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Modelling of Bonded Post-Tensioned Concrete Cantilever Beams under Flexural Loading

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Abstract

Prestressing is widely used technic all over the world for constructions of buildings, bridges, towers, offshore structures etc. due to its efficiency and economy for achieving requirements of long span with small depth. It is used for flexural strengthening of reinforced concrete structures for improving cracking loads and decreasing deflections due to service loads. There are two methods for prestressing (pre-tensioning and post-tensioning). In this paper, a three-dimensional nonlinear Finite Element (FE) method is used to determine the behaviour of Post-Tensioned (PT) concrete cantilever beams with different tendon profiles. Numerical analyses ANSYS package program is used for analysis of beams. The results from FE analysis is verified by experimental reference test result and good agreement is achieved. This paper is focused on the effect of different tendon profiles on the flexural behaviour of Bonded Post Tensioned (BPT) reinforced concrete cantilever beams. Six models with different tendon profiles are investigated. These models are without tendons, two tendons at the bottom, middle, top, parabolic tendons with one draped point and two draped points. Failure loads, deflections, and load versus deflection relationships for all models are examined and it is seen that the beam with one draped tendon profile shows a highest performance.

Keywords: Nonlinear Finite Element Analysis; Bonded Post Tensioned; Cantilever Concrete Beams.

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1. Introduction

Concrete is a widely used material, which is used in structural engineering constructions by different ways, it is very strong in compression but too weak in tension. Cracks appear in tension zones after application of loads due to weakness in tension zones. Some technics are presented to prevent or reduce such cracks from developing. Carrying out of huge compressive force longitudinally along or parallel to the structural element axis can prevent cracks growing by significantly removing or reducing the tensile stress. This longitudinal force increases shear, bending and torsional capacities of the beam element. After load applying, concrete has full capacity in compression which can be powerfully used in the entire depth of the concrete member sections, this moral technic is called prestressing technic [1]. Two different methods for prestressing concrete were developed: pre-tensioned and PT [2].

In the pre-tensioned method, stress are applied to tendons before concrete placing. Tendons are tensioned in a stressing bed. The formwork gathering placed with stressing beds and anchorages at the ends, then placing the concrete. After concrete reaches adequate strength, strands must be released at ends. This operation produces big internal forces, the forces produced by tendons are transmitted to the concrete member by bonds between tendon and concrete. This method is usually used in precast construction when many identical elements are required [2].

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The PT method is a widely used technic all over the world for structural members, to prevent cracks and to minimize deflections produced by externally applied loads. In this method, stress are applied after concrete placing and reach adequate hardening and strength. The tendons are covered with ducts before concrete casting, after concrete reach hardening, each tendons must be stressed by using stressing jacks to the desired load. To maintain the PT force, all tendons must be anchorage at the ends of the members. This tension stress counteracts with the external live and dead loads to reduce deflections and cracks. By PT method, greater loads, spans with the same member depth, crack controls, and smaller member sizes can be attained. Also PT method can be used in steel structures to improve seismic capacity. Furthermore, PT method can be used both internally and externally, as well as it is used for repair and strengthening of existing structures. Several structures are constructed by the PT method such as slabs, segmental bridges, continuous beams with long spans, parking, etc. The PT method can be constructed in precast and cast in place. Also by this method, a combination of both bonded and unbonded tendons can be used in the same structure, which increases clear spans without increasing the member depth. Such examples were constructed in the South Korea and in the US [3].

In BPT structures, bonds between surrounding concrete and tendons hold the stressing force in PT system. The adhesion between steel and concrete significantly affects the resistance of the PT members. The bond between the concrete and strands is mobilized by the transformed force from strands to the concrete [4]. In BPT system, there is a space between the tendons and ducts. Cement mortar is inserted in under pressure to fill this spaces, which significantly increases the bond and strain compatibility between the tendons and concrete, totally can protect the tendons against corrosion [2]. The tensile stress moves from nearby concrete to the reinforcements, due to the adequate bonds. The change in tendon strains at any section is equal to the change in strain in adjacent concrete section. Tendon strains depends on the strain changes in the concrete section.

Nusrath et al. (2015) studied the effect of different tendon profiles and the cable curvature on the construction of a structure to achieve greater strength and more economic structures. Hussien et al. (2012) presented an experimental program to study the behavior of bonded and unbonded prestressed beams with high strength and normal strength. Yapar et al. (2015) simulated the FE method to find out the behavior of PT prestressed concrete beams. Their results have a satisfactory agreement between the FE predictions and the test results under collapse load. [8, 9 and 10] studied the flexural behavior models of unbonded prestressed reinforced concrete members under the service load behavior before and after cracking, yield state, and ultimate state.

Most of the previous studies are focused on simply supported and continues PT reinforced concrete beams. In addition, modelling of bonded cantilever beams are needed more study especially the modelling of grouting the tendon after applied prestress load. This research presents a FE model using ANSYS software to model the bonded PT concrete cantilever beams. The proposed model was validated with previous available experimental study tested by Hussien et al. (2012) and was used to study the effects of the transverse and gravity loading on the flexural behavior of bonded PT concrete cantilever beams. A parametric study was conducted to investigate the effect of several selected parameters on the overall behaviour of PT concrete cantilever beam. These parameters include the effect of tendon profile and effect of loading type.

2. Finite Element Modeling

For modelling the concrete structural members several analytical and numerical methods are developed. FE analysis is a numerical modelling widely applied to the concrete structural members based on the use of the nonlinear behaviour of materials, FE analysis provides a tool that can simulate and predict the responses of reinforced and prestressed concrete structural members [11]. Complex geometries like PT reinforced concrete beams with tendons can be freely modelled by using the FE method. ANSYS package program is used for solid modelling and analysis, it is a highly acceptable and reliable commercial FE analysis program.

SOLID65 in ANSYS used for modelling concrete, this element has eight nodes and three degrees of freedom at each node, nodes in the x, y and z-directions are translated, it means concrete can crack in three orthogonal directions, plastic deformation and creep. The behaviour of this element works as a nonlinear isotropic material properties [12]. Figure 1 shows the geometry, node locations, and the coordinate system for this element.

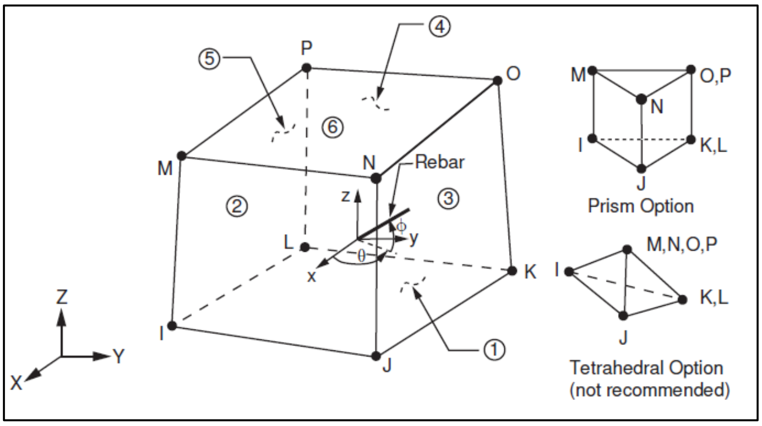

Figure 1. SOLID65 geometry [12]

LINK8 is a discrete and spare element, used for many engineering applications, this element used for model the trusses, links, sagging cables, and springs. This element is a 3-D spare and uniaxial tension-compression element. Link8 has three degrees of freedoms at each node, nodes in the x, y and z-directions are translated, and no bending of the element is considered [12]. Figure 2. shows the geometry, node locations, and the coordinate system for this element.

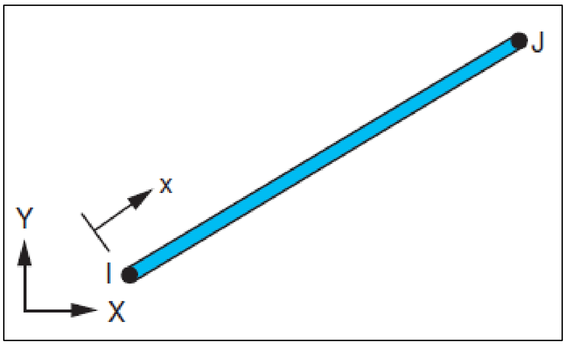

Figure 2. LINK8 geometry [12]

SHELL181 is suitable for analysing thin to moderately thick shell structures. Used to anchorage at both ends of the beam members to maintain the internal forces. It is four node elements with six degrees of freedom at each node, translations in the x, y, and z directions, and rotations about the x, y, and z-axes [12].

To avoid crushing at the point loads the bearing plates are used. SOLID45 is used for the 3-D modeling of solid structures. This element is defined by the eight nodes having three degrees of freedom at each node, translations in the nodal x, y, and z directions. This element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities [12].

The contact between concrete and tendon is modelled by the contact elements in ANSYS (CONTACT PAIR MANAGER). The method requires the definition of two surfaces that are target and contact surfaces. TARGE170 is used to represent various 3-D "target" surfaces for the associated contact elements CONTA175. This target surface is discretized by a set of target segment elements TARGE170 and is paired with its associated contact surface via a shared real constant set. It can impose any translational or rotational displacement, temperature, and magnetic potential on the target segment element. CONTA175 is used to represent contact and sliding between two surfaces (between a node and a surface, or between a line and a surface). This element is located on the surfaces of solid, beam, and shell elements. Figure 3. shows the node locations and the coordinate system for these elements [12].

