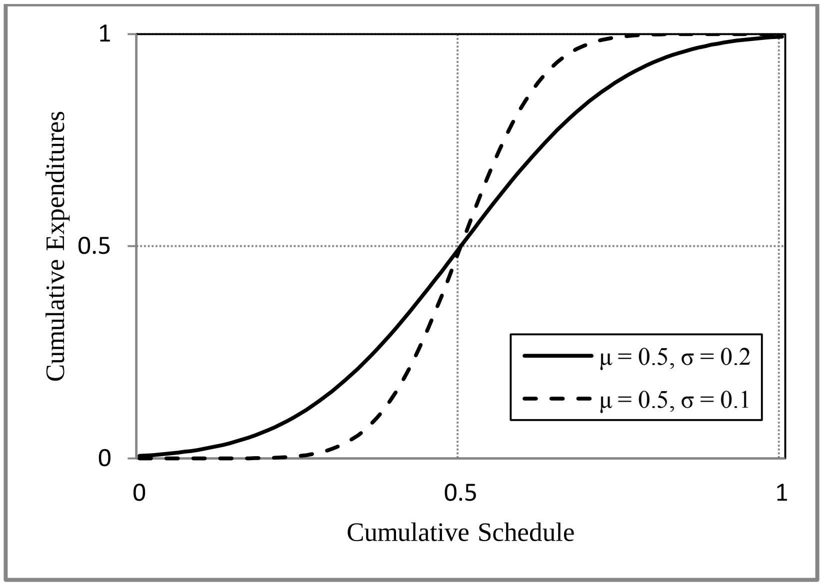


Figure 3: The Normal Distribution

Characteristics of the Weibull Distribution

The two-parameter Weibull distribution is named for the Swedish scientist Waloddi Weibull, who in 1951 used the distribution to model the breaking strength of materials (Johnson et al., 1994: 628). Mathematically, the Weibull CDF consists of a shape parameter, α, and scale parameter, β and is written as

$$\%\,Expenditures = 1 - e^{-(\frac{\%\,schedule}{\beta})^{\alpha}} \tag{4}$$

Within Excel, this same function is represented as

$$= Weibull(\%\,schedule, \alpha, \beta, true) \tag{5}$$

The effect of modifying the Weibull distribution's shape parameter, α, is shown in Figure 4. When α = 3.7 and β = 0.6, the distribution is approximately symmetric, or equivalent to the normal distribution, and expenditures are equivalent for the first half and last half of development. As we decrease the value of the shape parameter α below 3.7, while holding the scale parameter β constant, program spending increases during the first half of development and the distribution becomes positively skewed.

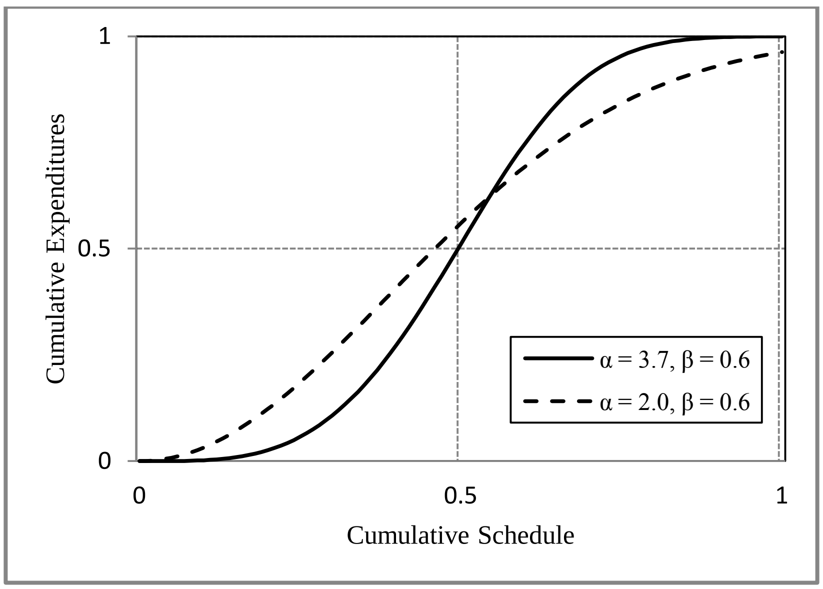


Figure 4: Effects of changing Weibull shape parameter α

Conversely, the effect of modifying the Weibull distribution's scale parameter, β, which controls statistical dispersion, is shown in Figure 5. Again we start with a Weibull distribution where α = 3.7 and β = 0.6, which is roughly equivalent to the normal distribution. As scale parameter β is decreased, the distribution becomes more highly peaked and spending varies more significantly from period to period. Therefore, a decrease in the value of β, while holding α constant, will result a less uniform, more "pointed" distribution of expenditures.

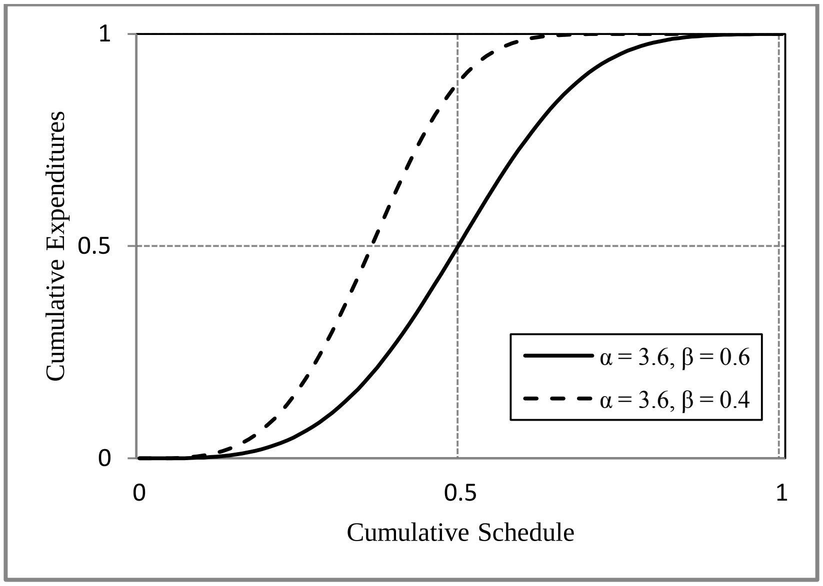


Figure 5: Effects of changing Weibull scale parameter β

Characteristics of the Rayleigh Distribution

The one-parameter Rayleigh is a special case of the Weibull distribution which assumes the shape parameter, α, is fixed at a value of 2, and only the scale parameter β is allowed to vary. As documented in Chapter 1, the Rayleigh forms the basis of early S-curve research (Norden, 1970; Putnam, 1978; Watkins, 1982; Abernathy, 1984). The Rayleigh's applicability to early research may be attributed to the characteristic that Rayleigh-modeled peak expenditures occur at 38 percent of program schedule, when program schedule is truncated at 97 percent of expenditures (to control for an infinite tail) (Lee et al., 1997). This characteristic is notable, as it results in a distribution in which approximately 60 percent of expenditures occur by 50 percent schedule, consistent with the 60/40 rule of thumb introduced earlier (Lee et al., 1997) (Burgess, 2006). Therefore, in all cases the Rayleigh distribution will be positively skewed, meaning that peak expenditures will occur during the first half of development.

Characteristics of the Beta Distribution

Compared to the Weibull distribution, which consists of only one higher-order shape parameter and one scale parameter, the Beta distribution contains two higher-order shape parameters, α and β. Therefore, the Beta is more versatile, and the distribution's "flexibility encourages its empirical use in a wide range of applications" (Johnson et al., 1995: 235). In CDF form, the Beta equation is written as

$$\%\,Expenditures = \frac{(1-\%\,schedule)^{\beta-1}(\%\,schedule)^{\alpha-1}\Gamma\,(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \qquad (6)$$

where Γ represents a factorial function for which $\Gamma(n) = (n-1)!$ Within Excel, this equation is represented by the command

$$= Betadist(\%\,schedule, \alpha, \beta) \qquad (7)$$

Although the relationship between the α and β parameters is mathematically very complex, two broad observations may be made relating to the Beta distribution's skew and kurtosis.

First, it is shown in Figure 6 that the skewness of the Beta distribution is influenced by the ratio between shape parameters α and β. When α = β, the distribution is symmetric and closely resembles a normal distribution, and expenditures are equivalent for the first half and last half of development. When α > β, the distribution is negatively skewed, resulting in peak spending during the last half of development. When α < β, the distribution is positively skewed, resulting in peak spending during the first half of development. Therefore, it is expected that α < β for most development programs.

Second, it is shown in Figure 7 that as the α and β parameters increase in value from zero, dispersion decreases (and kurtosis, or peakedness, therefore increases). When α = 1 and β = 1, the distribution is uniformly distributed, which assumes that expenditures are constant throughout development. By comparison, when α and β both increase in value to 3, dispersion decreases and the "S-curve shape" becomes visible.

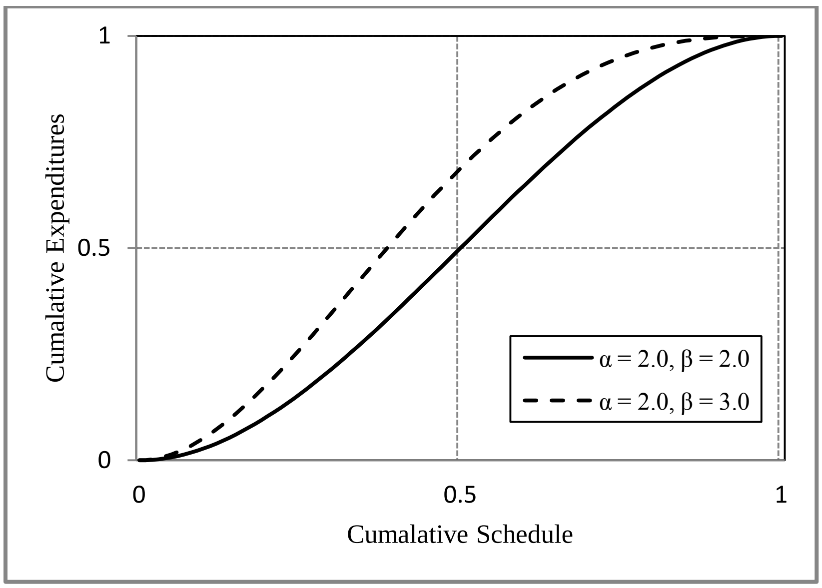


Figure 6: Effects of changing the ratio between Beta shape parameters

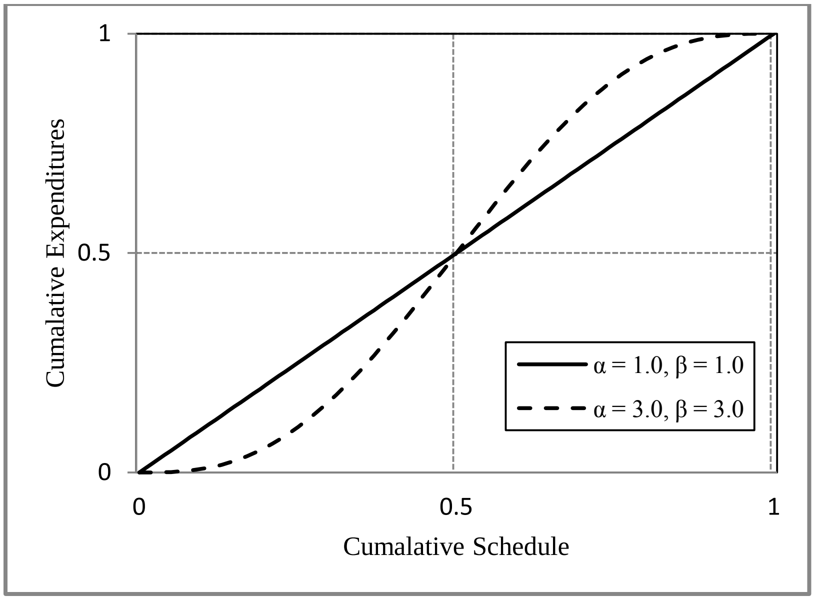


Figure 7: Effects of changing the magnitude of Beta shape parameters

Summary

Funding provided in the wrong fiscal year of a program may result in lost funding, schedule slips, and cost growth (Unger, 2001). As a result, the AFCAH, the primary guidebook for Air Force cost analysts, provides three methods to increase time phasing precision: schedule/milestone, analogy, and S-curve. For RDT&E applications, the S-curve is often the preferred method, as the S-curve does not require detailed program schedule or WBS data (AFCAH, 2007).

Initially, analysts applying the S-curve time phasing method primarily relied on the single parameter Rayleigh distribution. The Rayleigh distribution is a special case of the Weibull distribution which assumes a fixed shape parameter of 2. More recently, the Weibull and Beta distributions have become commonplace as Brown et al. (2002), Unger et al. (2004), and Burgess (2006) have demonstrated the increased precision available from utilizing multi-parameter distributions. Having defined the Weibull and Beta distributions and reviewed previous research, we will next define our methodology for constructing an aircraft-development specific S-curve model.

