

Cointegration and Error Correction Modelling in Time-Series Analysis: A Brief Introduction

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Cointegration and Error Correction Modelling in Time-Series Analysis: A Brief Introduction

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Criminological research is often based on time-series data showing some type of trend movement. Trending time-series may correlate strongly even in cases where no causal relationship exists (spurious causality). To avoid this problem researchers often apply some technique of detrending their data, such as by differencing the series. This approach, however, may bring up another problem: that of spurious non-causality. Both problems can, in principle, be avoided if the series under investigation are “difference-stationary” (if the trend movements are stochastic) and “cointegrated” (if the stochastically changing trend-movements in different variables correspond to each other). The article gives a brief introduction to key instruments and interpretative tools applied in cointegration modelling.

Criminologists often use time-series data to describe long-term developments of crime. Such data can also be used to identify and model assumed structural relationships between crime rates (treated as dependent variables) and factors like unemployment or divorce rates (treated as independent, explanatory variables). The adequacy of the specific analytical techniques and statistical models applied in such analyses has to be judged with regard to certain features – problems and possibilities – inherent in the given data. One of those features that need careful consideration is the absence or presence of trend components. Two or more time-series, each of them exhibiting a persisting upward or downward trend, will always correlate with each other (positively or negatively) even in cases where no causal relationship between them exists. On the other hand, if we eliminate the trend components the remaining series will likely be uncorrelated even in cases where their levels are structurally related to each other. Usually, however, there are more alternatives available than choosing between spurious causality and spurious non-causality. Often, level changes may proceed in a temporarily chang-

ing pattern, switching from upward to downward movements, speeding up or slowing down in this or that direction, in other words they might be “stochastic” (rather than “deterministic”). If two (or more) series that show such unsteady, stochastic trend movements still correlate with each other, then we can be quite confident that there is indeed a structural (causal) relationship between them; otherwise their unsteady trend movements would not be corresponsive across the series under inspection. This paper gives a brief introduction into certain statistical strategies and techniques that can be used (or should not be used) in testing and modelling structural relationships between time series exhibiting some type of trend development.

1. Deterministic versus Stochastic Trend Components

A trend component is usually represented in one of the following two ways: either “deterministically” as a linear or non-linear function of time or “stochastically” as a so-called *unit-root process*. A simple example of the first variant would be:

I wish to thank Christoph Birkel for his helpful comments on an earlier draft of this paper, which

also draws upon Thome (1996, 2005). Birkel’s own article (pp. ###–### in this issue) presupposes a

basic understanding of the concepts and procedures outlined in the present paper.

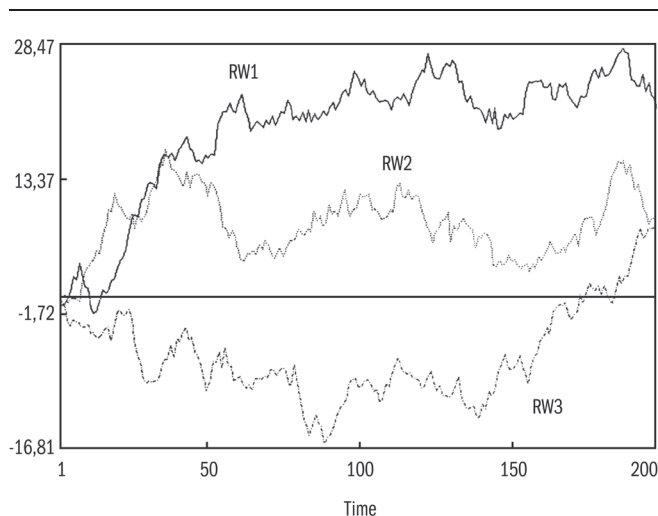
$$(1) \quad z_t = \alpha + \gamma t + \varepsilon_t, \quad t = 0, 1, 2, 3, \dots,$$

where t is a time-index, α the initial level of the time series $Z(t)$, and ε symbolizes a random (“error”) input with constant variance and an expected value $E(\varepsilon_t) = 0$. When the trend coefficient γ is known (estimated), the series can be detrended by calculating $z_t - \gamma t = \alpha + \varepsilon_t$. However, if the trend is in fact not deterministic but has been generated by a random process, this procedure of modelling and detrending the series would be inappropriate.¹ The simplest model of such a random process producing a trend is given by the following equation:

$$(2) \quad z_t = z_{t-1} + \varepsilon_t \leftrightarrow z_t = z_0 + \sum \varepsilon_t$$

