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Understanding the Instruments of National Power through a System of Differential Equations in a Counterinsurgency

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UNDERSTANDING THE INSTRUMENTS OF NATIONAL POWER THROUGH A SYSTEM OF DIFFERENTIAL EQUATIONS IN A COUNTERINSURGENCY

THESIS

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Abstract

Models that account for the progression of nation–building and the impacts of the instruments of national power – Diplomacy, Informational, Military, and Economic effects – are rare. This research proposes the development of such a model. Through the derivation of state indices for the operational variables: Political, Military, Economic, Social, Infrastructure, and Information, a functional form of a system of differential equations is developed to account for the interactions between the state indices and instruments of national power. The developed methodology is a mean–field inverse problem which solves for the coefficients of the differential equations in a data driven manner. Publicly–available data is used in the development of the indices and to describe the instruments of national power. Applying mean-field theory allows the indices to take on a mean value, in order to systemically solve the differential equations through a nonlinear program which derives minimum error producing coefficients. These models can then be used for a more in–depth analysis of historical events.

An application of the model is derived for Operation Iraqi Freedom to demonstrate the utility as well as effects of various alternate strategies, using the dynamics captured in the model. This modeling approach offers a potential significant capability when analyzing and planning for future Stabilization, Security, Transition, and Reconstruction Operations.

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To my wife

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I. Introduction

1.1 Background

> The United States has a significant stake in enhancing the capacity to assist in stabilizing and reconstructing countries or regions, especially those at risk of, in, or in transition from conflict or civil strife, and to help them establish a sustainable path toward peaceful societies, democracies, and market economies. The United States should work with other countries and organizations to anticipate state failure, avoid it whenever possible, and respond quickly and effectively when necessary and appropriate to promote peace, security, development, democratic practices, market economies, and the rule of law. Such work should aim to enable governments abroad to exercise sovereignty over their own territories and to prevent those territories from being used as a base of operations or safe haven for extremists, terrorists, organized crime groups, or others who pose a threat to US foreign policy, security, or economic interests. [10]
>
> – George W. Bush

A key interest of the United States are those states or regions which are in transition or reconstruction. As stated above the US seeks to aid and assist when needed in order to prevent states from becoming safe havens for threats. In the short history of the US there have been several military operations (Operation Urgent Fury – Grenada, Operation Just Cause – Panama, Operation Desert Storm – Kuwait, Operation Enduring Freedom – Afghanistan, Operation Iraqi Freedom – Iraq, *etc.*) which have been instances of these type of operations. While the operations vary in size, purpose, and intensity they remain examples of how the US employed its military capability to support its strategic goals.

The range of military support provided in foreign countries across the continuum from peace to conflict in order to assist a state or region that is under severe stress or has collapsed is called Stabilization, Security, Transition, and Reconstruction operations (SSTRO). These are not unilateral military operations but rather the synchronous effort of military and civilian, public and private, as well as US and international efforts to provide assistance to the state or region in need [49:i].

According to the Military Support to Stabilization, Security, Transition, and Reconstruction Operations Joint Operating Concept the definition for these four types of operations are:

Stabilization- involves activities undertaken to manage underlying tensions, to prevent or halt the deterioration of security, economic, and/or political systems, to create stability in the host nation or region, and to establish the preconditions for reconstruction efforts [49:2].

Security- involves the establishment of a safe and secure environment for the local populace, host nation military and civilian organizations as well as USG and coalition agencies, which are conducting SSTRO [49:2].

Transition- describes the process of shifting the lead responsibility and authority for helping provide or foster security, essential services, humanitarian assistance, economic development, and political governance from the intervening military and civilian agencies to the host nation. Transitions are event driven and will occur within the major mission elements (MMEs) at that point when the entity assuming the lead responsibility has the capability and capacity to carry out the relevant activities [49:2-3].

Reconstruction- is the process of rebuilding degraded, damaged, or destroyed political, socio-economic, and physical infrastructure of a country or territory to create the foundation for longer-term development [49:3].

The tools available to the US to conduct SSTRO are the instruments of national power (Diplomacy, Informational, Military, and Economic or DIME). These are the means through which the US applies its sources of power, to include– culture, industry, science and technology, academic institutions, and national will to achieve its national strategic objectives [17:I-8].

1.2 Problem Statement

With the threat of conflict or diaster striking anywhere in the world at a moments notice the ability to evaluate how specific packages of DIME assistance might be implemented and an assessment of their perceived impact would be a tool of great use in strategic level planning. Currently the Department of Defense (DOD) has tools to help determine, predict, and classify which states are instable, on a trajectory to become instable, or susceptible to instability in future years. None of theses tools are prescriptive in nature nor do they indicate what can be done to increase the level of stability in these states to reverse the current situation or trajectory [48].

Some assessments of SSTRO during Operation Iraqi Freedom (OIF) indicate the belief that the economic instrument of DIME was ineffective and perhaps even counter–productive. The point that leads to this assessment is that the impact of the economic effort is largely unknown and based upon anecdotal evidence which leads to the wrong lessons [15]. Even with a large focus on measuring and assessing, the impacts of the economic aid is largely unknown due to yearly turnover of units. Large portions of data are erased or destroyed as one unit departs and another arrives, thus making it difficult to evaluate anything over one year.

This research develops a methodology which considers the interactions of the nation providing SSTRO and the nation receiving SSTRO as a complex dynamical system. This system is then solved as in inverse problem with an application of mean–field theory. A solution methodology is presented and then an application using data from OIF demonstrates the dynamics captured from the actual data.

1.3 Research Objective and Scope

> As the Nation continues into this era of uncertainty and persistent conflict, the lines separating war and peace, enemy and friend, have blurred and no longer conform to the clear delineations we once knew. At the same time, emerging drivers of conflict and instability are combining with rapid cultural, social, and technological change to further complicate our understanding of the global security environment. Military success alone will not be sufficient to prevail in this environment. To confront the challenges before us, we must strengthen the capacity of the other elements of national power, leveraging the full potential of our interagency partners. [21:Forward]
>
> LTG William B. Caldwell, IV
> Commander, US Army Combined Arms Center

The nature of conflict is increasingly complex and uncertain. LTG Caldwell's words underscore both the importance of national power and the necessity to more clearly understand its effects on and within an uncertain military environment. The dynamic and potentially unpredictable nature of SSTRO is increasingly evident when the second and third order effects are considered. This indicates the need for dynamic models and data that can help make decisions when lives, money, and national security objectives are in jeopardy [16].

In order to understand the importance and effects of DIME on and within an uncertain military environment there must be a method to collect, analyze, and interpret data which provides insight on these instruments.

The objective of this research is to develop a methodology that provides insight on the application of the instruments of national power. This methodology will make use of unclassified data so that it may be applied in multiple situations under different conditions. The methodology is generic enough to apply to any nation yet resilient to changes in the data structure that are required to conduct multiple assessments.

This methodology entails–

- collecting and normalizing data to create indices which capture the current state.
- using numerical techniques to fit curves to the raw data and approximate derivative functions for the data.
- conjecturing a functional form as a system of differential equations which accounts for interactions between the indices and the impacts of the instruments of national power.
- formulating a nonlinear program to solve for the coefficients in the system of differential equations.
- using a numerical method with the results of the system of differential equations to gain insight on the system.

1.4 Summary

The relevance of this research was provided in this chapter as well as a brief description of the model methodology implemented in the research. The research presented here is based on unclassified information retrieved from the Brookings Institution collected during OIF and is representative of other conflicts. The results and findings will be presented to the Center for Army Analysis (CAA) as well as a graphical user interface which assists in evaluating various strategies.

The remainder of this document is organized as follows:

- Chapter II reviews the relevant literature that applies to the creation of the solution methodology.
- Chapter III discusses the development of the model and solution methodology.
- Chapter IV is an application of the model using the data from OIF as a test case and to demonstrate possible applications of the methodology.
- Chapter V presents a review of the significant insights, identifies topics for future research efforts, and conclusion.

II. Literature Review

This chapter examines the specific literature which applies to the creation of the model and solution methodology. Each area focuses on a specific concept and how that concept was applied in this methodology. The literature review provides the reader with the background context and justification of the methodology. Each section will specifically address the concepts selected in the creation of the methodology. A detailed description of how the concepts are applied is then addressed in Chapter III.

2.1 Instruments of National Power

The instruments of national power, sometimes referred to as the elements of national power, or categorized under the acronym DIME, are tools at the disposal of the US to aid in achieving its national strategic objectives. Through a coordinated interagency effort the employment of some or all of these instruments is an effective way for the US to apply its sources of power, to include its culture, industry, and national will [17:I-8]. All four instruments may be applied or a subset of the four, they are not necessarily mutually exclusive events. A detailed description of each DIME instrument–*Diplomacy, Informational, Military, and Economic* follows.

2.1.1 Diplomacy.

The diplomatic instrument is the primary means employed by the Department of State (DOS) to engage other states to advance the values, interests, and objectives of the US. The leaders of the US are responsible for having a clear understanding of the capabilities and consequences of military action. The threat of force can reinforce or enable the diplomatic process, however, just as easily that threat can destroy any progress. Within an area of responsibility (AOR) a combatant commander is responsible for integrating the military and diplomatic activities. Typically, the US ambassador and the corresponding country team are responsible for integrating the diplomatic-military activities abroad. In some specific instances, when directed by the President or Secretary of Defense (SecDef), the combatant commander will employ military forces as well as the other instruments of national power. In these instances, the US ambassador and team may complement the combatant commander by providing assistance with the diplomatic instrument activities that do not involve the military instrument. In these specific instances, planning should be complementary and coincidental, displaying a coordinated effort from the combatant commander and ambassador [17:I-9].

Diplomacy is not limited too but certainly may include actions such as providing mentors to a fledgling government, providing an interim structure to aid during a transition period, providing and conducting elections, and to both support and legitimize a government in the eyes of other states. Assessing and determining the level of diplomatic assistance is a complicated task that is often highly debated as well. As a result, diplomacy will not be included in this research.

2.1.2 Informational.

The informational instrument deals with both the protection of information and the distribution of information. The US has a free market for ideas, however, this does not mean that all ideas are distributed freely or shared openly. The US goes through great lengths to protect restricted information from unauthorized access. The distribution of information is a calculated step which must be executed exactly. The widespread distribution of an incorrect message would be disastrous. This leads to a US policy where the government will provide top-down guidance when it comes to employing the informational instrument. According to JP 1-0, the uses of the informational instrument are "...processes and efforts to understand and engage key audiences to create, strengthen, or preserve conditions favorable to advancing national interests and objectives through the use of coordinated information, themes, messages, and products synchronized with the actions of all instruments of national power." [17:I-9] A holistic effort is implemented through defense support to public diplomacy and military diplomacy activities [17:I-9].

The traditional military support to the informational instrument are:

- Information Operations (IO)– military operations which include offensive (attacking the adversary's information) and defensive (protecting our information) [17:I-9].
- Public Affairs (PA)– public information which is directed to both the adversary's public and to the US public [17:I-9].
- Defense Support to Public Diplomacy (DPSD)– measures taken by DOD components to support and facilitate public diplomacy efforts by the U.S [17:I-9].

The successful execution of the informational instrument requires integration into military plans and operations; as well as synchronization with all US efforts and agencies, and multinational partners. The informational instrument is perhaps one of the most complex instruments to manage because it is so difficult to measure. The ability to determine the impact or calculate the reception of a message is unknown and for this reason the informational instrument is not included in this research effort.

2.1.3 Military.

The military instrument is perhaps the most well–known, often used aspect of DIME. The US military can take on a wide range of operations in order to support the US national strategic objectives. In this instance the focus is on SSTRO and related operations. It is not unusual to see the military instrument used both home and abroad and that is why the military must strictly adhere to the values, principles, and standards set forth by the appropriate service organizations. The traditional purpose of the military is to fight and win the nation's wars. In the case of SSTRO and some other operations, the application of the military instrument continues well after the completion of a war or in some instances without conflict. The wide range of operations the military can conduct makes it a valuable commodity in the application of DIME. The military aspect must be integrated into the overall plan and synchronized with all other efforts [17:I-10]. In this research, the military instrument is defined as the number of troops on the ground in support of US mission.

2.1.4 Economic.

The economic instrument of national power is supported by the free market economy of the US. The backbone of this system is the Department of the Treasury, which is an influential participant in the international economy. As a part of the economic instrument of national power, the Department of the Treasury works with other federal agencies, other nations, and international financial institutions to encourage economic growth, raise standards of living, and predict and prevent, to the maximum extent possible, economic and financial crises [17:I-10]. The typical employment of the economic instrument in SSTRO is through aid packages and assistance to the nations economy, this is to aid in making a self–supportive nation, when aid is curtailed. In this research the economic instrument is defined as the amount of economic support provided to the nation.

2.1.5 DIME Summary.

According to doctrine, the application of DIME must always be a concerted effort executed by all agencies providing support. When executed in this manner the greatest effects can be realized. As demonstrated above, the DIME instruments are all actions provided or imposed by the US, in this case, they are exogenous or external variables to the state (system). In the system to be examined, there will be two instruments of national power considered, the acronym diME will be used to describe the military and economic instruments.

2.2 Operational Variables

Joint planning is in terms of six interrelated operational variables (*Political, Military, Economic, Social, Information, and Infrastructure* or PMESII), as shown in Figure 1 [20:1-5].

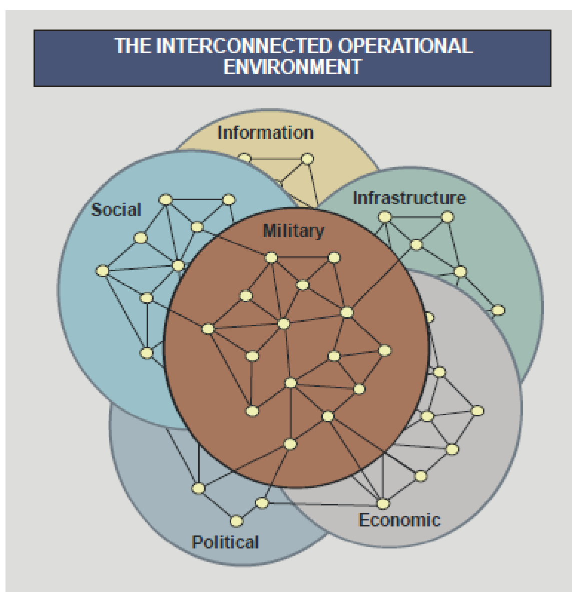


Figure 1. The Interconnected Operational Environment [18]

These variables encompass the broadest aspects of both the military and nonmilitary environment and will differ for each operation as well as different AORs within the same operation. The PMESII variables describe the military aspects of the operational environment as well as the influence of the population on the operational environment. As a result, they provide a view which emphasizes the human aspects of the operational environment. As the US becomes involved in more operations (with non-contiguous battlefields), understanding the human variables is crucial because the US forces live and operate amongst the population. A thorough understanding of PMESII helps combatant commanders to appreciate how the military instrument complements the other instruments of national power [20:1-5]. A description of the operational variable components used in this model are provided in Chapter III.

2.2.1 Political.

The political variable is a description of the distribution of responsibility and power across all levels of government. Whether the political structure has a strong degree of legitimacy with the population and international organizations it can strongly influence events. The political leaders will often use ideas, beliefs, and even violence to enhance their power and control over people and resources. There may be conflicting political groups and each may interact with the US or multinational force differently. Understanding the unique circumstances that motivate and drive these groups requires an understanding of all the relevant partnerships– *political, economic, military, religious, and cultural* [20:1-5,6].

2.2.2 Military.

The military variable describes the military capability of all armed forces in a specific operational environment. The armed forces of a state may include the role of providing both internal and external security. Additionally, influencing the military variable are paramilitary and guerilla forces, as well as the armed forces of neighboring or ally states. The organization's ability to field capabilities and use them locally, regionally, or globally is one way of assessing the military variable. These capabilities include: 1) Equipment, 2) Manpower, 3) Doctrine, 4) Training levels, 5) Resource constraints, 6) Leadership, 7) Organizational culture, 8) History, 9) and Nature of civil-military relations [20:1-6].

2.2.3 Economic.

The economic variable deals with the behaviors of individuals and groups pertaining to the production, distribution, and consumption of resources. The influence of industry, trade, development, finance, policies, capabilities, and legal constraints will play a significant part in the behaviors associated with economics. While a global economy is often described, the truth is economies differ by region and maybe affected by things such as the political environment. Factors associated with changes in the economic environment may include investments, price fluctuation, debt, and the existence of black markets. A deep understanding of the cost/benefit relationship in the political–economic realm leads to a better understanding of the interactions of the different groups within a state [20:1-6,7].

2.2.4 Social.

The society within a operational environment is the social variable. A society is "a population whose members are subject to the same political authority, occupy a common territory, have a common culture, and share a sense of identity" [20:1-7]. Societies include diverse structures that may only be observed by a member of that society or state. Cultures are composed of shared beliefs, values, customs, and behaviors that are used by society members to cope with one another and other societies. As with many things the attributes of a society may change over time, leading to a split within a society. The societies actions, opinions, or influences should be considered within the social environment, as they can have an effect on the mission [20:1-7].

Combatant commanders must develop an understanding of the social aspects within their AOR, as it allows for an understanding of the impact of operations and interactions with local leaders. Cultural disasters can often be avoided through understanding and cultural awareness [20:1-7].

2.2.5 Information.

The information variable describes the information environment, which is the network of people, organizations, and systems that collect, process, disseminate, or act on information. The information passed along this medium is often rapidly available and can be used by the US to shape the environment, used by an adversary to control and manipulate perception and understanding, and will be used to form opinions and make decisions. Not all states have a complex telecommunications network to share information, but nonetheless the information will be shared through less sophisticated methods [20:1-8]. Due to the complex nature and difficulty in measuring the effectiveness of information, it will not be considered it in this research.

2.2.6 Infrastructure.

The infrastructure variable describes the basic facilities, services, and installations required for a society to function. This also includes technological advances and development which can be applied to both civil and military purposes. All aspects of the environment must be considered when improving the infrastructure, certain actions can be perceived as favoring one group over another or may even offend some of the cultural ideologies of one group. Information operations are an important aspect of the infrastructure variable [20:1-8].

2.2.7 PMESII Summary.

Each operational environment is different and constantly evolving over time, the PMESII variables are used to help understand this complex adaptive environment. Through the PMESII variables, focus can be placed on specific elements within the operational environment which apply to the mission [20:1-5]. These operational variables are measures representing the internal or endogenous variables within the state (system). This research will use five of the six variables (excluding information) and refer to this set as PMESI.

2.3 Inverse Problems

The goal in solving several types of problems is to determine the set of parameters which describe the system and the laws relating the values of the parameters to the results of measurements. When some information is known about the values of the measurements, a theoretical relationship can be used to infer information on the values of the parameters. When the problem is posed in this manner, it is called an inverse problem. In inverse problems, the data are results of the measurements and the unknowns are the values of the parameters [52]. Partial information is given or known about a state function, $x(t)$ and the goal is to infer something about the laws governing state evolution, values of constant parameters, values of exogenous functions which characterize the system, or values of boundary conditions at certain points in time [24].

Tarantola and Valette propose that all problems, inverse or not, in order to be stated as well-posed problems are formulated as follows:

1. We have a certain state of information available on the values of the data set [52].
2. We also have a certain state of information on the values of the unknowns [52].
3. We have a certain state of information concerning the theoretical relationship that exists between the data and the unknowns [52].
4. Which is the final state of information on the values of the unknowns resulting from the combination of the three preceding states [52]?

Many experiments contain a finite amount of data in which can reconstruct a model with infinitely many degrees of freedom. The result is an inverse problem which is not unique in that there are many models that can explain the data. According to Tarantola and Valette, inversion really consists of two steps, from the traditional inverse problem there is the true model (m) and data (d). From the data, d an estimated model ($\hat{m}$) is constructed, this is an *estimation problem*. Additionally, the relationship between the estimated model, $\hat{m}$ and the true model, m must be investigated. This is called the *appraisal problem* [50]. The notion of this division of problems is demonstrated in Figure 2. The estimation problem is typically solved

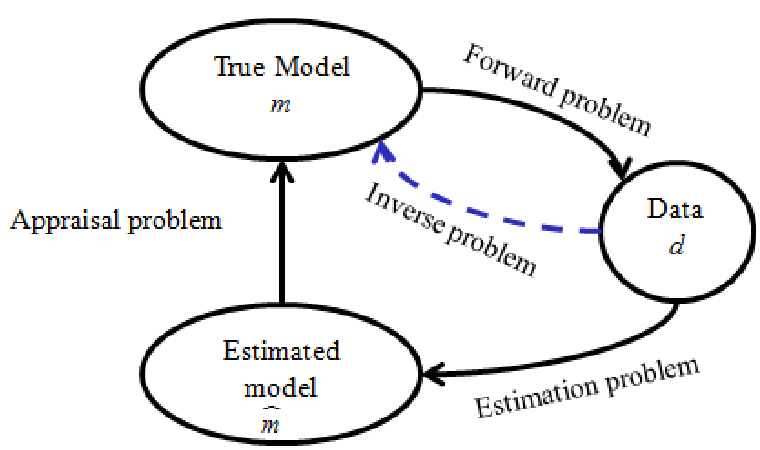


Figure 2. A problem divided into a forward problem, estimation problem, and appraisal problem for finite data sets, adapted from [50:389]

by fitting the model to the data, by letting the i^{th} data element d_i be related to the model m through the following relation

$$d_i = G_i(m) \tag{2.1}$$

where $G_i(m)$ is a nonlinear function and $G_i(m) \mapsto d_i$. The data fitting can be accomplished by minimizing the difference between the real data, d_i and estimated data $G_i(\hat{m})$ through a least-squares fit

$$S(\hat{m}) = \sum_i (d_i - G_i(\hat{m}))^2 \tag{2.2}$$

as a function of the estimated model $\hat{m}$[50].

The use of inverse problems in this research is to infer information about the dynamics of the system through a priori knowledge of the exogenous system variables, much like Lanchester equations in Section 2.4.

2.4 Lanchester Equations

Within military applications, one of the most famous treatments of inverse problems are the Lanchester equation solutions, published in F. W. Lanchester's *Aircraft in Warfare: The Dawn of the Fourth Arm in 1916*. While much of the focus of Lanchester's book centers on aircraft and their emerging use in World War I, his major contribution was to offer a set of differential equations to model combat power for both enemy (x) and ally (y) strength, by using existing data. Beginning with aerial combat he developed the Linear Law

$$\frac{dx}{dt} = -Axy \tag{2.3}$$
$$\frac{dy}{dt} = -Byx$$

where attrition is proportional to the attrition coefficients (A, B) and the size of both forces (x, y). This is associated with area fire such as indirect fire. To deal with direct or aimed fire, he develop the Square Law

$$\frac{dx}{dt} = -Ay \tag{2.4}$$
$$\frac{dy}{dt} = -Bx$$

where attrition is proportional to the strength (x, y) and effectiveness (A, B) [30]. Later he broadened his equations to apply in other types of conflicts. As an inverse problem, the coefficients are determined through knowledge of the data at time t. His work continues to serve as the basis and motivation for much research, to include this research.

Over time, several researchers have used Lanchester equations on prominent battles as the data became available. In addition to using new data, they also applied the equations to different types of conflicts, evaluated several different parameters, and different methods to solve the parameters. Bracken generalized Lanchester equations to model the Ardennes campaign in World War II, where he considered the performance of either opposing force at a point in time, with tactical parameters and attrition rates. Bracken solved for these parameters by implementing a *brute–force* method through a constrained grid search [9]. Extending Bracken's work, Fricker examined the same Ardennes campaign, but used a linear regression technique [22]. Clemens analyzed the same data set utilizing a nonlinear fit with the Newton-Rhapson algorithm [14]. Helmbold makes use of the Newton-Rhapson algorithm while examining the square law with scheduled reinforcements, as a direct problem and as an inverse problem [24]. Lucas and Turkes applied a response surface methodology to the Ardennes data set and solved for the parameters by regression through the origin. This method allowed them to use a contour plot and visually assess the optimal point for the parameters. Lucas and Turkes also advanced the idea of using R^2 (Equation 3.9) when using linear regression to compare models using weighted data [31]. Previous methods had primarily focused on the sum of squared residuals or sum of squared errors (SSE).

The conjugate gradient method is used by Chen to determine the coefficients for time dependent attrition in the nonlinear Lanchester square law inverse problem

$$\frac{dx_1(t)}{dt} = -D(t;x_1,x_2)x_2(t) + \frac{dR_1(t)}{dt}, \qquad t > 0; x_1(0) = x_{1,0} \tag{2.5}$$

$$\frac{dx_2(t)}{dt} = -A(t;x_1,x_2)x_1(t) + \frac{dR_2(t)}{dt}, \qquad t > 0; x_2(0) = x_{2,0} \tag{2.6}$$

where

A, D are force dependent attrition coefficients

R is the total reinforcement

x_1, x_2 are the estimated force strengths

by making use of the observed force strength data. In order to numerically solve these equations the fourth–order Runge–Kutta method was used. Chen found this method to be advantageous because there was no prior knowledge required to solve for the unknown parameters. This method allows for an arbitrary guess as an initial starting point [13].

The application of Lanchester Equations has been documented in other areas as well, such as irregular warfare. Schaffer, for example, used Lanchester Equations to model guerilla warfare and asymmetric engagements while employing and operationalizing an array of variables and coefficients (representing weapons strength, discipline, morale, *etc.*) to model an insurgent force in Phase II of an insurgency [47]. Richardson used a system of differential equations to model the arms race and instability of nation states based upon the current levels of its neighboring and/or *menacing* states [43]. Lanchester Equations and the types of models derived from them show the application of the inverse problem in many different fields.

2.5 Mean–Field Theory

Many problems involve a large number of independent variables where the exact calculation of such a problem is infeasible. In order to solve these problems, efficient approximation techniques are needed in order to better understand their dynamics. The method of using a Mean–Field Equation (MFE) to approximate these dynamics is an efficient approximation method to aid in solving problems dealing with uncertainty and complexity [40:ix,1].

