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Civil Engineering Journal

Vol. 4, No. 12, December, 2018

Element Free Galerkin Post-processing Technique Based Error Estimator for Elasticity Problems

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Received 15 September 2018; Accepted 25 November 2018

Abstract

The study present a Mesh Free based post-processing technique for asymptotically (upper) bounded error estimator for Finite Element Analyses of elastic problems. The proposed technique uses Galerkin Element Free procedure for recovery of the displacement derivatives over a patch of nodes in radial domains. The radial nodes patches are used to construct the trial shape functions utilizing the moving least-squares (MLS) techniques. The proposed technique has been tested on three benchmark elastic problems discretized using 4-node quadrilateral elements. The recovered nodal stresses are utilized to calculate the error in finite element solution in energy norm. The study also demonstrates adaptive analysis application of proposed error estimator. The performance of proposed error estimator based on mesh independent node patches has been compared with that of mesh dependent node patches based Zienkiewicz-Zhu (ZZ) error estimator on structured and unstructured mesh. The improved results of the proposed error estimator in terms of convergence rate and effectivity are obtained. It is shown that present study incorporates the superiority of the Mesh Free Galerkin method into finite element analysis environment.

Keywords: Error Estimation; Effectivity; Error Norm; Post-Processing Technique; Mesh Free Approach; Convergence; Moving Least Square Techniques.

1. Introduction

With advent of powerful computer system, finite element method has gained considerable prominence in industry. However, errors are introduced into the finite element method by the very process of subdividing the problem into sub regions. Much attention has been paid to develop error estimators to quantify the Finite Element solution errors. The error estimators can be categorized either based on its convergence through the effectivity index (ratio of the estimated error to the true error) or based on the procedures to obtain the estimates. The error estimators based on convergence are asymptotically exact, asymptotically (upper) bounded and asymptotically not bounded. The error estimators as per the estimation procedure are the residual-based error estimators [1], constitutive relation error (CRE) based error estimators [2] and the recovery-type ZZ error estimators [3]. A state-of-art of different error estimation technique developed to get the practical finite solution of linear, non-linear and transient problems analyses are presented by Gratsch and Bathe [4]. Nadal et al. [5] proposes the explicit-type recovery error estimator in energy norm for the linear elasticity problem using smooth solution. Hannukainen et al. [6] have developed a posteriori error estimate and showed improved convergence in non-coinciding meshes for problems of linear elasticity. A continuous estimated stress field based effective error estimator is presented by Ramsay et al. [7]. Riedlbeck et al. [8] have presented a posteriori error estimate based on an

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http://dx.doi.org/10.28991/cej-03091211

equilibrated stress reconstruction that is obtained from mixed finite element solutions of local Neumann linear elasticity problems. Yang and Deeks [9] present a node based error estimator for element free Galerkin (EFG) Method using mesh less domain and obtained optimum number and distribution of nodes in appropriate regions of domain for prescribed accuracy. Zhang and co-workers [10] developed adaptive finite element methods using the least-square point interpolation technique. Cai et al. [11] propose a hybrid error estimator, combination of the explicit residual and the improved ZZ, for the conforming finite element method and shown that the propose estimator is reliable on all meshes. Kumar et al. [12] propose several field transfer techniques that can be used to remap data on complete re-meshing of the computational domain or on mesh regularization resulting from an ALE (Arbitrary Lagrangian or Eulerian) formulation with super-convergence property on surface and volume.

The reliability of recovery estimation depends on the way to obtain the smoothed or post- processed continuous stress field. The post- processing procedures are based on the least square fitting of state variable by a higher order polynomial over elements or nodes patches. The various authors have proposed procedures for recovery of post- processed velocity (or the displacement) field or their derivatives (stress field). The super-convergent patch recovery scheme is proposed by Zienkiewicz and Zhu [13] in which post- processed stresses are obtained by interpolating from a stress surface fitted to the super-convergent stress points surrounding the node of interest. Ahmed et al. [14] evaluate the performance of a post- processing technique based upon the least square fitting of velocity field over an element patch. Parret-Fréaud et al. [15] presented a moving least squares (MLS) recovery-based procedure to obtain smoothed stresses field in which the continuity of the smoothed field is provided by the shape functions of the underlying mesh. The investigations is reported by González-Estrada et al. [16] for getting improved recovery of stress field using domain decomposition method in heterogeneous structures. From the literature review, it is clear that most of the proposed recovery procedures require the element mesh made from the nodes connectivity. The zone of influence or patch is dependent on elements and node connectivity to the elements. The proposed work is a contribution in post processing of the finite element analysis using the mesh less approach for recovery of state variable and their derivatives, in development of interpolation type error estimator. In this study, the post-processing of higher derivative of field variable utilize the moving least squares (MLS) procedure, which is robust and provide completeness and continuity [17], over a patch of mesh independent nodes, though nodes belongs to background mesh, for formulating a posteriori error estimator. The performance of proposed error estimator is compared on benchmark elastic problems in term of effectivity and rate of convergence with that of ZZ error estimator [13], which evaluates the error in energy nom using the post- processing technique based upon the least square fitting of stress field over a mesh dependent nodes patch.

2. Finite Element Formulation

Considering the two-dimensional linear elastic problem, a stress field (σ) and a unknown displacement field (u) are to be found satisfying over a domain Ω. The static equilibrium and boundary conditions are as follows,

$$LT\,\sigma + p = 0 \quad \text{in} \quad \Omega \tag{1}$$

$$\sigma \cdot n = \quad \text{on} \quad \Gamma t \tag{2}$$

$$u = \quad \text{on} \quad \Gamma u \tag{3}$$

Where, p is the force vector, LT is the derivative operator, and are prescribed tractions and displacements on Γt and Γu, respectively and n is the unit outward normal on the boundary $\Gamma = \Gamma t \cup \Gamma u$. The strain vector (ε) and corresponding stress-strain relation can be written as;

$$\varepsilon = L\,u \tag{4}$$

$$\sigma = D\,\varepsilon \tag{5}$$

Where, D is a constitutive relation matrix that relates the rate of stress to strain change.

