A Numerical Model to Predict Matric Suction Inside Unsaturated Soils

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The objective of this research is to introduce a numerical simulation model to predict approximate values of the matric suction inside unsaturated soils that have low water contents. The proposed model can be used to predict the relationship between the water content and the matric suction of a studied soil to construct the soil-water characteristic curve. In addition, the model can be utilized to combine the predicted matric suction with the soil parameters obtained experimentally, which enables us to explain how matric suction can affect the behaviour of unsaturated soils, without the need to utilize advanced measuring devices or special testing techniques. The model has given good results, especially when studying coarse-grained soils.

Keywords: Unsaturated soil, matric suction, surface tension, soil-water characteristic curve.

1 Introduction

The mechanical behaviour of unsaturated soils is greatly influenced by the degree of saturation and, consequently, by the matric suction. Matric suction is a function of many soil properties such as the grain size and the geometry of the pores constrained between the soil particles. In addition, matric suction depends on the pore fluid properties such as the interfacial forces, density, and the degree of saturation. Based on these relations, a simple numerical simulating model is introduced in this study to predict the relationship between the matric suction and water content inside unsaturated relatively dry samples (i.e., samples with low water contents). The suggested model, basically, makes use of the surface tension and the capillary action phenomena of the water between the particles in addition to the grain-size distribution curve of the soil under investigation. Then, the relationship between the water content and the matric suction is predicted and the corresponding soil-water characteristic curve can be constructed. However, the values of the predicted suction are approximate, and thus they can be used as a simple, quick indicator of how much (i.e., in which range) the matric suction will be inside the investigated soil.

2 Suction in unsaturated soils

The total suction, ψ, in an unsaturated soil is, generally, made up of two components; namely, matric suction and osmotic suction. The sum of these two components is called the total suction. Matric suction is defined as the difference between the pore-air pressure and the pore-water pressure (i.e., matric suction $= u_a - u_w$), while osmotic suction, η, is a function of the amount of dissolved salts in the pore fluid. Therefore, matric suction is attributed mainly to capillary actions in the soil structure, while osmotic suction is associated with physico-chemical interactions between soil minerals and pore water [13]. However, matric suction is of primary interest because many engineering problems involving unsaturated soils are commonly the result of environmental changes, which primarily affect the matric suction of the soil [9]. For most cases, environmental changes primarily affect the matric suction component, while osmotic changes are generally less significant. In other words, a change in total suction is essentially equivalent to a change in matric suction (i.e., $\Delta\psi \approx \Delta(u_a - u_w)$) [1]. In contemporary unsaturated soil mechanics theories, an element of soil is often considered as a simple three-phase system consisting of pore-air, pore-water and granular solid particles. Matric suction in such a system arises from capillary actions attributed to interactions between air-water menisci (which are generated from the surface tension phenomenon), and soil particles.

3 Role of surface tension

One of the most important properties that affect the matric suction is the surface tension. The phenomenon of surface tension results mainly from the intermolecular forces acting on molecules in the air-water interface, which is known as the contractile skin. A water molecule within the contractile skin experiences an unbalanced force towards the interior of the water. In order for the contractile skin to be in an equilibrium condition, a tensile pull is generated along it. The tensile pull is tangential to the contractile skin surface. Therefore, surface tension causes the contractile skin to behave like a stretched elastic curved membrane. If the contractile skin has a double curvature, (i.e., a three-dimensional membrane), the total excess in pressure acting on the membrane can be calculated as the sum of the components obtained in two principal directions as follows:

$$\Delta u = T_s \left(\frac{1}{R_1} + \frac{1}{R_2} \right). \tag{1}$$

Equation (1) is known as the Laplace equation of capillarity [1], where R_1 and R_2 are the radii of curvature of the membrane in two orthogonal principal planes, while T_s denotes the surface tension as illustrated in Fig. 1.

