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Using Multiattribute Utility Copulas in Support of

UAV Search and Destroy Operations

THESIS

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Using Multiattribute Utility Copulas in Support

of UAV Search and Destroy Operations

THESIS

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Degree of Master of Science in Operations Research

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Captain, USAF

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Using Multiattribute Utility Copulas in Support
of UAV Search and Destroy Operations

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Abstract

The multiattribute utility copula is an emerging form of utility function used by decision analysts to study decisions with dependent attributes. Failure to properly address attribute dependence may cause errors in selecting the optimal policy. This research examines two scenarios of interest to the modern warfighter. The first scenario employs a utility copula to determine the type, quantity, and altitude of UAVs to be sent to strike a stationary target. The second scenario employs a utility copula to examine the impact of attribute dependence on the optimal routing of UAVs in a contested operational environment when performing a search and destroy mission against a Markovian target. Routing decisions involve a tradeoff between risk of UAV exposure to the enemy and the ability to strike the target. This research informs decision makers and analysts with respect to the tactics, techniques, and procedures employed in UAV search and destroy missions. An ever increasing UAV operations tempo suggests such research becoming increasingly relevant to the warfighter.

Acknowledgements

First and foremost, I would like to thank my wonderful wife. Without her love and support during this time, my completion of this thesis would have been impossible. I would also like to think my amazing advisor, Dr. Robbins. He was willing to guide me through the research process, sometimes painfully.

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Using Multiattribute Utility Copulas in Support of UAV Search and Destroy Operations

1. Introduction

1.1 Background

The UAV Roadmap 2005-2030 states that information gathering has historically been a dangerous mission and as a result UAVs have become one of the primary mediums for information gathering activities in modern warfare. The UAV has proven itself to be an effective tool for not only information gathering but air to ground engagements as well. Indeed, the UAVs has proven to be better suited for "dull, dirty, or dangerous" missions. As a result, UAVs have become an integral part of planning activities in US operations, and substantial funds have been provided for the continued development and integration of UAVs into National Defense [1, 2].

Employing UAVs effectively on the battlefield requires proper planning and decision making [2]. Although continuous coverage over all areas of interest is ideal, limitations on the number of UAVs available to commanders and vulnerabilities to enemy anti-air activity make this infeasible. A fully operational Predator system consists of four aircraft, a ground control station, a satellite link, and crew [41]. Typically, 12 Predator systems are spread among three squadrons [1]. With those limited assets, a commander must decide where to send the aircraft with what payload. The analysis presented in this thesis assumes the commander seeks to maximize the number of enemies neutralized while simultaneously minimizing number of friendly UAVs destroyed. Defense experts state that UAV assignment is critical to the success of US intelligence operations [1].

This thesis presents an analysis of the decision situation faced by a commander assigning UAVs to information gathering activities and air to ground missions. The motivation is to examine the tradeoffs between achieving enemy disruption and exposing friendly aircraft to risk in an uncertain environment. Multiattribute Utility Theory (MAUT) provides a clear, transparent framework for investigating the issues. The research objective is to develop a decision model for commanders to use in the assignment of UAVs to targets that will improve the ability of the armed forces to counter asymmetric warfare operations, enhance global efforts in fighting terrorism, and an perform other US allied objectives.

Studies concerning the management of UAVs in conflict have been undertaken. The problem of UAV assignment and control is modeled as an integer linear program by Shima and Rasmussen [36], a stochastic continuous coverage problem by Ha [16], and a simulation of large numbers of micro UAVs by Corner [10]. An object oriented simulation to examine the use of UAVs in surveillance and active suppression of enemy air defenses was conducted by Walston [43]. The use of UAVs in special operations is examined by Howard [19] and aerial refueling techniques for extended UAV operations by Stephenson [38]. Longino [25] gives a thorough history of the development and use of UAVs from World War II onward. Kennedy [23] presents a plan for the future development of UAVs, specifically focusing on the development of High Altitude Endurance UAVs.

The distinguishing feature of this research is the ability of a commander to express risk aversion or risk seeking attitudes in the process of assigning UAVs to targeting operations. It approaches the assignment of UAVs to missions using decision analysis, the result of which is the development of a utility function linking enemy capability and the number of UAVs to assign to a mission.

1.2 Problem Statement

A commander is often faced with decisions with competing objectives. The commander has a desire to maintain the overall readiness of the fleet of aircraft by not unnecessarily losing UAVs, while simultaneously neutralizing enemy threats. This decision forces a commander to examine the risk attitudes associated with UAV loss and enemy disruption as well as tradeoffs between these two attributes. This research explores these tradeoffs in a multiattribute decision model that provides the framework for optimal decisions to be made under uncertainty.

1.3 Overview

Chapter 2 presents the framework for creation of a multiattribute utility decision model and proceeds to solve a decision using the model. The first model involves a stationary target and a set of 2 UAVs available to assign for sensor or strike duties. Chapter 3 provides a more complex model that introduces time dependent searches for a moving enemy using a fleet of 2 UAVs. This model allows for Bayesian updating of the decision space.

2. An Application of Copulas to UAV Mission Planning

2.1 Introduction

Most decision analysis problems are modeled with a multiattribute utility function combined with a joint probability distribution for the prospects of a decision situation [4]. When facing a situation where the variables are independent, this joint distribution is greatly simplified, becoming the product of their marginal distributions.

Similar to joint probability functions, the presence of utility independence between attributes in a decision situation greatly simplifies the construction of a multiattribute utility function [4]. When constructing a utility function over the attributes x and y where there is mutual utility independence, the multiattribute utility function is of the multilinear form [22].

The construction of utility functions with dependence between the attributes can be accomplished through several different approaches [4]. One such approach is to construct a value function that directly encodes the value of the attributes, then construct a single attribute utility function over the value function. Since the utility function is reduced to a single attribute case, this approach requires no assumptions of utility, preferential, or conditional independence [26]. Another approach to modeling dependence involves assumptions of preferential or conditional utility dependence. Farquhar [12] uses fractional hypercubes to model a higher-order utility function as a product of single attribute conditional utility functions. Bell [7] uses conditional utility independence to show that a utility function can be represented in two-attributes in an additive or multiplicative form.

Most of the recent work in attribute dependence focuses on direct assessment of utility without assumptions. One such example of this approach is seen in Kirkwood [24], who uses parametric dependence (i.e., a dependence condition requiring the use of an extra variable) to directly assess the utility function whenever the conditional

parametric dependence conditions hold. Harvey [17] uses a concept of risk linearity to derive a logarithmic form for a utility function from a linear relationship between values. Bell [8] develops a method for constructing conditional utility functions requiring no assumptions. Keeney [21] uses preferential independence to find a set of fundamental attributes.

The fundamental approach of this research is one of direct assessment of utility dependence. Since utility functions can be built based on the properties of probability distributions, properties of probability distributions are exploited to build dependence structures in utility space similar to those that already exist in probability space. This approach allows for the construction of a two attribute utility space from the marginal utility of each attribute with an assumption of non-existence of utility independence.

Abbas [3] develops the multiattribute utility copula, which links utility theory and the probabilistic copula structure. Abbas incorporates an Archimedean copula structure and more specifically, Frank's copula [13]. This structure allows for a coupling of two correlated distributions into a bivariate distribution using a parameter δ to relate the dependence between the distributions. For more information on copula structures, see Nelson [28].

In order to utilize the copula structure, a measurement of statistical dependence between distributions is required. One such measure is Spearman's rank correlation coefficient, Spearman's rho (ρ); it expresses how well the relationship between two variables can be described using a monotonic function [28]. The assessment of Spearman's rho is possible through direct interaction with the decision makers, as proposed by Reilly [32]. Once Spearman's rho has been assessed, Genest [15] proposes an approximation to convert the value of ρ to the required parameter δ used in Frank's copula.

The purpose of this research is to apply new techniques in decision analysis to an important UAV assignment problem. This construction informs decision makers

with respect to tradoffs and risk by exercising multiattribute utility copulas. This is the first research to propose a direct assessment of utility dependence for translation to δ, the parameter controlling utility dependence in an Archimedian multiattribute utility copula.

The rest of this chapter is organized as follows. Section 2.2 describes the decision space and the methodology for constructing the multiattribute utility copula. Section 2.3 contains the results of a notional proof of concept scenario analyzed using this approach. Section 2.4 presents a sensitivity analysis. Section 2.5 discusses conclusions and future work. Throughout this assessment, the decision maker is the commander of a fleet of UAVs and the terms are used interchangeably.

2.2   Methodology

The following section contains the definitions and functions used to create a multiattribute utility copula. Assessment procedures for the parameters used to model a decision maker's risk preferences are given. Analysis of a notional scenario demonstrates the use of the utility function. Section 2.2.1 outlines the definitions and functions used in this assessment. Section 2.2.2 details the parameter assessment methods for the decision maker's risk preferences. Section 2.2.3 details the notional scenario used to illustrate the multiattribute utility copula.

2.2.1   Definitions and Functions.   In this analysis, it is assumed the decision maker follows the axioms of normative utility theory developed by von Neumann and Morgentstern [42] and has a multiattribute utility function defined by $U(x, y)$ over attributes X and Y. Lower case x and y denotes an instance of attributes X and Y. Let x^0 (y^0) and x^* (y^*) denote the minimum and maximum values of the X (Y) attribute. The domain of the decision occurs on $[x^0, x^*]$ x $[y^0, y^*]$. The attribute X is defined as the number of enemy (or friendly) forces neutralized. The attribute Y is defined as the number of operational UAVs remaining after the engagement.

The methodology presented in this Chapter employs a Class 1 Archimedean utility copula, as introduced by Abbas [3]. Further description is warranted. A Class 1 utility copula requires conditional utility assessments at the maximum values of the component attributes and must satisfy

$$C_1(1, ..., 1, v_i, 1, ..., 1) = a_i v_i + b_i, \qquad i = 1, ..., n. \tag{2.1}$$

A Class 0 utility copula satisfies

$$C_0(0, ..., 0, v_i, 0, ..., 0) = a_i v_i + b_i, \qquad 1 \leq i \leq n, \tag{2.2}$$

and requires the utility assessments at the minimum values of its complement. A Class 1 copula is used in this analysis because there is a guarantee of strictly increasing marginal utility functions at the maximum margin for each attribute. The Archimedian functional form is a Class 1 utility copula that satisfies

$$C_1(1, ..., 1, v_i, 1, ..., 1) = a_i v_i + b_i, \qquad i = 1, ..., n. \tag{2.3}$$

where $a_i = a(1 - l_i)$ and $b_i = 1 - a_i = al_i + b$. This implies the same mathematical properties of a strictly increasing cumulative probability distribution or a strictly increasing normalized utility function on the domain [0,1]. This observation allows for all well-known functional forms of cumulative distributions (e.g. the Beta distribution) to be used [3].

