LARGE DROPLET IMPACT ON WATER LAYERS

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Abstract

The impact of large droplets onto an otherwise undisturbed layer of water is considered. The work, which is motivated primarily with regard to aircraft icing, is to try and help understand the role of splashing on the formation of ice on a wing, in particular for large droplets where splash appears to have a significant effect. Analytical and numerical approaches are used to investigate a single droplet impact onto a water layer. The flow for small times after impact is determined analytically, for both direct and oblique impacts. The impact is also examined numerically using the volume of fluid (VOF) method. At small times there are promising comparisons between the numerical results, the analytical solution and experimental work capturing the ejector sheet. At larger times there is qualitative agreement with experiments and related simulations. Various cases are considered, varying the droplet size to layer depth ratio, including surface roughness, droplet distortion and air effects. The amount of fluid splashed by such an impact is examined and is found to increase with droplet size and to be significantly influenced by surface roughness. The makeup of the splash is also considered, tracking the incoming fluid, and the splash is found to consist mostly of fluid originating in the layer.

Introduction

The high-speed impact of a single water droplet onto a previously undisturbed layer of water has a range of applications for example in the chocolate, spray-coating and aeronautics industries but in particular with regard to aircraft icing. For larger droplets splashing is thought to have a considerable influence on the amount of water collected on an aerofoil and therefore a great effect on the shape and quantity of ice produced. Despite this physical importance there has been relatively little previous work on droplet impact at high Reynolds number. Early interest in splashing and impact problems appeared in Worthington[1] whose book includes many images of splashing after either a droplet or a solid sphere impacts upon a fluid layer. To date, there has been relatively little previous direct theoretical input and suitable physical modeling on droplet impact, in particular concerning mass and consequent heat transfer and the relationship between input and rebound droplets. However, much work has been done on related aspects both analytically, for example Korobkin and Pukhnachov,[2] Howison et al.[3] who consider solutions at small times after impact (mostly for solid-water impacts), and numerically such as Weiss and Yarin[4] who use a Lagrangian-type approach to examine an inviscid droplet impact numerically. Much work has been done by Josserand and Zaleski[5] and references therein who have developed powerful three-dimensional techniques for capturing droplet impact but tend to examine impacts with Reynolds numbers one or two orders of magnitude less than is typical in an icing context. The motivation of the current work is to help to enhance the understanding of the influence of splashing on the formation of ice on a wing, in particular for super-cooled large droplets where splash is believed to have a significant effect.

Given that experiments isolating single droplet impacts and measuring overall splashed volume are difficult to perform, it seems desirable to develop a mathematical model which can describe the process and help guide predictions of mass loss due to splashing. Our approach is to start with a simple model, initially neglecting viscosity (the typical Reynolds number is large in the current practical regime, $Re \sim O(10^5)$), neglecting surface tension (high Weber number, $We \sim O(10^5)$), and neglecting the influence of air and pre-existing flow in order to concentrate on trends in the splashed water as the droplet size to layer depth ratio changes. The idea is to then include other relevant physical effects subsequently, such as those mentioned above as well as ice-surface roughness, air-cushioning, compressibility and oblique impacts. Analytical and computational approaches are used to investigate a single droplet impact onto a water layer. The flow for small times after impact is determined analytically, for direct and oblique

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impact and for very shallow layer depths. The impact is also examined numerically using the VOF (volume of fluid) method, initially treating the fluid as inviscid and incompressible. At small times there are promising comparisons with the analytical solution and with experimental work capturing the ejecta sheet. At larger times there is qualitative agreement with experiments and with related simulations.
The method is used to tackle various cases, such as altering the layer depth to droplet diameter ratio, the influence of surface tension and oblique impact, focusing on their effects on the form of splash produced, geometry of the crown formed and make-up of the ejected droplets. In particular the amount of rebounded fluid is examined. Although thermal effects are not included in the current work emphasis is also placed upon the exchange of fluid, tracking the pre-existing (in practice warmer) layer fluid and the (colder) incoming droplet fluid and considering the proportions of each in the splash. This exchange can have a substantial influence on the overall temperature of the water layer. Again the presence of an ice shape beneath the water layer is modelled and its effects on the rebound and the constituents of the ejected fluid are explored. Other aspects include the influence of an air layer, in particular pre-impact air cushioning and pre-existing airflow, and the influence of viscosity and compressibility.
In the first instance, then, we consider a single droplet impacting directly upon an otherwise undisturbed layer of water. We treat the behaviour as inviscid and water-only as a first step. The basic set-up is shown in figure 1. The Cartesian coordinates x,y, corresponding velocity components u, v and pressure p used here are non-dimensionalized with respect to a typical layer depth H_D (or typical droplet diameter D_D) and incoming droplet velocity v_D. The Reynolds number $Re \equiv v_D H_D/\nu_D$ is large, where ν_D is the kinematic viscosity of the fluid. Values for a typical icing situation are of the order $v_D \sim 100m/s$, $H_D \sim 30\mu m$ and droplet diameters ranging from $D_D \sim 40\mu m - 400\mu m$ for the large droplets of interest here, although conditions can vary dramatically through different stages and types of icing. The governing equations are the non-dimensionalized, two-dimensional, unsteady Navier-Stokes equations, namely

