

# A Differential Game Modeling Approach to Dynamic Traffic Assignment and Signal Control

Zhenlong Li and Songquan Shi  
The Laboratory of Complex Systems and Intelligence Science  
Institute of Automation  
Chinese Academy of Sciences  
Beijing  
China

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*This paper addresses a theoretical issue related to combined dynamic traffic assignment and signal control under conditions of congestion through a brief review of previous research and the discussion on interaction between dynamic traffic assignment and signal control. The dynamic characteristics of the interaction are approached using a differential game modeling approach here to formulate the decision-making process for solving the problem inherent in this combination. Specifically, the combined dynamic traffic assignment and signal control problem is formulated as a leader–follower differential game, where a leader and multiple followers engage interactively to finding optimal strategies under the assumption of an open-loop information structure. Discretization in time is used to find a numerical solution for the proposed game model, and a simulated annealing algorithm is applied to obtain optimal strategies. Finally, a simulation study is conducted on a simple traffic network in which numerical results demonstrate the effectiveness of the proposed approach.*

**Keywords:** dynamic traffic assignment, signal control, differential game theory

## 1. Introduction

Congestion is a daily occurrence on many stretches of traffic networks in urban areas. Building new roadways is no longer a feasible option due to the high costs, as well as the environmental and geographical limitations. Traditional traffic engineering technology is proving unequal to the rapidly growing traffic demand. Therefore, the focus is on developing Intelligent Transportation Systems (ITS) which apply emerging hard and soft information systems technologies to alleviate transportation congestion problems. Two important components of ITS are the Advanced Traffic Management Systems (ATMS) and the

Advanced Traveler Information Systems (ATIS). The success of ATMS and ATIS will depend on the availability and dissemination of current traffic network conditions. The most important problems in ATMS and ATIS occur in Dynamic Traffic Assignment and Signal Control. Dynamic traffic assignment systems are suites of software tools designed to support traffic management systems. Using current and historical data, dynamic traffic assignment systems estimate traffic flow patterns and determine appropriate signal control and route guidance strategies. Traffic signal control systems are designed to foster real-time sensing, communication, and control of urban networks. The primary objective of traffic signal control systems is to reduce congestion effects.

Conventional methods for setting traffic signals assume given flow patterns, and traffic flows are assigned to networks assuming fixed signal settings. This method is not fully satisfactory in the normal case in which traffic flows and signal settings are mutually interdependent (Yang and Yagar, 1995). This interdependence between assignment and control is more evident in a real-time dynamic situation. This interdependence forms a serious challenge to the deployment of ATMS and ATIS (Chen and Ben-Akiva, 1998). The combined traffic assignment and signal control problem has been a topic of substantial research. Allsop (1974) was the first to address the interaction between traffic control and traffic assignment; he suggested that the effects of signal settings on the traffic flow patterns should be taken into explicit account by combining traffic control and route choice.

There have been two approaches offered to address this problem: 1) the global optimization models, and 2) the iterative optimization and assignment procedure (Cantarella et al., 1991). The global optimization models seek the global optimality of the control policies when travelers adjust their route selections to the changes in signal settings. Sheffi and Powell (1983) formulated the optimal signal-setting problem as a mathematical program, in which traffic flow is constrained to become user equilibrium. Fisk (1984) described the global optimal signal-setting problem as a Stackelberg game between network users and a traffic agency. Smith et al. (1990) combined traffic assignment with responsive signal control into a static case. Yang and Yagar (1995) formulated the problem as a bi-level program, in which the optimal traffic control represents one level and the user equilibrium assignment, another level. The iterative optimization assignment procedure is to update the signal-setting for fixed flows by alternating the signal setting for fixed flows with solving the traffic equilibrium problem for fixed signal-setting until the individual solutions to the two problems are considered to be mutually compatible (Cantarella et al., 1991; Smith and Van Vuuren, 1993). Smith (1979) proposed a consistent control policy that ensures the existence of traffic equilibrium. Al-Malik (1991) investigated the Wardrop equilibrium under Webster control. Taale and Van Zuylen (2001) gave an extensive overview of the available literature. All the studies in the literature considered only the static traffic case, with three exceptions. The first is found in Gartner and Stamatiadis (1996), who proposed a framework to integrate dynamic traffic assignment with real-time control, but did not give any analytical model formulation. The second is Chen and Hsueh (1997), who formulated one particular model for the traffic-responsive signal timing scheme with user-optimal route choice. The third is Chen and Ben-Akiva (1998), who formulated the combined dynamic traffic assignment and dynamic traffic control as a one-level Cournot game between traffic authority and users. He also formulated the dynamic traffic assignment and dynamic traffic control as a Stackelberg game between the traffic authority and users. In his PhD thesis (Chen, 1998), Chen fashioned a set of

analytical models for the combined problem and developed new control strategies and new solution algorithms.

It is well-known that dynamic traffic assignment and signal control are two processes that influence each other. The two processes have different 'players' who may have different goals. The traffic manager will try to achieve a network optimum, while the road users will search for their own optimum, e.g. the fastest way to travel from origin to destination. Decisions taken by the traffic manager in controlling traffic in a certain way have an influence on the possibilities which travelers have to choose their route and time of departure. A change in traffic control may have as impact, changing traffic volumes. The influence of the type of control on route choice was the essence of the research carried out by Taale and Van Zuylen (2000). They extended it with more examples and showed that route choice did indeed depend strongly on the type of control used. At the same time, it is possible to follow an interactive approach, where the control scheme should be adjusted after each shift in traffic volume to achieve equilibrium. However, it can be shown that the process of traffic control adjustment, followed by a shift in traffic volumes, does not necessarily lead to a system optimum.