In the Class 1 Archimedian utility copula,

$$C(v_x, v_y) = -a \frac{1}{\delta} ln \left(1 - \frac{(1 - e^{-\delta(l_x + (1 - l_x)v_x)})(1 - e^{-\delta(l_y + (1 - l_y)v_y)})}{1 - e^{-\delta}} \right) + b, \tag{2.4}$$

The parameters v_x and v_y refer to the normalized utility functions for x and y. The parameters l_x, l_y, a, and b are defined by the following equations.

$$a(1 - l_x) = 1 - U(x^0, y^*), \qquad (2.5)$$

$$a(1 - l_y) = 1 - U(x^*, y^0), \qquad (2.6)$$

$$a = \frac{1}{1 + (1/\delta)ln\left(1 - (1 - e^{-\delta l_x})(1 - e^{-\delta l_y})/(1 - e^{-\delta})\right)}, \qquad (2.7)$$

$$b = a\frac{1}{\delta}ln\left(1 - \frac{(1 - e^{-\delta l_x})(1 - e^{-\delta l_y})}{1 - e^{-\delta}}\right). \qquad (2.8)$$

This approach requires the assumption that the decision makers utility functions v_x and v_y are exponential with risk aversion coefficients γ_x and γ_y respectively. The parameter δ represents the trade-off functions between attributes. An estimation of δ using Spearman's rank correlation parameter ρ is given by Genest [15].

$$\rho(\alpha) \simeq \frac{\alpha^{\frac{1}{2}}ln(\alpha) - \alpha + 1}{(\alpha^{\frac{1}{2}} - 1)^2}, \qquad (2.9)$$

where $\delta = -ln\alpha$.

In order to use this method of assessment, the parameters $U(x^0, y^*)$, $U(x^*, y^0)$, ρ, γ_x, and γ_y must be assessed from the decision maker.

2.2.2 *Parameter Assessment.*   The parameters $U(x^0, y^*)$ and $U(x^*, y^0)$ can be directly assessed from the decision maker. In the baseline model, x^0 is defined destroying friendly forces (thinking they are enemy) x^* is complete neutralization of enemy forces, y^0 is the destruction of two (all) UAVs, and y^* is survival of two (all) UAVs. An example of the lottery for the assessment of $U(x^0, y^*)$ would be $(x^0, y^*) \sim< (x^*, y^*); \pi_y; (x^0, y^0) >$, where $U(x^0, y^*) = \pi_y$. A similar lottery can be developed for $U(x^*, y^0)$.

The parameters γ_x and γ_y are the risk aversion coefficients of the utility functions v_x and v_y, respectively. Steps for the assessment of risk aversion coefficients

can be found in Keeney and Raiffa [22]. These parameters can be obtained from the decision maker through a series of lottery questions.

Reilly [32] suggests that, given proper training in variables of interest and training in the assessment method, rank correlation values can be meaningfully assessed by asking the decision maker for an estimate of the strength of the relationship between attributes. This would be accomplished by asking the question, "How would you characterize the strength and nature of the relationship between $U_x(x)$ and $U_y(y)$, using a scale from -7 to 7, where -7 represents a very strong negative relationship, 0 represents no relationship and 7 represents a very strong relationship?" In order to directly assess using this method, training is required on the definition of utility dependence as it relates to probabilistic dependence. This direct assessment method is one of the easiest to understand and provides a consistently low absolute error and low standard deviation in its results [32]. This step is integral to applying the multiattribute utility copula and combined with the Genest approximation provides the backbone for the dependence structure of the multiattribute utility copula..

2.2.3 Notional Scenario. In this section, a notional scenario is constructed to illustrate how the concepts of MUAT can be applied to solve a decision problem faced by a commanding officer of a squadron of UAVs. The rest of this section is organized as follows. Section 2.2.3.1 presents the decision context faced by a commander in the area of operations. Section 2.2.3.2 presents the information that the commander has at the time the decision must be made. Section 2.2.3.3 presents the alternatives faced by the commander and assumptions the commander's preferences for value and risk.

2.2.3.1 Decision Context. Consider a situation where a commander has two UAVs able to be dedicated to a mission in order to neutralize a stationary enemy threat. At the beginning of the mission, the commander has an intial belief

in the enemy presence. The commander must decide how many, if any, UAVs to
dedicate to the accomplishment of this mission.
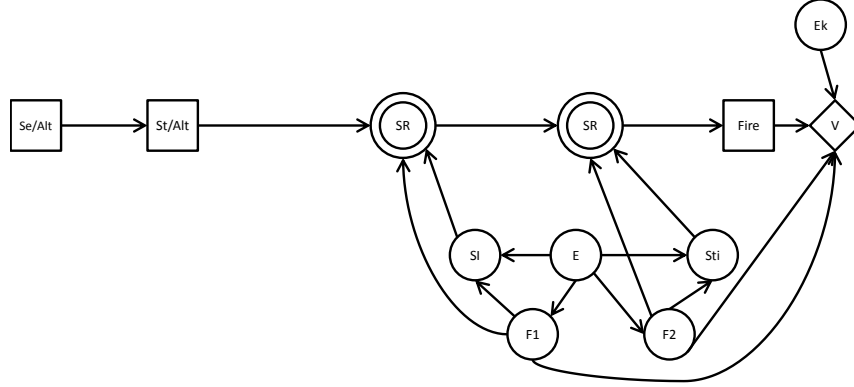
Figure 2.1   Assessed Decision Diagram

2.2.3.2   Information.   At this point in the decision the commander
has an initial belief, $P(E)$ that the enemy is in the area and that up to two UAVs
are available for the mission. Each UAV may be sent with a strike or sensor package
to determine if there is an enemy presence and order a strike if desired; however, if
a UAV package is sent, at least one must be loaded with a strike package in order
to accomplish the mission. It is assumed that the UAVs have sensor capabilities
that are dependent upon the altitude at which they are operating and the package
with which the UAV is loaded; UAVs with sensor packages are more capable than the
UAVs with strike packages at detecting the enemy. If a strike is ordered, the proposed
enemy threat is eradicated with a certain probability, depending on the altitude
and the presence of a sensor UAV. Each UAV acts as an independent sensor, with
independent results. Since the determination of enemy is dependent upon human
interaction in addition to sensor capability, this is not an unreasonable assumption.

In addition to the sensor report obtained from each UAV, the loss of a UAV can be viewed as an update of the probability that the target is actually an enemy. There is a probability that a UAV fails without any enemy activity and a probability that the UAV fails due to enemy activity.

The decision diagram corresponding to this decision situation is given in Figure 2.1. The notional probabilities associated with the the decision situation are given in Table 2.1.

Description	Probability Used
Enemy	0.7
Survival Given High Altitude, Enemy	0.9
Survival Given Low Altitude, Enemy	0.8
Survival Given High Altitude, No Enemy	0.98
Survival Given Low Altitude, No Enemy	0.98
Sensor "See's Enemy" Given High Altitude, Enemy	0.75
Sensor "See's Enemy" Given Low Altitude, Enemy	0.8
Belief that enemy caused failure given High Altitude, Enemy	0.9
Belief that enemy caused failure given Low Altitude, Enemy	0.95
Sensor "See's Enemy" Given High Altitude, No Enemy	0.35
Sensor "See's Enemy" Given Low Altitude, No Enemy	0.2
Belief that enemy caused failure given High Altitude, No Enemy	0.6
Belief that enemy caused failure given Low Altitude, No Enemy	0.7
Strike "See's Enemy" Given High Altitude, Enemy	0.6
Strike "See's Enemy" Given Low Altitude, Enemy	0.75
Strike "See's Enemy" Given High Altitude, No Enemy	0.4
Strike "See's Enemy" Given Low Altitude, No Enemy	0.35
Hit Probability Given High Altitude	0.7
Hit Probability Given Low Altitude	0.8

Table 2.1   Notional Probabilities to be Assessed from the DM or past data

2.2.3.3   Alternatives and Preferences.   The commander can send two strike UAVs, one strike and one sensor UAV, or one strike UAV at high or low altitude. Alternatively, the commander may take no action. Each alternative results in a measure for the attributes X and Y. The X attribute is defined on [-1, 1] with $-1 \prec 1$. A value of -1 represents a non-combatant destroyed by the UAV and a value

of 1 represents enemy neutralization. The Y attribute is defined as the number of UAVs surviving at the end of the encounter and is defined on $[0, 2]$ with $0 \prec 2$.

The notional assessed parameters for the commander in this decision situation are as follows. The utility of a friendly kill, but all UAVs returning is $U(x^0, y^*) = 0.2$. The utility of a succsessful strike but both UAVs lost is $U(x^*, y^0) = 0.3$. The assessed ranked correlation coefficient is $\rho = 0.7$. The respective risk aversion coefficients are $\gamma_x = -0.2$ and $\gamma_y = 0.3$. The risk aversion coefficients reflect a decision maker who is risk seeking in gains and risk averse in losses as prescribed by Tversky and Kahneman [40].

After assessing these parameters, the multiattribute decision copula parameters δ, l_x, l_y, a, and b are determined using Equations 2.5, 2.6, 2.7, 2.8, and 2.9. The following values are obtained:

$$\delta = -5.0899, \quad a = 1.0084, \quad b = -0.0084, \quad l_x = 0.3058, \quad l_y = 0.2066.$$

The multiattribute utility copula $C(x, y)$ is found by substituting into Equation 2.4,

$$C(x, y) = -\frac{1.01}{5.09} \ln \left(1 - \frac{\left(1 - e^{0.31 - 0.69 U_x(x)}\right)\left(1 - e^{0.207 - 0.713 U_y(y)}\right)}{1 - e^{-5.09}} \right) - 0.01.$$

Figure 2.2 shows the multiattribute utility function and Figure 2.3 shows the isopreference curves.
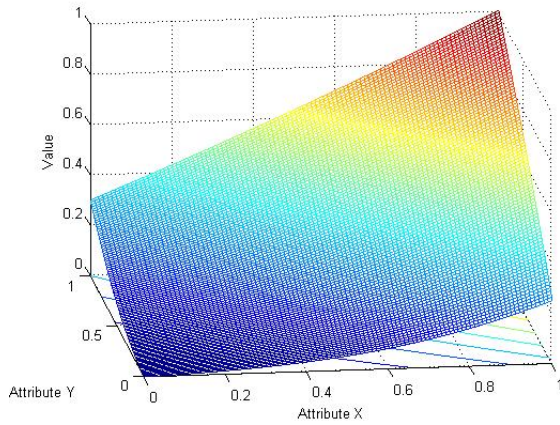
Figure 2.2 Multiattribute utility surface
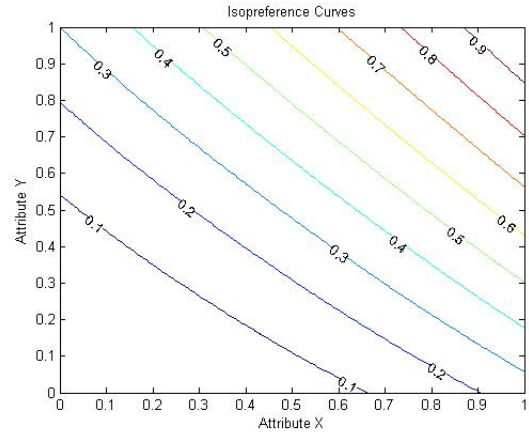
Figure 2.3 Multiattribute isopreference curves

2.3 Results

The alternative with the highest expected utility is to send one strike UAV at high altitude. The likelihood of misdiagnosing the enemy presence is low enough, due to the high prior probability of enemy presence, that benefit of sending a sensor UAV is outweighed by the risk of a sensor UAV being destroyed. For this reason, sending the strike at higher altitude is preferred as well. Table 2.2 shows the resulting expected utility of the 5 best alternatives.