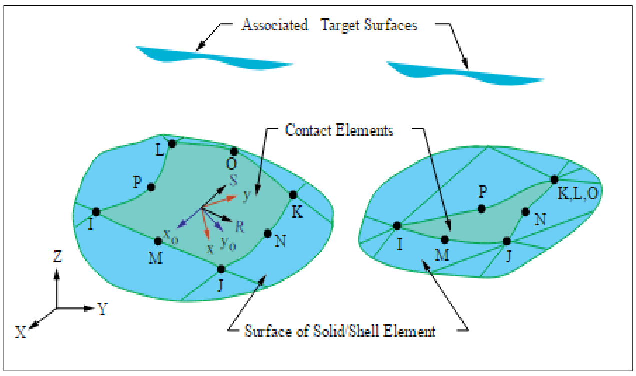

Figure 3. CONTA175 and TARGE170 geometry [12]

3. Material Properties

Material plays a significant role in ANSYS modelling. Real values of material properties should be given as an input in ANSYS. The stress-strain relationship for concrete in tension is almost linearly elastic up to the maximum tensile strength. Then, the concretes start cracking and the strength decreases continuously to zero. The multi-linear stress-strain relationship is considered for concrete in compression in this study. The adopted stress-strain relation is based on work done by Desayi and Krishnan (1964); as shown in Figure 4. The bilinear stress-strain relationship indicated in Figure 5. is considered for reinforcing steel bars in this study. Since the steel bars are slender, it could be assumed that bars transmit only axial force. On the other hand, the strands are considered as multilinear isotropic material in this study.

In the present study, the model to be used is capable of predicting failure of concrete materials. Both cracking and crushing failure modes are accounted. For the two input strength parameters (i.e. ultimate uniaxial tensile strength f_t and compressive strength $f_c^{'}$) are needed to define a failure surface for the concrete. Consequently, criterion for failure of the concrete due to a multiaxial stress state can be calculated by Willam and Warnke (1975).

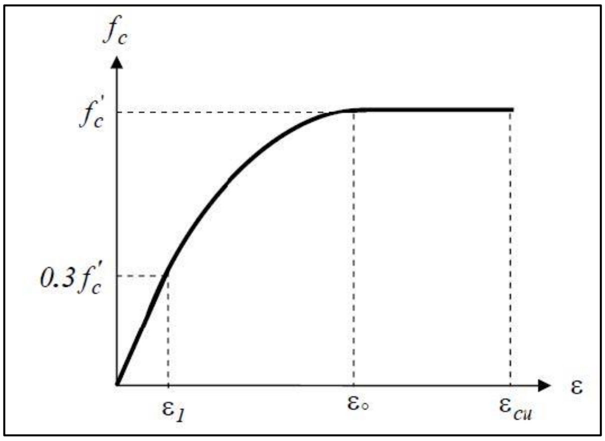

Figure 4. Stress- strain curve for concrete [13]

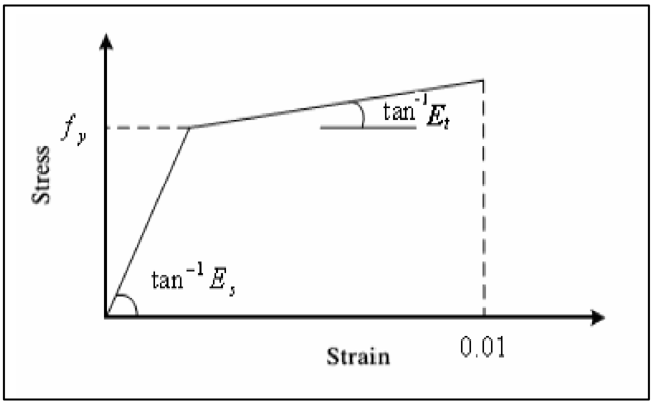

Figure 5. Stress- strain curve steel bars [15]

4. Analyses of Simply Supported Post-Tensioned Concrete Beam

The behaviour of bonded PT concrete beams is more complicated by reason the presence of ducts, the bond between concretes and ducts, the bond between ducts and grout, and the bond between grout and tendons. Two stages of analysis are needed for bonded beams. At the first stage the beam is analysed as unbonded PT beam under prestress loading and gravity loading only. Next the beam is analysed as bonded post tensioned beam. The two stages are needed because the grout is added after the prestressing of the strands.

The numerical results are verified by experimental results. To validate the proposed nonlinear models that explained in the Section 3, one of the experimentally simply supported prestressed concrete beam tested by Hussien et al (2012) is chosen. Beam B2-70-P-B has one strand with 12 mm diameter and two 10 mm diameter non-prestressed bars, overall dimensions are shown in Figure 6. The steel stirrups and non-prestressed steel bars are made of deformed high tensile steel with a yield stress of 470 MPa and ultimate strength of 610 MPa. The yield and ultimate stress of the prestressing steel strands are 1674 and 1860 MPa, respectively. The tendon is bonded and grouted with 36 MPa compressive strength. Figure 7. shows the meshing of reinforcements and tendons.

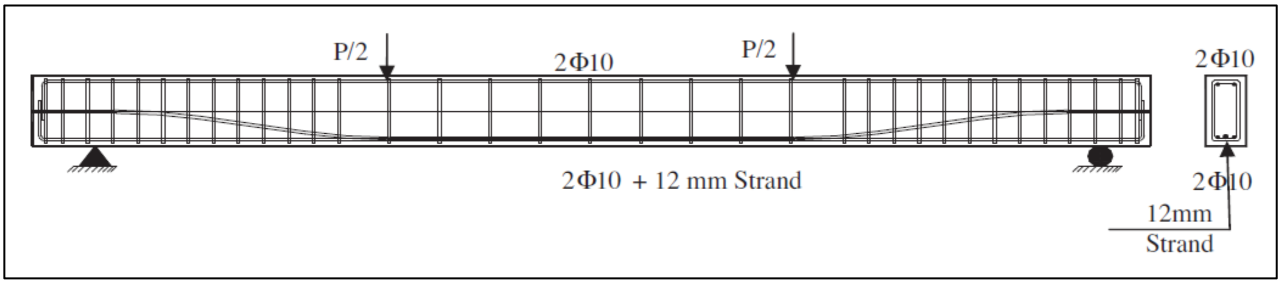

Figure 6. Reinforcement and dimension details of prestressed beam B2-70-P-B tested by [6]

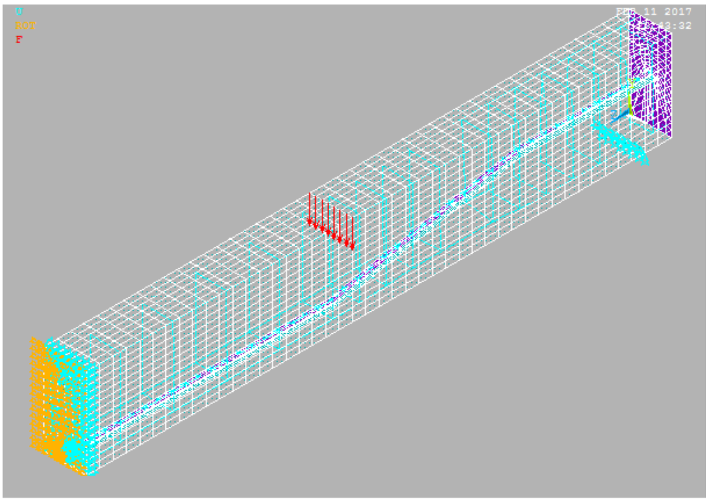

Figure 7. Meshing of reinforcements and tendons for half of beam

The experimental results by Hussien et al (2012) have good agreement with ANSYS computational FE method results as shown in Figure 8.