In soils, the surface of the grains tends always to adsorb water more strongly than air, while the air is compressed if it is completely encompassed by water. That is why, in unsaturated soils, the air pressure is always higher than that of the water. Thus, the contractile skin is assumed to be subjected to an air pressure, u_a, greater than the water pressure, u_w. The difference between these two pressures is the matric suc-

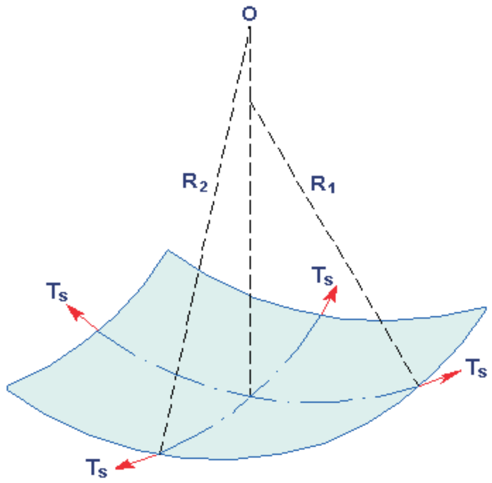


Fig. 1: Surface tension in the contractile skin

tion, $(u_a - u_w)$, and consequently, the pressure difference that causes the contractile skin to curve according to Eq. (1), can be formulated as:

$$(u_a - u_w) = T_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \quad (2)$$

Eq. (2) clearly shows that as the radius of curvature of the contractile skin decreases, the matric suction of a soil increases. This supports the fact that matric suction increases as a result of decreasing the water content of the soil. At the opposite extreme, when the pressure difference between the pore-air and pore-water goes to zero, the radius of curvature, R, goes to infinity. Thus, a flat air-water interface exists when the matric suction goes to zero, which is the case of a fully saturated soil. However, the contractile skin may be completely concave or a combination of concave and convex in orthogonal directions [3]. It is only necessary that the relative magnitudes of R_1 and R_2 are such that they balance equation (2). In fact, when the contractile skin spans between collections of soil particles, dependent on the geometry, it is necessary for continuity that the interface has curvature both concave and convex to the air phase [8]. This case arises obviously in unsaturated relatively dry soils, where the water menisci are concentrated only between the soil particles and there is no continuity between these menisci. In such a case, the matric suction can be calculated by converting Eq. (2) to the following equation:

$$(u_a - u_w) = T_s\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \quad (3)$$

4 Description of the numerical model

The numerical model introduced in the current study takes into consideration the effect of the capillary water on the behaviour of unsaturated soils. The soil is simulated by replacing its particles with a system of spheres, whereas water is supposed to exist at the contact points between the spheres as capillary water. Harr [6] stated that natural soil particles in the silt-size range, and coarser ranges generally, are bulky and fairly equidimensional. Therefore, there will be a small approximation in simulating the particles of coarse-grained soils by a system of spheres. Unfortunately, proposing a constitutive simulation model that takes into consideration the exact or even an approximate shape to simulate the real shape for clay particles is very complicated. Thus, the simulation model presented in the current study focuses on predicting the relationship between the water content and matric suction for coarse-grained soils. However, in the case of fine-grained soils, the spheres might be considered as packets of saturated clay particles (i.e., as aggregations of clay particles). In this case, the predicted values will be approximate ones, and thus they can be used only to indicate the range in which the matric suction will be inside such soils.

The suggested model takes into consideration the effect of both the packing pattern of the spherical particles and the ratio of the voids restricted between them. In addition, the matric suctions generated by the surface tension acting on the meniscus are calculated by taking into consideration the effect of the grain sizes, the water content, and the specific gravity of the particles. Fig. 2a represents a 3D illustration of four spherical particles and the pore-water menisci accumulated at points of contact as well as the surface tension forces acting on the water menisci. The shape of the capillary water at the points of contact between the particles can be considered as a center-pinched cylinder with two extracted segments of spheres, (one from its top and the other from the bottom), as shown in Fig. 2b.

