Journal of Mathematics Education
~JME~
Website: http://usnsj.com/index.php/JME
Email: editor.jme@usnsj.com

MATHEMATICS TEACHERS ASSESS INSTRUCTIONAL METHODS SUPPORTING KNOWLEDGE PROCESSES

AUTHORS INFO

Andreas M. Zendler
University of Education Ludwigsburg
zendler@ph-ludwigsburg.de

Cornelia Seitz
University of Education Ludwigsburg
seitz@ph-ludwigsburg.de

Dieter Klaudt
University of Education Ludwigsburg
dieter@klaudt.org

ARTICLE INFO

o-ISSN: 2528-2026
p-ISSN: 2528-2468
Vol. 4, No. 2, December 2019
URL: http://doi.org/10.31327/jomedu.v4i2.1096.

Suggestion for the Citation and Bibliography
Citation in text:
Zendler, Seitz, & Klaudt (2019)
Bibliography:
Zendler, A. M., Seitz, C., & Klaudt, D. (2019). Mathematics Teachers Assess Instructional Methods Supporting Knowledge Processes. *Journal of Mathematics Education*, *4*(2), 76-86 http://doi.org/10.31327/jomedu.v4i2.895

Abstract

Answers to the questions of which instructional methods are suitable for school, what instructional methods should be applied in teaching individual subjects and how instructional methods support the act of learning represent challenges to general education and education in individual subjects. This article focuses on the empirical examination of instructional methods supporting knowledge processes in the act of learning. A survey was conducted in which mathematics teachers evaluated 20 instructional methods in regard to the following knowledge processes: build, process, apply, transfer, assess and integrate. The results of the study demonstrate that certain instructional methods are especially predestined for mathematics education: problem-based learning, direct instruction, learning at stations, learning tasks, project work and discovery learning

Keywords: Mathematics education, instructional methods, teaching tools, knowledge processes, act of learning

A. Introduction

The wide range of instructional methods is almost incomprehensible. The associated literature describes a broad spectrum of instructional methods ranging from methods of conveying and acquiring knowledge to management methods for games, movement, emotions, groups, health, violence and conflicts. Answers to the questions of which instructional methods are suitable for school, what instructional methods should be applied in teaching individual subjects and how instructional methods support the act of learning represent challenges to general education and education in individual subjects.

A robust theory to instructional methods for mathematics education is missing (Hiebert & Grouws, 2007), which gives answers to the questions: (1) which teaching instructional methods should be used in mathematics education, (2) in what way do teaching methods in mathematics education support the learning process, and (3) which instructional methods are particularly suitable for which mathematical learning objects.

In view of the fact that there is little empirical material to date on instructional methods in mathematics education, three objectives have been at the forefront of the interest of a research project at the Institute of Mathematics and Computer Science of the University of Education Ludwigsburg: (1) An inventory of mathematics instructional methods: What instructional methods are currently in use in mathematics education?, (2) Instructional methods for the subject of mathematics: What instructional methods are appropriate for mathematics education?, and (3) specific application of instructional methods for the subject of mathematics: To what degree do instructional methods support the act of learning in mathematics education?

The following research hypothesis is linked with these three objectives: "Instructional methods for the subject of mathematics differ in supporting the act of learning".

B. Literature Review

1. Iinstructional Methods

Gugel (2011) cites more than 2,000 methods including their variations. The Internet provides well-prepared monographs of instructional methods, e.g. from *The Center for Teaching and Learning* (2015) cites 150 instructional method.

For mathematics education a number of good standard reference work is available which addresses the application of instructional methods (Zech, 1998; Heddens, Speer, & Brahier, 2008; Kidwell, & Ackerberg-Hastings, 2008; Barzel, Büchter, & Leuders, 2011; Reiss & Hammer; 2014; Cruickshank, Jenkins, Metcalf, 2011; Li, Silver, & Li, 2014; Ufer, Heinze, & Lipowsky, 2015).

Meyer (2002) is a source of a very general definition stating that instructional methods are the forms and procedures with which teachers and school pupils appropriate the natural and social reality surrounding them while observing the institutional framework conditions of the school. A stricter definition of method (than the one formulated above) which also represents the conceptual starting point for this article comes from Huber and Hader-Popp (2007): "The word method is understood to mean a clearly defined, conceptually perceivable and independent, if also integrated, component of teaching." (Huber & Hader-Popp, 2007, p. 3).

2. The Effectiveness Of Teaching Methods

Hattie (2009, chapter 9 and 10) informs about empirical results on the effectiveness of teaching methods in general. High effect sizes ($d > .50$) were demonstrated for microteaching ($d = .88$), reciprocal teaching ($d = .74$), feedback ($d = .73$), problem solving ($d = .61$), direct instruction ($d = .59$), mastery learning ($d = .58$), case study ($d = .57$), concept mapping ($d = .57$), peer tutoring ($d = .55$), cooperative (vs. competitive) learning ($d = .54$), and interactive instructional videos ($d = .52$). The empirical results presented by Hattie are kept very general and cannot be concretized for individual subjects. Particularly, for mathematics education empirical findings are lacking on the c with respect to the act of learning.