According to the Finite Element Method, the equilibrium equations are numerically solved in a domain that is discretized into several finite elements. The displacements and strains of any point within an element are calculated as follows;

$$u = N\,\nu \tag{6}$$

$$\varepsilon = L\,N\,\nu = B\,\nu \tag{7}$$

Where, ν is the nodal displacement matrix, u is the displacement of a certain point within an element, N is the matrix of the interpolation functions, also known as shape functions and B is the strain interpolation matrix.

The relationship between nodal forces and nodal displacements can be described as follows:

$$f = K\,v \tag{8}$$

In the above equation, f is the nodal force vector and K is known as the stiffness matrix. The global equations are obtained from assembly of elemental equations. The K and f matrices are found from following equations.

$$k_{ij} = \int_{\Omega} B_i^T\, D B_j d\Omega \tag{9}$$

$$f_{ij} = \int_{\Gamma_i} \phi_j\, \bar{t}_i d\Gamma + \int_{\Omega} \phi_j\, b d\Omega \tag{10}$$

Where b is the body force per unit volume.

3. Error Estimation and Mesh Improvement Strategy

The error can be evaluated in any appropriate norms. Since the finite element solution minimizes the error in the energy norm, the magnitude of the error in energy norm is a good measure of the overall quality of solution. The integral measure of the error in energy norm may be defined as follows.

$$(\|e\|) = \left[\int_{\Omega} e_{\sigma}^{*T} D e_{\sigma}^{*} d\Omega\right]^{\frac{1}{2}} \tag{11}$$

Where e_{σ}^{*} is the error in computed stress

The standard measure of the quality of error estimators i.e., effectivity index, ɵ, is defined as ratio of estimated error and true Error. An estimator is asymptotically exact for a particular problem if its effectivity index converges to one when the mesh size approaches to zero.

$$\theta = \frac{\|e_{ex}\|}{\|e\|} \tag{12}$$

Where $\|e_{ex}\|$ and $\|e\|$ denote the exact error and the evaluated error estimate in energy norm respectively.

The mesh improvement strategy based on the local and global error estimation to get the desired accuracy has been implemented using the procedure given in published literature Ahmed and Singh [18].

4. Interpolation Recovery Procedure

The adopted interpolation recovery procedure i.e. Moving Least Square (MLS) technique, consist of expressing the function u(x) in terms of the approximation uh(x), a weight function w(x) associated to each node, a basis P(x) and a set of coefficients, a(x). The domain nodes are defined by x1 ...xn where x1 = (x1, y1). The approximation function uh(x) can be defined as;

$$u^h(\mathrm{x}) = \sum_{i=0}^{m} \phi(x_i)u_i = \phi\,(x)u \tag{13}$$

Where ϕ (x) is shape function and m is the total number of terms in the basis. In the present study, the basis function is P(x) has adopted m as six for linear elements.

$$P^T(x) = [1, x, y, x^2, xy, y^2] \tag{14}$$

The, ϕ_i (x) is given as;

$$\phi_i(x) = P^T(x)a^{-1}(x)P(x_i)w(x - x_i) \tag{15}$$

The vector of coefficients a(x) can be obtained by minimizing a weighted residual as follows:

$$J = \sum_{i=0}^{n} w(x - x_i)\,[P^T(x_i)a(x) - u_i^h]^2 \tag{16}$$

Where n is the number of nodes *i* and $w(x-x_i)$ is a weight function in 2-D associated to each node (domain of influence of that node) which is usually built in such a way that it takes a unit value in the vicinity of the point where the function and its derivatives are to be computed and vanishes outside a region Ω_i surrounding the point x_i. The support of the shape function ϕ_i (x) is equivalent to the support of the weight function. The following cubic spline weight function with circular domain of influence is considered in the present study.

$$w(x-xi) = w(\bar{d}) = \begin{Bmatrix} \frac{2}{3} - 4\bar{d}^2 + 4\bar{d}^3 & for\ \bar{d} \le \frac{1}{2} \\ \frac{4}{3} - 4\bar{d} + 4\bar{d}^2 - \frac{4}{3}\bar{d}^3 & for\ 1 \le \bar{d} \le \frac{1}{2} \\ 0 & for\ \bar{d} > 1 \end{Bmatrix} \tag{17}$$

Where, $\bar{d} = \|x - x_i\| / d_w$ and d_w is the size of influence domain of the point x_i.

Minimization of weighted residual leads to the well-known form of shape function equation.

$$\phi(x) = P^T(x)a^{-1}(x)B(x)w(x-x_i)\phi^h = N^T(x)\phi^h \tag{18}$$

The coefficients a(x) and B(x) are given as;

$$a(x)\sum_{i=0}^{n} w_i(x-x_i)\,P(x_i)P^T(x_i)$$

$$B(x) = [w_i(x-x_i)P(x_i)\ \ldots .\, w_n(x-x_n)P(x_n)]$$

The derivatives of weight function with respect to x_i is computed using the chain rule of differentiation;

$$\frac{dw}{dx_i} = \begin{Bmatrix} (8\bar{d} + 12\bar{d}^2)\frac{dd}{dd_I} & for\ \bar{d} \le \frac{1}{2} \\ (-4 + 8\bar{d} - 4\bar{d}^2)\frac{d\bar{d}}{dd_I} & for\ 1 \le \bar{d} \le \frac{1}{2} \\ 0 & for\ \bar{d} > 1 \end{Bmatrix} \tag{19}$$

5. Research Methodology

The proposed research work methodology follows as under and Figure 1 shows flow chart of research methodology.

Step i. Read input data of problem and input data for error analysis.

Step ii. Generate an initial mesh of quadrilateral elements by discretizing the domain.

Step iii. Choose appropriate shape function for 4-node quadrilateral elements.

Step iv. Evaluate elemental stiffness matrices and compute the nodal load vector.