If the errors $\varepsilon(t)$ are distributed normally with constant variance² around the expected value $E(\varepsilon_t) = 0$, and if they are also uncorrelated with each other and with Z_{t-1} , (if they are “white noise”), this process is called a *simple random walk* (RW). Figure 1 represents three realizations of this type of processes exhibiting temporary upward and downward movements along the time axis.

Figure 1: Three realizations of a random walk



All these RW realizations start with the value $z_0 = 0$ and then successively add the accumulated random shocks $\sum \varepsilon_t$ ($t = 1, 2, 3, \dots, 200$) according to equation (2). Note that even a *simple* random walk (without “drift,” see below) may, within a limited period of time, appear to produce an overall trend component and/or a cyclical movement.

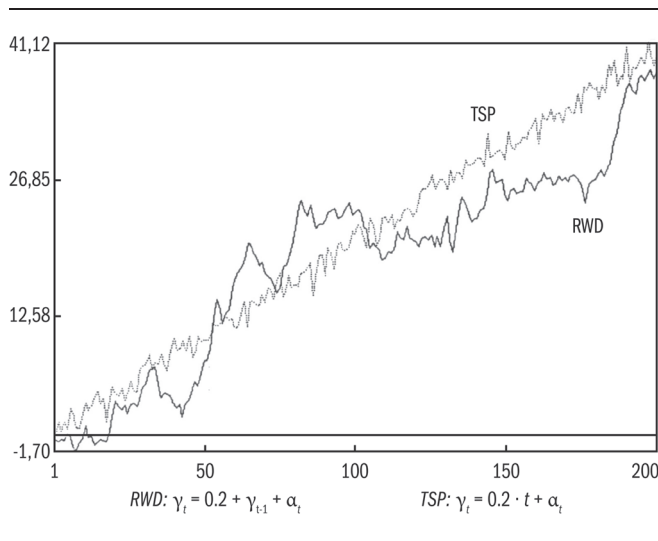
Stochastic trend components of this kind can be eliminated by calculating the first differences: $\Delta Z_t = Z_t - Z_{t-1}$. Consequently such a process is called a *difference-stationary process* (DSP) and contrasted with the *trend-stationary process* (TSP) given in equation (1). In some cases, the first-order differencing may not be sufficient to produce a stationary process, which however might still be achieved by differencing the series of first differences, and possibly the second differences as well and so forth, thereby leading to *second* or *higher order differences*, $\Delta^p Z_t$. Difference-stationary processes are also referred to as *Integrated Processes* of order p : $I(p)$ -processes. The more technical term “unit-root process” is derived from the mathematics of difference equations, which cannot be introduced here (a “unit-root” of 1 is the formal requirement of difference-stationarity).

Equation (2) can be extended by adding further components, in particular a constant term μ , a so-called *drift* parameter. This is a deterministic linear trend component (with μ as the slope coefficient), thus the time series moves more and more away from its original level, but since the trend component is embedded within a random walk process the fluctuations around this long-term trend line increase with time. Figure 2 represents such a *random walk with drift* (RWD), contrasted with a TSP according to equation (1).

¹ See Nelson and Kang (1981, 1984), Banerjee et al. (1993), and Raffalovich (1994).

² But note that the variance of the time series $Z(t)$ is $t \times \sigma^2$, so it increases with time.

Figure 2: Realization of a random walk with drift (RWD) and of a trend-stationary process (TSP)



Among the problems that arise when fitting a deterministic trend (according to equation 1) to a random walk process, we find the following (see Nelson and Kang 1981, 1984; Banerjee et al. 1983):

(1) If the series has been generated by a simple random walk (without *drift*), an OLS regression on the time index t produces a spurious coefficient of determination which does not decrease by increasing sample size. Standard tests of significance of the slope coefficient (based on Student's t statistic) tend to be largely biased in an upward direction. The correct null hypothesis (stating that the slope coefficient of the time index should be zero) will be rejected in a large majority of cases. (2) If the random walk contains a *drift* component, a coefficient of determination larger than zero (produced by an OLS regression on t) makes some sense, but it also tends to be overestimated. (3) The autocorrelations of the residuals resulting from such "spurious" regressions tend to exhibit an artificially cyclical pattern, whose period length and standard deviation depend (positively) on sample size (the length of the series observed).