III. Methodology

Chapter Overview

The purpose of this chapter is to define the methods used to gather and analyze data while constructing our model. First, we explain the source for aircraft RDT&E data and our decision to utilize total obligation authority (TOA) data instead of contract expenditures. Following data source selection, our method for converting TOA into estimated base year 2014 expenditures is explained. Next, we present our method for standardizing each expenditures data point into a percent schedule by percent expenditures data base. Finally, the procedure for estimating best-fit Rayleigh, Weibull, and Beta parameters is given, and the methodology for determining the significance of our resulting Rayleigh, Weibull, and Beta models is explained.

Data Source

The first step in any research methodology is determining the most credible and applicable source for data. Therefore, the efforts of previous S-curve researchers are reviewed. For their research, Brown et al. (2002), Porter and Gallagher (2004), and Unger et al. (2004) rely on the Select Acquisitions Report (SAR) to obtain the Total Obligation Authority (TOA), which is the amount of budget for each year of a development program. Using the OSD Comptroller outlay rates, the authors are able to use TOA to approximate annual expenditures. By comparison, Norden (1970), Weida (1977), Putnam (1978), Abernathy (1984), and Burgess (2006) obtain expenditure data directly from contractor cost reports; therefore, no further transformation of data is required to obtain actual expenditures. Although either method of data sourcing is acceptable, we determine that utilizing the SAR's TOA data are more desirable for two reasons.

First, we determine that SAR TOA data are more applicable to the "real-world" time phasing estimates that occur early within an aircraft program's development. Once transformed by outlay rates, TOA data directly estimate a development program's total expenditures across all development contracts. By comparison, contract expenditure data from a single contractor's Cost Data Summary Reports may only represent a portion of the total aircraft development effort. Furthermore, during the research process, it was discovered that the prime contractor and supporting contractors often submit their reoccurring annual expenditure reports in different months of the year. As a result, for programs with multiple development contracts, it is often not possible to sum expenditures across individual contracts to obtain total program expenditures. Due to this limitation, we therefore judge SAR data as more representative of the entire aircraft development effort.

Second, SAR data are more consistently reported and available. By law, SAR reports are updated and reported annually and include budgeted TOA amounts for every fiscal year of development. By comparison, a review of Cost Data Summary Reports within DACIMS revealed one or more missing annual reports for 19 of the 26 programs in our database. Furthermore, 8 of 26 programs considered have no Cost Data Summary Reports available for their entire development effort. Therefore, we determine that utilizing SAR data will result in a more robust database with greater explanatory power.

Data Selection

For the purposes of defining an aircraft-specific S-curve model and testing the 60/40 heuristic, only aircraft development programs are incorporated. Aircraft development programs are defined as any fixed-wing, manned aircraft developed for one or more of the U.S. DoD service branches and designated as DoD Acquisition Category 1 (ACAT 1). After applying these criteria, we arrive at an initial sample size of 28 programs with available SAR TOA data. Next, we further specify that programs must have a reported engineering and manufacturing development (EMD) contract award date, first flight date, and initial operational capability (IOC) date. After applying these criteria, 2 of the 28 aircraft development programs are deselected, leaving a remaining sample size of 26 aircraft as shown in Table 3. The C-130 AMP and KC-130J programs are both deselected for lacking a published first flight and IOC date.

Table 3: Selected Aircraft Development Programs

1	A-10	14	EA-6B ICAP
2	AWACS BLOCK 40	15	F-14A
3	AWACS RSIP	16	F-14D
4	B-1 CMUP	17	F-15A
5	B-2 EHF 1	18	F-16A/B
6	B-2 RMP	19	F-18E/F
7	B-2A	20	F-18A
8	C-17A	21	F-22
9	C-5 AMP	22	F-35 AF
10	C-5 RERP	23	F-5E
11	E-2D	24	JSTARS
12	E-6A	25	P-8A
13	EA-18G	26	T-46A

Converting TOA (Budget) to Annual Expenditures (Base Year)

RDT&E funding (appropriation 3600) is budgeted to execute in the first fiscal year of availability, but may be approved by the acquisition program's Chief Financial Officer to obligate in the second year of availability. Thus, RDT&E funding is considered "multiyear", and available for incurring obligation for more than one fiscal year (AFCAH, 2007: 15-4). Therefore, when estimating annual expenditures from TOA, we must account for this uncertainty by applying the OSD-Comptroller outlay rate to approximate multiyear spending. As an example, consider the published RDT&E Navy outlay rates for fiscal years 1992 through 2002 are shown in Table 4. From this table, we may interpret that, on average, a Navy RDT&E program will spend 55.95 percent of available FY92 funding during FY92 (first year), 33.01 percent of FY92 funding during FY93 (second year), and only 7.97 percent of FY92 funding during FY94 (third year).

Table 4: Navy RDTE Outlay Rates (FY92-02)

	Outlay Rates (Percentage)				
	First Year	Second Year	Third Year	Fourth Year	Fifth Year
FY92	55.95	33.01	7.97	1.43	1.64
FY93	54.36	33.88	7.91	1.32	2.53
FY94	54.46	33.8	7.89	1.32	2.53
FY95	47.73	38.8	9.06	1.51	2.9
FY96	50	35.2	9.8	2.2	2.8
FY97	51.77	34.45	8.97	1.2	3.61
FY98	49.23	36.6	9.15	2.18	2.84
FY99	50.9	38.69	6.31	2.34	1.76
FY00	53.77	38.6	5.29	1.17	1.17
FY01	53.3	38.41	5.25	1.17	1.87
FY02	52.33	36.1	5.79	3.93	1.85

This same methodology is applied to the F-18 E/F's TOA in Table 5. Using the outlay rates given in Table 3, we calculate in Table 4 that $195.6 Million of FY92's TOA will be expended during FY92. Furthermore, $115.4 Million of FY92's TOA will be expended during FY93, and $27.9 Million of FY92's TOA will be expended during FY94. Carrying this methodology from FY92 through FY02 reveals the estimated TY expenditures, shown in Table 4's bolded font. Additionally, Figure 8 compares the distribution of TOA and estimated expenditures. Due to the effect of multiyear funding, expenditures will always appear to "lag" behind TOA.

Table 5: F-18 E/F TOA to Then Year Expenditures Conversion

			Expenditures (TY $M)										
	FY	TOA	FY92	FY93	FY94	FY95	FY96	FY97	FY98	FY99	FY00	FY01	FY02
Budget (TY $M)	FY92	349.5	195.5	115.4	27.9	5.0	5.7	0.0	0.0	-	-	-	-
	FY93	842.1	-	457.8	285.3	66.6	11.1	21.3	0.0	0.0	-	-	-
	FY94	1396.2	-	-	760.4	471.9	110.2	18.4	35.3	0.0	0.0	-	-
	FY95	1246	-	-	-	594.7	483.4	112.9	18.8	36.1	0.0	0.0	-
	FY96	801.1	-	-	-	-	400.6	282.0	78.5	17.6	22.4	0.0	0.0
	FY97	345.4	-	-	-	-	-	178.8	119.0	31.0	4.1	12.5	0.0
	FY98	234.6	-	-	-	-	-	-	115.5	85.9	21.5	5.1	6.7
	FY99	195.6	-	-	-	-	-	-	-	99.6	75.7	12.3	4.6
	FY00	132.1	-	-	-	-	-	-	-	-	71.0	51.0	7.0
	FY01	13.9	-	-	-	-	-	-	-	-	-	7.4	5.3
	FY02	1.1	-	-	-	-	-	-	-	-	-	-	0.6
			195.5	**573.1**	**1073.5**	**1138.2**	**1011.0**	**613.4**	**367.1**	**270.2**	**194.7**	**88.3**	**24.1**

Following the application of outlay rates, the F-18 E/F's estimated then year annual expenditures must be converted to base year expenditures to control for the effects of inflation. As shown in Table 6, then year expenditures are divided by the OSD-Comptroller's raw inflation index to obtain base year dollars. FY2014 was selected as the base year, as the majority of our research was completed within FY2014.

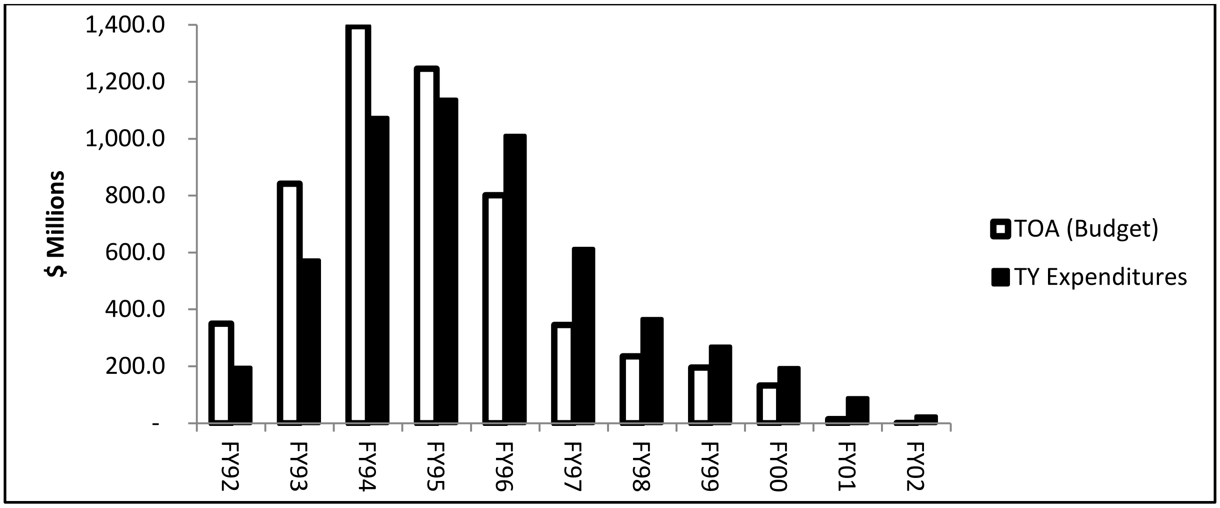


Figure 8: F-18 E/F Budget vs. TY Expenditures Profile

Table 6: F-18 E/F Then-Year Expenditures to Base-Year Expenditures Conversion

Fiscal Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
TY Expenditures ($M)	195.5	573.1	1073.5	1138.2	1011.0	613.4	367.1	270.2	194.7	88.3	24.1
Inflation Factor	0.675	0.693	0.707	0.721	0.735	0.750	0.756	0.762	0.772	0.786	0.793
BY14 Expend. ($M)	289.7	827.0	1518.4	1578.6	1375.5	817.9	485.6	354.6	252.2	112.3	30.4

Converting Annual Expenditures to Percent Schedule by Percent Expenditures

Before beginning analysis, the estimated expenditures for each aircraft program are standardized into a percent schedule by percent expenditure format by applying consistent development start and end criteria to each program. However, the selection of start and end criteria for development should not be made indiscriminately, as Burgess (2006) warns that:

> Models…cannot be more accurate than the underlying cost and schedule estimates, and they are meaningful only when the scope of both underlying estimates (cost and schedule) is precisely defined. In the case of schedule

> estimates, models must have precise definitions of start and end time, indexed to specific programmatic events that have common definitions across programs. It doesn't help to have an accurate model that predicts 60 percent spent at 50 percent time, for example, if the definition of "time" is ambiguous (Burgess, 2006:19).

Therefore, we determine that simply using the annual TOA funding or outlay-rate estimated expenditures to define the start or end of development would be inappropriate, as TOA is often distorted in a manner that disconnects funding from development milestones. For example, it is observed that many historical development SAR budgets do not distinguish between the initial and follow-on development efforts; specifically, the occurrence of follow-on development results in multiple funding inflection points that could significantly skew the right tail of the S-curve. It is determined that these follow-on development efforts are beyond the scope of the initial time phased estimate.

Engineering and Manufacturing (EMD) contract award is selected as our definition of schedule start, as the date of EMD contract award is commonly published and is synonymous with the primary contractor initiating aircraft development efforts. For most development efforts, EMD contract award coincides with the published Milestone B. By comparison, Initial Operating Capability (IOC) is selected as our definition of schedule end for two reasons. First, IOC is selected as development end because it is the latest milestone event that is consistently reported for all aircraft development programs within our database. By comparison, Full Operational Capability (FOC) was also considered as an alternate definition of schedule end; however, it is reported inconsistently as only four of 26 programs publish FOC. Second, IOC is selected as development end because AFCAH (2007) reports that the planned IOC date is highly influential on time phasing. "The IOC is a primary driver in determining the program's development and production schedules…the program's time phased estimate therefore must be consistent with the schedule…so that its budgetary inputs can support the achievement of the IOC" (AFCAH, 2007: 15-10).