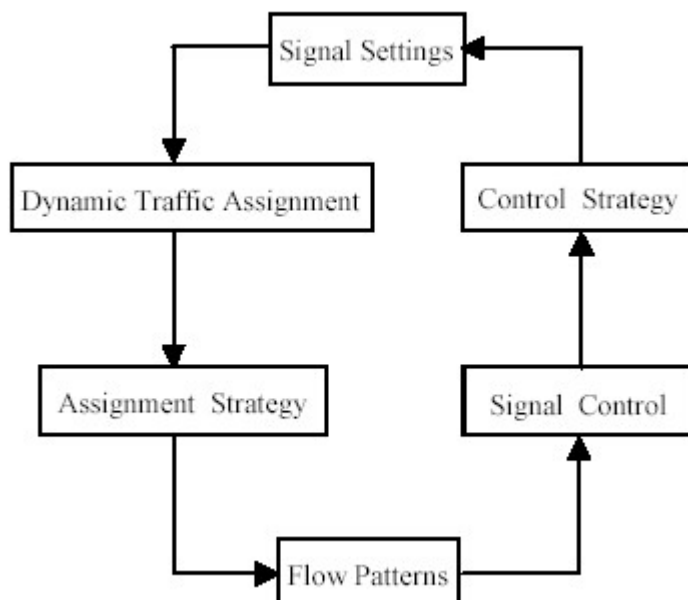
The system optimum is good for the network as a whole, but may be disadvantageous for some of the travelers in the network. Decision-makers, traffic managers and travelers strive to maximize their utility and each player's final utility will depend not merely on his/her own action but on the actions of others as well. It is obvious that the fundamental characteristic of the combined problem is formed by the dynamic characteristics of the interaction. The key problem posed is related to how traffic assignment and signal control interact dynamically when embedded within the same traffic network. Game theory provides a framework for modeling a decision-making process in which more players than one are involved. However, game theory is not congenial to problems involving dynamic phenomena because of the static nature in conventional game theory. Therefore, we need a dynamic game approach to study the interaction between dynamic traffic assignment and signal control. Differential game theory, one branch of this game theory (Isaacs, 1965; Lewin, 1994; Thoma and Wyner, 1991), provides a good framework for the combined problem, where it is assumed that the behavior of the system can be modeled as a system of ordinary differential equations. Differential game theory can be used to model situations where several interacting agents make strategic dynamic decisions. Differential game theory, which frequently deals with multiple performance indices, has been found applicable to the multi-objective control problem.

The application of differential game theory to the combined dynamic traffic assignment and signal control problem are considered here, with the focus on the dynamic characteristics of the interaction between dynamic traffic assignment and signal control. Inspired by the adaptability of Complex Adaptive Systems (CAS), We regard signal controllers as being at the intersections as the adaptive agents and model the combined problem on the basic idea that traffic flows are assigned in the proper road network by the traffic manager; these are then adapted by signal controllers by changing the signal settings. The traffic manager acts as the leader and signal controllers at the intersections act as the followers. A leader–follower differential game model of dynamic traffic assignment and signal control is then proposed. Discretization in time is used to find a numerical solution for the proposed game model, and a simulated annealing algorithm is applied to obtain optimal strategies. Finally, a simulation is conducted on a simple traffic network, with numerical results of this simulation demonstrating the effectiveness of the proposed approach.

This paper begins with an analysis of the interaction between dynamic traffic assignment and signal control, after which a differential game model of the combined problem is specified. The solution algorithm is then given, followed by an outline of a sample network and its characteristics. Analysis results, conclusions and a brief mention of further research round off the paper.

## 2. Model formulation

As mentioned above, dynamic traffic assignment and signal control are mutually interdependent. Because signal settings are usually determined by the flow patterns, these should be considered in the signal settings. Since flow patterns, described by the flow on each link, are influenced by signal settings, traffic assignment should take signal settings into account. Traffic assignment can cause a change in traffic flows, and the changed flows will subsequently make the signal settings non-optimum and thus require a corresponding change in signal settings. Of course, signal control can cause a change in travel time and the changed travel time will subsequently make the traffic assignment non-optimum and also require a corresponding change in traffic assignment. Traffic assignment can change the flow patterns from the macroscopic level, and signal control can change the flow patterns from the microscopic level. These two processes are simultaneously carried out and are highly interdependent, as shown in Figure 1.



*Figure 1. Interaction between traffic assignment and signal control*

Before presenting the combined dynamic traffic assignment and signal control problem, let us first address each dynamic traffic assignment and signal control individually.

## 2.1 Dynamic traffic assignment problem

Dynamic traffic assignment was a subject of recent studies in which various traffic assignment models were developed (Merchant and Nemhauser, 1978; Carey, 1992; Wie, Friesz and Tobin, 1990; Lu, 1996). Of these models, the dynamic system optimal model is usually employed for the criterion, its significance found in its provision of an upper limit in systematic search procedures for the optimal network design problem. This is why the dynamic system optimal model is used here.