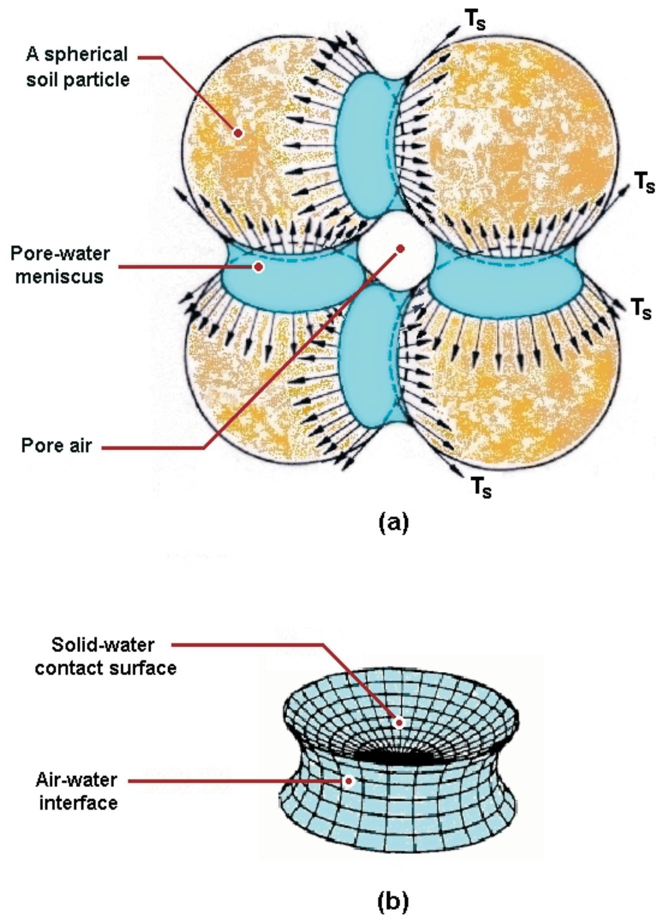
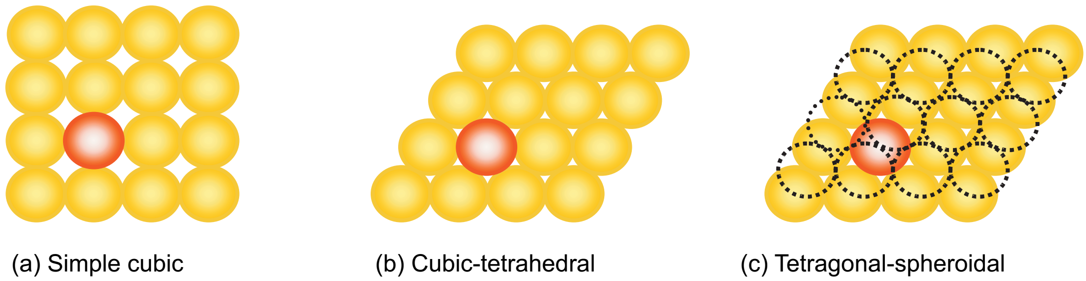
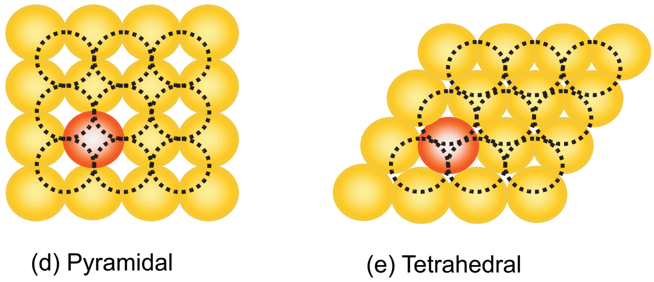


Fig. 2: 3D representation of the numerical model: (a) forces acting on spherical particles due to surface tension, (b) water meniscus at one point of contact between two particles





No.	(e)	$*(n)\%$	(n_c)
(a)	0.910	47.64	6
(b)	0.654	39.54	8
(c)	0.432	30.19	10
(d)	0.350	25.95	12
(e)	0.350	25.95	12

$* n\% = [e/(1+e)] \times 100$

Fig. 3: Theoretical packing of identical spheres

4.1 Numerical model geometry and calculations

A series of mathematical equations are adopted to introduce a simple constitutive relationship between matric suction and water content. The effect of the amount of water distributed at the contact points between the spheres is investigated. In the proposed model, matric suction can be evaluated as a function of the water content accumulated between the soil particles. Some studies have proposed similar models, but most of them have considered only the equal-sized particle to be the case, e.g., [7], and [12]. Therefore, the effect of considering non equal-sized spheres in the calculation of the matric suction and the water content is investigated in this study. Fig. (3) summarizes the possible theoretical packing of identical spheres where n and e are the porosity and the void ratio of the packing, respectively, while n_c denotes the number of points of contact for each particle.

4.1.1 Geometry of the spherical model

A simple representation of the proposed model can be introduced in two-dimensional space by considering the cross section of any two contacting identical spherical particles. The objective of studying the geometry of these two particles is to determine the main dimensions of the inter-particle water meniscus. Once these dimensions are known, the volume of the water accumulated between any two particles can be easily calculated. Then the water content can be evaluated as a function of the volume of the water and the volume of the particle. This can be achieved by taking into consideration the number of contact points for each possible packing pattern shown in Fig. 3. Fig. 4a shows two identical spheres, having equal radii of R, placed vertically and holding capillary water (contact moisture) between them. For simplification, the contact angle, β, between the water meniscus and the surface of the spherical particles is assumed to be zero.