The search through the magazines and conference reports on mathematics education shows that empirical studies on the effectiveness of instructional methods are rare. Current work provided findings related to direct instruction (Mainali, & Heck , 2015), comparisons of instructional methods (Code, Piccolo, & Kohler, 2014), problem-based learning (Karp, 2010; Schukajlow, Leiss, Pekrun, Blum, Müller, & Messner, 2012). Recently, Zendler, Seitz, and Klaudt (2018a, b) publish a validation study and a cross-contextual study on instructional methods in STEM subjects.

However, a variety of teaching examples with methodical focus are included in practice-oriented journals on mathematics education: e.g. *The Mathematics Educator*, *The Mathematics Enthusiast*, *Mathematics Teaching, Praxis der Mathematik in der Schule.* Zendler, Seitz, and Klaudt (2016) describe implementation steps for 20 different instructional methods (see Appendix A-1) and specify references which exemplify the instructional methods in mathematics classrooms.

Many theoretical learning/teaching approaches make a distinction between phases/processes/cycles for which instructional methodology aids are formulated; overviews of such are provided by Tennyson et al. (1997) and Petrina (2006). For instance, Merill (2002) suggests that the most effective learning environments are those that are problem-based and involve the student in four distinct phases of learning: (1) activation of prior experience, (2) demonstration of skills, (3) application of skills, and (4) integration or these skills into real world activities.

The theory from Collins et al. (1989), which has situated learning at its core, reveals four main phases: *modeling, scaffolding, fading,* and *coaching.* Cognitive oriented approaches (Bruner, 1966; Gagné et al., 2004) link instruction to the acquisition and processing of knowledge. They emphasize three (cognitive/knowledge) processes in the act of learning: *acquisition of new information, transformation (manipulating knowledge to make it fit new tasks), evaluation (checking whether the way we have manipulated information is adequate to the task)* (*see* Bruner, 1966, p. 48; Merriam & Caffarella, 2006, p. 46; Gowda, 2010)

C. Methodology

1. Research Design

Selection of instructional methods. The review of a series of instructional methods manuals (Ginnis 2001; Petrina 2006; Davis 2009; Joyce & Weil 2008; Peterßen 2009; Petty 2009; Brenner & Brenner 2011; Wiechmann 2011; Cruickshank et al. 2011) revealed more than 50 instructional methods to choose from. The review was characterized by the requirement that instructional methods had to pass the muster as being capable of being understood as clearly defined, conceptually perceivable and independent components of the instruction.

The following criteria were applied for the final selection of the instructional methods: (1) The actual application of the instructional methods in mathematics education, (2) the application of the instructional methods in STEM subjects (sciences, technology, engineering, mathematics) and (3) empirically examined instructional methods. The following 20 instructional methods (in alphabetical order) were able to be selected on the basis of these criteria.

Case study, computer simulation, concept mapping, direct instruction, discovery learning, experiment, jigsaw method, learning at stations, learning by teaching, learning tasks, Leittext method, models method, portfolio method, presentation, problem-based learning, programmed instruction, project work, reciprocal teaching, role-play, and web quest (*see* Appendix A-1 Instructional Methods).

Processes involved in the act of learning. The educational literature knows numerous variations relating teaching to learning as an act spread over time and to phases which can be distinguished during the course of learning (Bruner 1966; Petrina 2006; Olson 2007; Davis 2009). What all of the variations have in common is that learning (1) has a starting point, (2) a sequential form and (3) a (generally preliminary) end point. Educational literature describes this as the *classic three-step* pattern divided into the steps labeled *entry, work phase* and *graduation.* These three steps have particularly large distinctions in their educational functions and in the knowledge processes of the act of learning. Particularly in the work phase, important knowledge processes (Bruner 1966; Merriam and Caffarella, 2006; Gowda 2010) can be distinguished in the act of learning. This indicates the processes in the acquisition of knowledge (*build, process*), in the transformation of knowledge (*apply, transfer*) and in the evaluation of knowledge (*assess, integrate*) (*see* Appendix A-2 Knowledge processes).

Experimental design. An RBF-20×6 experimental design (Randomized Block Factorial design, 2-factor design with repeated measures, see Figure 1) is used to test the research hypothesis (Winer, Brown, & Michels, 1991; Kirk 2012).

Independent variables. Factor A comprises the p = 20 instructional methods with factor levels $a_1, ..., a_{20}$: case study, computer simulation, concept mapping, direct instruction, discovery learning, experiment, jigsaw method, learning at stations, learning by teaching, learning tasks, Leittext method, portfolio method, presentation, problem-based learning, programmed instruction, project work, reciprocal teaching, role-play, and web quest. Factor B represents the q = 6 knowledge processes with factor levels $b_1, ..., b_6$: *build, process, apply, transfer, assess* and *integrate.*

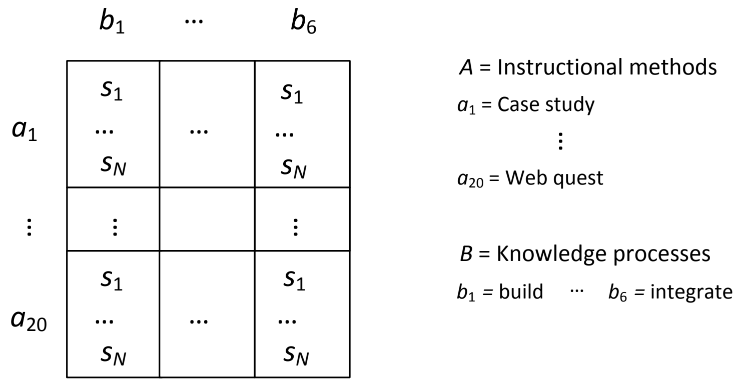

Figure 1. Layout of the used RBF-20×6 experimental design.