$$\begin{aligned} u_x + v_y &= 0, & (1)\\ u_t + uu_x + vu_y &= -p_x + \frac{1}{Re}\nabla^2 u, & (2)\\ v_t + uv_x + vv_y &= -p_y + \frac{1}{Re}\nabla^2 v; & (3) \end{aligned}$$

however we shall mostly consider inviscid flows in the current paper and so the viscous terms on the right-hand sides of (2) and (3) will be assumed negligible. We proceed by developing a mathematical solution for small times after impact and use this either as a starting point for a numerical scheme to consider $O(1)$ times or as a validation on a numerical scheme that can handle the change in topology as the two distinct bodies of fluid (droplet and layer) coalesce and become one. This paper outlines the small-time post-impact solution, describes the numerical scheme used for resolving the entire droplet impact at $O(1)$ times and presents samples of the trends and results.

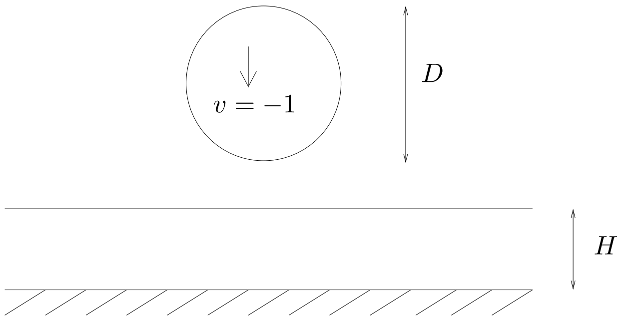

Fig. 1 The basic setup. A droplet of diameter D with incoming normalised velocity $v = -1$ impacts upon a layer of depth H.

The numerical method

The numerical method we have adopted is a volume of fluid (VOF) approach. One of the major difficulties with solving the current problem numerically is coping with the change in topology as the droplet enters the layer and secondary droplets are subsequently ejected. The VOF method can in principle handle such surface reconnection and breakup without any special treatment or catastrophic break down and so was thought suitable for our use. The method was first introduced by DeBar,[6] Hirt and Nichols[7] and Noh[8] and has been honed and improved upon since then by several authors, for example Rider and Kothe,[9] and in particular by Gueyffier et al.[10] who have applied it to droplet impacts. We shall give a brief overview here.
The main idea behind the VOF method is to track the position of the interface by use of a function F which represents the fraction of a given grid cell that is filled with fluid. In other words we introduce a function F which takes the values

$$F = \begin{cases} 1, & \text{if cell is full of fluid} \\ 0, & \text{if cell is empty} \\ \text{or} & \text{fraction of cell containing fluid} \end{cases} \qquad (4)$$

in each grid cell. As such we have $0 < F < 1$ in cells containing the free surface. This principle is easily extended to two fluids by regarding $F = 1$

as referring to fluid one and $F = 0$ representing a cell full of fluid two. To ensure the free-surface moves with the fluid the function F satisfies the evolution equation

$$\frac{\partial F}{\partial t} + \mathbf{u}.\nabla F = 0, \tag{5}$$

where $\mathbf{u}$ is the velocity vector determined from solving the flow in the main body of the fluid. Introducing F allows us to keep track of the free surface and follow it in time but in doing so we lose our knowledge of the exact position of the free surface; this needs to be inferred from the volume fractions. The reconstruction of the interface can be performed in a variety of ways, from the straightforward step-stair or SLIC methods,[7] which forces the free-surface to align with the x and y coordinates in each grid cell, to one of various piecewise linear (PLIC) methods. The latter constructs the interface by estimating the normal vector to the actual free surface in each grid cell and reconstructing the interface as the straight line which, with the same normal, encloses the correct amount of fluid F. The PLIC method gives much better resolution than the simple but relatively crude SLIC methods. See Rider and Kothe[9] for further details and comparisons. Equations (1)-(3) govern the flow in the fluid, subject to carefully applied boundary conditions at the free surface, and is solved using the SIMPLE algorithm. Surface tension is included using an approach called the continuous surface force (CSF) method (see Scardvelli and Zaleski[11] and references therein for details). Throughout this paper the Weber number is taken to be $We = 10^5$ which is fairly representative for practical icing situations.