The water menisci can be considered as a combination of concave and convex in orthogonal directions. The radii of curvature that define the geometry of the water meniscus in two orthogonal directions are "r_0" and "r_1". The volume of the water content accumulated between any two particles can be calculated as a function of the water retention angle, θ. From a geometrical point of view, increasing angle θ leads to an increase in the size of the water meniscus and simultaneously increases the volume of water. The main condition for the calculation to continue is the discontinuity between the regimes of the water menisci around the soil particles. Once the menisci begin to fuse with each other, the calculations cannot proceed because the shape of the meniscus will be irregular and very complicated to analyze. The menisci tend to fuse at an angle known as the critical water retention angle, θ_c [11]. From a geometrical consideration, this angle is equal to 45° for the simple cubic packing, while for the rest of the packing patterns, the fringes of the menisci begin to meet each other when the critical angle θ_c is equal to 30°.

According to Eq. (3), the matric suction generated by the water meniscus between the spherical particles depicted in Fig. 4a can be reformulated as:

$$(u_a - u_w) = T_s\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \tag{4}$$

From the geometry of the two contacting particles shown in Fig. 4a, the radii of curvature, r_0 and r_1, as well as dimensions x and z can be expressed using Eq. (5) through Eq. (8) as follows:

$$r_0 = R(\sec\theta - 1), \tag{5}$$

$$r_1 = R(1 + \tan\theta - \sec\theta), \tag{6}$$

$$x = R\sin\theta, \tag{7}$$

$$z = 2R(1 - \cos\theta). \tag{8}$$

Finally, it should be noted that "o_1" in Fig. 4a represents the distance from the centroid of the segment (a-b-c) to the

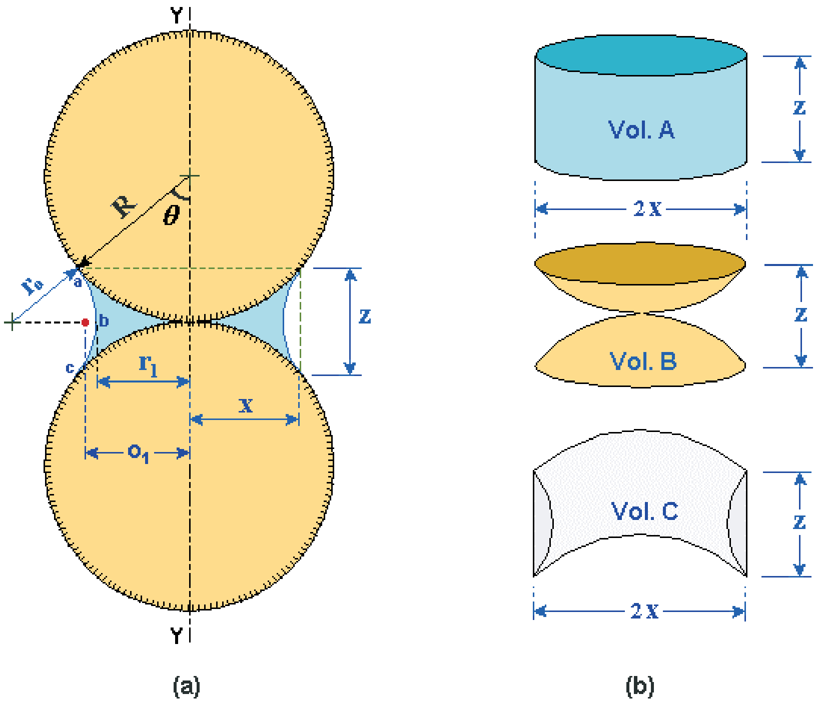


Fig. 4: Geometry of a spherical model for unsaturated soils: (a) main dimensions of water menisci between two particles, (b) volumes used to evaluate water content

axis of revolution (Y–Y). This distance can be calculated using the formulas described by Rektorys [10].