In this method, the values of the variables to be examined are replaced with the MFE. The variables of the dynamical system are used to determine some mean value; this is accomplished through an equation that provides the mean–field simplification. This allows the focus to be placed on one variable at a time by effectively holding the others constant. To think of this intuitively, envision a problem with multiple variables and only one is not represented by its mean value. This leaves the one free variable independent of the others, thus creating the ability to calculate the value of the free variable. The process is then repeated in the same fashion over all of the remaining values. Persson, Claesson, and Nordebo use this technique to conduct discrete adaptive filtering using a mean–field algorithm to minimize the Wienr-Hopf equations in a least–squares sense to produce comparable results without transient behavior and facilitate abrupt system changes [41].

The mean–field equations can be the mean value or an approximated probability distribution to represent the unknown variables. With a large number of variables that exhibit nonlinear behaviors fitting them with a nonlinear least–squares model is an effective method [41]. This concept was shown using an epidemic model based upon a system of differential equations by Kleczkowski and Grenfell [28].

This method injects a portion of generality into the process which still accounts for the noise in the system, yet simplifies the problem by using a constant in the place of an ever–changing variable. This method replaces some of the stochastic elements with deterministic elements, the result is stochastic system represented through its deterministic equivalent.

This method is not without error, as the number of estimated variables reduces the overall confidence level of the result by one degree of freedom each time. This does not indicate inaccuracy, but rather that the end result will be an overall estimation of the system based upon the previous interactions. It may downplay the effect of outliers in the generalization, but it does account for them. The error in the system is expected to be normally distributed. This is important to the least–squares fitting aspect and the independence of the variables in the system.

This method replaces an infinite dimension system with several dependent variables, with a series of independent variables in a finite dimension system, thus reducing the overall complexity of the problem and allowing us to understand the dynamics of complex problems.

2.6 Dynamical Systems

When creating models, a real world system to study is identified, all the aspects of that system are studied, and assumptions are made when and where they are needed. After studying the system, it is often translated into a mathematical relationship which can be modeled. A knowledge of mathematics is used to conduct analysis on this system, in hopes that the complicated interrelationships that exist in the real world system can be solved; then translate the knowledge gained from the model back to the real world system. Dynamical modeling is the science of modeling real world phenomena as it changes over time [46:3].

According to Boccara and Meiss, the definition of a dynamical system is a set or system of equations whose solution describes the evolution or trajectory, as a function of a parameter (time) along a set of states (phase space) of the system [33:105-106] [7:11]. The theory behind dynamical systems is primarily concerned with the qualitative properties of the system dynamics and gaining an understanding of the asymptotic properties, as $t \to \infty$.

A typical dynamical system is comprised of a phase space, $\mathcal{S}$, whose elements represent all possible states for the system; a time parameter, t, which may be discrete or continuous; and an evolution rule (a rule that governs the transition of states from t_i to t_{i+1} based upon knowledge of the states at prior times) [7:105-106]. A dynamical system is characterized according to these three elements. Systems with both discrete time space and time variables are often considered mappings. When the evolution rule is deterministic then for each time, t, it is a mapping from phase space to phase space

$$\varphi_t : \mathcal{S} \to \mathcal{S} \tag{2.7}$$

so that $x(t) = \varphi_t(x_0)$ indicates the state of the system at time t that begins at x_0. The value of t is assumed to only take on values in some allowed range, the set of nonnegative real numbers $\mathbb{R}^+$ and the initial value of $t = 0 \Rightarrow \varphi_0(x_0) = x_0$ [33:106].

Dynamical systems can be modeled by a finite number of coupled first-order ordinary differential equations

$$\begin{aligned} \dot{x_1} &= f_1(t; x_1, \ldots, x_n; u_1, \ldots, u_p) \\ \dot{x_2} &= f_2(t; x_1, \ldots, x_n; u_1, \ldots, u_p) \\ &\vdots \\ \dot{x_n} &= f_n(t; x_1, \ldots, x_n; u_1, \ldots, u_p) \end{aligned} \tag{2.8}$$

where $\dot{x_i}$ is the derivative of x_i with respect to time, t, and the set of variables $u_1, u_2, \ldots, u_p$ are control variables required for that system. The variables $x_1, x_2, \ldots, x_n$ are the state variables and represent the memory the dynamical system has of its past. In order to write these systems in compact form, vector notation is generally used. First, the vectors are defined as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix}, \qquad f(t, x, u) = \begin{bmatrix} f_1(t, x, u) \\ f_2(t, x, u) \\ \vdots \\ f_n(t, x, u) \end{bmatrix} \tag{2.9}$$

and then rewritten as a compact first-order vector differential equation

$$\dot{x} = f(t, x, u) \tag{2.10}$$

This is the state equation where x is the state and u is the control. Sometimes another equation

$$y = h(t, x, u) \tag{2.11}$$

defines an output vector comprised of variables of particular interest in the analysis of the system. The two together form the state space model or state model. Math models of finite dimensional systems do not always come in the form of a state model. However, physical systems can thoroughly be modeled in this form by carefully selecting the state variables [27:1-4].

While nonlinear systems are often more accurate models of real world systems than linear models, many of the linear models are actually linearizations of nonlinear models because it is often difficult to find a closed form solution of a nonlinear system. By using the appropriate techniques it is possible to determine qualitative behaviors of the solutions of a nonlinear system which is desired [46:367].

2.7 Nonlinear Programming

In data fitting, the goal is to find a model which best fits some observed data and prior information from a selection of potential models. The variables are the parameters of these models, and the constraints represent the prior knowledge or limits of the system. The objective function may be the measure of error between the observations and the predicted values from the model. The optimization problem is to find the vector of model parameters which minimizes the objective function and is consistent with prior information. When the optimization problem contains a nonlinear objective function or constraints and is not known to be convex, it is a nonlinear programming problem [8:9-11]. A typical nonlinear program is of the form:

$$\begin{aligned} \text{Minimize} \quad & f(\mathbf{x}) & & \\ \text{subject to} \quad & g_i(\mathbf{x}) \leq 0 & & \text{for } i = 1, \ldots, m \\ & h_i(\mathbf{x}) = 0 & & \text{for } i = 1, \ldots, \ell \\ & \mathbf{x} \in X & & \end{aligned} \tag{2.12}$$

where $f\, g_1, \ldots, g_m$, $h_1, \ldots, h_\ell$ are functions defined on $\mathbb{R}^n$, $X \subset \mathbb{R}^n$, and $\mathbf{x}$ is a vector of n components $x_1, \ldots, x_n$. The problem is solved for the variables $x_1, \ldots, x_n$ that satisfy the constraints and minimizes the function $f(x)$ [3:1-2].

Many nonlinear problems are extremely challenging, even ones that appear simple, may be unyielding. The field of nonlinear programming is filled with different approaches for specific problems, and even compromises in some instances. One common method of solving nonlinear programs, which is implemented in many commercial solver engines, is the generalized reduced gradient (GRG) method. This method uses a discrete sequence of positive step sizes and sequentially attempts to determine a corresponding $\mathbf{x}_{k+1}$ for each step size using a Newton-Raphson scheme. Then using the value of $f(\mathbf{x}_{k+1})$ at that point, a quadratic interpolation method is conducted, when the three–point pattern (TPP) is obtained then a quadratic fit is used to provide the next step size. The process is conducted until a feasible vector which yields the minimum objective function is found [3:612-613].

The GRG method is implemented in the *EXCEL* solver engine, and is used to solve the nonlinear program which calculates the coefficient values for the system of differential equations in this methodology.

2.8 Prior Related Research Efforts

Over the past decade, there have been several efforts which researched different aspects of SSTRO. The range of topics cover systems dynamics models, social network models, goal programming, optimization models, and classification models; each work contributing to the field of SSTRO research.

2.8.1 Modeling Efforts.

One attempt to model the dynamics involved in reconstruction operations was using systems dynamics modeling techniques to simulate the establishment of public order and safety by Richardson. The purpose was to help decision makers by providing insight regarding the possible policy alternatives presented to them. The main idea is to take complex problems and break them down into manageable subproblems, then aggregate assumptions about the simpler questions to estimate answers for the larger complex problem. This was demonstrated in a notional example at a national level [42].

Robbins then advanced the model by instituting a sub-national, regional level approach. This allowed the user to concentrate on potentially troublesome regions, by providing information specific to the dynamics within that AOR. The results help the user understand the significance of the dynamic relationship of forces involved during SSTRO and potentially gain insight to the successful completion of the SSTRO mission [44]. This model eventually was re-engineered by Air Force Research Laboratory–Rome Laboratory (AFRL-RL) and become the National Operational Environment Model (NOEM) currently maintained by the same organization.

A social network analysis study was conducted by Bernardoni using Ronald Burt's structural hole technique to facilitate nation–building in failing and failed states. Bernardoni applied Burt's technique at a national level to identify the structural gaps within a failing state by focusing on techniques that link professional and government community individuals [4].

2.8.2 Optimization Models.

The application of goal programming was conducted by Bang to formulate the Coalition Operation Planning Model which was based upon three different sub-models: the Coalition Mission-Unit Allocation Model (Shortest Path), the Coalition Mission-Support Model (Network Flow), and the Coalition Mission-Unit Grouping Model (Quadratic Assignment). This method was applied to notional humanitarian assistance scenario and showed that many of the decisions were directly influenced by the political nature of the coalition and the framework provided by the political situation [2].

A goal programming project scheduling approach was conducted by Chaney to prioritize and schedule activities to maximize the impacts in SSTRO. This was established through three goals: 1) restore essential services in a timely manner, 2) distribute employment equally throughout the state, and 3) meet standards for sustainable income in each region. This was applied in a notional scenario, and showed how to schedule activities to meet the three goals while still meeting the intent of the initial response. Chaney presented three main points in this work: 1) consider economic impacts of reconstruction activities, 2) quantitative project scheduling techniques can be applied to SSTRO, and 3) the establish of these techniques adds defensibility to the plan and can uncover potential shortfalls [12].

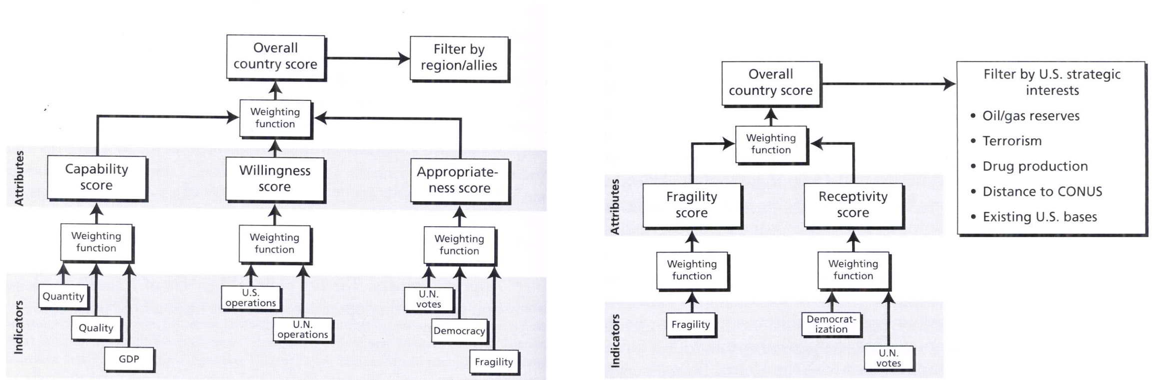
2.8.3 Classification Models.

With the number of failing or failed states on the rise, the ability to determine the indicators which lead to a failed state and identify states which are failing is a desirable feature. Nysether used factor analysis to identify the indicators and then apply discriminant analysis using the identified factors to classify states as stable, borderline, or failing. This was applied using open source data for 200 countries with 167 variables. This research is useful in identifying states which may require future SSTRO [38].

Understanding the factors which lead to war termination was researched by Robinson through the use of binary and multinomial logistic regression techniques. Robinson found that duration of conflict was the most relevant factor in predicting the winner of conflict and total casualties was the most relevant factor in predicting the manner in which an interstate war ends. This was examined in analysis of 19^{th} and 20^{th} century data [45].

Using the same methods as Nysether and Robinson as well as Canonical Correlation and Principal Component Analysis (PCA), Tannehill develop a mathematical model to forecast instability indicators in the Horn of Africa region using 54 variables over 32 years of observations. This model used indicators such as battle deaths, refugees, genocide deaths, and undernourishment to forecast instability. Tannehill found that a four–year forecast was possible while maintaining or improving the forecast error rate. This demonstrated the feasibility of longer term predictive models which would allow policy makers more time to develop plans [51].

In 2007, CAA initiated the Forecast and Analysis of Complex Threats (FACT) study. This study looked at predicting the potential for future conflict in select nation–states. The study found 13 features to measure and scaled the features on a $[0, 1]$ scale using the Euclidian distance. A PCA was conducted in an attempt to reduce the dimensionality of the data. The components then provided a proxy for similarity between states. A forecast was then generated using a Weighted Moving Average (WMA) and used both the k-Nearest Neighbor (KNN) and Nearest Centroid



(a) Coalition and Regional (b) Indigenous

Figure 3. RAND Study Models[32:98,115]

(NC) algorithms to classify future features. The study found that KNN performed better than NC with 85% or greater accuracy in all test cases. The methodology was adopted for use under the premise that it is predictive rather then prescriptive as discussed in Section 1.2 [48].

The RAND Arroyo Center conducted a study looking at the strategic elements to build partner capacity for stability operations in nations around the globe. The study focused on the elements which would align the security cooperation efforts of the US and building partner capacity. As part of this study they created two models; the Coalition and Regional model and the Indigenous model, shown in Figure 3.

The Coalition and Regional model was used to assess the capability of nations to be partners in stability operations: 28 countries fell into the high capability category, 5 of which were considered preferred. The study concluded that the high capability countries are unwilling to participate and/or are inappropriate for such operations. The preferred countries were Argentina, Czech Republic, Hungary, Poland, and South Africa [32:111].

The Indigenous model assessed how fragile a state was and the threat posed if they deteriorated or collapsed. The study found that this model also listed 28 of the 31 countries listed in the Fund for Peace's Failed States Index. Out of the 28, 16 were candidates based upon the US having two or more strategic interests with that country. The 16 are Afghanistan, Columbia, Cuba, Egypt, Indonesia, Iraq, Kuwait, Mexico, Nigeria, Pakistan, Peru, Qatar, Saudi Arabia, Turkey, United Arab Emirates, and Venezuela. Several of these nations are receiving aid already or are considered ineligible because of the current government in place [32:122-123].

The study concluded that it would be beneficial for the US to develop a selective strategy for partnership that nests with the security of the nation and the national military strategy [32:123-124].

2.8.4 Systems of Differential Equations Models.

A dynamic model of insurgency was created using Lanchester equations and Iraq war data by Blank et al. The model proposed a system of differential equations

$$\begin{aligned} \frac{dI}{dt} &= (r_i - \gamma_c)C \\ \frac{dC}{dt} &= (r_c - \gamma_i)I \end{aligned} \tag{2.13}$$

where

I is the number of insurgent attacks on the coalition

C is size of the coalition

r_i is the recruitment rate of the insurgents

r_c is the recruitment rate of the coalition

γ_i is the combat effectiveness coefficient of the insurgents

γ_c is the combat effectiveness coefficient of the coalition

The general solution to the system of differential equations is then used to plot the phase portraits of the system and deduce information based upon four cases: 1) the coalition increases in size and the number of attacks by the insurgents increases, 2) the size of the coalition decreases and the number of attacks by insurgents decreases, 3) the coalition increases and the number of insurgent attacks decreases, and 4) the coalition decreases and the number of insurgent attacks increases [6].

The relevance of the case is dependent upon the net recruitment rates $(r_i - \gamma_c)$ and $(r_c - \gamma_i)$ of the coalition and insurgents as well as the combat effectiveness of both sides. Using these plots the case where there is no coalition presence and the insurgent attacks are zero, the system is unstable, implying there is no amenable solution that leads to stability [6].

A nation–building model investigating the assimilation of different ethnicities into a single nation was developed by Yamamoto. This model was derived from the system of differential equations in the Deutsch Model for Nation–Formation by Karl Deutsch. Yamamoto derived two models and applied them to the Philippines [54].

The Modernism model is predicated upon the belief that a single underlying population (U) will mobilize into two different groups, assimilated (N) and differentiated(H). The Modernism model is formulated as

$$\begin{aligned}
\frac{dN}{dt} &= gN + \alpha mU \\
\frac{dH}{dt} &= gH + (1-\alpha)mU \\
\frac{dU}{dt} &= gU - mU
\end{aligned} \tag{2.14}$$

where

g is the natural population increase rate

m is the mobilization rate

α is the rate of integrating into the assimilated group (N)

$\alpha, m \in (0, 1]$

The Historicism model is predicated upon the belief that underlying population (U) is composed of two groups (Q, R) which will mobilize into the assimilated ($Q \mapsto N$) and differentiated ($R \mapsto H$). The Historicism model is formulated as

$$\begin{aligned}\frac{dN}{dt} &= gN + \alpha m Q \\ \frac{dH}{dt} &= gH + (1-\alpha) m R \\ \frac{dQ}{dt} &= gQ - \alpha m Q \\ \frac{dR}{dt} &= gR - (1-\alpha) m R \end{aligned} \tag{2.15}$$

where

g is the natural population increase rate

m is the mobilization rate

α is the rate of integrating into the assimilated group (N)

$\alpha, m \in (0, 1]$

These models investigate the effectiveness of the integration policies implemented by the Philippine government. The results suggest that the integration policy which involves the creation of an environment where multiple cultural groups can coexist is the most successful. Assimilating two groups into one culture in the Modernism model was unsuccessful [54].

A population model developed by Johnson and Madin was based upon the Logistic differential equation. This model makes use of population size (N), recruitment (r), carrying capacity (K), and mortality (m) to investigate the dynamics in the insurgent population. The discrete time logistic model takes on the following form

$$\Delta N = r\left(1 - \frac{N}{K}\right)N$$
$$N_{t+1} = Nt + r\left(1 - \frac{N_t}{K}\right)N_t - m_t \quad (2.16)$$

The model is applied to counterinsurgencies in Malaya (1948-1960) and Iraq (2003-2006) making use of data from United Kingdom Royal Air Force records and the Brookings Institution respectively. Given the available data, a least–squares optimization was implemented to estimate the unknown parameters (K, r), which are assumed to remain constant through the time period. After fitting the parameters, future trajectories were calculated using Equation 2.16. The results in the Iraq model suggested that 1) if sectarian violence had remained at low levels (such as 2006), the insurgency would have collapsed in 4-5 years based upon the US maintaining the trend of improving military performance, 2) moderate changes to the combination of K, r, or m may have led to the defeat of the insurgency in 6-12 months. Johnson and Madin suggest that increase in sectarian violence was the reason that the second case did not take place [25].

2.9 Summary

In this chapter, a review of relevant background literature was presented to the reader which provides context for the construction of the model and the solution methodology. The literature provided the theory and concepts which are behind the model and the solution methodology. In addition, a review of related SSTRO research efforts and systems of differential equations modeling was presented. Chapter III will describe the application of these concepts to the model and solution methodology.

III. Methodology

This research develops a solution methodology to solve the inverse problem with the following three steps:

1. Data collection and index formation.
2. Curve fitting and calculating derivatives.
3. Determining the coefficients of the system of differential equations.

The results of step 3 provide a means to conduct analysis on the system and gather insight on the dynamics of the relationship between the diME and PMESI in Chapter IV. The steps are graphically depicted in Figure 4 and described in the next three sections.

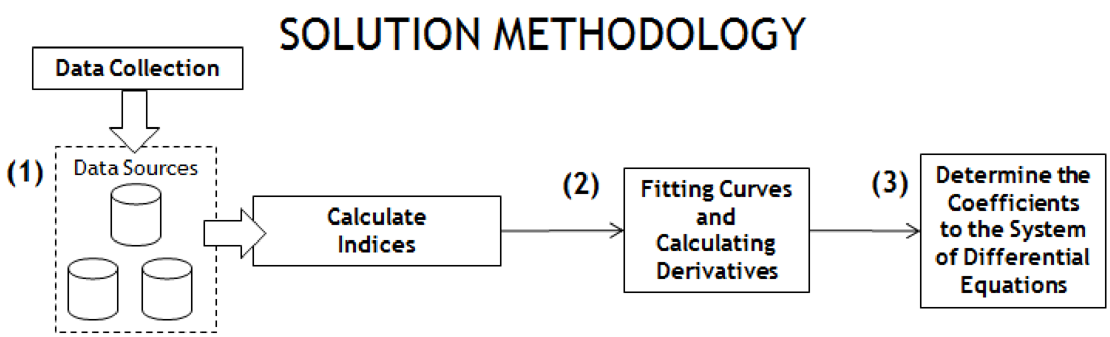


Figure 4. Solution Methodology

3.1 Data Collection and Index formation

The PMESI indices are composite indices; they represent a mathematical transformation (and aggregation) of different relevant indicators into one value. The use of such indicators to reflect country performance is widely practiced. A recent survey by Bandura details over 170 different composite country performance measurements used in practice [1]. The *Handbook on Constructing Composite Indicators*, published by the Organisation for Economic Co-operations and Development, notes composite indices:

- Can summarize complex, multi-dimensional realities for decisions makers [37:13].
- Are understood more easily than a list of their subcomponents and they reduce the set of indicators without dispensing underlying information [37:13].
- Can assess progress of countries over time [37:13].
- Enable comparisons of complex dimensions effectively [37:13].

Nardo et al warns that the justification and construction of composite indices lies in their fitness to the intended purpose and the acceptance of peers [37:14]. Following this concept, the composite indices are built following the PMESI operational variables outlined in the *Army Field Manual 3-0, Operations*. They are described abstractly in Section 2.2 and specifically in the next six sections.

3.1.1 Index construct.

The set of operational variables in the system is defined by $X = \{P, M, E, S, I\}$. Without loss of generality, each X can be described by the set of indicators, $X_t^{(j)}$, where j indicates the enumeration of the indicators $1, 2, \ \ldots, n$ and each indicator is a set of monthly observations, t given by:

$$\begin{aligned} X_t^{(1)} &= \left\{ x_1^{(1)}, x_2^{(1)}, \ldots, x_{70}^{(1)} \right\} \\ X_t^{(2)} &= \left\{ x_1^{(2)}, x_2^{(2)}, \ldots, x_{70}^{(2)} \right\} \\ &\vdots \\ X_t^{(n)} &= \left\{ x_1^{(n)}, x_2^{(n)}, \ldots, x_{70}^{(n)} \right\} \qquad (3.1) \end{aligned}$$

where

$$n = \text{the total number of indicators in } X$$

$$t = 1, 2, \ldots, 70$$

Each indicator, $x_t^{(j)} \in X$ will measure the same operational environment. However, some indicators in X will have observations with dissimilar units and frequency, and some will have missing data. Scale differences are compensated for by defining a common timeline for each indicator representing the overall time period $1, 2, \ldots, t$. Indicators with yearly data or missing monthly data are adjusted and gaps filled through linear interpolation. With a common observation frequency, the resulting data for each indicator is then normalized across the same $[0, 1]$ range, so that each observation has a common score scale. Using benchmarks within each indicator (maximum and minimum observations for the time period), the observations are normalized as follows:

$$Norm\left(x_t^{(j)}\right) = \begin{cases} \frac{\left(x_t^{(j)} - \max\left(x_t^{(j)}\right)\right)}{\left(\max\left(x_t^{(j)}\right) - \min\left(x_t^{(j)}\right)\right)} & \text{where } \max\left(x_t^{(j)}\right) \text{ is the best possible score} \\ 1 - \frac{\left(x_t^{(j)} - \max\left(x_t^{(j)}\right)\right)}{\left(\max\left(x_t^{(j)}\right) - \min\left(x_t^{(j)}\right)\right)} & \text{where } \min\left(x_t^{(j)}\right) \text{ is the best possible score} \end{cases} \tag{3.2}$$

Next, a weight, w, is assigned to each indicator representative of its perceived importance to the overall composition. Finally, the weight indicators are aggregated to achieve a composite index for each time step, t. For example, the final composite of the 1^{st} index is described by:

$$X_t^{(1)} = \sum_{j=1}^{n} (w_j) \left(Norm \left(x_t^{(1)} \right) \right) \tag{3.3}$$

where $X_t^{(1)}$ is the first composite index at time t.

The data used to create the indices are retrieved from the following open source data sets:

- The Brookings Institution (last accessed 06 December 2011) (www.brookings.edu/saban/iraq-index.aspx)
- The CIA Factbook (last accessed 06 December 2011) (https://www.cia.gov/library/publications/the-world-factbook/geos/iz.html)
- The United States Department of State (last accessed 06 December 2011) (http://www.state.gov/p/nea/ci/iz/)
- The United States Department of Defense (last accessed 06 December 2011) (http://www.defense.gov/home/features/iraq_reports/index.html)

The data used in this research is from sources considered credible and is readily available. A perfect model with unobservable data can only provide theoretical aspects at best, a good model with good data provides insight. Much of the literature reviewed concerning the Iraq war makes use of the same sources. In this research, exclusive use is made of data that is already collected or is continually collected.

The next five sections will describe each of the five PMESI indices and the two forcing functions in detail.

3.1.2 Political Index.

From the description in Section 2.2.1, the political variable is summarized as describing the distribution of responsibility and power at all levels of governance. The political variable, P_t, references the Political Index value at time t. The Political Index is made up of six data points:

Size of Iraqi Security Forces (ISF) on Duty – The total number of ISF on duty during a given month, this includes the Iraqi Police forces, Iraqi National Guard forces, Iraqi Border Patrol, and the Iraqi Armed Forces.

How confident are the Iraqi people in the Iraqi Army? – The percentage of Iraqi people that are confident in the ability of the Iraqi Army.

How confident are the Iraqi people in the Iraqi Police? – The percentage of Iraqi people that are confident in the ability of the Iraqi Police.

Elected Government Constant – A subjective score [0,0.5,1] based upon the current level of elected government officials.

Displaced Persons metric – The number of internally displaced persons per month.

