Project Selection in Traffic Accident Prevention and Mitigation

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This paper deals with two problems in relation to the accident treatment of urban street networks, i.e. accident prevention (AP) and accident mitigation (MP). These two problems are defined based on the concepts of suitable remaining trip-hours, link importance, and k-link connectedness of node-destinations in the network. The objective of problem (AP) is to upgrade the important links so as to maximize a measure of the performance of the network. This is a before-accident treatment of the network. The objective of problem (MP) is to mitigate the accident effects in the network so as to maximize a measure of network connectedness. This is an after-accident treatment of the network, which is done by equipping auxiliary links in the network to join the available set of links, in order to make important 1-link connected node-destination pairs, k-link connected $(k \geq 2)$*. Two algorithms have been proposed to solve these two problems. The reasonability of the solution results has been shown by applying these two algorithms on a small-sized (6 nodes, 10 links) example network. The feasibility of the application of these algorithms on larger networks has been investigated by applying them on the network of Sioux Falls (24 nodes and 76 links).*

Keywords: traffic accident, accident prevention, link importance, accident mitigation, network connectedness

1. Introduction

Accidents are events in transportation networks which hinder normal flows of traffic in the links, or halt some parts of the network for a short period of time (say, 10-15 minutes) or even longer. These events cost money, materials, take lives and even affect the environment, proportional to the severity of the events. Moreover, the delay caused by the accidents to those passengers not being involved in such events may be significant. The traffic in the accident-hit link backs up and the moment the drivers are informed of the event they are tempted to change their routes. Where there is no suitable alternative route, their detours would become unsuitable.

Attention has been recently paid to analysing such events from the perspective of network performance. Wakabayashi and Iida (1994), noting the importance of transportation networks in everyday life, emphasize the significance of having a reliable network that functions suitably despite the failure of parts of the network due to accidents, natural disasters, congestion, or closure of roads for maintenance. Asakura (1996) proposes to use a reliability measure defined as the probability of having the ratio of an origin-destination (O/D) travel time after the event over that time before it which is below a certain acceptable level. Du and Nicholson (1997a, 1997b), referring to transportation network as lifeline, define critical links as important yet weak links. An "important" link means that the link failure causes economic and social costs and by "weak" link is meant that the link is vulnerable to natural disaster. They propose identification of critical links by making a sensitivity analysis in the process of network performance improvement. Sanso and Soumis (1991) present a method for evaluating the performance of networks under uncertainties. As for transportation networks, they present a *3T* model for the analysis of traffic accidents which considers three periods of time for this analysis, i.e. before the accident, just after the accident, and after the information of accident occurrence has reached the users of the network. Sanso and Milot (1994) have used the *EMME/2* package to show the implementation of the *3T* model.

Literature reveals the need for more research to explore the effect of accidents upon the performance of the networks (Iida, 1999). This paper makes an attempt in this respect. First, as a preventive strategy this paper aims at solving a problem to identify a set of links which, when upgraded by certain actions under limited resources, would increase the network performance most. As a remedial strategy the paper then considers the problem of identifying a resource feasible subset of local access links in the network which, when equipped with certain means and introduced to the network, would create alternative routes for the users of accident-struck links and decrease the negative implications of weak connections of the network best.

Section 2 of the paper presents the definitions and assumptions made in this paper. Section 3 discusses the models and is followed by section 4, which presents applications of the models for two test networks. The paper is concluded with section 5.

2. Definitions, Assumptions, and Notations

Assume that, without loss of generality, accidents occur in network links. At any time, the state of the network may be represented by a state vector, $c = (..., c_{ij}, ...)$, where c_{ij} shows the state of link (i, j) of the network at that time: $c_{ij} = 0/1$, if an accident does/does not occur in link (i, j) (or, link (i, j) is not /is functioning). For example, $c^{\circ} = (1,...,1,...,1)$ shows that no link of the network is involved in an accident (or, that all links are functioning). This state is called a **prevalent** one.

Define remaining trip (rt^{es}) of a trip of a trip-maker in a new state c, is a trip with its origin being the point of the network where that trip-maker is, at the time state c starts, e, and its destination being the original destination of the trip, s. We call (rt^{es}) suitable if its travel time does not exceed that of the respective prevalent state beyond a certain level θ^{es} . Let $t^{es}(c)$ represent the rt^{es} travel time at state c. Let also $t_{\rho}^{es}(c)$ be the (rt^{es}) travel time through path ρ when the state of the network is c. Then, (rt^{es}) is suitable in state c if $\frac{t^{es}(c)}{t^{es}(c^{\circ})} \leq \theta^{es}$, and path ρ from e to s is a suitable path in state c if $\frac{t_{\rho}^{es}(c)}{t^{es}(c^{\circ})} \leq \theta^{es}$.

Node-destination (N/D) (j,s) is said to be suitable in state c, if there exists at least one suitable path from j to s in state c. N/D (j,s) is called k-link connected if destination s could be reached from j by k suitable paths with no links in common (see also Pierre and Elgibaout, 1997). Fig 1 shows 5 paths from node j to destination s, of which one path is longer than others and is not suitable. So, there are 4 suitable paths from j to s, but the N/D (j,s) is 2-link connected.

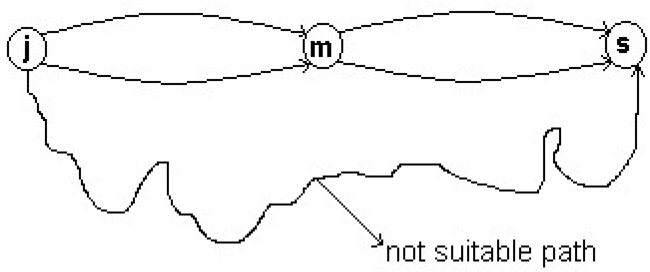


Fig.1. Suitable / not suitable paths, and k-link connected N/D (k=2)

Now let us turn to the assumptions made in this paper. For a street network, it is assumed that:

1. The probability of accident occurrence in a link during the analysis period (say, morning peak period) is known;
2. Accidents only occur in the links of the network (intersections may be represented by a set of links);
3. All network users will be informed of the accident immediately after the occurrence of an accident (say, by radio stations or variable sign messages);
4. Those travellers who have the accident-struck link on their paths to their respective destinations tend to change their paths to avoid that link, regardless of the severity of the accident;
5. What is important to the users of the network after the occurrence of the accident is to reach their destinations in a suitable time period;
6. The change of paths in (4) has a negligible effect upon the level of service offered to other travellers (because of the extent of the network or short duration of accident effect), so that they keep on using their usual paths and that their trips remain suitable;
7. Travellers who have to pass through the accident-struck link will experience unsuitable rt;
8. Accidents only affect the travellers who have already started their trips and those who have not started their trips would only do so if their trips become suitable.

Thus, the travellers in the network may be divided into three groups when an accident occurs in a link of the network: the first group are those travellers who do not have the accident-struck link on their paths from the origin to the destination. These travellers would, according to assumption (6), continue using their usual path to reach their respective destination and finish their trips suitably. The second group are those travellers who at the time of accident occurrence, are travelling within the accident-struck link and, if the accident is ahead of them, they have to pass through it and their rt's become unsuitable by assumption (7). Otherwise, if they are travelling past the accident point, they continue their trips and finish them suitably. The third group are those travellers who have the accident-struck link on their path from O to D and are informed right after the accident of its occurrence. Until they reach the link, they have time to change their paths to avoid the accident-hit link to finish their trips suitably. If there is no such alternative path from where they are to their destinations, then their rt's become unsuitable.