A traffic network is made up of different links and nodes. One OD pair can have different routes, with each route including different links and nodes. Let a directed graph  $G ( N, A )$  denote the road network and

- $N$  be the set of nodes in the network;
- $A$  be the set of links in the network;
- $A(k)$  be the set of links whose tail node is  $k$ ;
- $B(k)$  be the set of links whose head node is  $k$ ;
- $x_a(t)$  be the number of vehicles on link  $a$  at time  $t$ ;
- $x_a^n(t)$  be the number of vehicles arriving at destination  $n$  on link  $a$  at time  $t$ ;
- $u_a(t)$  be the entry flow into link  $a$  at time  $t$ ;
- $u_a^n(t)$  be the entry flow into link  $a$  arriving at destination  $n$  at time  $t$ ;
- $v_a(t)$  be the exit flow from link  $a$  at time  $t$ ;
- $v_a^n(t)$  be the exit flow from link  $a$  arriving at destination  $n$  at time  $t$ ;
- $q_{kn}(t)$  be the flow generated at the node  $k$  arriving at destination  $n$  at time  $t$ ;
- $\lambda_m(t)$  be the green ratio for the phase  $m$  at time  $t$ ;
- $t_a(x_a(t))$  be the travel time on link  $a$  ;
- $d_a(x_a(t), \lambda_m(t))$  be the signal delay and queuing delay at the intersection.

The travel time between the OD pair should be considered as a sum of the travel time and travel delay in a signaled network. Travel delay should be divided into two kinds, signal delay and queuing delay, the former being due to interruption of traffic by the traffic signal, and the latter to limited capacity (Yang and Yagar, 1995). Therefore the total travel time spent on the network,  $t_a(x_a(t), \lambda_m(t))$ , is the sum of flow-dependent running time,  $t_a(x_a(t))$ , and delay due to the signal and queuing delay at the intersection,  $d_a(x_a(t), \lambda_m(t))$ :

$$t_a(x_a(t), \lambda_m(t)) = t_a(x_a(t)) + d_a(x_m(t), \lambda_a(t)) \quad (1)$$

Friesz et al. (1989) modeled the dynamic traffic assignment problem as an optimal control formulation. A standard-system optimal control formulation that seeks to minimize the total system travel time can be represented as:

$$\min J_G = \sum_{a \in A} \int_0^T t_a(x_a(t), \lambda_m(t)) u_a(t) dt = \sum_{a \in A} \int_0^T t_a(x_a(t)) u_a(t) dt + \sum_{a \in A} \int_0^T \int_0^{x_a(t)} d_a(\omega, \lambda_m(t)) d\omega dt \quad (2-a)$$

s.t.

$$\dot{x}_a^n(t) = u_a^n(t) - v_a^n(t) \quad (2-b)$$

$$\sum_{a \in A(k)} u_a^n(t) = q_{kn}(t) + \sum_{a \in B(k)} v_a^n(t) \quad (2-c)$$

$$x_a^n(t) \geq 0, u_a^n(t) \geq 0, (a \in A, n \in N, t \in [0, T]) \quad (2-d)$$

## 2.2 Traffic signal control problem

Let us assume that each signal controller phase has been determined and the cycle length given. The problem for a signal controller is to find an optimal signal timing strategy, i.e. allocating green time to each signal phase during a time period. Consider the intersection shown in Figure 2, with the time period  $[0, T]$ . Let  $l_1(t)$  denote the number of queuing vehicles at time  $t$ , and  $u_1^w(t)$  the entry rate into the waiting area of link 1.

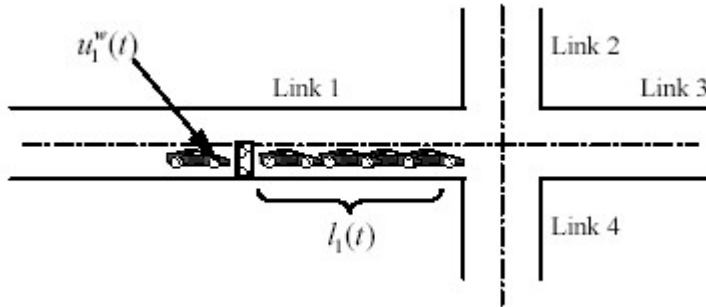


Figure 2. The intersection

In the congested situation, the equation for traffic flow state is:

$$\dot{l}_i(t) = u_i^w(t) - \frac{G_m(t)}{C(t)} S = u_i^w(t) - \lambda_m(t) S \quad (i = 1, 2, 3, 4.) \quad (3)$$

where  $C(t)$  is the cycle length,  $S$  the capacity of the intersection, and  $G_m(t)$  the green time for the phase  $m$ .

The objective of the intersection is to minimize the queuing delay, and the problem is to allocate green time to each signal phase during a cycle period to minimize the queuing delay (Jing, 1995). The objective function is then:

$$\min J_C = \min \int_0^T (l(t))^T Ql(t)dt \quad (4)$$

$$l(t) = (l_1(t), l_2(t), l_3(t), l_4(t)) \quad (5)$$

s.t.

$$G_{\min} \leq G_m(t) \leq G_{\max} \quad (6)$$

$$0 \leq l_i(t) \leq l_{\max}, \quad l_i(0) = 0, \quad i = 1, 2, 3, 4. \quad (7)$$

$$\sum_m G_m(t) = C(t) - L \quad (8)$$

where  $Q$  is weighting matrix.  $G_{\min}, G_{\max}$  is the respective minimum and maximum green times,  $l_{\max}$  the maximum queuing length, and  $L$  the lost time per cycle. According to Equation (8), the summation of green time over all phases and total lost time equals the cycle time.