Having selected appropriate start and end criteria for development, we next must truncate any expenditures that occur outside of development. Linear interpolation is utilized to estimate partial years which result from EMD contract award or IOC occurring during the fiscal year (any date other than 30 September). Following truncation, the standardization to percent schedule by percent expenditures occurs, as shown for the F-18 E/F in Figure 9. This procedure is repeated for all 26 aircraft development programs, until a dataset of standardized percent schedule and percent expenditure is gathered. The aggregated database generates a scatter plot with a visible S-curve, seen in Figure 10.

Outlay Rate-Estimated Expenditures (BY14 $) by Fiscal Year						
FY1992	FY1993	FY1994	FY1995	FY1996	FY1997	FY1998
289.70	827.04	1518.43	1578.70	1375.52	817.90	485.62
FY1999	FY2000	FY2001	FY2002	FY2003	FY2004	FY2005
354.55	252.27	112.37	30.44	7.63	2.17	0.36

Incremental and Cumulative Expenditures Controlling for RDTE Start and End										
Event	*EMD Awd*									*IOC*
Start	7/21/1992	9/30/1992	9/30/1993	9/30/1994	9/30/1995	9/30/1996	9/30/1997	9/30/1998	9/30/1999	9/30/2000
End	9/30/1992	9/30/1993	9/30/1994	9/30/1995	9/30/1996	9/30/1997	9/30/1998	9/30/1999	9/30/2000	9/15/2001
Incremental Expenditures (BY14 $)	56.35	827.04	1518.43	1578.70	1375.52	817.90	485.62	354.55	252.96	107.75
Cumulative Expenditures (BY14 $)	56.35	883.39	2401.82	3980.51	5356.03	6173.93	6659.55	7014.10	7267.05	7374.81

Percent Schedule by Percent Expenditures										
Schedule	0.0212	0.1304	0.2396	0.3488	0.4583	0.5675	0.6766	0.7858	0.8953	1.0000
Expenditures	0.0076	0.1198	0.3257	0.5397	0.7263	0.8372	0.9030	0.9511	0.9854	1.0000

Figure 9: F-18E/F Standardization to Percent Schedule by Percent Expenditures

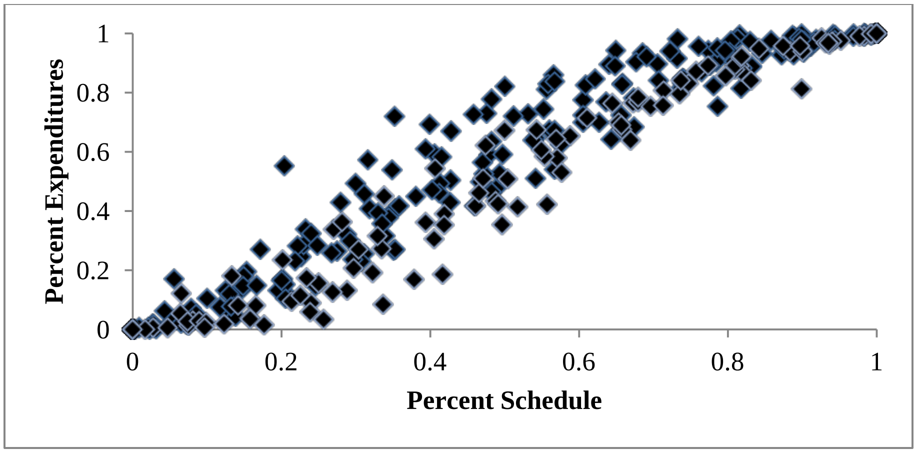


Figure 10: Standardized Percent Schedule by Percent Expenditures Database

Estimation of Weibull and Beta Distribution Parameters

After converting each program to a percent schedule by percent expenditures format, optimal parameters must be computed for each program. Nonlinear estimation techniques are utilized to determine the best-fit parameters for the Rayleigh (as a special case of the Weibull), Weibull, and Beta distributions. Analysis is completed separately for each program using the Microsoft Excel Solver add-in, with the parameter cells identified as changing cells and the sum-squared error (SSE) identified as the target cell. During analysis, Excel Solver allows the parameters of each distribution to vary until the SSE value between the actual expenditure percentage and the distribution-modeled expenditure percentage is minimized. As an example, Table 7 shows the results of optimal parameter analysis on the F-18E/F program, using the Weibull distribution. Changing the parameters to any value other than $\alpha = 1.76171$ and $\beta = 0.40402$ would increase error between actual and predicted expenditures for the F-18E/F. The estimated parameters by distribution type and program are provided in Appendix C.

Table 7: F-18E/F Minimized SSE for the Weibull Distribution

Cum. Schedule	Cumulative Expenditures		Squared Error
	Actual	Weibull Predicted	
0	0	0	0
0.021238	0.007641	0.005560	0.000004
0.130422	0.119784	0.127533	0.000060
0.239605	0.325678	0.328570	0.000008
0.348789	0.539744	0.537844	0.000004
0.458271	0.726260	0.713077	0.000174
0.567454	0.837165	0.837861	0.000000
0.676638	0.903013	0.916301	0.000177
0.785821	0.951089	0.960382	0.000086
0.895304	0.985389	0.982794	0.000007
1.000000	1.000000	0.992819	0.000052
		Sum Squared Error =	0.000572

Parameters	
Alpha	Beta
1.76171	0.40402

Tests for Linear Relationships between Parameters and Program Characteristics

Next, using linear regression, we test for the existence of relationships between an aircraft development program's attributes and its estimated parameters for the Rayleigh, Weibull and Beta models. For the linear regression model, the distribution's parameter becomes the dependent variable, while a development program's characteristics are treated as independent variables (also known as predictor variables). Based on previous S-curve research, it is postulated that the following program attributes are significant predictor variables: development program length (Brown et al., 2002; Burgess, 2006), development expenditure amount (Dibbly, 1988), branch of service (Brown et al., 2002), and scheduled time to first flight (Lee et al., 1997). In addition, several other program characteristics are considered as predictor variables for our linear model, as summarized in Table 8.

Table 8: Program Characteristics Considered as Predictor Variables

#	Predictor Variables	Previously Documented
1	Length of Development (EMD Award through IOC)	Brown et al. (2002), Burgess (2006)
2	Total Costs of Development (BY14 Dollars)	Dibbly (1988)
3	Branch of Service	Brown et al. (2002)
4	Aircraft Type (Attack/Fighter, Cargo, Bomber, or ISR)	-
5	Time to First Flight (% Schedule)	Lee et al. (1997)
6	RDT&E Prototypes	-
7	Upgrade Program (Not New Start)	-
8	Avionics-Specific Upgrade Program	-
9	Budget Threshold Breach	-
10	Schedule Threshold Breach	-
11	Concurrency (overlap between OT&E and Milestone C)	-

Although 11 predictor variables are considered for each distribution parameter, it is our goal to make each model as parsimonious, or simplistic, as possible. Therefore, the final linear regression for each distribution parameter will be limited to four or less predictor variables, with a goal of two or less. There are several reasons we strive for parsimony. First, the regression equation with four or fewer variables is less effort for cost analysts to apply and decreases the risk of a computational error. Second, the model with fewer variables is more easily explained and presented to program managers and other key decision makers. Third (and most importantly), DoD cost analyst often work with relatively small sample sizes of historic program cost data. An often cited “10-to-1 rule of thumb” within statistics recommends that a regression should have 10 or more observations per independent variable (Vittinghoff and McCulloch, 2007). Similarly, a linear modeling textbook by Neter et al. (1996) states that a ratio of six to 10 predictor variables per predictor variable is acceptable. Due to having only 26 useable observations within this thesis, we set the goal of 2 or fewer predictor variables per parameter, with an upper limit of 4 predictor variables per parameter.

Comparison of Final Rayleigh, Weibull, and Beta S-curve Models

Using the linear relationships between program characteristics and parameters, a final model will be developed for each distribution type. Hence, we will be directly comparing the final Rayleigh, Weibull, and Beta models, with the intention of defining the 'best' model. To determine the best model, the predicted TOA profile, derived from predicted annual expenditures, will be compared against the actual TOA profile. The best model will be defined as the model which provides for the highest average accuracy when predicting annual TOA, while controlling for model robustness across time and aircraft type. Therefore, we next present the Lee et al (1997) methodology for deriving a TOA profile from predicted annual expenditures, followed by definitions for model accuracy and robustness.

Convert Annual Expenditures to a TOA (Budget) Profile

The first step in this process of converting from base year 2014 annual expenditures to then year TOA is "escalating" each year's predicted base year 2014 expenditures, so that our prior controls for inflation are removed and current year expenditures are obtained. Escalation is accomplished by multiplying each year's predicted base year 2014 expenditures by the OSD-Comptroller's raw inflation index. It should be noted that this process is simply a reversal of the current year to base year methodology applied earlier in Table 6. The second step in our process involves transforming our predicted current year expenditures into a TOA profile through a application of the OSD-Comptroller's outlay rates. We elect to employ Lee et al.'s (1997) methodology of using a linear system of equations to calculate a TOA profile. Lee et al.'s (1997) methodology defines the linear equation

$$T_k = (O_k - s_2 T_{k-1} - s_3 T_{k-2} - \cdots - s_j T_{k-J+1})/s_1 \tag{8}$$

Where t represents the estimated TOA in a given year, s represents the outlay pattern value (value between 0 and 1), and O represents the outlay, also known as annual expenditure.

Model Accuracy

Our research will utilize the Pearson R^2 to judge each model's accuracy. We make the distinction that R^2 is computed separately for each of the aircraft development programs and then averaged to arrive at a mean model R^2. This technique ensures that aircraft development programs with a greater number of years of RDTE do not bias the reported R^2 value; hence, each aircraft development program is allotted equal weight in determining the model's accuracy.

Robustness

When selecting a best model, consideration will also be given to model robustness. As documented by Stingler (2010), robustness has many definitions that have changed throughout time. However, for this thesis, we will comprise a simple standard for S-curve model robustness: the best S-curve model should be able to accurately predict annual expenditures (as measured by R^2) for all aircraft types across all time periods. For example, an S-curve model which is only accurate in predicting time-phased expenditures for 1970's-era fighter aircraft would be useless for the cost analyst attempting to time phase a contemporary cargo aircraft.

Summary

This chapter explains the proposed methodology for estimating the best model for time-phasing aircraft RDT&E expenditures. We begin by first obtaining SAR budget data from DAMIR and estimating annual expenditures via outlay rates. After accounting for inflation through the application of the OSD-Comptroller inflation rates, the expenditure data are transformed into a percent expenditure by percent schedule format. This transformation is necessary to establish a database of standardized program data for analysis. Next, we estimate distribution parameters for each program using Excel's Solver; distribution parameters are adjusted until the SSE is minimized. Following the estimation of parameters, linear relationships between the distribution parameters and program characteristics are developed. These linear relationships are then used to construct our final Rayleigh, Weibull, and Beta models. Finally, using the R^2 for annual TOA and robustness checks, these final models are compared and a "best" model is defined. In the next chapter, we present the analysis of our data and model development.

IV. Analysis and Results

Introduction

This chapter provides the results from the methodology outlined in Chapter 3. First, using our database of 26 aircraft, we calculate the average percent of expenditures at the development program midpoint to determine the accuracy of the 60 percent expenditures by 50 percent schedule "rule of thumb". Second, linear relationships between development program characteristics and distribution parameters are developed. Finally, using the developed linear relationships, the best Rayleigh, Weibull, and Beta models are compared. The models are judged by their R^2 values in addition to robustness across time, aircraft type, and upgrade characteristic.

Accuracy of 60/40 "Rule of Thumb"

As introduced earlier, the 60 percent expenditures at 50 percent schedule heuristic (also known as the 60/40 rule of thumb) is well known to cost analysts (Lee at al., 1997). To test the accuracy of this rule, we calculate the estimated percent expenditures at 50 percent schedule for the 26 aircraft development programs in our database through linear interpolation. We discover that programs expend only 56.3 percent of their total expenditures at 50 percent schedule, on average, as summarized in Appendix B. More specifically, 7 programs expend more than 60 percent by the development midpoint, 16 expend less than 60 percent by the program midpoint, and 3 expend approximately 60 percent by the program midpoint.

However, a noticeable time trend emerges when programs are plotted across time in Figure 11. Expenditures for aircraft development programs have generally become more “front-loaded” over time, with a greater percentage of expenditures occurring by the development midpoint. The most significant change in development program spending is visible during the 1980s, the period during which the President’s Blue Ribbon Commission on Defense Management was preparing its final report. The report recommended sweeping changes to acquisitions management, to include increased usage of prototyping, limitations on concurrency, early development testing, and a multi-year appropriations cycle (Blue Ribbon Commission, 1986). We hypothesize that the President’s Blue Ribbon Panel had an influence on the distribution of aircraft development expenditures, and therefore elect to analyze separately the 14 “contemporary” programs which initiated development during the last 30 years. If the contemporary programs (from 1985 and later) are analyzed in isolation, we discover that programs expend a mean of 63.13 percent of total expenditures at 50 percent schedule, with a median of 59.83 percent expenditures at 50 percent schedule.