As the name suggests, calculating the first- or higher-order differences is the appropriate way of detrending a dif-

ference stationary process. However, two problems have to be considered before doing this. (1) The elimination of the respective trend components forestalls any possibility to identify and test the level relationship which might connect two or more series ("co-integration", see below). (2) Since differencing eliminates or reduces the weight of low-frequency components in general, it not only eliminates the trend, but also any cycles present in the series, no matter if they are deterministic or emanate from some stationary second- or higher-order autoregressive process.

With regard to causality, one has to be aware of additional problems. Imagine that we have two simple random walks, Y_t and X_t , according to equation (2), which have been produced independently from each other (by way of simulation experiments, for example). If we regress the Y -series on the X -series

$$(3) \quad y_t = \alpha + \beta x_t + \varepsilon_t$$

the theoretically expected slope coefficient is, of course, $\beta = 0$. But we are very likely to obtain a slope coefficient which departs significantly from zero, and this likelihood will increase with the length of the series (Banerjee et al. 1993, 74 ff.). This is another instance of "spurious regression" (Granger and Newbold 1974). And again this problem cannot be solved by detrending the series with a polynomial function of time before running the regression or by including the time index t in the set of regressors.

These observations taken together suggest the following approach: If one wants to identify or test structural (causal) relationships between seemingly trending time series, one should not start by detrending the data at all. Instead, one should first test the assumption that the series to be analyzed are *difference-stationary*, that the trend in each series is stochastic (this can be done by *unit-root* testing, as explained in section 3 below). A necessary (but not sufficient) condition for a structural relationship between such series is that they are integrated processes of the same order. If this turns out to be the case, an assumed structural relationship between the series can be identified and tested with the help of *cointegration models* (see section 2). When such a hypothesis has been confirmed, the temporal

dynamics in which the level of one series is adjusted to the changing level of the other series can be identified with the help of *error correction models*. Before illustrating the application of this strategy in section 3, the key concept of “cointegration” is briefly outlined in the next section.

2. The Concept of Cointegration

Two or more time series are said to be co-integrated if two conditions prevail: first, each of the series must be integrated to the same order; second, there must exist at least one linear combination among the series which is stationary. If there is only one such linear combination, it can easily be obtained by regressing one series upon the other(s):

$$(4) \quad y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

The residuals of this (static) regression are a linear combination of the Y- and X-series. Generally, linear combinations of two or more first-order integrated series are again integrated to the first order. But under specific conditions – if the stochastic trend components in each series evolve correspondingly (in “co-integration” with each other) – they will be stationary. Consequently, if the residuals of the estimated equation (4) prove to be stationary, we have a strong indicator for a causal relationship between the variables involved. Without engaging in formal derivations the following line of reasoning can be developed:

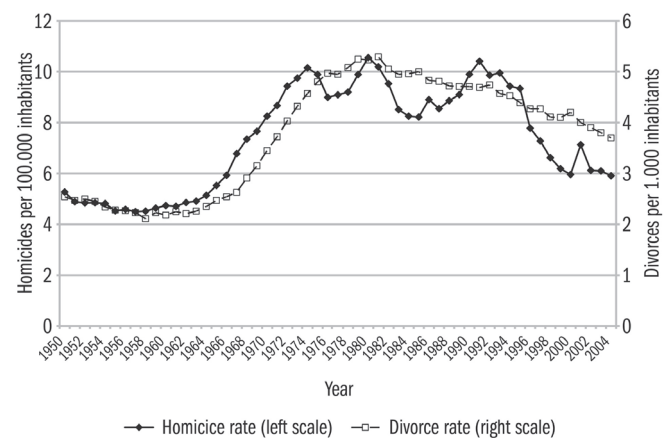
Imagine two time series, each dominated by stochastic trend components. If they are integrated to *different* orders they cannot be structurally related to each other in their long-term development.³ If they are integrated to the same order, their stochastically evolving trend components might be (causally) related to each other – or not. If we find a close correspondence between the trending up and down movements in different series, either positively or negatively, we can be quite confident that there is indeed a