Freedom of Press Index – The Reporters Without Borders Annual Freedom of Press Index extrapolated monthly.

The Political Index is decomposed in figure 5. The first three data points are aggre-

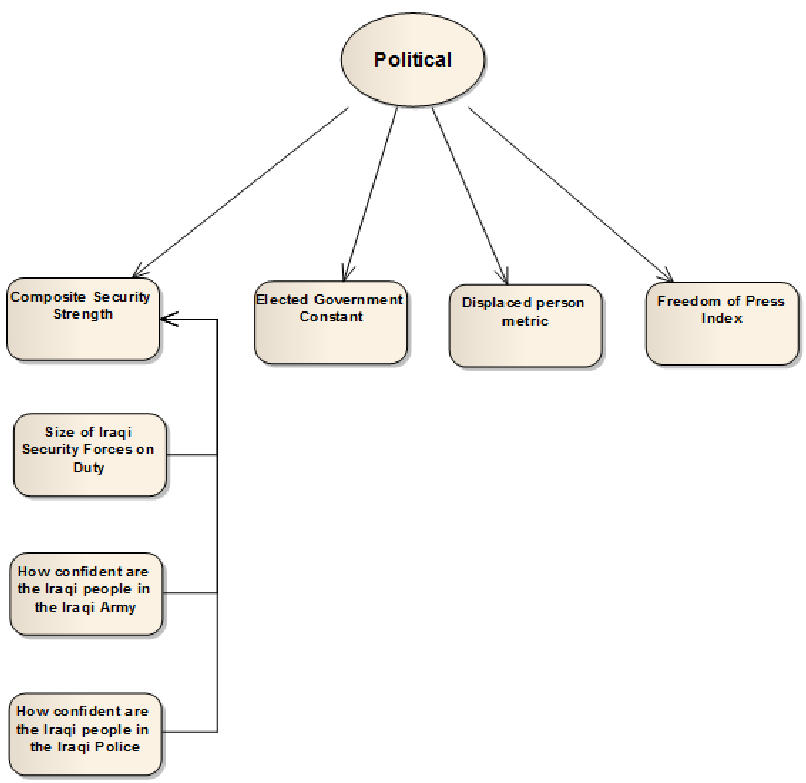


Figure 5. Decomposition of the Political Variable

gated into a composite security index which is then scaled by the Iraqi population and normalized on a [0,1] scale by the following expression

$$\min\left(\frac{(ISF_N)\left(\frac{Poll_1+Poll_2}{2}\right)}{Population(100k)}, 1\right) \tag{3.4}$$

where

$$ISF_N = \text{Size of Iraqi Security Forces}$$

$$Poll_1 = \text{Iraqi Army Confidence poll}$$

$$Poll_2 = \text{Iraqi Police Confidence poll}$$

The remaining three data points (Elected Government Constant, Displaced Persons Metric, and Freedom of Press Index) are then normalized by Equation 3.2 and the final composite index of P_t is calculated according to Equation 3.3.

3.1.3 Military Index.

From the description in Section 2.2.2, the military variable is summarized as the military capability of the armed forces in the operational environment. The military variable, M_t, references the Military Index value at time t. The Military Index is made up of five data points:

Number of ISF Killed in Action (KIA) – The total number of ISF that were KIA during a given month, this includes the Iraqi Police forces, Iraqi National Guard forces, Iraqi Border Patrol, and the Iraqi Armed Forces.

Number of Coalition Fatalities – The total number of coalition troops KIA during a given month, the number of coalition countries and forces may change throughout the war.

Number of Attacks on Iraqi Oil and Gas Infrastructure and Personnel – The number of attacks on oil and gas infrastructure and personnel during a given month.

Number of Iraqi Civilian Fatalities – The total number of Iraqi civilians KIA during a given month by acts of war or other violent means.

Multiple Fatality Car/Suicide Bombings – The number multiple fatality attacks carried out by car or suicide type bombings for a given month.

The Military Index is decomposed in Figure 6. The five data points are then normalized by Equation 3.2 and the final composite index of M_t is calculated according to Equation 3.3.

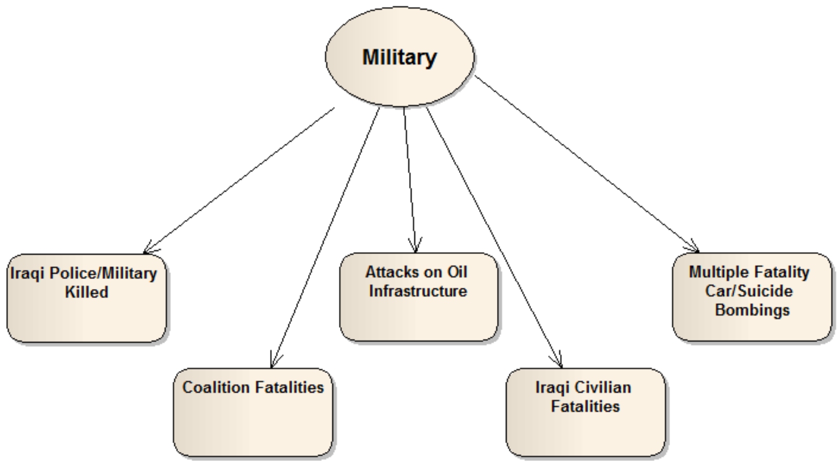


Figure 6. Decomposition of the Military Variable

3.1.4 Economic Index.

From the description in Section 2.2.3, the economic variable is summarized as the aspects which encompass individual and group behaviors related to producing, distributing, and consuming resources. The economic variable, E_t, references the Economic Index value at time t. The Economic Index is made up of four data points:

Nominal Gross Domestic Product (GDP) – The annual GDP for Iraq in terms of US dollars (billions), extrapolated monthly.

Consumer Price Index (CPI) Percent Change – The percent change of the CPI from year to year, extrapolated monthly.

Foreign Direct Investment (FDI) – The amount of monthly FDI attracted in millions of US dollars.

Oil Revenue – The amount of monthly revenue generated from oil production and distribution monthly in billions of US dollars.

The Economic Index is decomposed in Figure 7. The four data points are then

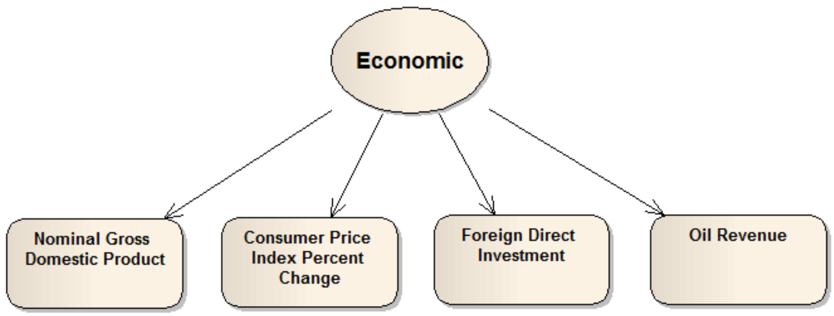


Figure 7. Decomposition of the Economic Variable

normalized by Equation 3.2 and the final composite index of E_t is calculated according to Equation 3.3.

3.1.5 Social Index.

From the description in Section 2.2.4, the social variable is summarized as the aspects that describe the societal groups and the factors influencing daily life within the operational environment and within the groups. The social variable, S_t, references the Social Index value at time t. The Social Index is made up of six data points:

Iraqi satisfaction level with the availability of clean water – Annual polling data regarding the percentage of Iraqi people satisfied with the current availability of clean water, extrapolated monthly.

Iraqi satisfaction level with the availability of medical care – Annual polling data regarding the percentage of Iraqi people satisfied with the current availability of medical care, extrapolated monthly.

Iraqi satisfaction level with the availability of fuel – Annual polling data regarding the percentage of Iraqi people satisfied with the current availability of fuel, extrapolated monthly.

Number of Doctors – The number of trained doctors in Iraqi for a given month.

Number of Trained Judges – The number of trained judges within Iraq for a given month.

Number of School-Age Children – The number of school-age children (primary and secondary schools) in billions for a given month.

The Social Index is decomposed in Figure 8. The first three data points are

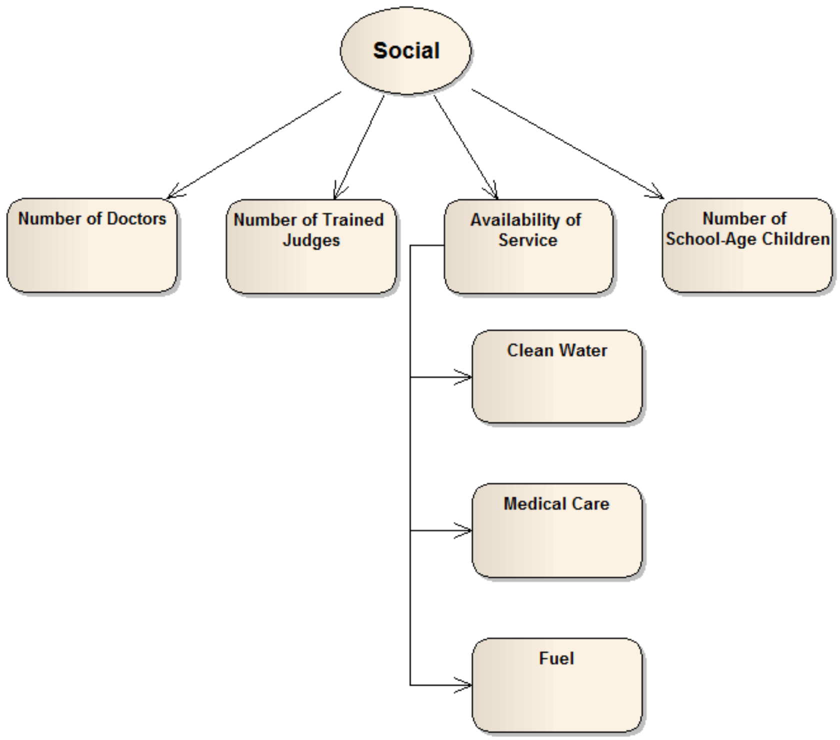


Figure 8. Decomposition of the Social Variable

aggregated into an overall availability of services index and normalized on a [0,1] scale by the following expression

$$\frac{1}{3}\sum_{j=1}^{3} Svc_j \tag{3.5}$$

where

$$Svc_j = \text{The score for poll j}$$

The remaining three data points (Number of Doctors, Number of Trained Judges, and Number of School-Age Children) are then normalized by Equation 3.2 and the final composite index of P_t is calculated according to Equation 3.3.

3.1.6 Infrastructure Index.

From the description in Section 2.2.6, the infrastructure variable is summarized as the basic facilities, services, and installations needed for a society to function. The infrastructure variable, I_t, references the Infrastructure Index value at time t. The Infrastructure Index is made up of three data points:

Number of Telephone Subscribers – The monthly number of Iraqi people who subscribe to telephone service, both cellular and land–line service.

Average Daily Hours of Electricity – The average hours of daily electricity service provided to the country of Iraq for a given month.

Number of Internet Subscribers – The monthly number of Iraqi people who subscribe to internet service.

The Economic Index is decomposed in Figure 9. The three data points are then

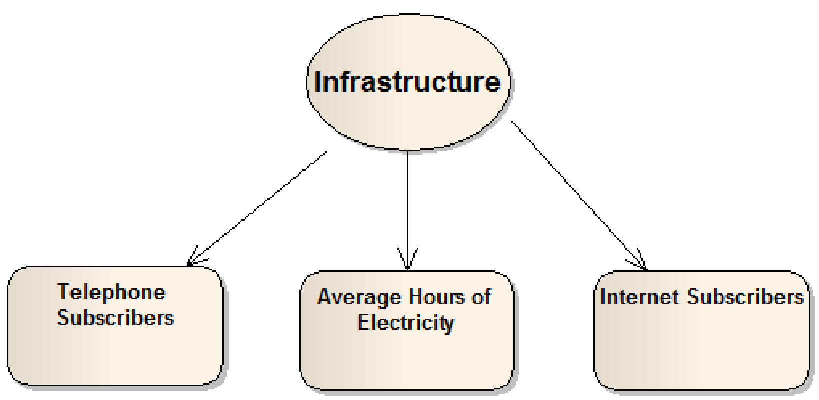


Figure 9. Decomposition of the Infrastructure Variable

normalized by Equation 3.2 and the final composite index of I_t is calculated according to Equation 3.3.

3.1.7 Military Forcing Function.

The military forcing function is another exogenous control variable to the system. In accordance with Section 2.1.3, the variable Mil_t indicates the total number of US armed forces with boots on the ground or BOG in Iraq during month t. The value of Mil_t can assume a real number value greater than 0 and is described in terms of 100,000 troops.

3.1.8 Economic Forcing Function.

The economic forcing function is the final exogenous control variable in this system. In accordance with Section 2.1.4, the variable Eco_t indicates the total amount of US monetary aid in billions of dollars distributed to Iraq during the fiscal year. This value is extrapolated to an amount per month t. The value of Eco_t can assume a real number value greater than 0.

3.2 Curve Fitting and Calculating Derivatives

Curve fitting is the process of using a mathematical function to express the relationship between a set of points. When using a least–squares approach, the goal is to find the curve which minimizes the sum of the squared error, the error associated between the known data point, and the fitted point. When the data points do not appear to be linear, an approach such as polynomial regression may be used. The problem of fitting a polynomial curve to data is a special case of the general linear regression where

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{3.6}$$

becomes

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \ldots + \boldsymbol{\epsilon} \tag{3.7}$$

and is widely used when the response is curvilinear and contains squared or higher-order terms. The following set of normal equations solve for $\boldsymbol{\beta}$ in a least–squares sense

$$\mathbf{X}^T\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}^T\mathbf{y} \tag{3.8}$$

The $\mathbf{y}$ vector are the responses or the set of known data points, $\mathbf{X}$ is the matrix of the predictor variables, and $\boldsymbol{\beta}$ is the vector of the unknown parameters [36:476-478][29:294-296].

In this research, the method of curve fitting is applied to approximate an equation as a function of time, t, to the calculated index values. This allows the calculation of an approximate first-order derivative of the index with respect to t. In terms of Figure 2 in Section 2.3 this represents the true model, m, and is used in the least-squares calculations. Typically, the use of the lowest–degree model possible that adequately explains the relationship is preferred. In order to determine the adequacy of the fit, the coefficient of determination or R^2 is used. This describes the ratio of the sum of squares

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T} \tag{3.9}$$

where

$$SS_R = \text{Sum of Squares Regression}$$

$$SS_T = \text{Sum of Squares Total}$$

$$SS_E = \text{Sum of Squares Error}$$

The R^2 statistic must be used with caution because it is possible to force R^2 to 1 by adding unnecessary terms to the model. A model with $n-1$ degrees will obtain a *perfect* fit, but may not be statistically superior to a model of a lesser degree [36:418].

After fitting the data to a polynomial function, the task of calculating the derivative is rather simple and makes use of the power rule

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \text{where } n \in \mathbb{R}. \tag{3.10}$$

and the result is a polynomial equation of degree $n - 1$ which describes the fitted curves rate of change as a function of time.

Through this process, observations of data over time are collected and compiled into a composite index, which summarizes complex data and assesses the progress over time as described by Nardo [37:13]. This develops a set of discrete points which can be used to describe the current state of the PMESI variables at a time, t. The general trend of the data helps to describe its progress, to capture this trend a polynomial regression technique is used. The time series plots show that the indices are not convex and are curvilinear which indicate a polynomial regression would be well suited to capture the trend. After selecting the proper fit, the PMESI indices can be expressed as a function of time. The derivative of the function is then calculated and used to determine the coefficients of the system of differential equations in the least–squares minimization. Through this series of steps, mean–field theory was applied to perform the necessary steps to determine the coefficients of the system of differential equations in the next section.

3.3 Determining the Coefficients of the System of Differential Equations

The final system of differential equations is a set of mean–field equations whose coefficients are derived by using a nonlinear least–squares method and the derivative information of the fitted curves. Similar to the index calculations in Section 3.1, each equation in the differential system corresponds to an operational variable. However, the two sets differ in that the system of differential equations expresses each point as a derivative function of itself (similar to Lanchester Equations), data from the other operational variables, and data from the US instruments of national power. In this way, each equation in the system of differential equations can be said to describe the *interrelatedness* of all variables in the operational environment while concurrently capturing the instruments of national power applied by the US. The functional form of the model represents the rate of change, of each state variable at time, t, as a function of both the state variables and the forcing functions demonstrated with $\dot{P}_t$ in Figure 10.

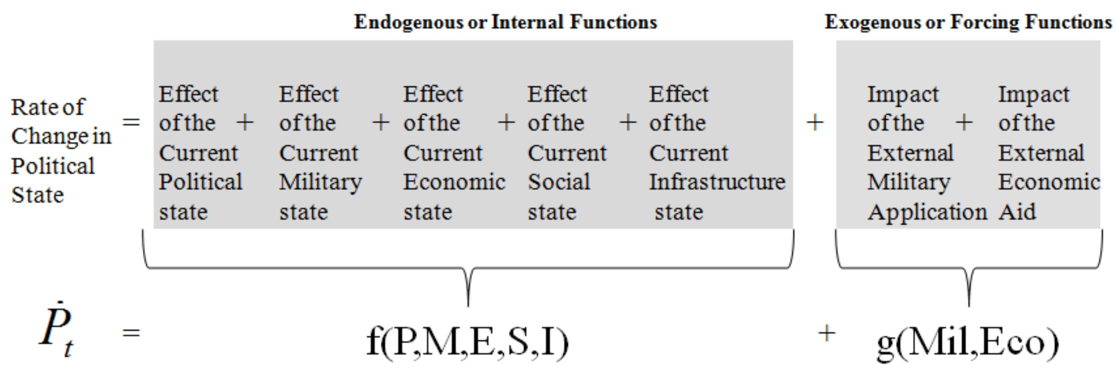


Figure 10. Depiction of the Functional Form

In differential equation form $\dot{P}_t$ is represented as

$$\begin{aligned} \dot{P}_t &= a_{11}\left(\frac{P_t}{b_{11}}-1\right)+a_{12}\left(\frac{M_t}{b_{12}}-1\right)+a_{13}\left(\frac{E_t}{b_{13}}-1\right)+a_{14}\left(\frac{S_t}{b_{14}}-1\right)+a_{15}\left(\frac{I_t}{b_{15}}-1\right) \\ &+ d_{11}Mil_t+d_{12}Eco_t \end{aligned} \tag{3.11}$$

To define the effect of the current state, two coefficients, a and b, are introduced for each of the PMESI variables.

$$\text{Effect of the current state} = a_{ij}\left(\frac{x_t}{b_{ij}}-1\right) \tag{3.12}$$

The a coefficient is the scaling factor, it represents the weight of the endogenous functions or the proportionality of the endogenous function to the rate of change for a PMESI variable. The b coefficient is the tipping point, it represents the point where a change in the parameter causes a change in the dynamical property of the system, much like a bifurcation [26:26-28]. Consider a saddle–node bifurcation in Figure 11, a saddle point is observed where the x and y axis meet, and from there it takes on

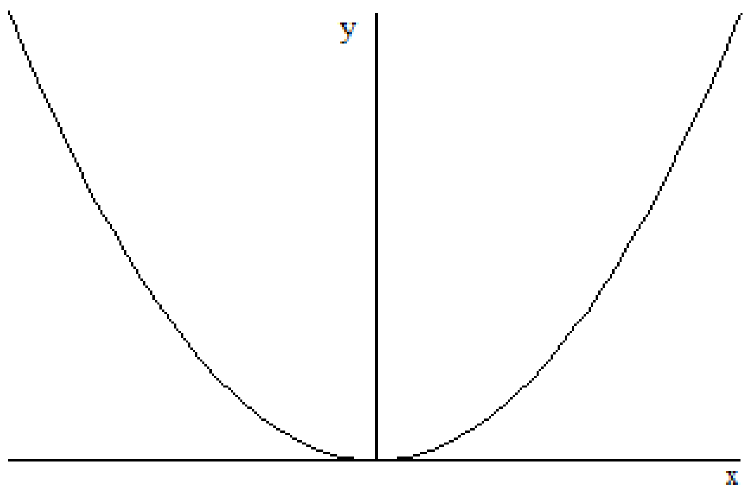


Figure 11. A Saddle Node Bifurcation

opposite trajectories. The tipping point represents a point when $x_t^{(j)} \approx b$ the effect of a variable is generally stable, when the value of $x_t^{(j)} > b$ there is a magnifying effect, and when $x_t^{(j)} < b$ there is a diminishing effect. The d_{ik} are the scaling factor coefficients for the diME forcing functions.

The a and b coefficients are similar to the proportional growth rate (r) and carrying capacity (K) in the Logistics Differential Equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

and in the Johnson and Madin model (Equation 2.16); except in those models the quantity in the parenthesis is subtracted from 1, and in Equation 3.12 it is the quantity minus 1. The reason is that in a population model, the population grows ($\frac{dP}{dt} > 0$) until it reaches its capacity ($\frac{dP}{dt} = 0$) and then the growth rate decreases ($\frac{dP}{dt} < 0$) because there is no capacity for the additional population. In Equation 3.12 the effect is monotonically increasing $\forall a, b,$ and $x_t^{(j)}$, the value of the coefficients and indices determine the contribution of each variable to the derivative of a specific variable.

This tipping point highlights the advantage of data driven model. Rather than looking for a subjective assessment from a subject matter expert (SME) the observed data determines the tipping point. By letting the data drive the value of the coefficients, the effect relative to the time period and the interrelatedness of the data can be evaluated. This means when evaluating the military of a country that is building its strength, the evaluation may be based upon the observations which demonstrate the development of the military, and how they interact with the other variables in the operational environment.

To build the functional form of the model the range of the variables must be defined. The range for $a_{ij}, b_{ij},$ and d_{ik} are

$$\begin{aligned} &a_{ij} \in \mathbb{R} && \text{for i,j=1,2,\ldots,5} \\ &b_{ij} \in \mathbb{R} \mid 0 \leq b_{ij} \leq 1 && \text{for i,j=1,2,\ldots,5} \\ &d_{ik} \in \mathbb{R} && \text{for i=1,2,\ldots,5} \\ & && \text{for k=1,2} \end{aligned}$$

and the resulting system of differential equations is

$$\begin{aligned}
\dot{P}_t &= a_{11}\left(\frac{P_t}{b_{11}}-1\right)+a_{12}\left(\frac{M_t}{b_{12}}-1\right)+a_{13}\left(\frac{E_t}{b_{13}}-1\right)+a_{14}\left(\frac{S_t}{b_{14}}-1\right)+a_{15}\left(\frac{I_t}{b_{15}}-1\right)\\
&+ d_{11}Mil_t+d_{12}Eco_t \qquad (3.13)
\end{aligned}$$

$$\begin{aligned}
\dot{M}_t &= a_{21}\left(\frac{P_t}{b_{21}}-1\right)+a_{22}\left(\frac{M_t}{b_{22}}-1\right)+a_{23}\left(\frac{E_t}{b_{23}}-1\right)+a_{24}\left(\frac{S_t}{b_{24}}-1\right)+a_{25}\left(\frac{I_t}{b_{25}}-1\right)\\
&+ d_{21}Mil_t+d_{22}Eco_t \qquad (3.14)
\end{aligned}$$

$$\begin{aligned}
\dot{E}_t &= a_{31}\left(\frac{P_t}{b_{31}}-1\right)+a_{32}\left(\frac{M_t}{b_{32}}-1\right)+a_{33}\left(\frac{E_t}{b_{33}}-1\right)+a_{34}\left(\frac{S_t}{b_{34}}-1\right)+a_{35}\left(\frac{I_t}{b_{35}}-1\right)\\
&+ d_{31}Mil_t+d_{32}Eco_t \qquad (3.15)
\end{aligned}$$

$$\begin{aligned}
\dot{S}_t &= a_{41}\left(\frac{P_t}{b_{41}}-1\right)+a_{42}\left(\frac{M_t}{b_{42}}-1\right)+a_{43}\left(\frac{E_t}{b_{43}}-1\right)+a_{44}\left(\frac{S_t}{b_{44}}-1\right)+a_{45}\left(\frac{I_t}{b_{45}}-1\right)\\
&+ d_{41}Mil_t+d_{42}Eco_t \qquad (3.16)
\end{aligned}$$

$$\begin{aligned}
\dot{I}_t &= a_{51}\left(\frac{P_t}{b_{51}}-1\right)+a_{52}\left(\frac{M_t}{b_{52}}-1\right)+a_{53}\left(\frac{E_t}{b_{53}}-1\right)+a_{54}\left(\frac{S_t}{b_{54}}-1\right)+a_{55}\left(\frac{I_t}{b_{55}}-1\right)\\
&+ d_{51}Mil_t+d_{52}Eco_t \qquad (3.17)
\end{aligned}$$

For compactness, this is defined in matrix notation with the following matrices. The $[A_{ij}]_{5\times5}$ matrix is made of the a coefficients. The $[B_{ij}]_{5\times5}$ is a matrix of the inverse of the b coefficients.