To present the models formally, let $N(V,A)$ be a network with V as the set of nodes and A as the set of links. Let n be the number of nodes, $n=|V|$, k and s represent the origin and destination respectively with O and D as the respective sets. Let also, P denote the set of O/D pairs, (k,s) , with demand d^{ks} from k to s . The (shortest) travel time from k to s is denoted by t^{ks} .

Moreover, let ρ denote a path in the network, and ρ^{ks} the set of paths from k to s . x_{ρ}^{ks} is the (user equilibrium) flow in path ρ from k to s in a prevalent (no-accident) situation, and x_{ij} one such flow in link (i,j) , which experiences the travel time t_{ij} .

Now let p_{ij} be the probability of non-occurrence of accident in link (i,j) . c represents the state of the network and c° the prevalent state. c°_{-ij} is the state of the network in which only link (i,j) is hit by an accident. Finally, let $t^{js}(c)$ denote the $N/D(j,s)$ travel time when the network is in state c .

We are now in a position to state the problem formally.

3. The Proposed Models

3.1 Choice of Accident Preventive Actions

This section is devoted to a model for choosing among a set of accident preventive actions under limited resources. First a link-importance index is introduced. The importance of a link in a network is related to its contribution to the performance of the network. The objective of this study is to improve the level of service offered to the travellers who are using the network at the time of accident event (assumption 8). It is appealing to use the number of suitable rt's after an accident event relative to that before this event as a measure of network performance. However, it is clear that, in order to differentiate between long rt's and short ones, the **remaining trip-hour (rt-hr)** is a better measure. Another appealing measure for network performance could have been suitable trip-hour. However, this measure fails to appropriately express the performance of the network in some cases. For example, consider a trip which usually takes one hour. Suppose this trip becomes involved in an accident 2 minutes just before it ends. How would the trip-maker express his feeling in such occasions? He would most probably say that the trip was fine just before it ends, but he was unfortunate in the last few minutes. That is to say the trip-maker is unsatisfied with the remaining (last few minutes) of the trip.

Let $E(c)$ be the sum of rt-hr's when the network is in state c. Let also $PI(c)$ be the **performance index** of the network in state c, defined as

$$PI(c) = \frac{E(c)}{E(c^{\circ})} \tag{1}$$

Now let us suppose that there are m passengers destined to s travelling in link (i, j) who are uniformly distributed over this link. Also assume that the occurrence of an accident in link (i, j) is uniformly distributed over this link. Then:

Proposition 1. The expected suitable rt-hr's for these travellers when the state of the network is c, $E_{ij}^{s}(c)$, is given by:

$$E_{ij}^{s}(c) \approx m(\frac{t_{ij}}{2} + t^{js})z^{js}(c) - m(\frac{t_{ij}}{3} + \frac{t^{js}}{2})\bar{z}_{ij}(c)z^{js}(c) \tag{2}$$

where $\bar{z}_{ij}(c)$ is a binary variable, which is 1 if an accident occurs in link (i, j) in state c, otherwise 0. Also, $z^{js}(c)$ is another binary variable, which is 1 if $rt^{js}(c)$ is suitable, 0 otherwise.

Now let us suppose that the users of the network are distributed over different portions of a path from origin k to destination s in proportion to the time taken to traverse those portions. In other words, a link that takes more time to travel through contains more travellers at any instant. With this assumption, we now state:

Proposition 2. Assume that users of the network are immediately informed of an accident occurrence in the network and that the number of passengers on any portion of the path is proportional to the travel time of that portion. Then we may write:

$$PI(c) = \sum_{j \in V} \sum_{s \in D} (\Phi^{js} - \Psi^{js}(c))z^{js}(c) \tag{3}$$

where,

$$\Phi^{js} = \frac{\sum_{k \in O} \sum_{\rho \in \rho^{ks}} \sum_{i \in B(j)} x_{\rho}^{ks} t_{ij} \delta_{ij,\rho}^{ks} (\frac{t_{ij}}{2} + t^{js})}{\sum_{(k,s) \in P} t^{ks} d^{ks} (\frac{t^{ks}}{2})} \tag{4}$$

$$\Psi^{js}(c) = \frac{\sum_{k\in O}\sum_{\rho\in\rho^{ks}}\sum_{i\in B(j)} x_{\rho}^{ks} t_{ij} \delta_{ij,\rho}^{ks} (\frac{t_{ij}}{3} + \frac{t^{js}}{2}) \bar{z}_{ij}(c)}{\sum_{(k,s)\in P} t^{ks} d^{ks} (\frac{t^{ks}}{2})} \tag{5}$$

where $B(j)$ is the set of tail nodes of the links of the network with head node j, and $\delta_{ij,\rho}^{ks}$ is a binary variable which takes the value of 1 if link (i,j) belongs to path ρ from origin k to destination s, and 0 otherwise.

It is worth noting that Φ^{js} is the performance index of the network from node j to the destination s, which is deducted by $\Psi^{js}(c)$ to represent the inefficiency caused by the occurrence of an accident in link (i,j) in this index in state c.

Definition. An important link in a network is a link such that reducing the probability of accident occurrence in that link would increase the performance index of the network significantly.

Define the importance index of link $(i,j), I_{ij}$, as the reduction of the performance index of the network without link (i,j) as compared with that of the prevalent state:

$I_{ij} = PI(c^{\circ}) - PI(c^{\circ}_{-ij})$

which is the change in performance of the network when (i,j) is excluded from the network. Clearly, $PI(c^{\circ}) = 1$, and according to Proposition 2,

$$PI(c^{\circ}_{-ij}) = \sum_{j\in V}\sum_{s\in D}[\Phi^{js} z^{js}(c^{\circ}_{-ij}) - \Psi^{js}(c^{\circ}_{-ij}) z^{js}(c^{\circ}_{-ij})] \tag{6}$$

Thus, one may write:

$$I_{ij} = [1 - \sum_{j\in V}\sum_{s\in D}\Phi^{js} z^{js}(c^{\circ}_{-ij})] + \sum_{j\in V}\sum_{s\in D}\Psi^{js}(c^{\circ}_{-ij}) z^{js}(c^{\circ}_{-ij}) \tag{7}$$

The first term of the above expression is a measure of the importance of link (i,j) in providing alternative suitable paths in the network, whilst the second term adds an amount to the importance of link (i,j), which an accident occurrence in link (i,j) would deduct.