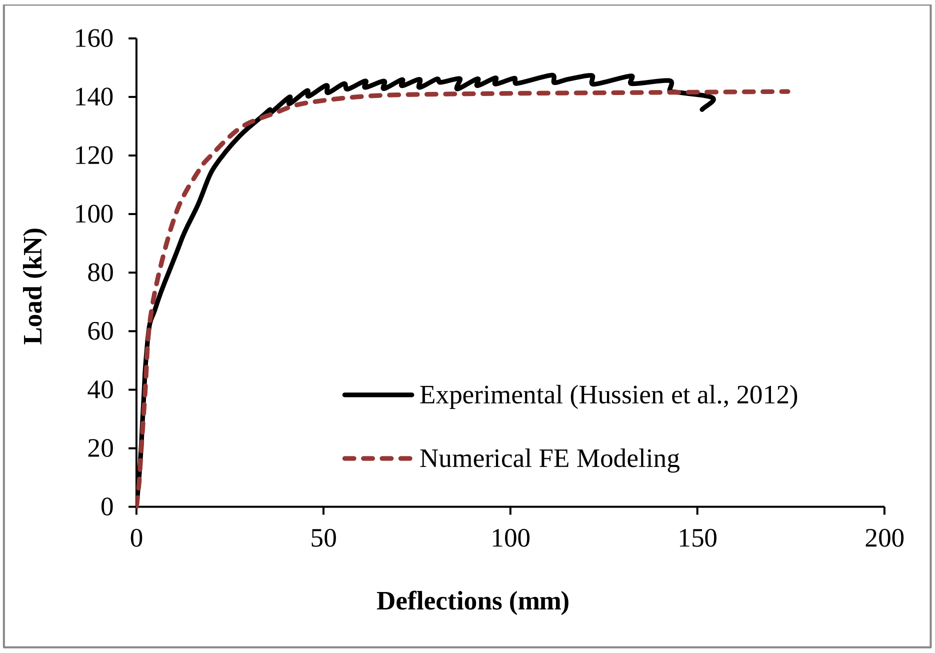

Figure 8. Load- deflection curves of experimental and numerical modelling

5. Parametric Study Related with Post Tensioned Cantilever Concrete Beam

The verified finite element model was used to investigate the effect of several selected parameters on the overall behaviour of PT bonded concrete cantilever beam. These parameters include the effect of tendon profile and the effect the loading type. The loading, geometry and boundary conditions considered for the proposed study for cantilever beam are shown in Figure 9. The prestressed tendons, beam dimensions, anchorages, load plates and supports are modelled in ANSYS by keypoints, lines, areas, and volumes. The modelled beam has 6300 mm long, with a 250 mm width and 450 mm height as illustrated in Figure 9. Two longitudinal reinforcements with diameter of 16 mm are used at the bottom and top of the beam, stirrups are arranged as 10 mm diameter of with 300 mm spacing.

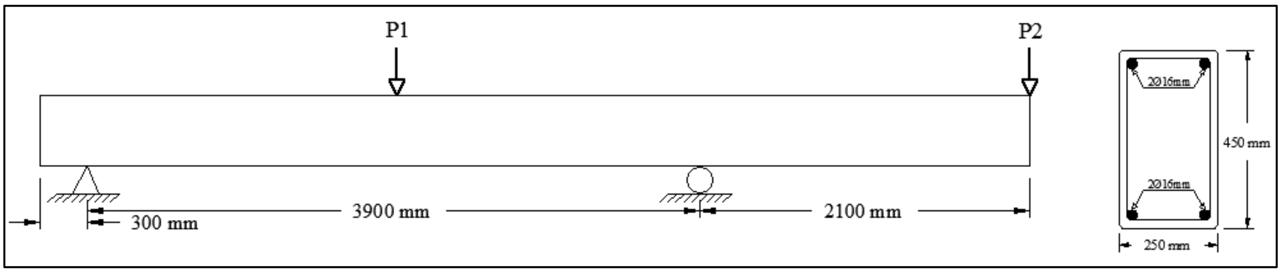

Figure 9. Proposed model with dimensions and cross-section details.

All material properties illustrated in Table 1.

Table 1. Material properties of concrete, reinforcement and strand.

Descriptions	Concrete	Steel	Strand
Ultimate compressive strength (MPa) (f'_c)	30	——	——
Ultimate tensile strength (MPa) (f_t)	3.8	——	——
Modulus of elasticity (MPa)	30000	200000	206290
Poisson's ratio (υ)	0.2	0.3	0.3
Yield strength (MPa)	——	400	1741

As an initial step of the FE analysis, the PT reinforced concrete beams are divided into a number of small elements [16]. This can be done by the meshing of concrete beams with appropriate mesh density in Z, Y, and X directions with mesh dimensions of 150, 25, 25 mm respectively, tendons have mesh with constant dimension of 150 mm in the Z-direction. The mesh density and boundary conditions are illustrated in Figure 10.

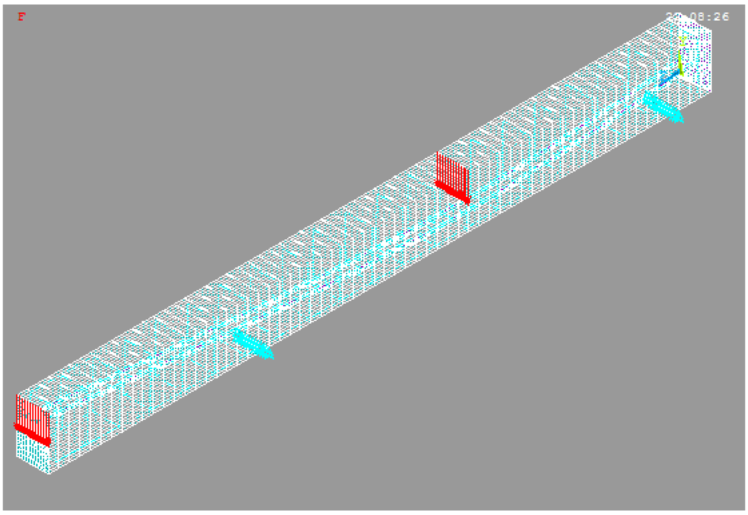

Figure 10. Meshing and boundary conditions

The boundary conditions include loads and supports are applied to constrain the model to get a unique solution. In hinge supports, all the nodes that lie on the middle line of the support are given a constraint value of zero in the Y, and X directions, in roller all the nodes that lie on the middle line of the support are given a constraint value of zero in the Y directions. The external load are applied incrementally up to failure by nodal forces on the steel plates on the top face of the beam. Six different tendon profile and three load cases are chosen as parameters of PT beam. These profiles are without tendons, straight tendons at bottom, middle, top, two draped points, and one draped point. Figure 11. shows the tendon profiles. These position and profile of tendons have been adopted in the present study because they are critical.

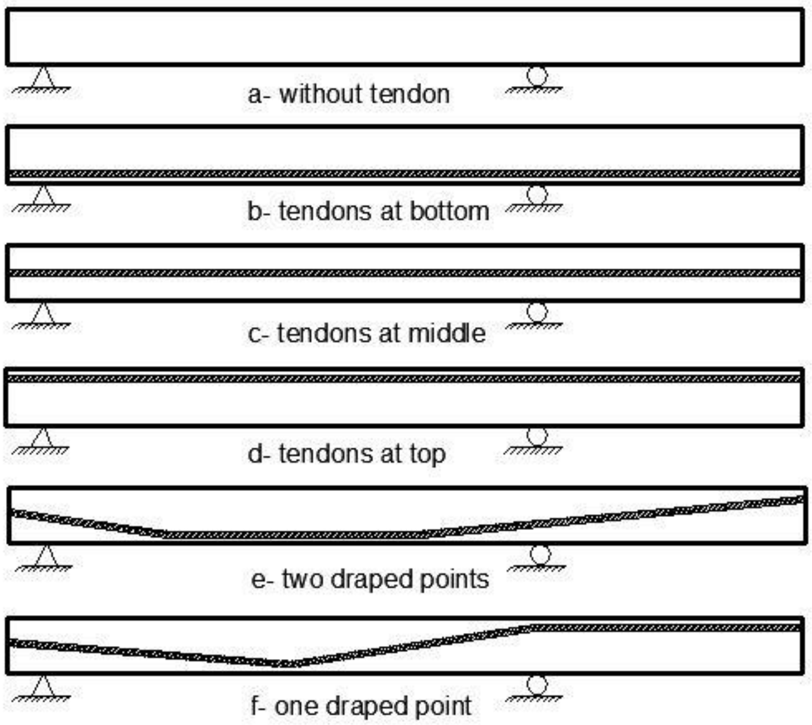

Figure 11. The proposed tendon profiles

Three different load cases are applied on the beams up to failure, Figure 12. illustrates the load cases.

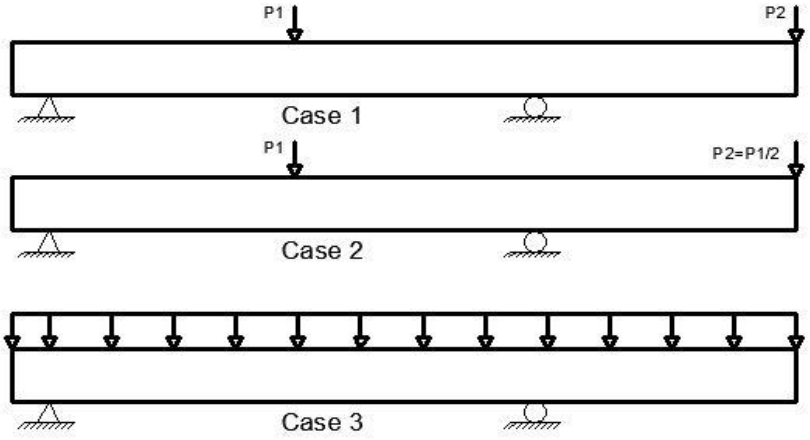

Figure 12. Load cases of parametric study

Applied loads are increased incrementally up to failure. Figure 13. shows the load-deflection curves at end point of cantilever beam for all load cases that was obtained from FE analysis, the curves showed that, the parabolic tendon profiles give maximum load capacity to failure and it has stiffer response compared to the beams with a straight tendon profile. Eccentricity is a distance between tendons and neutral axis of the member, which produces internal moments that act in opposition to moments induced by external loading. Increasing eccentricity will increase the tendon stress and decrease tensile stresses of the concrete. Parabolic profiles have the specified values of eccentricities, and this profile is similar to the beams bending moment. The use of parabolic tendons introduces transverse effects to carry more counteract the external loads, with both axial, bending and shear effects.