The traffic signal control model can be obtained by extending the above-mentioned traffic state equation, constraints and objective function to the  $k$  intersection, as shown in Equation (9), with the model meant to make total delay of each intersection minimal.

$$\min J_C^k = \min \int_0^T (l^k(t))^T Ql^k(t)dt \quad (9-a)$$

s.t.

$$l^k(t) = (l_{a_1}^k(t), l_{a_2}^k(t), \dots, l_{a_n}^k(t)) (a_1, a_2, \dots, a_n \in A(k)) \quad (9-b)$$

$$\dot{l}_a^k(t) = u_{a,k}^w(t) - \lambda_m^k(t)S (l_a^k(0) \geq 0) \quad (9-c)$$

$$\sum_m G_m^k(t) = C(t) - L (0 \leq G_{\min} \leq G_m^k(t) \leq G_{\max}), \quad k \in N, \quad t \in [0, T] \quad (9-d)$$

where

$k$  is the index of the intersection;

$G_m^k(t)$  the green time for phase  $m$  at intersection  $k$  at time  $t$ ;

$\lambda_m^k(t)$  the green ratio for phase  $m$  at intersection  $k$  at time  $t$ ;

$l_a^k(t)$  the number of queuing vehicle on link  $a$  at intersection  $k$  at time  $t$ ;

$u_{a,k}^w(t)$  the entry flow into the waiting area of link  $a$  at intersection  $k$  at time  $t$ ;

If the attraction of links to vehicle is not considered and the distance between intersections is not too great, the entry rate into the waiting area of link is equal to the delay of the entry rate into link (Ma, 1999):

$$u_{a,k}^w(t) = u_a^k(t - \tau) \quad (10)$$

where  $\tau$  is the travel time of freedom flow.

### 2.3 A model formulation for the combined problem

Traffic manager pursues the minimal total cost; in other words, it wants to minimize the total travel time. Each intersection aims at finding green time for each phase that minimizes its queuing delay. Traffic manager and intersections have different objectives. The goal of the manager is to find an optimal assignment strategy that drives the traffic flows to the system optimum. The optimal assignment strategy achieves the minimal total cost and leads to the most efficient utilization of system resources. Each intersection is mandated by the desire to minimize an individual cost function, namely, its queuing delay. The traffic manager and signal controllers at the intersection are regarded as the adaptive agents. We assume that the manager can guide travelers by disseminating information or providing route guidance, and most travelers take up the manager's suggestion. The travelers' routing and departure times are assumed to be fixed. Accordingly, traffic flows are assigned to the proper road network by the traffic manager, after which the flows are adapted by signal controllers at the intersections by changing the signal settings.

Traffic manager and intersections are considered the participants in the game, with the traffic manager acting as the leader (has the first mover advantage) and the intersections as followers. Using differential game theories, we consider a continuous time model. In this way dynamic traffic assignment and signal control can be formulated as a leader–follower differential game under the open-loop information structure, where  $u_a^n(t)$  is the control variable of traffic manager, and  $\lambda_m^k(t)$  is the control variable of intersection  $k$ .  $J_G$  is the payoff function of traffic manager, and  $J_C^k$  is the payoff function of intersection  $k$ . The leader–follower differential game model of dynamic traffic assignment and signal control follows:

$$\left\{ \begin{array}{l} \dot{x}_a^n(t) = u_a^n(t) - v_a^n(t) \\ \dot{j}_a^k(t) = u_a^k(t - \tau) - \lambda_m^k(t)S \\ \sum_{a \in A(k)} u_a^n(t) = q_{kn}(t) + \sum_{a \in B(k)} v_a^n(t) \\ j_a^k(0) \geq 0 \quad x_a^n(t) \geq 0 \quad u_a^n(t) \geq 0 \\ \sum_m G_m^k(t) = C(t) - L \\ J_G = \sum_{a \in A} \int_0^T l_a(x_a(t), \lambda_m(t)) u_a(t) dt \\ J_C^k = \int_0^T (l^k(t))^T Q l^k(t) dt \quad l^k(t) = (l_{a_1}^k(t), l_{a_2}^k(t), \dots, l_{a_n}^k(t)) \end{array} \right.$$

The aims of players, manager and intersections are to make their utility maximal by selecting strategies, i.e. control variables.

### 3. The solution algorithm

Discretization in time is used here to find a numerical solution for the proposed game model.



$$J_G = \min \sum_{a \in A} \sum_{i=1}^M x_a(i) t_a(x_a(i), \lambda_m(i))$$

$$x_a^n(i+1) = x_a^n(i) + u_a^n(i) - v_a^n(i)$$

$$\sum_{a \in A(k)} u_a^n(i) = q_{kn}(i) + \sum_{a \in B(k)} v_a^n(i)$$

$$u_a^n(i) \geq 0, \quad x_a^n(i) \geq 0, \quad a \in A, n \in N, \quad i = 0, 1, 2, \dots, M-1$$

$$J_C^k = \min \sum_{i=1}^M C(i) l^k(i) e$$

$$l^k(i) = (l_{a_1}^k(i), l_{a_2}^k(i), \dots, l_{a_n}^k(i))$$

$$l_a^k(i+1) = l_a^k(i) + u_{a,k}^w(i) - \lambda_m^k(i) S$$

$$\sum_m G_m^k(i) = C(i) - L$$

$$l_a^k(0) \geq 0, \quad k = 1, 2, \dots, N, \quad i = 0, 1, 2, \dots, M-1$$

where  $e$  is unit column vector.