Additionally, a second trend emerges between contemporary upgrade (to include modification programs) and contemporary new start programs, as shown in Table 9. On average, we find that the expenditures for contemporary upgrade programs are more “front loaded” than their contemporary new start counterparts. Contemporary upgrade programs expend 65.27 percent of total expenditures at 50 percent schedule, while contemporary new start programs expend 55.04 percent of total expenditures at 50 percent schedule. As a result, we assess that the 60/40 rule is generally not an accurate heuristic for the contemporary aircraft development program, given that the analyst can identify the development program as either a new start or upgrade.

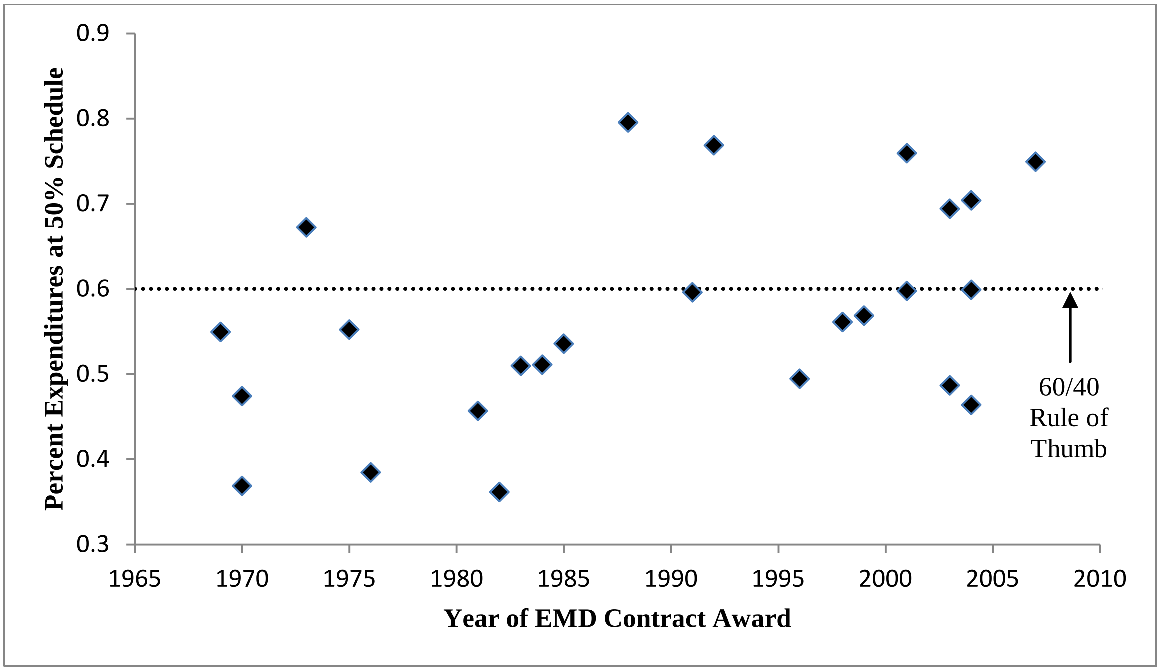


Figure 11: Percent Expenditures at 50% Schedule, by Year of Contract Award

Table 9: Percent Expenditures at 50% Schedule, New Start vs. Upgrade (1985-later)

New Start Program	Percent Expenditures at 50% Schedule
F-22	0.60
F-35 AF	0.60
JSTARS	0.54
P-8A	0.46
Mean:	**0.55**
Median:	**0.57**

Upgrade Program	Percent Expenditures at 50% Schedule
AWACS BLOCK 40	0.69
AWACS RSIP	0.80
B-1 CMUP	0.49
B-2 EHF 1	0.75
B-2 RMP	0.60
C-5 AMP	0.57
C-5 RERP	0.76
E-2D	0.70
EA-18G	0.49
EA-6B ICAP	0.56
F-18E/F	0.77
Mean:	**0.65**
Median:	**0.69**

Linear Relationships between Parameters and Program Characteristics

Utilizing unique program characteristics, linear relationships are developed to explain variation in the estimated distribution parameter values. In addition to the 11 program characteristics previously documented, we consider interactions between predictor variables and transformations of predictor variables, such as the application of natural log, to increase the explanatory power of our models. Predictor variables which are not statistically significant at the 5 percent level are excluded from the model outputs. Based on the discovery of the time trend observed earlier, we also consider the inclusion of an indicator variable, also known as a "dummy variable", for all aircraft with an EMD contract award date before 1985.

Additionally, it should be emphasized that the C-17A is excluded from the linear regression models for two reasons. First, the C-17A is an overly influential data point across all distribution parameters and consistently resulted in Cook's D values above 1. A Cook's D value above 0.5 indicates that a data point is significantly influential on the model estimates (Neter, Kutner, Nachtsheim, and Wasserman, 1996: 380). Second, a qualitative review of the acquisition history of the C-17A reveals that the C-17A aircraft suffered extensive development delays due to contracting difficulties, budget cuts, and management problems, as evidenced by a 1990 USAF Inspector General investigation for management improprieties (Saxer, 1995). Although schedule delays are not uncommon for aircraft development, we determine that the extent of the C-17A's delays make it a unique data point that should be separated from other aircraft within our sample.

Rayleigh Distribution – Scale Parameter β

Three predictor variables are significant at explaining variation in the Rayleigh's scale parameter β: time to first flight, upgrade program, and the interaction between the pre-1985 indicator and upgrade. Together, these three variables provide a R^2 of 0.535, and significance at the 5 percent level is maintained for all variables, as displayed in Table 10. Next, we test for influential data points by utilizing the Cook's D statistic. As shown in Appendix D, the absence of values over 0.5 indicates that no influential observations exist for the linear regression model. Finally, we use the residuals, which are the difference between the observed and predicted parameter values, to test separately for normality and constant variance. Appendix D displays the normal distribution fit to residual values; the Shapiro-Wilks test p-value greater than 0.05 indicates that the residual values are normally distributed. Residuals appear to have a reasonably uniform distribution across the predicted range, showing constant variance. Constant variance is further confirmed by the Breusch-Pagan test; the reported p-value of 0.5264 fails to rejects the null hypothesis of constant variance.

Weibull Distribution – Shape Parameter α

One predictor variable is significant at explaining variation in the Weibull's shape parameter α: the pre-1985 indicator. As shown in Table 11, this single variable provides a R^2 of 0.357, with statistical significance at the 0.05 level. As shown in Appendix D, the absence of Cook's D values over 0.5 indicates that no influential observations exist for either linear regression model. Additionally, Appendix D displays the normal distribution fit to residual values; the Shapiro-Wilk's p-value greater than 0.05 indicates that the residual values are normally distributed. Residuals appear to be uniformly distributed, with a Breusch-Pagan p-value of 0.6911.

Table 10: Summary of Fit – Rayleigh Scale Parameter β

Summary of Fit					
R^2					0.534592
R^2- Adjusted					0.468105
Model Parameter Estimates					
Term	Estimate	Std. Error	Prob. > |t|	Std. Beta	VIF
Intercept	0.450261	0.048829	**<0.0001**	-	-
First Flight (% Schedule)	0.240399	0.100000	**0.0255**	0.36211	1.02381
Upgrade Program	-0.078185	0.025229	**0.0054**	-0.48676	1.11316
Interaction	0.148797	0.045965	**0.0039**	0.50304	1.08960

Table 11: Summary of Fit -- Weibull Shape Parameter α

Summary of Fit					
R^2					0.356652
R^2- Adjusted					0.328681
Model Parameter Estimates					
Term	Estimate	Std. Error	Prob. > |t|	Std. Beta	VIF
Intercept	1.91047	0.049748	**<0.0001**	-	-
Pre-1985 Contract Award	0.503002	0.140866	**0.0016**	0.597204	1

Weibull Distribution – Scale Parameter β

As an indicator of robustness, it is discovered that the Weibull's scale parameter β is influenced by the same predictor variables as the Rayleigh's scale parameter β: time to first flight, upgrade program, and the interaction between the pre-1985 indicator and upgrade. Together, these three variables provide a R^2 of 0.547, and statistical significance is maintained at the 0.05 level for all variables, as displayed in Table 12. As shown in Appendix D, the absence of Cook's D values over 0.5 indicates that no influential observations exist for the linear regression model. Additionally, Appendix D displays the normal distribution fit to residual values; the Shapiro-Wilk's p-value greater than 0.05 indicates that the residual values are normally distributed. Residuals appear to be uniformly distributed, with a Bruesch-Pagan p-value of 0.5176.

Table 12: Summary of Fit -- Weibull Scale Parameter β

Summary of Fit					
R^2					0.546873
R^2- Adjusted					0.482141
Model Parameter Estimates					
Term	Estimate	Std. Error	Prob. > |t|	Std. Beta	VIF
Intercept	0.442652	0.049748	**<0.001**	-	-
First Flight (% Schedule)	0.258704	0.101884	**0.0191**	0.377406	1.0238
Upgrade Program	-0.079064	0.025703	**0.0057**	-0.47673	1.1132
Interaction	0.156363	0.04683	**0.0031**	0.51196	1.0896

Beta Distribution – Shape Parameter α

Three predictor variables are significant at explaining variation in the Beta's shape parameter α: the pre-1985 indicator, length of development, and the natural log of 1 divided by the length of development. Table 13 displays that these variables together provide a R^2 of 0.462, and significance at the 5 percent level is maintained for all variables. As shown in Appendix D, the absence of Cook's D values over 0.5 indicates that no influential observations exist for the regression model. Additionally, Appendix D displays the normal distribution fit to residual values; the Shapiro-Wilk's p-value greater than 0.05 indicates that the residual values are normally distributed.   Residuals appear to be uniformly distributed, with a Bruesch-Pagan p-value of 0.6473.

Table 13: Summary of Fit – Beta Shape Parameter α

Summary of Fit					
R^2					0.462269
R^2- Adjusted					0.385451
Model Parameter Estimates					
Term	Estimate	Std. Error	Prob. > |t|	Std. Beta	VIF
Intercept	-1.803242	0.362462	0.0988	-	-
Pre-1985 Contract Award	0.742528	0.873358	**0.0183**	0.745497	1.67176
Length of Development	-0.324556	0.112016	**0.0086**	-2.15116	21.5268
Ln(1/Length of Develop.)	-2.945843	0.91015	**0.0040**	-2.51231	23.5293

Beta Distribution – Shape Parameter β

Four predictor variables are useful in explaining variation in the Beta's shape parameter β: the pre-1985 indicator, length of development, upgrade, and the interaction between upgrade and the pre-1985 indicator. Together, these variables provide a R^2 of 0.593, and significance at the 5 percent level is maintained for all variables, as displayed in Table 14. As shown in Appendix D, the absence of Cook's D values over 0.5 indicates that no influential observations exist for the regression model. Additionally, Appendix D displays the normal distribution fit to residual values; the Shapiro-Wilk's p-value greater than 0.05 indicates that the residual values are normally distributed. The residuals plot appears to be uniformly distributed with a reported Breusch-Pagan p-value of 0.419.

Table 14: Summary of Fit – Beta Shape Parameter β

Summary of Fit					
R^2					0.593305
R^2- Adjusted					0.511966
Model Parameter Estimates					
Term	Estimate	Std. Error	Prob. > |t|	Std. Beta	VIF
Intercept	-0.361293	0.548769	0.5178	-	-
Pre-1985 Contract Award	1.426644	0.377201	**0.0012**	1.11796	4.29664
Length of Development	0.155172	0.039521	**0.0008**	0.80274	2.05559
Upgrade Program	1.492916	0.306782	**<.0001**	1.19306	2.95581
Interaction	-1.646058	0.45711	**0.0018**	-0.71431	1.93505

Functional Form of Final Models

Having established program characteristics to operate as predictor variables for the distribution parameters, we write the final functional forms of our Rayleigh, Weibull, and Beta non-linear equations. Additionally, we identify a baseline model for comparison: the 60/40 Rayleigh model, provided in Lee et al. (1997). It should be emphasized that the Lee et al. 60/40 model controls for the Rayleigh's infinite tail by assuming that schedule completion occurs at 97 percent of expenditures; therefore, the right side of the given equation is divided by 0.97.