structural, causal relationship between these series, precisely because of their stochastic nature. Without being causally connected, stochastic up and down movements could not be expected to move on in close correspondence across different series. And if we observe such a correspondence in stochastic movements, we can be quite confident that a causal relationship exists. This correspondence – or the lack of it – is revealed in the residuals of the (static) cointegration regression. If they are stationary, a *moving equilibrium* relationship must exist between the series under investigation. Externally induced departures from the equilibrium spawn more or less rapid readjustments. Differencing the series to level stationarity before performing the regression analysis would eliminate this long term co-movement, preventing it from being detected. One would thus fall victim not to spurious causality (or regression) but to spurious non-causality.

3. Modelling Cointegration: Two Examples

In our first example we look at the homicide and divorce rates of the United States from 1950 to 2005 (see Figure 3).⁴

Figure 3: U.S. homicide and divorce rates



³ However, a variable Y (such as confidence in the future) may remain level-stationary as long as there is a persistent upward or downward trend in another variable X, such as a steadily growing gross national product. In such a case, the structural relationship might be tested by regressing Y on the first dif-

ferences of X or the growth rates derived from X. Though a “co-integration” model can be formally specified only for two or more series integrated to the same order, a specific series might be differenced in advance, based on theoretical argument, before being included in the cointegration equation.

⁴ I am grateful to Steve Messner, who made these data available to me

In criminological research, divorce rates have repeatedly been treated as indicators of certain aspects of social disorganisation or institutional anomie considered to be conducive to criminal violence (Beaulieu and Messner 2010; Land et al. 1990; Pratt and Cullen 2005). Neither the homicide nor the divorce rates seem to follow a deterministic trend. So, we first check if they can be modelled as integrated processes of the same order. The unit-root tests we apply here is the *augmented* Dickey-Fuller Test (ADF Test), which takes into account the auto-correlation of residuals (Dickey and Fuller 1979, 1981).⁵ In its simplest form the equation to be estimated for testing-purposes is

$$(5) \quad y_t = \rho_a y_{t-1} + \varepsilon_t$$

The null hypothesis (that the Y-series is a first order integrated [difference-stationary] process) implies a coefficient $\rho = 1$, the alternative hypothesis (level stationarity) implies $\rho < 1$. The alternative of a stationary *autoregressive* process would usually need to be modelled with the inclusion of a constant term. So, in order to be “fair” against the alternative, in praxis equation (5) is usually expanded into equation (6):

$$(6) \quad y_t = \alpha_b + \rho_b y_{t-1} + \varepsilon_t$$

To be on the safe side, it is often recommended to also include the time index among the regressor variables, particularly if the series shows (or seems to show) a prevailing upward or downward movement within the observation period:

$$(7) \quad y_t = \alpha_c + \gamma t + \rho_c y_{t-1} + \varepsilon_t$$

In this way one can check if there is sufficient evidence for the presence of a stochastic trend component even though a deterministic trend component is given a chance to be

identified as well. If one wants to rule out the possibility that there is also a deterministic trend component (besides the stochastic trend component), one would have to test the combined hypothesis $\rho=1$ and $\gamma=0$.

Under the null hypotheses, the OLS estimates of the ρ parameters are not normally distributed. Dickey and Fuller (1981) have applied Monte Carlo simulations in order to establish the distribution of these estimates under various conditions: does the equation tested include a constant or a time index, does the true process include or not include a drift parameter, a deterministic trend component etc.? The ratio of the difference between observed and expected ρ -coefficient divided by the standard deviation is usually symbolized by the letter τ (to differentiate it from *Student's t*). The critical values of this test statistic and the significance level α associated with them, each specified for the varied conditions just mentioned, are available in the literature (see, for example, Hamilton 1994) and computer programs like *STATA*.