Rank	Sensor UAV/Altitude	Strike UAV/Altitude	Utility Copula
1	None	One High	0.688
2	One High	One High	0.688
3	One Low	One High	0.687
4	None	One Low	0.684
5	None	Two High	0.681

Table 2.2 Five best alternatives by utility copula

It is interesting to note that the multilinear model (discussed by Keeney and Raiffa [22]) provides a different order for the solutions. Not using any dependence structure would have resulted in the wrong decision being made. Since the multilinear utility requires the assumption of conditional independence among attributes, the resulting utility in one attribute would not update the utility of the other. The

copula form allows for a dependence assumption between the attributes, thus the utility of the number of enemy destroyed can change based on the utility of the number of UAVs returning to home base after the engagement, which more accurately reflects the commander's preferences. The result, shown in Table 2.3, is that the top four alternatives switch in order. The utility of the intermediate outcomes has changed so that the need to more accurately identify the enemy outweighs the the risk of UAV loss therefore the strike UAV must fly at low altitude (with a higher sensor accuracy).

Rank	Sensor UAV/Altitude	Strike UAV/Altitude	Utility Copula
1	None	One Low	0.697
2	One Low	One High	0.696
3	One High	One High	0.696
4	None	One High	0.694
5	None	Two High	0.684

Table 2.3   Five best alternatives by multilinear utility

2.4   Sensitivity Analysis

This section details the sensitivity on the assessed parameters for constructing the multiattribute utility copula. Section 2.4.1 details the sensitivity to the parameter ρ with an explanation of how this parameter can change the optimal alternative. Section 2.4.2 contains the sensitivity analysis of the model to $U(x^0, y^*)$ and $U(x^*, y^0)$. Section 2.4.3 details the sensitivity of this analysis to the parameters γ_x and γ_y.

2.4.1   Sensitivity to ρ.   The parameter ρ is translated to δ (for use in the copula by the Genest approximation in Equation 2.9). The parameter δ defines the corresponding tradeoffs curve between x and y. Because the structure used is a Class 1 utility copula, the maximal marginal utilities are assessed and used to build the utility function. This results in no change to the maximal marginal utilities despite any changes to ρ. Changes in the parameter ρ will result in changes to the shape

of the center of the utility copula. Figure 2.4 shows the utility isopreference curves with $\rho = -0.5$ and Figure 2.5 shows the utility isopreference curves with $\rho = -0.9$
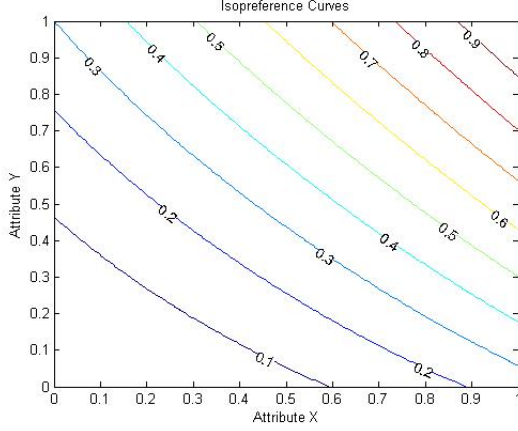
Figure 2.4 Isopreference with $\rho = -0.5$
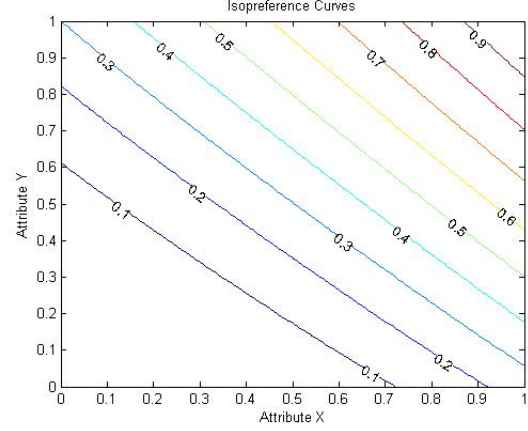
Figure 2.5 Isopreference with $\rho = -0.9$

The optimal decision changes between $\rho = -0.6$ and $\rho = -0.7$. The decision is still not the same as the one made by no utility dependence structure, in the multilinear model. This change occurred because the utility of points assessed away from the maximal margins changed enough to affect a change in expected utility derived from the probability distribution of outcomes. The result is that it is the need to positively identify an enemy outweighs the risk of loss to a sensor UAV. Table 2.4 shows the changes in the decision based on perturbations in ρ. In Table 2.4 Se/St represents the number of sensor and strike UAVs sent and the altitude. For example, 1H/1L represents one sensor package UAV sent at high altitude and one strike package UAV sent at low altitude.

2.4.2 Sensitivity to $U(x^0, y^*)$ and $U(x^*, y^0)$.   The parameter $U(x^0, y^*)$ defines the utility of the consequence in which all UAVs remain, but a non-enemy is destroyed (i.e, collateral damage occurs). The assessed parameter has a value of 0.3. Table 2.5 shows the top five optimal alternatives as a function of $U(x^0, y^*)$.

ρ=-0.5		ρ=-0.6		ρ=-0.7		ρ=-0.8		ρ=-0.9	
Se/St	U	Se/St	U	Se/St	U	Se/St	U	Se/St	U
1H/1H	0.691	1H/1H	0.689	N/1H	0.688	N/1H	0.687	N/1H	0.686
N/1H	0.690	N/1H	0.689	1H/1H	0.688	1H/1H	0.687	1H/1H	0.686
1L/1H	0.689	1L/1H	0.688	1L/1H	0.686	1L/1H	0.685	1L/1H	0.684
N/1L	0.687	N/1L	0.686	N/1L	0.684	N/1L	0.682	N/1L	0.681
N/2H	0.682	N/2H	0.681	N/2H	0.681	N/2H	0.681	N/2H	0.680

Table 2.4   Sensitivity to ρ

$U(x^0, y^*)$=0.1		$U(x^0, y^*)$=0.2		$U(x^0, y^*)$=0.3		$U(x^0, y^*)$=0.4		$U(x^0, y^*)$=0.5	
Se/St	U	Se/St	U	Se/St	U	Se/St	U	S/St	U
N/1H	0.681	N/1H	0.684	N/1H	0.688	1L/1H	0.697	1L/1H	0.707
1H/1H	0.675	1H/1H	0.681	1H/1H	0.688	1H/1H	0.695	1H/1H	0.702
N/1L	0.672	N/1L	0.678	1L/1H	0.686	N/1H	0.691	N/1L	0.698
N/2H	0.668	1L/1H	0.676	N/1L	0.684	N/1L	0.691	N/2L	0.696
1L/1H	0.667	N/2H	0.675	N/2H	0.681	N/2H	0.687	N/1H	0.695

Table 2.5   Sensitivity to $U(x^0, y^*)$

The optimal alternative changes based on sensitivity to $U(x^0, y^*)$ between 0.3 and 0.4. This parameter drastically changes the marginal utility on the upper margin for y, resulting in a pronounced change in the curvature of isopreference lines. Figure 2.6 shows the isopreference curve for a $U(x^0, y^*)$=0.1 and Figure 2.7 shows the isopreference curve for a $U(x^0, y^*)$=0.5. These figures clearly show that many of the outcomes of the decision situation would have drastic changes in utility based on perturbations to $U(x^0, y^*)$.

The parameter $U(x^*, y^0)$ is the utility of the consequence in which all enemy forces are neutralized, but the two UAVs are lost. In the notional model, this parameter has a value of 0.2. Table 2.6 shows the top five optimal alternatives as a function of $U(x^*, y^0)$.

The result of changing the parameter $U(x^*, y^0)$ was the opposite of changing $U(x^0, y^*)$. A change from the baseline occurred between 0.1 and 0.2, for similar reasons as the change in $U(x^0, y^*)$; the two parameters control a great deal of the overall shape of the utility function. This change is indicative of the higher utility
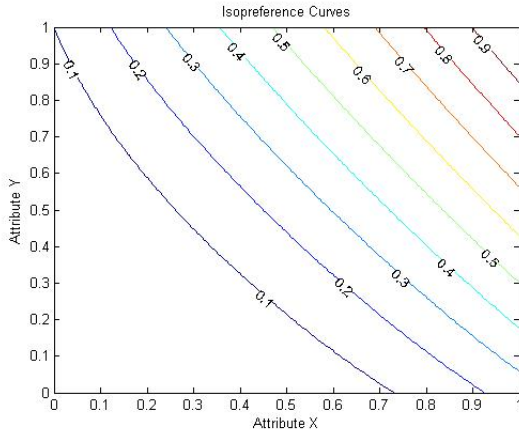
Figure 2.6 Isopreference with $U(x^0, y^*) = 0.1$
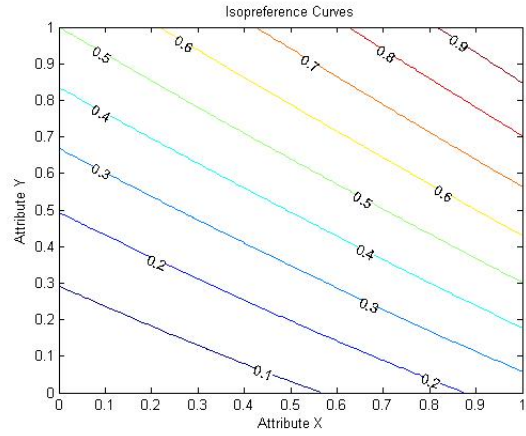
Figure 2.7 Isopreference with $U(x^0, y^*) = 0.5$

$U(x^*, y^0)$=0.0		$U(x^*, y^0)$=0.1		$U(x^*, y^0)$=0.2		$U(x^*, y^0)$=0.3		$U(x^*, y^0)$=0.4	
Se/St	U	Se/St	U	Se/St	U	Se/St	U	Se/St	U
1L/1H	0.631	1L/1H	0.659	N/1H	0.688	N/1H	0.723	N/1H	0.758
1H/1H	0.626	1H/1H	0.657	1H/1H	0.688	1H/1H	0.719	1H/1H	0.750
N/1L	0.621	N/1H	0.653	1L/1H	0.686	N/1L	0.716	N/1L	0.748
N/2H	0.620	N/1L	0.652	N/1L	0.684	1L/1H	0.714	N/N	0.744
N/1H	0.618	N/2H	0.650	N/2H	0.681	N/2H	0.712	N/2H	0.743

Table 2.6 Sensitivity to $U(x^*, y^0)$

for losing both UAVs associated with a higher probability of correct identification and neutralization of the enemy.