Step v. Apply boundary conditions to elemental stiffness matrices and nodal force matrix, and repeat step (iv) for all the elements.

Step vi. Assemble elemental stiffness matrices to form a global stiffness matrix and solve the global stiffness matrices for displacements.

Step vii. Calculate strain and stress tensor for each element.

Step viii. Calculate the post-processed stress using the recovery procedure and calculate error.

Step ix. Calculate the accuracy of the solution.

Step xiii. If the accuracy is not within the permissible limit, Calculate new element size to generate adaptive mesh.

6. Numerical Example

The performance of the proposed error estimator is investigated by three benchmark problems in elasticity, square test plate, plate with central hole and cylinder, for which exact solution is available. The linear quadrilateral elements have been used for the discretization of the domain.

6.1. (1×1) Square Benchmark Problem

The benchmark examples considers an infinite domain problem where it is extracted a 1×1 square domain, centred at the origin of coordinates with body loads (bx, by) over the domain. The corresponding Neumann boundary condition is imposed. The known exact displacement solution (u, v) and body loads in the form of 3rd order polynomials, are given in Equation 20 to 22 [13].

$$u = 0 \quad and \quad v = -x.y\ (1-x)\,(1-y) \tag{20}$$

$$bx = (\alpha + \beta).(1-2.x).(1-2.y) \tag{21}$$

$$by = -2.\beta\,.y.(1-y) - (\alpha + 2.\beta).2.x.(1-x) \tag{22}$$

The constants α and β are given as;

$$\alpha = E.\nu/((1-2\,.\nu)\,.(1+\nu)) \ and\ \beta = E/(2.(1+\nu))$$

Where E and ν are modulus of elasticity and poisson's ratio respectively with a value of 1.0 N/mm^2 and 0.3.

The benchmark problem domain is discretized using 4-node quadrilateral elements structured as well as unstructured mesh. The adaptive mesh improvement strategy is also used to study the local error distribution with desired solution accuracy having 3% and 8% global error. The refinement is guided through the error predicted by ZZ and EFG error estimators. The initial structured and unstructured mesh is shown in Figure 2. The variation of error in energy norm i.e. convergence rate and global effectivity of error estimators with order of refinement as uniform mesh are plotted in Figure 3 and Figure 4. The convergence rate of solution error with order of refinement in finite element solution, post- processed solution using ZZ recovery and EFG recovery are found respectively as 1.0017, 1.6560 and 2.0557. Table 1 present actual global error and global error predicted by the EFG and ZZ error estimators obtained using the uniform and non-uniform mesh. The variation error in energy norm and global effectivity of error estimators using unstructured mesh is shown in Figure 5 and 6.