Figures 13, 14. and 15. shows the load-displacement curves at end of cantilever beams for all load cases that was obtained from FEA, showed that the parabolic tendon profiles give maximum load capacity to failure and it has stiffer response compared to the beams with a straight tendon profile.

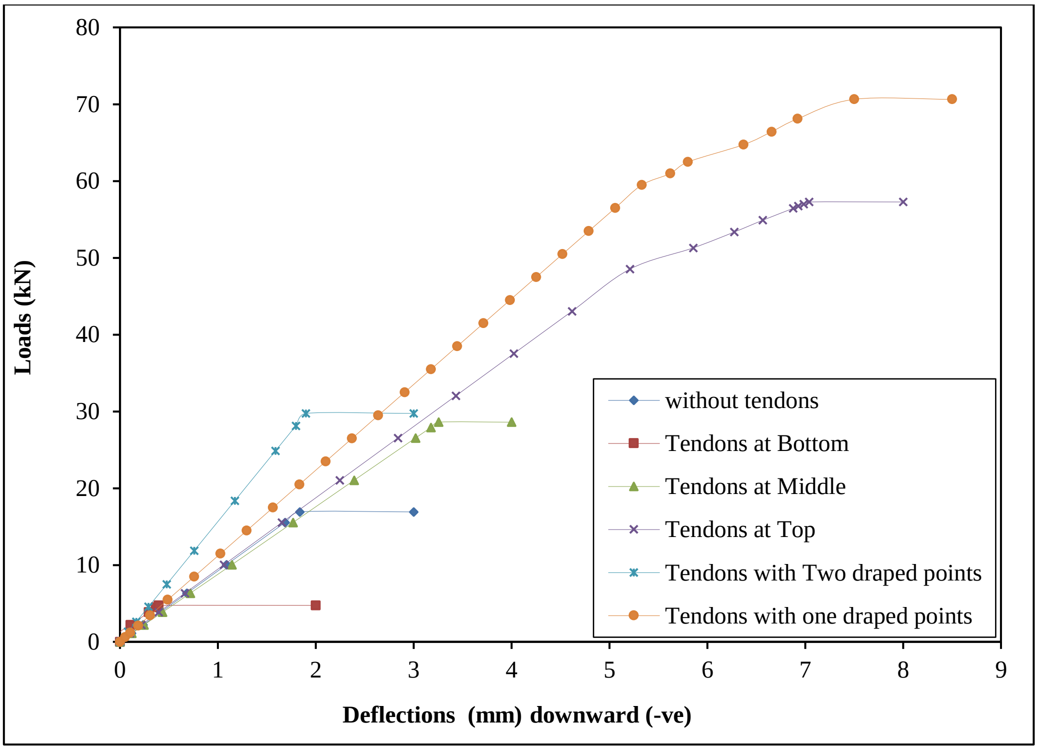

Figure 13 Load- displacement curves at the end for load case 1

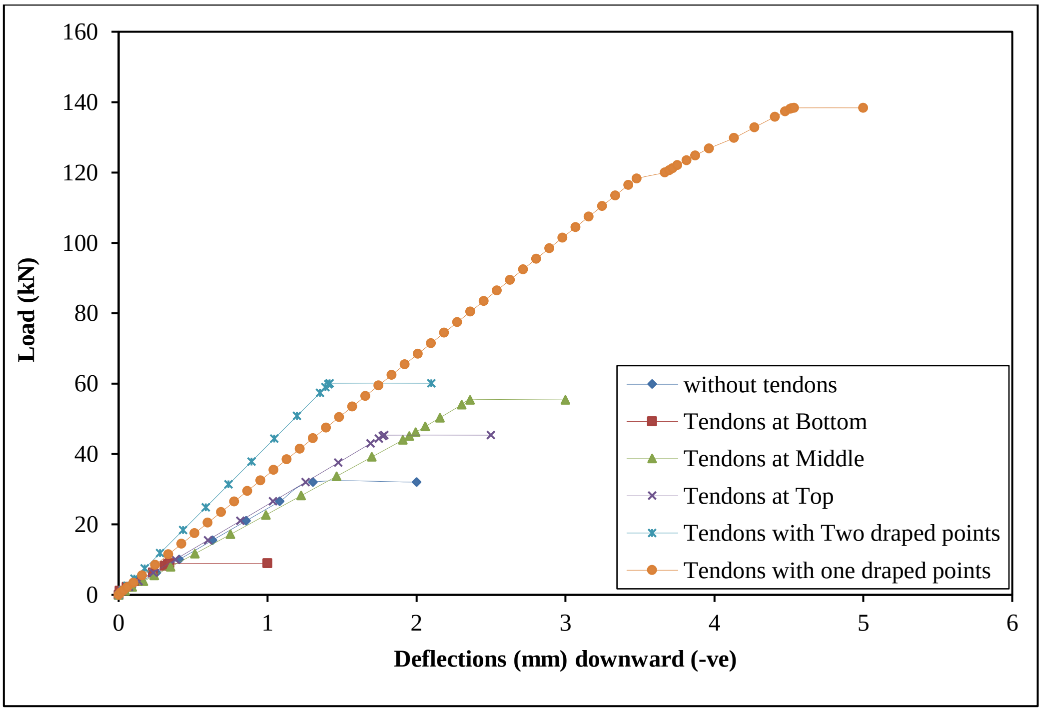

Figure 14. Load- displacement curves at the end for load case 2

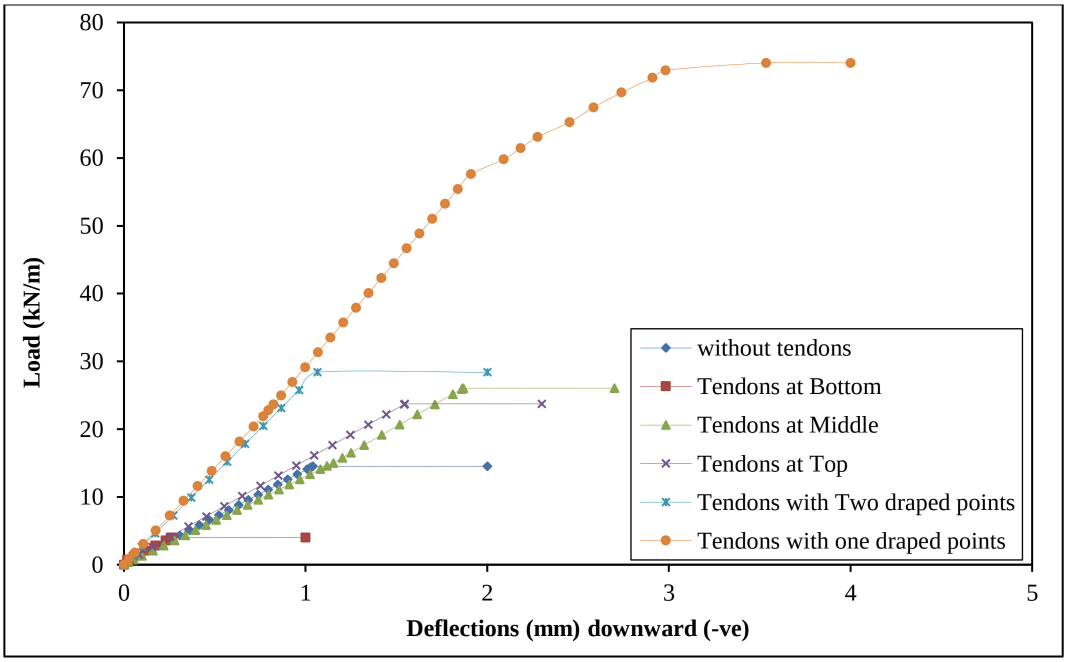

Figure 15. Load- displacement curves at the end for load case 3

Figures 16, 17 and 18 illustrates the load- displacement curves at the middle point of the simply supported parts of beams for all load cases. It shows that the ultimate load capacity increased with draped tendon profiles as compared with other undraped profiles. Straight profiles produced the lowest nominal resistance, before yielding no change occur in stiffness. Beam failures of all models were sudden failure above supports due to cantilever effect. After yielding point the draped tendon profiles was stiffer and more ductile than straight tendon profiles.

Figures 19, 20, and 21 illustrates the deformed shapes for all load cases with constant applied loads. When loads are applied and increases incrementally on the beam models, the member responses occur in the form of deformation. The initial beam conditions are attainable when the models undergo elastic deformation. Each material follows the constitutive properties considered by their stress-strain relationships. Each material provides the strain energy form to corresponding the deformation response of the members.

The parabolic tendon profile can undergo more deform without failure. The tendons with one draped point profile gives maximum deform and displacements with higher load caring capacity for all load cases because of the profile shapes which it is similar with its bending moments as compared with another profiles.