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25}\\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35}\\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55}\end{bmatrix} \qquad B=\begin{bmatrix} \frac{1}{b_{11}} & \frac{1}{b_{12}} & \frac{1}{b_{13}} & \frac{1}{b_{14}} & \frac{1}{b_{15}}\\ \frac{1}{b_{21}} & \frac{1}{b_{22}} & \frac{1}{b_{23}} & \frac{1}{b_{24}} & \frac{1}{b_{25}}\\ \frac{1}{b_{31}} & \frac{1}{b_{32}} & \frac{1}{b_{33}} & \frac{1}{b_{34}} & \frac{1}{b_{E35}}\\ \frac{1}{b_{41}} & \frac{1}{b_{42}} & \frac{1}{b_{43}} & \frac{1}{b_{44}} & \frac{1}{b_{45}}\\ \frac{1}{b_{51}} & \frac{1}{b_{52}} & \frac{1}{b_{53}} & \frac{1}{b_{54}} & \frac{1}{b_{55}}\end{bmatrix}$$

The PMESI variables make up the X matrix and the diME variables make up the M and E matrices.

$$X = \begin{bmatrix} P & M & E & S & I \\ P & M & E & S & I \\ P & M & E & S & I \\ P & M & E & S & I \\ P & M & E & S & I \end{bmatrix} \qquad M = \begin{bmatrix} Mil_1 \\ Mil_1 \\ Mil_1 \\ Mil_1 \\ Mil_1 \end{bmatrix} \qquad E = \begin{bmatrix} Eco_1 \\ Eco_1 \\ Eco_1 \\ Eco_1 \\ Eco_1 \end{bmatrix}$$

The d coefficients are 5×1 matrices defined as

$$C = \begin{bmatrix} d_{11} \\ d_{11} \\ d_{11} \\ d_{11} \\ d_{11} \end{bmatrix} \qquad F = \begin{bmatrix} d_{12} \\ d_{12} \\ d_{12} \\ d_{12} \\ d_{12} \end{bmatrix} \tag{3.18}$$

Finally, two matrices of 1's are required: the first is $[1]_{5\times 5}$ and the second is $[1]_{5\times 1}$. To represent these equations in matrix form and conduct the necessary operations, the Hadamard product ($\circ$) is required. The Hadamard product is defined for two matrices, $A, B \in \mathbb{R}^{m\times n}$ where $A \circ B \in \mathbb{R}^{m\times n}$. This product is also known as the element–wise product because the actual operations conducted are $[A]_{ij} \times [B]_{ij} = [A \circ B]_{ij}$ [34].

Using the Hadamard product the system of differential equations is

$$\underbrace{\left[A \circ \left((X \circ B) - [1]_{5\times 5}\right)[1]_{5\times 1}\right]}_{f(PMESI)} + \underbrace{(M \circ C) + (E \circ F)}_{g(diME)} = \begin{bmatrix} \dot{\hat{P}} \\ \dot{\hat{M}} \\ \dot{\hat{E}} \\ \dot{\hat{S}} \\ \dot{\hat{I}} \end{bmatrix} = \mathbf{v} \qquad (3.19)$$

In terms of Figure 2 from Section 2.3, this represents the estimated model, $\hat{m}$.

3.3.1 Solving for a and b.

So that the system of differential equations accurately describes the actual trends of our operational variables, the system is fit to the derivatives of the fitted–curves from the previous section. This method has been used by Kleczkowski and Grenfell to capture similar interactions [28]. In order to solve for the a and b coefficients, the following least–squares nonlinear minimization problem is formulated

$$\text{Minimize} \quad f(\mathbf{x}) = \sum_{i=1}^{5}\sum_{t=1}^{70}(m_t^{(i)} - \hat{m}_t^{(i)})^2 \qquad (3.20)$$

$$\text{subject to} \quad b_{ij} \leq 1 \qquad \text{for } i,j = 1,\ldots,5 \qquad (3.21)$$

$$b_{ij} > 0 \qquad \text{for } i,j = 1,\ldots,5 \qquad (3.22)$$

$$a_{ij}, b_{ij}, d_{ik} \in \mathbb{R} \qquad \text{for } i,j = 1,\ldots,5 \qquad (3.23)$$

$$\text{for } k = 1,2 \qquad (3.24)$$

$$t \in \mathbb{Z}^+ \qquad (3.25)$$

This method ultimately takes a measure from the space of a stochastic system, and approximates it through a deterministic set of equations, which determines a calculated derivative for each operational variable. The error (SSE) associated with the system is effectively minimized through the fitting of the a_{ij}, b_{ij}, and d_{ik} coefficients. This is accomplished by using the GRG method for solving nonlinear programs described in Section 2.7. There are two points to note in this case:

1. Just like many nonlinear problems, the solution for a_{ij}, b_{ij}, and d_{ik} is not unique. It is possible to get a similar answer with the different set of coefficients. It is typical that if there is more than one solution, then there is an infinite number of solutions that satisfy the equations. It is impossible to have exactly two solutions.

2. The solution provided is specific to the operational environment being studied. There is no master set of coefficients or parameters that can be used for all situations. The so–called "constant fallacy" described by Helmbold is often overlooked and leads researchers to believe they have found universal parameters when they have in fact found parameters specific only to their study [24]. The coefficients found must be updated when a new or expanded data set is used.

The results provide a mathematical expression of the operational environment, but with far more insight and capability than the original fitted curves from the previous section. At this point, the collected data from the operational environment used to calculate the composite indices cannot be changed without making a new model, as it reflects the internal environment and interactions between these internal environment variables. However, insight can be gained by testing changes in the external environment to see its impacts on the internal environment. Specifically, modifications to the instruments of national power used by the US in terms of military troops and economic aid can be implemented and then evaluated to see how these changes are reflected in the calculated indices.

3.4 Summary

This chapter described the model and the solution methodology. A general overview, description, and rationale was provided. The creation of the methodology allows for variation from the user as a means to evaluate the dynamics of a course of action.

This is a tool to glean insight from varying the diME inputs based upon the dynamics of the original environment. The methodology may be used to provide the decision maker with relevant information that will help provide clarity of action.

The strength of this model is capturing the interactions between the operational variables, as displayed in Figure 1. The same idea was applied in the functional form (Figure 10) and the system of differential equations (Equations 3.13-3.17). Each derivative calculated in the system of differential equations is composed of all operational variables as well as the diME forcing functions.

The data sources used in this research are the similar to other analysis efforts reviewed in Chapter II. The Brookings Institution data set is maintained monthly and retrieved from the Department of Defense, Department of State, as well as other reputable sources in their collection of public data. However, one could argue that better data exists in either unclassified or classified form. If there is in fact a superior data set, then that data can be applied as well. The indices can be adjusted to a new data set and then reapply the methodology. Regardless of the data, the interactions between the indices are captured, this is the intended design of the model, not the data. In a *perfect* scenario, the metrics are defined in advance with established collection methods and the result is perfect data. In practice this is rarely possible. Analysis is often initiated after the fact, or after conducting analysis the realization is that a different metric is needed. The adage *better data=better model* holds true, but in reality the best data available is used to create the best model possible because holding out for perfect is an unaffordable luxury.

The nature of generality is a limitation of this model. In the future sections different applications of diME are demonstrated to show the impact on the operational environment, but this will not show how it affected the components of each operational variable. The model demonstrates the effect, but does not indicate the cause. The model calculates a rate of change (derivative) based upon the relationships of the PMESI and diME, that can be used to determine a predicted index value at a certain time. This will not provide insight for the components which are used to create the index.

The methodology described in Chapter III is implemented using a data set from Operation Iraqi Freedom. The implementation method and results, as well as analysis from alternate diME strategies, are presented in Chapter IV.

IV. Implementation and Analysis

This chapter illustrates the utility of the model through an application of data from Operation Iraqi Freedom (OIF).

4.1 Implementation

In late 2002 and early 2003, the US began the initial planning for an operation to remove Saddam Hussein and overthrow his regime in Iraq. The US was looking for support to conduct this operation from the United Nations (UN) and the North Atlantic Treaty Organization (NATO) but was unable to garner support as they hoped. Even in the US government there were disagreements within Congress, the DOD, and the populace on whether or not the US should execute this plan. The size and makeup of the coalition was considerably smaller than that of the one charged with liberating Kuwait during Operation Desert Storm in 1991. The general perception was while the coalition forces were outnumbered there would be little resistance; this proved to be true. The prospects of the stabilization effort however were disputed by top officials. Then Army Chief of Staff General Eric Shinseki argued it would take several hundred thousand soldiers to stabilize post–war Iraq. Secretary of Defense (SecDef) Rumsfeld and Central Command (CENTCOM) commander General Tommy Franks disagreed and ultimately the invasion was planned under the assumption that the US would not lead the stabilization effort [35:23-24].

On the 20th of March 2003 the US–led coalition invaded Iraq to oust Saddam Hussein with roughly 130,000 US troops and 25,000 British troops. As expected, there was very little resistance. What happened after that was unexpected; the Iraqi military dissolved amongst the people and an insurgency evolved. The impacts from the lack of stabilization planning and shortage of troops had immediate consequences and long–term implications. As a result, the US has been involved in SSTRO through 2011, when the majority of troops were removed [35:24].

While the major combat operations, Dominate (Joint Phase III), were from March-May 2003, the first three months of OIF will be included in this analysis as this epoch includes the transition period from Dominate to Stabilize (Joint Phase IV). According to JP 3-0, also illustrated in Figure 12, while the level of effort in SSTRO

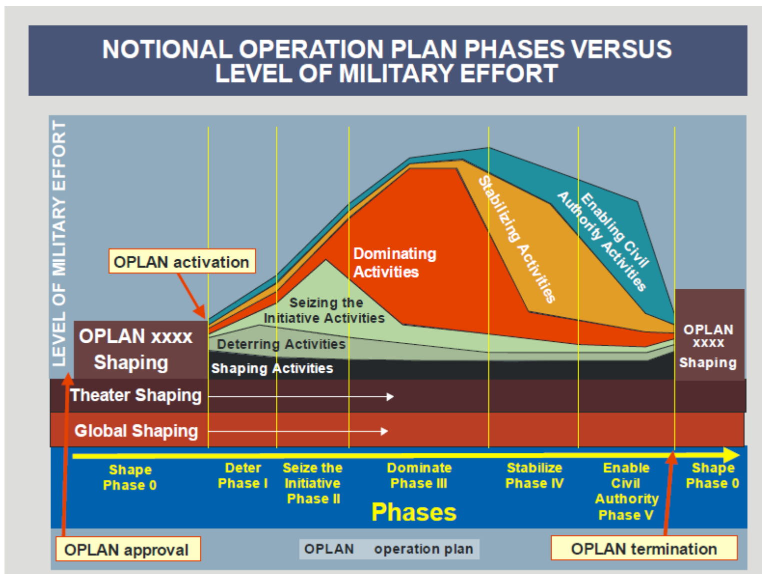


Figure 12. Notional Operation Plan Versus Level of Military Effort[18]

or stabilizing activities may be lesser in earlier phases, they are still present in planning and execution and therefore are included in this analysis. The time period covered in this analysis is the start of the war through the 70^{th} month, December 2008. This encompasses roughly five years of monthly data. For this application, the model is set with initial parameters from the available data sources for month 1, March 2003. An explanation of the application is covered in the following sections.

4.1.1 Data Collection and Index Formation.

Data collection started with the beginning of the war (March 2003) through December 2008. Each data point was collected monthly using the data sources listed in Section 3.1.1. The data in raw form is presented in Appendix A according to the five PMESI operational variables and the two diME forcing functions. In accordance with the guidance provided by the *Handbook on Constructing Composite Indicators*, the data selected assesses progress over time and summarizes the complex nature of the indices which they support [37]. Each data point is a monthly indicator, or has been extrapolated into a monthly indicator. While many aspects, such as economic aid, does not always have an impact felt at the time of distribution, by extrapolating the yearly data to monthly data the effect of the aid distributed is felt throughout the year. This effectively adds a time lag to the data. Any economic result (positive or negative) is assumed to have evenly distributed effects over the fiscal year.

Once the data has been collected, the indices are formulated and normalized according to Section 3.1.1. The weights used in this case are based upon a notional assessment. With the exception of the Political and Military indices, all the components are equally weighted ($w_i = \frac{1}{n}$) where n is the number of components. The Political and Military Index weights are shown in Table 1. The component weights

Table 1. Political and Military Index weights

Political Component	Weight (c_w)	Military Component	Weight(c_w)
Security Strength	2	ISF KIA	2
Elected Government	2	Coalition Fatalities	1
Displaced persons	1	Attacks on Iraqi Oil and Gas Infrastructure/Personnel	1
Freedom of Press	1	Iraqi Civilian Fatalities	2
		Multiple Fatality Bombings	3

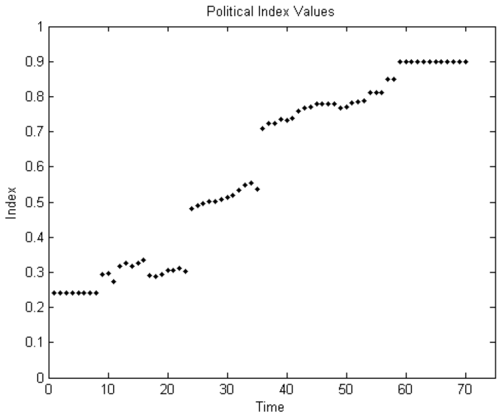
are then calculated in a similar manner ($w_i = \frac{c_w}{\sum c_w}$) where c_w is the weight of a component. The rationale for this weighting is the level of perceived strength and an elected government in place is twice as strong as the other two components in the Political Index. In the Military Index, multiple fatality events are extremely powerful events that sway opinion, thus have three times the impact as the coalition casualties and attacks on infrastructure. The number of ISF and civilians killed also has a stronger effect than coalition casualties and attacks on infrastructure, but not as much as multiple casualty events, and therefore, has a weight of two. The weighting scheme is ultimately up to the decision maker and must be adjusted accordingly by the analyst.

Each individual index is then plotted as a time series, as depicted in Figure 13. The calculated index values are presented in Appendix C. The data in this form is a series of discrete points. To calculate a derivative and determine the rate of change a continuous function is needed. Certain techniques are discussed Section 4.1.2 that deal with this issue.

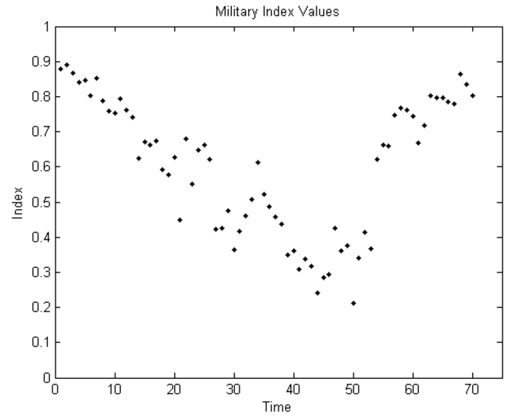
4.1.2 Curve Fitting and Calculating Derivatives.

Next, the normalized data is smoothed to capture the general trend of the index and any potential noise through the polynomial regression technique described in Section 3.2. This results in a generalized model, one which is not susceptible to extraordinary events, yet can still account for them. While any number of tools may be used to accomplish the curve fitting process, it is important to note the type of curve selected and its goodness–of–fit are important considerations that need to be addressed through statistical analysis and modeler experience.

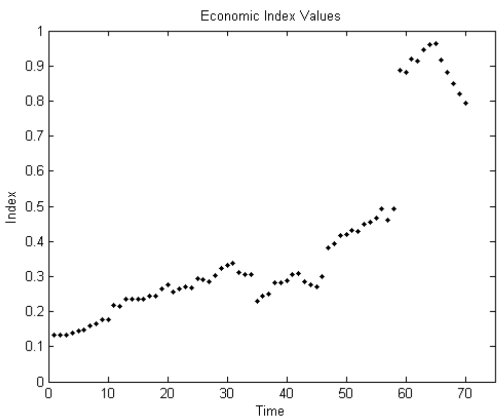
Several types of curves were fit to the data, including: Exponential, Logarithmic, Linear, and 2^{nd} through 6^{th} degree polynomials. The R^2 and end effects were observed for each curve fit to the data. The curve which best fit every index was selected. Based



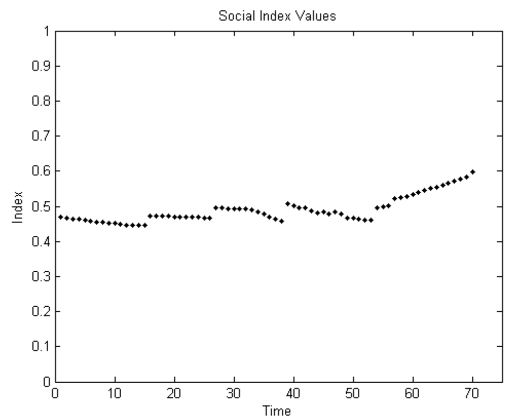
(a) Political



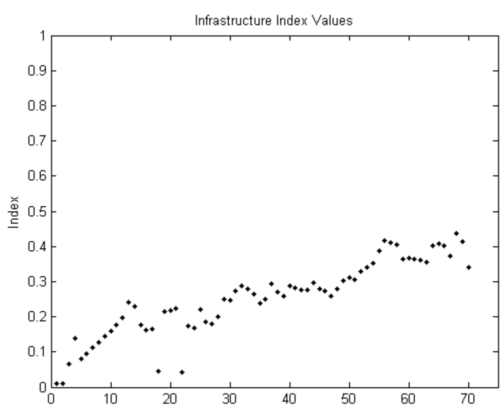
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 13. Index Time Series Plots

upon the Military Index, the polynomial function was most appropriate because of the end effects and shape of the data. The R^2 from the fitting process is shown in Table 2.

Table 2. R^2 **values for the curve fitting process**

	Expo	Log	Linear	Polynomial				
				2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Political	0.9118	0.7572	0.9450	0.9495	0.9719	0.9749	0.9765	0.9806
Military	0.0351	0.1713	0.0309	0.7378	0.7754	0.7971	0.8420	0.8532
Economic	0.8579	0.4514	0.7168	0.8484	0.8838	0.8968	0.9423	0.9571
Social	0.6346	0.3619	0.6265	0.7650	0.8364	0.8816	0.9079	0.9194
Infrastructure	0.5477	0.7694	0.8472	0.8498	0.8512	0.8677	0.8693	0.8802

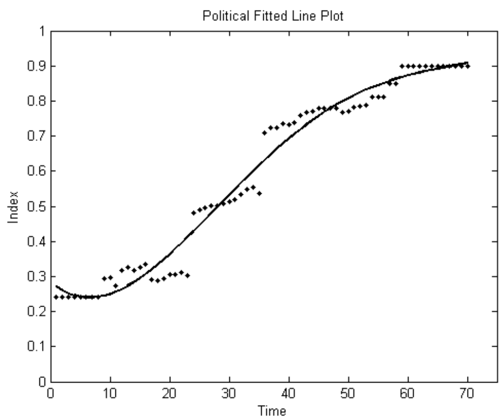
The fit selected was a 4^{th} degree polynomial because it minimized the SSE, was the lowest degree polynomial with all $R^2 \geq 0.8$, and by inspection the end effects provided a more adequate match than any of the curves listed above. The equations of the fitted polynomial curves are provided in Table 3. The values of the β coefficients are precise to a level of significance greater than 10 places, as a result the full set is not shown here but the full precision table is presented in Appendix B. The results of

Table 3. 4^{th} **degree polynomial equations**

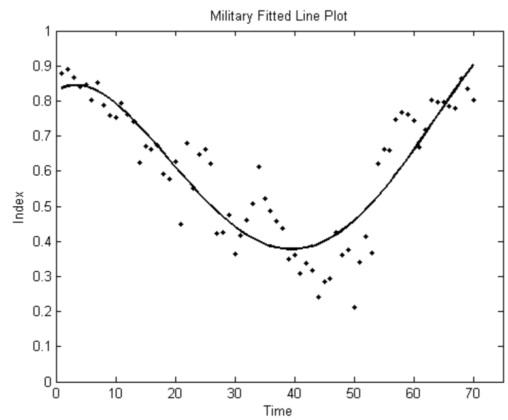
Index	**Equation**
Political	$y_P = 1.2e - 007t^4 - 2.3e - 005t^3 + 0.0013t^2 - 0.015t + 0.28$
Military	$y_M = -2.5e - 007t^4 + 4.1e - 005t^3 - 0.0018t^2 + 0.01t + 0.83$
Economic	$y_E = -2.5e - 007t^4 + 4.2e - 005t^3 - 0.0021t^2 + 0.04t + 0.0042$
Social	$y_S = 6.8e - 008t^4 - 8.2e - 006t^3 + 0.00032t^2 - 0.0038t + 0.47$
Infrastructure	$y_I = -1.2e - 007t^4 + 1.7e - 005t^3 - 0.00085t^2 + 0.02t + 0.013$

the fitting process are summarized in Table 4 and graphically depicted in Figure 14.

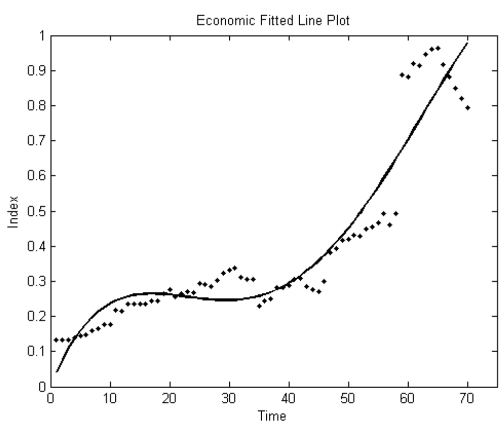
The idea of using curve fitting to approximate the indices accomplishes two things; a continuous function which describes the trend of the discrete index values is provided



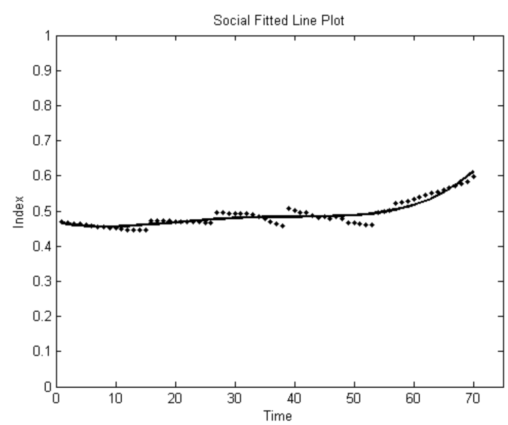
(a) Political



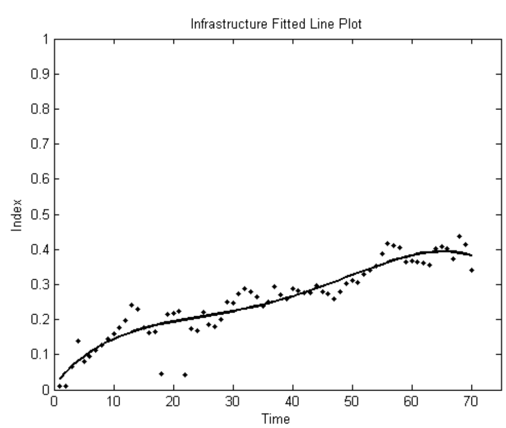
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 14. Index Fitted Polynomial Equation Plots

Table 4. 4^{th} degree polynomial fitting summary results

Equation	SSE	R^2
Political	0.104919	0.9749
Military	0.512325	0.7971
Economic	0.427826	0.8967
Social	0.011129	0.8816
Infrastructure	0.102024	0.8677

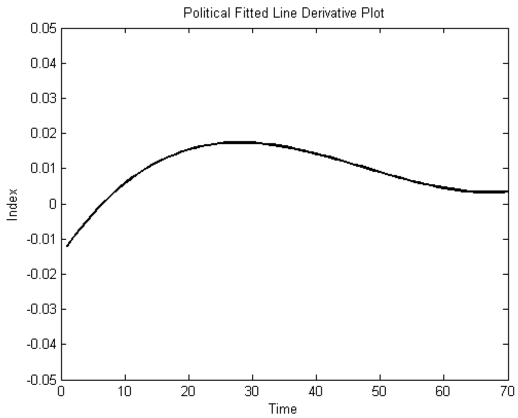
and the function can be used to calculate a rather simple derivative (many functions used to fit data–exponential, logarithmic, and polynomial functions have rather simple derivative functions). This captures the general trend of the data, accounts for the noise in the data, accounts for large deviations from the trend, but most importantly of all it is not highly reactive to sharp spikes and drops.

The derivatives of the equations in Table 3 are calculated with respect to t using Equation 3.10 and the result is provided in Table 5, and graphically in Figure 15. The values of the coefficients are precise to a level of significance greater than 10 places, as a result the full set is not shown here but is presented in Appendix B.

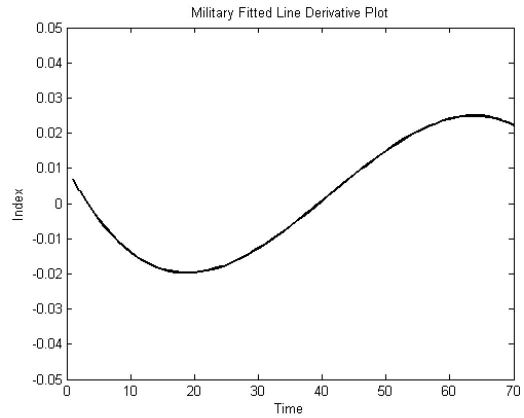
Table 5. Derivatives of the polynomial fitted equations

Index	Derivative
Political	$\frac{dP}{dt} = 4.74e-07t^3 - 6.75e-05t^2 + 2.66e-03t - 1.47e-02$
Military	$\frac{dM}{dt} = -9.82e-07t^3 + 1.22e-04t^2 - 3.54e-03t + 1.02e-02$
Economic	$\frac{dE}{dt} = -9.82e-07t^3 + 1.26e-04t^2 - 4.23e-03t + 4.03e-02$
Social	$\frac{dS}{dt} = 2.74e-07t^3 - 2.46e-05t^2 + 6.44e-04t - 3.76e-03$
Infrastructure	$\frac{dI}{dt} = -4.74e-07t^3 + 5.23e-05t^2 - 1.70e-03t + 2.01e-02$

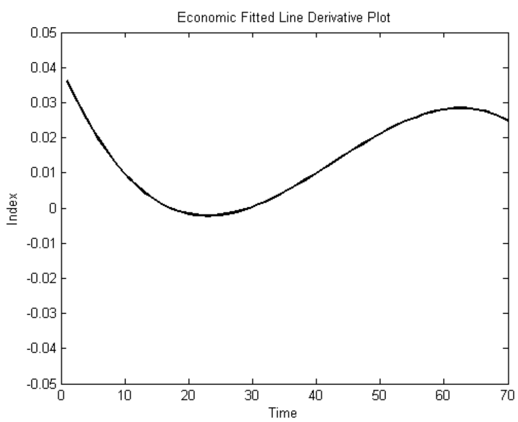
The derivatives of the fitted curve at each point, t, are used to approximate the derivative function of the operational variables. This is an integral part in calculating the coefficients of the final system of differential equations, completed in Section 4.1.3. In the next section, the rates of change for each variable at a given time, t, are found



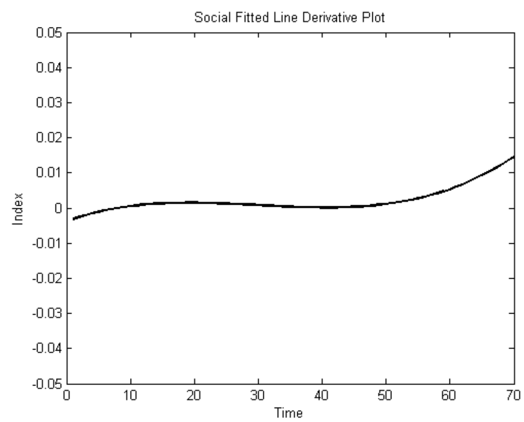
(a) Political



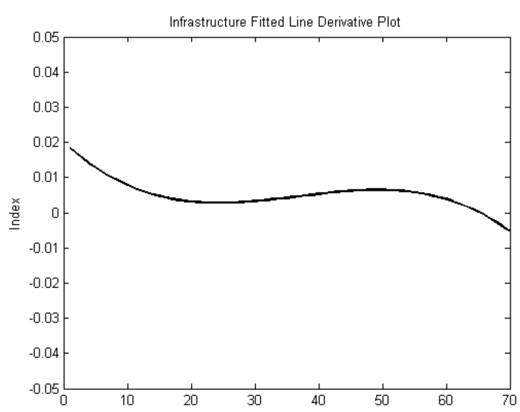
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 15. Fitted Polynomial Derivative Plots

by calculating the derivatives of the approximate functions through the system of differential equations.