Dependent variables. The dependent variable was the respondents´ evaluation of the instructional methods with respect to the six knowledge processes. Ratings were given on a six-point scale with ratings ranging from 0 ("not significant") to 5 ("very significant").

Power analysis The sample size for the RBF-20×6 experimental design (Mueller & Barton 1989; Mueller et al. 1992) is determined with a type II power analysis – N as a function of power (1-β), Δ and α . The desired power (1-β) is 0.80, and only large effects (Δ = 0.80) in relation to the dependent variable are classified as significant; the significance level is α = 0.05. Then a total sample of approximately N^* = 120 mathematics teachers is needed, based on the power calculations by Mueller and Barton (1989), respectively, by Mueller, LaVange, Ramey and Ramey (1992) for ε-corrected F-Tests.

Operational test hypothesis. Given the study design and the above specification of the independent and dependent variables, the operational hypothesis of the study can be formulated as follows:

"Instructional methods for the subject of mathematics differ in supporting the act of learning, as operationalized by mathematics teachers´ ratings on a six-point scale of the knowledge processes *build, process, apply, transfer, assess* and *integrate."*

2. *Instruments*

Sample. For the empirical study, a total of 120 mathematics teachers working at secondary schools in the German State of Baden-Württemberg were contacted in writing and asked to fill out a questionnaire on the application of instructional methods in mathematics education. The mathematics teachers who completed and returned the questionnaire taught mathematics in the grade levels 5 through 12/13. On average they had taught mathematics for more than 10 years; in addition to teaching mathematics, all of the mathematics teachers also taught a STEM subject.

Questionnaire. The questionnaire consisted of a short introduction listing the 20 instructional methods and the 6 knowledge processes. The questionnaire was accompanied by a booklet (Zendler, Seitz, & Klaudt, 2016) for the mathematics teachers describing the 20 instructional methods in accordance with a uniform scheme containing (1) a brief description and explanation, (2) concrete execution steps, (3) and examples from the relevant literature verifying the application of the instructional method in mathematics education.

Tasks. The p =20 instructional methods and the q = 6 knowledge processes were then presented in alphabetical order in a matrix with the instructional methods in the rows and the knowledge processes in the columns. Participants were asked to indicate the relevance of each of the 20 × 6 = 120 matrix cells: Each cell represents a combination of an instructional method and a knowledge process and requires an integer from 0 ("not significant") to 5 ("very significant") indicating the relevance of the combination (*see* Appendix A-3 Questionnaire).

Return rate. To maximize the return rate, we mailed the questionnaire in sealed, personalized envelopes enclosing a pre-addressed return envelope franked with stamps showing flower designs (see Dillman, 2000 for recommendations on increasing return rates). The return rate was 24,2 % (N =29 completed questionnaires of 40 received questionnaires), which can be regarded as a normal rate for surveys conducted by post (cf. Vaux & Briggs, 2005).

3. *Technique of Data Analysis*

The following procedure is recommended for the analysis of the experimental data (original data, *see* Appendix D. Data): (1) The data are initially analyzed descriptively. (2) Then a two-factor analysis of variance (ANOVA) with repeated measures was conducted in accordance with the RBF-20×6 experimental design (see Winer et al. 1991, Chapter 7). (3) Finally, a cluster analysis is calculated aimed at identifying groups of instructional methods which can be characterized by their support of similar knowledge processes in the act of learning.

The data on the RBF-20×6 experimental design were analyzed using IBM SPSS Statistics 22.0; the power analysis was calculated using PASS 13.

D. Findings and Discussion

1. Findings

1.1 Descriptive Findings

The heat map seen in Figure 2 contains means visualized for the 20 instructional methods in relation to the six cognitive/knowledge processes processes: *build, process, apply, transfer, assess*, and *integrate*. The figure also contains the grand means (N = 29) per instructional method. The instructional methods are sorted in accordance with these grand means.