4.1.2 Calculations of water content

The water content of the studied model can be calculated as the ratio between half the weight of water accumulated at the points of contacts of any spherical particle and the weight of the particle itself. The volume of the water meniscus, V_m, accumulated at one of the points of contact between two particles can be calculated according to the volumes depicted in Fig. 4b as:

$$V_m = (Vol.A) - (Vol.B) - (2 \times Vol.C). \tag{9}$$

Now, the dimensions described in Eq. (5) through (8), will be substituted in Eq. (9) to formulate the volume of water accumulated at one point of contact between two particles. The final resulting formula to calculate the volume of the center-pinched cylinder with the two extracted segments of spheres will be as follows:

$$V_m = 2\pi R^3\left((\sin^2\theta)(1-\cos\theta) - \frac{2}{3} + \cos\theta - \frac{(\cos^3\theta)}{3}\right) - \pi R^3\left(\tan\theta(\sec\theta - 1)^2\left(\pi\frac{1-\theta}{90} - \sin 2\theta\right) - \frac{4}{3(\cos\theta)^3}\right). \tag{10}$$

Then, the percentage water content, w_c %, accumulated around one particle at all points of contact can be expressed as:

$$w_c = \frac{37.5\, n_c V_m}{\pi R^3 G_s}, \tag{11}$$

where:

n_c the number of points of contact for each particle, which depends on the packing pattern of the spherical particles,

G_s the specific gravity of the soil particles.

In addition, the volumetric water content, θ_w, can be evaluated as:

$$\theta_w = \frac{0.375\, n_c V_m}{\pi R^3 (1+e)}. \tag{12}$$

In the simulation, the input data are the radius, R (m), the specific gravity, G_s, the surface tension of the air water interface, T_s (MN/m), the angle of water retention, θ (degree), the number of points of contact between particles, n_c, and finally the void ratio, e. It can be seen that all these inputs can be measured using relatively simple laboratory measurements. In addition, the implementation of the model is as simple as possible. At this point, it should be clarified that the foregoing equations can be applied easily in the case of soils that contain a uniform, homogeneous particle size. However, for soils composed of more than one particle size, the model can also be applied along with the grain-size distribution curve as described below.

4.1.3 Calculations for non equal-sized spherical particles

When dealing with a soil composed of non equal-sized spherical particles, it is necessary for the current model to use a combination of the capillary action in the water menisci along with the grain-size distribution curve. First, the grain-size distribution curve is divided up into divisions of uniform soil particles. Then, for each particle size, the individual relationships between water content and matric suction can be built up by means of equations (5) through (12). Once the whole grain-size distribution curve has been incrementally analyzed, the individual relations between water content and matric suctions are summed together using a superposition technique to get the soil-water characteristic curve (SWCC) for the whole investigated soil. However, in the assemblage of soil particles, the voids created between larger particles are assumed to be filled with smaller particles. It should be noted that the effect of the interaction among spheres of different sizes is ignored in this model. A similar technique was suggested by Fredlund et al., [4] and [5], who also used the grain-size distribution curve in their model.

5 Numerical simulations

The numerical simulations presented below are devoted mainly to checking the reliability of the proposed model. Some relationships between the matric suction and the water content were calculated numerically using this model. Then it was possible to introduce the graphical representations of these relationships (i.e., SWCC). The first simulation studies the effect of particle size on the predicted SWCC. In the second simulation, the effect of the packing pattern of the spherical particles, and as a result the effect of the soil's porosity, on the SWCC is investigated. The third simulation is presented to verify the proposed model in the case of constructing the SWCC for a soil with non equal-sized grain particles, making use of some of the experimental and numerical results available in the literature.

Simulation 1:

This simulation investigates the effect of the grain size of a soil on the relation between the matric suction and the water content, using the proposed numerical model. The input data needed to calculate the water content and the corresponding matric suctions for this simulation are: $T_s = 0.075$ N/m and $G_s = 2.65$, while the suggested radius of the particle, R, ranges from 0.001 mm to 1.0 mm, which is the common range for sand and silt grains. Fig. 5 shows the SWCC calculated for five different grain sizes, assuming pyramidal packing to be the packing of the simulating spherical particles (thus, $n_c = 12$, $e = 0.35$). As expected, at the same water content, the larger the size of the spheres, the lower is the predicted matric suction. It can be clearly seen from Fig. 5 that the matric suctions calculated from the diameter of the large particles are of the order of kilo-Pascals, whereas the suctions evaluated from the diameters of the small particles are of the order of mega-Pascals. In fact, this behaviour is similar to that known for real soils. However, it is well known that matric suction can develop only in the presence of both water and air at their interface surface (i.e., at the contractile skin). That is why in the case of completely dry soil, where the water content is equal to zero, there is no contractile skin; and thus, there is no matric suction. Once the water flows into the pores an