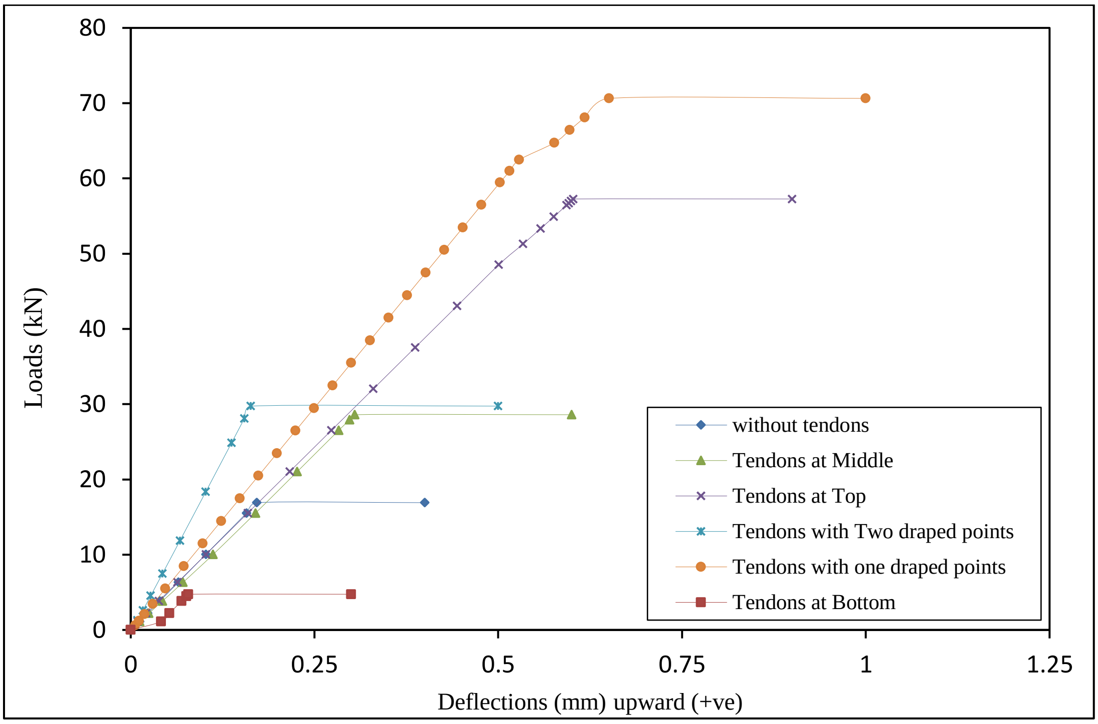

Figure 16. Load-displacement curves at middle of simply supported part for load case 1

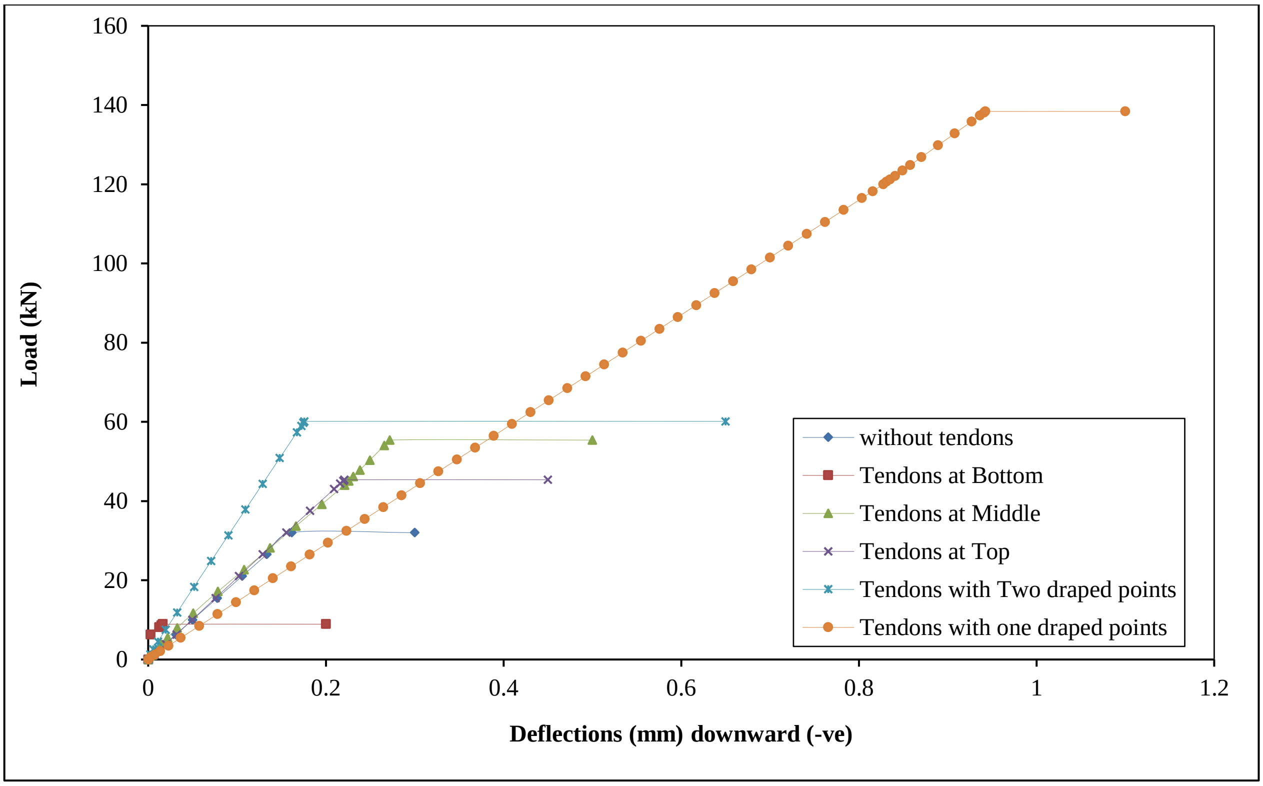

Figure 17. Load-displacement curves at middle of simply supported part for load case 2

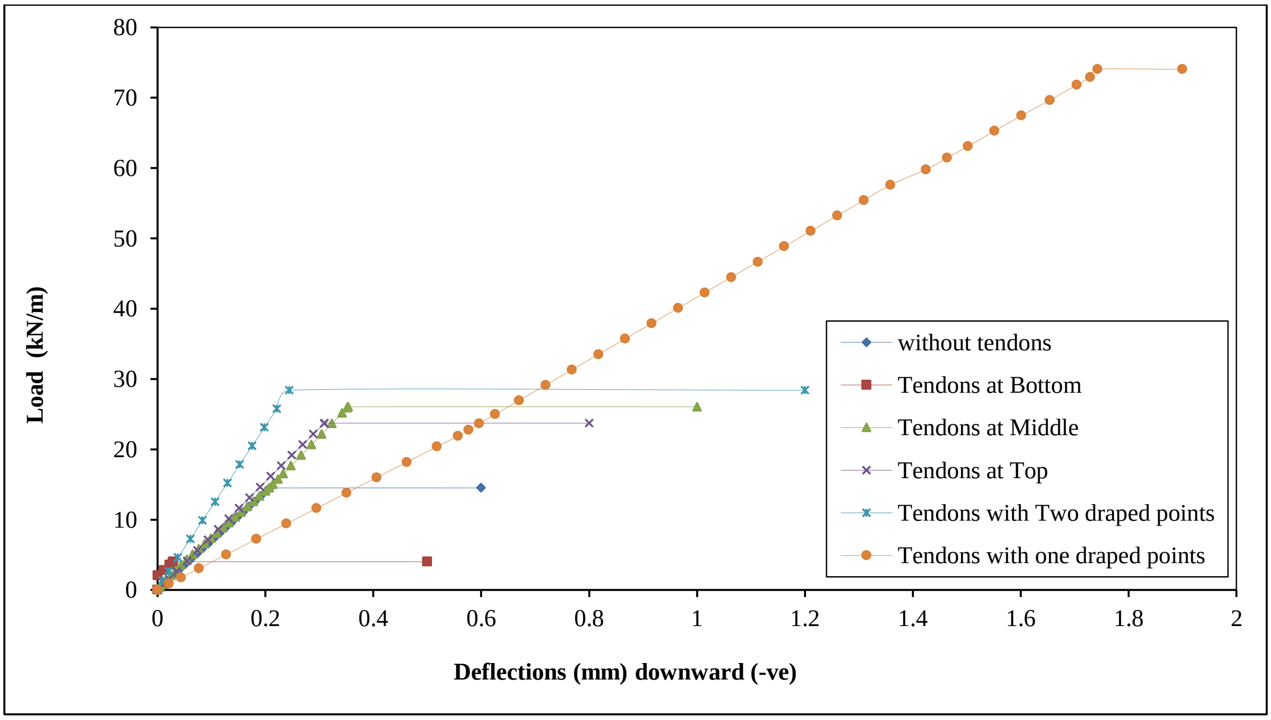

Figure 18. Load-displacement curves at middle of simply supported part for load case 3