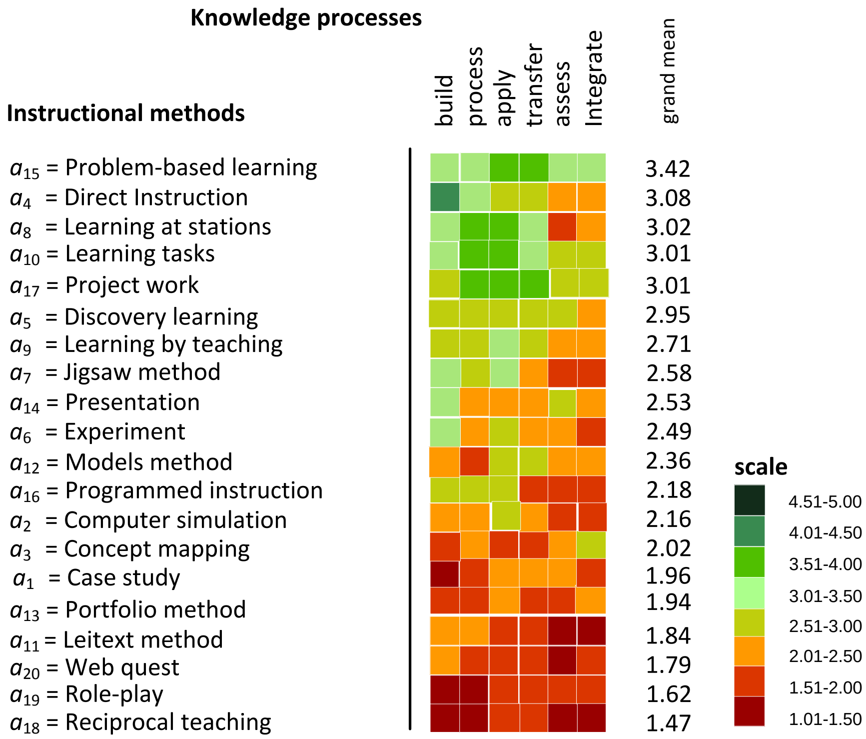

Figure 2. Means visualized for the cognitive/knowledge processes

Figure 2 shows initially that problem-based learning was assessed by the mathematics teachers as the best method for supporting the act of learning in mathematics education; this method is followed by five additional instructional methods: direct instruction, learning (at) stations, learning tasks, project work and discovery learning.

In a more detailed observation the heat map reveals that problem-based learning is distinguished by high values (> 3.00) for all knowledge processes. The instructional method direct instruction is characterized by a very high value (> 4.00) for the knowledge process *build.* The instructional method learning (at) stations is characterized for the knowledge processes of *process* and apply *build*. The learning tasks instructional method demonstrates high values for the knowledge process of *process* and *apply.* The project work instructional method is characterized by high values for the three knowledge processes of *process, apply* and *transfe*r.

The following instructional methods had relatively low values in all of the knowledge processes (< 3.00): models method, programmed instruction, computer simulation and concept mapping. The portfolio method, case study, Leittext method, Web quest, role-play and reciprocal teaching were rated as relatively poor (< 2.50) in all of the knowledge processes.

1.2 Analysis of Variance

To examine whether the combinations of instructional methods and knowledge processes within the act of learning differ, three statistical hypotheses are formulated which are tested at the significance level of $\alpha = 0.05$.

Statistical hypotheses. The three null hypotheses are as follows:

i) the means of the instructional methods $\mu_1, \mu_2, ..., \mu_{20}$ under the 20 levels of factor *A* are equal, i.e.:

$H_0: \mu_1 = \mu_2 = ... = \mu_{20}$ $(H_1: \mu_1 \neq \mu_2 \neq ... \neq \mu_{20})$;

ii) the means of the knowledge processes in the act of learning $\mu_1, \mu_2, ..., \mu_6$ under the 6 levels of factor *B* are equal, i.e.:

$H_0: \mu_1 = \mu_2 = ... = \mu_6$ $(H_1: \mu_1 \neq \mu_2 \neq ... \neq \mu_6)$;

iii) the means of the instructional methods with respect to the knowledge processes $\mu_{1\times1}, \mu_{1\times2}, ..., \mu_{20\times6}$ under the 20 × 6 levels of the factor combinations *A* × *B* are equal, i.e.:

$H_0: \mu_{1\times1} = \mu_{1\times2} = ... = \mu_{20\times6}$ $(H_1: : \mu_{1\times1} \neq \mu_{1\times2} \neq ... \neq \mu_{20\times6})$.

Testing the statistical assumptions. For an analysis of variance of an RBF-20×6 experimental design, the data must satisfy the condition of sphericity. This assumption was tested using Mauchly's *W* test for sphericity, with the test statistic *W* being compared to a chi-square distribution to assess the adequacy of the sphericity.

The assumption of sphericity must be discarded both for the instructional methods ($W < 0.001$, $\chi^2_{189} = 365.51$, $p < 0.001$) and also for the processes of the acquisition of knowledge ($W = 0.116$, $\chi^2_{14} = 45.53$, $p < 0.001$) at the α-level of 0.05. In the further analysis, we therefore applied the ε-correction of degrees of freedom proposed by Huynh and Feldt (1976).

Findinge. Table 1 contains the results of the ANOVA with the ε-correction of the degrees of freedom.