2.4.3 Sensitivity to γ_x and γ_y. The sensitivities to the risk parameters for x and y are given in Tables 2.7 and 2.8, respectively. These parameters describe the commander's risk tolerance in the single attribute space for x and y given complement attributes at the maximum (y^* and x^*). Changes in these parameters change the shape of the utility curve, especially at the margins.

As γ_x changes from -0.2 to 0.1, the optimal decision changes from sending only a strike UAV at high altitude to a strike UAV at high altitude and a sensor UAV at low altitude. This change is due to the change in risk attitude from risk seeking to risk averse. If the commander is risk averse in the enemy destruction attribute, the

γ_x=-0.8		γ_x=-0.5		γ_x=-0.2		γ_x=0.1		γ_x=0.4	
Se/St	U	Se/St	U	Se/St	U	Se/St	U	Se/St	U
N/1H	0.682	N/1H	0.685	N/1H	0.688	1L/1H	0.695	1L/1H	0.705
1H/1H	0.676	1H/1H	0.682	1H/1H	0.688	1H/1H	0.694	1H/1H	0.700
N/1L	0.673	N/1L	0.678	1L/1H	0.686	N/1H	0.691	N/1L	0.697
N/2H	0.670	1L/1H	0.678	N/1L	0.684	N/1L	0.690	N/1H	0.695
1L/1H	0.670	N/2H	0.675	N/2H	0.681	N/2H	0.687	N/2H	0.693

Table 2.7 Sensitivity to γ_x

utility of ordering a strike without higher sensor accuracy would be less likely due to uncertainty about the enemy presence.

γ_y=-0.3		γ_y=0.001		γ_y=0.3		γ_y=0.6		γ_y=0.9	
Se/St	U	Se/St	U	Se/St	U	Se/St	U	Se/St	U
N/1H	0.648	N/1H	0.668	N/1H	0.688	N/1H	0.721	N/N	0.768
N/2H	0.636	1H/1H	0.657	1H/1H	0.688	1H/1H	0.720	N/1H	0.755
1H/1H	0.631	N/2H	0.656	1L/1H	0.686	1L/1H	0.718	1H/1H	0.750
N/2L	0.629	1L/1H	0.653	N/1L	0.684	N/N	0.716	1L/1H	0.748
1L/1H	0.620	N/1L	0.651	N/2H	0.681	N/1L	0.716	N/1L	0.745

Table 2.8 Sensitivity to γ_y

As γ_y changes from 0.6 to 0.9 the risk of losing a UAV outweighs the benefit of possible enemy destruction according to the commander's risk attitude.

2.5 Conclusions

A notional decision situation consisting of UAV assignment involving two dependent attributes was assessed. A procedure for developing a multiattribute utility copula [3] was implimented and modified to include an approximation of the parameter δ, measuring the dependence of the attributes, to Spearman's rank correlation, ρ [15]. This is the first application of the multiattribute utility copula to a military decision problem.The utility dependence between attributes required a multiattribute model able to treat such dependence issues. The sensitivity analysis presented in this chapter shows the changes in the optimal alternatives based on perturbations of the assessed parameters. This is an important step in the decision process and cannot

be overlooked. This research can be used to inform decision makers of the impact of their risk attitudes and value tradeoffs to problems involving military decision analysis, and alter the tactics, techniques and procedures (TTPs).

3. An Application of Copulas to UAV Route Planning

3.1   Introduction

The previous chapter examines a commander's decision involving a single target with a fixed location. This allows for a thorough demonstration of the effect of the copula structure to the decision situation, but fails to account for the effect of enemy movement on the search and destroy capabilities of UAVs. This chapter explores a notional decision where a commander is trying to plan an optimal route for a pair of UAVs searching for a moving target through various types of terrain.

The search for a moving target appears in many sources of previous litera-ture. Stone and Kadane [39] classify the search for a target governed by Markovian movement as one of detection, where the searcher is attempting to maximize the probability of detection by time t, where-abouts, where the goal is to detect the target by time t or guess the whereabouts at time t, or surveillance, where the goal is to only be able to accurately guess the location of the target at time t. They argue that most research focuses on detection or whereabouts searches and continue to present solution techniques to the whereabouts search type problem.

A continuation of this research is the addition of multiple searchers by Santos [34] where the addition of multiple searchers for a single target exhibiting stochastic movement is shown to be computationally difficult and require the use of advanced heuristics, of which Genetic Algorithms (GAs) perform very well. Dell et al. [11] further examine the use of many heuristic decision types on a randomly moving enemy using multiple searchers along a constrained path. In their research, GA performs very well among the several heuristics examined. They also introduce the concept of Bayesian updates for the location of the enemy given a probability of non-detection at a discrete time step. Hollinger et al. [18] study the concept of coordinated versus non-coordinated search patterns through Bayesian updated target search for a non-adversarial enemy utilizing a partially observable markov

decision process to define the movement. Their research centers around a group of semi-autonomous robots searching an indoor space.

Optimal policies of active binary classification using Bayesian updates to label all data on a graph are studied extensively by Garnett et al. [14]. This research focuses on a one or two-step look ahead policy for determining the next location to investigate on the graph. In this approach, the utility is equal to the probability of detection, fitting with the detection search type presented by Stone and Kadane [39].

The search problem presents a similarity with the Vehicle Routing Problem (VRP), which has a large body of work focusing on the use of GAs to solve. Ombuki [29] and Nagata [27] present improvements on the GA for VRPs with time windows for delivery by modifying the crossover and mutation steps. A problem with the current GA types is the introduction (and subsequent deletion) of infeasible children for a VRP formulation, rectified through the use of a Hybrid GA presented by Prins [31] that does not create any infeasible children in the crossover or mutation steps. Yang and Yuan [46] present an improvement of the GA where mutations involve random swaps of genes and inversion of genes in specific chromosomes. This is largely the form used in the GA applied in this research.

Weinstein [45] and Schumacher [35] present the UAV cooperative search problem as a VRP with time windows formulation through the use of a Mixed Integer Linear Program. Each uses a set of tasks including location, classification, attack and battle damage assessment to be performed by multiple UAVs. Weinstein [45] allows the UAVs to avoid early detection by coordinating movement into specific areas of the map.

Research into UAV routing is also not lacking. Russel and Lamont [33] present an improved GA specific to UAV mission planning that performs better on standard test problems than previous GA. Bertuccelli et al. [9] examines a UAV task assignment problem with uncertainty, where there is Bayesian updating of target locations

and tasks include location classification and strikes. Shima et al. [37] introduce a matrix representation of chromosomes to reduce computing time in a multi-UAV task assignment GA. This problem is viewed as a combinatorial optimization problem that is NP-Hard. Jin et al. [20] incorporates a target response to the cooperative search and task assignment problem.

Pohl and Lamont [30] examine the routing of UAV swarms, or large groups of semi-autonomous UAVs. Employing GA, they force several UAVs to track several targets while staying within distance constraints.

Terrain covering is another search type presented by Ablavsky [5] where distribution of effort between multiple UAVs provides for a complete covering of a 2D space using geometric movement capabilities of the aircraft. In this form, multiple coverage on the same set of UAVs may be performed, but penalized through the objective function. Another example of the coverage of a 2D map is presented by Wang [44]; this dissertation includes Bayesian updates and DTMC defined target movement.

Abdelhafiz et al. [6] present several instances of the multi-objective UAV mission planning problem where the introduction of risk to UAVs as a deterrent to scheduling actions creates a more complex solution space. This tradeoff between goals of coordinated search and destruction and risk mitigation provides the backbone of the use of decision analysis in this research.

The rest of this chapter is organized into sections. Section 3.2 discusses the methodological approach for the heuristic to solve this problem. Section 3.3 examines a scenario where an enemy is attempting to evade detection. Section 3.4 examines an enemy trying to travel from one area of the map to another. Section 3.5 discusses relevant conclusions to be drawn from this analysis.

3.2   Methodology

In this model, enemy movement is assumed to be Markovian. The distribution of enemy location, $E(t)$, is entirely characterized through the use of an initial belief in location and a transition matrix P. A discrete time markov chain serves to model the enemy movement at each time period. As the model progresses through time periods, Bayesian updates define a new vector of probabilities for location of the enemy based on input from previous time epochs as described in Section 3.2.1 and demonstrated in Section 3.3.4. The probability of enemy location is considered a state, s, and the decision maker can send UAVs to a location, changing the subsequent utility and probability distributions of enemy locations. This is a Markov Decision Process with a utility function to characterize the outcomes.

The construction of the utility copula used to measure a commander's risk preferences among alternatives uses the same procedures as outlined in Chapter 2. This section introduces the notion of time into the decision situation through the use of Bayesian updating and provides an outline of the GA used to search the alternatives for the solution with the highest expected utility. The utility function used for this research is Equation 2.4.

In this analysis a maximum of five time periods is examined and it is assumed that a UAV has a standard operating window of sufficient length to allow unconstrained flight through the area of operations without a distance consideration. A specific area on the map may be searched multiple times, providing that there is an increased utility associated with doing so. If the probability of enemy detection is sufficiently low, based on Bayesian updates, this is a way to avoid the enemy entirely, avoiding UAV damage and reducing the probability of firing at a low-probability target. In this manner, the model has many similarities to the standard VRP, but without the normal constraints on distance or number of visits.

3.2.1 Bayesian Updates. The process of Bayesian updating is seen throughout the literature as a way to update belief in enemy locations based on known information after each time epoch. In this manner, the future assignment of UAVs to search areas on a map are influenced by searches in past time epochs. This is done using the conditional prior belief in enemy location and the observations for the locations using equation 3.1

$$P(E|V) = \frac{P(V|E)}{\sum_n P(E|V_n)P(V_n)} P(V).$$

(3.1)

This calculates the posterior belief in enemy presence at that node. Subsequent nodes are calculated, and updates occur on the nodes that were not observed based on these calculations. UAVs seeing the enemy or being destroyed each have a different effect in updating the probability of enemy presence in an area on the map, and subsequent decision tree nodes reflect this probabilistic belief. Sample calculations of the Bayesian updates are shown in Section 3.3.4.