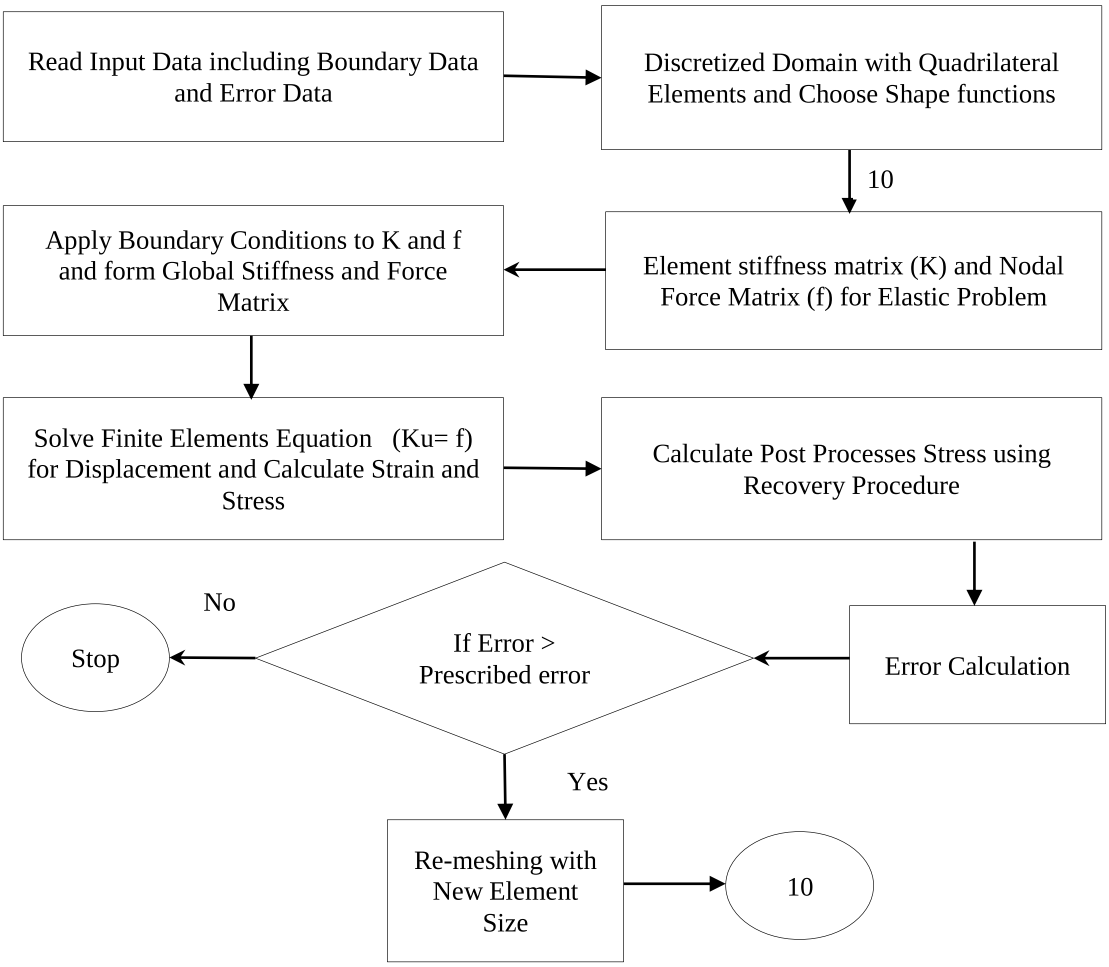


Figure 1. Flow chart for Post-processing Based Finite Element Analysis

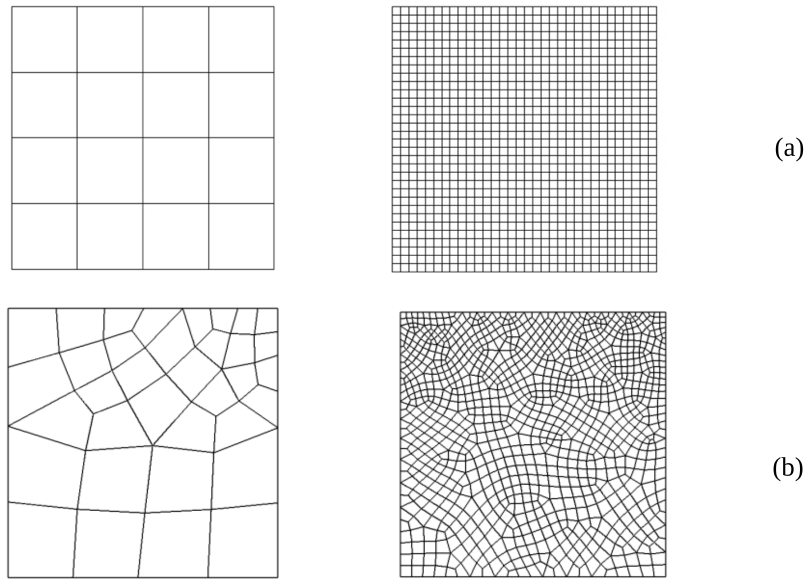


Figure 2. Domain discretization with quadrilateral elements, (a) Structured mesh and (b) Unstructured mesh

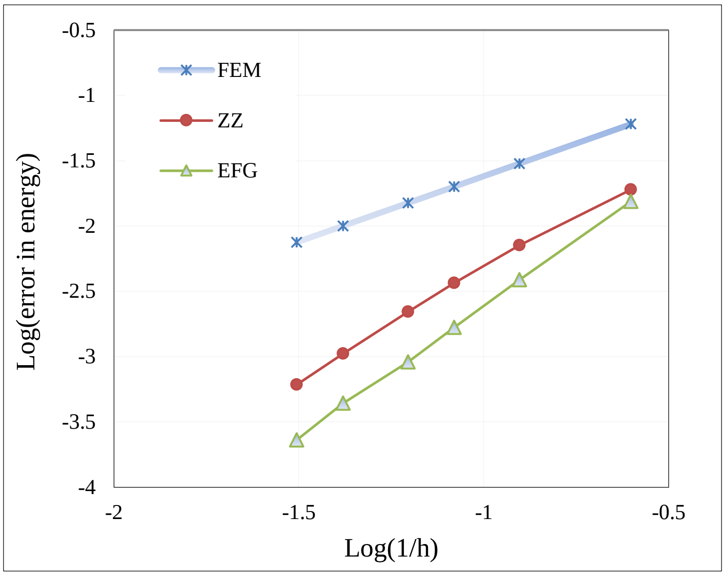


Figure 3. Log-Log Plot of error in energy verses mesh size (h) for benchmark problem (uniform mesh)

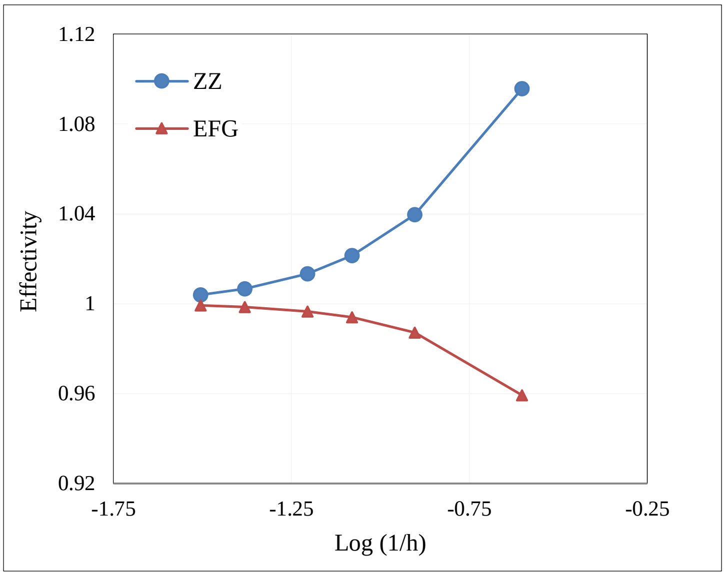


Figure 4. Global Effectivity plot of uniform mesh size (h) (benchmark problem)

Table 1. Actual and predicted global errors in energy norm under different post-processing techniques using structured and un-structured mesh (benchmark problem)

Global Error % (Structured Mesh)					Global Error % (Un-structured Mesh)				
No. of element	**DoF**	**Actual**	**ZZ**	**EFG**	**No. of element**	**DoF**	**Actual**	**ZZ**	**EFG**
16	50	25.1	27.1	23.7	35	94	23.9	26.9	24.0
144	338	8.3	8.6	8.2	155	358	9.3	9.6	9.3
Rate of conv.		1.0017	1.6560	2.0557					