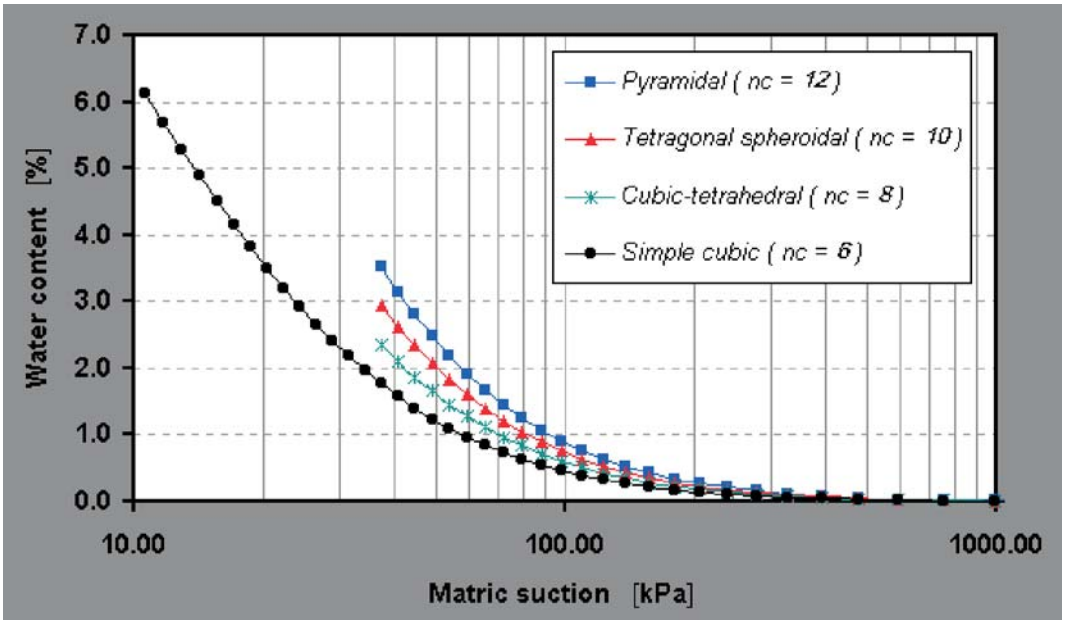


Fig. 6: Effect of the packing type on the behaviour of the simulated soil-water characteristic curve

the fact that the numbers of contacts between the particles in the case of a dense soil are innumerably large. In this case, the numbers of water menisci accumulated at the points of contact are also large. Consequently, the matric suction for a dense soil is believed to be extremely high.

Simulation 3:

The main objective of this simulation is to verify the possibility of using the suggested model to predict the SWCC in the case of a soil that comprises non equal-sized grain particles. This simulation makes use of one of the numerical and experimental results reported in 1997 by Fredlund et al., [4], who predicted a SWCC for sand using the grain-size distribution curve shown in Fig. 7. The data drawn out from this curve were input by Fredlund et al., [4] into the computer program "Soilvision", which is mainly based on the theoretical method suggested by Fredlund and Xing [2] to predict the SWCC. It can be seen from Fig. 7 that the studied sand has a relatively uniform particle size distribution (i.e., a low range of different grain sizes). However, such a relatively uniform distribution is speculated to be the ideal case for examining the reliability of the proposed model. The simulation processes were done by dividing the grain size distribution curve into small divisions of uniform soil particles as illustrated in Fig. 7. The SWCC is estimated for each soil division, and then the final SWCC is built up by the summation of all the divisional soil-water characteristic curves. The predicted results of the current model as well as both the laboratory measured and the analytically predicted SWCC