In this game, there are many decision-makers at the lower level and one decision-maker at the upper level. The game is played as follows: the leader chooses a strategy that will affect the followers' restraint-set and objective function, so as to achieve system optimum with respect to the distribution of the traffic flows in the network. The followers then react by modifying their behavior. The aim of each player is to minimize his/her own objective function which will depend on players' strategies. Both the leader's and the followers' strategies will affect the overall system performance. The strategy  $(u^*(k), \lambda^*(k))$  makes the overall system performance optimal;  $u^*(k)$  and  $\lambda^*(k)$  can be obtained by solving the model.

The simulated annealing algorithm (SA) presents an optimization technique that can process objective functions possessing quite arbitrary degrees of nonlinearities, discontinuities and process quite arbitrary boundary conditions and constraints imposed on these objective functions. Simulated annealing is an optimization strategy invented by Kirkpatrick et al.(1983). The SA algorithm does not require derivative information, but merely needs to be supplied with an objective function for each trial solution it generates. Compared with conventional, iterative search techniques, the SA algorithm is robust. Compared with genetic algorithms (GA), the SA is, according to some suggestions in the literature, a 'quick starter', arriving at good solutions in a short time. However, the SA is not able to improve on that given more time. Furthermore GA algorithm is reported to be a 'slow starter', i.e. able to improve the solution consistently when given more time. In comparing SA and GA for a traffic routing problem, Mann and Smith (1996) reported GA to give slightly better solutions than SA, but also noted that the SA achieved its solutions much quicker. Lahtinen et al. (1996) provided a good discussion on how a meaningful empirical comparison should be

done. Lahtinen et al.(1996) compared several algorithms, including SA and GA ; their results indicated that given the same amount of time, SA consistently gave better solutions than GA. For this reason, the simulated annealing algorithm is applied here to obtain optimal strategies. Consider an analysis period divided into equal intervals  $j = 1, 2, \dots, H$ . here, we can implement the simulated annealing algorithm to the model at the start of each interval. The algorithm for solving the model can be summarized as follows:

**Step 1:** Determine an initial temperature  $T_1$  and the initial solution  $(u_a^n(j))^0$ . Solve the lower problem for a given  $(u_a^n(j))^0$  to obtain  $(\lambda_m^k(j))^0$ . Set the iteration counter at  $p=1$ .

**Step 2:** New trial solutions can be generated according to:

$$(u_a^n(j))^p = (u_a^n(j))^{p-1} + \Delta u$$

where  $\Delta u$  is a random number in neighbourhood of  $(u_a^n(j))^{p-1}$ . Compute the new objective function value  $F((u_a^n(j))^p)$  and the change  $\Delta F$  in the objective function,  $\Delta F = F((u_a^n(j))^p) - F((u_a^n(j))^{p-1})$ .

**Step 3:** If  $\Delta F \leq 0$ , then  $(u_a^n(j))^p = (u_a^n(j))^{p-1} + \Delta u$ ; if  $\Delta F > 0$ , compute the probability of accepting new trial solutions

$$p(\Delta F) = \exp\left(-\frac{\Delta F}{p \cdot T_p}\right)$$

and generate a random number  $r$ ,  $0 < r < 1$ . If the Metropolis criterion is satisfied, i.e.  $p(\Delta F) \geq r$ , then

$$(u_a^n(j))^p = (u_a^n(j))^{p-1} + \Delta u. \text{ Otherwise } (u_a^n(j))^p = (u_a^n(j))^{p-1}.$$

**Step 4:** If a stopping criterion is satisfied, then  $(u_a^n(j))^p$  is the optimal solution of the upper problem and solves the lower problem for given  $(u_a^n(j))^p$ , yielding

$$(\lambda_m^k(j))^p.$$

$((u_a^n(j))^p, (\lambda_m^k(j))^p)$  is the approximate global optimal solution. Otherwise, go to Step 5.

**Step 5:** A new temperature is generated according to:

$$T_{p+1} = \alpha T_p$$

Let  $p=p+1$  and go to Step 2.

#### 4. A numerical example

In this section, a numerical example is presented to illustrate the application of the model and algorithm above. The test network is indicated in Figure 3 with seven nodes and eight links. Detailed link characteristics are shown in Table 1.