3.2.2 Genetic Algorithm. GA is a heuristic approach to searching the alternatives for the best solution to a fitness function. This is done through the use of a chromosome representing each alternative. The locations of the nodes visited at a certain time are genes along this chromosome. Mutations are random replacement of these genes (nodes) with another feasible gene. Crossovers are meant to mimic natural reproduction and provide a splicing of the genes associated with two chromosomes. The population size of a genetic algorithm ensures that only the best solutions from each iteration are kept and allowed to mutate or crossover.

To effectively search for the alternatives with the highest expected utility, a GA is used. Use of this heuristic approach is shown in the literature to be one of the most efficient approaches to solving the VRP [27, 29, 33, 34, 37]. For this specific application of GA to the VRP, mutation procedures as outlined by Yang

and Yuan [46] is used, in conjunction with standard crossover procedures. A matrix representation provides computational simplicity [37].

The expected utility is used as the fitness function for the GA. Chromosomes are sorted by the expected utility of their paths and only the best chromosomes are kept for the next iteration of the GA.

Since there are no constraints on the number of times that a node may be visited, the crossover operation is simplified to a switching of genes along the chromosomal representation for each path. Figures 3.1 and 3.2 show an example of a crossover operation performed on two chromosomes.

A | 1 | 2 | 1 | 2 | **1** | **2** | **1** | **2** | 1 | 2

B | 3 | 4 | 3 | 4 | **3** | **4** | **3** | **4** | 3 | 4

Figure 3.1   Chromosomes A and B prior to crossover

A | 1 | 2 | 1 | 2 | **3** | **4** | **3** | **4** | 1 | 2

B | 3 | 4 | 3 | 4 | **1** | **2** | **1** | **2** | 3 | 4

Figure 3.2   Chromosomes A and B after crossover

Two types of mutations are used. The inversion operation selects a section of chromosome and reverses the order of the genes along that section. Figure 3.3 shows a chromosome before and after inversion.

The second type of mutation is the random replacement. In this mutation a random number (between 1 and 3) of genes are replaced with randomly chosen areas

1	2	3	4	**5**	**6**	**7**	**8**	9	1

1	2	3	4	**8**	**7**	**6**	**5**	9	1

Figure 3.3 Chromosome which has undergone inversion mutation

of the map to test for better solutions. Figure 3.4 shows a random replacement on 3 genes in a chromosome.

1	**2**	3	4	**5**	6	7	8	**9**	1

1	**5**	3	4	**2**	6	7	8	**8**	1

Figure 3.4 Random replacement on 3 genes

Due to the lack of constraints and the subsequent lack of infeasible children, a GA performs well in searching the decision space for this model. As the complexity of the model increases, through scope or constraints, other heuristics should be considered.

3.3 Notional Scenario 1

3.3.1 Decision Context. A notional 2D map space is comprised of three different terrain types. Each terrain type provides bonuses or penalties in the form of modified probabilities for the enemy forces and UAV searchers capabilities (e.g. survivability, accuracy). Table 3.1 and Table 3.2 represent the terrain types and adjacency list for the fully connected hexagonal map space. Figure 3.5 shows a graphical representation of the area map. Node numbers are the areas of the graph, and the letters represent the terrain type associated with that area.

Node	Terrain	Node	Terrain	Node	Terrain	Node	Terrain
1	Hill	10	Mountain	19	Plains	28	Plains
2	Mountain	11	Mountain	20	Hill	29	Hill
3	Mountain	12	Mountain	21	Hill	30	Hill
4	Mountain	13	Hill	22	Hill	31	Plains
5	Mountain	14	Hill	23	Hill	32	Plains
6	Mountain	15	Mountain	24	Hill	33	Plains
7	Hill	16	Mountain	25	Plains	34	Plains
8	Hill	17	Mountain	26	Plains	35	Plains
9	Mountain	18	Mountain	27	Plains	36	Plains

Table 3.1   Terrain type by node

Node	Adjacency	Node	Adjacency
1	1, 2, 7	19	13, 14, 19, 20, 25
2	1, 2, 3, 7, 8	20	14, 15, 19, 20, 21, 25, 26
3	2, 3, 4, 8, 9	21	15, 16, 20, 21, 22, 26, 27
4	3, 4, 5, 9, 10	22	16, 17, 21, 22, 23, 27, 28
5	4, 5, 6, 10, 11	23	17, 18, 22, 23, 24, 28, 29
6	5, 6, 11, 12	24	18, 23, 24, 29, 30
7	1, 2, 7, 8, 13	25	19, 20, 25, 26, 31
8	2, 3, 7, 8, 9, 13, 14	26	20, 21, 25, 26, 27, 31, 32
9	3, 4, 8, 9, 10, 14, 15	27	21, 22, 26, 27, 28, 32, 33
10	4, 5, 9, 10, 11, 15, 16	28	22, 23, 27, 28, 29, 33, 34
11	5, 6, 10, 11, 12, 16, 17	29	23, 24, 28, 29, 30, 34, 35
12	6, 11, 12, 17, 18	30	24, 29, 30, 35, 36
13	7, 8, 13, 14, 19	31	25, 26, 31, 32
14	8, 9, 13, 14, 15, 19, 20	32	26, 27, 31, 32, 33
15	9, 10, 14, 15, 16, 20, 21	33	27, 28, 32, 33, 34
16	10, 11, 15, 16, 17, 21, 22	34	28, 29, 33, 34, 35
17	11, 12, 16, 17, 18, 22, 23	35	29, 30, 34, 35, 36
18	12, 17, 18, 23, 24	36	30, 35, 36

Table 3.2   Adjacency listing for scenario 1

The commander must plan the UAV flight path for two UAVs through four time periods. Each aircraft has a list of four nodes to visit and the subsequent utility is calculated using the probabilities and utility as described in the later sections.

3.3.2   Information.   In this notional scenario, assume that the enemy begins at node 21 and has movement described by a P matrix created with the following
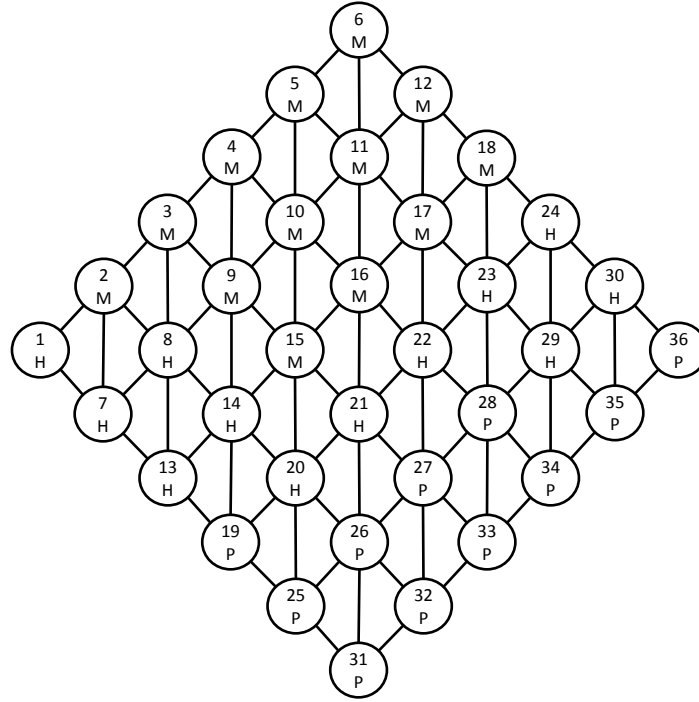
Figure 3.5   Graphical representation of Area Map

arbitrary mechanism to define belief in enemy movement; each adjacent node is coded as a 1 if plains, 2 if hills, and 3 if mountains. This serves as a mechanism for differing probabilities of enemy movement, UAV survivability, and UAV accuracy. The sum of all adjacencies is added in this manner and the probability of traveling to each adjacent node is the number coded divided by the sum of all adjacent codes. In this manner, the calculation of the transition vector associated with node 21, which is adjacent to 15, 16, 20, 21, 25, and 26, is calculated as follows; each node is assigned a number associated with its terrain type, which sums to 14. The probability of the enemy staying at node 21 would be 2/14. This method of coding transition probabilities is notional.

Probabilities for the UAV performance are dependent on location and presence of enemy. A list of these probabilities can be found in Table 3.3. If there is an enemy, a strike results in enemy neutralization with a probability of 0.8. If there is no enemy, a strike results in collateral damage with the same probability.

Distinction	Terrain	Probability
Accuracy	Mountains	0.85
Accuracy	Hills	0.9
Accuracy	Plains	0.95
Survivability Given Enemy	Mountains	0.7
Survivability Given Enemy	Hills	0.8
Survivability Given Enemy	Plains	0.9
Survivability Given No Enemy	Mountains	0.9
Survivability Given No Enemy	Hills	0.95
Survivability Given No Enemy	Plains	0.98

Table 3.3 Probability adjustments for terrain type

3.3.3 Alternatives and Preferences.   The commander must plan the flight paths for both UAVs and the alternatives are all possible flight plans. An assigned restriction is that UAVs cannot operate in the same map location in the same time period. The attributes X and Y are similar to those described in Chapter 2, however, the scaling on the attributes is slightly different. The X attribute is defined as enemy neutralization and is defined on $x \in \{0, 0.5, 1\}$. A 0 represents a friendly fire incident, a 0.5 represents an unsuccessful strike or no strike, and a 1 represents complete enemy neutralization. The Y attribute represents the number of UAVs returning after the mission and is defined on $y \in \{0, 0.5, 1\}$. A 0 represents no UAVs returning, a 0.5 represents 1 UAV returning, and a 1 represents 2 UAVs returning after the mission is complete.

The notional assessed parameters for the commander in this decision situation are as follows. The value of no strike but all UAVs returning is $U(x^0, y^*) = 0.15$. The value of a succsessful strike but both UAVs lost is $U(x^*, y^0) = 0.4$. The notional assessed ranked correlation coefficient is $\rho = 0.5$. The respective risk aversion coefficients are $\gamma_x = -1.2$ and $\gamma_y = 1.3$. The risk aversion coefficients reflect a decision maker who is risk seeking in gains and risk averse in losses as prescribed by Tversky and Kahneman [40].

