

Introducing the concept of infinite series: Preliminary analyses of curriculum content and pedagogical practice

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Infinite series is a complex, often counter-intuitive but significant mathematical concept with a wide range of applications in mathematics and science. The few studies on the learning and teaching of this concept (e.g. Alcock and Simpson 2004; Mamona 1990) suggest that students appear to have little conceptual understanding of infinite series; have no visual imagery associated with it; and see little or no relevance to it in mathematical and other situations. The work we present in this report is the first phase of a study currently in progress in the UK and Canada that investigates the learning and teaching of this concept.

Here we report preliminary analyses, carried out by the first two authors, of *curriculum content* and *pedagogical practice* in the UK. Our analysis of curriculum content consists of the analysis of seven texts named by UK lecturers as frequently recommended to undergraduates. Our analysis of pedagogical practice is based on interviews with lecturers who have substantial experience of teaching the topic to mathematics and other undergraduates. Our text analysis and interviewing addressed questions that had emerged from the literature and the third author's analysis of texts used in the Canadian context. The Canadian analyses suggested that, even though the concept enjoys substantial coverage in most texts, its presentation is largely a-historical and decontextualised, almost exclusively in the algebraic register and with few graphical representations and applications (Nardi, Biza and González-Martín 2008).

In our analysis of the above mentioned UK texts we found no historical references and only three applications outside the field of mathematics (of which only one was of substantial extent). We also identified a tendency for intra-mathematical connections, namely references to uses of the concept that span across several mathematical topics (Nardi et al. 2008). Regarding visual representations, only three texts included figures (eleven in total). These figures were used mainly for a visual description of the series terms or the partial sums as: points on the number line or areas of rectangles. The scarcity as well as the often decorative, inconsequential nature of these figural representations led us to the preliminary conclusion that contrary to recommendations by the relevant literature (e.g Alcock and Simpson 2004), the texts do not seem to encourage the dynamic interplay between the algebraic and the graphic registers in their introductions to the concept of infinite series.

Following our interviewing of experienced lecturers, it seems that the above issues (scarcity of visual representations, minimal or no contextualisation through applications and historical references) warrant further investigation. One interviewee, for example, attributed the visual scarcity of the texts partly to the typical apprehension of the mathematics community towards visual representations that they may divert students from the generality of a mathematical argument. Despite this risk, he claimed, figures are 'a good heuristic'; can be 'persuasive'; and, can help students overcome limited or misleading perceptions concerning infinite series. Thus he recommended acknowledging their presence in student written work both with encouraging verbal commentary and 'certainly some credit, though not full, some marks'. The same interviewee stressed that students come to university studies having limited experience of infinite series that may consist entirely of working with the geometric series and rarely with cases of non-convergence. This limited experience often entails a misleading perception that an infinite series with decreasing elements always converges. He elaborated the example of the harmonic series (decreasing elements, divergent) for altering student misunderstandings and he offered a figural-concretised version of the algebraic proof of the non-convergence of the harmonic series that he uses often in his lectures.

In the above we have begun to explore how curriculum content and pedagogical practice concerning the introduction to the concept of infinite series can address substantial student learning needs. Our results so far indicate that privileging an almost exclusively formal-algebraic register denies students the insight that can be gained from engaging with appropriate visual representations, examples and applications of the concept. We are now exploring the conjectures emerging from these indications.

References

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