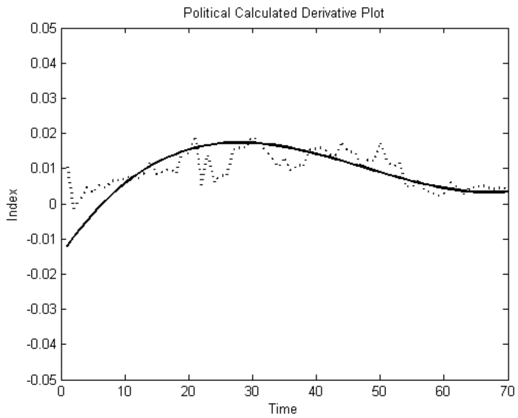
4.1.3 Determining the Coefficients of the System of Differential Equations.

To fit the system of differential equations, the coefficients are derived through the nonlinear least–squares method. The derivative functions in Table 5 are used to calculate $m_t^{(i)}$ in the minimization (Equation 3.20). Using the nonlinear program, the values of a_{ij}, b_{ij}, and d_{ik} coefficients are calculated which minimize the SSE as described in Equations 3.20-3.25 through the GRG method. The resulting a_{ij}, b_{ij}, and d_{ik} are then used in Equations 3.13–3.17 in Section 3.3 which calculate the $\hat{m}_t^{(i)}$ in Equation 3.20. The a, b, and d coefficients are provided in Tables 6–8. The values of these coefficients are precise to a level of significance greater than 10 places. As a result, the full tables are not shown here, but are presented in Appendix B.

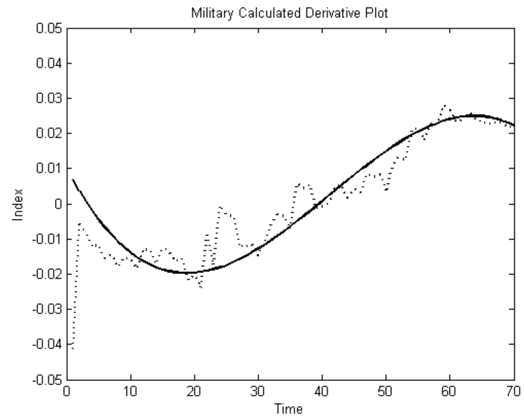
Table 6. a **coefficients**

i \ j	**Political**	**Military**	**Economic**	**Social**	**Infrastructure**
Political	-0.0108181	-0.0200317	0.0000199	0.0040096	0.0035656
Military	0.0743136	0.0168815	0.0002164	-0.0101848	-0.0241455
Economic	0.0287454	0.0155291	-0.0008631	-0.0100445	-0.0156717
Social	-0.0005479	0.0001096	0.0004305	0.0246105	0.0021363
Infrastructure	0.0034676	0.0008256	-0.0010147	-0.0264765	-0.0054859

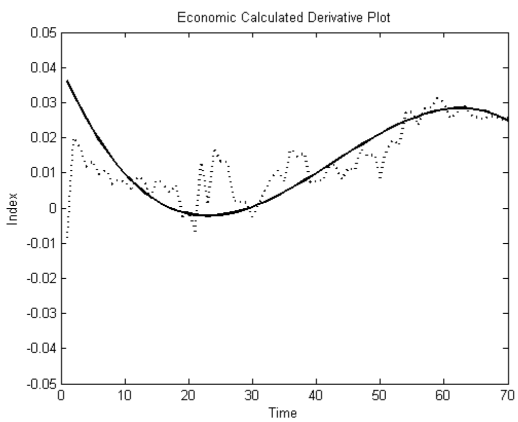
The coefficients are calculated by minimizing the SSE; Table 9 shows the SSE attributed to each calculated derivative as well as the overall SSE that was minimized by the nonlinear program. The derivatives (calculated and fitted) are then plotted to see how good of an approximation the system of differential equations provides, as shown in Figure 16. In applying mean–field theory, the errors associated



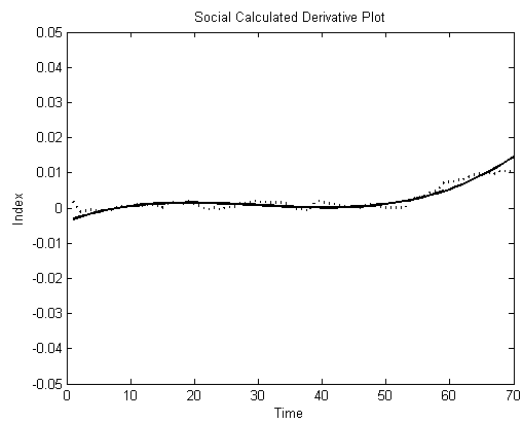
(a) Political



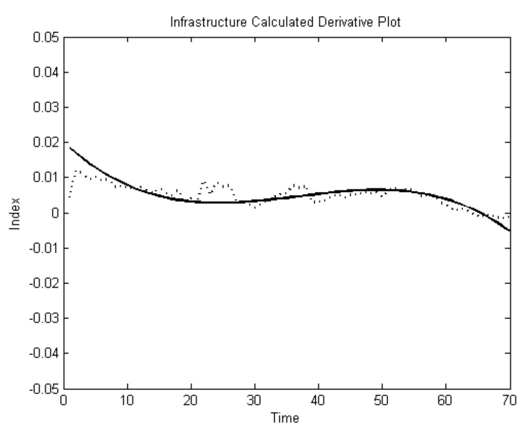
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 16. Calculated Derivative Plots– The solid lines represent the derivatives from the polynomial function and the dashed lines are those calculated from the system of differential equations.

Table 7. b coefficients

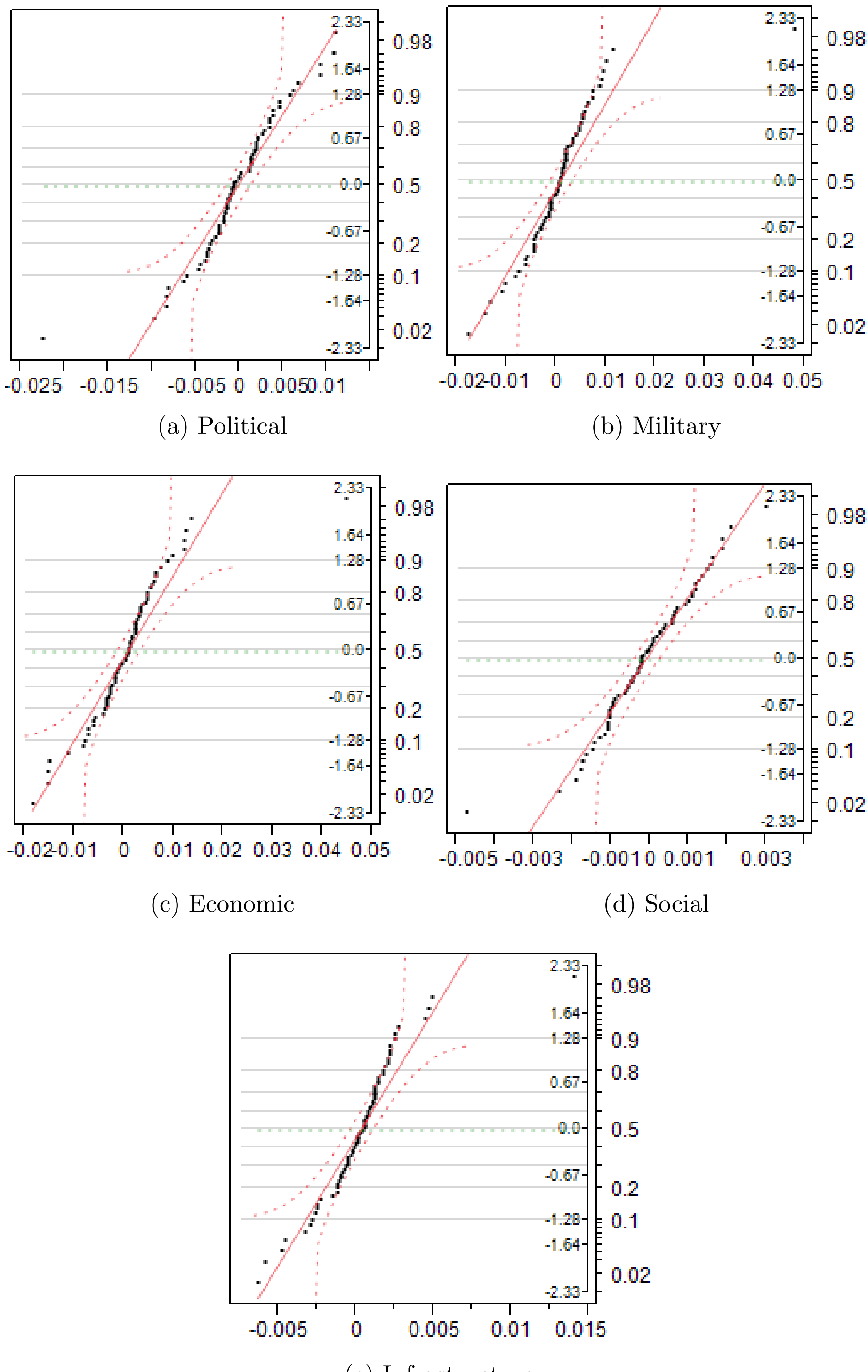
i \ j	Political	Military	Economic	Social	Infrastructure
Political	0.3980158	0.6319466	0.0061526	0.0894118	0.1010944
Military	0.9093994	0.5026149	0.5234950	0.1443637	0.6481461
Economic	0.4439128	0.3606699	0.1873238	0.1093262	0.3084922
Social	0.1788101	0.0781656	0.0700705	0.4343948	0.2960607
Infrastructure	0.2387622	0.0769009	0.1342082	0.3479762	0.2603047

Table 8. d coefficients

i \ k	Military	Economic
Political	-0.0074973	-0.0008035
Military	0.0230743	0.0007235
Economic	0.0184431	0.0007638
Social	-0.0017058	0.0008385
Infrastructure	0.0049494	-0.0005529

with the approximations can be expected to be normally distributed, as mentioned in Section 2.5. To determine if the errors are approximately normally distributed, a Normal Probability Plot is conducted. Based on mean-field theory, the data are plotted against a theoretical normal distribution and should fall along the diagonal if they are normally distributed, deviations from this trend indicate a departure from normality. A normal probability plot is shown in Figure 17 and depicts the error between the calculated and fitted derivative. The plots in Figure 17 indicate there are some deviations from normality, but the data are contained within the 95% confidence bands and one can assume normality in the errors associated with approximating derivatives through the system of differential equations.

The calculated derivatives indicate the rate of change in the system for each operational variable. The derivatives provide useful information; however, if the rate of change and a starting point are known, then a calculated index $(\hat{P}, \hat{M}, \hat{E}, \hat{S}, \hat{I})$ can



(a) Political

(b) Military

(c) Economic

(d) Social

(e) Infrastructure

Figure 17. Normal Probability Plots (Derivative)– The data should plot along the diagonal if they are normal, if the data are contained within the dashed confidence bands then one can reasonably assume normality.

Table 9. System of differential equations SSE

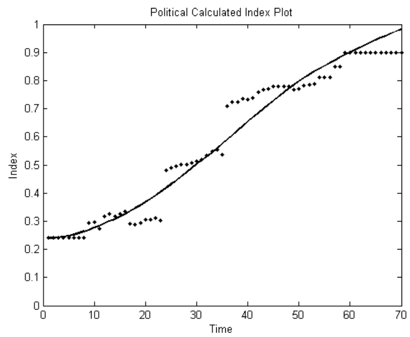
Derivative	SSE
Political	0.00122978
Military	0.00222477
Economic	0.00282604
Social	0.00009662
Infrastructure	0.000337669
Total	0.00671488

be used to gain more insight. This is an initial value problem. One method of solving first-order differential equations with a numerical method is the Euler method, which uses the derivative and the initial value to estimate the solution. The Euler method is

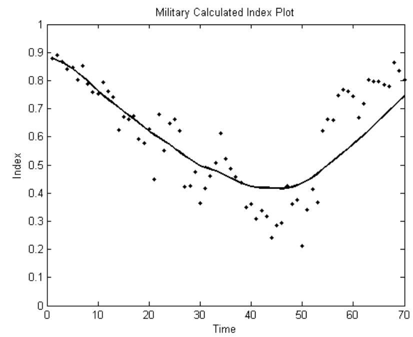
$$u_{k+1} = u_k + ha_k \qquad \text{for } k = 0, 1, \ldots, n \tag{4.1}$$

where $a_k = u'_k$ and h is the step size [23:394-397]. Numerical methods like this are especially insightful when an exact solution cannot be found, as in this case. Through an application of this method, the index values can be estimated through the knowledge of an initial value and the derivatives provided from the system of differential equations. Using this method, with $h = 1$, the approximate index is calculated and compared to the original index values in Figure 18. While the calculated index captures the general trend of the index values, it loses some accuracy as it is only given an initial value from data and then calculated purely from estimated data over the entire period. This estimation can be established as the solution with the most error; if additional values from other points in time are available to provide trajectory corrections, it would decrease the error.

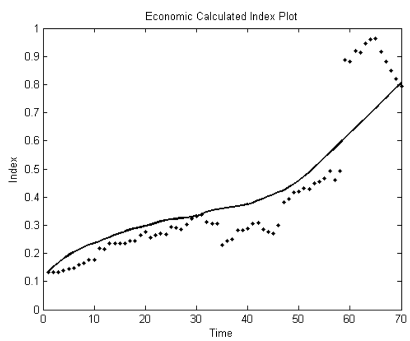
The plots in Figure 18 indicate that some accuracy has been lost in the process. Once again, the normal probability plots are used to determine if the error is normally distributed. From the plots in Figure 19, normality can only be assumed with the



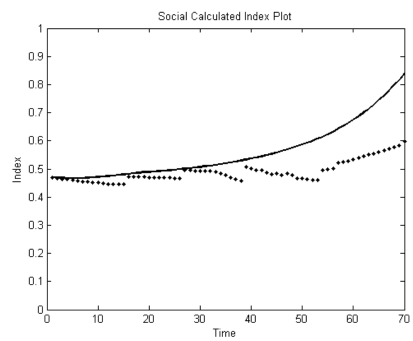
(a) Political



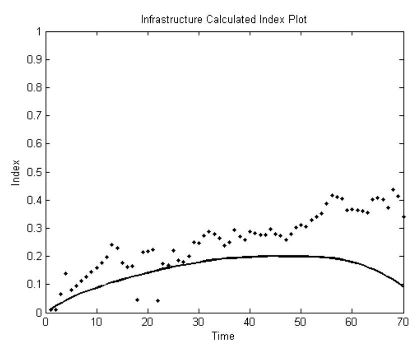
(b) Military



(c) Economic

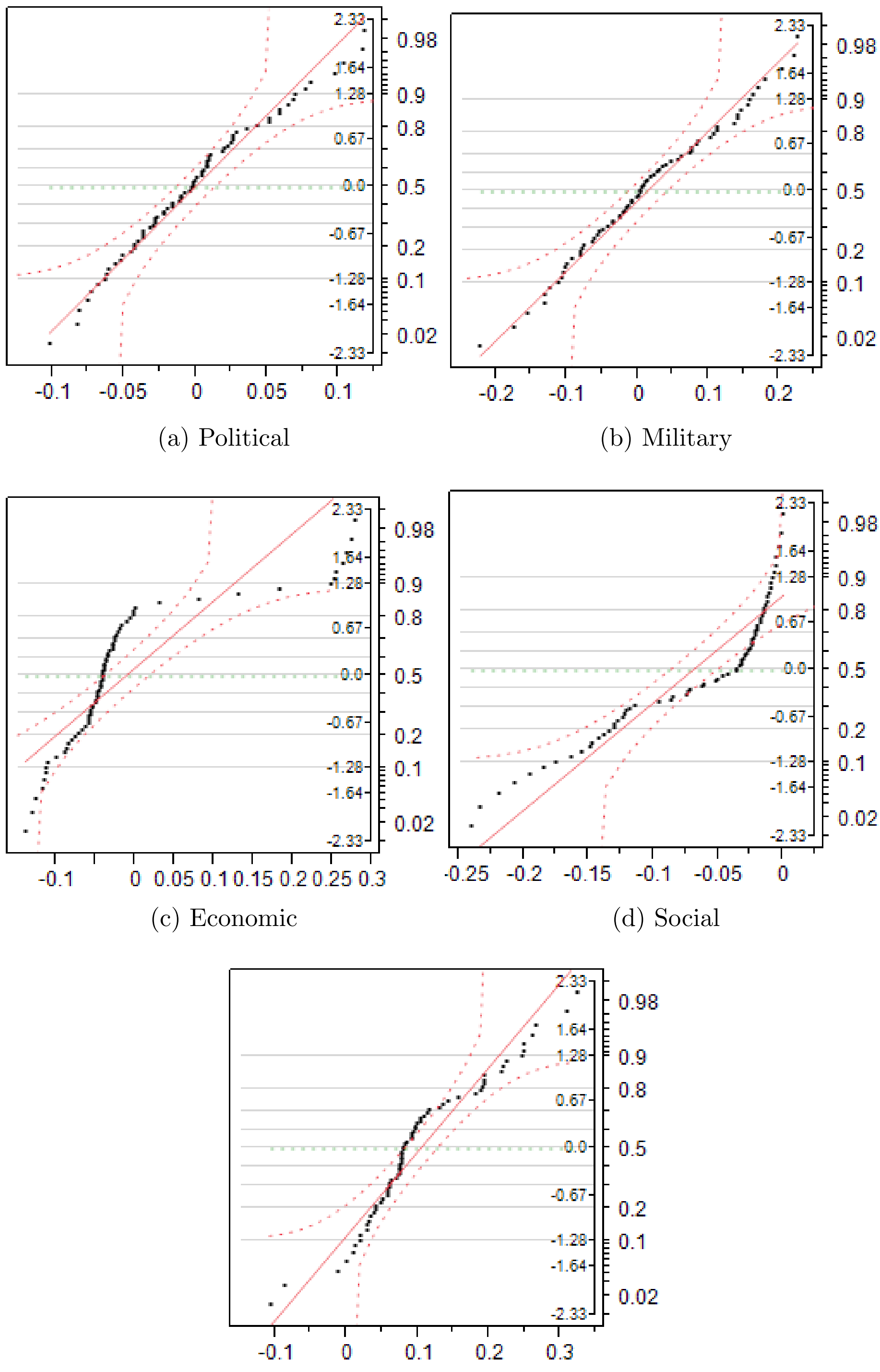


(d) Social



(e) Infrastructure

Figure 18. Calculated Indices versus Actual Indices– The solid lines represent the calculated indices from the Euler method and the points are the actual monthly index values.



(a) Political (b) Military

(c) Economic (d) Social

(e) Infrastructure

Figure 19. Normal Probability Plots (Euler Method)– The data should plot along the diagonal if they are normal, if the data are contained within the dashed confidence bands then one can reasonably assume normality.

Political and Military Index. The remaining three indicate departures from normality and have values outside the 95% confidence level band. This is not completely unexpected as the second estimated data point is based upon the initial value and an estimated derivative, the third is based upon the second estimated point and the estimated derivative, and so on. It is not difficult to see how method which builds upon estimations can lead to an over or underestimation. Based upon Figure 19, it can be observed that Figure 19c and Figure 19e are positively skewed and Figure 19d is negatively skewed asserting they are non-normal. A Shapiro-Wilk Goodness-of-Fit test was conducted and the results are in Table 10. From this table and Figures 19a and

Table 10. Shapiro-Wilk goodness-of-fit test

Calculated Index	p-value
Political	0.2873
Military	0.6141
Economic	< 0.0001
Social	< 0.0001
Infrastructure	0.0021

b, the conclusion can be made that the Political and Military error is normally distributed, indicating the successful application of the mean-field approximation. When the Shapiro-Wilk Goodness-of-Fit test is applied ($\alpha = 0.05$) one fails to reject the error is normally distributed. In this case, the Political Index has a p-value of 0.2873 and the Military Index has a p-value of 0.6141 while the other three have p-values less than $\alpha = 0.05$. This indicates the mean-field method captures the general trend of the variable it is approximating when the data that provides the approximation is appropriate. The inference here is that the data which builds the Economic, Social, and Infrastructure indices may be biased, collection methods may have changed, or the data needs to be improved, indicating an area for future research.

4.2 Analysis of Alternate Strategies

In this section the methodology is applied to evaluate alterative strategies which reflect possible modifications to the military and economic (diME) influence as applied by the US in Iraq. The following sections organize modifications to economic and military data (on the side of the US) into different strategies. These modifications are derived from actual implemented plans, proposed legislation from Congress, or are hypothetical and demonstrate the what-if analysis feature of the model.

4.2.1 No Surge.

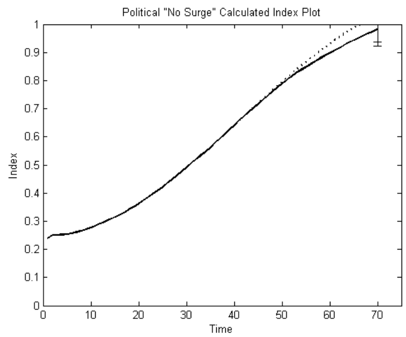
On the 10th of January 2007, President George W. Bush delivered a speech to the American Public outlining a new strategy in Iraq. As part of that strategy he called for the additional deployment of $20,000$ troops, the majority deploying to Baghdad to help Iraqis clear and secure neighborhoods, help them protect the population, and help ensure that the Iraqi forces left behind are capable of providing the security needed [11]. The first deployment of troops was in January 2007 and in July 2007 all surge troops had been deployed. The surge would last to July 2008 and was roughly an increase of $28,000$ troops [39].

The actual troop numbers for the Mil_t variable represent the surge strategy and serve as the base case for evaluating the alternate strategies. In order to evaluate the no surge strategy, the Mil_t are adjusted under the assumption that if the surge was not implemented, the number of troops would have remained the same during for the time period. Therefore, the number of troops are held constant from the December 2006 level through February 2009 (when the troop level returned to near the pre–surge level). The adjusted levels of Mil_t are listed in Appendix D. All other variables remain the same, specifically the coefficients are not changed as the goal is to evaluate the alternative strategy under the conditions that took place. In other words, if everything else remained the same how would the indices have been affected by the no surge policy? The results are shown in Figure 20.

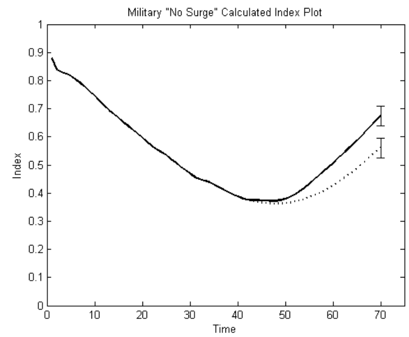
The observation here is how the indices change over time based upon a different troop level, while keeping all other variables constant. The change in the Mil_t variables did have impacts on all the variables, some more than others. The plots are exactly identical up until month 47 (there is no change in the inputs until month 48) where the surge began, and then takes a different trajectory based upon the interactions from the different data. As a result the trajectory, of the Political and Social indices slightly improved over time; the Military, Economic, and Infrastructure trajectories decreased over time. A t–test on the difference of the means is established with the following hypothesis:

$$H_0 : \mu_{basecase} - \mu_{nosurge} = 0$$
$$H_A : \mu_{basecase} - \mu_{nosurge} \neq 0$$
$$\alpha = 0.05$$

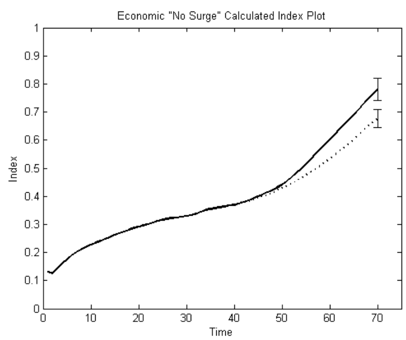
This tests the hypothesis that there is no difference between the alternate strategy (no surge) and the actual strategy (base case). The results from the tests are in Table 11 and shows in all cases H_0 is rejected because all p-values are less than alpha, as shown in the first row of the table. This confirms changing the Mil_t has a significant effect on the indices at the $\alpha = 0.05$ level. The second line of the table shows the Military, Economic, and Infrastructure Index difference means are significantly greater then 0, indicating the base case indices were significantly higher at the $\alpha = 0.05$ level. The third line indicates the opposite, the Political and Social Index difference means were significantly less than 0 and the no surge indices where higher than the base case, all at the $\alpha = 0.05$ level.