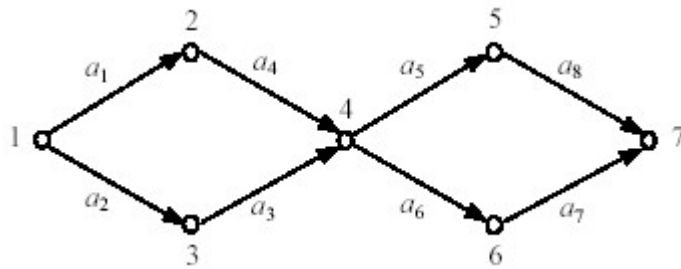


Figure 3. Simulation network

Table 1. Information on links

Link number	Start-End node	Length (m)	Capacity (# of Vehicle)	No. of lanes	Speed of freedom flows (km/h)
a1	1-2	500	600	1	80
a2	1-3	400	600	1	80
a3	3-4	300	600	1	80
a4	2-4	400	400	1	60
a5	4-5	300	600	1	80
a6	4-6	400	600	1	80
a7	6-7	400	400	1	60
a8	5-7	300	600	1	80

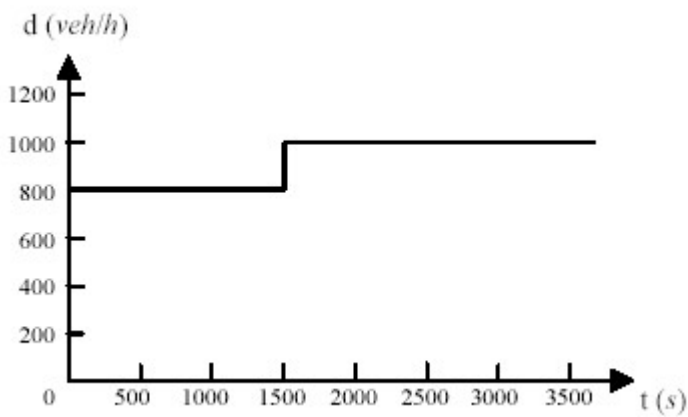


Figure 4. The OD demand

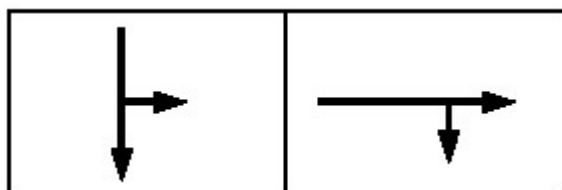


Figure 5. Two-phase signal

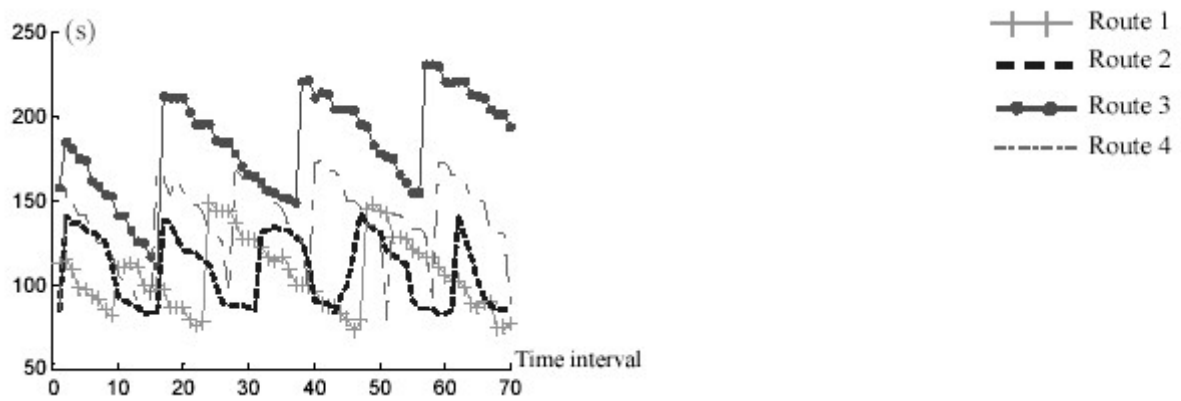
One origin–destination pair from node 1 to node 7 is assumed. The OD pair has four alternative routes available: Route1(a1/a4/a5/a8), with a length of 1500 m, Route 2 (a1/a4/a6/a7), with a length of 1700 m, Route 3 (a2/a3/a5/a8), with a length of 1300 m and Route 4 (a2/a3/a6/a7), with a length of 1500 m. The OD demand is shown in Figure 4. During the simulation, it is necessary to specify the link travel-time function, also known as the link performance function. A widely used link travel-time function is the formulation suggested by the US Bureau of Public Roads (BPR) and used in the paper. Both assignment cycle and control cycle are equal to 100s. The sample time is 50s. We assume that the obedience rate, the percentage of drivers following the manager's route guidance, to be 80%. The signal phases used in the paper are shown in Figure 5.

It is important to note that a high initial temperature will be necessary to find a global minimum and reduce the possibility of being trapped in a local optimum. Therefore the number of iterations will have to be very large and too much computation time will have to be spent. In a practical situation, however, there is a trade-off between finding the global minimum and a point near the global minimum, depending on the time one is prepared to wait until a global minimum has been found. Therefore, it may be more efficient to use a stopping criterion after a certain number of iterations with only minor changes of the energy function or even without any improvements of the energy function (Salomons et al.,1995). The stopping criterion greatly affects the SA's performance. For reasons of efficiency, our implementation uses the following parameters:

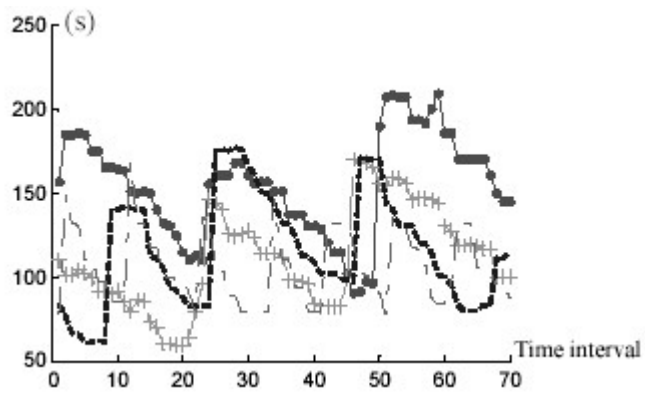
- initial temperature  $T_1 = 1000$
- final temperature  $T_f = 0.1$
- cooling factor  $\alpha = 0.98$

The stopping criterion used in the paper represents only minor changes in the objective function or the current temperature less than the final temperature.