Table 1. ANOVA with Huynh-Feldt ε-corrections of the degrees of freedom

Source of variation (*within*)	***SS***	***df***	***MS***	***F***	***p***	***η2***
A (instructional methods)	1024.01	9.18	111.59	10.32	< .001	.269
error (A)	2777.39	256.93	10.81			
B (knowledge processes)	156.29	4.19	37.28	9.71	< .001	.257
error (B)	450.78	117.38	3.84			
A × B	448.45	34.07	13.16	4.64	< .001	.142
error (A × B)	2707.15	953.84	2.83			

The main effect *A* (instructional methods) is significant ($F_{9.18, 256.93} = 10.32$, $p < 0.001$) at the α-level of 0.05, i.e., the corresponding H_0 is rejected: the instructional methods differ from one another.

The main effect *B* (knowledge processes) is significant ($F_{4.19, 117.38} = 9.71$, $p < 0.001$) at the α-level of 0.05, i.e., the corresponding H_0 is rejected: the knowledge processes differ from one another.

The interaction effect *A* × *B* (instructional methods × knowledge processes) is significant ($F_{34.07, 953.84} = 4.64$, $p < 0.001$) at the α-level of 0.05, i.e., the corresponding H_0 is rejected: the instructional methods differ from one another with respect to knowledge processes.

1.3 Cluster Analysis

The alphabetically sorted 20 × 6 data matrix (see Appendix D. Data) with the means of the instructional methods in regard to the knowledge processes is taken as the data basis for the cluster analysis. The cluster analysis has been done using the method of Ward (1963) with squared Euclidean distance as distance measure (Everitt et al., 2001) For the termination of the algorithm, the C-Index (Hubert & Levin, 1976) has been taken into consideration (this is visualized as "cut" in the following figure).

Figure 3 shows the results of the cluster analysis for the instructional methods. The dendrogram reveals that seven clusters with instructional methods emerged. Cluster 1, 2, 3, and 4 contain instructional methods with (relatively) high values for many knowledge processes.

Cluster 5, 6, and 7 contain instructional methods with (relatively) low values for many knowledge processes.

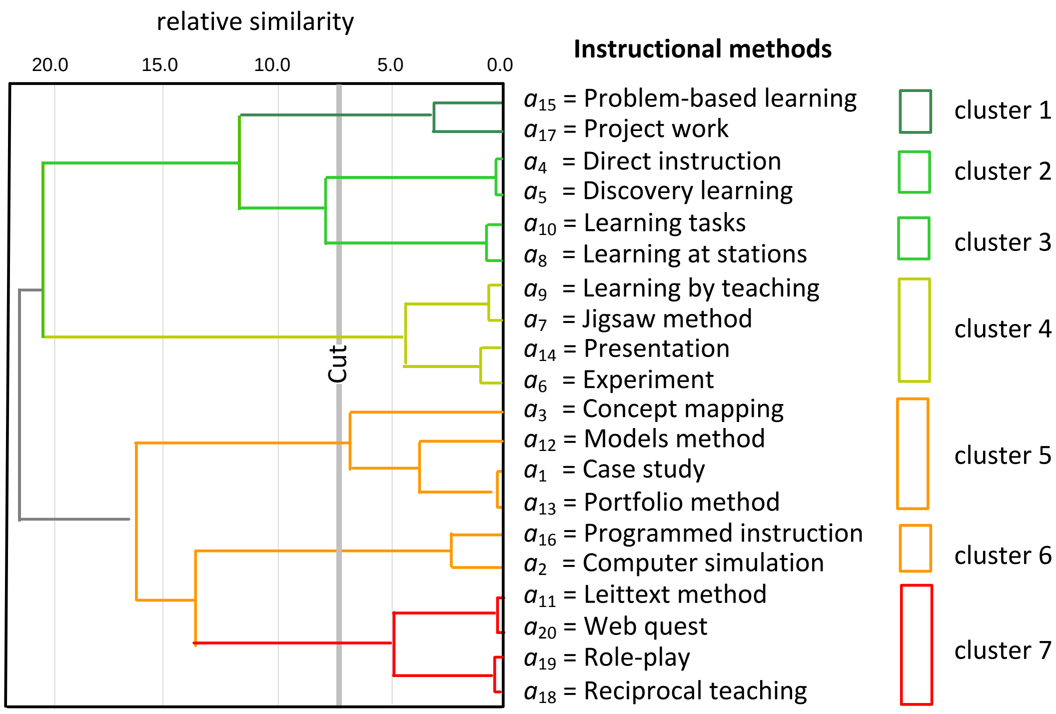

Figure 3. Dendrogram and clusters of instructional methods (N = 29)

Cluster 1. This cluster contains the two instructional methods problem-based learning und project work. They are characterized by similarily high values for the knowledge processes *apply* und *transfer.* From the dendrogram and the heatmap it is obvious that this cluster is not homogeneous as shown by the relative similarity index. Moreover, the dendrogram shows that these instructional methods cannot be merged with other instructional methods due to the value for the C-index (see "Cut" in Figure 3).