Small-time solution

As discussed earlier we have developed a mathematical solution to the problem for small times after impact which can be used either as a check/validation or a starting point for the numerical scheme. We shall give brief details here. If we consider a region where x, y are $O(t^{\frac{1}{2}})$ around the neck of the impact, expand the velocities, pressure and free-surface as

$$[u, v, p, f] \sim [u, v, t^{-\frac{1}{2}}p, tf] \tag{6}$$

and substitute into the governing Euler equations, we obtain the Cauchy-Riemann equations for f_{tt} and $-p_x$ (extended into the $x-y$ plane) namely

$$f^{\pm}_{ttx} = -p^{\pm}_{xy}, \tag{7}$$
$$f^{\pm}_{tty} = p^{\pm}_{xx}, \tag{8}$$

where $\pm$ indicate quantities above (in the drop) and below (in the layer). See figure 2.

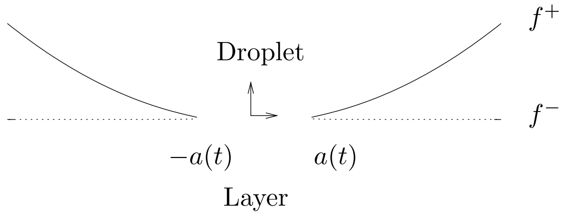

Fig. 2 Small time after impact problem in the neck region of the impact. + denotes quantities in the drop and − quantities in the layer.

These must be solved subject to the boundary conditions

$$f^+ \sim x^2 - t, \quad f^- \to 0 \quad \text{as} \quad |x| \to \infty, y = 0, \tag{9}$$

to match with the circular droplet impacting with velocity -1 above and with the undisturbed horizontal layer below in the outer $O(1)$ region, and

$$p^+ = p^- = 0 \quad \text{for} \quad |x| > a(t), y = 0, \tag{10}$$

on the free surface as surface tension has no effect to leading order on this scale. We also require pressure continuity and identical free surface values in the middle, so that

$$p^+ = p^-, \quad f^+ = f^-, \quad \text{for} \quad |x| < a(t). \tag{11}$$

Finally the contact points $x = \pm a(t)$ where the above and below free surfaces meet is also unknown in advance and must be determined as part of the solution.
The problem now is equivalent to a mixed boundary condition one along the real axis and using a complex analysis method leads to the solutions

$$f^{\pm} = \pm\frac{1}{2}|x|(x^2 - 2t)^{\frac{1}{2}} + \frac{x^2}{2} - \frac{t}{2}, \quad x^2 > 2t, \tag{12}$$

$$f^{\pm} = \frac{x^2}{2} - \frac{t}{2}, \quad x^2 < 2t. \tag{13}$$

As can be seen the contact points are found to be at $x = \pm(2t)^{\frac{1}{2}}$. The small-time solution (12), (13) is shown in figure 3 for two times; as time increases the point in the neck region where the free-surfaces meet moves outwards and upwards.
To smooth out the square-root behaviour exhibited by (12) and the inverse root pressure p near the contact point we must consider a smaller region where $x - \sqrt{2t} = t^{\frac{3}{2}}\bar{x}$. We omit the details of this inner region here (see Purvis and Smith[12] for a complete description. A similar problem in a different, solid-water context has previously been considered[3]). This subsequent region demonstrates that the root behaviour is smoothed out locally by an initial fast moving horizontal jet forming in the neck region (e.g. where the three lines meet in figure 3, for each case (a), (b) there).

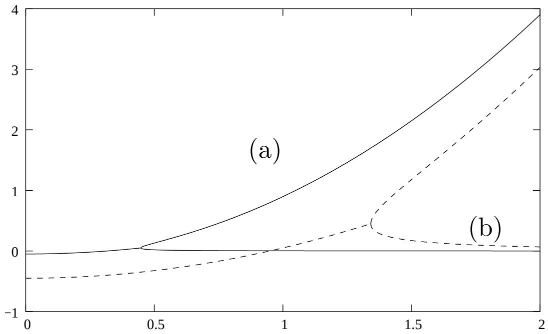

Fig. 3 The small time solution in the neck region showing f^+ and f^- at (a) t=0.1 (solid) and (b) t=0.9 (dashed).