3.2 Accident Proofing of the Network

Suppose that several accident preventive (AP) measures or actions may be undertaken for each link in the network. Suppose that for each street (or link), (i,j) there are k_{ij} alternative actions, and that alternative k would reduce the probability of accident occurrence in link (i,j) by α_{ij}^{k} percent. Moreover, suppose that there are L types of resources needed to undertake the projects of interest and that B^{l} is the amount of resource l. Let e_{ij}^{kl} be the amount of resource l required for implementing alternative k over link (i,j). The question is, under the limitation of the available resources, which alternative action of each street should be implemented so as to maximize the performance of the network under study. The following is a model to answer this question:

$$(AP) \quad Max \sum_{(i,j)\in A} \sum_{k=1}^{k_{ij}} [I_{ij}(1-p_{ij})\alpha_{ij}^{k}]z_{ij}^{k} \tag{8-0}$$

$$s.t.: (1) \quad \sum_{k=1}^{k_{ij}} z_{ij}^{k} \le 1 \quad ;(i,j)\in A \tag{8-1}$$

$$(2) \sum_{(i,j)\in A} \sum_{k=1}^{k_{ij}} c_{ij}^{kl} z_{ij}^{k} \le B^{l} \quad ;l=1,...,L \tag{8-2}$$

$$(3) \quad z_{ij}^{k} = 0/1 \quad ;(i,j)\in A, k=1,...,k_{ij} \tag{8-3}$$

In this model, $1-p_{ij}$ is the probability that an accident occurs in link $(i,j)\in A$ and hence $(1-p_{ij})\alpha_{ij}^{k}$ the reduction of this probability by alternative action k for this link, which is weighted by the importance of the link to form $I_{ij}(1-p_{ij})\alpha_{ij}^{k}$. This " benefit " of alternative k of link (i,j) accrues if action k of link (i,j) is chosen $(z_{ij}^{k}=1)$, otherwise not $(z_{ij}^{k}=0)$, and thus obtaining the expression $I_{ij}(1-p_{ij})\alpha_{ij}^{k}z_{ij}^{k}$ which is summed over all links (i,j) in the network. The first constraint ensures that at most one alternative is chosen for each link. The second constraint is the resource constraint, and the third constraint limits z_{ij}^{k} to 0 (rejected) or 1 (accepted). The following algorithm is presented to solve problem (AP).

Algorithm (AP): To identify links to be improved with an ultimate goal of Accident Prevention.

Step 0. *Initialization.* Prepare the following information: $V \,\&\, A$ for $N(V,A)$; p_{ij} ,$\forall(i,j)\in A$; d^{ks} ,$\forall(k,s)\in P$; t_{ij} function $\forall(i,j)\in A$; θ^{js},$\forall(j,s)\in V\times S$; and B^{l},$\forall l\in L$.

Step 1. *Equilibrium Flow Computation.* Solve a user equilibrium flow problem for the network $N(V,A)$ and demand $\{d^{ks}, \forall(k,s)\in P\}$ to find the path flows and link travel times.

Step 2. *Finding N/D Information.* For all node-destinations $(j,s)\in V\times D$, using the information obtained from step 1 above, compute Φ^{js} from Eqn. (4), and $\Psi^{js}(c_{-ij}^{\circ})$ from Eqn. (5). For all links $(i,j)\in A$, exclude link (i,j) and compute the shortest travel time from j to s, $t^{js}(c_{-ij}^{\circ}), \forall(j,s)\in V\times D$ using the equilibrium link travel times obtained in step 1 above. Identify the suitability of the shortest path from j to s $(i.e., z^{js}(c_{-ij}^{\circ})=1)$ by verifying that for node-destination (j,s) $\dfrac{t^{js}(c_{-ij}^{\circ})}{t^{js}(c^{\circ})} \le \theta^{js}$, for all $(j,s)\in V\times D$.

Step 3. *Compute Link Importance Index.* Compute I_{ij} by Eqn. (7), using the information obtained in step 2.

Step 4. *Solve Problem (AP).* Solve problem (AP) by a suitable algorithm, using I_{ij} obtained in step 3 above.

3.3 A Method for Accident Mitigation

The previous section has dealt with the case of improving the important links so as to increase the performance of the network. In this respect, problem (AP) is a before-accident problem. This section, however, deals with the after-accident problem, i.e. the case of mitigating the negative impacts of accidents in the network. The objective here is to find the best action for each selected link from among a set of candidate links so that they are collectively resource feasible, and when implemented would strengthen the weakness of the network by offering alternative paths to the travellers to reach their destinations after the occurrence of an accident.

After an accident occurrence, travellers who have the accident-struck link on their paths to their destinations, would seek alternative paths to avoid this link. For those travellers with no such an alternative path, the rt would become unsuitable. Suppose that some local roads/streets may be equipped in such a way that they become available to the traffic as alternative path creators or bypass streets. These auxiliary streets may make some of the unsuitable rt's, suitable; however, they are designed in such a way that they are virtually closed to through traffic in case they are not expected to bypass a related troubled zone of the network.

In the following a method is presented to choose among investments which aim to prepare auxiliary links for cases of accident occurrence. The objective of the network is to equip those auxiliary links that make the "important" and 1-link connected node-destinations at least 2-link connected. The "importance" of a node-destination pair (j,s) is evident from Φ^{js}. Referring to Eqn. (4), Φ^{js} is the proportion of the remaining trip-hours that are destined to s and j is the first node to be reached right after the accident occurrence.

Now suppose that there is a set of candidate links proposed to be equipped as auxiliary links, each requiring certain costs to be implemented (e.g. for meeting safety standards, installing control measures, increasing capacity, regulating speed etc.). A choice of these links is subject to budget constraint. Let e_{ij} be the cost of preparing auxiliary link (i,j) for joining the existing network $N(V,A)$ when needed, and B the budget. Let A_y be the set of auxiliary links, and y the vector of choice with elements y_{ij} which takes values of 1 or 0 depending on accepting auxiliary link $(i,j) \in A_y$ or rejecting it, respectively. Let A'_y represent the set of links of a chosen network corresponding to the decision $y: A'_y = A \cup \{(i,j) \mid (i,j) \in A_y, y_{ij} = 1\}$ Finally, let z_y^{js} be a binary variable which is equal to 1 if with decision y, $N/D(j,s)$ in the network $N(V,A'_y)$ is $k_{k\geq 2}$-link connected, and 0 otherwise (i.e. if it is 1- link connected). Let us show this variable for the existing network, $N(V,A)$, by $z_{\circ}^{js}$. The following is a network design (ND) problem for choice of accident mitigation (AM) measures:

$$(AM) \quad Max \sum_{j \in V} \sum_{s \in D} \Phi^{js} (z_y^{js} - z_\circ^{js}) \tag{9-0}$$

$$s.t.: (1) \quad \sum_{(i,j) \in A_y} e_{ij} y_{ij} \le B \tag{9-1}$$

$$(2) \quad y_{ij} = 0/1, \quad \forall (i,j) \in A_y \tag{9-2}$$

$$(3) \quad z_y^{js} = 1, \; if \; (j,s) \; is \; k_{k \ge 2} - link \;\; connected \; in \; N(V, A'_y)$$

$$0, \; otherwise. \tag{9-3}$$

This problem may be approached by any of the suitable set of existing algorithms to solve ND problem. The following is then a general procedure to solve this problem:

Algorithm (AM): To choose links to be prepared to join the network when needed.