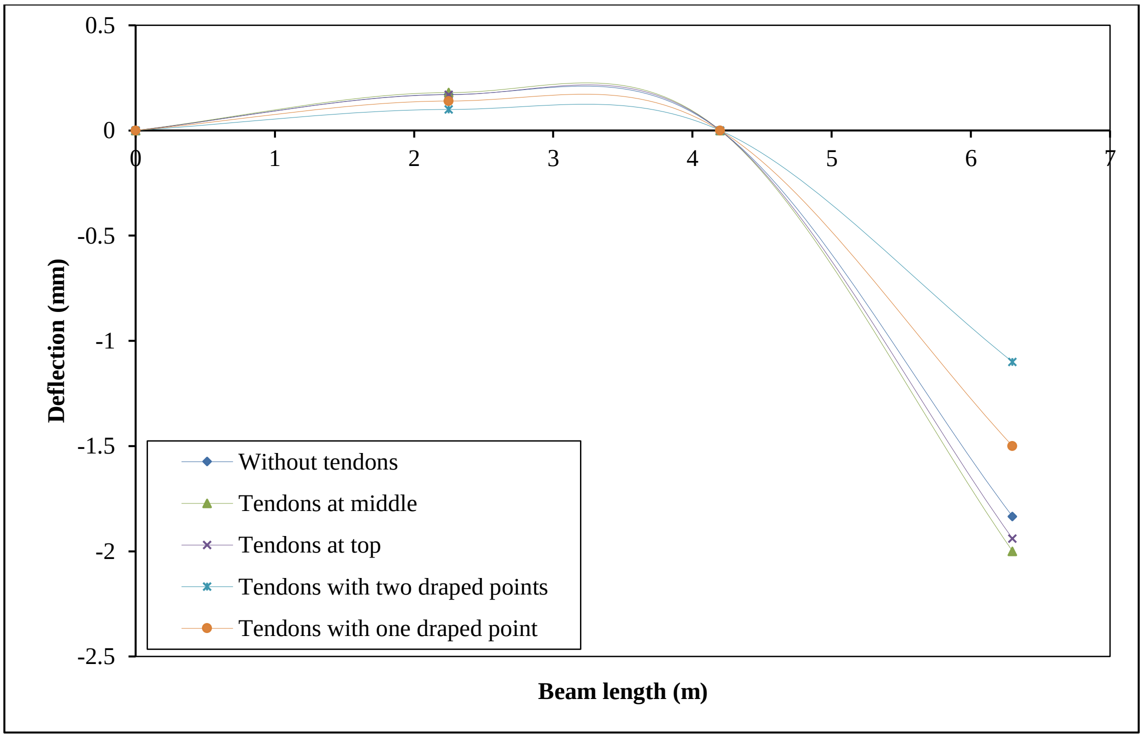

Figure 29. Deformed and deflection shapes for load case 1 with constant load 17 kN

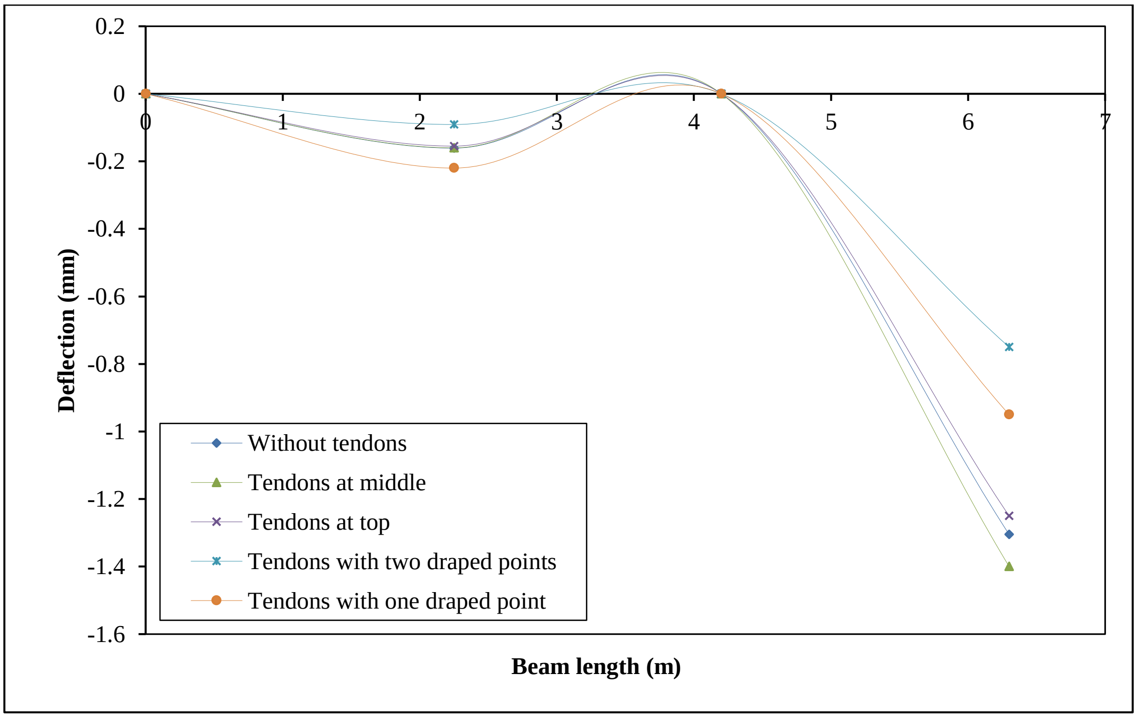

Figure 20. Deformed and deflection shapes for load case 2 with constant load 32 kN

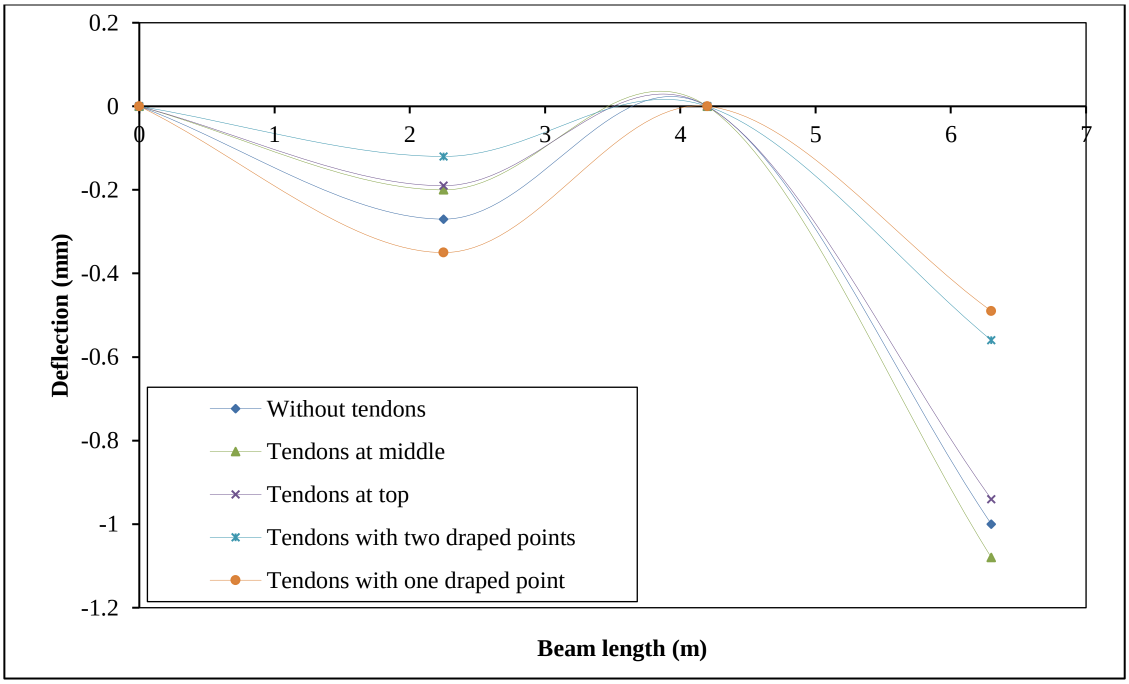

Figure 21. Deformed and deflection shapes for load case 3 with constant load 14 kN

Table 2. indicates the failure loads and deflections at end and middle points of PT concrete cantilever beams for the three types of loadings. The total deflections calculated supersite of the upward displacement due to PT force and downward displacement due to the gravity and applied loads.

Table 2. Failure loads and deflections at the end and middle points

Loading cases	Tendon profile	Failure loads (kN)	Mid deflections (mm)	End deflections (mm)
Case 1	Without tendons	17.0	0.171	-1.835
	Tendons at bottom	4.7	0.078	-0.396
	Tendons at middle	28.6	0.304	-3.256
	Tendons at top	57.0	0.601	-7.039
	Tendons with two draped points	29.7	0.163	-1.899
	Tendons with one draped point	70.6	0.650	-7.501
Case 2	Without tendons	32.0	-0.161	-1.305
	Tendons at bottom	9.0	-0.016	-0.342
	Tendons at middle	55.0	-0.272	-2.354
	Tendons at top	45.0	-0.22	-1.784
	Tendons with two draped points	60.0	-0.175	-1.417
	Tendons with one draped point	138.0	-0.942	-4.534
Case 3 (Failure loads are in kN/m)	Without tendons	14.5	-0.208	-1.039
	Tendons at bottom	4.0	-0.029	-0.263
	Tendons at middle	26.0	-0.353	-1.868
	Tendons at top	23.7	-0.309	-1.546
	Tendons with two draped points	28.4	-0.243	-1.064
	Tendons with one draped point	74.0	-1.742	-3.535

Table 2. shows that the maximum failure loads will be gained when one draped tendon profile is used, the maximum failure loads are 70.6 kN, 138.0 kN, and 74.0 kN/m for load cases 1, 2, and 3 respectively.

Minimum failure loads occur when bottom tendon profile is used, the failure loads are 4.7 kN, 9.0 kN, and 4.0 kN/m for load cases 1, 2, and 3 respectively. From Table 2, can be obsreved that the failure load in beams with one draped point of tendon profile increases about 315 % , 331 % and 410 % as compared with beams without tendons profile for the load case1, case2 and case3 respectively.