Final Rayleigh Distribution Model

$$\%\,Expenditures = 1 - e^{-(\frac{\%\,schedule}{\beta})^2} \tag{9}$$

$$\beta \; = \; 0.45026 + 0.2404\,(\%\,Schedule\ at\ First\ Flight)$$

$$-\,0.07819\,(Upgrade) + 0.1488\,(Pre - 1985\ \times Upgrade)$$

Final Weibull Distribution Model

$$\% \, Expenditures = 1 - e^{-\left(\frac{\% \, schedule}{\beta}\right)^{\alpha}} \tag{10}$$

$$\alpha = 1.91047 + 0.503 \, (Pre - 1985)$$

$$\beta \; = \; 0.44265 + 0.2587 \, (\% \, Schedule \; at \; First \; Flight)$$

$$- \, 0.07906 \, (Upgrade) + \; 0.15636 \, (Pre - 1985 \; \times Upgrade)$$

Final Beta Distribution Model

$$\% \, Expenditures = \; \frac{(1 - \% \, schedule)^{\beta - 1} (\% \, schedule)^{\alpha - 1} \Gamma \, (\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \tag{11}$$

$$\alpha = \; -1.80324 + 0.74253 \, (Pre - 1985) - \; 0.32456 \, (Length) -$$

$$2.94584 \, (\ln \frac{1}{Length})$$

$$\beta = -0.36129 + 1.42664 \, (Pre - 1985) \; + \; 0.155517 \, (Length)$$

$$+ \, 1.49292 \, (Upgrade) - \; 1.64606 \, (Pre - 1985 \; \times Upgrade)$$

Baseline 60/40 "Rule of Thumb" Model

$$\% \, Expenditures = \frac{1 - e^{-3.52 (\% \, schedule)^2}}{0.97} \tag{12}$$

Comparison between Final Models' Predictive Capability

Annual TOA R^2

Next, using our written equations, we compare the predictive accuracy of our Rayleigh, Weibull, and Beta models against the baseline 60/40 model. As previously explained in Chapter 3, accuracy is defined as R^2, or the percentage of total variability in annual TOA that is explained by the model. After applying our methodology for calculating R^2, we find that the Weibull model offers the highest nominal predictive ability across all programs, on average, as shown in Table 15. When only contemporary programs with a contract award of 1985 or later are considered, the Weibull again outperforms the Rayleigh and Beta distributions, although by a less significant percentage. As a result, we are not yet satisfied the Weibull model is superior, and next turn to robustness checks to validate if any model is consistently superior (or inferior) for predicting a particular time period, aircraft type or upgrade program.

Table 15: R^2 Comparison between Final Models

	60/40	Rayleigh	Weibull	Beta
Mean R^2	0.6807	0.7365	**0.7463**	0.6988
Mean R^2 (CY85 –14 contract award only)	0.6813	0.7149	**0.7152**	0.6648
Median	0.7240	0.8002	**0.8011**	0.7163
Max R^2	0.9488	0.9538	**0.9707**	0.9138
Min R^2	0.2785	0.3173	0.3715	**0.3784**

R^2 Range (All Historical Data)	60/40	Rayleigh	Weibull	Beta
< 0.5	4	3	1	2
0.5 < 0.6	4	3	5	5
0.6 < 0.7	4	4	2	4
0.7 < 0.8	7	1	4	7
0.8 < 0.9	4	10	9	6
0.9 < 1.0	3	4	4	1
Total	25	25	25	25

Robustness Check 1

As our first test of robustness, we examine each model's predictive ability as a function of time in Figure 12 and Table 16. Although it is observed that no model is radically superior for predicting budget requirements for more contemporary aircraft, the Weibull model holds as the most predictive during two of the five 10 year periods.

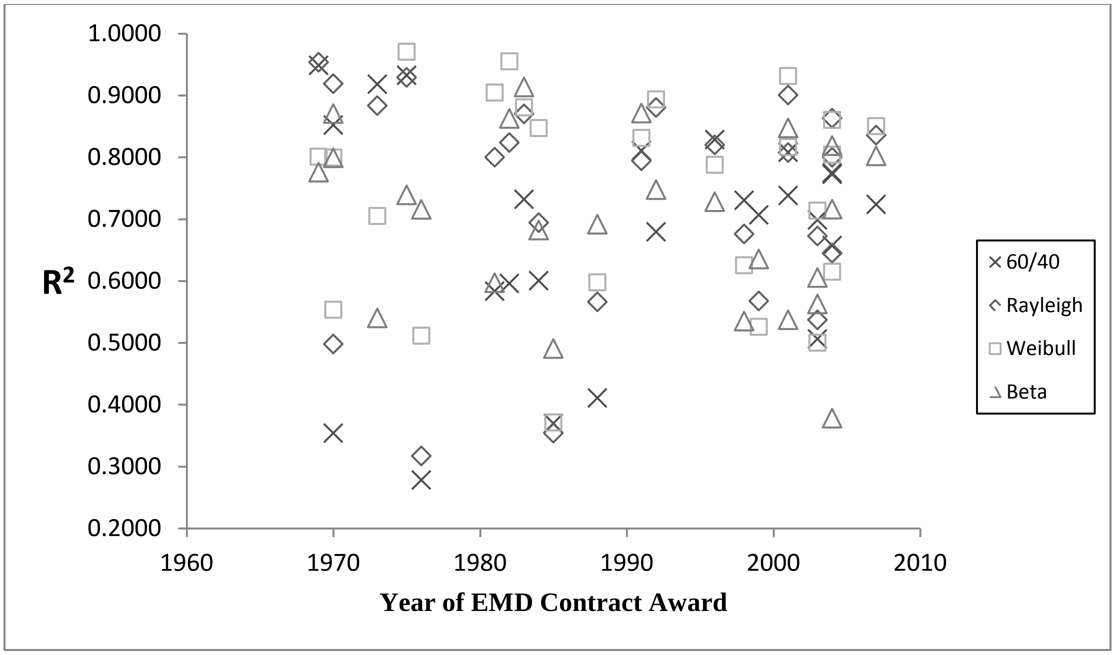


Figure 12: Models' Predictive Ability over Time

Table 16: Models' Predictive Ability over Time

Year of EMD Contract Award	Sample (n)	60/40	Rayleigh	Weibull	Beta
CY65-74	4	0.7684	**0.8137**	0.7149	0.7466
CY75-84	6	0.5583	0.7012	**0.8200**	0.7545
CY85-94	4	0.5679	0.6489	0.6737	**0.7006**
CY95-04	10	0.7223	**0.7291**	0.7182	0.6367
CY05-14	1	0.7240	0.8359	**0.8504**	0.8024

Robustness Check 2

Next, as a second check of robustness, the mean R^2 value for each aircraft type is computed. Aircraft are grouped into four broad development categories: attack/fighter, bomber, cargo, and ISR/electronic. For both fighter (Table 17) and bomber (Table 19) aircraft, it is observed that the Weibull model generally outperforms both the Rayleigh and Beta model, as measured by the mean. Conversely, for cargo (Table 18), the Beta outperforms the Rayleigh and Weibull; however, with a sample size of only two cargo aircraft, our inference is limited. Finally, it is seen in Table 20 that we are least successful at predicting the annual TOA for ISR/Electronic aircraft, as the best performing model (the Rayleigh) offers a mean R^2 of only 0.65.

Table 17: Attack/Fighter Aircraft R^2 Comparison

Program	60/40	Rayleigh	Weibull	Beta
A-10	**0.9182**	0.8836	0.7053	0.5404
F-14A	0.9488	**0.9538**	0.8011	0.7761
F-14D	0.6007	0.6944	**0.8473**	0.6825
F-15A	0.3543	0.4983	0.5537	**0.7993**
F-16A	0.9327	0.9293	**0.9707**	0.7394
F-18 E/F	0.6796	0.8806	**0.8937**	0.7479
F-18A	0.2785	0.3173	0.5118	**0.7160**
F-22	0.8109	0.7944	0.8314	**0.8716**
F-35 (USAF)	0.8082	0.8077	**0.8167**	0.5375
F-5E	0.8523	**0.9191**	0.7996	0.8708
Mean:	0.7184	0.7678	**0.7731**	0.7281
Median:	0.8096	**0.8442**	0.8089	0.7436

Table 18: Cargo Aircraft R^2 Comparison

Program	60/40	Rayleigh	Weibull	Beta
C-5 AMP	**0.7070**	0.5679	0.5261	0.6357
C-5 RERP	0.7384	0.9007	**0.9316**	0.8476
Mean	0.7227	0.7343	0.7288	**0.7417**
Median:	0.7227	0.7343	0.7288	**0.7417**

Table 19: Bomber Aircraft R^2 Comparison

Program	60/40	Rayleigh	Weibull	Beta
B-1 CMUP	**0.8287**	0.8202	0.7880	0.7283
B-2 EHF 1	0.7240	0.8359	**0.8504**	0.8024
B-2 RMP	0.7750	0.8000	**0.8044**	0.7163
B-2A	0.5837	0.8002	**0.9048**	0.5974
Mean	0.7278	0.8141	**0.8369**	0.7111
Median:	0.7495	0.8102	**0.8274**	0.7223

Table 20: ISR/Electronic Aircraft R^2 Comparison

Program	60/40	Rayleigh	Weibull	Beta
AWACS Block 40	0.5065	0.6728	**0.7139**	0.6058
AWACS RSIP	0.4111	0.5664	0.5980	**0.6919**
E-2 D	0.7726	**0.8633**	0.8605	0.8190
E-6A	0.7323	0.8703	0.8808	**0.9138**
EA-18G	**0.6989**	0.5373	0.5004	0.5630
EA-6B ICAP	**0.7307**	0.6761	0.6256	0.5353
JSTARS	0.3700	0.3543	0.3715	**0.4910**
P-8A	**0.6575**	0.6452	0.6151	0.3784
Mean	0.6099	**0.6482**	0.6457	0.6248
Median:	**0.6782**	0.6590	0.6203	0.5844

Robustness Check 3

As a third, and final, check of robustness, the ability of each model to predict annual TOA for both "new starts" and "upgrade" programs is compared. In Table 21 and Table 22, it is shown that, on average, we predict the distribution of TOA for new starts and upgrade programs with equal success. Across both new starts and upgrades, the Weibull model generally outperforms the Rayleigh and Beta models.

Table 21: New Start Aircraft R^2 Comparison

Program	60/40	Rayleigh	Weibull	Beta
A-10	**0.9182**	0.8836	0.7053	0.5404
B-2A	0.5837	0.8002	**0.9048**	0.5974
E-6A	0.7323	0.7803	0.8808	**0.9138**
F-14A	0.9488	**0.9538**	0.8011	0.7761
F-15A	0.3543	0.4983	0.5537	**0.7993**
F-16A	0.9327	0.9293	**0.9707**	0.7394
F-18A	0.2785	0.3173	0.5118	**0.7160**
F-22	0.8109	0.7944	0.8314	**0.8716**
F-35 AF	0.8082	0.8077	**0.8167**	0.5375
JSTARS	0.3700	0.3543	0.3715	**0.4910**
P-8A	**0.6575**	0.6452	0.6151	0.3784
T-46A	0.5962	0.8339	**0.9552**	0.8628
Mean:	0.6659	0.7232	**0.7432**	0.6853
Median:	0.6949	0.8040	**0.8089**	0.7277

Table 22: Upgrade Aircraft R^2 Comparison

Program	60/40	Rayleigh	Weibull	Beta
AWACS Block 40	0.5065	0.6728	**0.7139**	0.6058
AWACS RSIP	0.4111	0.5664	0.5980	**0.6919**
B-1 CMUP	**0.8287**	0.8202	0.7880	0.7283
B-2 EHF 1	0.7240	0.8359	**0.8504**	0.8024
B-2 RMP	0.7750	0.8000	**0.8044**	0.7163
C-5 AMP	**0.7070**	0.5679	0.5261	0.6357
C-5 RERP	0.7384	0.9007	**0.9316**	0.8476
E-2 D	0.7726	**0.8633**	0.8605	0.8190
EA-18G	**0.6989**	0.5373	0.5004	0.5630
EA-6B ICAP	**0.7307**	0.6761	0.6256	0.5353
F-14D	0.6007	0.6944	**0.8473**	0.6825
F-18 E/F	0.6796	0.8806	**0.8937**	0.7479
F-5E	0.8523	**0.9191**	0.7996	0.8708
Mean:	0.6943	0.7488	**0.7492**	0.7113
Median:	0.7240	**0.8000**	0.7996	0.7163

Summary

Chapter 4 provides the results of applying our methodology to a sample of 26 ACAT 1 aircraft development programs. First, it is shown that the accuracy of the 60/40 "rule of thumb" is limited, as it does not account for the differences in time phasing between new start and upgrade development programs. Next, we provide goodness-of-fit results that show that three unique aircraft development characteristics may be used to estimate the Rayleigh, Weibull, and Beta distribution parameters. Specifically, it is shown that time to first flight, length of development, and upgrade are statistically significant at the 0.05 level for estimating one or more distribution parameters. Finally, we use the estimated linear relationships to construct a final Rayleigh, Weibull, and Beta model. Through a series of accuracy and robustness checks, these final models are compared against the baseline 60/40 model provided by Lee et al. (1997). We discover that the Weibull model outperforms other proposed models for the majority of robustness checks. In the following chapter, Chapter 5, we conclude our research effort by answering Chapter 1 research questions, recognizing model limitations, and identifying areas for future research.

V. Conclusions and Recommendations

Introduction

In this chapter, we revisit our initial research questions to validate that our research accomplished its intended goal. Additionally, we review the limitations of findings, identify areas for future research, and conclude by summarizing the significance of this research.