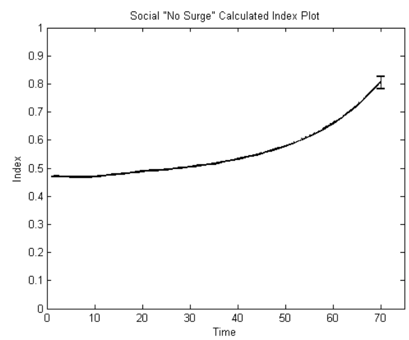
(a) Political



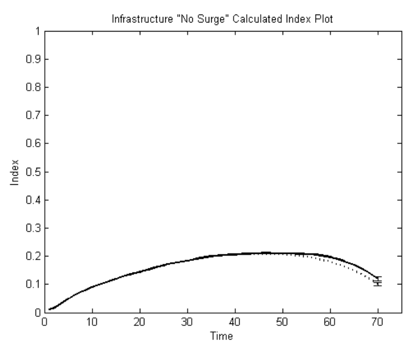
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 20. Calculated Index Plots (No Surge)– The solid lines represent the calculated indices of the Euler solution and the dashed lines represent the indices under the no surge alternative. The error bars show the 95% Confidence Interval associated with the end effect.

Table 11. t-test on the difference of means between the base case and no surge

Test	Political	Military	Economic	Social	Infrastructure
Prob $>\mid t \mid$	< 0.0001	< 0.0001	< 0.0001	< 0.0016	< 0.0001
Prob $> t$	1	< 0.0001	< 0.0001	0.9992	< 0.0001
Prob $< t$	< 0.0001	1	1	< 0.0008	1

Additionally there is 95% Confidence Interval (CI) on the $t = 70$ observation which shows the error associated with the end effects. This is calculated by

$$x_{70}^{(j)} \pm \ t_{\left(n-1,1-\frac{\alpha}{2}\right)}\sqrt{\frac{S^2(n)}{n}} \qquad (4.2)$$

where

$$\begin{aligned} \alpha &= 0.05 \\ n &= \text{the number of observations} \\ S^2 &= \text{the sample variance} \\ 1.9959 &= t_{\left(n-1,1-\frac{\alpha}{2}\right)} \end{aligned}$$

This CI is built around the $x_{70}^{(j)}$ observation versus $\bar{x}^{(j)}$ to compare the end effects vice the mean. Even with the associated error, there is still a significant difference in the Military, Economic, and Infrastructure indices between the two strategies in the final time period. Based on the results from the model, it can be concluded that the no surge policy implemented had impacts on all of the PMESI indices. The results in Table 11 show the difference of means for the Political and Social Index is greater than 0 indicating the mean of the no surge index was greater than the mean of the base case index. In the others, the mean of the no surge was less than 0, indicating the mean of the base case was greater than the no surge policy. Based on Table 12, the end effects of the Political, Social, and Infrastructure indices have

Table 12. 95% **confidence interval on the end effects (base case vs. no surge)**

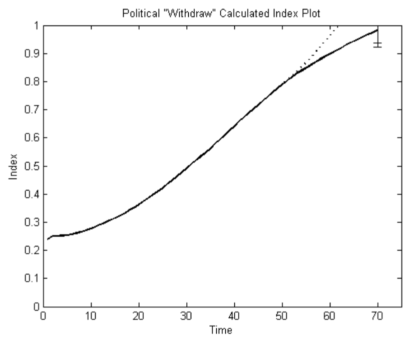
	Base Case	**No Surge**	**Overlap?**
Political	(0.922384,1)	(0.93787,1)	Yes
Military	(0.639915,0.709656)	(0.524145,0.59624)	No
Economic	(0.741091,0.819947)	(0.643858,0.7086)	No
Social	(0.786461,0.82957)	(0.783501,0.826717)	Yes
Infrastructure	(0.107126,0.133476)	(0.087413,0.11328)	Yes

overlapping CIs indicating the change is insignificant. The remaining plots do not have overlapping CIs which confirm the results of the t–test and demonstrate that the alternative application of the no surge diME affected the trajectory of the index in a significant manner based on the dynamics of the operational environment.

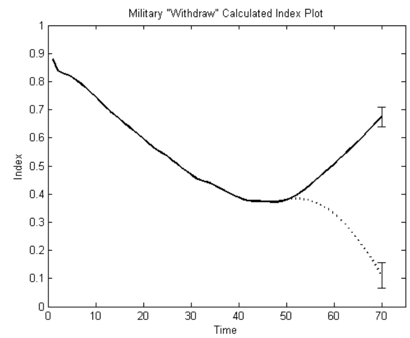
4.2.2 Complete Reduction by 2008.

In March–July of 2007, Congress proposed a series of resolutions that would lead to the removal of US troops in Iraq. The first one, House Resolution (H.R.) 1951 was passed by Congress and vetoed by President Bush; H.R. 2956 was passed by the House and required the SecDeF to initiate the reduction of troops in Iraq immediately through April 1, 2008. This resolution was then sent to the Senate where it was narrowly defeated 52-47. This was never introduced again and the current surge plan continued as outlined in January of 2007 [53].

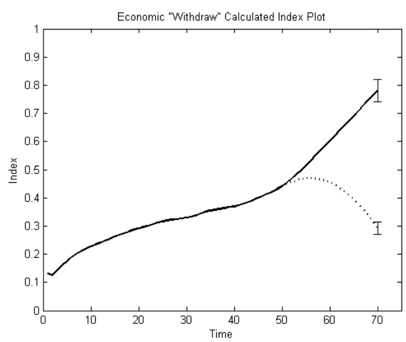
To evaluate the withdrawal strategy, assume that the March Resolution passed and the number of troops were reduced in even increments over the next year leading to no troops in April 2008. The adjusted levels of Mil_t are listed in Appendix D. All other variables remain the same, as in the previous case. In other words, if everything else remained the same how would a phased withdrawal implemented in 2007 affect the indices? The results are shown in Figure 21.



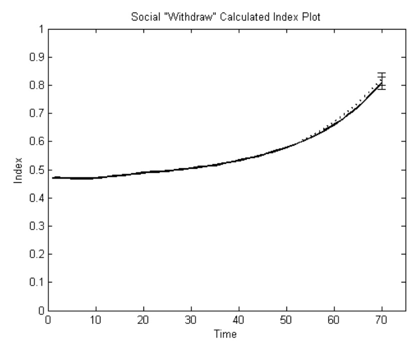
(a) Political



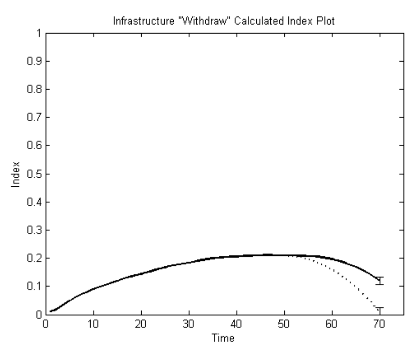
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 21. Calculated Index Plots (Withdrawal)– The solid lines represent the calculated indices from the Euler solution and the dashed lines represent the indices under the withdrawal alternative. The error bars show the 95% Confidence Interval associated with the end effect.

The observation here is how the indices changed over time based upon the withdrawal troop level and timeline, with all other variables constant. The change in the Mil_t variables did have effects on all the variables, some more than others. The plots are exactly identical up until month 50 where the withdrawal began and then takes on a different trajectory, based upon the interactions from the different data. As a result, the trajectory of the Political and Social indices slightly improved over time; the Military, Economic, and Infrastructure trajectories decreased over time. The results from Table 13 indicate this a little more clearly. A t–test on the difference of the means, $H_0 : \mu_{basecase} - \mu_{withdraw} = 0$ and $H_A : \mu_{basecase} - \mu_{withdraw} \neq 0$ at $\alpha = 0.05$ shows in all cases H_0 is rejected because all p-values are less than alpha, as shown in the first row of the table. This confirms changing the Mil_t has a significant effect on the indices at the $\alpha = 0.05$ level. The second line of the table shows the Military, Economic, and Infrastructure Index difference means are significantly greater then 0, indicating the base case indices were significantly higher at the $\alpha = 0.05$ level. The third line indicates the opposite, the Political and Social Index difference means were significantly less than 0 and the withdraw indices where higher than the base case, all at the $\alpha = 0.05$ level.

Table 13. t-test on the difference of means between the base case and withdraw

Test	Political	Military	Economic	Social	Infrastructure
Prob $>\mid t \mid$	< 0.0001	0.0002	0.0003	< 0.0001	0.0002
Prob $> t$	1	0.0001	0.0001	1	0.0001
Prob $< t$	< 0.0001	0.9999	0.9999	< 0.0001	0.9999

After interpreting the results in Table 13, it can be seen that the withdraw policy had impacts on all of the PMESI indices. Based on the results, the difference of means for the Political and Social Index is greater than 0 indicating the mean of the withdraw index was greater than the mean of the base case index. In the others,

Table 14. 95% **confidence interval on the end effects (base case vs. withdraw)**

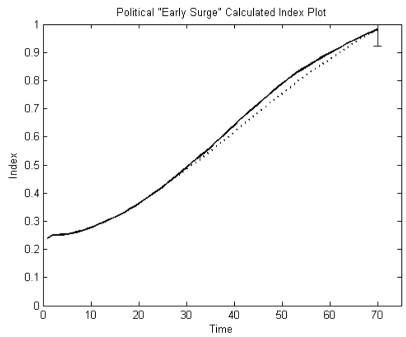
	Base Case	**Withdraw**	**Overlap?**
Political	(0.922384,1)	(0.93678,1)	Yes
Military	(0.639915,0.709656)	(0.066189,0.155942)	No
Economic	(0.741091,0.819947)	(0.271422,0.315198)	No
Social	(0.786461,0.82957)	(0.798613,0.843939)	Yes
Infrastructure	(0.107126,0.133476)	(-0.00573,0.023497)	No

the mean of the withdraw was less than 0, indicating the mean of the base case was greater than the no surge policy. However, the end effects in Table 14 demonstrate that the alternative application of the withdraw diME affected the trajectory of the index in a significant manner based on the dynamics of the operational environment. The overlapping CI on the end effects of the Political and Social Index indicate the change on those trajectories is insignificant.

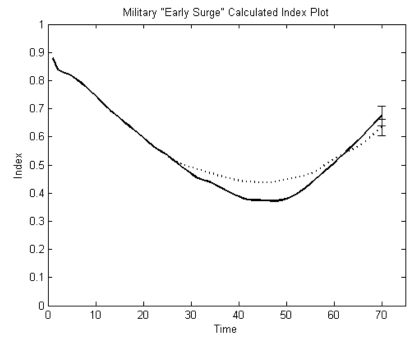
4.2.3 Two Year Earlier Surge.

A look at some scenarios which may be alternatives in future operations can demonstrate the wargaming ability of this model. One which will be examined is the notion of executing the surge approximately two years earlier and after two years of conflict. In order to evaluate this strategy, the assumption is the same troop numbers were used in the exact same manner only two years earlier. The numbers of troops prior to and after the surge were adjusted slightly so that the progression continued in a logical manner with the flow of troops. The adjusted levels of Mil_t are listed in Appendix D. All other variables remain the same, as in the previous case. In this scenario, an earlier surge implemented in 2004 is examined to see how it impacted the indices. The results are shown in Figure 22.

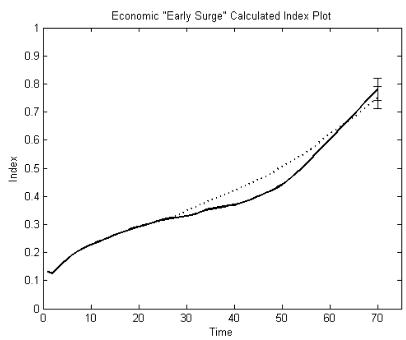
The observation here is how the indices change based upon the early surge and with all other variables constant. The change in the Mil_t variables had similar effects on



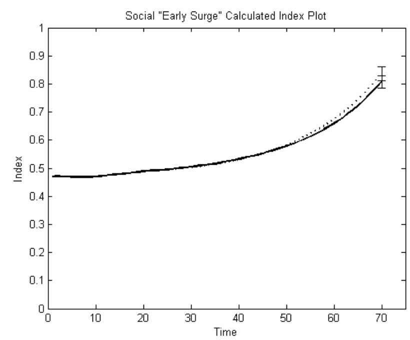
(a) Political



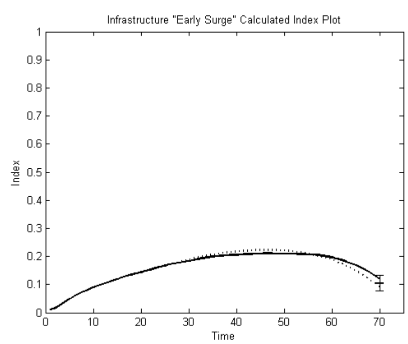
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 22. Calculated Index Plots (Early Surge)– The solid lines represent the calculated indices from the Euler solution and the dashed lines represent the indices under the Early Surge alternative. The error bars show the 95% Confidence Interval associated with the end effect.

all the variables, much like the previous two cases. The plots are exactly identical up until month 24 where the early surge plan begins, then takes on a different trajectory, based upon the interactions from the different data. Unlike the previous two cases the early surge indices deviate from the base case and then intersect the base case plots again in four out of the five indices. In this case, only the Political Index initially worsened, however it did return to the path of the original plot as it approached month 70. The remaining four indices all increased initially and then dropped below the base case plots towards the end of the time period with the exception of the Social variable which remained at the increased rate. The results from Table 15 indicate this a little more clearly.

Table 15. t-test on the difference of means between the base case and early surge

Test	Political	Military	Economic	Social	Infrastructure
Prob $>\mid t \mid$	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.7363
Prob $> t$	< 0.0001	1	1	1	0.6319
Prob $< t$	1	< 0.0001	< 0.0001	< 0.0001	0.3681

Table 16. 95% **confidence interval on the end effects (base case vs. early surge)**

	Base Case	Early Surge	Overlap?
Political	(0.922384,1)	(0.923514,1)	Yes
Military	(0.639915,0.709656)	(0.602759,0.66226)	Yes
Economic	(0.741091,0.819947)	(0.710834,0.790809)	Yes
Social	(0.786461,0.82957)	(0.81284,0.85985)	Yes
Infrastructure	(0.107126,0.133476)	(0.076573,0.104759)	No

A t–test on the difference of the means, $H_0 : \mu_{basecase} - \mu_{earlysurge} = 0$ and $H_A : \mu_{basecase} - \mu_{earlysurge} \neq 0$ at $\alpha = 0.05$ shows that in four out of five cases H_0 is rejected because all p-values are less than alpha. In the case of the Infrastructure Index, H_0 cannot be rejected, as shown in the first row of the table. This confirms changing the Mil_t has a significant effect on the all except the Infrastructure Index at the

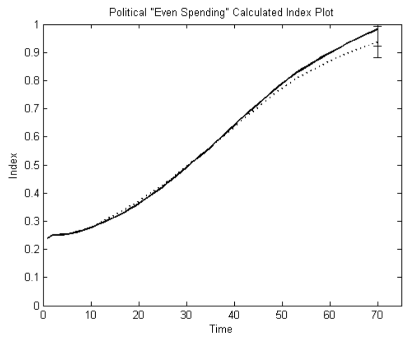
$\alpha = 0.05$ level. The second line of the table shows the Political Index difference mean is significantly greater then 0, indicating the base case indices were significantly higher at the $\alpha = 0.05$ level. The third line indicates the Military, Economic, and Social Index difference means were significantly less than 0 and the early surge indices where higher than the base case, all at the $\alpha = 0.05$ level.

Based on the results in Table 15, it can be concluded that there were significant impacts in all indices except Infrastructure, Table 16 confirms this with overlapping CIs at $t = 70$ in all except Infrastructure and the overall trajectory was not affected. This may indicate an alternate strategy of an early surge versus a surge in 2007 would have both had similar impacts with the exception of Infrastructure. The trajectory of the early surge index was impacted minimally based on the dynamics of the operational environment.

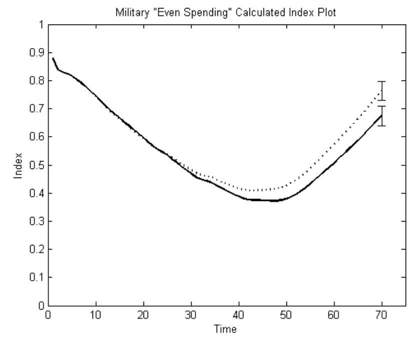
4.2.4 Even–Spending Strategy.

In addition to varying the troop levels, the ability to look at some scenarios involving the economic aid and explore the impact of the Eco_t variable are also possible. One examined is the notion of executing a flat or even-spending strategy. In this case, a constant spending level of \$4.822 billion per year is assumed, which is the average spending for the first five years. The adjusted levels of Eco_t are listed in Appendix D. All other variables remain the same, troop numbers remain at the base case level. In this scenario how an even–spending plan implemented would affect the indices is examined. The results are shown in Figure 23.

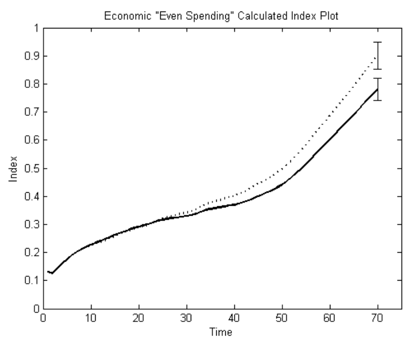
This instance shows the indices changed based upon the evenly distributed spending strategy with all other variables constant is observed. The change in the Eco_t variables did affect each variable, much like the previous cases. The changes are observed almost immediately as the spending strategy has been altered from the very



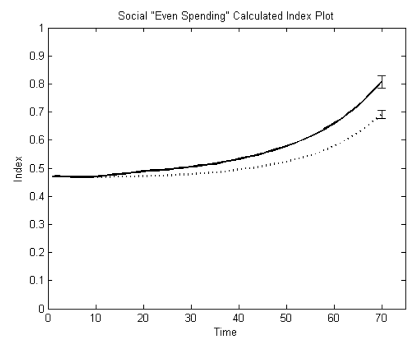
(a) Political



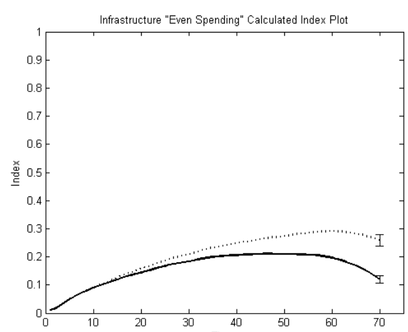
(b) Military



(c) Economic



(d) Social



(e) Infrastructure

Figure 23. Calculated Index Plots (Even–Spending)– The solid lines represent the calculated indices from the Euler solution and the dashed lines represent the indices under the Even–Spending alternative. The error bars show the 95% Confidence Interval associated with the end effect.

beginning. In this scenario, a difference exists in every plot, the Political and Social appear to decrease and the remaining all show increases. Once again, the significance of the differences is tested. The results are listed in Table 17.

Table 17. t-test on the difference of means between the base case and even-spending

Test	Political	Military	Economic	Social	Infrastructure
Prob $>\mid t \mid$	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Prob $> t$	< 0.0001	1	1	< 0.0001	1
Prob $< t$	1	< 0.0001	< 0.0001	1	< 0.0001

A t–test on the difference of the means, $H_0 : \mu_{basecase} - \mu_{earlysurge} = 0$ and $H_A : \mu_{basecase} - \mu_{earlysurge} \neq 0$ at $\alpha = 0.05$ shows that in all cases H_0 is rejected because all p-values are less than alpha. This confirms changing the Eco_t has a significant affect on the indices at the $\alpha = 0.05$ level. The second line of the table shows the Political and Social indices difference means are significantly greater then 0, indicating the base case indices were significantly higher at the $\alpha = 0.05$ level. The third line indicates the Military, Economic, and Infrastructure Index difference means were significantly less than 0 and that the even–spending indices where higher than the base case, all at the $\alpha = 0.05$ level.

Table 18. 95% **confidence interval on the end effects (base case vs. even-spending)**

	Base Case	Even-Spending	Overlap?
Political	(0.922384,1)	(0.880909,0.993017)	Yes
Military	(0.639915,0.709656)	(0.730415,0.795904)	No
Economic	(0.741091,0.819947)	(0.852733,0.949412)	No
Social	(0.786461,0.82957)	(0.677516,0.705383)	No
Infrastructure	(0.107126,0.133476)	(0.238472,0.277943)	No

Interestingly, this is the only strategy which had a noticeable change in the social environment. Based upon the result, one may deduce that an even-spending strategy impacts all of the PMESI indices. Based on the results in Table 17, it can be seen the difference of means for the Political and Social indices are less than 0 indicating the mean of the base case index was greater than the mean of the even-spending index. In the others, the mean of the difference was greater than 0 indicating the mean of the base case was less than the even–spending policy. As shown in Figure 23 and Table 18 the end effects of the Political Index have overlapping CIs indicating the change is insignificant. The remaining plots do not have overlapping CIs which confirm the results of the t–test and demonstrate the alternative application of the even–spending diME affected the trajectory of the index in a significant manner based on the dynamics of the operational environment. It may also be inferred that varying the economic aid may demonstrate a more prominent impact than the troop level based on the four cases presented here.

4.2.5 Summary.

As hypothesized, the model provides a means to evaluate various strategies through changing the inputs which are available to the US In all but one instance, changing an exogenous variable resulted in a statistically significant difference which could then be determined to have an increasing or decreasing affect as well. However, the differences did not always lead to a changed trajectory or differing end effects. While no one variable resulted in improving the indices in each case, it is clear to see there are rewards and costs associated with changing the inputs. Once again, it is important to note this set of coefficients and equations is currently based upon the data provided to build the model. Changing one of the inputs will not have the same effect in all cases, as mentioned previously in Section 3.3.1.

As previously mentioned, the strength of this model is the interactions between the operational variables, the interrelatedness between the PMESI variables within the operational environment. This is demonstrated in the analysis as well; in each of the previous four sections a change in one forcing function led to changes in each index. The changes were not linear (increase in $X =$ increase in Y), but rather nonlinear and a result of all the variables changing. A change in one forcing function changes all equations in the system, thereby affecting the entire system (operational environment). The resulting changes often produced a measurable difference in the index. This ability to collect, analyze, and interpret data provides insight on the diME functions, ultimately helping to understand its effects within and on the operational environment.

The interpretation of the results may not always be easy to understand; some of the cases in the previous sections may be interpreted differently by analysts and decision makers. The result is an impact, one that is without complete understanding of the cause. This is due to the generality of the model, the decision maker is provided with insight, and information regarding the strategy, the results are not omniscient.

4.3 Application to SSTRO

In order to apply this to additional SSTRO there is one requirement– data. The indices are created in such a manner that they are not dependent upon a specific data element nor are they required to be fashioned in the exact same manner as this application. The primary driver in the Iraqi economy is considered to be revenues from oil exports so it is considered in the development of these indices. This may not be the case in other states, but the same type of information must be captured. For example, if this were applied to a country within US Africa Command (AFRICOM), oil revenues may be a piece of the economy but revenue from mining operations would be an important aspect as well. In addition, the instruments of national power may differ slightly as well based on the application. The DIME forcing functions and the PMESII variables construct the basic framework, which forms the basis for measurement.

One possible use of this model is to provide wargamers a tool which will allow for the evaluation of strategies. The analyst can *turn the dials* of certain aspects of war and see how such changes affect the operational variables of the host nation. While the application presented in this research was post-analysis, the ability to provide current and future analysis is present. History has a way of repeating itself and this can be an advantage in analysis. If there was a growing sentiment that SSTRO was needed in country X, planners could use this tool to wargame certain strategies. For instance, historical data from country X can be used to derive the coefficients, or even data from similar or neighboring countries may provide useful insights to appropriately plan SSTRO. An interested analyst could then, for example, observe the affects of adding 50,000 troops and a 10% increase in economic aid to a host nation. The result would demonstrate the causal relationship in how the instruments of national power serving as inputs affect the operational environment output. Furthermore, the analyst could test and evaluate different strategies and consider the positive and negative results from each. This capability, to conduct tradeoff analysis between competing strategies, could save money, time, and lives.

4.4 Summary

This chapter served to illustrate the application of the model and solution methodology. Actual data from OIF provided the framework for the application and possible (actual and notional) strategies demonstrated the usefulness of this research effort. The insights gathered may be used by planners to determine force strategy and conduct campaign analysis. The end state is to provide the decision maker with the proper framework to achieve clarity of action in the decision.

This research provides a good step forward to evaluate the complexities of SSTRO, however, there remains areas which can be addressed and further the research. Chapter V presents a discussion on the overall conclusions and recommendations for future research.

V. Conclusions and Recomendations

This chapter reviews the significant insights, identifies topics for future research efforts, and presents conclusions.

5.1 Review

In support of the US Army Center for Army Analysis efforts to develop analytical methods for use in Irregular Warfare operations, this research develops a methodology which evaluates the effects of leveraging the instruments of national power in the operational environment. The developed model attempts to capture the interrelatedness and complexities reflective of an actual operational environment. In so doing, it captures each moment of the operational environment as the total contribution of each internal and external variable– the sum of its parts.

The shift in warfare as described in FM 3.0 has gone from *around the people* to *among the people* [20]. This change marks a new paradigm beyond how we fight and into how we form our operations. The ability to measure the PMESII variables parallels the work performed by military planners. If operations are planned in this context, it makes sense to measure and evaluate them in the same context. This model provides insight to the analyst on the application of the instruments of national power in terms of the operational environment to be considered.