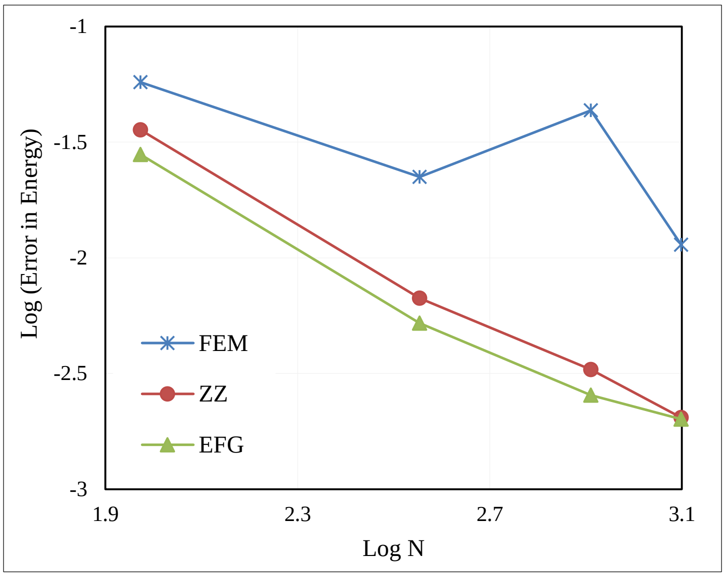


Figure 5. Log-Log Plot of error in energy verses degree of freedom (N) using unstructured mesh (benchmark problem)

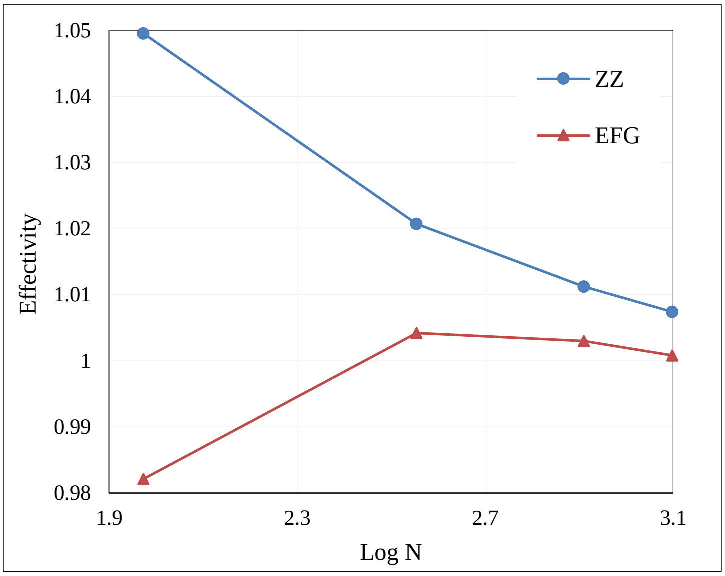


Figure 6. Global Effectivity plot verses degree of freedom (N) using unstructured mesh (benchmark problem)

6.2. Plate Problem

The infinite elastic square plate with a rigid central circular inclusion, a typical stress concentration problem, having radius (a) of 1 unit is considered for performance study of error estimator. The one quarter of square domain (4X4) is modeled because of symmetry of plate problem. The plate discretized with quadrilateral elements is shown in Figure 7 (a). The boundary conditions are as follows. Along the circular arc, both displacement components are zero. Along the symmetry line, the normal displacement component and shear stress are zero. Static boundaries conditions are imposed from the traction computed from the given equations. The exact stress field for the plate are given in following equations

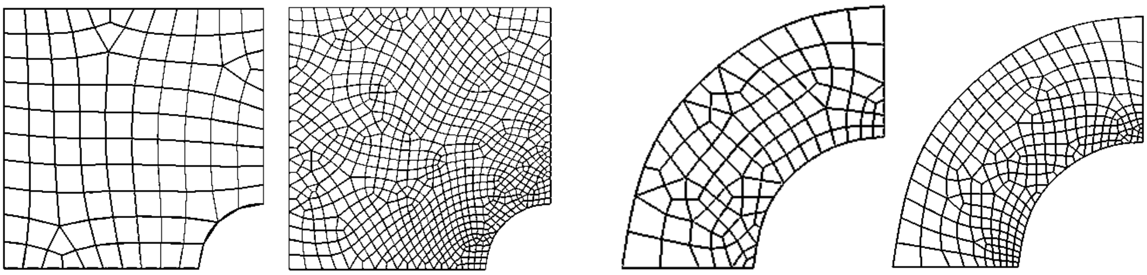
[13]. Figure 8 and 9 depicts the variation of error in energy norm and effectivity of post processing error estimators namely ZZ and EFG, with refinement in plate problem.

$$\sigma_x = \sigma_\infty \{1-[(a^2/r^2)(1.5\cos 2\theta + \cos 4\theta)]+(1.5a^4/r^4 \cos 4\theta)\} \quad (23)$$

$$\sigma_y = \sigma_\infty \{0-[(a^2/r^2)(1.5\cos 2\theta - \cos 4\theta)]+(1.5a^4/r^4 \cos 4\theta)\} \quad (24)$$

$$\sigma_{xy} = \sigma_\infty \{0-[(a^2/r^2)(1.5\sin 2\theta + \sin 4\theta)]+(1.5a^4/r^4 \sin 4\theta)\} \quad (25)$$

Where $r = y^2 + x^2$.



(a) Plate with Hole Problem **(b) Cylinder Problem**

Figure 7. Plate and cylinder problems domain discretization with quadrilateral elements

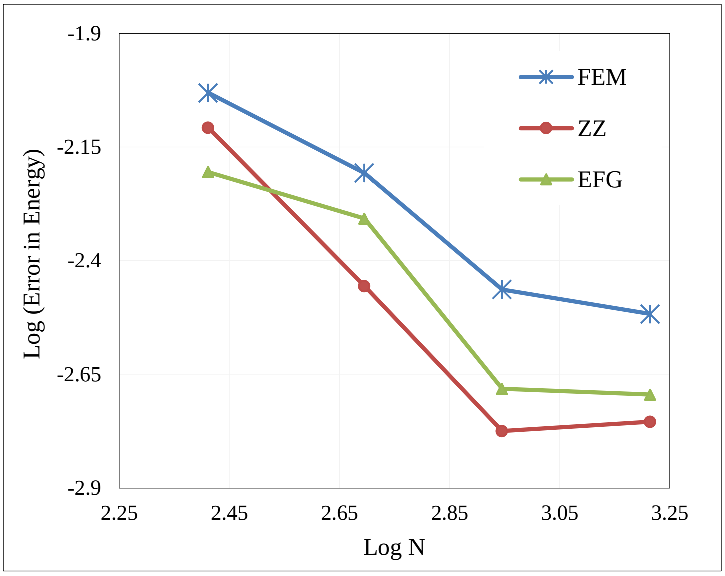


Figure 8. Log-Log Plot of Error in Energy verses Degree of Freedom (N) for Plate Problem

