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# On the Beliefs off the Path: Equilibrium Refinement due to Quantal Response and Level- $k$

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### Abstract

This paper studies the relevance of equilibrium and nonequilibrium explanations of behavior, with respects to equilibrium refinement, as players gain experience. We investigate this experimentally using an incomplete information sequential move game with heterogeneous preferences and multiple perfect equilibria. Only the limit point of quantal response (the limiting logit equilibrium), and alternatively that of level- $k$  reasoning (extensive form rationalizability), restricts beliefs off the equilibrium path. Both concepts converge to the same unique equilibrium, but the predictions differ prior to convergence. We show that with experience of repeated play in relatively constant environments, subjects approach equilibrium via the quantal response learning path. With experience spanning also across relatively novel environments, though, level- $k$  reasoning tends to dominate.

### JEL classification codes

C72, C91, D62

### Keywords

Incomplete information, equilibrium refinement, logit equilibrium, rationalizability, quantal response, level- $k$ , inequity aversion, experiment



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# 1 Introduction

Leading contemporary theories of bounded rationality explain behavior in strategic interactions by specifically modeling both noisiness of payoff maximization and formation of beliefs about opponents' strategies. In quantal response equilibrium (QRE) models (see McKelvey and Palfrey, 1995, 1998), players correctly anticipate their co-players' strategies but they respond noisily. Nonequilibrium models, such as level- $k$  reasoning, e.g. Stahl and Wilson (1994), Nagel (1995), Ho et al. (1998), Costa-Gomes et al. (2001), or cognitive hierarchy, see Camerer et al. (2004) and Rogers et al. (2009), additionally model how players form conjectures about their co-players' strategies and thereby deviate from equilibrium. In these models, different player types have different levels of reasoning, and players respond to the belief that their co-players are lower level types. Much of this literature focuses on and succeeds in explaining initial play in novel situations where equilibration is least likely. For example, Crawford and Iriberri (2007) show that the initial behavior in the auction experiment of Goeree et al. (2002) is better explained by level- $k$  than QRE. Kübler and Weizsäcker (2004) find a similar result in their analysis of information cascades.

Our study investigates the complementary relevance of QRE and level- $k$  reasoning as players gain experience. The analysis is based on the tenet that experience gained from initial play, which is not in equilibrium, shapes beliefs in “eventual play” along the approach to equilibrium; the experience gained from “eventual play” shapes beliefs in equilibrium. Besides applying concepts such as QRE and level- $k$  to explain actions and beliefs during initial play, we apply them to explain actions and beliefs in eventual play, and by extrapolating the individual transitions from level- $k$  reasoning to QRE reasoning and vice versa, we can tell which model most suitably guides belief restriction in equilibrium. We investigate this in a game where the limit points of both QRE and level- $k$  uniquely restrict the beliefs that players hold in equilibrium, while other established refinement concepts such as perfection (Selten, 1975), properness (Myerson, 1978), and sequentiality (Kreps and Wilson, 1982) induce a multiplicity of equilibria and hence are insufficient to explain our observations.<sup>1</sup>

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<sup>1</sup>Both limiting logit equilibria and perfect equilibria are obtained as limit points when “bounded rationality” disappears, but in our context their respective equilibrium predictions are differently precise. This difference occurs, because perfection is not based on a specific model of bounded rationality, i.e. because trembles are unrestricted under perfection. QRE models the form of “bounded rationality” explicitly, which our experimental design exploits.

The game is an extension of the club game described by Dixit (2003).<sup>2</sup> The players have heterogeneous preferences for joining a club, and the payoffs are supermodular in that one's incentive to join the club increases with the number of other players who join the club. Each player moves exactly once, and Dixit assumed complete information on the exogenously randomized order of moves. This game is dominance solvable under mild conditions, e.g. if player 1 strictly prefers joining in general, if player 2 is best off joining if at least one opponent joins, if player 3 is best off joining if at least two opponents join, and so on. In any such case all players will join the club—even if the majority is better off when no club forms at all.<sup>3</sup>

Our study extends Dixit's game by introducing incomplete information on the order of moves. A player knows his position in the order of moves and the actions of players in preceding moves, but does not know which co-player has moved or is to move. The equilibrium outcome is uniquely characterized if players obey sequential rationality and Bayes' Rule whenever possible—all players join the club. The equilibrium strategies, however, are not uniquely characterized under these assumptions. In particular, beliefs and actions are unrestricted in information sets where some co-player had deviated. Neither sequentially rational nor perfect (or proper) players can uniquely derive "who did not join" from the definition of the game. In their eyes, it is "unclear" which co-player/s have deviated.

By intuition this is very clear, however. Players with weaker incentives to join are more likely to tremble and hence they deviate from "all join" with higher probability. The limit point of QRE (limiting logit equilibrium, LLE) and the limit point of level- $k$  reasoning (extensive form rationalizability, EFR) both restrict relative tremble probabilities between players and uniquely induce these beliefs. By the construct of this game, both of these explanations for deviations from equilibrium are therefore ex ante equally intuitive candidate explanations for behavior—even those close to

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<sup>2</sup>The club game models peer effects due to social ties. "Joining a club" can be interpreted as a metaphor for joining in an activity, e.g. riots (Schelling, 1960) or the usage of alcohol and drugs (Gaviria and Raphael, 2001), or even "ethnification", i.e. adopting a marker for identifying membership to a certain ethnic group such as by wearing a certain color of hat (Kuran, 1998). In our investigation of equilibrium refinement, we focus on it as an abstract model of such peer effects.

<sup>3</sup>Player 1's strategy is fully characterized after one iteration of eliminating dominating strategies, 2's strategy is so after two iterations, and so on. All games that are dominance solvable in this manner have unique extensive-form rationalizable strategy profiles under incomplete information on the order of moves, as explained below, and can be used as the basis of an analysis similar to ours.

equilibrium—while the predictions of most other concepts differ. In our experiment, we will observe whether subjects first learn to predict the co-players’ strategies accurately or first learn to maximize their payoffs accurately. In the former case, bounded rationality close to equilibrium is shaped by noisy maximization (the QRE path toward equilibrium), and in the latter case it is shaped by level- $k$  reasoning (the level- $k$  path).

The incomplete information variant of the club game thus allows us to ascertain if subjects approach equilibrium via the QRE or the level- $k$  learning path as they gain experience. Our paper introduces a framework to examine this point econometrically and derives rather conclusive results (with respect to the club game). Our analysis is based on “mixture-of-types” models, see Stahl and Wilson (1994, 1995) or Crawford and Iriberri (2007), where level- $k$  and equilibrium types co-exist in the population. First, the maximum likelihood estimates of the model parameters, e.g. population shares of the various types, are determined. Thanks to Bayes’ Rule we can then classify the subjects as level- $k$  and QRE types individually. By tracing the individual type transitions across different periods of the experiment, we estimate the subjects’ learning path (in an autoregressive multinomial logit regression model) and the fixed point of the learning path shows us where behavior converges as subjects gain experience.

On a related note, there is a concern for how asymmetric welfare distributions in games such as the one we study can influence behavior. The club favors some but not others, and for this reason our analysis controls for inequity aversion, see Fehr and Schmidt (1999), which constitutes an alternative explanation for deviations from the equilibrium path. Specific player types might refuse to join simply because the outcome that results from equilibrium play is inequitable. To our knowledge, the present study is novel in segregating level- $k$  and QRE types while controlling for social preferences. As it turns out, the initial behavior of the subjects does indeed exhibit significant traces of inequity aversion, but inequity aversion vanishes as subjects gain experience (i.e. the respective coefficient becomes insignificant).

The main results can be summarized as follows. Sequential rationality is not violated 90% of the time, and in particular in later stages of the experiment, the observations are “close to” equilibrium. The subjects that deviate from “joining” tend to be those with the weaker incentives to join, which relates to the results of Battalio et al. (2001) and Goeree et al. (2002), who report an “optimization premium” to a similar effect. While inequity aversion is by itself powerless in explaining the data, controlling for it improves explanatory power overall significantly, and since its relevance depends

on the subjects' level of experience, neglecting inequity aversion altogether biases the estimation of the individual learning paths drastically. According to the learning paths, subjects transit across level- $k$  and QRE types over time. The population tends to converge toward a steady state comprising mostly QRE player types if subjects gain experience by repeatedly playing the same game. That being said, the population tends to converge toward a steady state with most subjects being level-2 types if subjects also gain experience by being exposed to different experimental treatments as time goes on.

Section 2 sets up the model. Section 3 presents the equilibrium analysis. Section 4 describes and motivates the experimental games, procedure and logistics. Section 5 reports the experimental results. Section 6 analyzes the modes of reasoning. Section 7 discusses the results and Section 8 concludes.

## 2 The model

The set of players is  $N = \{1, \dots, n\}$ , with typical elements  $i, j \in N$ . In every information set belonging to player  $i$ , he has to decide whether to join the club ( $a_i = 1$ ) or not ( $a_i = 0$ ). The players move sequentially, as described below, and payoffs accrue when all players have moved. Player  $i$ 's eventual payoff from joining the club is denoted as  $p_i(1, k)$ , where  $k$  denotes the number of opponents that joined, and his respective payoff from not joining is denoted as  $p_i(0, k)$ . The payoffs are asymmetric between players, supermodular, see Eq. (1), and non-degenerate, see Eq. (2).

$$\forall i \in N \quad \forall k > l: \quad p_i(1, k) - p_i(0, k) > p_i(1, l) - p_i(0, l) \quad (1)$$

$$\forall i \in N \quad \forall k < n: \quad p_i(1, k) - p_i(0, k) \neq 0 \quad (2)$$

Without loss of generality, label players such that player 1 is most interested in joining the club, player 2 is second-most interested in doing so, and so on, as defined next.

$$\forall k < n \quad \forall i < j: \quad p_i(1, k) - p_i(0, k) > p_j(1, k) - p_j(0, k) \quad (3)$$

Now, to obtain a *club game* as considered here, assume that player 1 is generally best off joining the club, player 2 is best off joining *iff* at least one other player joins, player 3 is best off joining *iff* at least two other players join, and so on (*connectedness*).

$$\forall i \in N: \quad \min \{k \in \mathbb{N} \mid p_i(1, k) - p_i(0, k) > 0\} = i - 1 \quad (4)$$

The players move sequentially under a random order. Every player is informed on his own position in the move sequence and on the number of opponents that had chosen to join the club prior to his move, but on nothing else. The set of possible move sequences is denoted as  $\mathcal{R}$ , and any particular move sequence  $R \in \mathcal{R}$  is a one-to-one function  $R : N \rightarrow (1, \dots, n)$  where  $R(i)$  denotes the position of  $i \in N$ . The move sequence is chosen randomly by Nature according to a probability distribution  $P \in \Delta(\mathcal{R})$  which is common knowledge and assigns a positive probability to every  $R \in \mathcal{R}$ .