This research has proposed a methodology to solve a problem currently facing strategic planners. In researching this problem, it is discovered that there are models which look at operational effectiveness, models which predict instability, and models which determine which countries are candidates to provide SSTRO. However, there are no models which look at the tools available to conduct SSTRO and evaluate their impact through the variables which describe the operational environment.

5.2 Insights

This model was proposed as an inverse problem; through the available data an estimated model was derived. Following some of the extensive work in the field of Lanchester equations, a system of differential equations was developed to capture the interrelatedness of the PMESI variables. The number of independent variables makes this complex system of differential equations difficult to solve; however, the application of mean-field theory reduces the complexity and facilitates the solution. This is accomplished by applying a composite index value to approximate the variables. The errors of this approximation method are assumed to be normally distributed and after applying the approximation, Figure 17 demonstrates that with an $\alpha = 0.05$ the assumption of normality of errors is valid.

After formulating the problem, the coefficients of the system of differential equations are derived by applying a nonlinear least–squares minimization problem. This minimizes the error between the fitted line derivatives and the system of differential equations while solving for the coefficients. The result is a model which describes the effects of applying diME through the PMESI variables. This methodology demonstrates a procedure using common software, available on most computers, and data that is available to anyone.

The application conducted in this research demonstrates some of the abilities of this model, such as what-if analysis. This allows the analyst to detect a statistical difference in indices as a result of adjusting the diME levels, as well as determining if it was a positive or negative change.

This methodology provides an analyst means to collect, analyze, and interpret data which helps to better understand the importance and effects of DIME and its relationship with the operational environment. Understanding the dynamics that make up the operational environment is paramount to understanding the SSTRO mission. The methodology proposed in this research allows analysts and decision makers the ability to investigate varying applications of the instruments of national power in a SSTRO environment.

The methodology encompasses:

- A data collection effort which leads to the development of normalized PMESI indices which describe the general trend of the operational environment with respect to that variable.
- A numerical technique which helps to understand how the general trend of a variable changes as a function of time with respect to the dynamics of the operational environment.
- A tool which integrates the data and provides a graphical depiction of how differing applications of diME respond given an established set of dynamics for the operational environment.

Within a specific environment, this model may be used to provide relevant information to a decision maker. This is limited by certain factors which are critical aspects of the methodology. The reliance on actual data underscores a salient limitation– the model is robust to the extent of available, quality data. As with many research efforts, better data makes for better models. Furthermore, the weights assigned in the composite index creation and different curve-fitting techniques will affect the final output. The decision makers vision of the weighting hierarchy must be captured to reflect his or her true beliefs. Analysts must be rigorous in their application of methods when applying the presented solution methodology to ensure the output is credible. Of utmost importance is what Helmbold refers to as the constant fallacy, the coefficients derived are not constants which can then be applied in all applications [24]. The coefficients are specific to environment (time and space) whose dynamics they describe. No model is perfect; but the initial results from this research are promising.

5.3 Potential Future Research

While conducting this research several ideas for improvement came up, some were implemented right away, others fall in the category of outside the scope of this effort and are tagged for future research endeavors. These topics for future research are discussed within this section.

5.3.1 Data Refinement.

The data collection and weighting scheme are an area for future research. As stated earlier, one of the features of this model is the use of the data we have, not the data we wish we had. This indicates the model is quite capable with limited data; however, with better data the full potential of the model can be realized. Numerous open sources of data, such as that provided by the World Bank, collect several of the metrics currently used and several more metrics that may be used. A robust analysis which compiles these data sets and uses a multivariate analysis technique such as principal components analysis or factor analysis to determine what set of components or factors can provide information on the operational variables would be of use. This would identify the smallest subset of variables which explain the greatest amount of variance, and would make logical candidates to form the indices.

The current baseline in the model normalizes the index components by the maximum and minimum observations. This may not always indicate the true benchmark for success or failure of that component. One method to remedy this is the establishment of a common baseline. This common baseline would take into account several countries and establish levels which indicate success or failure based upon the current levels of those countries. This group of countries may include similar countries with respect to their geographic location, culture, religion, language, economy, and government. This would allow the normalization of the component scores to consider more than just the best and worst observations for the evaluated time period.

The normalization method is a strictly linear function; many of the components do have a point of diminishing returns, indicating that a sigmoid function and its S shape may be a better way to express the normalization function. For instance, the difference between 20 and 24 hours of electricity a day will have more impact with a linear function versus a sigmoid function. The four hours make up one-eighth of a day, but truly may be insignificant. The heart of this problem will be determining what is *good enough*.

The weighting scheme can often be a point of contention amongst decision makers. The weights in this analysis represent a subjective proportionality assessment. In order to tailor the weights to allow the decision maker the most flexibility and ease of use, a Rank Order Centroid method can be implemented

$$W_i = \frac{1}{M} \sum_{n=1}^{M} \frac{1}{r_n} \tag{5.1}$$

where M is the number of items, r_n is the rank of the n^{th} item and W_i is the weight of the i^{th} item. This allows the analyst to easily turn the decision maker's preference into weights. This also allows the analyst to implement equal weighting, if that is desired as well. If there is a group of decision makers, the analyst can make use of their individual weights to develop an overall weighting scheme based on the group. There are several methods in multi-attribute utility theory which may be applied as well, however, many of them can be difficult to translate and implement with a group of people unfamiliar with the theory.

5.3.2 Retribution.

The current model only makes use of the actions that the US can impact. While planning is often well thought and meticulous down to the last detail, a common adage is that no plan survives first contact. So for every US action, it can be assumed there may be an adversary reaction, intended to counteract the plan. The inclusion of factors which are contrary to objectives of the US in the model would provide an additional level of fidelity and provide more insight to the application of the instruments of national power. This would require detailed threat analysis and would likely be different for every country, and perhaps at regional or lower levels.

This may be instituted through the inclusion of stochastic shocks, ω_t, to the system. The ω_t can be a random occurrence which impacts the derivative of an index during the time period. This would capture events which are unpredictable, both good and bad, and would be an after–the–fact adjustment to the derivative of the index.

The addition of an *insurgent action* term in the functional form might be a plausible action to accomplish this aspect. This could be similar to the DIME aspect and represent insurgent leadership, information capabilities, strength, and economic status. This data would not likely be found in a public database; however, there are force-to-population models which estimate insurgent strength, the leadership (legitimate and perhaps illegitimate) structure is often available for an AOR, and a simple ordinal ranking system could be used to provide a metric on other components.

5.3.3 Effects of the Operational Environment.

In FM 3-24, *Counterinsurgency*, the idea of describing the effects of the operational environment is a part of the Intelligence Preparation process that takes place after defining the operational environment. There are six socio–cultural factors listed to evaluate the people: (1) Society, (2) Social Structure, (3) Culture, (4) Language, (5) Power and Authority, and (6) Interests [19:3-3,3-14].

The implementation of some or all of these factors in this model would lead to a better understanding of how the operational environment is affected by the application of the US instruments of national power. Very few nation-states are exactly alike and understanding the complexities that make them unique is paramount to understanding the operational environment. This is similar to the work by Yamamota which looked at the assimilation of different ethnicities into a single nation considering the aspects of Language and Power, in Section 2.8.4. A better understanding of the operational environment would in–turn lead to a better application of the instruments of national power.

5.3.4 Portfolio of Models.

As currently developed, the only application of this methodology has been Iraq, and as of December 2011 the operations in Iraq have come to close. While the data for Iraq exists and is easy to locate, the true power of this model would be in the application for a country that is on the verge of instability. A portfolio of models would provide the availability to conduct a quick–turn analysis on a country of interest. Additionally, this would aid in the development of the indices, be useful in countries where data is limited, or potentially be used to conduct *back of the envelope* analysis to determine an estimate before in-depth analysis can be conducted. This could be accomplished through models based on a combination of countries, specific similar countries, or even regional models to represent the country of interest. Planners can then realize the impacts and plan applications of the instruments of national power to achieve desired effects. The common baseline referred to in Section 5.3.1 is required to achieve this.

5.3.5 Dynamic Programming.

The application of DIME is done in stages (sequentially in discrete time), both yearly and monthly. The outcome of the application decision is not always known, but an estimate can be determined. The goal is to then apply DIME (an additive cost function) in a manner which maximizes the positive change in the PMESII indices of a nation–state. Dynamic programming is sequential decision making under uncertainty and has two principal features, the model has an underlying discrete-time dynamic system and a cost function which is additive over time. The dynamic system is expressed by

$$x_{k+1} = f_k(x_k, u_k, \omega_k) \qquad k = 0, 1, \ldots, N-1 \tag{5.2}$$

where

k indexes discrete time

x_k is the state of the system and summarizes past information which is relevant for future optimization

u_k is the control or decision at time k

ω_k is a stochastic element or noise in the system

N is the planning horizon (may be infinite)

f_k is a function that describes the system and update mechanism

The cost function $g_k(x_k, u_k, \omega_k)$ is additive over time and due to the presence of the stochastic element the problem is formulated as the optimization of an *expected cost*

$$E\left\{g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, \omega_k)\right\} \tag{5.3}$$

In this research, the elements needed to implement this approach are available: x_k is the PMESI index at time k, u_k is the application of the diME instruments, and ω_k would be an instance of the stochastic shock or noise in the systems to replicate the true unknown element in the system. Solving this as a Dynamic Programming problem, the goal is to obtain the optimal policy for the time horizon by optimizing the controls over each time index [5:2-3].

5.4 Conclusion

This research provides a methodology to understand the application of the instruments of national power through the operational variables. This provides the ability to conduct tradeoff analysis and wargame. The most prominent aspect of this research is not the evaluation of the progress in Iraq specifically, but rather how the application of the instruments of national power may be applied more effectively and can be applied in a future time period, for a unknown country. The understanding of the dynamics from the system can then be implemented in a strategic plan; measured, updated, and reevaluated periodically if needed. Through an increased awareness and deeper understanding of operational environment dynamics, the goal is to conduct SSTRO which minimizes time, resources, and casualties, all while achieving national strategic objectives.

Appendix A. Data Tables

Table 19. Data to Calculate the Political Index

Political						
Time (month)	ISF Strength	Iraqi Army (Poll)	Iraqi Police (Poll)	Freedom of Press Index	Displaced Persons	Elected Gov't Constant
1	0	0	0	37.5	44444	0
2	0	0	0	37.5	44444	0
3	8000	0	0	37.5	44444	0
4	19000	0	0	37.5	44444	0
5	30000	0	0	37.5	44444	0
6	37170	0	0	37.5	44444	0
7	44200	0	0	37.5	44444	0
8	66800	0	0	37.5	44444	0
9	94800	39	45	37.5	44444	0
10	99600	39	45	37.5	44444	0
11	108800	39	45	58.5	33333	0
12	125000	56	69	58.5	33333	0
13	134991	56	69	58.5	33333	0
14	124253	56	69	58.5	33333	0
15	135712	56	69	58.5	33333	0
16	145317	56	69	58.5	33333	0
17	95088	56	69	58.5	33333	0
18	91468	56	69	58.5	33333	0
19	98708	56	69	58.5	33333	0
20	110998	56	69	58.5	33333	0
21	113506	56	69	58.5	33333	0
22	118009	56	69	58.5	33333	0
23	125373	56	69	67	33333	0
24	141761	56	69	67	33333	0.5
25	151618	56	69	67	33333	0.5
26	159493	56	69	67	33333	0.5
27	168227	56	69	67	33333	0.5
28	168674	56	69	67	33333	0.5
29	173900	56	69	67	33333	0.5
30	182900	56	69	67	33333	0.5
31	192100	56	69	67	33333	0.5
32	211000	56	69	67	33333	0.5
33	214000	66	67	67	33333	0.5

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Table 19 – Continued

Political						
Time (month)	ISF Strength	Iraqi Army (Poll)	Iraqi Police (Poll)	Freedom of Press Index	Displaced Persons	Elected Gov't Constant
34	223700	66	67	67	33333	0.5
35	227300	66	67	66.8	66667	0.5
36	232100	66	67	66.8	66667	1
37	250500	66	67	66.8	66667	1
38	253700	66	67	66.8	66667	1
39	265600	66	67	66.8	66667	1
40	264600	66	67	66.8	66667	1
41	269600	66	67	66.8	66667	1
42	298000	66	67	66.8	66667	1
43	307800	66	67	66.8	66667	1
44	312400	66	67	66.8	66667	1
45	323000	66	67	66.8	66667	1
46	323000	66	67	66.8	66667	1
47	323000	66	67	67.8	61667	1
48	323180	66	67	67.8	61667	1
49	329800	61	64	67.8	61667	1
50	333100	61	64	67.8	61667	1
51	348700	61	64	67.8	61667	1
52	353100	61	64	67.8	61667	1
53	359700	61	64	67.8	61667	1
54	359700	67	69	67.8	61667	1
55	359700	67	69	67.8	61667	1
56	359700	67	69	67.8	61667	1
57	429630	67	69	67.8	61667	1
58	439678	67	69	67.8	61667	1
59	441779	67	69	59.4	2500	1
60	425345	65	67	59.4	2500	1
61	444502	65	67	59.4	2500	1
62	444502	65	67	59.4	2500	1
63	478524	65	67	59.4	2500	1
64	478524	65	67	59.4	2500	1
65	513506	65	67	59.4	2500	1
66	513506	65	67	59.4	2500	1
67	531000	65	67	59.4	2500	1
68	542125	65	67	59.4	2500	1
69	558279	65	67	59.4	2500	1
70	589054	65	67	59.4	2500	1

Table 20. Data to Calculate the Military Index

Military					
Time (month)	ISF KIA	Coalition KIA	Attacks on Oil and Gas Infrastructure and Personnel	Iraqi Civilian KIA	Multiple Casualty Car/Suicide Events
1	65	92	0	0	0
2	65	80	0	0	0
3	65	41	0	866	0
4	65	36	6	1026	0
5	65	49	2	935	1
6	65	43	5	1292	4
7	65	33	2	860	3
8	65	47	4	825	13
9	70	110	9	677	6
10	80	48	9	817	14
11	90	52	2	831	9
12	100	23	2	938	17
13	110	52	6	1190	9
14	120	140	4	2014	9
15	130	84	7	1627	9
16	140	50	12	1021	19
17	150	58	17	932	11
18	160	75	21	1517	13
19	160	87	20	1434	16
20	160	68	11	1329	17
21	160	141	30	2638	11
22	50	76	17	1333	17
23	109	127	13	1448	28
24	103	60	13	1599	18
25	176	39	10	1333	13
26	199	52	5	1200	21
27	259	88	10	1777	36
28	296	83	10	1517	34
29	304	58	9	1658	26
30	282	85	9	3303	27
31	233	52	9	1964	46
32	215	99	9	1376	39
33	176	86	0	1640	41
34	193	68	3	1348	21
35	189	64	10	1778	30
36	158	58	9	2165	39
37	191	33	15	2378	37

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Table 20 – Continued

			Military		
Time (month)	ISF KIA	Coalition KIA	Attacks on Oil and Gas Infrastructure and Personnel	Iraqi Civilian KIA	Multiple Casualty Car/Suicide Events
38	201	82	6	2284	40
39	150	79	13	2669	56
40	132	63	8	3149	57
41	217	46	7	3590	53
42	233	66	2	3009	52
43	150	77	10	3345	57
44	224	110	4	3709	56
45	123	78	11	3462	65
46	123	115	5	2914	69
47	91	86	5	3500	44
48	150	85	8	2700	56
49	215	82	3	2400	51
50	300	117	19	2500	53
51	198	132	14	2600	42
52	197	108	14	1950	39
53	232	89	12	2350	43
54	76	88	0	2000	28
55	96	69	1	1100	30
56	114	41	1	950	34
57	89	40	1	750	22
58	72	24	1	750	23
59	69	40	1	600	24
60	110	30	2	700	21
61	161	40	1	750	28
62	113	52	0	950	21
63	110	21	0	550	14
64	77	31	0	490	19
65	98	13	0	500	19
66	85	23	0	450	22
67	98	25	0	400	22
68	48	14	0	350	14
69	27	17	0	270	24
70	74	16	0	350	23

Table 21. Data to Calculate the Economic Index

		Economic		
Time (month)	Gross Domestic Product in US Billions $	Consumer Price Index Change (%)	Foreign Direct Investment US Millions $	Revenue from Oil US Millions $
1	1.13	2.83	0.00	0.00
2	1.13	2.83	0.00	0.00
3	1.13	2.83	0.00	0.00
4	1.13	2.83	0.00	0.20
5	1.13	2.83	0.00	0.36
6	1.13	2.83	0.00	0.44
7	1.13	2.83	0.00	0.73
8	1.13	2.83	0.00	0.89
9	1.13	2.83	0.00	1.21
10	1.13	2.83	0.00	1.26
11	1.22	2.67	10.00	1.26
12	1.30	2.67	10.00	1.10
13	1.39	2.67	10.00	1.61
14	1.47	2.67	10.00	1.50
15	1.55	2.67	10.00	1.36
16	1.64	2.67	10.00	1.28
17	1.72	2.67	10.00	1.40
18	1.81	2.67	10.00	1.24
19	1.89	2.67	10.00	1.75
20	1.97	2.67	10.00	1.99
21	2.06	2.67	10.00	1.25
22	2.14	2.67	10.00	1.44
23	2.20	2.67	10.00	1.49
24	2.26	2.67	10.00	1.34
25	2.33	2.67	10.00	1.99
26	2.39	2.67	10.00	1.83
27	2.45	2.67	10.00	1.57
28	2.51	2.67	10.00	2.03
29	2.57	2.67	10.00	2.47
30	2.63	2.67	10.00	2.63
31	2.69	2.67	10.00	2.74
32	2.75	2.67	10.00	1.90
33	2.81	2.67	10.00	1.67
34	2.88	2.67	10.00	1.60
35	2.97	4.17	10.00	1.84
36	3.06	4.17	10.00	2.16
37	3.14	4.17	10.00	2.25

Continued on Next Page…

Table 21 – Continued

	Economic			
Time (month)	Gross Domestic Product in US Billions $	Consumer Price Index Change (%)	Foreign Direct Investment US Millions $	Revenue from Oil US Millions $
38	3.22	4.17	10.00	3.02
39	3.29	4.17	10.00	2.92
40	3.35	4.17	10.00	3.03
41	3.41	4.17	10.00	3.41
42	3.46	4.17	10.00	3.44
43	3.51	4.17	10.00	2.73
44	3.55	4.17	10.00	2.45
45	3.59	4.17	10.00	2.19
46	4.04	4.17	10.00	2.46
47	4.09	2.51	10.00	1.89
48	4.14	2.51	10.00	2.11
49	4.19	2.51	10.00	2.75
50	4.23	2.51	10.00	2.75
51	4.28	2.51	10.00	3.05
52	4.33	2.51	10.00	2.87
53	4.38	2.51	10.00	3.39
54	4.43	2.51	10.00	3.49
55	4.47	2.51	10.00	3.79
56	4.52	2.51	10.00	4.44
57	4.57	2.51	10.00	3.47
58	4.62	2.51	10.00	4.27
59	4.65	0.23	100.00	5.21
60	4.69	0.23	100.00	4.94
61	4.73	0.23	100.00	5.94
62	4.77	0.23	100.00	5.77
63	4.81	0.23	100.00	6.65
64	4.85	0.23	100.00	6.99
65	4.88	0.23	100.00	7.01
66	4.92	0.23	100.00	5.65
67	4.96	0.23	100.00	4.64
68	5.00	0.23	100.00	3.68
69	5.04	0.23	100.00	2.77
70	5.08	0.23	100.00	1.99

Table 22. Data to Calculate the Social Index

Social						
	Polls: Access to Services					
Time (month)	Clean Water	Medical Care	Fuel	Number of Doctors	Number of Trained Judges	Percent of School Age Children in School
1	0.35	0.50	0.51	23748	0	0.73
2	0.35	0.50	0.51	23496	0	0.72
3	0.35	0.50	0.51	23244	0	0.72
4	0.35	0.50	0.51	22993	0	0.72
5	0.35	0.50	0.51	22741	0	0.72
6	0.35	0.50	0.51	22489	0	0.72
7	0.35	0.50	0.51	22237	0	0.72
8	0.35	0.50	0.51	21985	0	0.72
9	0.35	0.50	0.51	21733	0	0.72
10	0.35	0.50	0.51	21481	0	0.72
11	0.35	0.50	0.51	21229	0	0.72
12	0.35	0.50	0.51	20978	0	0.72
13	0.35	0.50	0.52	20726	0	0.72
14	0.36	0.51	0.52	20474	0	0.72
15	0.36	0.51	0.53	20222	0	0.72
16	0.37	0.52	0.53	19970	175	0.72
17	0.37	0.52	0.54	19718	175	0.72
18	0.38	0.52	0.54	19466	175	0.72
19	0.38	0.53	0.55	19214	175	0.72
20	0.39	0.53	0.55	18963	175	0.72
21	0.39	0.53	0.56	18711	175	0.72
22	0.40	0.54	0.56	18459	175	0.72
23	0.40	0.54	0.57	18207	175	0.72
24	0.41	0.55	0.57	17955	175	0.72
25	0.41	0.55	0.58	17703	175	0.72
26	0.42	0.55	0.58	17451	175	0.72
27	0.42	0.56	0.59	17199	351	0.72
28	0.43	0.56	0.59	16948	351	0.72
29	0.43	0.56	0.60	16696	351	0.72
30	0.44	0.57	0.60	16444	351	0.72
31	0.44	0.57	0.61	16192	351	0.72
32	0.45	0.58	0.61	15940	351	0.72
33	0.45	0.58	0.62	15688	351	0.72
34	0.43	0.56	0.60	15436	351	0.72
35	0.41	0.55	0.58	15184	351	0.72
36	0.39	0.53	0.56	14933	351	0.72

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Table 22 – Continued

	Social					
	Polls: Access to Services					
Time (month)	Clean Water	Medical Care	Fuel	Number of Doctors	Number of Trained Judges	Percent of School Age Children in School
37	0.37	0.51	0.54	14681	351	0.71
38	0.35	0.49	0.52	14429	351	0.71
39	0.33	0.48	0.50	14177	700	0.71
40	0.31	0.46	0.48	13925	700	0.71
41	0.29	0.44	0.47	13673	700	0.71
42	0.26	0.42	0.45	13421	740	0.71
43	0.24	0.41	0.43	13169	740	0.71
44	0.22	0.39	0.41	12918	740	0.71
45	0.20	0.37	0.39	12666	800	0.71
46	0.18	0.35	0.37	12414	800	0.71
47	0.16	0.34	0.35	12396	870	0.71
48	0.14	0.32	0.33	12378	870	0.71
49	0.12	0.30	0.31	12360	870	0.71
50	0.11	0.29	0.31	12343	870	0.71
51	0.10	0.28	0.32	12325	870	0.71
52	0.10	0.27	0.32	12307	870	0.71
53	0.09	0.26	0.33	12289	870	0.71
54	0.08	0.25	0.33	12271	1100	0.71
55	0.09	0.26	0.34	12253	1100	0.71
56	0.09	0.27	0.35	12236	1100	0.71
57	0.10	0.28	0.36	12218	1200	0.71
58	0.11	0.29	0.36	12200	1200	0.71
59	0.11	0.30	0.37	12433	1200	0.71
60	0.12	0.31	0.38	12667	1200	0.71
61	0.14	0.32	0.38	12900	1200	0.71
62	0.16	0.32	0.38	13133	1200	0.71
63	0.19	0.33	0.39	13367	1200	0.71
64	0.21	0.33	0.39	13600	1180	0.71
65	0.23	0.34	0.39	13833	1180	0.71
66	0.25	0.35	0.39	14067	1180	0.71
67	0.27	0.35	0.39	14300	1180	0.71
68	0.29	0.36	0.39	14533	1180	0.70
69	0.32	0.36	0.40	14767	1180	0.70
70	0.34	0.37	0.40	15000	1225	0.70

Table 23. Data to Calculate the Infrastructure Index

Infrastructure			
Time (month)	Monthly Telephone Subscribers	Average Hours of Electricity	Monthly Internet Subscribers
1	833000	0.00	4500
2	733000	0.00	4557
3	683000	0.00	4614
4	633000	4.00	4671
5	6000000	4.00	4729
6	600000	5.13	4786
7	600000	6.25	4843
8	600000	7.38	4900
9	600000	8.50	11038
10	600000	9.63	17175
11	600000	10.75	23313
12	600000	11.88	29450
13	900000	13.00	35588
14	984225	16.00	41725
15	1095000	15.00	47863
16	1220000	11.00	54000
17	1200000	10.00	59000
18	1331574	10.00	73000
19	1463148	1.00	87000
20	1579457	13.00	95000
21	1753000	13.00	102978
22	2135000	13.00	110000
23	2152000	11.00	117147
24	2449139	9.00	124293
25	2569110	8.50	135685
26	2982115	11.80	147076
27	3172771	9.00	151839
28	3450000	8.40	156603
29	3801822	9.40	161366
30	4100000	12.60	166130
31	4590398	12.00	170893
32	4911320	13.50	175656
33	5232243	14.30	180420
34	5553165	13.30	185183
35	5874087	12.00	189946
36	6195009	9.80	194710
37	6515932	10.30	199473
38	6836854	13.10	204237

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Table 23 – Continued

Infrastructure			
Time (month)	Monthly Telephone Subscribers	Average Hours of Electricity	Monthly Internet Subscribers
39	7400000	10.90	209000
40	7575000	9.90	206078
41	7750000	11.94	203155
42	7925000	11.42	200233
43	8100000	10.87	197310
44	8200000	10.80	196347
45	8200000	12.30	195704
46	8500000	10.90	194420
47	9800000	9.20	212210
48	9800000	8.00	230000
49	9810000	9.30	240333
50	9820000	10.90	250667
51	9830000	11.60	261000
52	10117500	10.10	355417
53	10405000	10.60	449833
54	10692500	10.40	544250
55	10980000	10.30	638667
56	11267500	11.80	733083
57	11555000	12.90	827500
58	11842500	12.30	818227
59	12130000	11.70	808955
60	12417500	8.70	799682
61	12705000	8.70	790409
62	12992500	8.40	781137
63	13280000	8.00	771864
64	13450000	7.60	762591
65	13620000	10.90	753319
66	13790000	11.30	744046
67	13960000	10.80	734773
68	14130000	8.70	725501
69	14300000	13.40	716228
70	14900000	11.20	706955