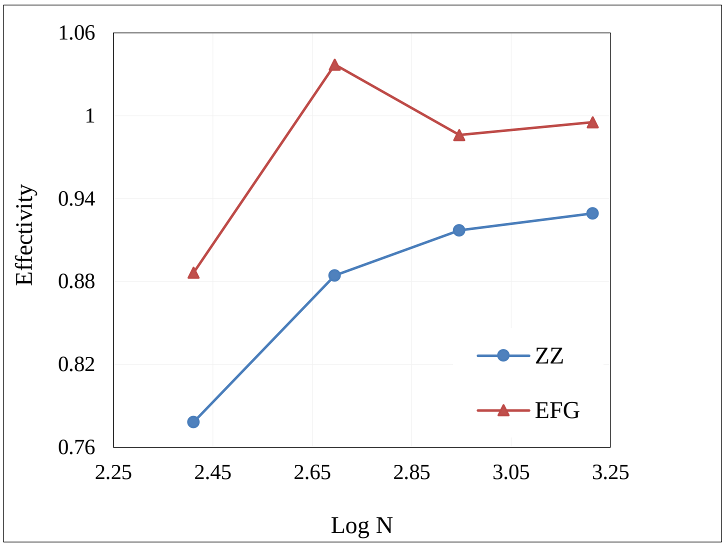


Figure 9. Global Effectivity plot verses Degree of Freedom (N) for plate problem

6.3. Cylinder Problem

The discretized geometry of the cylinder problem, quarter part modelled because of symmetry, is shown in Figure 7 (b). The internal and external surfaces are of radius "a" and "b". The close form solution for the cylinder problem is given by the following expressions [19]. Figure 10 and 11 shows the convergence of errors and global effectivity obtained, with decreasing mesh size in energy norm, using post processing ZZ and EFG error estimators.

$$u_r = [P(1+\nu)]/[E(c^2 - 1)][\, r(1-2\,\nu) + (b^2/r)] \quad (26)$$

$$\sigma_r = P/[(c^2 - 1)][1- (b^2/r)] \quad (27)$$

$$\sigma_\theta = P/[(c^2 - 1)][1+ (b^2/r)] \quad (28)$$

Where c = (b/a), P=1, E=1000 and ν = 0.3.

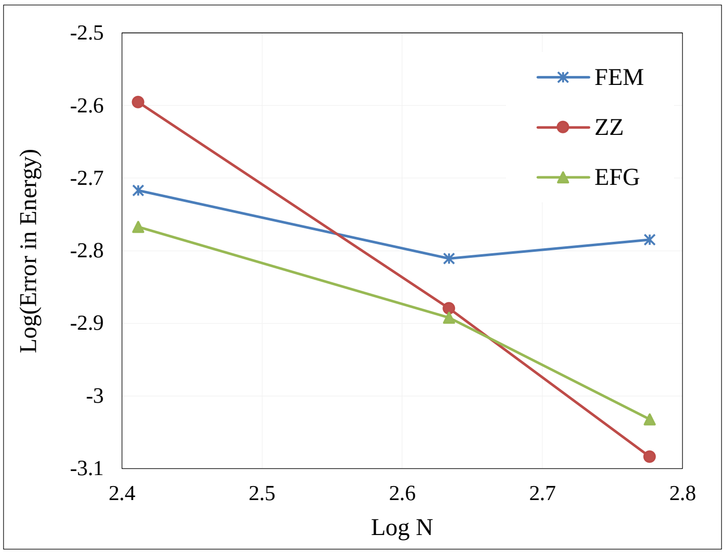


Figure 10. Log-Log Plot of error in energy verses degree of freedom (N) for cylinder problem

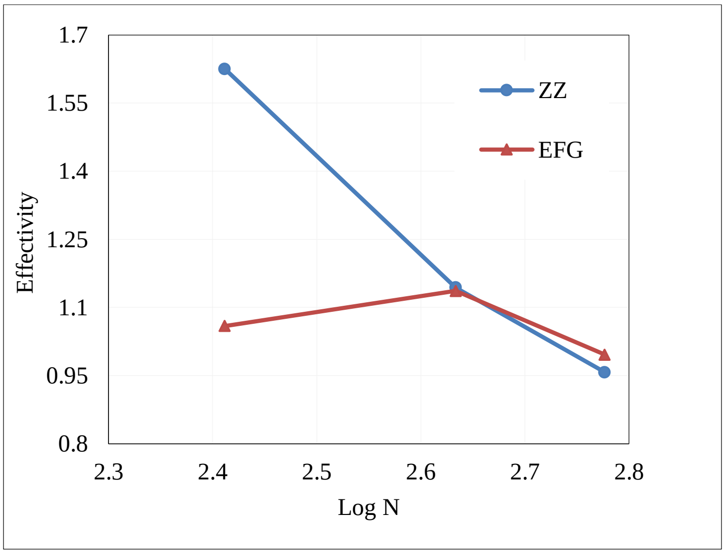


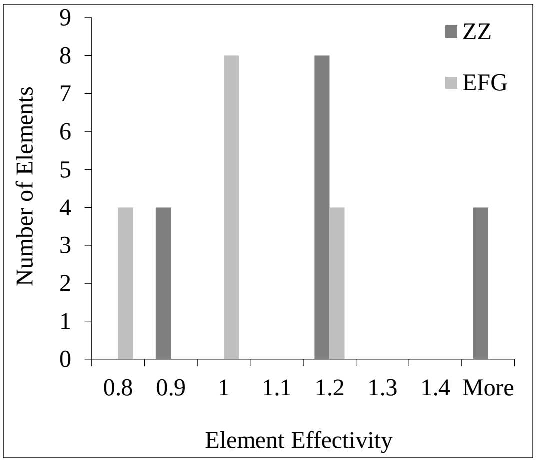
Figure 11. Global Effectivity plot verses degree of freedom (N) for cylinder problem