The remaining definitions are standard. Histories of actions are duples  $(R, h) \in \mathcal{R} \times H$  for  $H := \cup_{t=0}^4 \{0, 1\}^t$ , where  $R \in \mathcal{R}$  is the move of nature and  $h \in H$  is the possibly empty list of the decisions of the players who moved already. Information sets are denoted as triples  $(i, t, k)$ , where  $i$  is the player to move,  $t \in \{1, 2, 3, 4\}$  is his position in the move sequence, and  $k \in \{0, 1, \dots, t-1\}$  is the number of players who have joined already. Typical information sets are denoted as  $I \in \mathcal{I}$ , and the subset of  $I$  where player  $i$  is to move is  $I_i$ . The strategy of player  $i$  is a function  $\sigma_i : I_i \rightarrow [0, 1]$ , and  $\sigma_i(t, k)$  denotes the probability that  $i$  joins in the information set  $(i, t, k)$ .

Players cannot distinguish histories of actions that lead to the same information set. The belief of player  $i$  that the history  $(R, h)$  applies when he finds himself in the information set  $(i, t, k)$  is denoted as  $\mu_i(R, h|t, k)$ . Beliefs are updated according to Bayes' rule, and in our case they can be expressed as follows. Fix a strategy profile  $\sigma$ . If the probability that  $j \in N$  chooses action  $a_j \in \{0, 1\}$  in  $(j, t, k)$  is denoted as  $\sigma_j(t, k)(a_j)$  for the moment, the probability that history  $(R, h)$  results is (a priori)

$$\Pr(R, h) = P(R) * \prod_{t' < t} \sigma_{R^{-1}(t')} (t', \sum_{\tau < t'} h_\tau) (h_{t'}). \quad (5)$$

The belief system of  $i \in N$  satisfies Bayes' Rule if for all  $R \in \mathcal{R}$ ,  $t \in \{1, 2, 3, 4\}$ ,  $k < t$ , and all  $h \in \{0, 1\}^{t-1}$  such that  $\sum_{t' < t} h_{t'} = k$ ,

$$\mu_i(R, h|t, k) * \sum_{\tilde{R} \in \mathcal{R}} \Pr(\tilde{R}, h) = \Pr(R, h). \quad (6)$$

Given  $\sigma$ , let  $\pi_i(\sigma)$  denote the expected payoff of  $i \in N$ . In addition, let  $\pi_i(a_i, \sigma|R, h)$  denote the expected payoff of  $i$  if he chooses action  $a_i \in \{0, 1\}$  after history  $(R, h)$ , where everything else is played according to  $\sigma$ . Sequential rationality is satisfied if, for all information sets  $(i, t, k)$ ,

$$\sigma_i(t, k) \in \arg \max_{s \in [0, 1]} \sum_{(R, h)} \mu_i(R, h|t, k) * [s * \pi_i(1, \sigma|R, h) + (1 - s) * \pi_i(0, \sigma|R, h)]. \quad (7)$$

### 3 Equilibrium analysis

Our first result shows that, by sequential rationality, all players join the club under all move sequences if their beliefs satisfy Bayes' Rule. The incompleteness of the players' information and the fact that all players but one may prefer not having a club at all are thereby shown to be outcome irrelevant.

**Proposition 3.1.** *Fix a strategy profile  $\sigma$  that is sequentially rational for some system of beliefs satisfying Bayes' Rule. All players choose to "join" with probability 1 regardless of the move order.*

To gain intuition, assume there would be an equilibrium where some players  $N' \subseteq N$  do not join along the equilibrium path. The remaining players  $N'' = N \setminus N'$  necessarily join in all information sets along the equilibrium path (regardless of the move sequence). Along the path, the players' beliefs are correct in that they do not assign positive probability to some  $i \in N''$  not having joined the club when it could have been (according to  $\sigma$ ) some  $j \in N'$ . In any information set compatible with  $\sigma$ , the player to move therefore believes that all  $i \in N''$  will still join if he sticks to  $\sigma$ . Now assume said player is  $j = \min N''$ , i.e. the one with the lowest index under the assumed ordering Eq. (3), and the information set considered is one where he does not join. Since all move sequences have positive probability, all  $i \in N''$  join along the path of play even if  $j$  joins in this information set, and by "connectedness" Eq. (4) this implies that  $j$  is best off joining in this information set (a contradiction). It follows that all players join along the path of play, i.e. for all information sets  $(i, t, k)$ ,

$$k = t - 1 \quad \Rightarrow \quad \sigma_i(t, k) = 1. \quad (\text{all join})$$

The remainder of this section investigates actions off the path. This is straightforward in information sets where non-iterative dominance arguments suffice. In information sets where the requisite number of opponents have joined already, the respective players are best off joining. In information sets where the requisite number of opponents cannot be reached anymore, they are best off not joining.

$$\begin{aligned} k \geq i - 1 & \quad \Rightarrow \quad \sigma_i(t, k) = 1 & (\text{dominant}) \\ k + (n - t) < i - 1 & \quad \Rightarrow \quad \sigma_i(t, k) = 0 & (\text{dominated}) \end{aligned}$$

In the remaining information sets  $(i, t, k)$ , the following applies: (i) at least one player has chosen not to join, (ii) the number of players who have joined already does not

suffice to trigger “dominance” in  $i$ ’s eyes, and (iii) the number of players who have decided not to join does not suffice to trigger “dominatedness” in  $i$ ’s eyes. In such information sets,  $i$ ’s optimal action depends on whom he believes is (are) the player(s) who decided not to join. For example, assume that one player had decided not to join, one player is left to move after  $i$ , and  $i$ ’s decision depends on whether the subsequent player joins. If  $i$  believes that a move order applies according to which the subsequent player is the least enthusiastic player (with respect to joining the club, i.e. player  $n$ ), then  $i$  should not join. For, in this case, the least enthusiastic player would not join regardless of  $i$ ’s decision. However, if  $i$  believes that a move order applies according to which say player 1 moves last, then  $i$  is best off joining.

Bayesian updating does of course not restrict beliefs off the path. Thus, player  $i$  may well believe that the most enthusiastic player did not join while the least enthusiastic player is still to move. Similarly, under perfection, if player 1 trembles toward “not join” with probability  $\epsilon$  in all information sets on the path, player  $n$  trembles with probability  $\epsilon^3$ , and all other opponents tremble with probability  $\epsilon^2$ , then (as  $\epsilon$  approaches 0) player  $i$  believes that player 1 did not join and player  $n$  is still to move. This holds true regardless of the ex-ante probabilities of the various move orders.

Since player 1 enthusiastic and  $n$  is not, the belief that 1 would be more likely to tremble than  $n$  is counter-intuitive, however. Such beliefs can be sustained under say perfection only because relative tremble probabilities between players are unrestricted. More restrictive concepts such as properness restrict relative tremble probabilities “within players” (i.e. between the various actions of individual players), and concepts such as strategic stability require robustness with respect to all tremble patterns—including counter-intuitive ones as those discussed above. Similarly, the global games approach to equilibrium refinement (Carlsson and van Damme, 1993; Frankel et al., 2003) and analyses of “robust equilibria” (Morris et al., 1995; Kajii and Morris, 1997; Ui, 2001) do not restrict relative payoff perturbations between players, and hence the implied restrictions on relative tremble probabilities are very weak.

Solution concepts derived from models of bounded rationality tend to induce the intuitive restrictions between players. For example, equilibrium refinement based on choice-theoretic models following e.g. McFadden (1976, 1984) generally restricts relative payoff perturbations between players, and hence we obtain falsifiable restrictions of relative tremble probabilities between players. Note that, while the original definition of quantal response equilibrium is not restricted in this manner, see McKelvey



and Palfrey (1995), most game-theoretic as well as choice-theoretic studies actually assume the logit model implied by the extreme value distribution (see also Anderson et al., 1998). The following result establishes uniqueness of limiting logit equilibria in club games (see Def. A.1) and characterizes the solution.

**Proposition 3.2** (Logit equilibrium). *The unique limiting logit equilibrium satisfies, for all  $i \in N$  and  $k < t \leq n$ ,*

$$\sigma_i(t, k)(1) = \begin{cases} 1, & \text{if } k \geq i - (n - t) \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

In words, players join if and only if this pays off when all subsequent players will choose to join. This rule-of-thumb applies both on and off the equilibrium path in the limiting logit equilibrium. Implicitly, the players have the most optimistic beliefs about the move order of their opponents. If opponents did not join, then this must have been the least enthusiastic opponents. For, the variances of payoff perturbations are constant between players and hence the payoffs of the least enthusiastic players are most likely to be perturbed so that they do not join. By “connectedness” Eq. (4), players with such beliefs are best off acting according to said rule-of-thumb.<sup>4</sup>

In turn, players may also deviate from equilibrium strategies when they fail to anticipate their opponents’ strategies correctly. We analyze this possibility using the “level- $k$ ” model of reasoning.<sup>5</sup> Accordingly, the players believe their opponents act non-strategically, or that they believe that their opponents believe they would respond to non-strategic players, and so on. This is related to assuming rationality, mutual knowledge of rationality, and so on, up to common knowledge of rationality at level infinity. The following result shows that extensive form rationalizability (EFR, see Pearce, 1984, and Battigalli, 1997) induces the same refinement effect as limiting logit equilibrium. That is, as level- $k$  players become more rational, their beliefs approach the ones described above—without imposing any restrictions of the actions at level 0.

**Proposition 3.3** (Rationalizability). *The unique extensive form rationalizable strategy profile (see Def. A.2) is the limiting logit equilibrium Eq. (8).*

<sup>4</sup>Experimental evidence on the assertion that players that have less to lose are more likely to tremble is mixed, however (see e.g. Battalio et al., 2001, and Cooper and Van Huyck, 2003).

<sup>5</sup>Following for example Ho et al. (1998), Stahl (1998), (Gneezy, 2005), Costa-Gomes and Crawford (2006), and (Crawford and Iriberry, 2007).

Pearce defined EFR as an iterative elimination procedure, and in each iteration, the players' beliefs are restricted to those that are compatible with strategies that are not yet eliminated. In the first iteration, all players realize that player 1 generally joins, whereas players  $2, \dots, n$  may or may not join as first movers. As for the beliefs in the second iteration, this implies that in information sets where some player did not join in the first round, no player believes this would have been player 1 (since there are more "plausible" explanations in the sense that other explanations do not violate rationality). Iteratively applied, this induces the aforementioned refinement effect. Alternative refinements of rationalizability, e.g. perfect rationalizability (Bernheim, 1984) or proper rationalizability (Schuhmacher, 1999; Asheim, 2002), do not induce this refinement effect.