Step 0. *Initialization*. Prepare the original network, $N(V,A)$; set of auxiliary links A_y , with cost e_{ij} for all $(i,j) \in A_y$; O/D demand d^{ks} for all $(k,s) \in P$; volume-delay functions $t_{ij}(x_{ij})$ for all $(i,j) \in A$; suitability standards θ^{js} for all $(j,s) \in V \times D$; the travel time of auxiliary links $(i,j), \bar{t}_{ij}$, for all $(i,j) \in A_y$.

Step 1. Assign O/D demand, d^{ks}, to the original network $N(V,A)$ and find the UE path flows and link travel times.

Step 2. For all $N/D\,(j,s)$ find Φ^{js} from Eqn. (4), using the results of step 1.

Step 3. Solve problem (AM) using a method to identify 1-link connected N/D , and any conventional ND algorithm. ∘

Remark 1. A procedure to identify $k_{k \ge 2}$-link-connected $N/D(j,s)$ in network $N(V,A)$ is as follows.

Let A_{-ij} denote the set of links A excluding link $(i,j) \in A$. We call $N/D(j,s)$, $k_{k \ge 2}$-link connected if it remains suitable for $N(V, A_{-ij})$, for all $(i,j) \in A$. Otherwise, i.e. if it becomes unsuitable for at least one such network, the N/D is called 1-link connected.

Remark 2. If we assume that using auxiliary links by some travellers would have a negligible effect on the flows of the rest of the links of the original network, then one may easily find the 1-link connected NDs by the procedure mentioned in remark 1. Moreover, one need not worry about Braess' paradox in such a case and hence use simpler algorithms of ND like Ochoa-Rosso and Silva(1968).

4. Numerical Examples

4.1 AP Problem 1

Consider the Example Network 1 in Fig. 2 with 6 nodes and 10 links. Suppose that link travel time functions are of the usual type: $t_{ij}(x_{ij}) = a_{ij} + b_{ij} x_{ij}^4$. Table 1 presents the parameters of this function for each link as well as the respective probability of non-occurrence of accident in the link. O/D trips from origins 1 and 4 to destinations 3 and 6 are assumed to be 7 thousands of trips / day. Assume an average occupancy rate of 1 person per vehicle for all O/Ds. Also assume that $\theta^{js} = 1.3$, i.e. any node–destination travel time after the occurrence of an accident which exceeds the respective value of travel time in a prevalent state (c°) by 30% , would be considered unsuitable. Consider 3 alternative actions for reduction of accident rates for each link in the network: (1) police presence to enforce laws more positively, (2) enhancing geometrics, signs, markings etc. of links in addition to the action mentioned in alternative 1 above, and (3) prompt response to accident occurrence to remove disabled vehicles and cleaning up the accident site quickly in addition to alternative 2 above. Let us assume that 1 unit of police would reduce the accident occurrence by 1/2 , 1 unit of police and 1 unit of expenditure in link enhancement would reduce this probability by 1/4 and finally, let us assume that 1 unit of police , plus 1 unit of expenditure in links, plus 1 unit of accident scene management workforce would remove either the probability of accident occurrence or the effect of an accident on the rest of the traffic. Assume that there are $B^1 = 4$ units of police, $B^2 = 3$ units of financial resources, and $B^3 = 2$ units of accident scene clearance workforce. The question is what link should be treated by which alternative to obtain the best result.

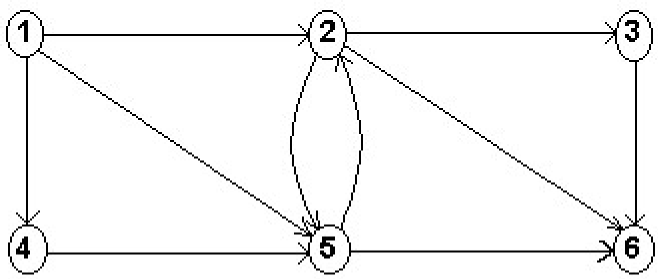


Fig.2 . Example Network 1. (node numbers are written on them)

Table 1. Specifications of the links of Example Network 1.

Link (i, j)	Free flow time (a_{ij}) $(\times 10^{-2}\, hr)$	Congestion parameter (b_{ij}) $(10^{-4} \times hr/(1000veh/day)^4)$	Prob. of non-occurrence of accident (p_{ij})
(1,2)	5	0.030	0.96
(1,4)	3	0.090	1.00
(1,5)	18	0.030	1.00
(2,3)	10	0.100	0.96
(2,5)	9	0.070	1.00
(2,6)	2	0.050	0.98
(3,6)	3	0.100	1.00
(4,5)	1	0.050	0.96
(5,2)	4	0.060	0.98
(5,6)	4	0.120	0.98

Applying algorithm AP to the problem resulted in the following. In step 1, at user equilibrium, each of the paths $\rho_1 = \{(1,2),(2,3)\}$, $\rho_2 = \{(1,2),(2,6)\}$, $\rho_3 = \{(4,5),(5,2),(2,3)\}$, and $\rho_4 = \{(4,5),(5,6)\}$ get a flow of 7 units and the link flows and travel times become as shown in Table 2. In step 2 of the algorithm, using Eqn.(4), one may compute Φ^{js} for all $(j,s) \in V \times D$. Also, using Eqn. (5), one may compute $\Psi^{js}(c^{\circ}_{-ij})$, and thus $\sum_{j\in V}\sum_{s\in D}\Psi^{js}(c^{\circ}_{-ij})$, for all $(i,j) \in A$. The former quantity is given in Table 3, and the latter in Table 2. Now we are in a position to find the suitability of the remaining trip from j to s, $(j,s) \in V \times D$, and thus determine the values of $z^{js}(c^{\circ}_{-ij})$.

Having had Φ^{js}, $\Psi^{js}(c^{\circ}_{-ij})$ and $z^{js}(c^{\circ}_{-ij})$, for all $(j,s) \in V \times D$ and all $(i,j) \in A$, one may then calculate in step 3 the importance of link (i,j), for all $(i,j) \in A$, by using Eqn. (7) as shown in Table 2. Step 4 of the algorithm may now be performed by solving problem (AP) using information regarding I_{ij} from step 3, and the input information p_{ij}, α^k_{ij}, and B^l $(k = 1,2,3,\quad l = 1,2,3 \quad and \quad (i,j) \in A)$. This is done here by the effective gradient method as presented by Nazim (1983). The optimal decisions are presented in Table 2, as the chosen alternative for each link.

Table 2. Results of applying algorithm (AP) on Example Network 1.

Link (i,j)	t_{ij} **(hr)**	x_{ij} $(1000veh/hr)$	$\sum_{j\in V}\sum_{s\in D}\Psi^{js}(c^{\circ}_{-ij})$	I_{ij}	$z^{\#}_{ij}$
(1,2)	0.1652	14	0.1131	0.1131	2
(1,4)	0.0300	0	0	0	-
(1,5)	0.1800	0	0	0	-
(2,3)	0.4842	14	0.1215	0.5050	3
(2,5)	0.0900	0	0	0	-
(2,6)	0.0320	7	0.0009	0.1043	-
(3,6)	0.0300	0	0	0	-
(4,5)	0.2021	14	0.1436	0.1436	3
(5,2)	0.0544	7	0.0199	0.2077	1
(5,6)	0.0688	7	0.0031	0.0031	-

\# z_{ij} =k shows that alternative k is chosen for link (i, j) . “-“means do nothing alternative.