7. Discussion

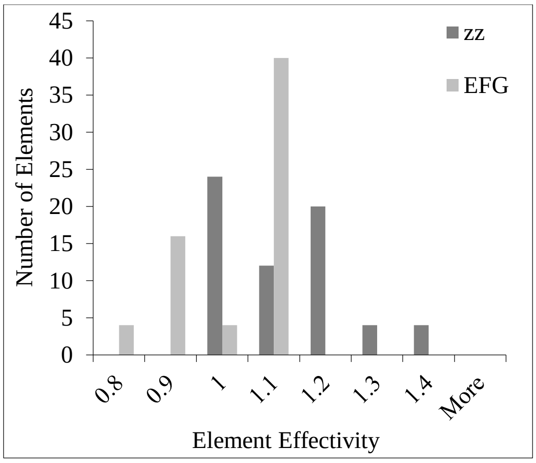
The performance of the two post processing based error estimators namely Element Free Galerkin (EFG) and Zienkiewicz- Zhu (ZZ) error estimators have been studied by analyzing three benchmark problems, i.e. square test plate, plate with central hole and cylinder, using structured and unstructured 4-node quadrilateral mesh. The EFG based error estimation procedure considers mesh independent patch of nodes in a circular region for post processing of the displacement derivatives by a higher order polynomial while, Zienkiewicz- Zhu (ZZ) error estimator considers mesh dependent patch of nodes. The background meshing scheme and their node locations used by the Element Free Galerkin (EFG) as well as Zienkiewicz- Zhu (ZZ) post processing procedure are the same. The Figure 3, 5, 8 and 10 shows that

that proposed EFG based recovery technique predict higher convergence over the ZZ recovery technique. The Figure 4, 6, 9 and 11 shows that that proposed EFG based recovery technique achieve efffectivity very nearly one over the ZZ recovery technique with increasing refinement. It is evident from the analysis results that the error convergence obtained with the help of proposed element free Galerkin (EFG) error estimator is found to be superior to that for the ZZ super convergent recovery procedure based error estimator considering various mesh schemes. The order of error of proposed error estimator is also smaller. The Figure 12 shows element effectivity frequency in problem domain with decreasing element size. The figure depicts that local effectivity is converging to around one for most of the elements with finer mesh size. It is also seen that element free approach based post processing technique is more effective than the element dependent ZZ super convergent recovery technique especially in problems with singularity. The proposed mesh less error estimation scheme improves the recovery of derivatives especially near the boundary as compared to ZZ recovery scheme because of lesser number of nodes at boundary due to element dependency.

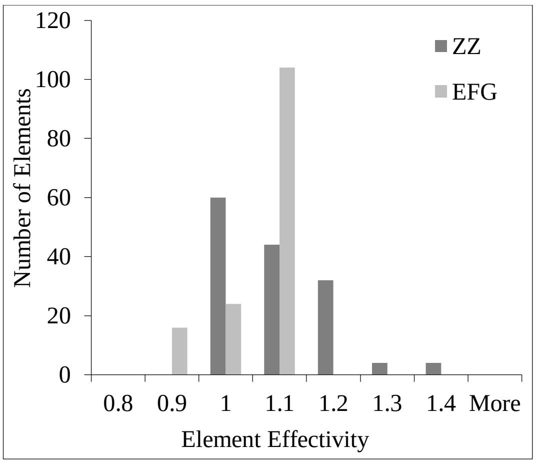
Adaptive finite element application of the proposed error estimator is also demonstrated using the benchmark problem with predefined target error. The initial structured and unstructured meshes are adaptively refined to achieve the specified target error. The adaptively refined meshes obtain through the guidance of ZZ and EFG error estimators depict the distribution of local error in the solution domain. The adaptive refines meshes at target error of 8%are given in Figures 13 to 14. Table 2and Table 3 provide the number of element and DOF in adaptively refined mesh at prescribed target error using ZZ error estimator and EFG error estimator with uniform and non-uniform mesh. The refined mesh plots show that the meshes become finer in areas of high local error to get a uniform accuracy throughout the domain. The lesser number of element and DOF are required with initial structured meshing than initial un-structured meshing to get the same accuracy of the finite element solution, and number of element and DOF are lesser with proposed error estimator as an error controller than ZZ error estimator. It concludes from the results that the initial domain sub divisions greatly affects the quality of the refined meshing and propose error estimator compute the error faithfully at local and global levels.



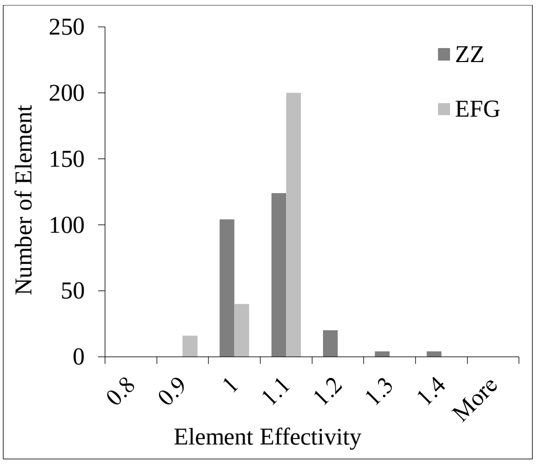
a. Mesh size (1/4)



b. Mesh Size (1/8)