When we apply this testing strategy to our series of homicide and divorce rates, the assumption that both series are difference-stationary (integrated) processes of order 1 is confirmed.⁶ When estimating the ρ -coefficient on the basis of equation (7) we get $\hat{\rho}_c = 0.95$ for homicide rates and $\hat{\rho}_c = 0.98$ for divorce rates. These observed coefficients depart only $\tau = 1.21$ and $\tau = 1.19$ standard deviations from their expected value of $\rho = 1.0$; consequently, the error risk for rejecting the null hypothesis would be $\alpha \geq 10$ percent.⁷ In addition, we tested the combined hypothesis $\rho=1$ and $\gamma=0$; it was confirmed for both series.

Since the two series are apparently integrated processes of the same order, they are also candidates for co-integration. We thus regress the homicide series on the divorce series according to equation (3). The estimated slope coefficient is highly significant, the coefficient of determination is

5 There is quite some discussion in the literature about the appropriateness of unit-root testing in general and of specific drawbacks or advantages of alternative testing strategies (for example DeJong et al. 1992; Hamilton 1994). Generally, one can say that these tests get more problematic the shorter the

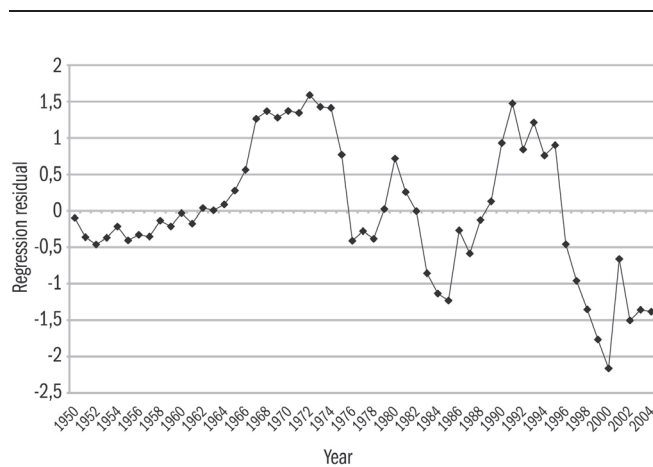
length of the inspected time series is (some authors even argue that they are useless with less than 100 observations).

6 I am grateful to Christoph Birkel, who carried out these tests with the help of the *STATA* computer program.

7 The critical value chosen to accept or reject the null hypothesis depends on whether one would like to reject the null hypothesis (which is made more convincing the lower the α -value) or rather not reject it (which is made more convincing the higher the α -value; commonly recommended in this case are values of $\alpha \geq 10$ percent).

$R^2 = 0.80$. However, the estimated residuals do not represent a stationary process. As shown in Figure 4, there is a clear upward trend between 1950 and the mid-1970s, and a slight and fuzzy decline afterwards. The results of the ADF Tests applied to the residual series confirm this impression. The conclusion thus is that the changing level of homicide rates is not causally related to the changing level of divorce rates. Apparently, what we have here is an example of *spurious regression*.

Figure 4: Residuals of cointegration model with homicide and divorce rates



A word of caution however is in order at this point. If there are strong theoretical arguments in support of a structural relationship between the two variables (against the detected spuriousness), one might consider alternative testing procedures before giving up the hypothesis. For example, one might apply models that combine stochastic and deterministic trend conceptions by assuming that deterministic trends change their functional form stochastically over time (see Perron 1989; Perron and Vogelsang 1992). Alternative testing procedures have also been used by Christoph Birkel in his article published in our present issue. A very helpful overview on various testing and modelling strategies concerning cointegration is presented by Enders (2010).

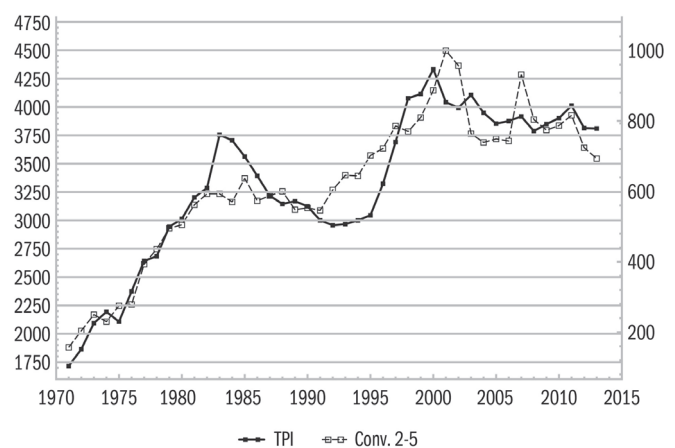
8 I am grateful to Rainer Metz (GESIS-Leibniz Institute, Cologne) who made these data available to me and also carried out the statistical analyses (selec-

tively) reported below. For an extended analysis and theoretical discussion of law enforcement and incarceration practices see Metz (2013)