Table 3. Quantity Φ^{js} for Example Network 1.

j \ s	3	6
1	0	0
2	0.1957	0.1034
3	0.3646	0
4	0	0
5	0.1878	0.1364
6	0	0.0122

4.2 Discussion

The contribution of the improvement in link (i, j) to the objective function of problem (AP) may be written as:

$$f_{ij}^{k} = I_{ij}.\Delta p_{ij} = \left[1 - \sum_{j\in V}\sum_{s\in D}\Phi^{js} z^{js}(c_{-ij}^{\circ})\right]\Delta p_{ij} + \left[\sum_{j\in V}\sum_{s\in D}\Psi^{js}(c_{-ij}^{\circ}) z^{js}(c_{-ij}^{\circ})\right]\Delta p_{ij} \tag{10}$$

where $\Delta p_{ij} = (1 - p_{ij})\alpha_{ij}^{k}$ is the percent of reduction by alternative action k in the probability of accident occurrence in link (i, j). This improvement in p_{ij} results in an improvement of suitable rt- hr's of the passengers in the accident-struck link (i, j) by an amount equivalent to the second part of the expression on the right-hand side of Eqn. (10) . The first part of Eqn. (10) regards the rt-hr's of the network which becomes unsuitable because of the unavailability of an alternative path for the accident-struck link. Improvement in link (i, j) would improve these rt-hr's by an amount equal to $\sum_{j\in V}\sum_{s\in D}\Phi^{js} z^{js}(c_{-ij}^{\circ})$.

Table 2 shows that link $(2,3)$ has the most importance, because the passengers going from nodes 1 and 4 to node 3 have no other option, except passing through link $(2,3)$. By accident occurrence in this roadway almost all remaining trips to destination 3 become unsuitable.

It is worth noting that link $(5,2)$ is on the one and only path that leads passengers of origin 4 to destination 3 . With an accident in this link, some of the remaining trips in the network become unsuitable. There is an alternative path to replace link $(5,6)$. Path $\rho = \{(5,2),(2,6)\}$ is such a path. So, passengers destined for node 6 may take this alternative path in case of an accident occurrence in link $(5,6)$, and keep their rt's suitable. However, link $(2,6)$ is an important link, because there is no suitable alternative path to replace this link for those passengers who reach node 2 on their way to destination 6. Path $\rho = \{(2,5),(5,6)\}$ is an alternative, but not with suitable travel time. Links $(1,2),(1,5),(1,4)$ and $(4,5)$ are links with tail nodes as origins. According to assumption 8, passengers who have not yet started their trips, would not do so in case there is no suitable path.

In passing, we note that the second term of Eqn.(10) is the benefit accrued due to the reduction of accident chance for those passengers in link (i, j), experiencing an accident in this link. This quantity is a function of the number of passengers in this link (which is in turn a function of the link flow, link length etc.) and their travel times to their respective destinations. Links $(4,5),(2,3)$, and $(1,2)$ are links with high values of this quantity.

4.3 AM Problem 1

Consider once again the Example Network 1 in Fig. 2, but this time for an accident mitigation example. Suppose that local streets (links) a, b, and c, with constant travel times 0.60, 0.04, and 0.50 hours may be equipped to bypass links $(5,3),(2,6)$, and $(2,3)$, at a cost of 1 unit each, respectively. Given a budget B, the problem is to identify the (best) projects which increase the objective function (9-0) most.

Solving the resulting problem by a suitable algorithm (e.g. a branch-and-bound algorithm similar to Ochoa-Rosso and Silva (1968)), results in the following solution: choose “c” for B=1, and choose “a” and “c” for B=2. It is quite clear why local street “c” has been chosen for B=1. This street may be a suitable alternative for link $(2,3)$, the failure of which would make a significant number of rt’s in the network unsuitable. Similarly, street “a” may offer an alternative path for those passengers who reach node 5 and intend to go to destination 3.

4.4 Problem 2

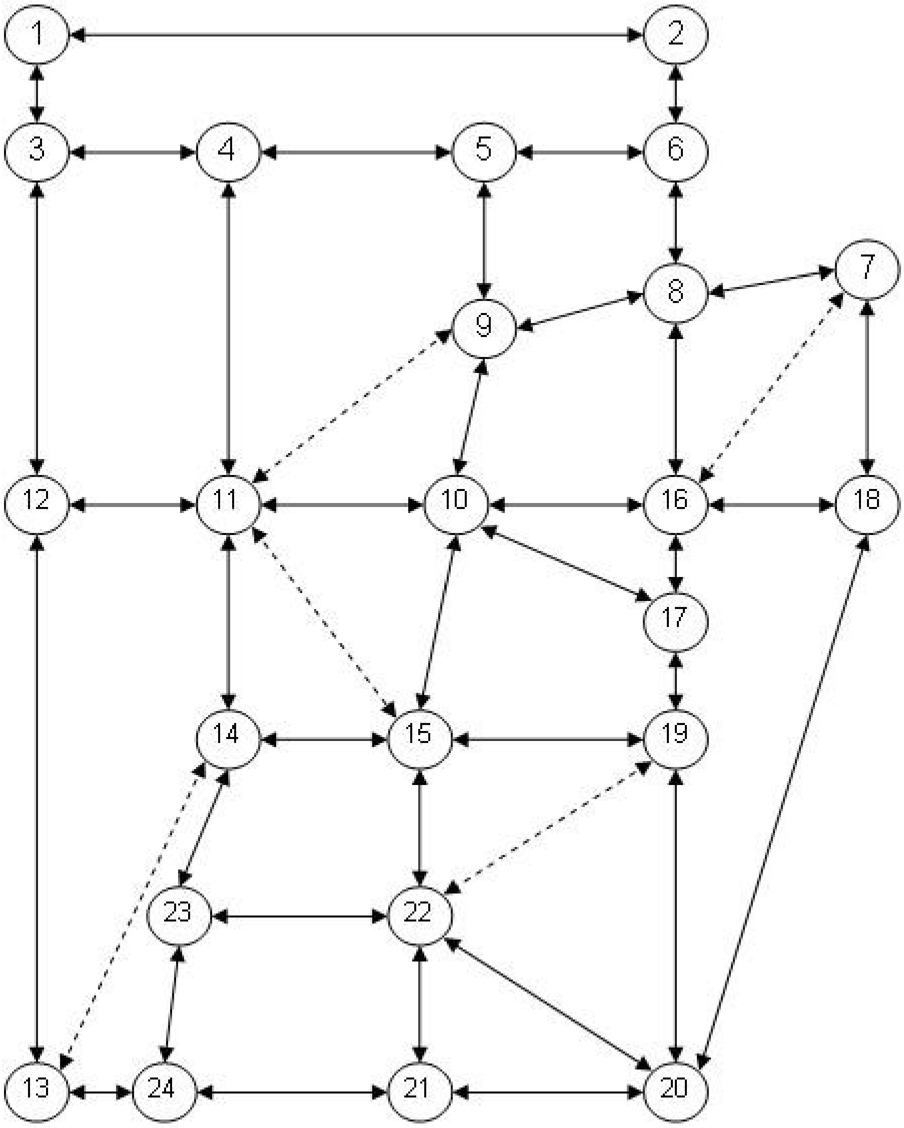
To show the applicability of the algorithms presented on large networks, Sioux Falls’network has been chosen as an Example Network 2. This network, with 24 nodes and 76 links, is shown in Fig.3. The network specifications and O/D demands are given in Tables 4 and 5, respectively. Again, for ease of computation, suppose that the average vehicle occupancy is 1 person. The probability of non-occurrence of accident for each link is given in Table 4. Finally, assume that $\theta^{js} = 1.15$ for all $j \in V$ and $s \in D$.