The uniaxial tensile stress of concrete is 3.8 MPa, during the analysis process the cracks began when the uniaxial tensile stress of concrete was over 3.8 MPa. Figures 22, 23, and 24. shows the stress distributions in z-direction for one draped point profile for all load cases at failure. Beam failures for all models were sudden failure above the supports due to cantilever part. Figures 25, 26 and 27. shows cracks pattern at failure of the beams with one draped point of tendon profile for all load cases.

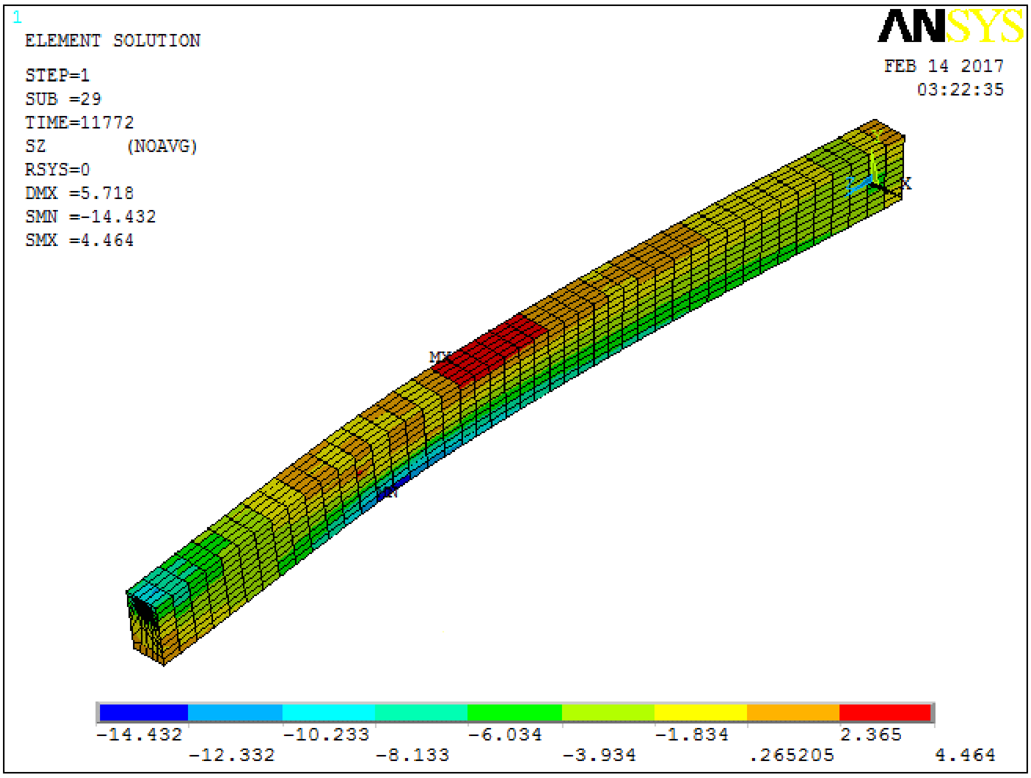

Figure 22. Longitudinal stress of concrete in z-direction for the beams with one draped point of tendon profile for load cases 1 at failure

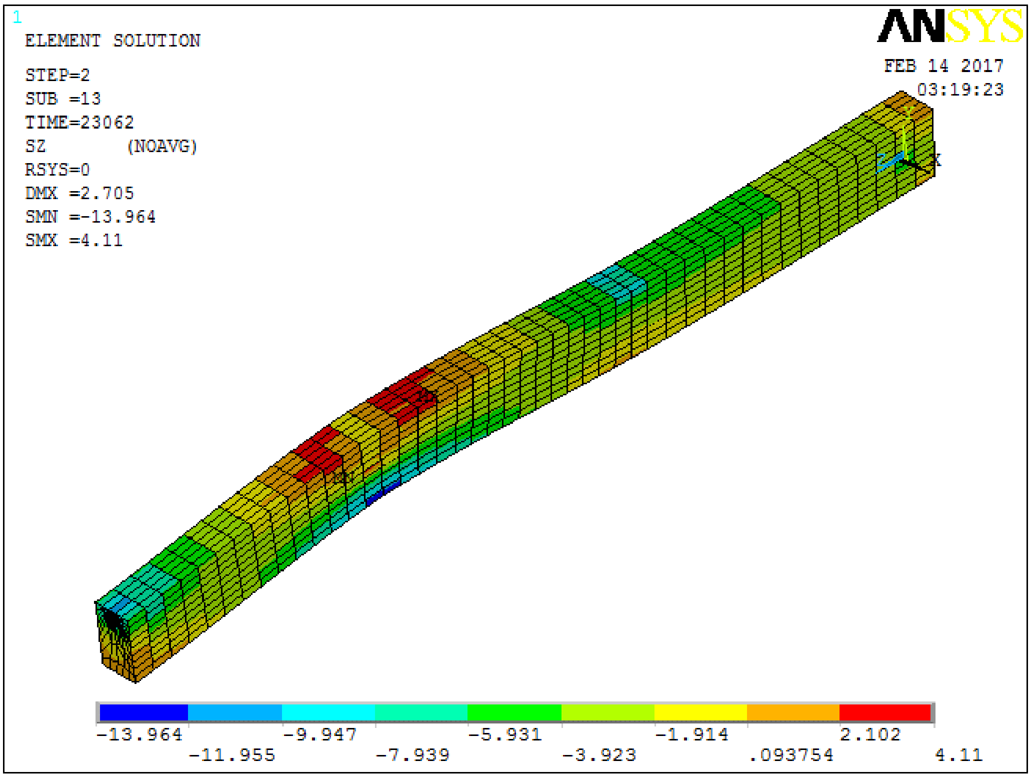

Figure 23. Longitudinal stress of concrete in z-direction for the beams with one draped point of tendon profile for load cases 2 at failure

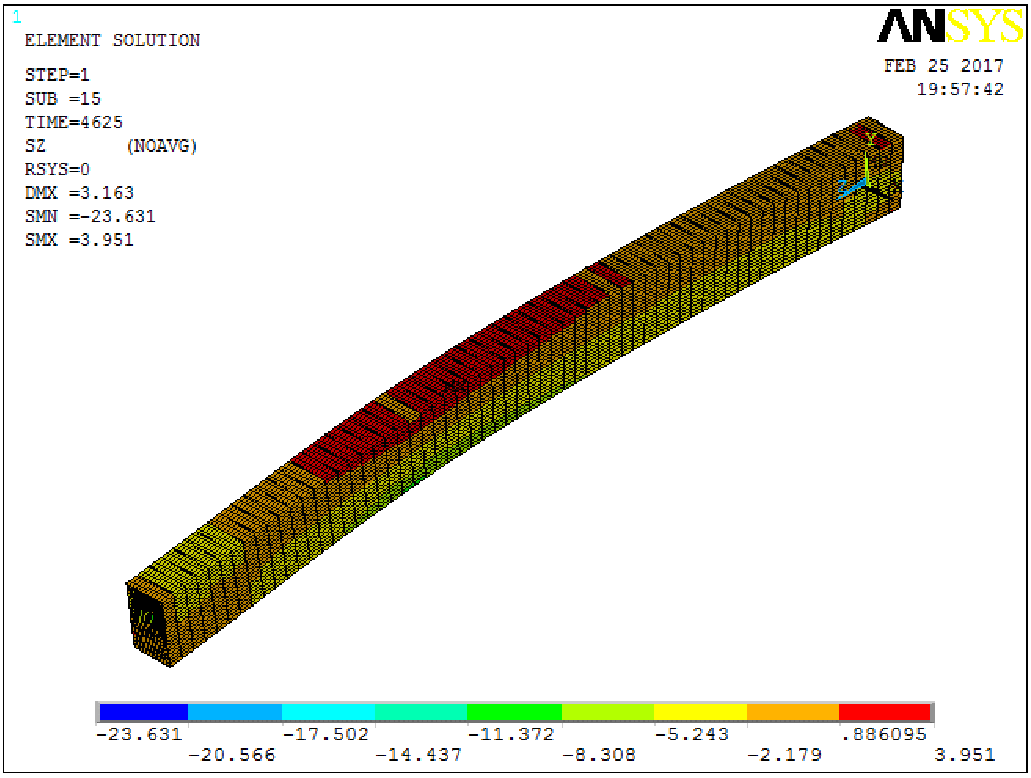

Figure 24. Longitudinal stress of concrete in z-direction for the beams with one draped point of tendon profile for load cases 3 at failure