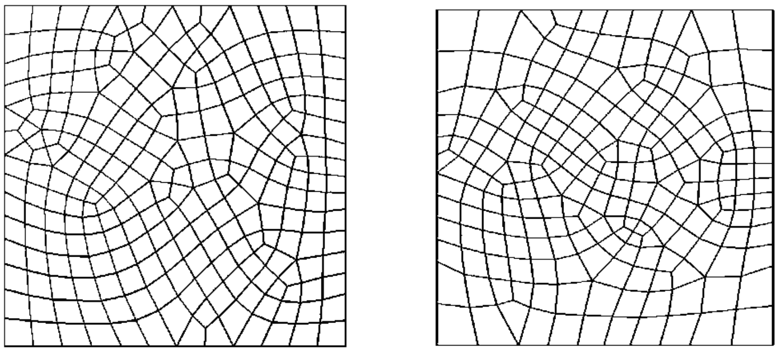


c. Mesh size (1/12)

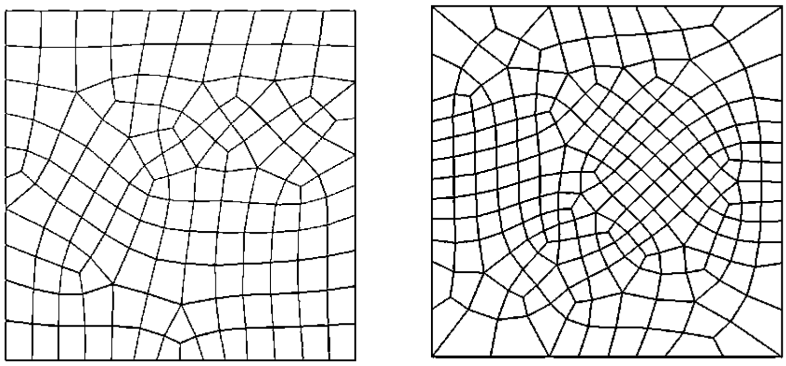


d. Mesh Size (1/16)

Figure 12. Histogram: Element (local) effectivity with number of elements using uniform mesh and with different error estimators

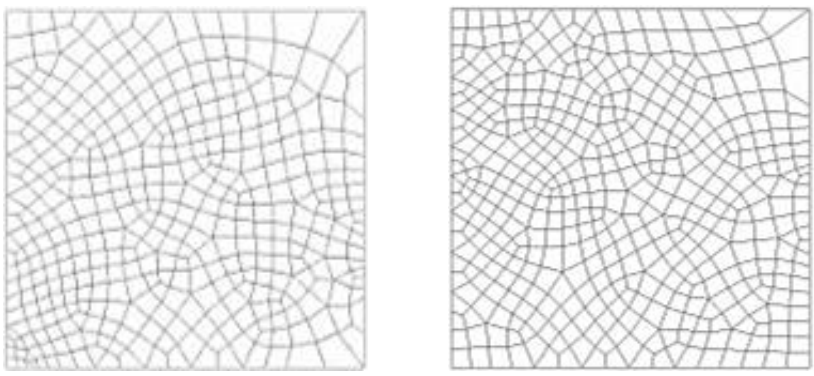


(a). ZZ error estimator

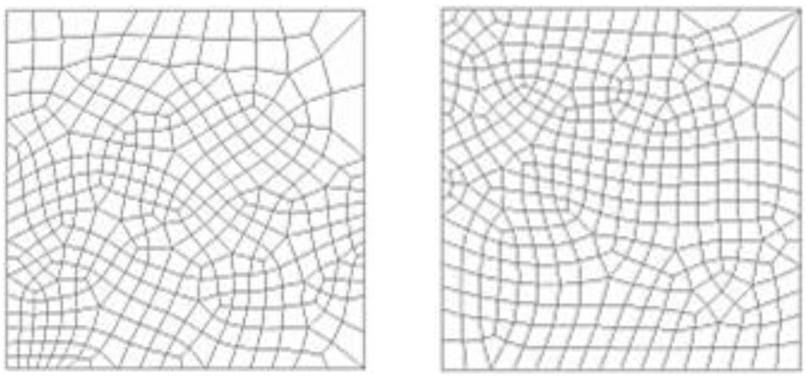


(b). EFG error estimator

Figure 13. Adaptively refined mesh using uniform initial sub-division with different error estimators



(a). ZZ error estimator



(b). EFG error estimator

Figure 14. Adaptively refined mesh using unstructured initial mesh with different error estimators

Table 2. Number of element and DOF in adaptive refined mesh with predefined accuracy under different post-processing techniques (uniform initial mesh and 8% target error)

Initial Mesh		ZZ Recovery		EFG Recovery	
No. of element	DoF	No. of element	DoF	No. of element	DoF
16	50	251	558	144	334
144	338	217	482	222	486

Table 3. Number of element and DOF in adaptive refined mesh with predefined accuracy under different post-processing techniques (un-structured initial mesh and 8% target error)

Initial Mesh		ZZ Recovery		EFG Recovery	
No. of element	DoF	No. of element	DoF	No. of element	DoF
35	94	455	986	356	774
155	358	390	850	362	798

8. Conclusion

The post processing type error estimators namely Element Free Galerkin (EFG) and Zienkiewicz- Zhu (ZZ) error estimators are analyzed using structured and unstructured 4-node quadrilateral mesh by applying proposed post processing scheme to benchmark problems in elasticity. The application of proposed post processing type error estimator to Adaptive finite element analysis is also shown. The adaptively refined meshes, based on the guidance of EFG error estimators and ZZ error estimators are compared. The proposed Galerkin Mesh Free based error estimation procedure is based on post processing of the displacement derivatives by a higher order polynomial over a patch of nodes in a circular boundary. The performance of proposed EFG error estimator in term of effectivity and rate of convergence has been compared with that of Zienkiewicz-Zhu (ZZ) error estimator. Results show that the performance of the EFG error estimator is better than the ZZ error estimator. It can be concluded that propose EFG error estimator is effective in predicting the errors at local and global levels in FE domains.

9. Funding and Acknowledgements

Authors thanks Deanship of Scientific Research, Ministry of Higher Education, KSA for providing financial support to carrying out the research work. The authors also acknowledge to the Dean, Faculty of Engineering for his valuable support and help.

10. Conflict of Interest

The author declares no conflicts of interest.

11. References

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