Using Algorithm AP to solve problem (AP) results in the importance of links as given in Table 6. For a vector of resources as (30,15,5) = (police, financial resources, accident scene clearance units), the solution of the problem is given in Table 6.

Applying Algorithm (AM) on the Example Network 2, proposes the solution given below for various budget levels for the set of candidate local links given in Table 7:

Budget (B)	**Proposed local streets**
1	d
2	d, f
5	c, d, e, f, j

There is an interesting point to note here. Because of the existence of many links in an urban network, like the Example Network 2, it seems that hardly any N/D is 1-link connected; hence, the question arises as to how useful and effective it is to solve an AM problem to implement the actions chosen.. Table 8 is an answer to this question. In this table the ratio of the number of 1-link connected N/Ds to the total number of N/Ds in the Example Network 2 (of Sioux Falls) for various values of θ^{js} are presented. The ratio in this table shows that it is quite significant for this seemingly connected network. Thus, solution to problem (AM) could be valuable information.



Legend:

________ Main streets

_ _ _ _ _ _ local streets

Fig. 3. Example Network 2: The Sioux Falls network.

Table 4. Specifications of the links of Example Network 2.

Link $(i, j)^*$	**Free flow time** (a_{ij}) $(\times 10^{-2}\, hr)$	**Congestion parameter** (b_{ij}) $(10^{-4} \times hr/(1000 veh/day)^4)$	**Prob. of non-occurrence of accident** (p_{ij})
(1,2)	5.96	0.00023	0.99
(1,3)	4.34	0.00017	0.99
(2,6)	5.17	0.12408	0.98
(3,4)	4.31	0.00069	0.99
(3,12)	4.14	0.00016	0.99
(4,5)	2.16	0.00035	0.99
(4,11)	6.46	0.15504	0.99
(5,6)	4.17	0.10008	0.99
(5,9)	5.03	0.00755	0.98
(7,18)	2.18	0.00008	0.98
(8,9)	9.61	0.23064	0.99
(8,16)	4.82	0.11568	0.98
(10,11)	5.00	0.00750	0.98
(10,15)	5.87	0.00265	0.98
(10,17)	8.04	0.19296	0.98
(11,12)	6.46	0.15504	0.98
(11,14)	4.42	0.10608	0.97
(12,13)	2.98	0.00011	0.99
(14,15)	4.52	0.10848	0.98
(15,19)	3.50	0.00104	0.98
(15,22)	3.50	0.00525	0.98
(16,17)	1.67	0.04008	0.95
(16,18)	2.69	0.00025	0.99
(17,19)	2.31	0.05544	0.96
(18,20)	4.46	0.00017	0.99
(19,20)	3.99	0.09576	0.97
(20,21)	5.72	0.13728	0.98
(20,22)	4.71	0.11304	0.98
(21,22)	1.67	0.04008	0.98
(21,24)	3.29	0.07896	0.97
(22,23)	4.00	0.09600	0.97
(14,23)	4.25	0.10200	0.98
(23,24)	1.88	0.04512	0.98
(9,10)	2.75	0.00124	0.98
(6,8)	2.17	0.05208	0.95
(13,24)	3.72	0.08928	0.96
(7,8)	2.50	0.01185	0.98
(10,16)	4.50	0.10800	0.98

* The information for (j,i) is the same as that of (i, j)

Table 5. O/D travel demand for Example Network 2 (thousands of veh./day)*

O-D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0.3	0.3	1.2	0.5	0.7	1	1.7	1.2	2.7	1.2	0.5	1.1	0.6	1	1.2	0.9	0.3	0.6	0.6	0.3	0.8	0.6	0.3
2	0.2	0	0.3	0.5	0.3	1	0.5	0.9	0.5	1.2	0.5	0.4	0.7	0.2	0.3	0.9	0.5	0.1	0.3	0.3	0.2	0.3	0.2	0.1
3	0.3	0.3	0	0.6	0.2	0.7	0.2	0.5	0.3	0.7	0.7	0.5	0.4	0.3	0.2	0.5	0.3	0.1	0.1	0.2	0.1	0.3	0.2	0.2
4	1.2	0.5	0.6	0	1	1	1	1.4	1.5	2.5	3	1.4	1.2	1.1	1	1.6	1.1	0.3	0.5	0.8	0.5	0.9	1	0.5
5	0.5	0.3	0.2	1	0	0.6	0.5	1.2	1.7	2.1	1.1	0.5	0.4	0.4	0.6	1.1	0.6	0.1	0.3	0.4	0.2	0.5	0.3	0.2
6	0.7	1	0.7	1	0.6	0	0.8	1.7	0.9	1.6	0.9	0.6	0.5	0.3	0.5	2	1.1	0.2	0.6	0.7	0.3	0.6	0.3	0.2
7	1	0.5	0.3	0.9	0.5	0.8	0	2.1	1.2	3.8	1.1	1.5	0.9	0.6	1.1	2.9	2.1	0.4	0.9	1.2	0.6	1.1	0.5	0.3
8	1.7	0.9	0.5	1.4	1.2	1.7	2.1	0	1.7	3.3	1.8	1.3	1.3	0.8	1.3	4.5	2.8	0.6	1.5	1.9	0.8	1.2	0.8	0.5
9	1.2	0.5	0.4	1.5	1.7	0.9	1.2	1.7	0	5.7	2.9	1.3	1.2	1.2	2	3	1.9	0.4	1	1.4	0.7	1.5	1.1	0.5
10	2.7	1.2	0.7	2.5	2.1	1.6	3.8	3.3	5.7	0	8.1	4.1	3.9	4.3	8.1	8.9	7.9	1.4	3.7	5.1	2.6	5.4	3.7	1.7
11	1.2	0.5	0.6	3	1.1	0.9	1.1	1.8	2.9	8	0	2.9	2.1	3.2	2.9	2.9	2.1	0.4	1	1.4	0.9	2.3	2.7	1.2
12	0.5	0.4	0.5	1.4	0.5	0.6	1.5	1.3	1.3	4.1	2.9	0	2.8	1.4	1.6	1.4	1.3	0.5	0.7	1	0.8	1.5	1.5	1.1
13	1.1	0.7	0.4	1.2	0.4	0.5	0.9	1.3	1.2	3.8	2.1	2.8	0	1.3	1.5	1.3	1.2	0.2	0.7	1.4	1.3	2.6	1.7	1.6
14	0.6	0.2	0.2	1.1	0.4	0.3	0.6	0.8	1.2	4.3	3.2	1.4	1.3	0	2.7	1.4	1.4	0.3	0.7	1	0.9	2.5	2.2	0.9
15	1	0.3	0.2	1	0.5	0.5	1.1	1.3	2	8.1	2.9	1.5	1.5	2.7	0	2.5	3.1	0.5	1.7	2.2	1.7	5.2	2	0.9
16	1.2	0.9	0.5	1.6	1.1	2	2.9	4.5	3	8.9	2.9	1.4	1.3	1.4	2.5	0	5.7	1	2.7	3.4	1.3	2.5	1.1	0.7
17	0.9	0.5	0.3	1.1	0.6	1.1	2.1	2.8	1.9	7.8	2.1	1.3	1.2	1.4	3.1	5.7	0	1.3	3.5	3.5	1.3	3.5	1.3	0.6
18	0.3	0.1	0.1	0.3	0.1	0.2	0.4	0.6	0.4	1.4	0.4	0.5	0.2	0.2	0.5	1	1.3	0	0.7	0.9	0.3	0.7	0.2	0.1
19	0.6	0.3	0.1	0.5	0.3	0.6	0.9	1.5	1	3.7	1	0.7	0.7	0.7	1.7	2.7	3.5	0.7	0	2.5	0.9	2.5	0.8	0.4
20	0.6	0.3	0.2	0.8	0.4	0.7	1.2	1.9	1.4	5.1	1.4	1	1.4	1	2.2	3.4	3.5	0.9	2.5	0	2.5	5	1.4	1
21	0.3	0.2	0.1	0.4	0.2	0.3	0.6	0.8	0.7	2.6	0.9	0.8	1.3	0.9	1.7	1.3	1.3	0.3	0.9	2.5	0	3.7	1.5	1.2
22	0.8	0.3	0.3	0.9	0.5	0.6	1.1	1.2	1.5	5.4	2.3	1.5	2.6	2.5	5.2	2.5	3.5	0.7	2.5	5	3.7	0	4.4	2.4
23	0.6	0.2	0.3	1	0.3	0.3	0.5	0.8	1.1	3.7	2.7	1.5	1.7	2.2	2	1.1	1.3	0.2	0.8	1.4	1.5	4.4	0	1.6
24	0.3	0.1	0.2	0.5	0.2	0.2	0.3	0.5	0.5	1.7	1.2	1.1	1.6	0.9	0.9	0.7	0.6	0.1	0.4	1	1.2	2.4	1.5	0