Simulation is conducted using the proposed approach of this paper. CPU times are measured on PII366, and the run time is about 6.8 seconds on average, which is far less than the cycle of assignment and control, indicating that the solution algorithm is available and the algorithm performance acceptable. A fixed-time control policy based on the Webster method is also carried out for comparison. The route travel time and the number of vehicles on the route can be obtained through two cases at the sample time. The results of the simulation are shown in Figures 6 and 7.

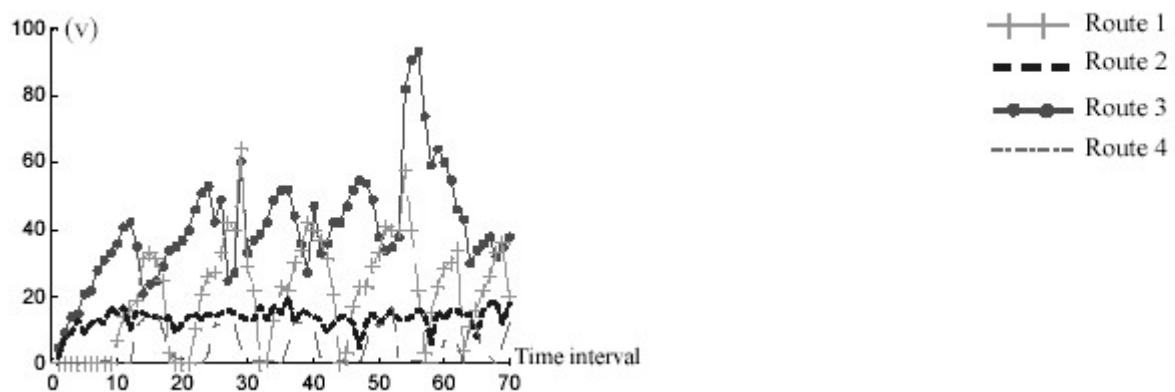


(6-a) In the case of fixed-time control

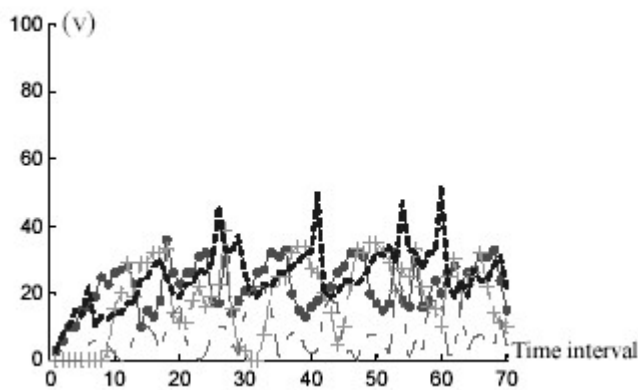


(6-b) In the case of the proposed approach

Figure 6. Travel time



(7-a) In the case of fixed-time control



(7-b) In the case of the proposed approach

Figure 7. The number of vehicles

In the case of ‘fixed-time control’, the average travel times for the four routes are 101.38s, 103.25s, 174.46s, 129.27s, respectively, and total average travel time is 127.09s. In the case of ‘the proposed approach’, the average travel time of the four routes are 107.47s, 108.12s, 147.36s, 101.15s, respectively, and total average travel time is 116.03s, a reduction of 8.70%. In Figures 6 and 7, we see that the vehicles on Route 3 increase sharply in the ‘fixed-time control’ case, since most of drivers choose Route 3, which is the shortest route in a static situation. Consequently, congestion occurs on Route 3 and the network performances decline. While, in the case of ‘the proposed approach’, the flows are adjusted by the traffic manager and the signal timing is correspondingly changed when the flows on Route 3 increase. Therefore congestion can be alleviated to a certain extent. However, it is important to note that the global optimum can not always be obtained; sometimes, a point may be found that is near the global optimum.

## 5. Conclusions and future research

In this paper, differential game theory has been applied to the modeling of the combined dynamic traffic assignment and signal control problem. The simulation was conducted on a simple sample network using two cases: the fixed-time control and the approach proposed in this paper. The numerical results demonstrate the validity and effectiveness of the proposed approach.

There is a great need for future research on this topic. Firstly, demand is assumed to be fixed in the model of this paper, while the demand for transportation is, in practice, variable. A subsequent study will deal with the elastic demand in the traffic-assignment problem. In a practical situation, the assumption that traffic manager can assign routes to travelers is not practical, the dynamic user-equilibrium would be more realistic. Secondly, the paper proposes a differential game modeling approach to the combined problem, but differential game theory is not used for solving and analysing the problem. How the differential game theory can be used to solve the combined problem needs further study. Finally, there are plans to perform parallel simulated annealing to improve the performance of algorithm.