Table 24. **Military and Economic Inputs (diME)**

diME Inputs		
Time (month)	Military #Troops	Economic US Billions $
1	0.00	0.4286
2	1.50	0.4286
3	1.50	0.4286
4	1.49	0.4286
5	1.39	0.4286
6	1.32	0.4286
7	1.31	0.4286
8	1.23	1.6250
9	1.22	1.6250
10	1.22	1.6250
11	1.15	1.6250
12	1.30	1.6250
13	1.37	1.6250
14	1.38	1.6250
15	1.38	1.6250
16	1.40	1.6250
17	1.40	1.6250
18	1.38	1.6250
19	1.38	1.6250
20	1.38	0.1667
21	1.48	0.1667
22	1.50	0.1667
23	1.55	0.1667
24	1.50	0.1667
25	1.42	0.1667
26	1.38	0.1667
27	1.35	0.1667
28	1.38	0.1667
29	1.38	0.1667
30	1.38	0.1667
31	1.52	0.1667
32	1.60	0.2667
33	1.60	0.2667
34	1.36	0.2667
35	1.33	0.2667
36	1.33	0.2667
37	1.32	0.2667
38	1.32	0.2667

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Table 24 – Continued

diME Inputs		
Time (month)	Military # Troops	Economic US Billions $
39	1.27	0.2667
40	1.30	0.2667
41	1.38	0.2667
42	1.44	0.2667
43	1.44	0.2667
44	1.40	0.2667
45	1.40	0.2667
46	1.32	0.2667
47	1.35	0.2667
48	1.42	0.2667
49	1.46	0.2667
50	1.50	0.2667
51	1.57	0.2667
52	1.60	0.2667
53	1.62	0.2667
54	1.68	0.2667
55	1.71	0.2667
56	1.62	0.2250
57	1.60	0.2250
58	1.57	0.2250
59	1.57	0.2250
60	1.55	0.2250
61	1.53	0.2250
62	1.50	0.2250
63	1.48	0.2250
64	1.48	0.2250
65	1.48	0.2250
66	1.48	0.2250
67	1.48	0.2250
68	1.48	0.1833
69	1.45	0.1833
70	1.42	0.1833

Appendix B. Regression and Coefficient Tables

Table 25. Table of Regression Coefficients

	Political
β_1	0.00000011844073546663800
β_2	-0.00002250086842663090000
β_3	0.00133143041495831000000
β_4	-0.01466377441557400000000
β_5	0.28469247737427800000000
	Military
β_1	-0.00000024547253720522900
β_2	0.00004055754475489060000
β_3	-0.00176755522389604000000
β_4	0.01024206711313980000000
β_5	0.82902520191780100000000
	Economic
β_1	-0.00000024547487951500600
β_2	0.00004199360863928890000
β_3	-0.00211451002742180000000
β_4	0.04034090075512610000000
β_5	0.00418144507801799000000
	Social
β_1	0.00000006843872139960920
β_2	-0.00000820453113023478000
β_3	0.00032188358380080800000
β_4	-0.00375731840927074000000
β_5	0.46888394672716100000000
	Infrastructure
β_1	-0.00000011837715959962800
β_2	0.00001742272861532740000
β_3	-0.00085108685547952100000
β_4	0.02008144509396460000000
β_5	0.01251049720667650000000

Table 26. Table of Derivative Coefficients

	Political
c_1	0.0000004737629418665520
c_2	-0.0000675026052798927000
c_3	0.0026628608299166200000
c_4	-0.0146637744155740000000

	Military
c_1	-0.0000009818901488209160
c_2	0.0001216726342646720000
c_3	-0.0035351104477920800000
c_4	0.0102420671131398000000

	Economic
c_1	-0.0000009818995180600240
c_2	0.0001259808259178670000
c_3	-0.0042290200548436000000
c_4	0.0403409007551261000000

	Social
c_1	0.0000002737548855984370
c_2	-0.0000246135933907043000
c_3	0.0006437671676016160000
c_4	-0.0037573184092707400000

	Infrastructure
c_1	-0.0000004735086383985120
c_2	0.0000522681858459822000
c_3	-0.0017021737109590400000
c_4	0.0200814450939646000000

Table 27. a **Coefficients (full precision)**

i	$j = 1$
1	-0.01083501615326630000
2	0.07431361157128300000
3	0.02874407545100780000
4	-0.00054775677185041400
5	0.00346700297334305000

i	$j = 3$
1	-0.02005569187569890000
2	0.01688088305261940000
3	0.01552892230140780000
4	0.00010964415239150800
5	0.00082537241058253200

i	$j = 3$
1	0.00002031568921243150
2	0.00021608926013287400
3	-0.00086319629349991100
4	0.00043050900543419400
5	-0.00101436755553736000

i	$j = 4$
1	0.00400525576986245000
2	-0.01018366931286610000
3	-0.01004271724607080000
4	0.02461020169169960000
5	-0.02647687739702560000

i	$j = 5$
1	0.00356203566192572000
2	-0.02414472846748650000
3	-0.01567071616639290000
4	0.00213555037894761000
5	-0.00548575949780422000

Table 28. b **Coefficients (full precision)**

i	$j = 1$
1	0.398015141750026000
2	0.909401079911930000
3	0.443913560641917000
4	0.178810101884673000
5	0.238762358092223000

i	$j = 2$
1	0.631945615708440000
2	0.502615618698794000
3	0.360670421790453000
4	0.078165558991190800
5	0.076900990198610300

i	$j = 3$
1	0.006152639745050480
2	0.523495025889631000
3	0.187323699465370000
4	0.070070513180899000
5	0.134208115831946000

i	$j = 4$
1	0.089412319856625600
2	0.144362348198396000
3	0.109325323093760000
4	0.434394881937596000
5	0.347975417823036000

i	$j = 5$
1	0.101094797180729000
2	0.648145314969934000
3	0.308491646319828000
4	0.296060693010735000
5	0.260304480244896000

Table 29. d **Coefficients (full precision)**

i	$k=1$	$k=2$
1	-0.0074938163170398700	-0.0008034283214846400
2	0.0230729470057021000	0.0007235929449071280
3	0.0184395682832470000	0.0007634774704140440
4	-0.0017063445940387500	0.0008384700491929670
5	0.0049510481717654300	-0.0005527356198253540

Appendix C. PMESI Index Values

Table 30. Normalized Index Values

	Normalized Index Values				
Time	Political	Military	Economic	Social	Infrastructure
1	0.2401	0.8800	0.1327	0.4696	0.0108
2	0.2403	0.8894	0.1327	0.4671	0.0100
3	0.2405	0.8683	0.1327	0.4645	0.0648
4	0.2407	0.8404	0.1398	0.4620	0.1376
5	0.2408	0.8456	0.1455	0.4595	0.0799
6	0.2409	0.8033	0.1484	0.4577	0.0955
7	0.2410	0.8530	0.1587	0.4559	0.1112
8	0.2410	0.7884	0.1644	0.4540	0.1276
9	0.2944	0.7593	0.1758	0.4522	0.1440
10	0.2970	0.7538	0.1776	0.4503	0.1604
11	0.2745	0.7926	0.2165	0.4485	0.1769
12	0.3176	0.7631	0.2147	0.4467	0.1973
13	0.3257	0.7416	0.2368	0.4460	0.2408
14	0.3167	0.6230	0.2368	0.4453	0.2292
15	0.3260	0.6719	0.2357	0.4446	0.1761
16	0.3337	0.6609	0.2367	0.4731	0.1625
17	0.2922	0.6727	0.2449	0.4724	0.1661
18	0.2891	0.5925	0.2431	0.4716	0.0446
19	0.2949	0.5772	0.2652	0.4709	0.2137
20	0.3048	0.6270	0.2777	0.4701	0.2170
21	0.3067	0.4496	0.2552	0.4694	0.2228
22	0.3102	0.6786	0.2659	0.4686	0.0433
23	0.3018	0.5501	0.2705	0.4693	0.1730
24	0.4816	0.6465	0.2680	0.4686	0.1690
25	0.4894	0.6609	0.2940	0.4678	0.2215
26	0.4955	0.6217	0.2911	0.4670	0.1856
27	0.5023	0.4239	0.2847	0.4955	0.1813
28	0.5024	0.4260	0.3039	0.4948	0.2001
29	0.5063	0.4738	0.3225	0.4940	0.2489
30	0.5132	0.3652	0.3310	0.4931	0.2472
31	0.5202	0.4155	0.3378	0.4923	0.2725
32	0.5349	0.4607	0.3107	0.4914	0.2881
33	0.5478	0.5073	0.3053	0.4906	0.2787
34	0.5555	0.6120	0.3056	0.4838	0.2651
35	0.5374	0.5229	0.2287	0.4770	0.2389
36	0.7077	0.4873	0.2443	0.4703	0.2502
37	0.7227	0.4576	0.2513	0.4635	0.2935

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Table 30 – Continued

Normalized Index Values					
Time	Political	Military	Economic	Social	Infrastructure
38	0.7250	0.4361	0.2822	0.4567	0.2703
39	0.7344	0.3495	0.2818	0.5081	0.2579
40	0.7332	0.3602	0.2887	0.5013	0.2878
41	0.7369	0.3081	0.3049	0.4945	0.2821
42	0.7599	0.3388	0.3084	0.4945	0.2760
43	0.7675	0.3169	0.2854	0.4878	0.2759
44	0.7709	0.2420	0.2775	0.4810	0.2963
45	0.7791	0.2864	0.2701	0.4843	0.2802
46	0.7786	0.2930	0.3005	0.4776	0.2745
47	0.7795	0.4249	0.3819	0.4843	0.2597
48	0.7792	0.3614	0.3920	0.4793	0.2788
49	0.7680	0.3769	0.4170	0.4666	0.3021
50	0.7700	0.2123	0.4193	0.4650	0.3129
51	0.7814	0.3407	0.4322	0.4634	0.3067
52	0.7843	0.4138	0.4280	0.4618	0.3283
53	0.7888	0.3673	0.4488	0.4603	0.3400
54	0.8122	0.6200	0.4546	0.4970	0.3531
55	0.8117	0.6609	0.4675	0.4995	0.3884
56	0.8111	0.6595	0.4929	0.5020	0.4180
57	0.8499	0.7485	0.4605	0.5212	0.4115
58	0.8500	0.7687	0.4913	0.5237	0.4050
59	0.8995	0.7624	0.8886	0.5281	0.3652
60	0.8995	0.7452	0.8807	0.5324	0.3670
61	0.8995	0.6669	0.9182	0.5384	0.3646
62	0.8995	0.7181	0.9139	0.5444	0.3609
63	0.8995	0.8025	0.9470	0.5505	0.3557
64	0.8995	0.7981	0.9609	0.5532	0.4019
65	0.8995	0.7964	0.9634	0.5592	0.4079
66	0.8995	0.7865	0.9167	0.5652	0.4013
67	0.8995	0.7784	0.8825	0.5712	0.3726
68	0.8995	0.8653	0.8500	0.5772	0.4382
69	0.8995	0.8347	0.8193	0.5833	0.4131
70	0.8995	0.8012	0.7933	0.5968	0.3394

Appendix D. Alternate diME Figures

Table 31. Alternate diME Figures

Alternate diME Figures								
Time	No Surge		Withdrawal		Early Surge		Even-Spending	
1	0.00	0.4286	0.00	0.4286	0.00	0.4286	0.00	0.4019
2	1.50	0.4286	1.50	0.4286	1.50	0.4286	1.50	0.4019
3	1.50	0.4286	1.50	0.4286	1.50	0.4286	1.50	0.4019
4	1.49	0.4286	1.49	0.4286	1.49	0.4286	1.49	0.4019
5	1.39	0.4286	1.39	0.4286	1.39	0.4286	1.39	0.4019
6	1.32	0.4286	1.32	0.4286	1.32	0.4286	1.32	0.4019
7	1.31	0.4286	1.31	0.4286	1.31	0.4286	1.31	0.4019
8	1.23	1.6250	1.23	1.6250	1.23	1.6250	1.23	0.4019
9	1.22	1.6250	1.22	1.6250	1.22	1.6250	1.22	0.4019
10	1.22	1.6250	1.22	1.6250	1.22	1.6250	1.22	0.4019
11	1.15	1.6250	1.15	1.6250	1.15	1.6250	1.15	0.4019
12	1.30	1.6250	1.30	1.6250	1.30	1.6250	1.30	0.4019
13	1.37	1.6250	1.37	1.6250	1.37	1.6250	1.37	0.4019
14	1.38	1.6250	1.38	1.6250	1.38	1.6250	1.38	0.4019
15	1.38	1.6250	1.38	1.6250	1.38	1.6250	1.38	0.4019
16	1.40	1.6250	1.40	1.6250	1.40	1.6250	1.40	0.4019
17	1.40	1.6250	1.40	1.6250	1.40	1.6250	1.40	0.4019
18	1.38	1.6250	1.38	1.6250	1.38	1.6250	1.38	0.4019
19	1.38	1.6250	1.38	1.6250	1.38	1.6250	1.38	0.4019
20	1.38	0.1667	1.38	0.1667	1.38	0.1667	1.38	0.4019
21	1.48	0.1667	1.48	0.1667	1.48	0.1667	1.48	0.4019
22	1.50	0.1667	1.50	0.1667	1.48	0.1667	1.50	0.4019
23	1.55	0.1667	1.55	0.1667	1.48	0.1667	1.55	0.4019
24	1.50	0.1667	1.50	0.1667	1.48	0.1667	1.50	0.4019
25	1.42	0.1667	1.42	0.1667	1.50	0.1667	1.42	0.4019
26	1.38	0.1667	1.38	0.1667	1.57	0.1667	1.38	0.4019
27	1.35	0.1667	1.35	0.1667	1.60	0.1667	1.35	0.4019
28	1.38	0.1667	1.38	0.1667	1.62	0.1667	1.38	0.4019
29	1.38	0.1667	1.38	0.1667	1.68	0.1667	1.38	0.4019
30	1.38	0.1667	1.38	0.1667	1.71	0.1667	1.38	0.4019
31	1.52	0.1667	1.52	0.1667	1.62	0.1667	1.52	0.4019
32	1.60	0.2667	1.60	0.2667	1.60	0.2667	1.60	0.4019
33	1.60	0.2667	1.60	0.2667	1.57	0.2667	1.60	0.4019
34	1.36	0.2667	1.36	0.2667	1.57	0.2667	1.36	0.4019
35	1.33	0.2667	1.33	0.2667	1.55	0.2667	1.33	0.4019
36	1.33	0.2667	1.33	0.2667	1.53	0.2667	1.33	0.4019
37	1.32	0.2667	1.32	0.2667	1.50	0.2667	1.32	0.4019

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Table 31 – Continued

Alternate diME Figures								
Time	No Surge		Withdrawal		Early Surge		Even-Spending	
38	1.32	0.2667	1.32	0.2667	1.48	0.2667	1.32	0.4019
39	1.32	0.2667	1.27	0.2667	1.48	0.2667	1.27	0.4019
40	1.32	0.2667	1.30	0.2667	1.48	0.2667	1.30	0.4019
41	1.32	0.2667	1.38	0.2667	1.48	0.2667	1.38	0.4019
42	1.32	0.2667	1.44	0.2667	1.48	0.2667	1.44	0.4019
43	1.32	0.2667	1.44	0.2667	1.48	0.2667	1.44	0.4019
44	1.32	0.2667	1.40	0.2667	1.45	0.2667	1.40	0.4019
45	1.32	0.2667	1.40	0.2667	1.42	0.2667	1.40	0.4019
46	1.32	0.2667	1.32	0.2667	1.40	0.2667	1.32	0.4019
47	1.32	0.2667	1.35	0.2667	1.50	0.2667	1.35	0.4019
48	1.32	0.2667	1.42	0.2667	1.55	0.2667	1.42	0.4019
49	1.32	0.2667	1.46	0.2667	1.50	0.2667	1.46	0.4019
50	1.32	0.2667	1.35	0.2667	1.42	0.2667	1.50	0.4019
51	1.32	0.2667	1.24	0.2667	1.38	0.2667	1.57	0.4019
52	1.32	0.2667	1.12	0.2667	1.35	0.2667	1.60	0.4019
53	1.32	0.2667	1.01	0.2667	1.38	0.2667	1.62	0.4019
54	1.32	0.2667	0.90	0.2667	1.38	0.2667	1.68	0.4019
55	1.32	0.2667	0.79	0.2667	1.38	0.2667	1.71	0.4019
56	1.32	0.2250	0.67	0.2250	1.52	0.2250	1.62	0.4019
57	1.32	0.2250	0.56	0.2250	1.60	0.2250	1.60	0.4019
58	1.32	0.2250	0.45	0.2250	1.60	0.2250	1.57	0.4019
59	1.32	0.2250	0.34	0.2250	1.36	0.2250	1.57	0.4019
60	1.32	0.2250	0.22	0.2250	1.33	0.2250	1.55	0.4019
61	1.32	0.2250	0.11	0.2250	1.33	0.2250	1.53	0.4019
62	1.32	0.2250	0.00	0.2250	1.32	0.2250	1.50	0.4019
63	1.32	0.2250	0.00	0.2250	1.32	0.2250	1.48	0.4019
64	1.32	0.2250	0.00	0.2250	1.27	0.2250	1.48	0.4019
65	1.32	0.2250	0.00	0.2250	1.30	0.2250	1.48	0.4019
66	1.32	0.2250	0.00	0.2250	1.38	0.2250	1.48	0.4019
67	1.32	0.2250	0.00	0.2250	1.44	0.2250	1.48	0.4019
68	1.32	0.1833	0.00	0.1833	1.44	0.1833	1.48	0.4019
69	1.32	0.1833	0.00	0.1833	1.40	0.1833	1.45	0.4019
70	1.32	0.1833	0.00	0.1833	1.40	0.1833	1.42	0.4019

Appendix E. Quad Chart

The Quad Chart for this research is found below.

UNDERSTANDING THE INSTRUMENTS OF NATIONAL POWER THROUGH A SYSTEM OF DIFFERENTIAL EQUATIONS IN A COUNTERINSURGENCY

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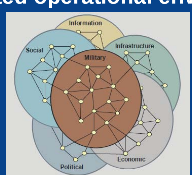
Readers: Dr. James Chrissis and Dr. John Colombi
Department of Operational Sciences (ENS)
Air Force Institute of Technology

Introduction

- In 2011 the Vice Chairman of Joint Chiefs of Staff directed the Center for Army Analysis (CAA) to pursue the development of Irregular Warfare models to help fill several gaps
- CAA expressed a need for a model to understand the application of the instruments of national power

Definitions:

- DIME: Diplomacy, Informational, Military, and Economic – the instruments of national power.
- PMESII: Political, Military, Economic, Social, Information, and Infrastructure – the interconnected operational environment.



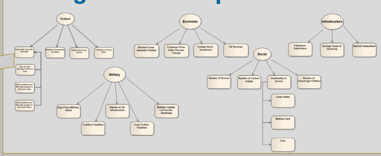
- SSTRO: Stabilization, Security, Transition, and Reconstruction Operations.

Research Goals

- Develop a methodology that provides metrics to measure the state of a country undergoing SSTRO, captures the interactions between the state variables, captures the effects of external influences, and allows for exploratory analysis

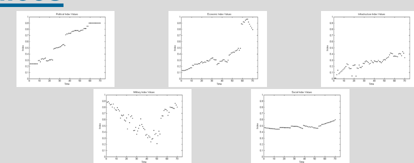
Data Collection

Normalize the Data
Weight the Components



Calculate Indices

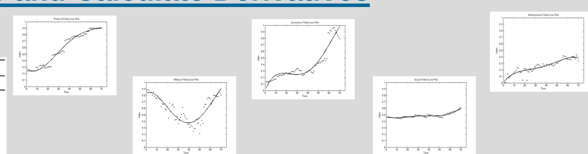
Calculate the Index



Fit Curves and Calculate Derivatives

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \ldots + \varepsilon$$

	Expo	Log	Linear	Polynomial 2nd	Polynomial 3rd	Polynomial 4th	Polynomial 5th	Polynomial 6th
Political	0.9118	0.7572	0.9450	0.9495	0.9719	0.9749	0.9765	0.9806
Military	0.0351	0.1713	0.0309	0.7378	0.7754	0.7971	0.8420	0.8532
Economic	0.8579	0.4514	0.7168	0.8484	0.8838	0.8968	0.9423	0.9571
Social	0.6346	0.3619	0.6265	0.7650	0.8364	0.8816	0.9079	0.9194
Infrastructure	0.5477	0.7694	0.8472	0.8498	0.8512	0.8677	0.8693	0.8802



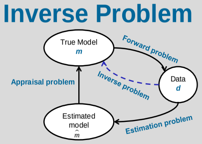
Determine Coefficients to the System of Differential Equations

Least-square Minimization

$$\text{Minimize} \quad f(\mathbf{x}) = \sum_{i=1}^{5} \sum_{t=1}^{70} (m_i^{(t)} - \hat{m}_i^{(t)})^2$$

$$\begin{aligned} \text{subject to} \quad & b_{ij} \le 1 && \text{for } i,j = 1,\ldots,5 \\ & b_{ij} > 0 && \text{for } i,j = 1,\ldots,5 \\ & a_{ij}, b_{ij}, d_{ik} \in \mathbb{R} && \text{for } i,j = 1,\ldots,5 \\ & && \text{for } k = 1,2 \\ & t \in \mathbb{Z}^+ \end{aligned}$$

Inverse Problem



Calculate *a, b, and d* coef.

$$\dot{P} = a_{1,1}\left(\frac{P}{b_{1,1}} - 1\right) + a_{1,2}\left(\frac{M}{b_{1,2}} - 1\right) + a_{1,3}\left(\frac{E}{b_{1,3}} - 1\right) + a_{1,4}\left(\frac{S}{b_{1,4}} - 1\right) + a_{1,5}\left(\frac{I}{b_{1,5}} - 1\right) + d_{11} Mil + d_{21} Eco$$

$$\dot{M} = a_{2,1}\left(\frac{P}{b_{2,1}} - 1\right) + a_{2,2}\left(\frac{M}{b_{2,2}} - 1\right) + a_{2,3}\left(\frac{E}{b_{2,3}} - 1\right) + a_{2,4}\left(\frac{S}{b_{2,4}} - 1\right) + a_{2,5}\left(\frac{I}{b_{2,5}} - 1\right) + d_{12} Mil + d_{22} Eco$$

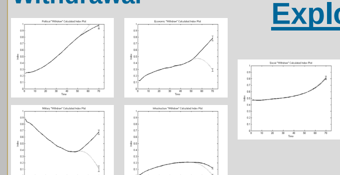
$$\dot{E} = a_{3,1}\left(\frac{P}{b_{3,1}} - 1\right) + a_{3,2}\left(\frac{M}{b_{3,2}} - 1\right) + a_{3,3}\left(\frac{E}{b_{3,3}} - 1\right) + a_{3,4}\left(\frac{S}{b_{3,4}} - 1\right) + a_{3,5}\left(\frac{I}{b3} - 1\right) + d_{13} Mil + d_{23} Eco$$

$$\dot{S} = a_{4,1}\left(\frac{P}{b_{4,1}} - 1\right) + a_{4,2}\left(\frac{M}{b_{4,2}} - 1\right) + a_{4,3}\left(\frac{E}{b_{4,3}} - 1\right) + a_{4,4}\left(\frac{S}{b_{4,4}} - 1\right) + a_{4,5}\left(\frac{I}{b_{4,5}} - 1\right) + d_{14} Mil + d_{24} Eco$$

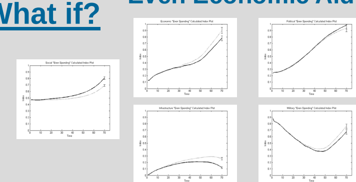
$$\dot{I} = a_{5,1}\left(\frac{P}{b_{5,1}} - 1\right) + a_{5,2}\left(\frac{M}{b_{5,2}} - 1\right) + a_{5,3}\left(\frac{E}{b_{5,3}} - 1\right) + a_{I,4}\left(\frac{S}{b_{5,4}} - 1\right) + a_{5,5}\left(\frac{I}{b_{5,5}} - 1\right) + d_{15} Mil + d_{25} Eco$$

Exploratory Analysis – What if?

Withdrawal



Even Economic Aid



Also:
- No Surge
- Earlier Surge

Conclusions

- Repeatable methodology that makes use of available open source data and common software.
- Captures the interactions of PMESI and the influences of diME. Measures the changes in PMESI through the application of diME.
- Demonstrates the capability to conduct what-if analysis, possible near-term forecasting.
- Developing models of historical SSTRO examples may inform decision makers on strategies for future operations.

Impact

- Fills a gap, first methodology to accomplish this.
- Ability for the analyst to "turn the dials" of the DIME and see impact to the PMESII

Future Research

- Expand data collection to find the "best" data available. Refine index normalization and weighting.
- Future operations – a portfolio of models.
- Apply as a dynamic programming problem and investigate the optimal policy/controls.

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14. ABSTRACT Models that account for the progression of nation-building and the impacts of the instruments of national power – Diplomacy, Informational, Military, and Economic effects – are rare. This research proposes the development of such a model. Through the derivation of state indices for the operational variables: Political, Military, Economic, Social, Infrastructure, and Information, a functional form of a system of differential equations is developed to account for the interactions between the state indices and instruments of national power. This methodology is a mean–field inverse problem which solves for the coefficients of the differential equations in a data driven manner. Publicly–available data is used to develop the indices and describe the instruments of national power. Applying mean–field theory allows the differential equations to be solved through a nonlinear program which derives minimum error producing coefficients. An application of the model is derived for Operation Iraqi Freedom to demonstrate the utility as well as effects of various alternate strategies, using the dynamics captured in the model. This modeling approach offers a potential significant capability when analyzing and planning for future Stabilization, Security, Transition, and Reconstruction Operations.

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