The solution is valid for both direct impacts and impacts at an angle. In the latter case the speed of the contact point is generally large compared to the horizontal velocity of the droplet and the small-time solution is unaffected to leading order.

Computational results

Basic cases

We have applied the method described earlier to a wide variety of cases, a snapshot of which we include here. Figures 4-7 show the free-surface shapes at four times (t=0.5,1.5,3.5,5.5) for different values of droplet diameters $D = 0.125, 0.5, 1, 4$ with the layer depth remaining constant, $H = 0.5$. The droplet starts from a height above the layer such that in each case impact occurs when t=0.1. Notice the behaviour at small times remains largely unchanged; after the initial impact fast moving horizontal jets appear in the neck region. These then re-impinge and upward jets (the birth of the corona or crown) appear from the layer. This initial jetting is interesting and appears to be a continuation of the jetting found in the small-time post-impact solution discussed earlier. In fact the VOF method seems to capture the small-time solution reasonably well and results from the numerical method with a droplet started before impact and one started from our small-time solution are indistinguishable.

Further validation of the accuracy, at least qualitatively, comes from comparison with the experimental work of Thoroddson.[13] He examines a single droplet impacting upon an undisturbed water layer, and in particular visualises the ejecta sheet that forms at small time. The comparisons are favourable both with the jetting motion and the reconnection with the layer.

Obviously the overall amount of splash produced in a realistic icing situation is dependent upon many factors excluded here, not least three-dimensional effects and the presence of a strong airflow in practice. In the present work however

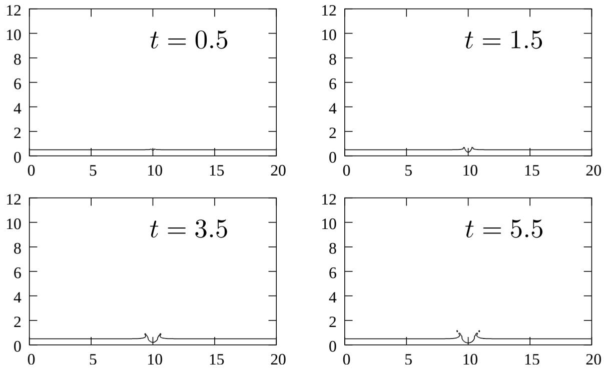

Fig. 4 Free-surface shapes at non-dimensional times t=0.5,1.5,3.5,5.5 for a droplet diameter to layer depth ratio $D/H = 0.25$.

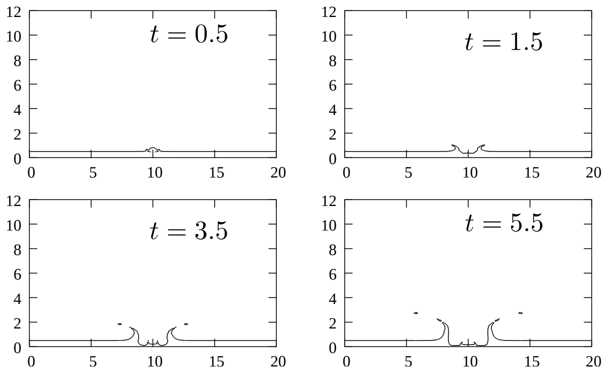

Fig. 5 Free-surface shapes at non-dimensional times t=0.5,1.5,3.5,5.5 for a droplet diameter to layer depth ratio $D/H = 1$.

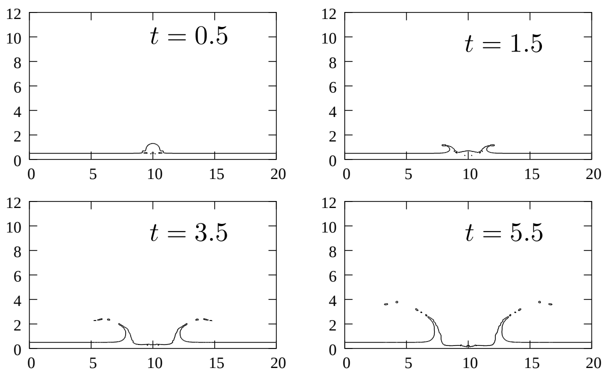

Fig. 6 Free-surface shapes at non-dimensional times t=0.5,1.5,3.5,5.5 for a droplet diameter to layer depth ratio $D/H = 2$.