## 4 Experimental design

This section has two parts. The first part presents and motivates the parameters chosen for the experimental games. We describe the variety of information sets and discuss their significance in terms of our tests. We then state some theoretical expectations that will be tested in Section 5. The second part reports the procedure and logistics.

### 4.1 Experimental games

This linear utility functions adopted in the experiment are special cases of the framework considered above and are expressed using the notation of Dixit (2003). A game has four types of players, each corresponding to a preference index coefficient  $\iota = 1, 2, 3, 4$ , thus capturing the heterogeneity of preferences. The higher the  $\iota$ , ceteris paribus, the lower will be the player's benefit from joining the club. The fixed benefit from joining the club is denoted by  $\beta$ , and  $\gamma$  and  $\tau$  capture the positive and negative network externality parameters, respectively. All coefficients and parameters are non-negative. We can then write down the payoff functions as

$$S_i = 20 \cdot [1 + \iota - \tau \sum_{j \neq i} a_j] \quad \text{payoff for player } i \text{ from NOT joining the club,} \quad (9)$$

$$B_i = 20 \cdot [1 + \beta + \gamma \sum_{j \neq i} a_j - \iota] \quad \text{payoff for player } i \text{ from joining the club.} \quad (10)$$

The games were parameterized to obtain integer valuations to simplify the presentation of the games to the subjects. Table 1 shows the values of  $S_i$  and  $B_i$  used in the experi-

Table 1: Experimental games (payoffs of the various player types in response to 0...3 co-participants joining the club)

Treatment A						Treatment B					
		0	1	2	3			0	1	2	3
Type 1	<i>Yes</i>	42	52	62	72	Type 1	<i>Yes</i>	62	72	82	92
	<i>No</i>	40	10	-20	-50		<i>No</i>	40	10	-20	-50
Type 2	<i>Yes</i>	22	32	42	52	Type 2	<i>Yes</i>	42	52	62	72
	<i>No</i>	60	30	0	-30		<i>No</i>	60	30	0	-30
Type 3	<i>Yes</i>	2	12	22	32	Type 3	<i>Yes</i>	22	32	42	52
	<i>No</i>	80	50	20	-10		<i>No</i>	80	50	20	-10
Type 4	<i>Yes</i>	-18	-8	2	12	Type 4	<i>Yes</i>	2	12	22	32
	<i>No</i>	100	70	40	10		<i>No</i>	100	70	40	10

  

Treatment C						Treatment D					
		0	1	2	3			0	1	2	3
Type 1	<i>Yes</i>	42	62	82	102	Type 1	<i>Yes</i>	62	82	102	122
	<i>No</i>	40	20	0	-20		<i>No</i>	40	20	0	-20
Type 2	<i>Yes</i>	22	42	62	82	Type 2	<i>Yes</i>	42	62	82	102
	<i>No</i>	60	40	20	0		<i>No</i>	60	40	20	0
Type 3	<i>Yes</i>	2	22	42	62	Type 3	<i>Yes</i>	22	42	62	82
	<i>No</i>	80	60	40	20		<i>No</i>	80	60	40	20
Type 4	<i>Yes</i>	-18	2	22	42	Type 4	<i>Yes</i>	2	22	42	62
	<i>No</i>	100	80	60	40		<i>No</i>	100	80	60	40

ment. There is complete information on the payoff structure and type distribution, but players do not know the order of moves as discussed above.

We chose a game with four players as it is a club game with sufficient complexity to discriminate level- $k$  reasoning from QRE.<sup>6</sup> The payoff structures in all treatments result from parameter shifts that preserved the same “topology” of information sets across treatments (see below), and needless to say, the same (unique) equilibrium predictions. We varied only the strengths of incentives and externalities across treatments. The choice of parameter values for the individual incentive parameters  $\beta$  and the externality parameters  $\gamma$  and  $\tau$  interacted to yield a  $2 \times 2$  factorial design. Treatments A and B share the same externality effects, as do treatments C and D. The only thing that varies is the strength of incentives. Incentives in treatments B and D are stronger

<sup>6</sup>Keeping the precision  $\lambda$  constant, level- $k$  players are more likely to join than QRE-players if they are of low type (1 or 2) in the club game, and they are less likely to join if they are of high type (3 or 4) in the club game. This effect is less pronounced if there are only three players overall.

in absolute terms than in treatments A and C. Treatments A and C share the same incentive effects, as do treatments B and D. The only thing that varies is the strength of externalities. Externalities in treatments C and D are stronger in absolute terms than in A and B. This design allows for direct cross-comparability, to test the effects of each exogenously determined variable, versus the robustness of the theory across different parameter sets. Shifts in the latitude of inequity across players in outcomes naturally result from these manipulations, and in particular our design varies the proportion of players who would have been better off had the club not been started.<sup>7</sup> These variations allow us to examine to which degree social preferences affect behavior.

Finally, we characterize the various information sets in the game. Based on iterated dominance, four types can be distinguished. All treatments contained the same combination of information set types per player, i.e. the type of a given information set  $(i, t, k)$  was constant across treatments. The information set types are as follows.

*Joining is non-iteratively dominant.* In these information sets, the requisite number of co-players that must join to make a player prefer to join is reached, and joining is individually optimal regardless of one's conjecture concerning the remaining players.

*Joining is non-iteratively dominated.* The requisite number of co-players is unreachable and joining is individually dominated.

*Joining is iteratively dominant (on path).* These information sets are on the equilibrium path (i.e. no player has deviated from joining yet) and the requisite number has not yet been reached. According to EFR, the respective player should join, but iterated elimination of non-rationalizable strategies is required.

*Joining is iteratively dominant (off path).* These information sets are off the equilibrium path, and the requisite number has not yet been reached, although it can still be reached. According to EFR, the players are best off joining, but actions are largely unrestricted under perfection and properness. We refer to these information sets as “unclear” information sets.

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<sup>7</sup>In treatment A, clubs are welfare reducing for three players out of four. In treatments B and C, clubs are welfare improving for two players out of four, and welfare reducing for the other two. In treatment D, clubs are welfare improving for three players out of four.

## 4.2 Procedure and logistics

The experiment was conducted in the experimental economics laboratory at the European University Viadrina, Frankfurt (Oder), Germany. The experimental instructions are in Appendix B. Subjects were recruited from an email list consisting of students from the faculties of Cultural Science, Business and Economics, and Law. We conducted six sessions with twelve subjects each. Each session consisted of four stages,<sup>8</sup> each of eight rounds.

Each round consisted of a game where subjects were matched into groups of four and asked in sequence if they wanted to “become a member of a club,” and to choose *yes* or *no*. This phrasing was chosen as being intuitive and simple to understand. Each group contained player types 1, 2, 3, and 4 (corresponding to preference index coefficient  $\iota = 1, 2, 3, 4$  labeled as P, Q, R, and S in the experiment), and this was known to the subjects. The payoff tables for all four players in a group were shown on the computer display. For each round, subjects were randomly allocated to a position in the order of moves. When it came to a subject’s turn to choose, he/she was informed of how many had chosen before his/her move, and how many had joined the club (chosen ‘yes’). At the end of each round, subjects were given feedback of the total number of subjects in their group who had joined, and their own earning.

Each stage involved different treatments. We presented treatments either in the sequence A-C-B-D or in the inverse sequence D-B-C-A to control for order effects. Subjects were randomly allocated a type at the start of the experiment; a subject’s type was maintained throughout the experiment. After each round, subjects were re-matched into new groups of four. This allows for experience, and eliminates reputation effects.

At the beginning of the experiment, subjects were randomly allocated to their seats. Then, they were asked to read the experimental instructions, provided on printed sheets, and to answer a short control questionnaire for us to check their understanding. Subjects in doubt were verbally advised by the experimental assistants before being allowed to begin. Each computer terminal was partitioned, so that subjects were un-

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<sup>8</sup>We also had an additional brief stage to try to measure level of reasoning by the subjects using a different task (a ‘*p*-beauty contest’ game), as in principle finding the rationalizable equilibrium in the clubs game may depend on different levels of reasoning for different types. However, this measure was not related to behavior in the earlier part of the experiment and it will not be referred to further.

Table 2: Frequency of rationalizable actions

Treatment	A	B	C	D
Type 1	0.99	0.99	1.00	0.99
Type 2	0.89	0.94	0.91	0.97
Type 3	0.67	0.90	0.76	0.94
Type 4	0.44	0.71	0.49	0.79

able to communicate via audio or visual signals, or to look at other computer screens. Decisions were thus made in privacy. At the end of the experiment, subjects were informed of their payments, and asked to privately choose a codename and password. This was used to anonymously collect their payments from a third party one week after the experiment. Each subject was given a €6 participation fee and the earnings from two randomly chosen “winning rounds” from stages 1-4, and one from stage 5. Each experimental point was worth €0.10. The average payout per subject was €20.83 for approximately 1.5-2 hours per session.<sup>9</sup>

## 5 Basic results

Table 2 shows the frequency of rationalizable actions for each of the information sets, treatments, and types. Generally, the refined theoretical prediction performs well: 84% of all actions in the experiment were rationalizable according to EFR. We therefore believe that our average observation is rather close to equilibrium, and in this sense it constitutes “eventual play” as discussed in the Introduction. In order to inform us on how to model “eventual play” this section describes the deviations from EFR and LLE in some detail. Most deviations from the predictions concern decisions where subjects who “should have” joined did not join. In treatments A, B, C and D respectively, of those who chose not to join, in aggregate 20%, 13%, 18%, and 18% were rationalizable, for types 3 11%, 6%, 8% and 18% were rationalizable, while for types 4 28%, 18%, 24% and 22% were rationalizable. It appears that the less the incentive to join the club, the lower the joining rate.

<sup>9</sup>The monetary incentives provided in our experiment are substantial by local standards. Our mean payment of about €12 per hour is, for example, 50% more than the mean wage of a research assistant at Frankfurt (Oder).

Table 3: Joining rates in information sets “joining is non-iteratively dominant”

$(i, t, k)$	joining is ...	$i = 1$	$i = 2$	$i = 3$	$i = 4$
$k \geq i - 1$	dominant	573/576	356/368	211/215	103/122
$k + (n - t) < i - 1$	dominated	n.a.	0/0	1/12	11/80

Table 13 reports the joining rates per information set. The shaded cells denote information sets where not joining is uniquely rationalizable, i.e. joining is non-iteratively dominated. The data suggests that: (i) for specific cells, the higher the player type, the smaller the joining rate; (ii) for specific types and given numbers of players that would maximally join the club, the earlier the mover position, the smaller the joining rate, and; (iii) for specific types and mover positions, the higher the number of (“required”) players that are yet to join, the smaller the joining rate is. We evaluate these observations against the theoretical expectations set out above.