Research Questions Answered

1 – *Is the 'rule of thumb' that 60 percent of expenditures occur by 50 percent schedule (60/40 S-curve) accurate for contemporary aircraft development programs?*

Lee et al. (1997) trace the origin of the 60/40 rule of thumb to a circa-1980 aircraft development study completed by the OSD Cost Analysis Improvement Group (CAIG). However, the OSD-CAIG study could not be recovered, so we instead review all available studies on time phasing aircraft development.

During the 1970's, both Weida (1976) and General Research Corporation (1979) find that aircraft development contracts have 57 percent expenditures at 50 percent schedule. Using the Weibull parameters located in the appendix from Brown (2001), we separately estimate that aircraft development programs have 57.1 percent expenditures at 50 percent schedule, on average. Similarly, the 26 historical aircraft development programs from this thesis report 56.3 percent expenditures at 50 percent schedule, on average. These results would likely lead the cost analyst to erroneously select a 56/44 or 57/43 "rule of thumb" over the 60/40.

However, attempting to identify a single rule of thumb that summarizes all aircraft across time would be a mistake, as two significant trends emerge. Firstly, as shown earlier in Figure 13, a significant time trend is observed within our data. Aircraft programs which began development during calendar year 1985 or later have expenditure distributions which are more "front loaded" (skewed right) than programs from before 1985. As a result, when the 14 "contemporary" programs from 1985 and later are analyzed in isolation, we find that programs expend a mean of 63.1 percent of total expenditures at 50 percent schedule, with a median of 59.8 percent expenditures at 50 percent schedule.

Next, we further delineate contemporary aircraft as either new starts or upgrades in Table 9. It is discovered that contemporary new starts (n=4) expend a mean of 55.0 percent expenditures at 50 percent schedule, while contemporary upgrades (n=11) expend a mean of 65.3 percent expenditures at 50 percent schedule. Therefore, as a heuristic, we determine that the 60/40 rule of thumb generally overestimates early expenditures for new starts, while underestimating early expenditures for upgrade programs.

2 – *What program and/or schedule characteristics best predict distribution parameters for the Rayleigh, Weibull, and Beta distributions?*

As summarized previously in Table 8, eleven unique attributes were considered initially as predictor variables for the distribution parameters. However, only three of eleven characteristics were statistically significant. The time to first flight and program upgrade characteristics are predictive for the Rayleigh and Weibull distribution parameters, while the time to first flight, upgrade, length of development characteristic are predictive for the Beta distribution parameters. Total development costs, branch of service, aircraft type, prototyping, budget/schedule threshold breaches, and concurrency were not statistically significant predictor variables (at the 0.05 level) for any of the distribution parameters.

3 – *Which distribution (Rayleigh, Weibull, or Beta) provides the best S-curve model for time phasing contemporary aircraft development programs?*

The Weibull distribution model, which explained 74.6 percent of the total variation in annual expenditures, is marginally more accurate than our Rayleigh and Beta models, which explained 73.7 percent and 69.9 percent of variation, respectively. All three models proved more accurate than the 60/40 model, which explained only 68.0 percent of total variation. Robustness checks reveal that the Weibull model is superior across most aircraft types and sampled time periods, with exceptions.

Due to these exceptions, we recognize that no time phasing model is perfect, and recommend that the cost analyst consider the application of two or more S-curve models as a "cross check" of their time phased estimate. To facilitate this cross-check, we again provide the final functional forms for our Rayleigh, Weibull, and Beta models in Appendix E using Microsoft Excel notation. For clarity, those predictor variables which are only applicable to historical aircraft with contract award dates prior to 1985 are removed.

Limitations

We recognize several major limitations that could potentially limit the applicability of this research to "real world" cost analysis applications. First, continuing a methodology previously applied by Brown et al. (2002), Porter and Gallagher (2004), and Unger et al. (2004), we utilize the published OSD-Comptroller outlay rates to estimate expenditures from budgeted TOA. Therefore, we recognize that the estimated program expenditures to which we fit our models are not "actuals", but instead approximations of expenditures. For this reason, we elect to apply the Lee et al. (1997) methodology to transform our predicted expenditures into predicted TOA before measuring R^2.   Similarly, when applying the models presented in this thesis to "real world" estimates, we recommend that the cost analyst also apply the Lee et al. methodology to convert their predicted expenditures into predicted TOA.

Second, in constructing our model, we utilize budget and schedule data from the latest SAR available for each aircraft development program. We do not account for any cost or schedule growth which exists between the aircraft program's first and latest SAR. Therefore, the reported Weibull time phasing model R^2 of 0.746 assumes absolutely accurate inputs for the cost, schedule, and program characteristics. Once again, this assumption contrasts with the "real world", where we recognize that cost and schedule estimates are rarely clairvoyant. As a result, if any of estimated model inputs are less than accurate, the model accuracy will decrease, on average, below 0.746.

Third, we recommend that the cost analyst should observe caution when using our models to "extrapolate", or estimate beyond the original range of historical observations. Due to the usage of an indicator variable which distinguishes between pre-1985 contract awards and more contemporary programs, we further restrict this recommendation by asserting that the cost analyst should avoid going beyond the range of inputs for contemporary aircraft given in Table 23, particularly for the Beta model. As a demonstration of the danger of interpolation, the reader should recognize that our data set includes only four new starts which began development during 1985 or later. For each of these new starts, the length of development was between 9.3 and 14.8 years. If the Beta model is extrapolated against a theoretical new start program with a first flight at 40 percent schedule and a comparatively short 6 year development length, we discover that it estimates only 20 percent expenditures at 50 percent schedule, which is unlikely. However, it should be distinguished that we observe that our Weibull and Rayleigh models are generally more stable (compared to the Beta) when extrapolated outside of the observed range.

Table 23: Upper and Lower Bounds for Model Inputs

	New Start				Upgrade			
	All Historic		Contemporary		All Historic		Contemporary	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
First Flight (% Schedule)	0.29	0.72	0.34	0.42	0.28	0.66	0.39	0.66
Length of Development	4.2	14.8	9.3	14.8	5.7	12.2	5.7	12.2

Recommendations for Future Research

We recommend two areas for future research. First, as addressed within limitations, our thesis assumes accurate cost and schedule inputs without growth. Therefore, for future research, we recommend the creation of a time phasing model with the capability to account for expected cost and schedule growth. Secondly, we assert that it would be valuable for future researchers to expand upon the time phasing investigation completed by Unger et al. (2004). Unger et al. find that a statistically significant relationship exists between the shape of the initial budget profile for a development program and observed cost and schedule growth. However, as a potential limitation of their study, Unger et al. do not account for the effect of budget curtailments, specifically budget "cuts" within the early years of development. We therefore hypothesize that both 1) the initial shape of the budget (as measured by Weibull parameters) and 2) whether that initial budget is fully funded are predictive of future cost and schedule growth.

Summary

Past research documents that funding provided in the wrong fiscal years of a development program result in productivity inefficiencies, schedule slips and increased program costs (Belcher et. al, 1999; Unger, 2001). Therefore, it is in the best interest of the system program office to submit an appropriate time phased estimate to the budget formulation process. To assist in the estimation process, our research first tests the applicability of the 60/40 "rule of thumb" to contemporary aircraft programs. We discover that, as a heuristic, the 60/40 does not account for the difference is spending patterns between new start and upgrade development programs. Contemporary new start programs expend a mean of 55 percent expenditures at 50 percent schedule (55/45), while contemporary upgrade programs expend a mean of 65 percent expenditures at 50 percent schedule (65/35). Next, we construct a methodology for estimating distribution parameters for the Rayleigh, Weibull, and Beta using characteristics common to all aircraft developing programs. We find that time to first flight, years of development, and upgrade are all statistically significant predictors of our distribution parameters. Finally, using our proposed methodology, we identify three final models which are estimated to improve time phasing accuracy when compared to the 60/40 "rule of thumb".

Appendix A: Weida (1977) Expenditure Inflection Points

	% Expenditures	% Schedule
B-1	0.56	0.36
F-105	0.58	0.34
C-141	0.54	0.51
C-5	0.52	0.34
XB-70	0.47	0.44
Tug R	0.53	0.55
Tug C	0.50	0.46
Tug E	0.64	0.51
Tug G	0.66	0.51
C-5 Quality Assurance Hours	0.44	0.46
C-5 Production Hours	0.52	0.48
A-10 System	0.49	0.47
A-10 Engine	0.69	0.62
A-10 Gun	0.56	0.45
A-10 Milestones	0.54	0.42
AGM-65 A	0.62	0.43
AWACS	0.70	0.51
Mean:	0.56	0.46
Median:	0.54	0.46
Std. Deviation:	0.08	0.07

Appendix B: Estimated Percent Expenditures at 50% Schedule, by Program

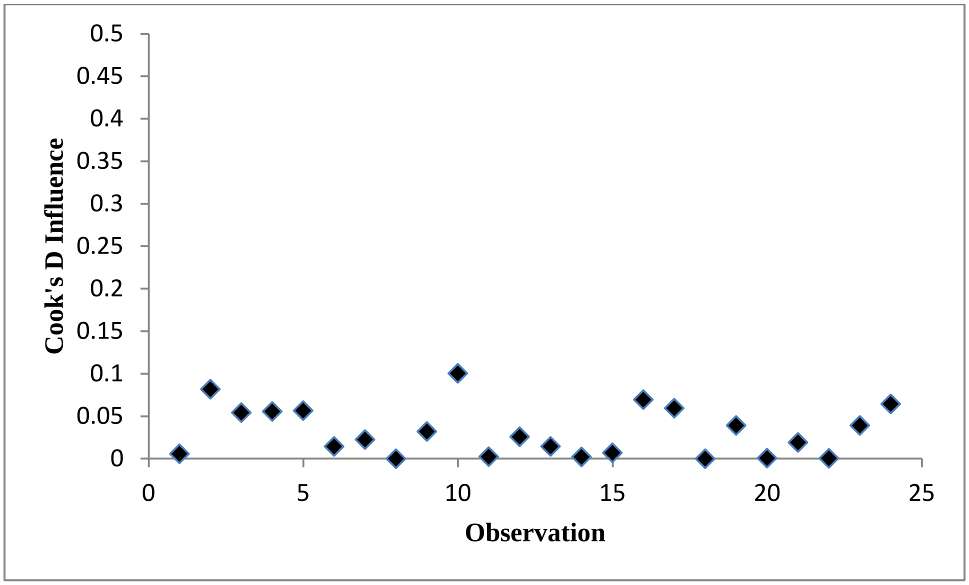
Program	Year of EMD Contract Award	Percent Expenditures at 50% Schedule		
		Linear Interpolation	Weibull	Beta
A-10	1973	0.67	0.68	0.66
AWACS BLOCK 40	2003	0.69	0.71	0.69
AWACS RSIP	1988	0.80	0.82	0.82
B-1 CMUP	1996	0.49	0.51	0.50
B-2 EHF 1	2007	0.75	0.78	0.76
B-2 RMP	2004	0.60	0.62	0.59
B-2A	1981	0.46	0.46	0.45
C-17A	1982	0.36	0.35	0.35
C-5 AMP	1999	0.57	0.59	0.58
C-5 RERP	2001	0.76	0.77	0.76
E-2D	2004	0.70	0.71	0.69
E-6A	1983	0.51	0.54	0.52
EA-18G	2003	0.49	0.50	0.48
EA-6B ICAP	1998	0.56	0.56	0.54
F-14A	1969	0.55	0.57	0.54
F-14D	1984	0.51	0.50	0.49
F-15A	1970	0.47	0.49	0.47
F-16A/B	1975	0.55	0.55	0.54
F-18E/F	1992	0.77	0.77	0.76
F-18A	1976	0.38	0.38	0.38
F-22	1991	0.60	0.63	0.60
F-35 AF	2001	0.60	0.61	0.58
F-5E	1970	0.37	0.35	0.35
JSTARS	1985	0.54	0.59	0.56
P-8A	2004	0.46	0.47	0.46
T-46A	1982	0.43	0.45	0.43
	Mean:	**0.56**	**0.58**	**0.56**
	Median:	**0.55**	**0.57**	**0.54**
	Std. Deviation:	**0.13**	**0.13**	**0.13**