After assessing these parameters for this decision situation, the multiattribute utility copula parameters δ, l_x, l_y, a, and b are determined by using Equations 2.5, 2.6, 2.7, 2.8, and 2.9. The following values are obtained:

$$\delta = 3.2646, \quad a = 1.6363, \quad b = -0.6363, \quad l_x = 0.4805, \quad l_y = 0.6333.$$

The multiattribute utility copula $C(x, y)$ is found by substituting into Equation 2.4. Figure 3.6 shows the multiattribute utility function and Figure 3.7 shows the isopreference curves.
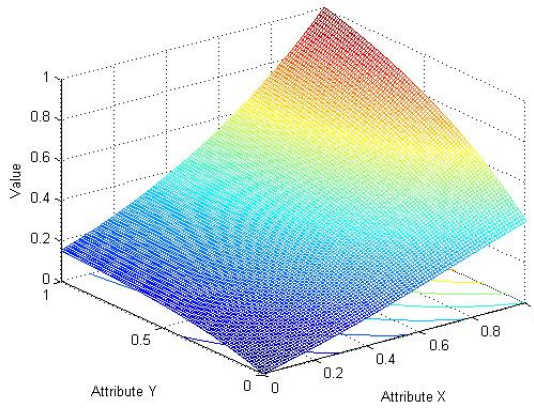

Figure 3.6 Multiattribute utility surface
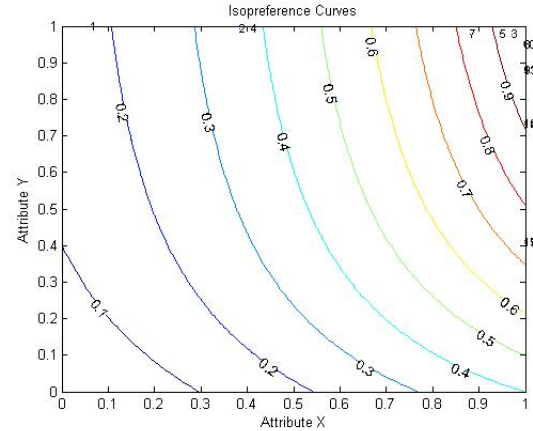

Figure 3.7 Multiattribute isopreference curves

3.3.4 Sample Calculations.   The calculations for utility and Bayesian updates for a specific path are demonstrated in this section. Using a sample alternative from the notional scenario, the calculations are performed for the first time period for the path where UAV1 visits nodes 27, 21, 20, and 15 and UAV2 visits nodes 22, 20, 26, and 10 at each time period respectively. The posterior probabilities for the first time period are calculated using the following equations:

For UAV 1:

$$P(S(1)) = P(E)P(S|E) + P(\bar{E})P(S|\bar{E}) =$$

$$0.071 * 0.9 + 0.929 * 0.98 = 0.9743,$$

$$P(V(1)|S) = P(E)P(S|E)P(V|E)/P(S) + P(\bar{E})P(S|\bar{E})P(V|\bar{E})/P(S) =$$

$$0.071 * 0.95 * 0.9/0.9743 + 0.929 * 0.98 * 0.05/0.9743 = 0.1094,$$

$$P(E(1)|VS) = P(E)P(S|E)P(V|E)/(P(S)P(V|S))$$

$$= 0.071 * 0.9 * 0.95/(0.9743 * 0.1094) = 0.5703.$$

For UAV 2:

$$P(S(2)) = 0.143 * 0.8 + 0.857 * 0.95 = 0.9286,$$

$$P(V(2)|S) = 0.143 * 0.8 * 0.9/0.9286 + 0.857 * 0.95 * 0.1/0.9286 = 0.1986,$$

$$P(E(2)|VS) = 0.143 * 0.8 * 0.9/(0.9286 * 0.1986) = 0.5581.$$

The utility of the first time period is calculated for each outcome. For example, the utility in the case where both survive and see the enemy would be given by:

$$P(S(1))P(S(2))(P(V(1)|S)P(V(2)|S) *$$

$$\max(P(E(1)|VS)(P(Hit|E)UM(3,3) + P(NoHit|E)UM(2,3)) +$$

$$P(\bar{E}(1)|VS)(P(Hit|E)UM(1,3) + P(NoHit|E)UM(2,3)),$$

$$P(E(2)|VS)(P(Hit|E)UM(3,3) + P(NoHit|E)UM(2,3)) +$$

$$P(\bar{E}(2)|VS)(P(Hit|E)UM(1,3) + P(NoHit|E)UM(2,3))).$$

The other outcomes are calculated similarly and the resulting cumulative expected utility for time period 1 is found. After this calculation, Bayesian updates are calculated for all outcomes where the enemy was not found (either by UAV destruction or lack of sensor accuracy). The outcome where both aircraft survived but did not see the enemy would be calculated in the following manner. Node 27 would be updated to

$$P(E(1)) = \frac{P(E)P(S|E)P(\bar{V}|E)}{P(E)P(S|E)P(\bar{V}|E) + P(\bar{E})P(S|\bar{E})P(\bar{V}|\bar{E})} =$$

$$\frac{0.071 * .9 * .05}{0.071 * .9 * .05 + 0.929 * 0.98 * .95} = 0.0037.$$

The resulting new enemy probability vector is given by:

```
PE=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.2675 0.2675 0 0

    0 0.1784 0.1784 0.0154 0 0 0 0.0892 0.0037 0 0 0 0 0 0 0 0 0].
```

This vector is then multiplied by the transition matrix P to give the enemy locations for the next time period for the subsequent outcomes. This process is repeated for all possible outcomes at each time period.

3.3.5 Results. The alternative with the highest expected utility is to send UAV 1 to area 16, 21, 15, and 15 and send UAV 2 to 22, 20, 21, and 21. Table 3.4 shows the resulting expected utility of the five best alternatives found by the GA.

Rank	UAV 1	UAV 2	Utility
1	16, 21, 15, 15	22, 20, 21, 21	0.425
2	16, 21, 15, 16	22, 20, 21, 15	0.424
3	16, 21, 15, 21	22, 20, 21, 15	0.423
4	16, 21, 15, 21	22, 20, 21, 20	0.422
5	16, 21, 15, 21	22, 20, 21, 19	0.420

Table 3.4   Best alternatives for Notional Scenario 1

The best result is shown in Figure 3.8. In this figure the red nodes represent those visited by UAV 1 and the blue nodes represent those visited by UAV 2.
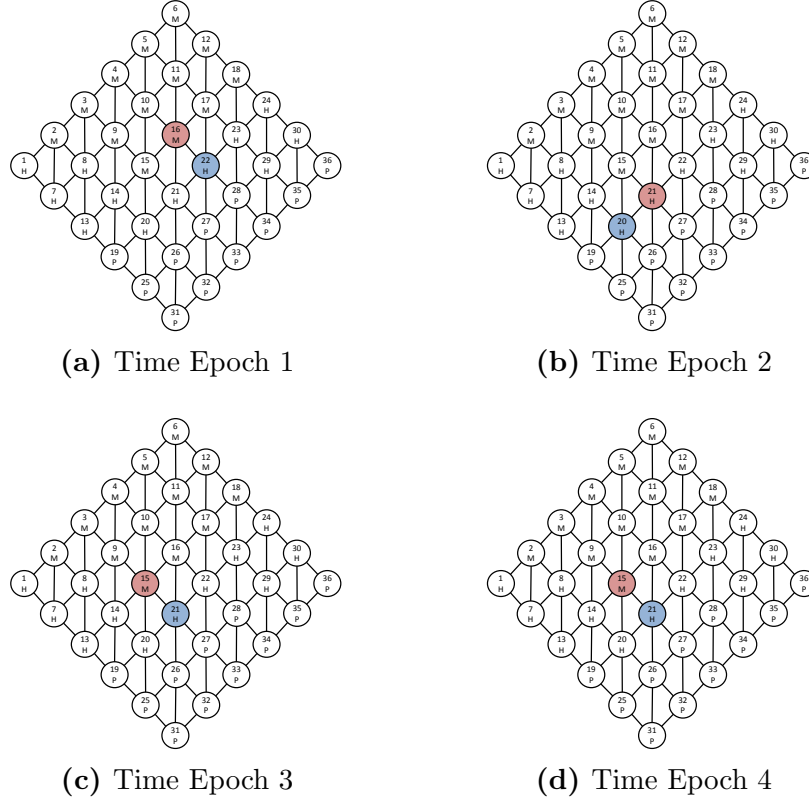

(a) Time Epoch 1

(b) Time Epoch 2

(c) Time Epoch 3

(d) Time Epoch 4

Figure 3.8   Nodes visited by each UAV at each time period in scenario 1

There are multiple visits to several nodes on this list. As time advances, the enemy location probability in any area of the map decreases. Thus, in the later time epochs, the UAVs may search an area multiple times to avoid searching an area with a high chance of target misidentification as a result of a low prior probability of enemy presence.

3.3.6 Sensitivity Analysis.   The following section details the sensitivity analysis to the assessed parameters for constructing the multiattribute utility copula as well as the probabilities of survival and accuracy for the UAV sensors. The

sensitivites to ρ, $U(x^*, y^0)$, $U(x^0, y^*)$, γ_x, γ_y, $P(S|E)$, $P(V|E)$, and the number of time steps evaluated by the utility model are examined.

3.3.6.1 *Sensitivity to ρ.*   The paramater ρ is translated into δ to describe the corresponding tradeoffs curve between x and y. Changes in the parameter act in the same way on the utility function as in Chapter 2.

The model is not very sensitive to changes in the correlation between the utilities of X and Y. The areas searched by each UAV does not change for $\rho \in [-0.4, 1.0]$. At $\rho = -0.41$ the paths change to send UAV 1 to 16, 21, 21, and 15 and to send UAV 2 to 22, 20, 20, and 10. This change is due to the utility of the intermediate outcomes (those not at the extreme margins) changing. The parameter ρ affects the areas in the center of the utility copula, thus the outcome of $x = 0.5$, and $y = 0.5$ will be changed. As this change propegates through the decision tree, this will change the overall utility of the alternatives, and can change the answer. If these outcomes are unlikely, no changes in the optimal alternative will appear.

3.3.6.2 *Sensitivity to $U(x^*, y^0)$ and $U(x^0, y^*)$.*   The parameters $U(x^*, y^0)$ and $U(x^0, y^*)$ define the corner points of the maximum marginal utility functions and are directly assessed from the decision maker. $U(x^*, y^0)$ defines the utility of finding and destroying the enemy target without the destruction of a UAV. The optimal alternative does not change for small perturbations of this value; it remains constant for $U(x^*, y^0) \in [0, 0.5]$. For $U(x^*, y^0) > 0.5$, changes in the areas of the map searched by the UAVs change slightly. UAV 1 still visit 16, 21, 15, and 15. However, UAV 2 now visits 22, 20, 20, and 16. This change is due to rather large changes in the shape of the utility function at the maximum margin for $U(y)$. This changes the utilities associated with $y = 0$ and $y = 0.5$.

The parameter $U(x^0, y^*)$ is the utility of eradicating a friendly force, but retaining both UAVs. This paramater causes more changes in the optimal alternative. The results of changes are shown in Table 3.5.

Parameter	UAV 1	UAV 2
$U(x^0, y^*)$=0.05	16, 21, 15, 15	22, 20, 20, 16
$U(x^0, y^*)$=0.15	16, 21, 15, 15	22, 20, 21, 21
$U(x^0, y^*)$=0.25	16, 21, 21, 15	22, 20, 20, 10
$U(x^0, y^*)$=0.35	16, 21, 21, 15	22, 20, 20, 21
$U(x^0, y^*)$=0.45	16, 21, 21, 21	22, 20, 20, 20

Table 3.5 Sensitivity to $U(x^0, y^*)$ for scenario 1

Changing $U(x^0, y^*)$ alters the assignment of the later searched nodes, where the probability of UAV loss is greater relative to the probability of finding the enemy. UAV survival is dependent upon natural flight problems and enemy presence.