The demand values should be divided by 2 and then be used

Table 6. Result of applying algorithm (AP) on Example Network 2.

Link (i, j)	$\sum_{j\in V}\sum_{s\in D}\Psi^{js}(c^{\circ}_{-ij})$	I_{ij}	$z^{\#}_{ij}$	**Link** (i, j)	$\sum_{j\in V}\sum_{s\in D}\Psi^{js}(c^{\circ}_{-ij})$	I_{ij}	$z^{\#}_{ij}$
(1,2)	0.0019	0.0064	-	(2,1)	0.0008	0.0143	-
(1,3)	0.0026	0.0045	-	(3,1)	0.0006	0.0223	-
(2,6)	0.0045	0.0081	-	(6,2)	0.0012	0.0238	1
(3,4)	0.0041	0.0137	-	(4,3)	0.0019	0.0306	-
(3,12)	0.0015	0.0239	-	(12,3)	0.0028	0.0152	-
(4,5)	0.0036	0.0130	-	(5,4)	0.0013	0.0432	-
(4,11)	0.0022	0.0071	-	(11,4)	0.0019	0.0093	-
(5,6)	0.0055	0.0128	-	(6,5)	0.0048	0.0159	-
(5,9)	0.0100	0.0209	-	(9,5)	0.0072	0.0289	1
(7,18)	0.0021	0.0238	-	(18,7)	0.0013	0.0306	-
(8,9)	0.0077	0.0136	-	(9,8)	0.0062	0.0151	-
(8,16)	0.0056	0.0056	-	(16,8)	0.0068	0.0068	-
(10,11)	0.0093	0.0332	1	(11,10)	0.0112	0.0267	1
(10,15)	0.0180	0.0367	2	(15,10)	0.0120	0.0506	2
(10,17)	0.0022	0.0155	-	(17,10)	0.0044	0.0080	-
(11,12)	0.0039	0.0039	-	12,11)	0.0089	0.0089	-
(11,14)	0.0084	0.0178	1	(14,11)	0.0070	0.0201	1
(12,13)	0.0017	0.0270	-	(13,12)	0.0022	0.0270	-
(14,15)	0.0081	0.0138	-	(15,14)	0.0058	0.0180	-
(15,19)	0.0052	0.0291	1	(19,15)	0.0055	0.0402	2
(15,22)	0.0090	0.0373	2	(22,15)	0.0143	0.0271	1
(16,17)	0.0119	0.0419	3	(17,16)	0.0177	0.0296	3
(16,18)	0.0023	0.0281	-	(18,16)	0.0018	0.0275	-
(17,19)	0.0114	0.0234	2	(19,17)	0.0091	0.0242	2
(18,20)	0.0042	0.0405	-	(20,18)	0.0037	0.0454	1
(19,20)	0.0058	0.0158	1	(20,19)	0.0090	0.0197	1
(20,21)	0.0033	0.0089	-	(21,20)	0.0023	0.0186	-
(20,22)	0.0029	0.0088	-	(22,20)	0.0033	0.0095	-
(21,22)	0.0025	0.0078	-	(22,21)	0.0019	0.0117	-
(21,24)	0.0090	0.0192	1	(24,21)	0.0071	0.0226	1
(22,23)	0.0062	0.0134	-	(23,22)	0.0076	0.0106	-
(14,23)	0.0042	0.0161	-	(23,14)	0.0061	0.0151	-
(23,24)	0.0022	0.0129	-	(24,23)	0.0017	0.0138	-
(9,10)	0.0083	0.0367	2	(10,9)	0.0080	0.0359	2
(6,8)	0.0127	0.0286	3	(8,6)	0.0116	0.0333	3
(13,24)	0.0180	0.0233	2	(24,13)	0.0087	0.0408	3
(7,8)	0.0068	0.0219	-	(8,7)	0.0044	0.0351	2
(10,16)	0.0206	0.0206	1	(16,10)	0.0162	0.0162	1

\# z_{ij} =k shows that alternative k is chosen for link (i, j) . “-“ means do nothing alternative.

Table 7. Candidate local links for problem (AM) for the Example Network 2.

Local street name	Link (i, j)	$\bar{t}_{ij}$ (hr)	Cost of local st. preparation
a	(7,16)	0.20	1
b	(16,7)	0.20	1
c	(13,14)	0.22	1
d	(14,13)	0.22	1
e	(11,15)	0.24	1
f	(15,11)	0.24	1
g	(9,11)	0.24	1
h	(11,9)	0.24	1
i	(19,22)	0.26	1
j	(22,19)	0.26	1

Table 8. Ratio of 1-link connected N/Ds to total N/Ds

θ^{js}	Ratio	θ^{js}	Ratio
1.10	0.7708	1.50	0.4167
1.15	0.6806	1.70	0.3160
1.20	0.6076	2.00	0.2083
1.30	0.5104		

5. Summary and Conclusions

Accident events in transportation networks have three distinct adverse effects: (a) the effects upon those that are directly involved in the accident, which may be a combination of monetary costs, injuries or deaths. These are important and sometimes tragic consequences of traffic accidents that are the primary objectives of most analysts to deal with; (b) the effects upon the users of the network at large, which are usually in the form of traffic delays, opportunity losses and even induced accidents, and (c) the effects upon the environment which may be in the form of air pollution, environmental damages resulting from toxic material spillage, fumes etc.