Appendix C: Estimated Distribution Parameters by Program

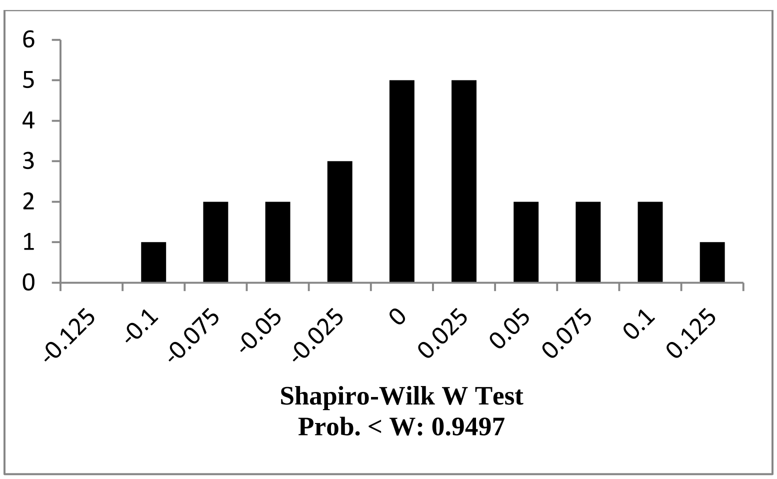
Program	Rayleigh – Scale β	Weibull – Shape α	Weibull – Scale β	Beta - Shape α	Beta - Shape β
A-10	0.46	1.65	0.46	1.24	1.84
AWACS Block 40	0.44	1.45	0.43	1.09	1.80
AWACS RSIP	0.37	1.76	0.37	1.78	3.73
B-1 CMUP	0.57	2.46	0.57	2.21	2.20
B-2 EHF 1	0.39	1.52	0.38	1.25	2.46
B-2 RMP	0.51	1.87	0.51	1.47	1.85
B-2A	0.60	2.54	0.60	2.25	2.01
C-17A	0.64	3.67	0.63	4.47	3.46
C-5 AMP	0.52	2.39	0.52	2.30	2.71
C-5 RERP	0.41	1.86	0.41	1.84	3.30
E-2D	0.44	1.71	0.44	1.46	2.33
E-6A	0.56	2.19	0.56	1.76	1.83
EA-18G	0.59	2.18	0.59	1.67	1.57
EA-6B ICAP	0.55	2.10	0.55	1.81	1.99
F-14A	0.55	1.99	0.55	1.53	1.69
F-14D	0.58	2.63	0.58	2.66	2.61
F-15A	0.59	2.42	0.59	1.98	1.86
F-16A/B	0.54	2.40	0.55	2.31	2.51
F-18A	0.64	3.10	0.64	3.09	2.41
F-18E/F	0.40	1.76	0.40	1.71	3.13
F-22	0.51	1.75	0.50	1.32	1.70
F-35 AF	0.52	1.87	0.52	1.51	1.86
F-5E	0.68	2.59	0.69	1.31	0.85
JSTARS	0.54	1.68	0.54	1.14	1.33
P-8A	0.60	2.29	0.61	1.72	1.55
T-46A	0.61	2.63	0.61	2.24	1.93
Mean:	**0.53**	**2.17**	**0.53**	**1.86**	**2.13**
Median:	**0.55**	**2.10**	**0.55**	**1.72**	**1.93**
Std. Deviation:	**0.08**	**0.50**	**0.09**	**0.72**	**0.70**

Appendix D: Tests for Influence, Normality, and Constant Variance

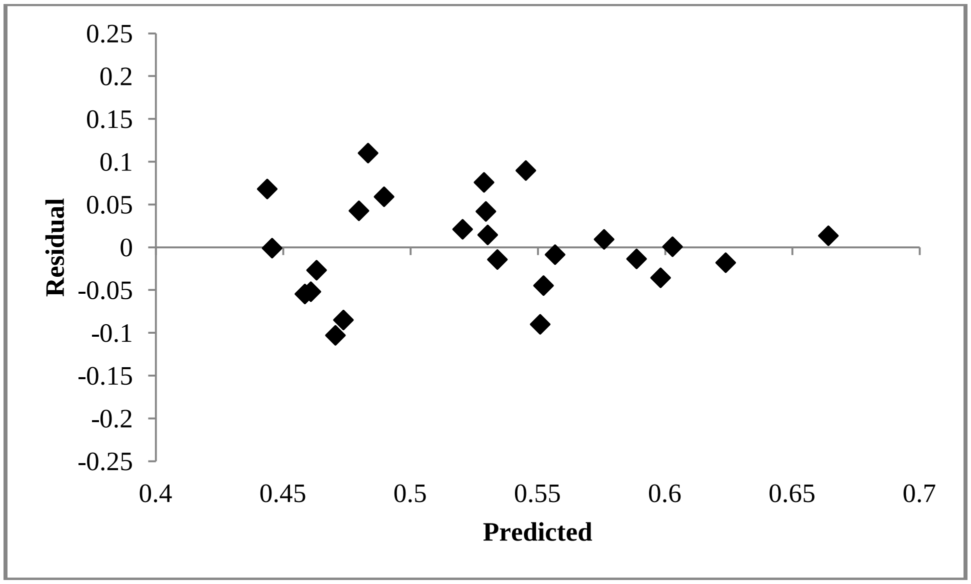
Rayleigh Scale Parameter* β *– Influential Data Points



Rayleigh Scale Parameter* β*– Normality of Residuals

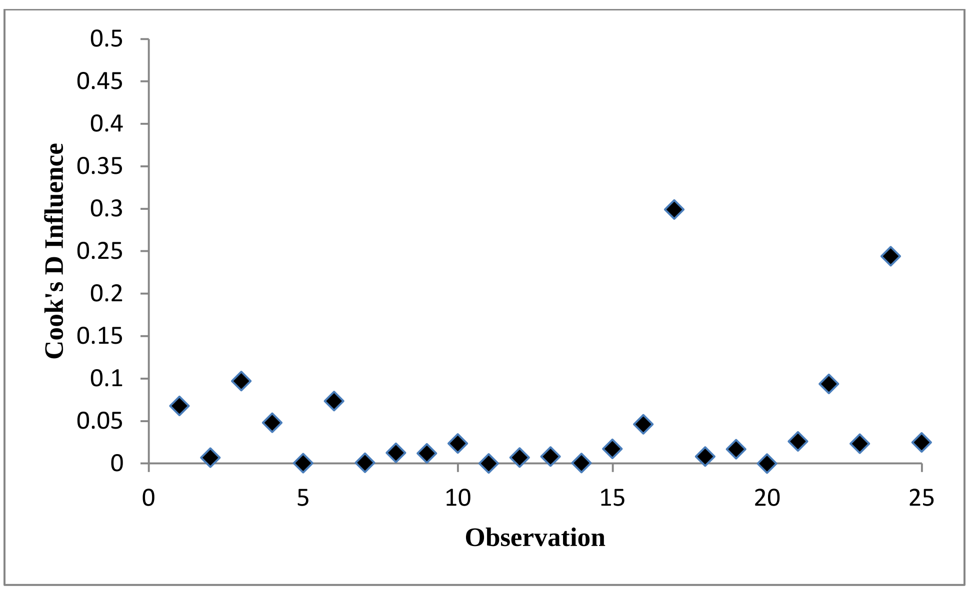


Rayleigh Scale Parameter β – Constant Variance

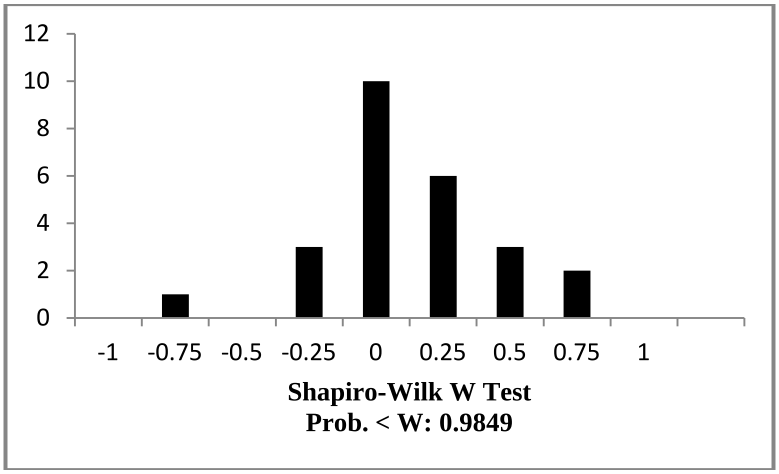


Breusch-Pagan Test:
P-value – 0.5264

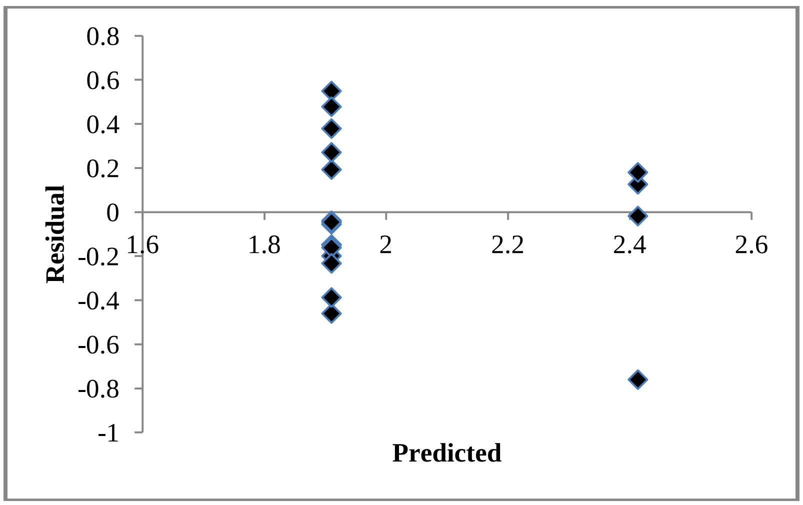
Weibull Shape Parameter α – Influential Data Points



Weibull Shape Parameter α – Normality of Residuals

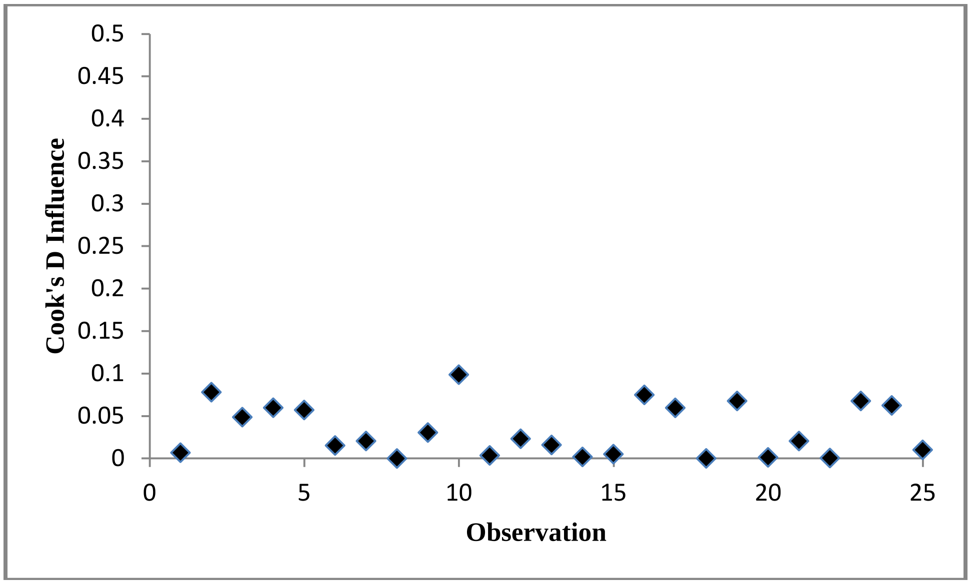


Weibull Shape Parameter α – Constant Variance

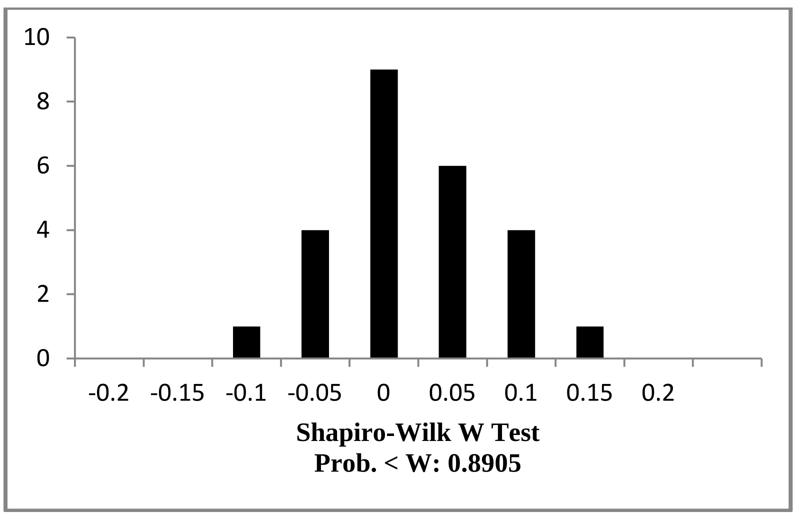


<u>Breusch-Pagan Test:</u>
P-value – 0.6911

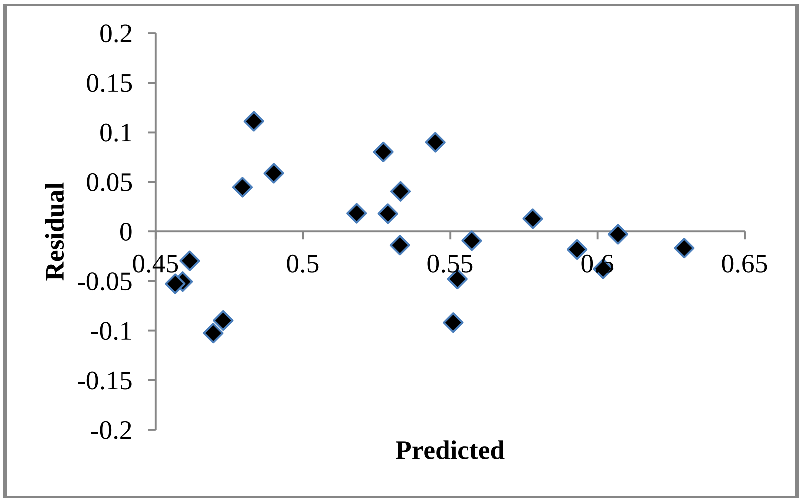
Weibull Scale Parameter β – Influential Data Points