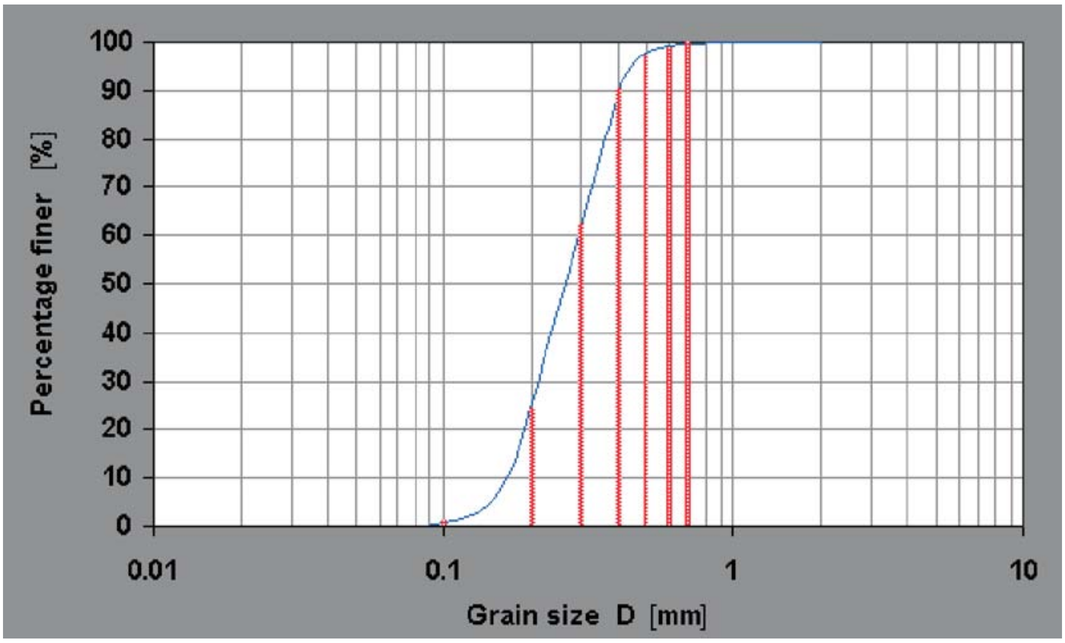


Fig. 7: Grain-size distribution curve of sand showing the dividing method used for the current study

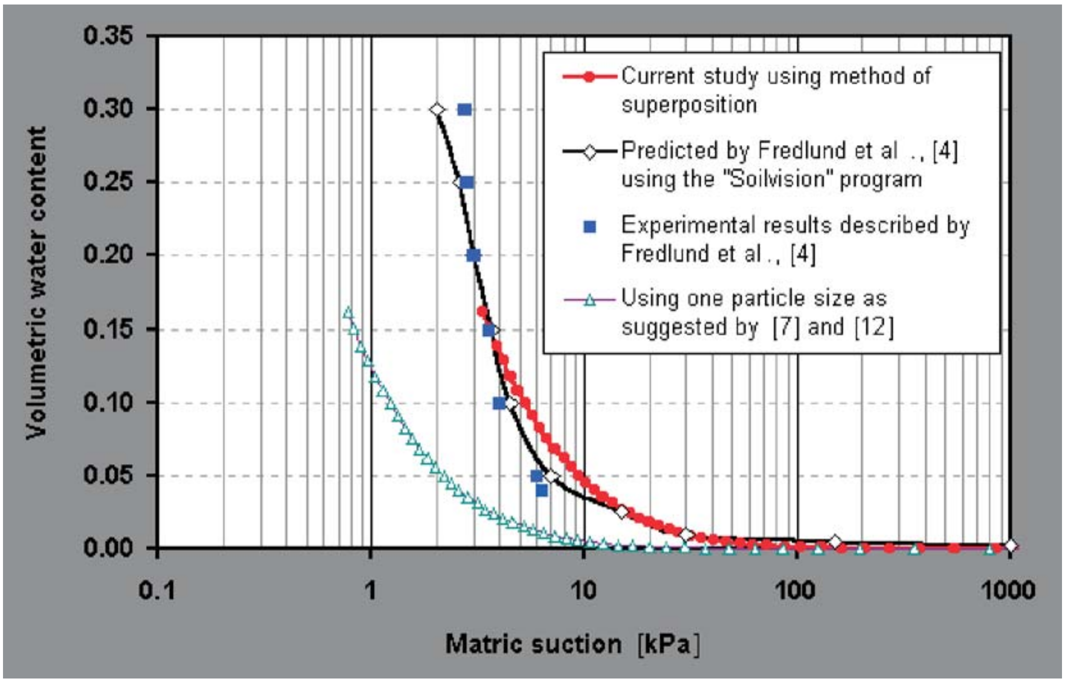


Fig. 8: A comparison between the SWCC of sand using the current numerical model and other methods

reported by Fredlund et al., [2] are all depicted in Fig. 8. As indicated in this figure, the predicted SWCC for uniform sand using the current model compared well to both measured and predicted curves introduced by Fredlund et al., [2]. It should be noted that some studies have proposed similar models, but most of them have considered only the equal-sized particle to be the case, e.g., [7], and [12]. Therefore, to illustrate the importance of considering different particle sizes in the simulation of a soil, an SWCC was also drawn in Fig. 8, taking into account only one particle size from the grain-size distribution curve shown in Fig. 7. In this case, an average particle size of 0.35 mm was taken into consideration (i.e., $R = 0.175$ mm). It is clear from the figure that considering only one particle size has under-predicted the SWCC for the simulated sand.