First, we look at behavior in information sets where joining is either non-iteratively dominant or non-iteratively dominated. This is a test of individual rationality abstract of conjectures about opponents, since rational decision making does not rely on iterated arguments in these cases, and it gives us an indication of the sensitivity of subjects to the salience of pecuniary payoffs. Most of the time, subjects do (not) join when joining is non-iteratively dominant (dominated) as shown by the Table 3.

**Result 5.1** (non-iterative dominance). *Subjects act almost as predicted when joining is either non-iteratively dominant or non-iteratively dominated.*

Trust in the rationality of co-players is required in information sets  $(i, t, k)$  on the equilibrium path ( $k = t - 1$ ) where the requisite number of players has not joined yet ( $k < i - 1$ ). Here, joining is uniquely predicted in weak PBEs, but iterated reasoning and confidence in the co-players’ rationality is required. As shown in the proof of Prop. 3.3, the structure of reasoning modeled in extensive form rationalizability implies that type 2 requires two levels of reasoning in these information sets, type 3 requires three levels of reasoning, and type 4 requires four levels. From the perspective of EFR, the required level of reasoning does not depend on one’s position in the move sequence (e.g. on whether type 3 moves first or second). Table 4 shows that while most subjects join in such information sets, one’s position in the order of moves also seems to play a role.

Table 4: Joining rates when “joining is iteratively dominant (on path)”

$(i, t, k)$	(2, 1, 0)	(3, 1, 0)	(3, 2, 1)	(4, 1, 0)	(4, 2, 1)	(4, 3, 2)
join	138/150	114/157	90/114	43/135	62/119	73/120

Table 5: Joining rates when “joining is iteratively dominant (off path)”

(2, 1, 0) vs. (2, 2, 0)	(3, 1, 0) vs. (3, 2, 0)	(3, 2, 1) vs. (3, 3, 1)
$\frac{138}{150} \approx \frac{36}{44}$	$\frac{114}{157} > \frac{19}{39}$	$\frac{90}{114} > \frac{25}{39}$

**Result 5.2** (iterative dominance on path). *Most players join the club in information sets on the equilibrium path where it is iteratively dominant to join.*

The average joining rates decrease with player type and increase with one’s position in the order of moves. The former effect, the relation to player type, suggests that subjects themselves do not best respond to their beliefs or that they have heterogeneous beliefs. The latter effect, the relation to one’s position in the order of moves, suggests that subjects do not believe their opponents are rational, or that they believe that their opponents believe their opponents are not rational. Both of these effects are captured in QREs, where all players are boundedly rational, and know that they are, and know that they know, and so on.<sup>10</sup> Level- $k$  models can explain these observations only if we consider a level- $k$  structure based on iterated logit response (as we will do below).

To complete our preliminary view of behavior in the different information sets, we look at the decisions of subjects in the “unclear” information sets off the equilibrium path (see Section 4.1 for the explanation of information sets where “joining is iteratively dominant (off path)”). In these information sets, the subjects should join according to EFR and LLE, but equilibrium refinement requires belief restrictions in terms of relative tremble probabilities. Table 5 shows that this prediction does not always meet the experimental data.

**Result 5.3** (iterative dominance off path). *Not all subjects join the club in information sets off the equilibrium path where it is iteratively dominant to join.*

<sup>10</sup>A subject playing a logit response to its belief, for example, joins with a probability  $p < 1$  in general; his expected payoff from joining the club is higher ex post, i.e. after a co-participant has joined, than ex ante, i.e. prior to a co-participant joining with high probability.



In unclear information sets, LLE and EFR predict that all will join, whereas perfection predicts only a “monotonicity” effect: keeping the circumstances constant, types with higher incentives are not less likely to join in these information sets. Type 2 should not join less often in  $(2,2,0)$  than type 3 in  $(3,2,0)$ . Since type 2 subjects indeed join significantly more often than type 3 subjects,<sup>11</sup> monotonicity in this sense applies, which suggests that subjects’ beliefs do not violate consistency (Kreps and Wilson, 1982). In addition, our results show that the subjects join more often than they would with uniformly distributed beliefs in two out of four of the unclear information sets (again using Wilcoxon tests using session-level data).<sup>12</sup> This is predicted, for example, by level- $k$  and QRE models of bounded rationality.

Let us next look at the joining rates in “unclear” information sets in relation to the joining rates in the respectively preceding information sets on the equilibrium path. That is, we compare the unclear  $(2,2,0)$  with  $(2,1,0)$ , which is its closest relative on the equilibrium path, the unclear  $(3,2,0)$  with  $(3,1,0)$ , and the unclear  $(3,3,1)$  with  $(3,2,1)$ . This comparison (see Table 5) shows that the differences within these pairs are significant in two out of three cases.<sup>13</sup> However, this significance is not necessarily an effect of “uncertainty”; an explanation based on logit response or level- $k$  reasoning may explain this observation as well.

Finally, let us look at the sensitivity of the joining rates with respect to the treatment parameters. In particular, the joining rates should not be independent of the incentives parameter  $\beta$  if subjects play logit responses. Since this affects both the own incentives and the opponents’ incentives, the effect should reinforce itself and hence it should be significant. Table 6 shows that, as predicted, joining rates are higher in incentive treatments for all player types (with the exception of player type 1, who joins in any case).<sup>14</sup> The externality effect, in turn, is insignificant for all player types, as it is predicted by concepts based on logit response discussed so far.

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<sup>11</sup>This prediction is supported at the 6% level ( $p=0.052$ ) in Wilcoxon paired-sample tests using session-level data.

<sup>12</sup>Consider for example the information set  $(2,2,0)$ . We understand “uniformly distributed beliefs” in the sense that a third of the subjects believes that type 1 was the not-joining first mover, another third of them believes it was type 3, and the final third believes it was type 4 (all such beliefs are compatible with perfection). Hence, type 2 should join with probability  $2/3$  in this information set.

<sup>13</sup>At the 10% level,  $p = 0.094$ , in Wilcoxon paired-sample tests using session level data.

<sup>14</sup>This prediction is supported at the 5% level in Wilcoxon paired sample tests (the  $p$ -value is about 0.018 in all cases).

Table 6: Significance of externality and incentive effects

Player type	$i = 1$	$i = 2$	$i = 3$	$i = 4$
$A + C$ vs. $B + D$	$\frac{287}{288} \approx \frac{285}{288}$	$\frac{259}{288} < \frac{275}{288}$	$\frac{198}{288} < \frac{261}{288}$	$\frac{86}{288} < \frac{206}{288}$
$A + B$ vs. $C + D$	$\frac{286}{288} \approx \frac{286}{288}$	$\frac{264}{288} \approx \frac{270}{288}$	$\frac{219}{288} \approx \frac{240}{288}$	$\frac{136}{288} \approx \frac{156}{288}$

Our results suggest that while behavior is much in line with that predicted by LLE and EFR, a better explanation of the data would be achieved by considering models that weaken the assumption of best response. The observations made here, on the basis of descriptive statistics and univariate tests, provide us with an idea of the data. The analysis to follow investigates these effects in multi-variate frameworks, using multinomial probit models with nested random effects at the levels of subjects nested in sessions. The first model explains joining decisions based on DOMINANT (= 1 when joining is dominant), DOMINATED (= 1 when joining is dominated), UNCLEAR (being in one of the four unclear information sets), LEVREAS (the level of reasoning required according to EFR in this information set), INCENTIVE (= 1 in treatments B and D where incentives are strengthened in absolute terms), and EXTERNALITY (= 1 in treatments C and D where externalities are strengthened). Table 7 reports the results.

The probability of joining depends significantly on the required level of reasoning, in that more “correct” choices are made when the required level of reasoning is lower. This indicates the existence of a bounded degree of iterative reasoning, i.e. a limited belief in others’ rationality. In relation to this effect, the INCENTIVE effect corresponds with the effect of about  $-1$  level of reasoning required, and UNCLEAR corresponds with the effect of about  $+1$  level of reasoning required. The externality effect is fairly small, about a quarter of the effect of a level of reasoning, and insignificant after controlling for TYPE. DOMINANT is not statistically significant, but DOMINATED has a strong negative effect on joining.

The observation that UNCLEAR “costs” and INCENTIVE “gains” the effect of about one level of reasoning is compatible with the notion of logit response. For example, if subjects are more confident that enough co-participants join in INCENTIVE treatments, they will then be more likely to join themselves. The incentives of type 1 are strengthened (as everybody else’s incentives are), hence type 2 is more confident in type 1’s joining and thus more likely to join herself, hence type 3 is more confident

Table 7: Results of probit regression with random effects (Subject and Session)

(\* and \*\* denote significance at .05 and .01 level, resp., and the  $t$ -values are given in parentheses)

Intercept	DOMINANT	DOMINATED	UNCLEAR	LEVREAS	INCENTIVE	EXTERNALITY	TYPE
2.257** (6.77)	-0.038 (-0.16)	-2.880** (-9.11)	-0.607** (-4.35)	-0.651** (-6.69)	0.870** (10.64)	0.174* (2.28)	—
2.646** (6.93)	0.320 (1.01)	-2.023** (-4.538)	-0.573** (-3.099)	-0.352* (-2.511)	0.855** (7.839)	0.179 (1.737)	-0.459** (-4.426)

and more likely to join, and therefore also type 4.

Controlling for TYPE (= preference index  $t$ ) in the above regression model changes the results only slightly. TYPE captures the relative gains from joining: the lower the type, the higher one's incentives. The existence of information sets with non-iteratively dominant or dominated actions implies that TYPE is not perfectly correlated with the required level of reasoning (see again Table 7). Controlling for TYPE, EXTERNALITY becomes statistically insignificant. The following summarizes the above observations.

**Result 5.4** (level of reasoning, treatment effects and information sets). *The probability of joining is positively related to the relative gains from joining the club, and negatively related to the required level of reasoning.*

Decisions are sensitive to the relative gains, which vary across treatments and subjects, and the level of reasoning required to make the “correct” choices. Both of these observations are compatible with strategic equilibrium and level- $k$  reasoning, provided we assume quantal response. Furthermore, subjects with weaker preferences for joining the club join less often than they should when joining is non-iteratively dominant. In the next section, we analyze which model of bounded rationality explains these deviations from EFR and LLE best.