Cluster 2. This cluster consists of the two instructional methods of direct instruction and discovery learning. These methods are characterized by values that are similarily high in regard to the knowledge processes *apply, transfer, assess, and integrate.*

Cluster 3. This cluster contains the two instructional methods Learning tasks and learning at stations. These are characterized by similarily high values for the knowledge processes *build, process, apply,* and *transfer.*

Cluster 4. This cluster consists of four instructional methods*:* Learning by teaching, jigsaw, presentation, and experiment. Whereas learning by teaching and jigsaw are characterized by similar values with respect to the knowledge processes of *process and apply,* presentation and Experiment are characterized by similar values for the knowledge processes of *build, process,* and *transfer.*

Cluster 5. This cluster comprises four instructional methods: Concept mapping, models method, case study, and portfolio method. The last-mentioned instructional methods case study and portfolio method have very similar values with respect to all knowledge processes. The two other instructional methods are assigned to this cluster consecutively: first the model method, then concept mapping.

Cluster 6. This cluster contains the two instructional methods Programmed instruction and computer simulation. They are characterized by many similar values for most of the knowledge processes (see the data in Appendix A-4).

Cluster 7. This cluster consists of two very homogeneous sub clusters with the instructional methods Leittext method and Web quest as well as role-play and reciprocal teaching, respectively. The four instructional methods have (very) low values for almost all knowledge processes. The instructional methods in the second sub cluster show very low, but similar values.

2. Discussion

It must first be noted that the findings support the research hypothesis formulated in this paper's Introduction that instructional methods for the subject of mathematics differ in supporting the act of learning.

In the opinions of the mathematics teachers, problem-based learning seems to perform best in relation to almost all of the knowledge processes. Direct instruction is best suited for the knowledge processes of *build*; moreover, it is appropriate for the knowledge process of *process*. Learning at stations and learning tasks are useful for two knowledge processes (process, *apply*). Project work is the instructional method that is suitable for the knowledge processes of *process, apply and transfer*. The instructional models discovery learning and learning by teaching method can be used with respect to four and five knowledge process, respectively. In addition, the instructional methods jigsaw, presentation and experiment are useful in relation to at least two knowledge processes. The instructional methods that are unsuitable for mathematics education are case study, portfolio method, Leittext method, role-play, and reciprocal teaching.

Conversely, these findings also answer the question regarding what knowledge processes are adequately supported by which instructional method. It must first be noted that the knowledge processes in the act of learning are supported by the instructional methods in wholly different ways. The knowledge process *build* is supported by the instructional methods problem-based learning, direct instruction, learning at stations, learning tasks and project work. The situation is similar for the knowledge process of *process* which is positively influenced by the instructional methods problem-based learning, direct instruction, learning at stations, learning tasks, and project work. The knowledge processes *apply* and *transfer* are supported in particular when the instructional methods problem-based learning, direct instruction, learning at stations, learning tasks, and project work are applied. The knowledge processes *assess* and *integrate* are supported by problem-based learning, by the instructional methods learning tasks, project work, and concept mapping.

The first four knowledge processes (*build, process, apply, transfer*) receive significantly greater support from the instructional methods than the last two knowledge processes (*assess, integrate*). The knowledge processes *assess* and *integrate* are only relatively well supported by one instructional method: problem-based learning.

E. Conclusion

Based on the opinions of the mathematics teachers, the following recommendations can be expressed for the application of instructional methods in mathematics education: (1) For the knowledge process of *build* direct instruction should be used in combination with problem-based learning and augmented by learning tasks in order to initiate the knowledge process of *process*. (2) For the knowledge process of *apply*, earning tasks, learning at stations, and project work should be used.(3) For the knowledge process of *apply,* problem-based learning should be used in combination with learning tasks, learning at stations, and project work. (4) To support the knowledge processes of *transfer, assess* and i*ntegrate*, problem-based learning should be used and supported by learning tasks. (5) To introduce diversity into mathematics education and to increase the motivation of the learners it is recommended to use instructional methods in a substituting role to the extent that they support similar knowledge processes. It can for instance be derived from the cluster analysis and the heat map that learning tasks and learning at stations are similar in their relation to the knowledge processes, as are – to some extent– earning by teaching and jigsaw in their relation to the knowledge processes *build, process* and *apply.*

The findings determined in this examination on the application of instructional methods in mathematics education confirm the recommendations made in standard works on the subject of mathematics education. This applies for the instructional methods problem-based learning, learning tasks, learning (at) stations, and project work (cf. Zech, 1998; Reiss & Hammer, 2013; Barzel, Büchter, & Leuders, 2011; Heddens, Speer, & Brahier, 2008; Kidwell & Ackerberg-Hastings; Li, Silver, & Li, 2014). In contrast, the positive findings cited by the literature on the instructional method of programmed instruction, models method, and portfolio method are not applicable.

The data from mathematics teachers who teach at secondary schools was able to be included in the study. In order to verify and validate the results of these findings an examination should take place in authentic teaching and learning settings, and should not be based on subjective opinions. Instructional methods assessed in this study as being very unfavorable for mathematics education such as reciprocal teaching, role-play, Web quest and the Leittext method (Leittext method) do not need to be observed further.

Moreover, the findings in this study showed that the knowledge processes *assess* and *integrate* are only adequately supported by one instructional method, namely problem-based learning. As such, the field of developing methods for mathematics education is faced with the task of developing instructional methods which support these knowledge processes in the act of learning. In the authors' opinion, the starting point for the development of such instructional methods can be found within the context of competence-based learning tasks and in cross-curricular instruction.