we try to gain some appreciation for the overall trends in droplet splashing from our simplified model. To do this we consider two measures of the splash for various ratios of droplet diameter to layer depth D/H, namely the maximum height of the splash and the volume of the splash or how much fluid is ejected above a given threshold. The amount of splash is shown in figure 8. It measures the sum of the function F, multiplied

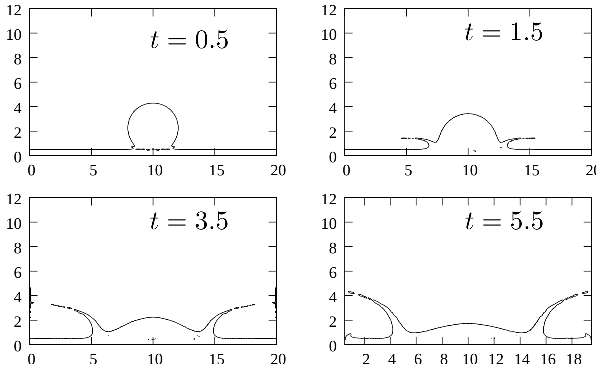

Fig. 7 Free-surface shapes at non-dimensional times t=0.5,1.5,3.5,5.5 for a droplet diameter to layer depth ratio $D/H = 8$.

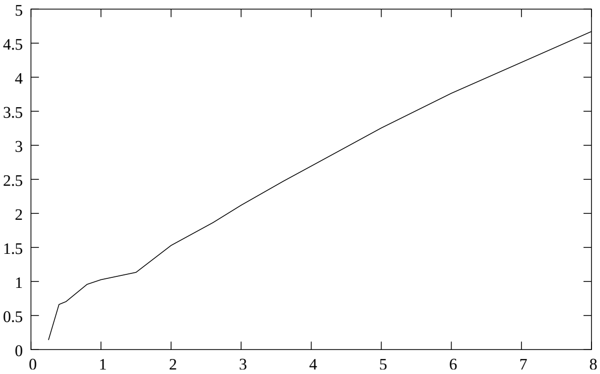

Fig. 8 Amount of fluid ejected above the original layer against D/H ratio at non-dimensional time t=0.5.

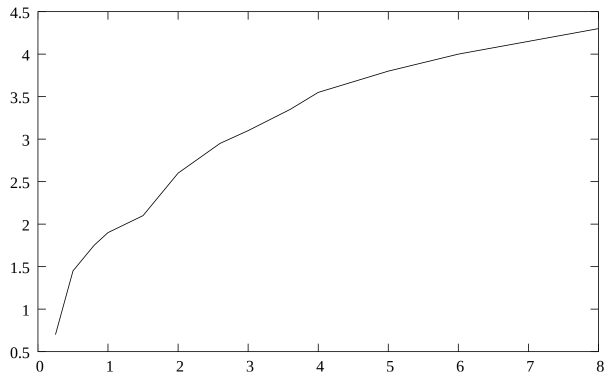

Fig. 9 Maximum height attained by fluid ejected above the original layer against D/H ratio at non-dimensional time t=5.

by the size of the grid cell, in grid cells above the original layer height which have positive velocity v (so as to exclude fluid in the incoming droplet). Notice that there is very little splash for $D/H < 0.5$, the quantities then seem to plateau to some extent before another change at roughly $D/H = 1.5$. Figure 9 shows the maximum height obtained by the splash, which is taken to be the highest droplet rather than the top of the crown. Once again similar trends appear with distinct changes at around the same D/H ratios as in the volume ejected case. These results certainly suggest that for larger droplets splash becomes an important consideration, with droplets that are smaller than the layer depth producing little if any splashing whearas the largest droplets examined here are creating significant splashes. Of course in reality not all fluid that gets splashed above the free-surface will be ejected and not return. Over longer times surface tension will pull much of the crown back into the layer. Also in reality other effects such as air flow will influence what escapes. Ejected droplets may also escape completely or reimpinge at another location on the layer (possibly with another splash although the droplets then will be small and so subsequent splashes might be insignificant). However, the current model still acts as a potentially useful guide to the overall trends.