Also, the bivariate cointegration model might be underspecified in our example: there might be additional input factors (apart from divorce rates) that should be included in the model (see Messner et al. 2011). In such cases, not all of the input factors included in a cointegration model need to be integrated processes of the same order. Greenberg (2001) even finds a cointegrating relationship between US homicide and divorce rates using yearly data for the period between 1946 and 1997. He notes, however, that “the parallel movement may have weakened some in recent years” (ibid., 302). This weakening apparently continued in the following years until the end of our observation period (2005), thus tilting the results toward “spurious regression” (for the bivariate relationship).

We now take a look at two other series: (a) the total number of prison inmates (TPI) who, in their great majority, serve short-term sentences, and (b) the (smaller) number of perpetrators sentenced to relatively long periods of imprisonment between two and five years (CONV2-5) in the German state of Hesse between 1971 and 2013 (see Figure 5).⁸

Figure 5: Total number of prisoners (left scale) and number serving two to five years (right scale)



Law-making as well as law enforcement and sentencing practices are influenced by public opinion and political opportunities following short- and long-term fluctuations. Public discussions often focus on more severe and cruel criminal acts, but such discussions may also increase the readiness and determination to prosecute and incarcerate people for minor crimes as well. On the other hand, they may lead to a redirection of policing and prosecution resources to concentrate on more serious crimes. In Germany we have no detailed and comprehensive statistics on length of imprisonment actually served compared to original sentence. So one might ask (among other things) if the (relatively small) number of convictions covering some limited range of severity (like, as in our example, two to five years of incarceration) will in the long run closely correspond (or not correspond) to the development of the total number of prison inmates. How indicative or representative are these convictions for the long-term development of the total number of imprisoned persons? As it turns out in our particular example, eliminating the seemingly linear trend component in both series before doing any kind of regression analysis does not lead to zero-correlations between the two residual series. Nonetheless, it is advisable not to eliminate the trend or drift component right at the beginning of the analysis, but to check if the two series are in fact cointegrated, whether they have a long term equilibrium relationship with regard to their (stochastic trend) levels.

Thus, we first apply ADF Tests to both series according to equation (7). Their results support the assumption that they incorporate first-order integrated (difference-stationary) processes.

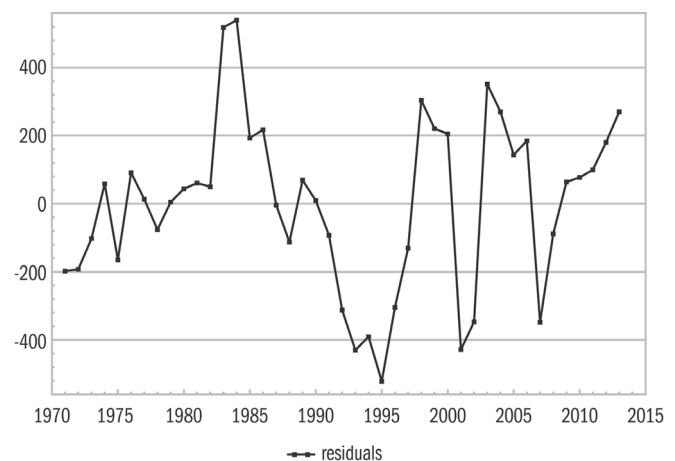
In the next step we run the co-integration regression (see equation 3 above) with the following result:

$$(8) \text{TPI}_t = 1437.7 + 3.033(\text{CONV2to5})_t + e_t$$

The slope coefficient of 3.033 implies that an increase in the number of people sentenced to (relatively) long-term

imprisonment will uplift the total number of prisoners by a factor of 3.03 (with a coefficient of determination $R^2 = 0.86$). For example, an addition of ten more persons convicted in this category will be followed by an increase of thirty in the total number of prisoners (whatever the mechanism in this process might be). But before we accept this hypothesis, we must check whether or not the residuals of this regression are stationary (see Figure 6)

Figure 6: Residuals of cointegration model with total numbers of prisoners and those serving two to five years



The results show that they are; the unit-root test confirms the impression received from visual inspection of the residuals: the null hypothesis of a unit-root can be rejected with a risk of $\alpha < 0.001$.