## 6 Understanding behavior and modes of reasoning

This section relaxes the assumptions made in the theoretical analysis in three distinctive steps, in order to illustrate their incremental relevance in relation to our data and to motivate the modeling decisions made below. First, we relax the assumption that subjects play mutual best responses, by considering mutual logit responses following McKelvey and Palfrey (1998). Second, we relax the assumption that subjects are concerned with pecuniary payoffs by considering interdependent preferences (in addition

Table 8: Goodness-of-fit of the basic models

	uniform	EFR	QRE	Maximum
LL	-1597	$-\infty$	-807.8	-584.1
QS	1152	1550	1805.7	2070

to logit response). Third, we will additionally relax the assumptions of homogeneity across subjects and equilibration of behavior (in mixture-of-types level- $k$  models).

Table 8 reports the improvement in the goodness-of-fit obtained in the first step (the parameter estimates are reported below). Besides the log-likelihood, we also report the quadratic score (QS, see e.g. Selten, 1998) which allows comparisons with models of zero likelihood (EFR being one in our case). Table 8 reports these measures for the rational equilibrium prediction (“EFR”), the fitted logit equilibrium (“QRE”), the naive prediction of uniform randomization in all information sets (“Uniform”), and the hypothetically optimal fit denoted as “Maximum” in Table 8.<sup>15</sup> Relating the QRE prediction ( $LL = -807.8$ ) to the distance between the naive “uniform” prediction ( $LL = -1597$ ) and the hypothetical “maximum” prediction ( $LL = -584.1$ ), it can be seen that QRE covers 77.9% of this distance. That is, the QRE models explains more than three quarters of what can be explained in terms of log-likelihood.

Second, we investigate the relevance of modeling social preferences in addition to logit response. We consider six models of the following family of utility functions. Let  $u_i(\mathbf{p})$  denote  $i$ ’s utility if the profile of payoffs is  $\mathbf{p} \in \mathbb{R}^N$ , and define

$$u_i = p_i - \sum_{j:p_i < p_j} \alpha * |p_i - p_j| - \sum_{j:p_i \geq p_j} \beta * |p_i - p_j| - \sum_{j:p_i < p_j} \alpha' * p_j - \sum_{j:p_i \geq p_j} \beta' * p_j \quad (11)$$

conditional on the parameters  $(\alpha, \beta, \alpha', \beta')$ . This family contains many well-known utility functions as special cases. If  $\alpha = \beta = 0$ , on the one hand, then linear altruism results through  $\alpha' = \beta' > 0$ , linear spite results through  $\alpha' = \beta' < 0$ , and conditional altruism/spite results through  $\alpha' \neq \beta'$ . On the other hand, if  $\alpha' = \beta' = 0$ , then the Fehr-Schmidt preferences of inequity aversion are obtained. Our analysis distinguishes three models of inequity aversion, namely symmetric inequity aversion ( $\alpha = \beta$ ), asymmetric inequity aversion ( $\alpha \neq \beta$ ), and uni-directional inequity aversion ( $\alpha \neq 0, \beta = 0$ ), and the three corresponding models of conditional altruism/spite ( $\alpha' = \beta'$ ,  $\alpha' \neq \beta'$ , and  $\alpha' \neq 0$ ,

<sup>15</sup>This “maximum model” uses 40 free parameters to “predict” the actually observed relative frequencies of joining in all information sets.

Table 9: Estimation results for models of spite and inequity aversion

Model	$\lambda$	$\alpha$	$\beta$	$\alpha'$	$\beta'$	$LL$
Plain	0.0575					-807.79
Symm. IA	0.0463	0.2025				-791.96
Uni-dir. IA	0.216	0.216				-777.16
Asymm. IA	0.0413	0.0625	-0.5538			-766.09
Symm. Spite	0.0675			0.126		-770.95
Uni-dir. Spite	0.060			0.150		-751.39
Asymm. Spite	0.0488			0.1225	-0.25	-742.54

$\beta' = 0$ ).

Table 9 reports the results. The uni-directional model of conditional spite (“Uni-dir. Spite”) seems most appropriate to model social preferences in the club game. That is, subjects are envious with respect to better-earning co-players, but they are concerned with their co-players payoffs rather than the payoff differences. The model is parsimonious, it is the best one-parametric model overall, significantly better than any of the Fehr-Schmidt models, and almost as predictive as the respective two-parametric model of conditional spite. In relation to the distance between plain “QRE” ( $LL = -807.8$ ) and “Maximum” ( $LL = -584.1$ ), this model scores the log-likelihood  $LL = -751.4$  and thus covers 25.2% of what remained to be explained (and 83.5% overall). We refer to this model as “QRE+Ineq” in the following.

Table 10 reports the results of a regression analysis investigating in which informations sets the subjects deviate significantly from the two QRE predictions (with and without “Ineq”). We do so by controlling for the joining rates derived from the QRE models in the regression models underlying Table 7 discussed before. The plain QRE model overestimates joining when joining is dominated (DOMINATED is significant), and it underestimates joining of high types and when high levels of reasoning are required (due to the significance of LEVREAS and TYPE). Overall, low types join too rarely and high types join too often, and the levels of reasoning required seem significant. Controlling for inequity aversion, as in QRE+Ineq accounts for all these type-related biases, but in relation to this model, subjects join too rarely when joining is non-iteratively dominant (now DOMINANT is significant, which is independent of type). To summarize, controlling for social preferences eliminates type-related biases rather effectively, but a bias related to the level of reasoning continues to exist.

In order to account for this, we next relax the assumptions of equilibration and homogeneity. Following the literature, we consider a mixture-of-types model allowing for the coexistence of equilibrium types and level- $k$  types. Primarily, our model allows for the following four types. L0 randomizes uniformly ( $\lambda_0 = 0$ ), L1 logit responds ( $\lambda_1 \geq 0$ ) to level 0, L2 logit responds ( $\lambda_2 \geq 0$ ) to level 1, and the equilibrium types stick to the logit equilibrium (QRE) corresponding with some  $\lambda_e \geq 0$ . Let the set of types be denoted as  $K$ , e.g.  $K = \{0, 1, 2, e\}$ , and define the respective precision parameters as  $\lambda = (\lambda_k)_{k \in K}$  and the type's population shares as  $(\rho_k)_{k \in K}$  with  $\sum_k \rho_k = 1$ . Given  $(\lambda, \rho)$ , the corresponding predictions are computed straightforwardly.<sup>16</sup> Let  $o_{s,t}$  denote the decision of subject  $s = 1, \dots, 72$  in round  $t \in T = \{1, \dots, 32\}$  and define as  $\Pr(o_{s,t}|k, \lambda)$  as the theoretical probability of  $o_{s,t}$  conditional on  $s$  being of type  $k$  with precision  $\lambda$ . The likelihood of observing  $o_s = (o_{s,t})_t$  unconditional of type, where errors are independent conditional on types, is

$$L(o_s|\lambda, \rho) = \sum_{k \in K} \rho_k \prod_{t \in T} \Pr(o_{s,t}|k, \lambda). \quad (12)$$

Accumulating over all subjects, the likelihood of  $o = (o_s)$  given  $(\lambda, \rho)$  therefore is

$$L(o|\lambda, \rho) = \prod_{s=1}^{72} \sum_{k \in K} \rho_k \prod_{t \in T} \Pr(o_{s,t}|k, \lambda). \quad (13)$$

We consider models allowing for up to three levels of reasoning, up to two equilibrium types, and with or without controlling for social preferences. The ML estimates and log-likelihoods as well as Bayesian information criteria (BICs) are reported in Table 11. The BIC improves significantly (in the sense of Vuong, 1989) at the .05 level while levels 0, 1, and 2 are added (models A – C in Table 11), but not significantly if we go beyond this threshold by adding a level-3 type or a second equilibrium type (D and E in Table 11, respectively). As before, controlling for social preferences improves log-likelihood and BIC significantly (model F), toward  $LL = -677.2$ . As an incremental assumption beyond “QRE+Ineq” ( $LL = -751.4$ ), allowing for the mixture of types explains 44.4% of the noise that remained to be explained (in relation to the “Maximum”  $LL = -584.1$ ), and it covers 90.8% of the overall distance between the naive prediction “Uniform” and the hypothetical prediction “Maximum.”

**Result 6.1** (bounded rationality and social preferences). *The data are explained fairly well by a mixture of level- $k$  and QRE types with conditional spite.*

<sup>16</sup>We compute QREs using Gambit, see McKelvey et al. (2007).

Let us briefly look at the difference in the estimated type shares between the two-level model without social preferences (Model *C* in Table 11) and its counterpart with social preferences (*D* in Table 11). Without controlling for social preferences, the aggregate population share of L1 and L2 types is estimated as 78.1%, and the share of QRE types is estimated as 17.4%. After controlling for inequity aversion, 16.7% are L1 or L2 while 78.5% are “in equilibrium.” This inversion of the estimated type shares corresponds closely with the regression results from Table 10 reported above, where type and level of reasoning was significant without social preferences, but not after controlling for social preferences. That is, level- $k$  reasoning and social preferences provide alternative explanations for certain aspects of the data—that low types join too rarely and high types join too often—and, overall, such deviations from logit equilibrium are better explained through controlling for social preferences in our case.

In order to investigate the robustness of this interpretation, we will next consider the types’ population shares and the degree of social preferences in different stages of the experiment. We distinguish experience along two dimensions. On the one hand, we consider *vertical experience* when subjects gain a deep understanding of more similar games by repetition (i.e. by comparing rounds 1-4 with rounds 5-8 per stage). On the other hand, we consider *horizontal experience* when subjects gain experience in a breadth of differently parameterized, less similar games (i.e. by comparing a given set of rounds from stages 1 and 2 with stages 3 and 4). By distinguishing phases of low and high levels of each, vertical and horizontal experience, our data set is segregated into four quadrants. For example, in rounds 1-4 of stages 1 and 2, the subjects have relatively low degrees of both horizontal and vertical experience. In rounds 1-4 of stages 3 and 4, the subjects have low vertical experience in the respective games (we observe their initial actions in new games), but they have a comparably high degree of horizontal experience (being in stages 3 and 4).

Table 12 reports the estimated shares of population types and social preference parameters in these four quadrants. It also extrapolates the individual type transitions as reported shortly. It is found that social preferences affect decision making primarily when subjects have low vertical experience. The social preference parameters are  $\alpha = 0.223$  and  $\alpha = 0.0925$  for subjects with low vertical experience (for low and high horizontal experience, respectively), and  $\alpha = 0.0253$  and  $\alpha = 0.0066$  for subjects with high vertical experience. The respective differences are highly significant, and in particular, subjects are concerned purely with pecuniary payoffs when they are experi-

enced in both dimensions.