Make-up of the splash

Another issue we address is the origin of the fluid contained in the splash. Was it originally in the droplet or in the layer? This can be important in terms of thermal effects where a typical droplet is considerably cooler than the layer fluid and, depending on the make-up of the splash, could have a significant influence on the rate of ice formation and overall layer temperature. Assuming there is no mixing of the two fluids we can track the two fluids separately using the VOF approach discussed earlier by using a function F_1, defined as F but for just the droplet fluid. Using this approach we can see that the initial horizontal jet consists almost exclusively of layer fluid, a property which again compares well experimental results;[13] see our figure 10 which shows the initial jetting and the separate fluids. Figure 11 shows a typical example of the free surface and constituent parts with different droplet sizes. The original droplet fluid tends to form a pool near the base of the impact with relatively little droplet fluid entering the crown, certainly in the smaller droplet examples. Figure 12 compares the volume ejected into the splash with that originating in the layer and the droplet. For $D/H < 1$ almost all of the ejected fluid originates in the layer. Significant amounts from the droplet start to increase for $D/H > 1.5$. In all the cases considered so far the majority comes from the layer, with a roughly $2 : 1$ split in the $D/H = 8$ case.

Surface roughness

The next issue of likely importance is ice surface roughness. In practical applications the surface beneath the layer is not flat as in the cases considered so far. In reality it will exhibit roughness

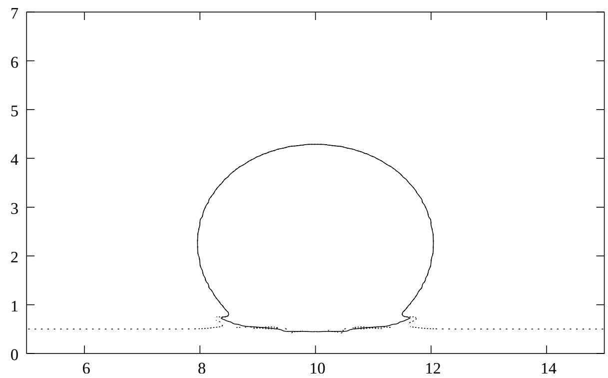

Fig. 10 Small time jetting for $D/H = 8$ showing the free-surface and the divide between droplet and layer fluid.

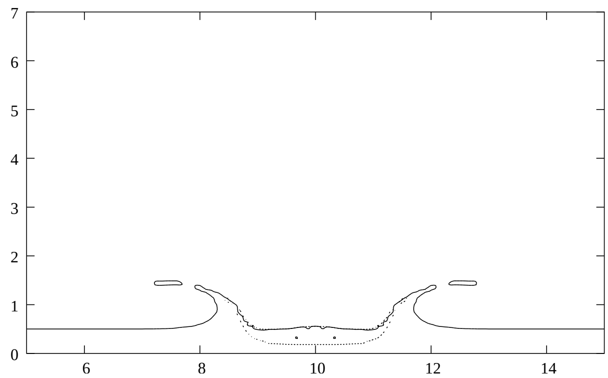

Fig. 11 An example of tracking droplet fluid for a case $D/H = 2$ at time t=2. Note the pool of droplet fluid and that little has entered the crown.

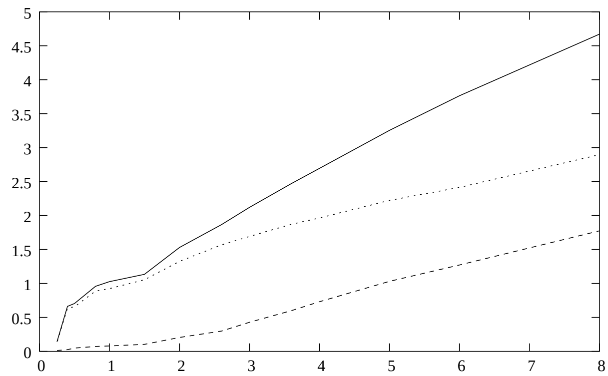

Fig. 12 Volume ejected against D/H ratio at time t=5. The graph shows the total fluid ejected (top line) and the amounts that originated in the layer (middle) and the droplet (bottom).

on various scales both of the order of the layer depth and considerably larger. In an attempt to obtain guidance as to the importance this might have, we have considered several idealized cases of surface roughness to investigate their overall effect on the splashing trends seen earlier. The simple model we adopt is to include rectangular blocks either side of the point of impact with varying heights. The presence of such blocks tends to reduce the size of, and to some extent the angle of, the crown formed with the overall effect of reducing the volume of liquid ejected. The reduction only becomes a significant amount when the height of the bumps approached the layer depth. If the blocks are placed further apart the overall effect is diminished, as might be expected. See figure 13, which shows a sample of the crown shape for various block heights with the block separated by a single droplet diameter, and figure 14 which shows the same but with a $4D$ gap in between. Figure 15 compares the ejected amounts of fluid. For larger droplets the same trends can be seen.