Although great attention has been paid to reducing the adverse effects mentioned above in (a) and (c) , the authors are not aware of any previous work in the area of adverse effects in (b) above (See Iida,1999). This paper endeavours to formulate and solve the design problems of accident prevention and mitigation of (urban) road networks, basically with the objective of reducing the adverse effects of accidents upon the users of the network at large. This is in accordance with upgrading the performance of the network to function properly in cases of accidents or other similar events. In this respect, a network performance index has been defined based on the concept of suitable remaining trip-hours in a network after an accident occurrence in a link. This index led us to a measure of the importance of a link in the network, based on which the problem of accident prevention is defined and an algorithm is presented to solve it. The proposed (AP) algorithm incorporates the following important factors into the decision-making process: the link importance (topology characteristic, flow and length), the probability of accident occurrence in the link (congested and unsuitable traffic behaviour), and the cost of link improvement. A usual approach to select a street for

improvement in accident prevention is based on only some of the above-mentioned factors including congestion, traffic flow and accident probability.
The results of this algorithm have been shown for the design of two example networks. The smaller example network has 6 nodes and 10 links and is used to show the reasonability and suitability of the solution results. The second example network with 24 nodes and 76 links is used to show the applicability of the algorithms on large or real-sized networks.
In this case, the accident mitigation problem is defined based on the concept of remaining trip-hours and k-link connectedness. The objective of this problem is to make the important 1-link connected node-destination pairs at least 2-link connected. Again, an algorithm has been proposed to solve this problem and the results of this algorithm have been shown for the two example networks mentioned above. Contrary to the general belief that 1-link connected node-destinations are rare in an urban street network, it has been shown in this paper that a seemingly connected Sioux Falls' network has a surprisingly high proportion of 1-link connected node-destinations. Thus, (AM) algorithm may be an effective means for improving network connectivity.
Several assumptions have been made in the definition of the problem (AP) and (AM). Some of them are made to avoid undue complexities in the presentation of the problem and may be relaxed. For example, assumption 2 (accidents only occur in the links) may be relaxed by representing an intersection by several links. Research is under way to relax assumption 3 (users are informed about accidents immediately), and assumption 6 (the change of paths has negligible effects upon other users of the network). Moreover, the concept of suitable remaining trip-hours is related to a measure of welfare of the travellers, e.g. consumer surplus.

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Appendix

A.1 Proof of Proposition 1

There are $(\frac{m}{t_{ij}})dt$ number of travellers in a dt time interval at a point e in link (i, j) destined to s, by assumption of uniform distribution of the travellers over links, as shown in Figure A1. If t is the travel time from e to j, the time for these travellers to reach destination s is $t+t^{js}$, and the expected suitable rt-hr's is approximated by $(\frac{m}{t_{ij}}dt)(t+t^{js})(1-\frac{t}{t_{ij}}\bar{z}_{ij}(c))z^{js}(c)$ [1], where $(1-\frac{t}{t_{ij}}\bar{z}_{ij}(c))$ is the probability of having no accident from e to j in state c, and $z^{js}(c)$ accounts for the rest of the trips from j to s to be suitable. Thus,

$$E_{ij}^{s}(c) \approx \int_{0}^{t_{ij}} (\frac{m}{t_{ij}})(t+t^{js})(1-\frac{t}{t_{ij}}\bar{z}_{ij}(c))z^{js}(c)dt$$

which leads to the stated expression.

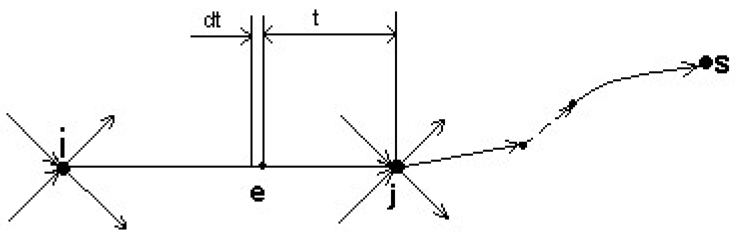

Fig.A1. Accident occurrence and determination of expected suitable rt-hr's.

1 Assuming that rt^{es} is suitable if and only if there is no accident in link segment (e, j) and rt^{js} is suitable.

A.2 Proof of Proposition 2

By the proportionality assumption, for x_{ρ}^{ks} number of travellers per unit of time on path ρ from k to s, there will be $x_{\rho}^{ks}t_{\rho}^{ks}$ travellers on path ρ at any instant of time, with $x_{\rho}^{ks}t_{\rho}^{ks}(\frac{t_{ij}}{t_{\rho}^{ks}})$ being the share of link (i, j) of that path hosting these travellers at that instant. Thus, in general $x_{\rho}^{ks}t_{ij}\delta_{ij,\rho}^{ks}$ is the number of travellers in link (i, j) due to those travelling in path ρ from k to s. Then, $\sum_{k\in O}\sum_{\rho\in\rho^{ks}} x_{\rho}^{ks}t_{ij}\delta_{ij,\rho}^{ks}$ would be the total number of travellers in link (i, j) heading to destination s. So, according to Proposition 1, the expected rt-hr's of all travellers in link (i, j) destined to s in state c may be written as:

$$E_{ij}^{s}(c) = \sum_{k\in O}\sum_{\rho\in\rho^{ks}} [x_{\rho}^{ks}t_{ij}\delta_{ij,\rho}^{ks}(\frac{t_{ij}}{2} + t^{js})z^{js}(c) - x_{\rho}^{ks}t_{ij}\delta_{ij,\rho}^{ks}(\frac{t_{ij}}{3} + \frac{t^{js}}{2})\bar{z}_{ij}(c)z^{js}(c)]$$

Then, this quantity in state c for all links with the head node j, $E^{js}(c)$, is:

$$E^{js}(c) = \sum_{i\in B(j)} E_{ij}^{s}(c)$$

For the network it is:

$$E(c) = \sum_{j\in V}\sum_{s\in D} E^{js}(c)$$

On the other hand, for the prevalent state c° the rt-hr's of the network may be computed as follows: d^{ks} is the rate of demand per unit of time from k to s, which takes t^{ks} unit of time to reach s from k. Then, at any instant, $d^{ks}t^{ks}$ would be the total number of travellers from k to s. On the average, these travellers are half-way through their path from k to s, so that the average remaining time for them would be $\frac{t^{ks}}{2}$, and hence:

$$E(c^{\circ}) = \sum_{(k,s)\in P} d^{ks}t^{ks}(\frac{t^{ks}}{2})$$

Thus,

$$PI(c) = \frac{E(c)}{E(c^{\circ})} = \frac{\sum_{j\in V}\sum_{s\in D} E^{js}(c)}{\sum_{(k,s)\in P} d^{ks}t^{ks}(\frac{t^{ks}}{2})}$$

which leads to the stated expression.