3.3.6.3 Sensitivity to γ_x and γ_y.   Changes to γ_x and γ_y change the shape of the utility curve at the maximum margins, since this is a Class 1 utility copula. The result is to change the entire shape of the utility function and can change interior as well as marginal utility values. The corner points remain fixed, as those are only changed through the parameters $U(x^*, y^0)$ and $U(x^0, y^*)$.

The parameter γ_x is stable, with no changes occurring in the range of $\gamma_x \in [-0.2, -2.2]$. At $\gamma_x = 0.8$, as the decision maker changes to being risk prone in the attribute x, the search path for UAV 2 changes to 22, 20, 21, and 10, a very slight change. At $\gamma_x = -3.2$, the search patterns for both UAVs change. UAV 1 now searches 16, 21, 21, and 15 and UAV 2 searches 22, 20, 20, and 10. This is because the utility of the intermediate values for attribute x are higher, resulting in less penalty for missing the enemy with a strike.

The parameter γ_y is also stable, with only changes to the last nodes searched being made. The optimal alternative for $\gamma_y = 0.3$ is UAV 2 searching 22, 20, 20, and 16, with no change to UAV 1, and the optimal alternative for $\gamma_y = 3.3$ is UAV 2 searching 22, 15, 21, 20, again with no changes for UAV 1. This reflects the result that the priority nodes do not change based on changes to the parameter γ_y and that the model is not very sensitive to changes in this parameter.

3.3.6.4 Sensitivity to Assessed Probabilities. The assessed probabilities for survival and accuracy affect the likelihood of outcomes from the decision tree for each alternative, and thus, can have large effects on the expected utility. As survivability decreases, UAVa tend towards avoiding enemies with larger penalties (like mountainous terrain). As accuracy decreases, the posterior probability of enemy location, and the accuracy of the Bayesian updates for subsequent time epochs is diminished.

The probability of survival is subject only to small changes at a lower projected value. Changing the probability of survival given enemy and mountains to 0.6, 0.7 for hills and 0.8 in plains resulted in a UAV 2 path of 22, 20, 20, and 16, with no change in the path to UAV 1, indicating that only the last time epoch was changed. However, upon increasing the survival to 0.8 for mountains, 0.9 for hills, and 0.99 for plains, the path with the highest expected utility for UAV 2 was 22, 15, 21 and 20, with no change to UAV 1. This shows that increasing the probability of survival allowed for a more thorough search of mountain terrain (area 15) and hills (area 20).

The accuracy of the sensors for UAV is stable as it is decreasing, but results in a small change as it is increasing. At an accuracy of 0.9 for mountains, 0.95 for hills and 0.99 for plains, the path for UAV 2 shifted to node 16 for the fourth time epoch, ensuring a more thorough search of mountainous terrain.

3.3.6.5 Sensitivity to the Planning Horizon. This section describes the sensitivity of the model to the number of time steps that the commander has to plan for UAV movement. In longer planning horizons, it is possible for the UAV to use Bayesian updating and the properties of Markovian movement to ensure that mountainous areas of the map do not have to be searched with high probability of enemy activity (where survival probabilities are lower), whereas in shorter planning horizons, it may be more beneficial to go to the locations with the highest probability of enemy activity.

Shortening the planning horizon to 2 time steps results in the alternative with the highest expected utility being sending UAV 1 to 16 and 21 and UAV 2 to 15 and 20. This is the first sensitivity analyzed in this paper to effect a change in the locations searched in the first time epoch. The UAVs are sent, in this instance, to the locations with the highest probability of enemy activity in each time epoch.

At a planning horizon of 3 time steps, the path of UAV 1 changed to 20, 21, and 15 and UAV 2 became 16, 16, 21. Again, drastic changes from the 2 and 4 time step planning horizons are present, and some avoidance (area 15) of enemy is taking place in earlier time steps to ensure a higher probability of UAV survival.

At a planning horizon of 5 time steps, the optimal path of UAV 1 is 16, 21, 21, 15 and 15 and UAV 2 is 22, 20, 20, 10, and 21. This shows some change from the model at 4 time steps, but the changes occur in later time epochs. It is clear that as the planning horizon is made longer it becomes advantageous in utility to not directly search areas with the highest enemy probability as soon as possible in every case.

3.4   Notional Scenario 2

3.4.1   Decision Context.   In the second notional scenario considered by this analysis, the enemy is considered to have a goal. After starting at area 31, two time steps have passed and it is assumed that the enemy will attempt to reach a safe zone at node 6. Table 3.6 represents adjacency list for the directed hexagonal map space.

The adjacency list for this space is greatly reduced, as the enemy will only move towards their goal, and no longer backtrack or stay in an area for more than one time unit. The probability of enemy movement into any area is unknown, so it is assumed that they will move into any adjacent node (according to the adjacency list) with equal probability to all other adjacent nodes. The information that was received on the enemy was delayed, so two time steps have taken place before the

Node	Adjacency	Node	Adjacency
1	2	19	13, 14, 20
2	3	20	14, 15, 21
3	4	21	15, 16, 22
4	5	22	16, 17, 23
5	6	23	17, 18, 24
6	–	24	18
7	1, 2, 8	25	19, 20,26
8	2, 3, 9	26	20, 21, 27
9	3, 4, 10	27	21, 22, 28
10	4, 5, 11	28	22, 23, 29
11	5, 6, 12	29	23, 24, 30
12	6	30	24
13	7, 8, 14	31	25, 26, 32
14	8, 9, 15	32	26, 27, 33
15	9, 10, 16	33	27, 28, 34
16	10, 11, 17	34	28, 29, 35
17	11, 12, 18	35	29, 30, 36
18	12	36	30

Table 3.6   Adjacency listing for scenario 2

search can begin. The search will be conducted for three time steps, with sensitivities performed on increasing or decreasing that planning horizon.

3.4.2   Results.   The alternative with the highest expected utility is to send UAV 1 to area 27, 14, and 16 and send UAV 2 to 26, 15, and 22. Table 3.7 shows the resulting expected utility of 3 of the best alternatives found by the GA.

Rank	UAV 1	UAV 2	Utility
1	27, 14, 16	26, 15, 22	0.530
2	27, 15, 16	26, 14, 22	0.529
3	27, 15, 15	26, 14, 22	0.528

Table 3.7   Best alternatives for Notional Scenario 2

The best result is shown in Figure 3.9. In this figure the red nodes represent those visited by UAV 1 and the blue nodes represent those visited by UAV 2.
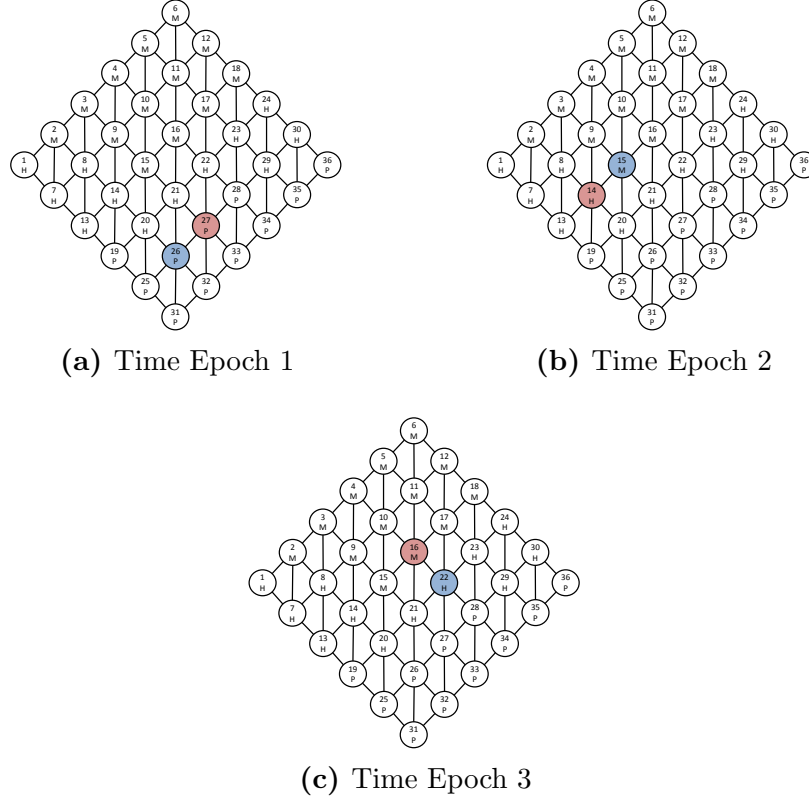
(a) Time Epoch 1

(b) Time Epoch 2

(c) Time Epoch 3

Figure 3.9   Nodes visited by each UAV at each time period in scenario 2

It should be noted that there are multiple ways to get to certain areas of the map while taking different amounts of time. For example, area 26 can be reached by moving directly from 31 or moving from 31 to 25 to 26. Area 26 is visited at time step 2, while there is still a probability of finding the enemy there.

3.4.3   Sensitivity Analysis.   The following section details the sensitivity analysis to the assessed parameters for constructing the multiattribute utility copula as well as the probabilities of survival and accuracy for the UAV sensors. The sensitivites to ρ, $U(x^*, y^0)$, $U(x^0, y^*)$, γ_x, γ_y, $P(S|E)$, $P(V|E)$, and the number of time steps evaluated by the utility model are examined.

3.4.3.1   Sensitivity to ρ.   The parameter ρ is used to calculate the parameter δ describing the corresponding tradeoffs curve between x and y. This measure defines the shape of the utility surface, and as seen in Chapter 2, can greatly impact a decision.

In this scenario, there are no changes to the decision made based on an increase to the parameter ρ; however, the optimal paths change if ρ is decreased even slightly. At $\rho = 0.4$, the optimal paths become 27, 21, and 16 for UAV 1 and 26, 14, and 10 for UAV 2. This change is due to changes in the intermediate utility values rippling through the probability calculations in the decision tree.

3.4.3.2   Sensitivity to $U(x^*, y^0)$ and $U(x^0, y^*)$.   The parameters $U(x^*, y^0)$ and $U(x^0, y^*)$ are the utility corner points for the maximum/minimum value of x and minimum/maximum value of y. Changes to these parameters can change the entire shape of the utility curve.

In this scenario, the optimal decision does not change with changes to $U(x^*, y^0)$, even extreme changes. As $U(x^0, y^*)$ is increased, to 0.35, the optimal paths change to 27, 21, and 16 for UAV 1 and 26, 14, 28 for UAV 2. This change reflects a greater importance on UAVs returning, and thus, an larger avoidance of mountainous areas of the map than the general result.