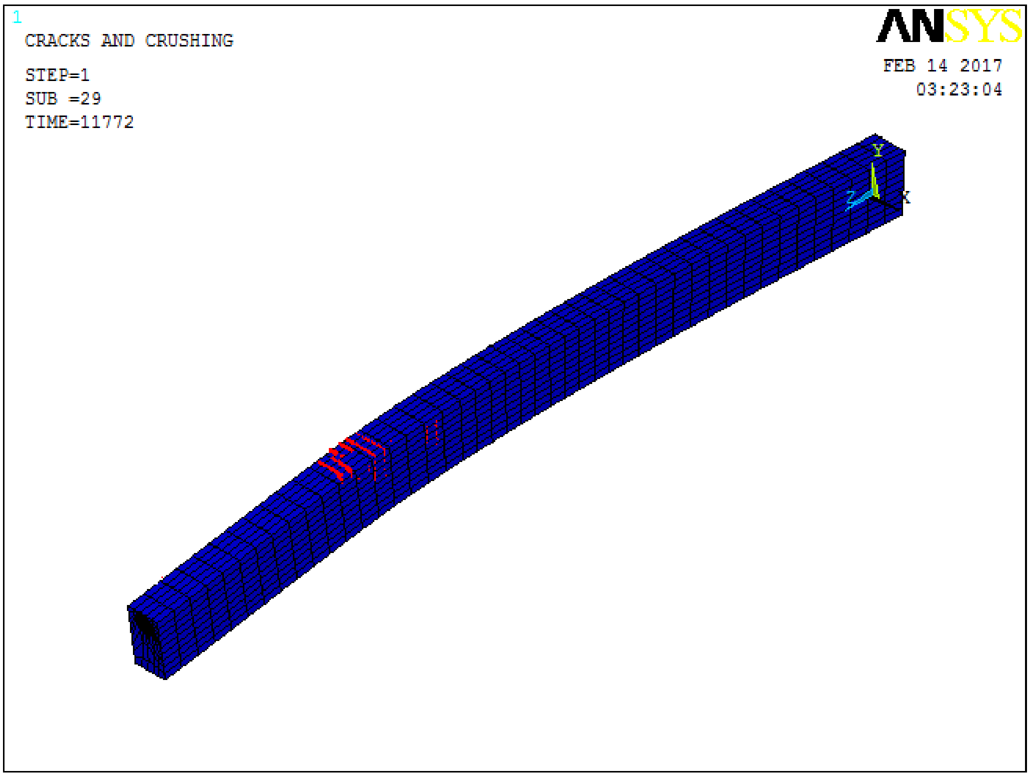

Figure 25. Cracks pattern at failure in concrete for the beams with one draped point of tendon profile for load cases 1

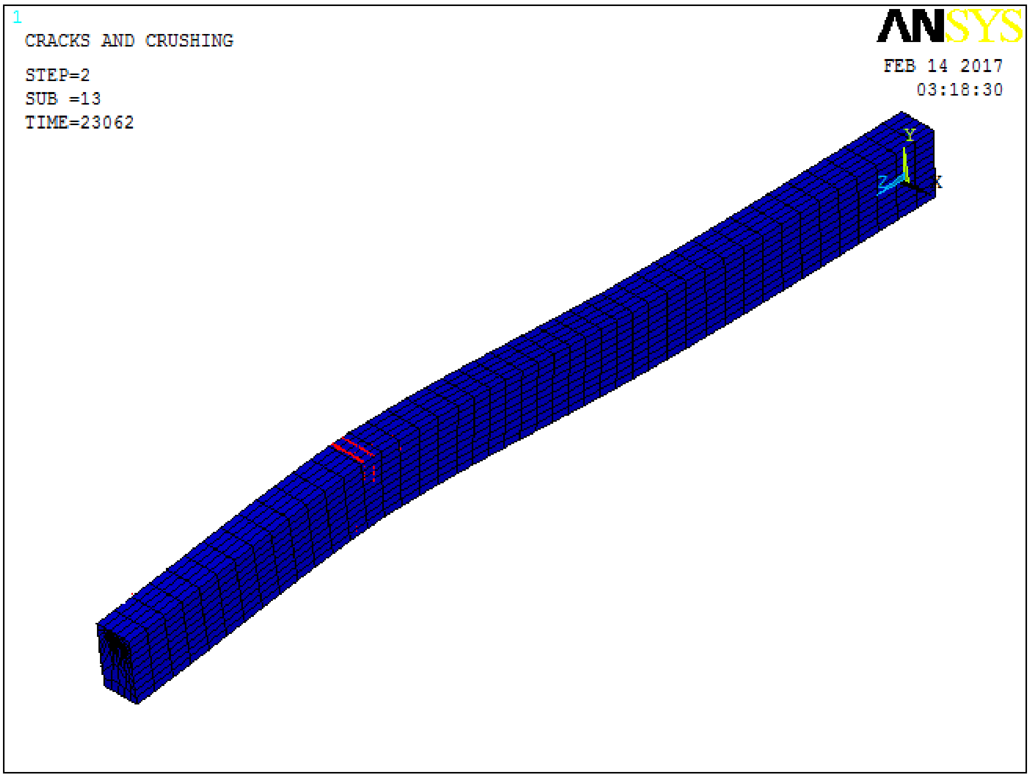

Figure 26. Cracks pattern at failure in concrete for the beams with one draped point of tendon profile for load cases 2

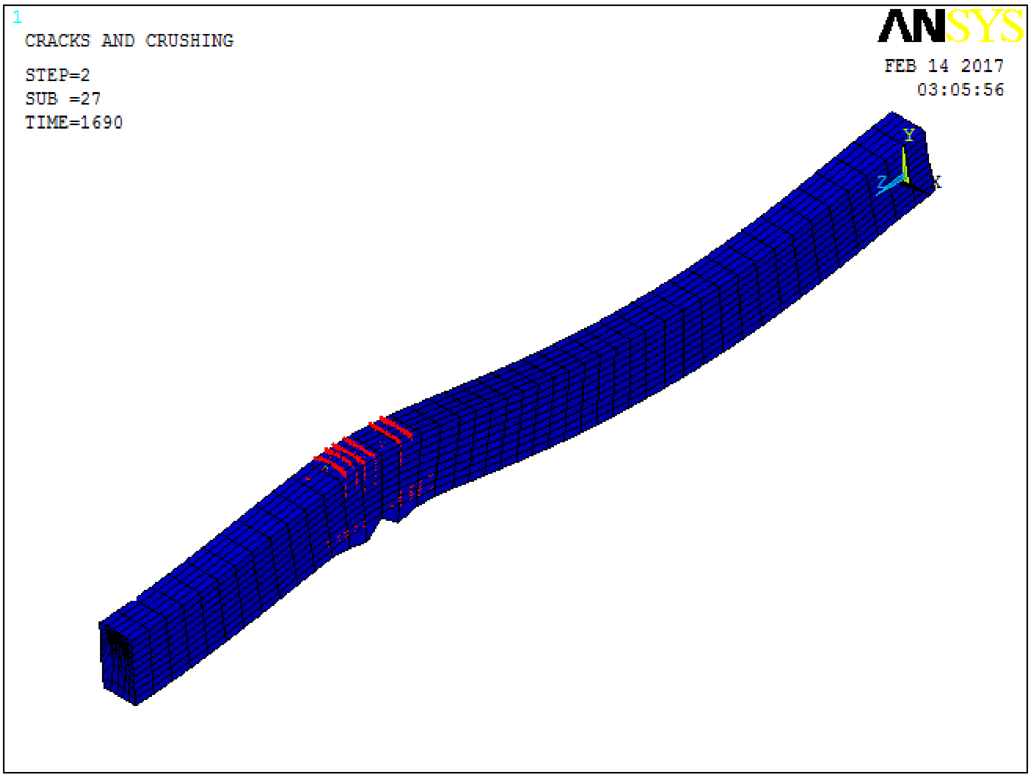

Figure 27. Cracks pattern at failure in concrete for the beams with one draped point of tendon profile for load cases 3

6. Conclusion

In this paper, a 3D thermo-mechanical nonlinear finite element model is developed to study the behavior of bonded PT concrete beams. The contact between the concrete and the tendon is modelled by contact elements. Contact elements allowing the post-tensioned tendon to retain its profile during the deformation of the slab and modelling the bond behaviour between the grout and tendon. Experimental bonded PT concrete beam from literature were chosen for numerical analyses verification, and good agreement was achieved between numerical and test results.

This study investigates the effect of several factors on the load-deflection response throughout the entire range of behavior using the nonlinear analysis of the ANSYS finite element program. The verified finite element model was used to investigate the effect of several selected parameters on the overall behavior of PT concrete cantilever beam. These parameters include the effect of tendon profile and the effect of loading type. It was observed that the beams with tendon profile have a great effect on the ultimate load capacity. The ultimate load capacity increases by using parabolic tendon profiles. The beams with one draped tendon profile shows a higher ultimate load capacity and a stiffer response as compared to the beams with straight tendon profiles for all load cases. Because parabolic profiles have the specified values of eccentricities, and this profile is similar with the beams bending moment.

Beams with tendons at the bottom gives smaller failure loads and it is very weak. Because in parabolic profiles, the curvature of the cable exerts a force on the concrete to counterbalance the forces causing tension. The tendons are located with eccentricities to counteract the sagging bending moments due to transverse loads. Consequently, the prestressed beams deflect upwards on the application of prestress. Since the bending moment is the product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of the bending moment diagram. For the effect of draped profile, it was observed that the ultimate load capacity increased with draped tendon profiles as compared with other undraped profiles. Straight profiles produced the lowest nominal resistance. The application of load types on each model greatly affects the load-deflection response and failure loads. When the applied loads are distributed uniformly, the load carried by beam will increase. The failure load in beams with one draped point profile about 315 %, 331 % and 410 % increases as compared with beams without tendon profile for the load case1, case2 and case3 respectively.

7. References

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