Finally, we consider the subjects' mode of reasoning (level- $k$  or equilibrium) depending on quadrant, and how the type transitions can be extrapolated to understand their implications with respect to equilibrium refinement. To understand the individual transitions, we first need to derive individual classifications. We derive them using Bayes' Rule. Recall  $\Pr(o_{s,t}|k, \lambda)$  as defined above, prior to Eq. (12), consider the set of types  $K = \{0, 1, 2, e\}$  as before, and assume the population parameters are  $(\rho_k, \lambda_k)_{k \in K}$ . The ex-post probability that subject  $s$  (characterized by its choices  $o_s$ , see above) is of type  $k$  is

$$\Pr(k|o_s) = \frac{\Pr(k \cap o_s)}{\Pr(o_s)} = \frac{\rho_k \prod_{t \in T} \Pr(o_{s,t}|k, \lambda)}{\sum_{k' \in K} \rho_{k'} \prod_{t \in T} \Pr(o_{s,t}|k', \lambda)}. \quad (14)$$

Classifications obtained in this way are probabilistic. Let  $x_{i,k}^{rs}$  denote the probability that subject  $i$  is of type  $k$  in quadrant  $(r, s)$  of the experiment. For example,  $\mathbf{x}_i^{21}$  is  $i$ 's classification profile in quadrant 21 (high horizontal experience, low vertical experience). The individual type transitions can now be modeled using an auto-regressive multinomial logit model as follows.

$$\mathbf{x}_i^{21} \triangleq \alpha + \beta_1 \mathbf{x}_i^{11} \quad (15)$$

$$\mathbf{x}_i^{22} \triangleq \alpha + \gamma_1 + \beta_1 \mathbf{x}_i^{12} \quad (16)$$

$$\mathbf{x}_i^{12} \triangleq \alpha + \beta_2 \mathbf{x}_i^{11} \quad (17)$$

$$\mathbf{x}_i^{22} \triangleq \alpha + \gamma_2 + \beta_2 \mathbf{x}_i^{21} \quad (18)$$

$$\mathbf{x}_i^{22} \triangleq \alpha + \beta_1 \mathbf{x}_i^{11} + \beta_2 \mathbf{x}_i^{11} + \beta_3 \mathbf{x}_i^{11} \quad (19)$$

For example, Eq. (18) describes the transition from low to high vertical experience for subjects with a high level of horizontal experience. The fixed points of these type transitions are the projected steady states of the populations if the subjects gain experience in the respective direction. Table 12 reports the fixed points derived from this analysis.<sup>17</sup> The results can be summarized as follows.

**Result 6.2** (transition of types). *With experience, L1 types vanish, the proportion of L2 types increase, the proportion of QRE types decrease, and conditional spite becomes irrelevant. QRE types dominate if subjects gain vertical experience, and L2 types dominate if subjects gain horizontal experience or if they gain both.*

<sup>17</sup>To verify robustness, we have conducted a second analysis where the transition matrix  $\beta_1$  ( $\beta_2$ ) for accumulating horizontal (vertical) experience depends on the current level of vertical (horizontal) experience. The results largely concur.



The projected implications of gaining experience, toward equilibrium in vertical transitions and toward level-2 in horizontal transitions, closely correspond with our intuition—although these results are novel in relation to the existing literature. As a side note, the projected steady states differ notably if we neglect controlling for social preferences. This may not be surprising, since the estimated relevance of social preferences has been shown to depend on experience, but it illustrates the importance of controlling for it. Finally, note which effect dominates as subjects gain both horizontal and vertical experience, i.e. the fixed point of Eq. (19). According to our results, vertical experience dominates horizontal experience in the club game, and the subject pool converges to level-2 types along the diagonal. This result seems rather surprising to us. Further research may investigate to which degree it continues to hold if we consider games other than club games.

## 7 Discussion

This paper applied two leading theories of bounded rationality to equilibrium refinement in extensive form games with incomplete information. Our analysis of level- $k$  thinking in extensive form games contributes to a literature that has been focused on normal form games (e.g. Costa-Gomes et al., 2001; Crawford and Iriberri, 2007) and recently also two-stage games (Stahl and Haruvy, 2008) and signaling games (Kawagoe and Takizawa, 2008). Kawagoe and Takizawa (2008) find that level- $k$  explains their signaling game data better than equilibrium concepts including QRE. In their games, competing refinement concepts are theoretically plausible, and the limit points of QRE and level- $k$  predict equilibria that are either different from those of the refinement concepts or from each other.

In our games, in contrast, the limit points of QRE and level- $k$  (LLE and EFR) yield the same unique equilibrium prediction, while other refinement concepts have no bite. Apart from demonstrating the refinement capabilities of LLE and EFR, we can thus study the relevance of each model, and their complementarity, by observing how subjects' behavior follows their respective convergent paths when these models are given equal chances to perform. Our basic results underscore the qualitative applicability of both concepts and show that the sensitivity of behavior to incentives and mover positions are largely consistent with QRE and iterated logit response. The observed

sensitivity of overall behavior to the required level of reasoning, however, indicates that an explanation of behavior that neglects level- $k$  reasoning is incomplete.

Econometric modeling that combines bounded iterative thinking and noisy responses is intuitive and has proven to be a suitable explanation of experimental behavior. Goeree and Holt (2004) extended the notion of rationalizability (Bernheim, 1984; Pearce, 1984) by adding noise to iterative reasoning to explain initial responses in  $2 \times 2$  normal form games. Crawford and Iriberri (2007) analyzed the initial rounds of play in the private value auction experiment of Goeree et al. (2002) with a model of level- $k$  types playing alongside QRE types. Our finding that pluralistic models including logit equilibrium and level- $k$  types outperformed singular equilibrium models (EFR and QRE) relates closely to these studies.

To date, the complementarity of limited iterative reasoning and social preferences is largely unexplored. This approach is promising, though, as exemplified by Gneezy (2005) who added altruism to the cognitive hierarchy model of Camerer et al. (2004) to better explain behavior in first price auctions. In our experiment, as for the subjects with less incentive to join, the actions of L1 and L2 types are similar to those of conditionally spiteful equilibrium types. These subjects would have been erroneously misclassified as L1 and L2 types had social preferences not been controlled for. Social preferences should not be neglected by default when trying to model bounded rationality, as that might result in a misspecified model resulting in biased estimation and misclassified types. In this case, a one- or two-parametric specification of social preferences is a small price to pay for properly modeling behavior and its dynamics.

McKelvey and Palfrey (1995) show that QRE limits outcomes, and besides applying it to understanding initial play, it can serve as a reduced form learning model, e.g. one where precision increases over time (see also Turocy, 2005). Along these lines, using QRE and level- $k$  as equilibrium refinement tools, they can be used to analyze not only initial responses but even behavior given experience, as we have done. Stahl (1996), for example, estimated learning effects of rules and precision transitions over time using beauty contest data over four rounds of Nagel (1995). Our experimental design provided subjects with actual learning opportunities spanning both the vertical dimension, where subjects repeatedly played games where the relevant treatment effect of incentives was kept constant, and the horizontal dimension, where the incentive effect changed over time. We found that with vertical experience, subjects converged to equilibrium reasoning, whereas with horizontal experience, subjects increasingly

adopted level- $k$  reasoning. The former result points in a direction similar to the intuition that “in all but the simplest games, equilibrium concepts will have the most explanatory power when people have the opportunity to learn about others’ decision probabilities through experience” (Goeree and Holt, 2004, p. 369). But, it must be noted that there is another form of “experience,” one which spans across different subsets of repeated games novel relative to one another. With such experience, level- $k$  reasoning sets in over time.

In this sense, it is shown that both QRE and level- $k$  are relevant modes of reasoning that can co-exist, and that each model is more relevant with respects to different paths of learning. The applicability of level- $k$  reasoning extends to learning, notwithstanding that subjects gain experience, when exposed to a diversity of games. Previous applications analyzed the relevance of level- $k$  reasoning to either novel situations in terms of one-shot games (e.g., Costa-Gomes et al., 2001; Costa-Gomes and Crawford, 2006), or initial play, i.e. blocks of games (Crawford and Iriberri, 2007). Our results show that the relevance of level- $k$  reasoning to the notion of “initial responses” is robust to the accumulation of vertical experience (the dynamics of which are better captured by QRE), i.e. to the level of vertical experience as long as it is kept constant. A possible interpretation is that QRE serves as a means to adapting to an environment, whereas level- $k$  serves to transfer knowledge across varying environments. Finally, it is interesting to note that, in our setting, the role of social preferences observed at the outset diminished over time; the population converged to one where strategic reasoning dominated. In this sense, subjects learned to be “reasonable.”

## 8 Conclusions

This paper modeled strategic choice in an experiment where subjects played varyingly parameterized extensive-form games with incomplete information. In these games, beliefs off the equilibrium path are not restricted by standard refinement concepts, but they are in the limiting logit equilibrium and under extensive form rationalizability (i.e. limiting level- $k$  reasoning). This allowed us to investigate the complementary relevance of logit equilibrium and level- $k$  reasoning in the learning dynamics. Throughout the analysis, we considered social preferences, namely a one-parametric model of conditional spite, jointly with quantal response and level- $k$  reasoning in order to avoid

mis-specification of the learning process. Our key findings are that with accumulation of experience in relatively constant environments, subjects approach equilibrium via the QRE learning path; with experience spanning also across relatively novel environments, though, level- $k$  reasoning is increasingly adopted.