F. References

Abell, S. K., & Lederman, N. G. (2007). *Handbook of research on science education*. New York: Lawrence Erlbaum.

Aldrich, C. (2009). *Simulations & serios games*. San Francisco: Pfeiffer.

Aronson, E. (1978). *The jigsaw classroom*. Beverly Hills: Sage Publications.

Barzel, B., Büchter, A., & Leuders, T. (2011). *Mathematik. Methodik. Handbuch für die Sekundarstufe I und II.* Berlin: Cornelsen Scriptor.

Branom, M. E. (1918). *The project method in education*. London: BiblioBazaar (Reprinted 2008).

Brenner, G., & Brenner, K. (2011). *Methoden für alle Fälle*. Berlin: Cornelsen.

Brown, A. L., & Campione, J. C. (1990). Communities of learning and thinking, or a context by any other name. *Human Development, 21*, 108–129.

Bruner, J. S. (1966). *The process of education*. Cambridge: Harvard University Press.

Büchter, A., & Leuders, T. (2005). *Mathematikaufgaben selbst entwickeln: Lernen fördern-Leistung überprüfen*. Cornelsen Scriptor.

Code, W. Piccolo, C., & Kohler, D. (2014). Teaching methods comparison in a large calculus class*.* *ZDM Mathematics Education*, *46,* 589–601.

Collins, A., Brown, & Newman, S.E. (1989). Cognitive apprenticeship. Teaching the crafts of reading, writing, and mathematics. In L.B. Resnick (Ed.), *Knowing, learning and instruction* (pp. 453–494). Hillsdale: Erlbaum.

Cruickshank, D. R., Jenkins, D. B., & Metcalf, K. K. (2011). *The act of teaching*. Boston: McGraw Hill.

Davis, Gross B. (2009). *Tools for teaching*. San Francisco: Jossey-Bass.

Dillman, D. A. (2000). *Mail and Internet surveys: the tailored design method*. New York: Wiley.

Everitt, B.S., Landau, S., & Leese, M. (2001). *Cluster analysis*. London: Arnold.

Flewelling, G., & Higginson, W. (2003). *Teaching with rich learning tasks: A handbook*. Sydney: Australian Association of Mathematics Teachers.

Gartner, A., Kohler, M. C., & Riessman, F. (1971). *Children teach children. Learning by teaching*. New York: Harper & Row.

Ginnis, P. (2001). *The teacher's toolkit. Classroom achievement*. Carmarthen: Crown House Publishing.

Gugel, G. (2011). *2000 Methoden für Schule und Lehrebildung*. Weinheim: Beltz.

Hattie, J. (2009). *Visible learning*. New York: Routledge.

Heddens, J. W., Speer, W. R., Brahier, D. J. (2008). *Today's Mathematics: Concepts, Methods, and Classroom Activities.* New York: Wiley.

Hiebert, J. und Grouws, D.A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte, NC: Information Age.

Huber, S.G., & Hader-Popp, S. (2007). Unterrichtsentwicklung durch Methodenvielfalt im Unterricht fördern: das Methodenatelier als schulinterne Fortbildung. In A. Bartz,. J. Fabian, S.G. Huber, Kloft, C. H. Rosenbusch, H. Sassenscheidt (Eds.), *PraxisWissen Schulleitung* (30.31). München: Wolters Kluwer.

Hubert, L. J., & Levin, J. R. (1976). A general statistical framework for assessing categorical clustering in free recall. *Psychological Bulletin 83*, 1072–1080.

Huynh, H., & Feldt, L. S. (1976) Estimation of the Box correction for degrees of freedom from sample data in randomised block and split-plot designs. *Journal of Educational Statistics, 1*, 69–82.

Karp, A. (2010). Analyzing and attempting to overcome prospective teachers' difficulties during problem-solving instruction. *Journal of Mathematics Teacher Education, 13*, 121–139.

Katz, H. (2009). Figuren – ein Mandala-Workshop mit EuklidDynaGeo. *Praxis der Mathematik in der Schule, 30,* 40–43.Kidwell, P. A., & Ackerberg-Hastings, A. (2008). *Tools of*

American Mathematics Teaching, 1800-2000. Baltimore: Johns Hopkins University Press.

Kirk, E. (2012). *Experimental design*. Belmon: Wadsworth.

Kron, F. W. (2008). *Grundwissen Didaktik*. Stuttgart: UTB.

Lester, F.K. (Ed.) (2007), Second handbook of research on mathematics teaching and learning. Charlotte, NC: Information Age.

Li, Y., Silver, E.A., & Li, S. (2014). *Transforming Mathematics Instruction: Multiple Approaches and Practices*. Berlin: Springer.

Lucas, C. A. (2000). Jigsaw lesson for operations of complex numbers. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 10*(3), 219–224.

Mainali, B. B., & Heck, A. (2015). Comparison of traditional instruction on reflection and rotation in a Nepalese high school with an ICT-rich, student-centered, investigative approach. *International Journal of Science and Mathematics Education.* Published online, November 9.