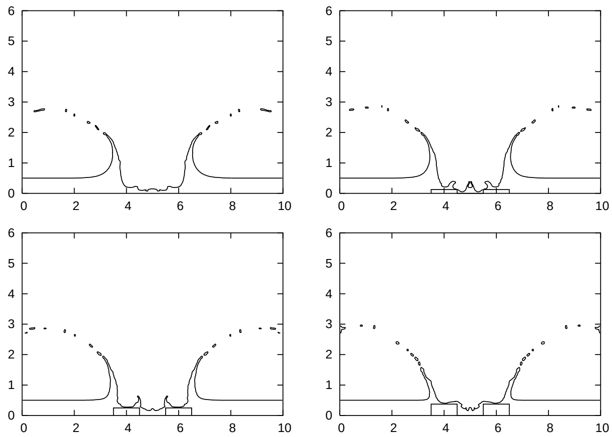

Fig. 13 Free-surface shapes at time t=4 for cases with surface roughness with the blocks having heights 0,0.125,0.25,0.375 respectively. The ratio $D/H = 1$ here.

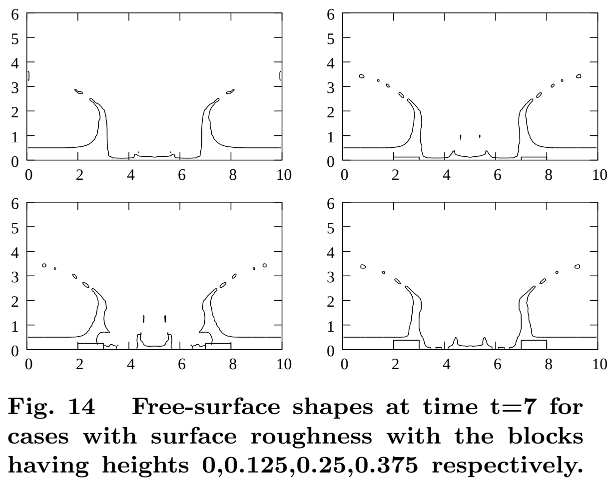

Fig. 14 Free-surface shapes at time t=7 for cases with surface roughness with the blocks having heights 0,0.125,0.25,0.375 respectively. The blocks are further apart here than in figure 13 but H/D is again 1.

The overall constitution of the splash is largely unchanged and most of the reduction comes from the layer. It should also be noted that, even for high

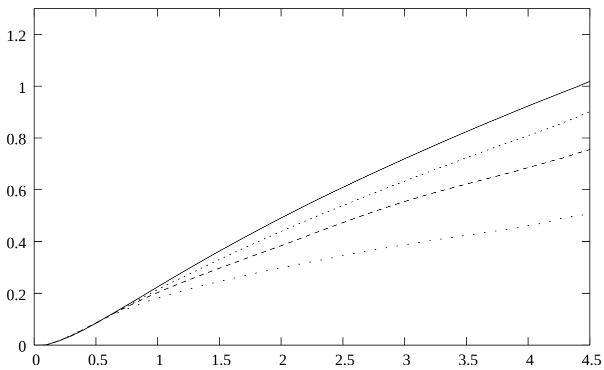

Fig. 15 The amount of fluid ejected for cases in figure 13. The lines, from top to bottom, represent the bumps of height 0,0.125,0.25,0.375.

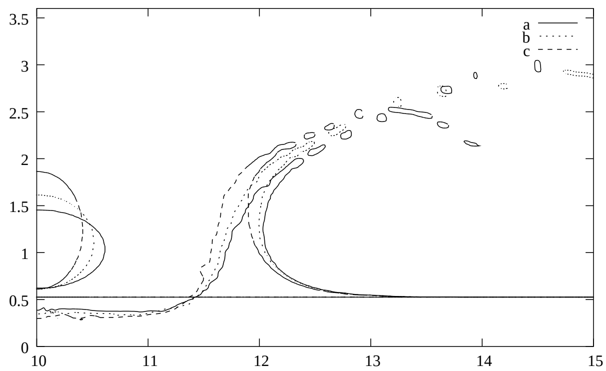

Fig. 16 Free-surface shapes for deformed droplets at time t=3. The cases shown are (a) stretched in x direction, (b) circular droplet, (c) stretched in y-direction.