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## References

- Allsop, R.E. (1974) Some possibilities for using traffic control to influence trip distribution and route choice. *Proceedings of the 6th International Symposium on transportation and Traffic Theory*, Elsevier, Amsterdam, pp. 345-374.
- Al-Malik, M.S. (1991) *An investigation and development of a combined traffic signal control-traffic assignment model*, PhD Thesis, Georgia Institute of Technology.
- Cantarella, G.E., Improta, G. and Sforza, A. (1991) Iterative procedure for equilibrium network traffic signal setting, *Transportation Research*, Vol. 24A, pp.241-249.
- Carey, M. (1992) Nonconvexity of the dynamic traffic assignment problem, *Transportation Research*, Vol. 26B, No.2, pp.127-133.
- Chen, H.K. and Hsueh, C.F. (1997) Combining signal timing plan and dynamic traffic assignment. Presented at the *76th annual transportation Research Board Meeting*, Washington D.C, January, pp.12-16.
- Chen, O.J. (1998) *Integration of dynamic traffic control and assignment*, PhD Thesis, Massachusetts Institute of Technology.
- Chen, O.J. and Ben-Akiva, M.E. (1998) Dynamic traffic control and assignment: a game-theoretic approach. Presented in the *77th Annual Meeting of Transportation Research Board*, Washington, DC, January 11-15.

- Fisk, C.S. (1984) Game theory and transportation systems modeling, *Transportation Research*, Vol.18B, pp.301-313.
- Friesz, T.L., Luque, F.J., Tobin, R.L. and Wie, B.W. (1989) Dynamic network traffic assignment considered as a continuous time optimal control problem, *Operations Research*, Vol. 37, pp.893-901.
- Gartner, N. and Stamatiadis, C. (1996) Framework for the integration of dynamic traffic assignment with real-time control. In *Proceedings of the 3rd Annual World Congress on Intelligent Transportation Systems*, Orlando, Florida.
- Isaacs, R. (1965) *Differential games*. John Wiley: New York.
- Jing, B.SH. (1995) *Roads Traffic Control Engineering*, China Communications Press.
- Kirkpatrick, S., Gelatt, Jr.C.D. and Vecchi, M.P. (1983) Optimization by Simulated Annealing, *Science*, Vol. 220, pp. 671-680.
- Lahtinen, J., Myllymaki, P., Silander, T. and Tirri, H. (1996) Empirical comparison of stochastic algorithms in a graph optimization problem. *Proceedings of the Second Nordic Workshop on Genetic Algorithms and their Applications (2NWGA)*, Vaasa, Finland, pp.45-59.
- Lewin, J. (1994) *Differential Games*, Springer-Verlag, London Limited .
- Lu, H.P. (1996) Dynamic traffic assignment model - a review and future, *Journal of Highway and Transportation Research and Development*, Vol. 13, No. 2, pp.34-43.
- Ma, SH.F. (1999) *Study on traffic control and vehicle route guidance intelligent transport systems*, PhD Thesis, Tianjin University.
- Maan, J.W. and Smith, G.D. (1996) A Comparison of Heuristics for Telecommunications Traffic Routing, *Modern Heuristic Search Methods*. John Wiley and Sons, pp.235-254.
- Merchant, D.K. and Nemhauser, G.L. (1978) A model and an algorithm for the dynamic traffic assignment problems, *Transportation Science* ,Vol. 12, No.3, pp.183-199.
- Salomons, O.W., et al. (1995) Collaborative Product Development in CAD and CAPP. *Proceedings of the IFIP WG 5.2 Workshop on Knowledge Intensive CAD*, Espoo, Finland, pp. 81-104
- Sheffi, Y. and Powell, W.B. (1983) Optimal signal settings over transportation networks, *Journal of Transportation Engineering*, Vol. 109, No. 6, pp. 824-839.
- Smith, M.J. (1979) Traffic control and route choice: a simple example, *Transportation Research*, Vol.13B, No.4, pp.289-294.
- Smith, M.J. and Van Vuren, T. (1993) Traffic equilibrium with responsive traffic control, *Transportation science*, Vol. 27, pp.118-132.
- Smith, M.J., et al. (1990) The dynamic of traffic assignment and traffic control: a theoretical study, *Transportation Research*, Vol. 24B, No. 6, pp.409-422.
- Taale, H. and van Zuylen, H.J. (2000) Traffic control types and route choice: some simple examples. *Proceedings of the 7th World Congress on Intelligent Transport Systems*, Turin.



Taale, H. and van Zuylen, H.J. (2001) The combined traffic assignment and control problem. an overview of 25 years of research. *Proceedings of the 9th World Conference on Transport Research*, Seoul.

Thoma, M. and Wyner, A. (1991) *Differential Games-Developments in Modeling and Computation*, Springer-Verlag, Berlin, Heidelberg .

Wie, B.W., Friesz, T.L. and Tobin, R.L. (1990) Dynamic user optimal traffic assignment on congested multideestination network, *Transportation Research*, Vol. 24B, pp.431-442.

Yang, H. and Yagar, S. (1995) Traffic assignment and signal control in saturated road networks, *Transportation Research*, Vol. 29A, No. 2, pp.125-139.