6 Conclusion

The proposed numerical model has given good results when studying coarse-grained soils. In general, this model can be used as a simple quick indicator of how much (i.e., in which range) the matric suction will be inside an investigated soil. However, the model has proved to be effective for sand that has a somewhat uniform grain-size distribution. In addition, it has demonstrated that, when a soil is nearly dry, the remaining water in the voids may sustain very high negative pore pressure, and thus very high matric suction develops inside this soil. Moreover, applying the proposed model taking into account the effect of non equal-sized grain particles has given more accurate results than those obtained using only one particle size, when compared to the results available in the literature, since the latter case under-predicts the SWCC for a simulated soil. However, this model needs to be developed to simulate the behaviour of unsaturated fine-grained soils more precisely.

Notations

e	Voids ratio
G_s	Specific weight (specific gravity)
n	Porosity
n_c	Number of points of contacts between soil particles
T_s	Surface tension of water
u_a	Pore-air pressure
u_w	Pore-water pressure
$(u_a - u_w)$	Matric suction
β	Contact angle between the water meniscus and surface of the soil particle
θ	Water retention angle
θ_c	Critical angle of water retention
η	Osmotic suction
ψ	Total suction

References

[1] Fredlund D. G., Rahardjo H.: *Soil mechanics for unsaturated soils*. John Wiley and Sons, Inc., New York, NY 10158-0012, ISBN 0-471-85008-X, 1993.

[2] Fredlund D. G., Xing A.: "Equations for the soil-water characteristic curve." *Canadian Geotechnical Journal*, Vol. **31** (1994), p. 521–532.

[3] Fredlund M. D., Wilson G. W., Fredlund D. G.: Indirect procedures to determine unsaturated soil property functions. Proceedings of the 50th Canadian Geotechnical Conference, Ottawa, Ontario, Canada, 1997.

[4] Fredlund M. D., Fredlund D. G., Wilson G. W.: Prediction of the soil-water characteristic curve from grain-size distribution and volume-mass properties. Proceedings of the 3rd Brazilian Symposium on Unsaturated Soils,

NONSAT, Rio de Janeiro, Brazil, Vol. **1** (1997), p. 13–23.

[5] Fredlund M. D., Wilson G. W., Fredlund D. G.: "Use of the grain-size distribution for estimation of soil-water characteristic curve." *Canadian Geotechnical Journal*, Vol. **39** (2002), p. 1103–1117.

[6] Harr M. E.: *Mechanics of particulate media: a probabilistic approach*. McGraw-Hill International Book Co., New York, ISBN 0-07-26698-6, 1977.

[7] Likos W. J., Lu N.: Hysteresis of capillary cohesion in unsaturated soils. 15th ASCE Engineering Mechanics Conference, Columbia University, New York, June 2–5, 2002.

[8] Murray E. J.: "An equation of state for unsaturated soils." *Canadian Geotechnical Journal*, Vol. **39** (2002), No. 1, p. 125–140.

[9] Nishimura T., Hirabayashi Y., Fredlund D. G., Gan J. K.: "Influence of stress history on the strength parameters of an unsaturated statically compacted soil." *Canadian Geotechnical Journal*, Vol. **36** (1999), p. 251–261.

[10] Rektorys K.: *Survey of applicable mathematics*. Second revised edition. Vol. 1, Kluwer Academic Publishers, MAIA 280, ISBN 0-7923-0680-5, 1994.

[11] Shimada K., Fuji H., Nishimura S., Nishiyama T.: Change of shear strength due to surface tension and matric suction of pore water in unsaturated sandy soils. Proceedings of the 1st Asian Conference on Unsaturated Soils (UNSAT-ASIA 2000), Singapore, ISBN 90-5809-139-2, 2000, p. 147–152.

[12] Tateyama K., Fukui Y.: Understanding of some behaviour of unsaturated soil by theoretical study on capillary action. Proceedings of the 1st Asian Conference on Unsaturated Soils (UNSAT-ASIA 2000), Singapore, ISBN 90-5809-139-2, 2000, p. 163–168.

[13] Wan A. W. L., Gray M. N., Graham J.: On the relations of suction, moisture content, and soil structure in compacted clays. Proceedings of the 1st International Conference on Unsaturated Soils, Paris, France, ISBN 90 5410 584 4, Vol. 1, 1995, p. 215–222.

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