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Table 10: Extension of Table 7 by using QRE predictions as additional regressors

(\* and \*\* denote significance at .05 and .01 level, resp., and the  $t$ -values are given in parentheses)

	Intercept	DOMINANT	DOMINATED	UNCLEAR	LEVREAS	INCENTIVE	EXTERNALITY	TYPE	EQ-PREDICTION
QRE	0.771 (1.840)	0.352 (1.497)	-0.890* (-2.502)	-0.153 (-1.053)	-0.205* (-0.1970)	0.065 (0.469)	0.022 (0.290)	-0.278** (-3.228)	2.192** (6.939)
QRE + Ineq	-0.973 (-1.722)	0.809** (3.374)	-0.141 (-0.358)	-0.212 (-1.529)	-0.016 (-0.145)	0.011 (0.083)	0.006 (0.081)	-0.062 (-0.646)	2.842** (7.629)

Table 11: Estimates and BICs for various model specifications, using  $BIC = -LL + k/2 \cdot \ln(\#obs)$ 

Model	Level 1			Level 2		Level 3		QRE 1		QRE 2		$\alpha$	$BIC_{(LL)}$
	$\rho_0$	$\rho_1$	$\lambda_1$	$\rho_2$	$\lambda_2$	$\rho_3$	$\lambda_3$	$\rho_e$	$\lambda_e$	$\rho_f$	$\lambda_f$		
<i>A</i>	.067							.933 (0.031)	0.0675 (0.0043)				773.04 (-768.76)
<i>B</i>	.061	.280 (0.059)	1.142 (0.298)					.658 (0.063)	0.061 (0.003)				741.20 (-732.64)
<i>C</i>	.045	.182 (0.066)	1.651 (0.677)	.599 (0.089)	0.102 (0.0085)			.174 (0.072)	0.044 (0.007)				707.28 (-694.45)
<i>D</i>	0	.179 (0.066)	2.754 (3.215)	.328 (0.075)	0.053 (0.005)	.462 (0.090)	0.107 (0.011)	0.031 (0.021)	0 (-)				707.44 (-690.33)
<i>E</i>	.044	.114 (0.077)	1.168 (0.457)	.576 (0.089)	0.097 (0.008)			.144 (0.073)	0.041 (0.010)	.122 (0.073)	1.915 (0.121)		704.54 (-687.44)
<i>F</i>	.047	0 (-)	- (-)	.167 (0.072)	0.045 (0.008)			.785 (0.069)	0.092 (0.0026)			0.142 (0.0016)	692.20 (-677.23)





# Supplementary material

## Appendix A: Relegated proofs

**Proof of Proposition 3.1** Assume an equilibrium exists where some  $i \in N$  does not join with probability 1 in an information set that is reached with positive probability along the path of play. Then there also exists an equilibrium where some  $i \in N$  does not join with probability 1 in all information sets along the path but all  $j < i$  do so (note that  $i = 1$  may apply, in which case the set of  $j < i$  is empty). Fix any information set along the equilibrium path where  $i$  does not join with probability. By assumption, all  $j < i$  join with probability 1 if  $i$  does not join in this information set. This implies, by Eq. (4), that  $i$  is best off not to join only if he believes that less than  $i - 1$  opponents join overall if he joins in that information set, which can be satisfied only if some player  $j < i$  does not join if  $i$  joins in that information set. Hence  $i > 1$ ,  $i$  joins with probability 0 in this information set (according the assumed equilibrium), and at least one  $j < i$  has not moved yet (otherwise,  $i - 1$  opponents would have joined already). First, consider the case that  $i$ 's information set is reached through a history of actions where exactly one  $j < i$  has not moved yet. In this case, if  $i$  joins, then  $j$  will have to move in an information set where at least  $i - 1$  opponents have already joined, which implies that  $j$  joins by sequential rationality (contradicting the assumption that he does not join with probability 1). Second, consider the case that  $i$ 's information set is reached through a history of actions where more than one  $j < i$  has not moved yet. In this case, by the assumption that all move sequences have positive probability,  $i$ 's joining in this information set does not imply that all of the succeeding opponents find themselves in information sets off the equilibrium path. Regardless of the actual move sequence, only the lastly moving player of the  $j < i$  can find himself in an information set off the path. All other  $j < i$  join because they are in information sets compatible with the equilibrium path, and the lastly moving  $j < i$  joins by sequential rationality. Hence, all  $j < i$  join even if  $i$  joins, which implies a contradiction.  $\square$

**Definition A.1** (Limiting logit equilibrium). A strategy profile  $\sigma$  is a limiting logit equilibrium if there exists a sequence  $(\lambda^r)$  converging to  $\infty$  and a sequence  $(\sigma^r)$  converging to  $\sigma$  such that for all  $r$  and all  $(i, t, k)$ :  $\sigma_i^r(t, k) = \exp\{\lambda^r * \pi_i(1, \sigma|t, k)\} / (\exp\{\lambda^r * \pi_i(0, \sigma|t, k)\} + \exp\{\lambda^r * \pi_i(1, \sigma|t, k)\})$ , where  $\pi_i(a_i, \sigma|t, k)$  is  $i$ 's expected payoff under

$\sigma$  if he deviates to  $a_i \in \{0, 1\}$  with probability 1 in  $(i, t, k)$  conditional on reaching the information set  $(i, t, k)$ .

**Proof of Proposition 3.2** By Theorem 2 of McKelvey and Palfrey (1998), all limiting logit equilibria (LLEs) correspond with strategies of sequential equilibria as  $\lambda$  approaches infinity. By Prop. 3.1 this implies that all players join in all information sets along the path of play, and by dominance arguments, it implies that they join with probability 1 in information sets  $(i, t, k)$  where  $k \geq i - 1$  and with probability 0 in information sets  $(i, t, k)$  where  $k + n - t < i - 1$ . The following shows that they join with probability 1 in the remaining information sets. Let  $\sigma$  be a LLE as defined above (i.e. the limit of QREs as  $\lambda$  approaches infinity). First, we show that all players  $i < n$  join with probability 1 in all information sets  $(i, t, k)$  where  $k = t - 2$  (i.e. where exactly one opponent had chosen not to join). All players join with probability 1 along the path of play, i.e. in all information sets  $(i, t, k)$  where  $k = t - 1$ . Hence, as  $\lambda$  tends to infinity, the expected payoff of  $i$  in such an information set from joining equates with  $p_i(1, n - 1)$ , and the expected payoff from not joining equates with  $p_i(0, n - 1)$ . Hence, in the limit, the probability that  $i$  does not join when  $k = t - 1$  is

$$1 - \sigma_i(t, k) = \frac{\exp\{\lambda \cdot p_i(0, k)\}}{\exp\{\lambda \cdot p_i(0, k)\} + \exp\{\lambda \cdot p_i(1, k)\}} = \frac{\exp\{\lambda \cdot 0\}}{\exp\{\lambda \cdot 0\} + \exp\{\lambda \cdot [p_i(1, k) - p_i(0, k)]\}}$$

and thus for all  $i < j \leq n$ , the following is a consequence of Eq. (3) as  $\lambda$  tends to infinity.

$$\frac{1 - \sigma_i(t, k)}{1 - \sigma_j(t, k)} = \frac{\exp\{\lambda \cdot 0\} + \exp\{\lambda \cdot [p_j(1, k) - p_j(0, k)]\}}{\exp\{\lambda \cdot 0\} + \exp\{\lambda \cdot [p_i(1, k) - p_i(0, k)]\}} \xrightarrow{\lambda \rightarrow \infty} 0 \quad (20)$$

Speaking loosely, player  $i$  is infinitely less likely (as  $\lambda \rightarrow \infty$ ) to deviate from the equilibrium path than  $j$  if  $i < j$ . Hence, in all information sets  $(i, t, k)$  with  $k = t - 2$  and  $i < n - 1$ ,  $i$  believes that with probability 1 it was player  $j = n$  who deviated from the equilibrium path, and hence  $i$  is best off joining, and uniquely so by Eq. (2).

In a similar manner, we can show that all  $i < n - 1$  are uniquely best off joining in all information sets  $(i, t, k)$  where  $k = t - 3$ , i.e. when two opponents have already chosen not to join. All players  $i < n - 1$  believe that with probability 1 this must have been the players  $n - 1$  and  $n$ , and hence all  $i < n - 1$  are uniquely best off joining. By logical induction, the proposition can thus be proved for all information sets.  $\square$

**Definition A.2** (Extensive form rationalizability). Let  $I_i(\sigma_{-i})$  denote the set of  $I \in I_i$  that are reached with positive probability under a given  $\sigma$  (which is independent of his own strategy). Similarly, for any  $\Sigma'_{-i} \subseteq \Sigma_{-i}$ , let  $I_i(\Sigma'_{-i})$  denote the  $I \in I_i$  such that there exists  $\sigma_{-i} \in \Sigma'_{-i}$  for which  $I \in I_i(\sigma_{-i})$ . The payoff of  $i \in N$  conditional on information set  $I \in I_i$  being reached is denoted  $\pi_i(\sigma|I)$ . Now define  $\Sigma_i^1 = \Sigma_i$  for all  $i \in N$ , and for all  $t$ ,

$$\Sigma_i^{t+1} = \left\{ \sigma_i \in \Sigma_i^t \mid \forall I \in I_i(\Sigma_{-i}^t) \exists \sigma_{-i} \in \Sigma_{-i}^t : \right. \\ \left. I \in I_i(\sigma_{-i}) \text{ and } \sigma_i \in \arg \max_{\sigma'_i \in \Sigma_i^t} \pi(\sigma'_i, \sigma_{-i}|I) \right\}. \quad (21)$$

The strategies in  $R_i = \cap_{t \geq 1} \Sigma_i^t$  are extensive form rationalizable for player  $i$ . Note that this definition has been simplified suitably to fit our context. In relation to Pearce's Definition 9, point (vi) simplifies to a conditional payoff calculation. Besides, as each player moves exactly once along any path of play, points (ii) and (iii) become redundant (which allows us to skip the explicit notation of conjectures).

**Proof of Proposition 3.3** For any information set  $(i, t, k)$ , let  $\bar{n}(t, k) = k + (n - t)$  denote the number of opponents that will have joined eventually if all later players join with probability 1. After the first round of eliminating strategies, player  $i$  joins with probability 1 in all information sets  $(i, t, k)$  where  $k \geq k_i^*$ , and he joins with probability 0 if  $\bar{n}(t, k) < k_i^*$ . As for player 1, this is equivalent to the claimed strategy Eq. (8). As for players  $i > 1$ , this includes all information sets where  $i$  is claimed not to join. We have to show that all  $i > 1$  join with probability 1 in the remaining information sets as the number of iterations  $\tau$  approaches infinity. Following Def. A.2, let  $\Sigma_i^\tau$  denote the set of  $i$ 's strategies that are not eliminated prior to round  $\tau \geq 2$ . Moreover, for all information sets  $(i, t, k)$ , let  $A_{\tau, i}(t, k)$  denote  $i$ 's set of rationalizable actions in  $(i, t, k)$  and round  $\tau$ . Assume that the following assumptions hold true in round  $\tau$  of the induction (noting that they are satisfied in round  $\tau = 2$ , i.e. after round 1). (A1)  $i$ 's strategy set is the product set of the action sets  $A_{\tau, i}$ , i.e.  $\Sigma_i^\tau = \times_{I \in I_i} A_{\tau, i}(I)$ . (A2) For all players  $i < \tau$  and all  $(t, k)$ ,  $A_{\tau, i}(t, k) = \{1\}$  if  $\bar{n}(t, k) \geq k_i^*$  and  $A_{\tau, i}(t, k) = \{0\}$  if  $\bar{n}(t, k) < k_i^*$ . (A3) For all  $i \geq \tau$  and all  $(t, k)$ ,  $A_{\tau, i}(t, k) = \{1\}$  if  $k \geq k_i^*$ ,  $A_{\tau, i}(t, k) = \{0\}$  if  $\bar{n}(t, k) < k_i^*$ , and either  $A_{\tau, i}(t, k) = \{1\}$  or  $A_{\tau, i}(t, k) = [0, 1]$  if  $\bar{n}(t, k) \geq k_i^* > k$ . We claim that if the induction assumptions (A1)-(A3) are satisfied in round  $\tau$ , then they are also satisfied in round  $\tau + 1$ . It follows that (A1)-(A3) hold true for all  $\tau \geq 2$ , hence that (A2) holds true for  $\tau \geq n$ , which proves the proposition.