Weibull Scale Parameter β – Normality of Residuals

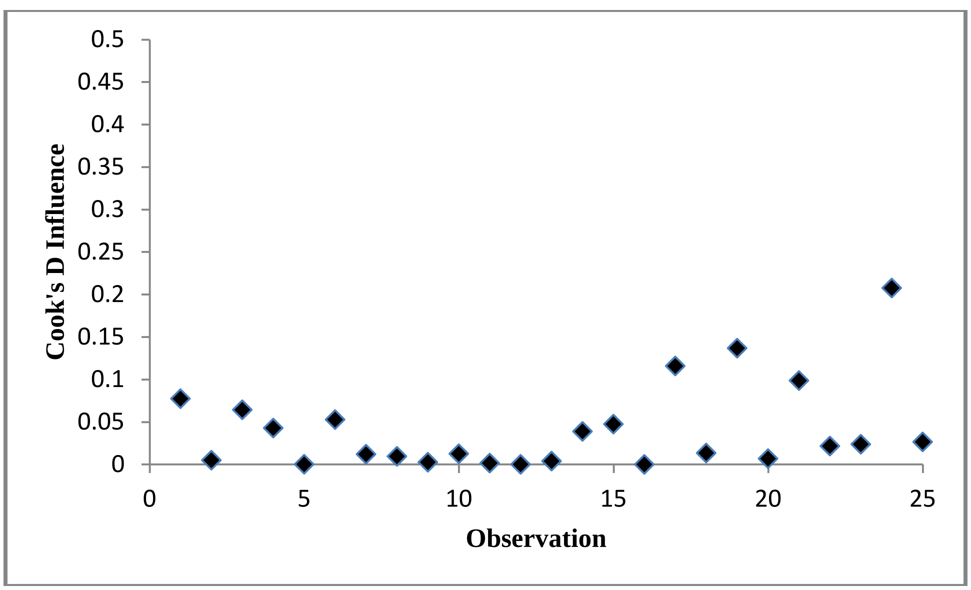


Weibull Scale Parameter β – Constant Variance

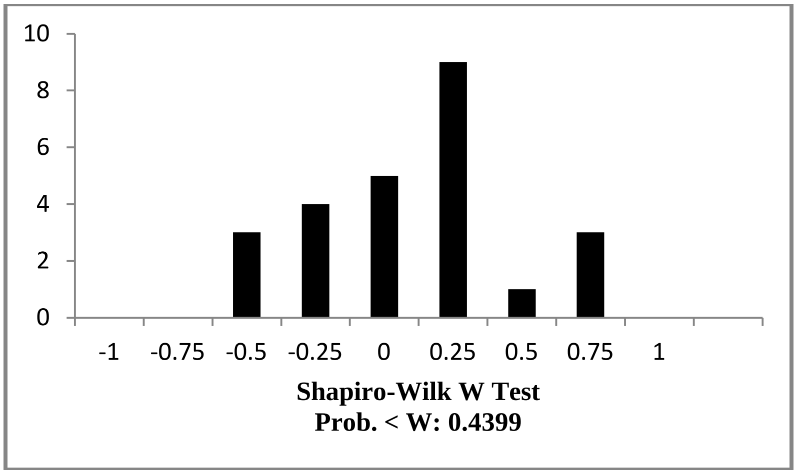


<u>Breusch-Pagan Test:</u>
P-value – 0.5176

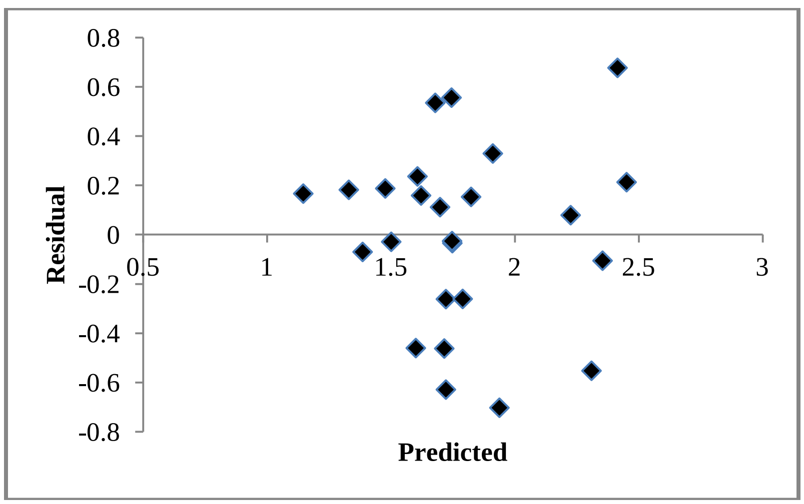
Beta Shape Parameter α – Influential Data Points



Beta Shape Parameter α – Normality of Residuals

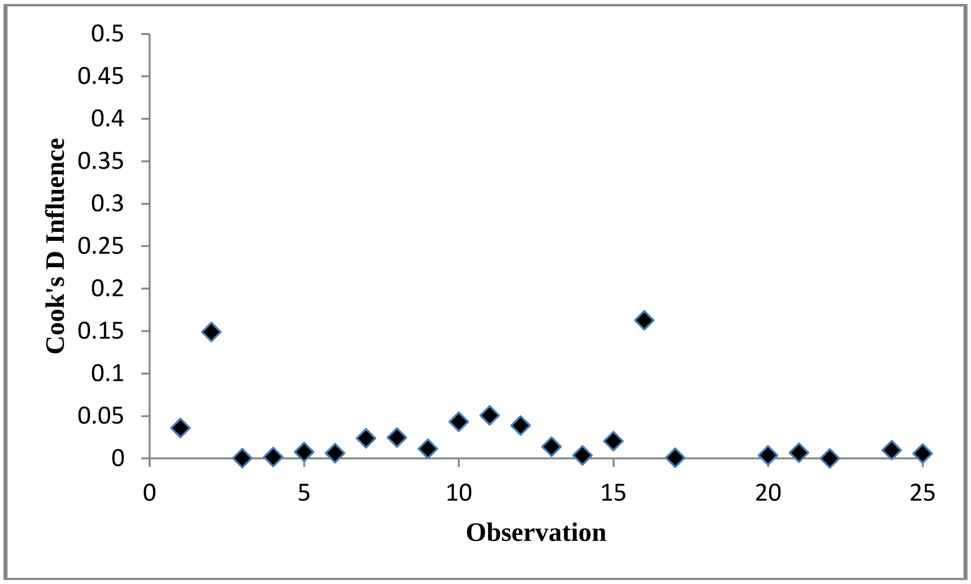


Beta Shape Parameter α – Constant Variance

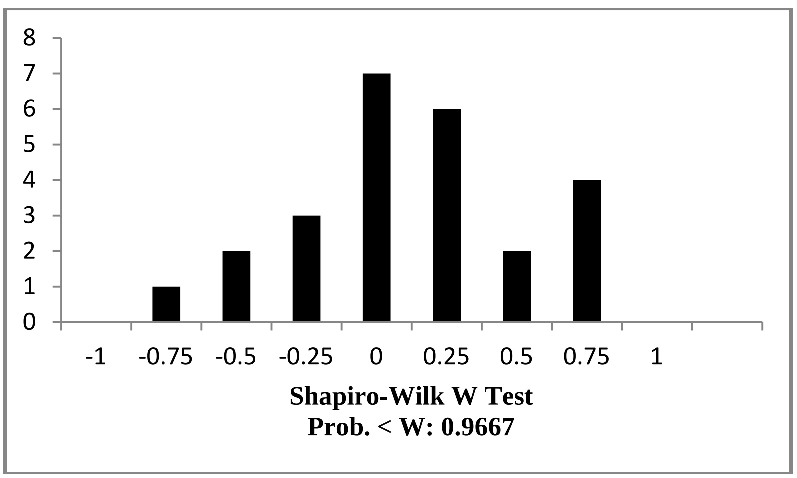


<u>Breusch-Pagan Test:</u>
P-value – 0.6473

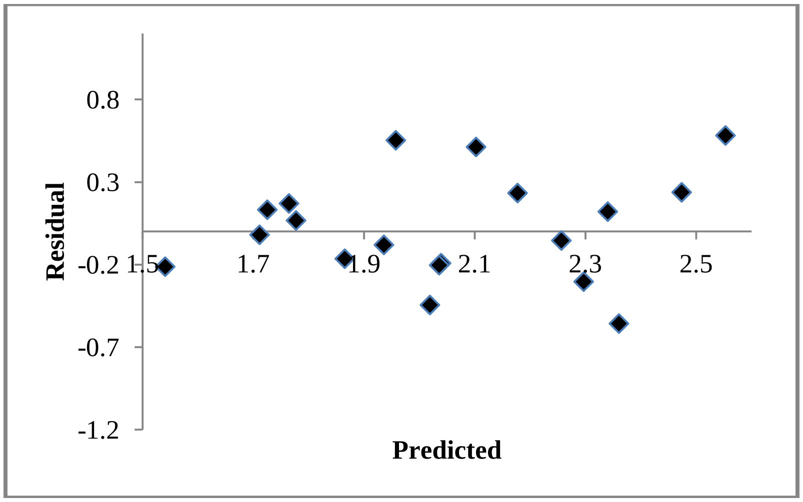
Beta Shape Parameter β – Influential Data Points



Beta Shape Parameter β – Normality of Residuals



Beta Shape Parameter* β *– Constant Variance



<u>Breusch-Pagan Test:</u>
P-value – 0.4188

Appendix E: Functional Forms of Final Models (Excel Notation)

Note: For easier application to contemporary aircraft, input variables which are applicable only to aircraft with pre-1985 contract awards have been removed from models.

Final Rayleigh Distribution Model

$$= Weibull(\%\ schedule, \alpha, \beta, true)$$

$$\alpha = 2$$

$$\beta \ = \ 0.45026 + 0.2404\ (\%\ Schedule\ at\ First\ Flight)$$
$$-\ 0.07819\ (0\ if\ New\ Start; 1\ if\ Upgrade)$$

Final Weibull Distribution Model

$$= Weibull(\%\ schedule, \alpha, \beta, true)$$

$$\alpha = 1.91047$$

$$\beta \ = \ 0.44265 + 0.2587\ (\%\ Schedule\ at\ First\ Flight)$$
$$-\ 0.07906\ (0\ if\ New\ Start; 1\ if\ Upgrade)$$

Final Beta Distribution Model

$$= Betadist(\%\ schedule, \alpha, \beta)$$

$$\alpha = \ -1.80324\ -\ 0.32456\ (Length) -\ 2.94584\ (\ln \frac{1}{Length})$$

$$\beta = -0.36129\ +\ 0.155517\ (Length)$$
$$+\ 1.49292\ (0\ if\ New\ Start; 1\ if\ Upgrade)$$

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Vita

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14. ABSTRACT

Early research on time phasing primarily focuses on the theoretical foundation for applying the continuous distribution function, or S-curve, to model the distribution of development expenditures. Minimal methodology is provided for estimating the S-curve's parameter values. Brown, White, and Gallagher (2002) resolve this shortcoming through regression analysis, but their methodology has not been widely adopted by aircraft cost analysts, as it is judged as overly broad and not specific to aircraft. Instead, analysts commonly apply the 60/40 "rule of thumb" to aircraft development, assuming 60 percent expenditures at 50 percent schedule. It is currently unknown if the 60/40 heuristic accurately describes contemporary aircraft development programs. Therefore, using a sample of 26 DoD aircraft programs, we first test the accuracy of 60/40, discovering that, as a heuristic, the 60/40 cannot account for differences between new start and upgrade programs. Next, we improve upon prior research by using program characteristics to construct an aircraft-specific methodology for estimating parameters. Finally, we conclude our research by comparing the accuracy of our Rayleigh, Weibull, and Beta S-curve models. Our Weibull model explains 74.6 percent of total variation in annual budget, improving the estimation of budgets by 6.5 percent, on average, over the baseline 60/40 model.

15. SUBJECT TERMS

Time Phasing, S-curve, 60/40, Aircraft, Rayleigh, Weibull, Beta, Continuous Distribution Function

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