3.4.3.3   Sensitivity to γ_x and γ_y.   The parameters γ_x and γ_y define the shape of the utility surface at the maximum margins for Class 1 utility copulas. This shape change affects the rest of the utility surface shape as well, but is more pronounced at the maximal marginal utility.

This analysis showed no changes to even extreme values in γ_x. The optimal path remained fixed for all $\gamma_x \in [-2.8, 4.2]$. The parameter γ_y is stable in increases, and the optimal decision only change for extreme decreases. At $\gamma_y = -2.7$ the paths

chang to UAV 1 to 26, 21, and 16 and UAV 2 to 27, 14, and 10. This reflects a more cavalier attitude by the commander in UAV loss.

3.4.3.4 Sensitivity to Assessed Probabilities. As in notional scenario 1, the assessed probabilities define the likelihood of the resulting outcomes for each alternative. Changes in survivability and accuracy are examined in this analysis.

Survivability affects the likelihood of UAV to be shot down by enemy forces based on the probability of enemy activity and the terrain type that they are flying in. At a survivability of 0.6 for mountains, 0.7 for hills, and 0.8 for plains, the optimal alternative changes to sending UAV 1 to 26, 21 and 16 and UAV 2 to 27, 14, and 10. The optimal alternative does not change with increases to the survivability of the drones.

3.4.3.5 Sensitivity to the Planning Horizon. As seen in scenario 1, the planning horizon can effect how the UAVs search the area, effectively ensuring that probabilities of enemy presence in later time epochs are higher at specific areas, or sacrificing this to find the enemy more quickly. This scenario is no different.

At a 2-time step planning horizon, the optimal alternative become sending UAV 1 to 26 then 21 and sending UAV 2 to 27 then 14. This represents the areas with the highest probability of enemy presence and lowest probability of UAV loss (no mountainous terrain is searched). This changes for the 4-time step planning horizon, where UAV 1 goes to 26, 14, 22, 10 and UAV 2 goes to 27, 15, 28, 11. In this scenario, there is a distinct "funneling" of enemy activity into nodes 10 and 11, to increase the chance of finding the enemy at later time epochs.

3.5 Conclusions

The research presented in this chapter is a natural MAUT extenstion to the classic search and destroy theoretical problem addressed in Section 4.1. This research

applies current trends in Decision Analysis to address a commander's risk preferences associated with the problem.

In this chapter, two notional decision situations were evaluated consisting of a time dependent UAV search for a markovian moving target. The attributes of enemy neutralization and UAV survival were used to create a multiattribute utility copula, and a GA was used to search the alternatives for the alternative with the highest expected utility.

The concept of time as it applies to a search using multiattribute utility is an interesting prospect. This provided an ability for the UAV to route the enemy into areas where the search would be either more profitable or less dangerous. This aspect of model shows where the utility copula formulation allows for tradeoffs to dictate the paths the UAVs take in such a way to increase the overall expected utility of the search.

4. Conclusion

This research provides an original approach to the UAV search and destroy problem by employing multiattribute utility theory. Multiattribute utility theory is useful in any situation where a decision involves competing objectives. In the case where these these objectives are utility dependent on each other, multiattribute utility copulas provide a mechanism to model the dependence.

This thesis has demonstrated a new class of multiattribute utility functions with a Class 1 Archimedian copula structure. Two scenarios provided applications of the decision model and various operational results are derived. The work of this thesis is the initial step in examining the decision process facing commanders in a UAV search and destroy situation.

Several examples of future research in the application of multiattribute utility copulas to UAV search and destroy problems exist. As the number of UAVs at the commanders disposal increase, so does the size and complexity of the alternatives and outcomes. This research limited the number of UAVs to 2; however, there are often more UAVs available, it would there fore be worthwhile to examine the use of more than two UAVs. Another possible extension is to add anti-air defenses, enemy safe zones, or structures to the area map in scenario 2. These adjustments add to the complexity of the problem, but allow for a more realistic interpretation of the decision situation. These variables would require careful analysis as to the benefit and cost to target destruction or UAV survival before implementation. Other extensions to consider are the introduction of more terrain features, such as roads or towns and the separation of collateral damage into a separate attribute.

This research is useful in situations where there is a long term planning objective that requires careful consideration of risk attitdues. The use of this research on many common decisions could serve to update the TTPs associated with those problems. Other applications that may benefit from the use of multiattribute utility

copulas include the search for a downed pilot, where resources and success serve as attributes, or the area of cyber warfare, where risk of detection is competing against infiltration of an enemy network.

Using Multiattribute Utility Copulas in Support of UAV Search and Destroy Operations

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Research Goal

The goal of this research is to apply an emerging form of utility function, the multiattribute utility copula, to UAV search and destroy mission planning operations. This research informs decision makers and analysts with respect to tactics, techniques, and procedures employed in such missions. Increasing UAV operations tempo suggests such research becoming increasingly relevant to the warfighter.

Stationary Enemy
Utility copula is used to help decision makers determine the type, quantity, and altitude of UAVs to send to strike a stationary target. A commanders risk attitude helps to determine the tradeoff between UAV exposure to enemy and the ability to strike a target.

Moving Enemy
Utility copula examines the impact of attribute dependeence on the optimal routing of UAVs in a contested environment when performing a search and destroy mission against a Markovian target with Bayesian updates.

Stationary Enemy Model Formulation

Decision Diagram
Commander must decide whether to send 2, 1 or 0 UAVs at a target. The UAVs may be loaded with a sensor package or a strike package, and act as independent sensors. Upon reaching the enemy the commander must make a decision on whether to fire or not. The outcomes of these decisions each have a utility as prescribed by a multiattribute utility copula.
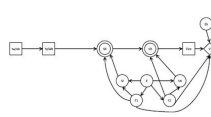
Multilinear Utility

Alternative Rank	Sensor UAV/Altitude	Strike UAV/Altitude
1	None	One/Low
2	One/Low	One/High
3	One/High	One/High
4	None	One/High
5	None	Two/High

Multiattribute Utility Copula

Alternative Rank	Sensor UAV/Altitude	Strike UAV/Altitude
1	None	One/High
2	One/High	One/High
3	One/Low	One/High
4	None	One/Low
5	None	Two/High

Assumptions of utility independence (as is typically done) does not provide the correct answer. Since the multilinear utility requires the assumption of conditional independence among attributes, the resulting utility on one attribute would not update the utility of another. The copula form allows for a mutual utility dependence structure, which more accurately reflects the decision maker's preferences.

Sensitivity Analysis
A sensitivity analysis is performed on all assessed parameters showing a robust decision model .

Conclusions
This research can be used to inform decision makers of the impact of their risk attitudes and value tradeoffs to problems involving military decision analysis, and alter the tactics, techniques, and procedures associated with these decisions.

Multiattribute Utility Copulas

Functional Form

$$C(v_x, v_y) = -a\frac{1}{\delta}\ln\left(1 - e^{-\delta\left(1 - \frac{\left(1 - e^{-\delta(I_x + (1-I_x)v_x)}\right)\left(1 - e^{-\delta(I_y + (1-I_y)v_y)}\right)}{1 - e^{-\delta}}\right)}\right) + b$$

Parameter Definitions
v_x – Normalized Utility function for x
v_y – Normalized Utility function for y
Parameters solved using the following system of equations
$$a(1 - I_x) = 1 - U(x^0, y^s)$$
$$a(1 - I_y) = 1 - U(x^s, y^0)$$
$$a = \frac{1}{1 + (1/\delta)\ln(1 - (1 - e^{-\delta})(1 - e^{-\delta\nu})/(1 - e^{-\delta})}$$
$$b = a\frac{1}{\delta}\ln\left(1 - \frac{(1 - e^{-\delta})(1 - e^{-\delta\nu})}{(1 - e^{-\delta})}\right)$$
ρ – Ranked correlation coefficient, used to solve for δ
$$\rho(\alpha) = \frac{\alpha^2\ln(\alpha) - \alpha + 1}{\left(\alpha^2 - 1\right)}$$
$$\delta = -\ln\alpha$$

Multiattribute Utility Function
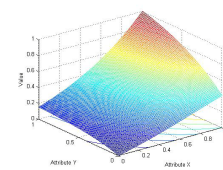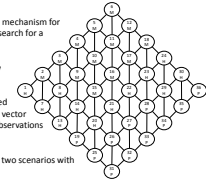

Isopreference Curves
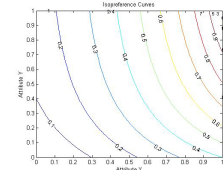
Moving Enemy Model Formulation

Area Map
This research uses the concept of terrain as an arbitrary mechanism for providing benefits and costs to UAV operations as they search for a moving enemy.

Enemy movement is modeled as a discrete time markov chain through the hexagonal grid shown.

As the enemy moves through the area, location is tracked through a probability vector. The use of this probability vector allows for Bayesian updating of the location based on observations of the enemy.

The commander must plan a flight path for two UAVs in two scenarios with random and goal oriented enemies.

Genetic Algorithm
A genetic algorithm is developed to search the decision space for the alternative with the highest utility. The location of each UAV at a specific time are represented as genes along the chromosome that represents the paths of both UAVs. Crossover and Mutation operations are developed to speed the convergence.

Scenario 1
This scenario involved the search for an enemy with random movement, starting in node 21. Movement was defined using ratios with movement to mountains being most likely followed by hills and plains.

Scenario 2
This scenario involved the search for an enemy with goal oriented movement, starting in node 31, the enemy is attempting to reach node 6. Movement is equally likely to any of the nodes above the current location.

Results
UAV 1 visits nodes 16, 21, 15, and 15
UAV 2 visits nodes 22, 20, 21, and 21

Results
UAV 1 visits nodes 27, 14,and 16
UAV 2 visits nodes 26, 15,and 22

Conclusions

This research provides an original approach to the UAV search and destroy problem by employing multattribute utility theory.

Future Work

Increase the number of UAVs actively searching the decision space

Add anti-air defense, enemy safe zones, structures, or other terrain features to the area map.

Applications

This research can be applied to several problems including the search for a downed pilot, long term attack scenarios, and cyber warfare.

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14. ABSTRACT
The multiattribute utility copula is an emerging form of utility function used by decision analysts to study decisions with dependent attributes. Failure to properly address attribute dependence may cause errors in selecting the optimal policy. This research examines two scenarios of interest to the modern warfighter. The first scenario employs a utility copula to determine the type, quantity, and altitude of UAVs to be sent to strike a stationary target. The second scenario employs a utility copula to examine the impact of attribute dependence on the optimal routing of UAVs in a contested operational environment when performing a search and destroy mission against a Markovian target. Routing decisions involve a tradeoff between risk of UAV exposure to the enemy and the ability to strike the target. This research informs decision makers and analysts with respect to the tactics, techniques, and procedures employed in UAV search and destroy missions. An ever increasing UAV operations tempo suggests such research becoming increasingly relevant to the warfighter.

15. SUBJECT TERMS
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