So far, we have produced evidence that these two series are cointegrated, but the static regression says nothing about the dynamics of the re-equilibration processes. Engle and Granger (1987) have suggested that this process can be specified in a so-called error correction model, which we present here in the form adapted to our example⁹ and with parameter estimates obtained by OLS regression:¹⁰

⁹ In other cases, lagged terms of the dependent and/or the independent variable might need to be included as well.

¹⁰ We skip here any discussion about diagnostic statistical tests concerning the adequacy of the model specification and the estimation procedure (for example Greene 1993, 216 ff.). The long-term

effect parameters of equation (8) and the short-term effect parameters of equation (9) can also be estimated simultaneously in one equation (see Wolters 1995, p. 153; Wagner and Hlouskova 2007).

$$(9) \Delta TPI_t = 0.876 \times \Delta(\text{CONV2to5})_t - 0.314 \times Z_{t-1} + e_t$$

As already mentioned, the symbol Δ indicates differencing, $\Delta TPI_t = TPI_t - TPI_{t-1}$. Consequently, the dependent variable and the regressor variables in this model are stationary. The variable Z_{t-1} is given by the estimated Residuals $TPI_t - 1437.7 - 3.0333(\text{CONV2to5})_t$ derived from equation (8). These residuals represent departures from the equilibrium relationship. The negative coefficient of -0.314 thus gives the rate of readjustment, year by year, towards the new equilibrium level (*re-equilibration*), no matter whether the disequilibrium has been induced by a change in the independent variable or by a change in the dependent variable caused by some (non-specified) intervention impacting directly the dependent variable. The system is in a state of equilibrium if $Z_t = 0$, i.e., $TPI_t - 3.03(\text{CONV2to5})_t = 1437.7$. Let us suppose that at some year t the number of people sentenced to between two and five years will increase from 200 to 220. Then, according to equation (8), the equilibrium level in TPI would change from $3.03 \times 200 + 1437.7 = 2044$ to $3.03 \times 220 + 1437.7 = 2104$, an increase of 60. The level change in the number of convicted persons (in this category) is immediately (in the same year) answered (according to equation 9) by an expected increase of $0.876 \times 20 = 17.5$ in the total number of prisoners (TPI),¹¹ thus reducing the dis-equilibrium from 60 to $(60 - 17.5) = 42.5$. This remaining disequilibrium will (according to equation 9) in the next year be reduced to $(42.5 - 0.314 \times 42.5) = 29.15$, in the subsequent year to $(29.15 - 0.314 \times 29.15) = 20$ and so forth.

4. Concluding Remarks

Within criminological research, time series data are often used to depict the long-term development of crime and to test hypotheses seeking to explain such developments. In order to identify and test structural relationships that may exist among two or more theoretically relevant time-series, one has to take into account the specific components and features inherent in such data. The point of departure in this paper has been the distinction between deterministic and stochastic trend-components and the danger (related to these components) of falling victim to either spurious regression or spurious non-causality. Subsequently, some of the basic features and practical steps in cointegration modelling have been outlined as a strategy which helps to identify and test structural relationships between trending time series without getting entrapped into spurious causality or non-causality. The practical application of this strategy has been illustrated by analysing American homicide and divorce rates given for the years 1950 to 2005, and German data on the number of sentenced and imprisoned people in the years 1971 to 2013. The purpose of this paper has been to outline the basic ideas behind the concepts of unit-root testing and cointegration modelling, which are useful instruments in analysing time-series data relevant to criminological research. The analyses presented here could have been extended both with regard to substantive as well as methodological and technical issues. This however would have gone beyond the (didactically defined) scope of the article.

¹¹ Note that the interpretation of the regression coefficient (here: 0.876) does not change when the two variables have been transformed by the same filter, here by taking first differences.

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