Meyer, H. (2002). Unterrichtsmethoden. In H. Kiper, H. Meyer, & W. Topsch (Eds.), *Einführung in die Schulpädagogik* (pp. 109-121). Berlin: Cornelsen.

Moust, J., Bouhuijs, P., & Schmidt. H. (2007). *Introduction to problem-based learning*. London: Chapman & Hall.

Mueller, K. E., & Barton, C. N. (1989). Approximate power for repeated-measures ANOVA lacking sphericity. *Journal of the American Statistical Association, 84*(406), 549–555.

Mueller, K. E., LaVange, L. E., Ramey, S. L., & Ramey, C. T. (1992). Power calculations for general linear multivariate models including repeated measures applications. *Journal of the American Statistical Association, 87*(420), 1209–1226.

NCTM (National Council of Teachers of Mathematics). (2000). P*rinciples and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Novak, J. D. (1990). Concept mapping: A useful tool for science education. *Journal of Research in Science Teaching, 27*, 10, 937–949.

Novak, J. D., & Cañas, A. J. (2008). *The theory underlying concept maps and how to construct and use them.* Technical Report Institute for Human and Machine Cognition, CmapTools, Pensacola Fl.

Olson, D. R. (2007). *Jerome Bruner: The cognitive revolution in educational theory*. New York: Continuum.

Petty, G. (2009). *Teaching today: a practical guide*. Cheltenham: Nelson Thornes.

Reinmann-Rothmeier, G., & Mandl, H. (2001). Unterrichten und Lernumgebungen gestalten. Forschungsbericht. In A. Krapp & B. Weidenmann (Eds.), *Pädagogische Psychologie* (pp. 613–658). Weinheim: Beltz.

Reiss, K. & Hammer, C. (2013). *Grundlagen der Mathematikdidaktik: Eine Einführung für den Unterricht in der Sekundarstufe*. München: Birkhäuser.

Mainali, B. B., & Heck, A. (2015). Comparison of Traditional Instruction on Reflection and Rotation in a Nepalese High School with an ICT-Rich, Student-Centered, Investigative Approach. *International Journal of Science and Mathematics Education.* Published online, November 9.

Schukajlow, S., Leiss, D., Pekrun, R., Blum, W., Müller, M., & Messner, M. (2012). Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations. *Educational Studies in Mathematics, 79*, 215–237.

Straka, G. A., & Macke, G. (2006). *Lern-lehrtheoretische Didaktik*. München: Waxmann.

The Center for Teaching and Learning (2016). *150 Teaching Methods*. Retrieved April, 2016 from http://teaching.uncc.edu/learning-resources/articles-books/best-practice/instructional-methods/150-teaching-methods

Tomlinson, C. A. (1999). *The differentiated classroom: Responding to the need of all learners*. Alexandria, VA: Association for Supervision and Curriculum Development.

Ufer, S., Heinze, A. und Lipowsky, F. (2015). *Unterrichtsmethoden und Instruktionsstrategien.* In Bruder, R., Hefendehl-Hebecker, L., Schmidt-Thieme, B., Weigand, H.-G. (Eds.), Handbuch der Mathematikdidaktik S. 411-434. Berlin: Springer Spektrum.

Vaux, A., & Briggs, C. S. (2005). Conducting mail and Internet surveys. In F. T. L. Leong, & J. T. Austin, *The psychology research handbook* (pp. 186–209). Thousand Oaks, CA: Sage.

Wankel, C., & Blessinger, P. (2012). *Increasing student engagement and retention using online learning activities: Wikis, blogs and webquests*. London: Emerald Group.

Ward, J. H. (1963). Hierarchical groupings to optimize an objective function. *Journal of the American Statistical Association 58*, 236–244.

Wiechmann, J. (2011a). Frontalunterricht. In J. Wiechmann (Ed.), *Zwölf Unterrichtsmethoden* (S. 24–38). Weinheim: Beltz.

Wiechmann, J. (2011b). Direkte Instruktion. In J. Wiechmann (Eds.), *Zwölf Unterrichtsmethoden* (pp. 39–35). Wein

Wiechmann, J. (Ed.) (2011). *Zwölf Unterrichtsmethoden.* Weinheim: Beltz.

Williams, H. (2011). To what extent might role play be a useful tool for learning mathematics? In C. Smith (Ed.), *Proceedings of the British Society for Research into Learning Mathematics, 31*(3), 161–166.

Winer, B. J., Brown, D. R., & Michels, K. M. (1991). *Statistical principles in experimental design.* Boston: McGraw-Hill.

Zech, F. (1998). *Grundkurs Mathematikdidaktik: Theoretische und praktische Anleitungen für das Lehren und Lernen von Mathematik*. Berlin: Beltz.

Zendler, A., Seitz, C., & Klaudt, D. (2016). *Booklet II Instructional Methods to Mathematics Education*. Berlin: epubli.

Zendler, A. Seitz, C., & Klaudt, D. (2018a). Instructional methods in STEM and English Subjects: a Validation Study. *International Journal of Research in Education and Science, 4*(2), 686–702.

Zendler, A., Seitz, C., & Klaudt, D. (2018b). Instructional methods in STEM education: A cross-contextual study. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(7), 2969–2986.