blocks, the small-time solution remains unchanged as the blocks do not impede upon the small $O(t^{\frac{1}{2}})$ region discussed earlier. High blocks placed closer to the impact point could enter this region and change the small-time jetting behaviour. A similar situation is found if the layer depth is very small since then the small time problem includes the wall layer and to leading order it is as if the droplet is hitting the wall as the thin layer is unable to react quickly enough to the incoming droplet.[12]

Deformed droplets

There is also interest in whether real droplets are spherical on impact or instead have deformed significantly on approaching the aerofoil. To examine if any potential deformation could be significant we have run several cases where the incoming droplet is an oval rather than a circle (in our two-dimensional context). Each droplet has the same area and incoming velocity, and therefore momentum, but in one case the droplet is stretched along the x-axis and in the other along the y-axis. Figure 16 shows the free-surface shapes for an undeformed droplet and two stretched droplets. The crown shape formed by such impacts is slightly different in each case but the differences are not particularly significant and comparison of the amount of ejected fluid as shown in figure 17 would appear to confirm that a slightly deformed initial droplet shape has relatively little impact on the overall longer term behaviour.

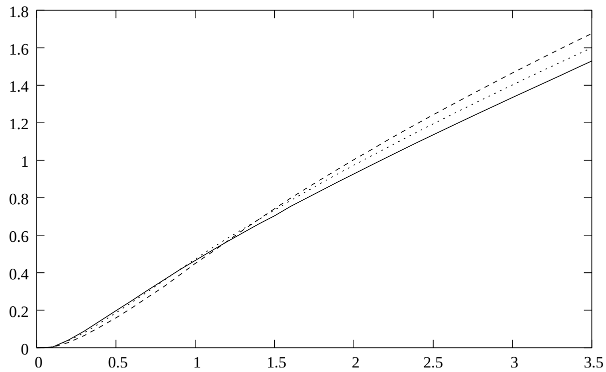

Fig. 17 Amount of fluid ejected against time for deformed droplets. The cases shown are (dashed) stretched in x direction, (dotted) circular droplet, (solid) stretched in y-direction

Air effects

The numerical method has also been modified to include air effects. So far this has mostly been for the inviscid regime, as in the example in figure 18 . Air effects become significant near impact despite the small air-to-water ratio of of densities. Similar phenomena are predicted analytically in related works Smith et al.,[14] Purvis and Smith[12,15] which incorporate also the ratio of viscosities through part of the viscous terms in (2), (3) and numerical studies by us on the inclusion of the full viscous terms are under way. These studies aim to capture both air-cushioning and pre-existing-flow effects.

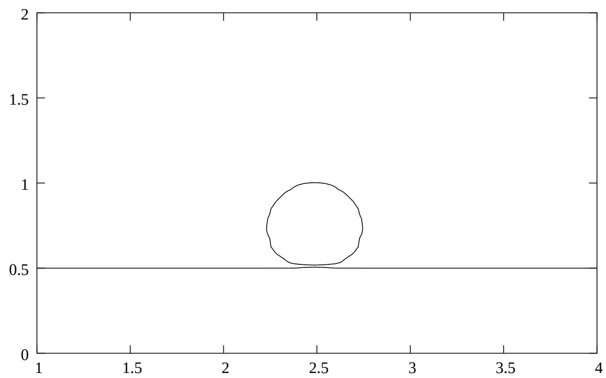

Fig. 18 Free surface shape just before impact when the effects of air are included. Note the deformation of the droplet and layer due to air cushioning effects.

Further comments

The impact of a single droplet onto a layer of fluid has been examined for various droplet sizes. This appears to confirm that larger droplets produce more splash. The make-up of the ejected fluid was also considered. The crown formed by a droplet impact consists mostly of layer fluid being splashed away rather than fluid originating in the droplet, although an increasingly large percentage comes from the drop as the ratio of droplet diameter to layer depth is increased. Also examined were the influence of surface roughness which, in our idealised model, does have a significant effect on the amount of fluid ejected especially when the height of the roughness is comparable with the depth of the layer. The initial droplet shape was shown to be less crucial, with slight deformations making little difference to the overall solution. Finally pre-impact cushioning has been observed in cases where the influence of air is included.

There are many other issues that need to be addressed. These include the influence of viscosity, a more detailed examination of the influence of air layers, pre-existing fluid flow in the air, in the water or in both. Also of interest would be three-dimensional effects both in the small-time solution and in the full volume of fluid problem.

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