Let  $I'_i \subseteq I_i$  denote the subset of  $i$ 's information sets where  $A_{\tau,i}(I) = [0, 1]$ . By (A3),  $\bar{n}(r, h) \geq k_i^*$  holds true in all  $(i, t, k) \in I'_i$ . Fix  $(i, t, k) \in I'_i$ , and define an ordering  $R \in \mathcal{R}$  such that  $R(i) = t$  and  $R^{-1}(t') > i$  for as many  $t' < t$  as possible, given  $(t, k)$ , and where these players  $j > i$  move initially. As a consequence of  $(i, r, h) \in I'_i$  and  $A_{\tau,i}(i, t, k) = [0, 1]$ , it can be shown that for all players  $j > i$  that move prior to  $i$  under the ordering  $R$ ,  $A_{\tau,j}(t', k') \ni \{0\}$  applies in their respective information sets. By (A1), this implies that exists  $\sigma_{-i} \in \Sigma_{-i}^t$  such that  $\sigma_j(t', k') = 0$  for all these  $j > i$  and the respective information sets. Hence, there exists some  $\sigma_{-i}$  such that information set  $(i, t, k) \in I'_i$  will be reached if ordering  $R$  is chosen. The ordering  $R$  is chosen with positive probability, and thus, any  $(i, t, k) \in I'_i$  can still be reached along the path of play (given  $\Sigma^\tau$ ).

An implication of (A2) is that all players  $j < \tau$  join with probability 1 along all paths of play that are compatible with  $\Sigma^\tau$ . Hence, for any  $\sigma \in \Sigma^t$  and any move order  $R$ , at least  $\tau - 1$  players join. By (A1) this applies regardless of how player  $i = \tau$  moves in any information set  $(i, t, k) \in I'_i$ . For player  $i = \tau$ , this implies by Eq. (4) that player  $i$  is uniquely best off joining in any  $(i, t, k) \in I'_i$ . Since all of those information sets can be reached along the path of play under  $\Sigma^\tau$  in round  $\tau$ , this confirms (A2) for  $\tau + 1$ . (A3) follows, because for all  $j > \tau$  in information sets  $(j, t, k)$  where  $\bar{n}(t, k) \geq k_j^* > k$ , the equilibrium prediction  $A_{\tau,j}(t, k) = \{1\}$  is obviously robust to elimination, and if any other strategy is eliminated in such an information set, then all but the equilibrium prediction get eliminated (since the action sets are binomial in every information set). Finally, (A1) applies also for  $\tau + 1$  since every player moves exactly once for every move order.  $\square$

## Appendix B: Experimental Instructions

This is the English translation of the German experimental instructions for stages 1–4.

### General Instructions

You are about to participate in an experiment on decision-making. The experiment is divided into five stages, and each stage is divided into rounds. During the experiment you will earn *experimental points*.

At the start you are assigned an *initial endowment* of 60 experimental points. Each experimental point you earn in the experiment is worth 10 cents.

At the end of the experiment, three *winning rounds* will be randomly chosen by the computer, two from stages 1 through 4, and one from stage 5. What you earn in the winning rounds will be added or subtracted from the initial endowment to determine your final winnings. You will not know which rounds are the winning rounds until the end of the experiment.

There are twelve participants in the experiment. There are four *types of participants*, which we label as P, Q, R and S. There are three participants of each type. You can find out which type you are by looking at the right-hand corner of your computer display. Participant types are only relevant for stages 1 through 4. Your participant type will stay the same throughout the experiment.

### **Your Decision in Stages 1 Through 4**

There are eight rounds in each of stages 1 through 4. Each round you are *matched* with three other participants (*coparticipants*) in the room, *one of each participant type*, as a *set of participants*. Therefore, each set of participants is always made of four people and always has a P participant, a Q participant, a R participant and a S participant. If for example you are a S participant, this means that your coparticipants will always be a P participant, a Q participant and a R participant; a similar reasoning applies if you belong to one of the other participant types.

Coparticipants are chosen randomly each round from each of the other participant types, and so it is very unlikely that you will be matched in the same set of participants as you move from one round to the next.

In each round *you, and your matched coparticipants, are each asked in turn to decide whether you would like to become a member of a club*. You have to choose between either “yes” or “no”. How much you earn from the round depends on three factors:

- (a) your action;
- (b) how many of your matched coparticipants choose “yes”;
- (c) your participant type.

Information is provided each round in the form of decision tables. An illustrative example of decision table (**not** used in the experiment) is as follows:

	Final number of other participants who choose “yes”			
	0	1	2	3
Yes	30	40	50	60
No	60	50	40	30

If this decision table were to apply to you, it would tell you the amount of experimental points that you would earn from choosing “yes” or “no”, depending on the number of other participants who choose “yes”. For example, if you chose “yes” and two other participants also chose “yes”, you would earn 50 experimental points; similarly, you would get 50 points if one other participant chose “yes” but you chose “no”.

Decision tables vary according to the participant type, and the computer screen is currently displaying those that are applicable to stage 1 for the P participant, Q participant, R participant and S participant. Everyone is being given exactly the same set of information as that provided in this sheet and on the computer screen.

Decisions are taken in turns. The order in which you and your coparticipants make decisions is determined at random each round. Click on Check to find out if it is your turn. A message will appear when it is your turn to make your decision. It will inform you of how many coparticipants have made their decisions and chosen “yes” so far in the round.

When it is your turn to decide, and you have made your decision, if you choose “yes”, click the “yes” button; if you choose “no”, click the “no” button. To confirm your decision, click on Confirm, and a message box will appear asking if you are sure of your decision. If so, click on OK in the message box, and then on Confirm again. If not, you may change your decision either by clicking on Cancel, and then entering your new decision. You may change your decision anytime before the 2nd click on Confirm.

After everyone has made their decision for the round, you will be told the final number of other participants who have chosen “yes” and your corresponding earning for the round. You may then move on to the next round by clicking on Continue.

The same set of decision tables is used for all the rounds in a stage; however,

decision tables change across stages.

Before starting stage 1, we ask you to answer a brief questionnaire, with the only purpose of checking whether you have understood the instructions. Raise your hand when you have completed the questionnaire.

Many thanks for your participation to the experiment, and good luck!

**Please raise your hand if you have any questions.**



Table 13: Joining rates (numbers of observations) for all information sets

Treatment A					Treatment B				
Type 1	Mover 1	Mover 2	Mover 3	Mover 4	Type 1	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	0.97(35)	1(16)	1(15)	1(5)	0 joined	0.97(31)	1(9)	1(3)	1(2)
1 joined		1(16)	1(22)	1(10)	1 joined		1(22)	1(16)	1(2)
2 joined			1(8)	1(12)	2 joined			1(21)	1(11)
3 joined				1(5)	3 joined				1(27)
Type 2	Mover 1	Mover 2	Mover 3	Mover 4	Type 2	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	0.84(31)	0.71(14)	0.43(7)	(0)	0 joined	0.92(37)	1(10)	0.33(3)	(0)
1 joined		1(23)	1(19)	0.95(19)	1 joined		1(33)	1(11)	0.75(4)
2 joined			0.9(10)	1(18)	2 joined			0.96(25)	1(9)
3 joined				0.67(3)	3 joined				0.92(12)
Type 3	Mover 1	Mover 2	Mover 3	Mover 4	Type 3	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	0.59(39)	0.27(15)	0(2)	0(1)	0 joined	0.86(43)	0.6(10)	(0)	(0)
1 joined		0.56(27)	0.42(12)	0(3)	1 joined		0.83(24)	0.89(9)	1(1)
2 joined			1(14)	0.96(24)	2 joined			1(24)	1(13)
3 joined				1(7)	3 joined				1(20)
Type 4	Mover 1	Mover 2	Mover 3	Mover 4	Type 4	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	0.15(39)	0.1(10)	(0)	(0)	0 joined	0.36(33)	0(2)	(0)	(0)
1 joined		0.13(23)	0.1(10)	0.5(2)	1 joined		0.71(34)	0.33(3)	(0)
2 joined			0.32(25)	0(11)	2 joined			0.86(29)	0.5(8)
3 joined				0.71(24)	3 joined				0.94(35)
Treatment C					Treatment D				
Type 1	Mover 1	Mover 2	Mover 3	Mover 4	Type 1	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	1(28)	1(15)	1(8)	1(6)	0 joined	1(40)	1(7)	(0)	0.5(2)
1 joined		1(24)	1(14)	1(4)	1 joined		1(27)	0.9(10)	1(2)
2 joined			1(17)	1(18)	2 joined			1(29)	1(9)
3 joined				1(10)	3 joined				1(18)
Type 2	Mover 1	Mover 2	Mover 3	Mover 4	Type 2	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	0.93(45)	0.83(12)	0(4)	(0)	0 joined	0.97(37)	0.75(8)	(0)	(0)
1 joined		0.95(19)	1(22)	0.92(12)	1 joined		1(30)	1(12)	1(1)
2 joined			1(13)	0.92(12)	2 joined			0.96(23)	1(4)
3 joined				0.8(5)	3 joined				0.97(29)
Type 3	Mover 1	Mover 2	Mover 3	Mover 4	Type 3	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	0.62(45)	0.5(10)	0(2)	(0)	0 joined	0.87(30)	1(4)	0(2)	(0)
1 joined		0.87(31)	0.5(12)	0(1)	1 joined		0.88(32)	1(6)	(0)
2 joined			1(13)	0.91(23)	2 joined			0.96(28)	1(13)
3 joined				1(7)	3 joined				1(29)
Type 4	Mover 1	Mover 2	Mover 3	Mover 4	Type 4	Mover 1	Mover 2	Mover 3	Mover 4
0 joined	0.08(26)	0(7)	(0)	(0)	0 joined	0.62(37)	(0)	(0)	(0)
1 joined		0.31(26)	0.2(5)	0(1)	1 joined		0.75(36)	0(2)	(0)
2 joined			0.35(34)	0.08(12)	2 joined			0.88(32)	0.14(7)
3 joined				0.76(33)	3 joined				0.93(30)

(white) joining is the dominant action, without iterative eliminations

joining is uniquely EF-rationalizable, but not uniquely trembling-hand perfect

joining is not rationalizable