

## An Investigation of the Fundamental Period of Vibration for Moment Resisting Concrete Frames

Ahmed N. Mohamed <sup>a\*</sup>, Khaled F. El Kashif <sup>b</sup>, Hamed M. Salem <sup>c</sup>

<sup>a</sup> M.Sc. Student, Structural Engineering Department, Cairo University, Giza, Egypt.

<sup>b</sup> Assistant Professor, Structural Engineering Department, Cairo University, Giza, Egypt.

<sup>c</sup> Professor, Structural Engineering Department, Cairo University, Giza, Egypt

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### Abstract

The determination of fundamental period of vibration for structures is essential to earthquake design. The current codes provide empirical formulas to estimate the approximated fundamental period and these formulas are dependent on building material, height of structure or number of stories. Such a formulation is excessively conservative and unable to account for other parameters such as: length to width ratios, vertical element size and floors area. This study investigated the fundamental periods of mid-rise reinforced concrete moment resisting frames. A total of 13 moment resisting frames were analyzed by ETABS 15.2.2, for gross and cracked eigenvalue analysis and Extreme Loading for Structures Software® or ELS, for non-linear dynamic analysis. The estimated periods of vibration were compared with empirical equations, including current code equations. As expected, the results show that building periods estimated based on simple equations provided by earthquake design codes in Europe (EC8) and America (UBC97 and ASCE 7-10) are significantly smaller than the periods computed using nonlinear dynamic analysis. Based on the results obtained from the analyzed models, equations for calculating period of vibration are proposed. These proposed equations will allow design engineers to quickly and accurately estimate the fundamental period of moment resisting frames with taking different length to width ratios, vertical element size, floors area and building height into account. The interaction between reduction factor and the reduced period of vibration is studied, and it is found that values of maximum period of vibration can be used as an alternative method to calculate the inelastic base shear value without taking reduction factors in consideration.

*Keywords:* Fundamental Period of Vibration; Moment Resisting Frames; Stiffness and Mass of Building.

### 1. Introduction

Determination of fundamental period is essential to earthquake design and assessment. The accurate estimation for this property will improve the estimation of global seismic demands. Since this property is dependent on mass and stiffness, it is affected by many factors such as building material, structural regularity, the number of stories and bays and the section properties including dimensions and extent of cracking. Cracking of RC members decreases its stiffness significantly and so this reduction should be considered in analysis to determine the expected period of vibration.

As this property cannot be analytically computed for a structure before design process, building codes provide empirical formulas that is depend on material, steel or concrete, and height of structure or number of stories. It is also allowed using finite element with assumed mass and stiffness to determine this property during the preliminary design stage and limit the estimated period with an upper bound factor.

\* Corresponding author: [ahmednader727@gmail.com](mailto:ahmednader727@gmail.com)

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The main objectives of this study are: carrying out a parametric study on moment resisting frames (MRF) in terms of height of structure, the ratios of length to width of plan dimensions ( $L/W$ ), various sections of vertical elements and plan area, comparing the resulted periods with current codes formulae, deriving a new equation to estimate the fundamental period of moment resisting frames by using non-linear regression analysis.

## 2. Period Height Relationships in Seismic Design Codes

ATC3-06 presented the first simplified equation that relates the period to the height of the building and, for many years, this equation has been used to design reinforced concrete structures with this form [1]:

$$T = C_t \times H^{0.75} \quad (1)$$

Where,  $T$ : fundamental period of vibration;  $C_t$ : regression coefficient (0.025 for RC moment resisting frames);  $H$ : the height of the building.

SEAOC-88 [2] re-evaluated the value of regression coefficient and it was found that the value of  $C_t=0.03$  was more valid than the value given by ATC3-06 [1]. Generally, the coefficient  $C_t$  was calibrated such that the derived fundamental period would underestimate the period by approximately 10-20% at the first yield of building to obtain conservative estimate for the base shear.

Bertero et al. (1988) studied the time histories of building responses for 18 RC Moment Resisting Frames. Two substantial increases were found in the results. The cause of the first increase was identified to be the onset of structural and non-structural damage. The reason for the second increase in period was non-structural components are no longer contributing significantly to the stiffness of the structure, causing the building to vibrate as a bare structural frame. They concluded that the regression coefficient value 0.04 is more reliable than the coefficient value proposed by SEAOC-88. For a lower bound estimate of the period, Bertero et al. (1988) recommended the use of  $C_t = 0.035$  [2, 3].

Bertero et al. (1988) found that the contribution of non-structural elements in stiffness of buildings used in Bertero et al. study is trivial if it is compared with structural frame. It is also noted that non-structural elements do not have a considerable effect beyond the first 5 seconds of earthquake motion. As a result, the equation obtained from Bertero et al. study for design of reinforced concrete moment resisting frames appears to be justified [3, 4].

The use of period-height form with SEAOC-88 [2] recommended 0.03 ( $C_t=0.075$  m) coefficient has been included in many design codes, for example, UBC-97 [5] and EC-8 [6]. In UBC-97 [5], It is permitted to calculate the fundamental period by using rational analysis but the calculated period shouldn't exceed 1.3 T for zone 4 and 1.4T for zone 1,2 and 3; where "T" is the approximated fundamental period determined by Equation 1.

Goel and Chopra (1997) gathered data from 8 Californian earthquakes, starting from the 1971's San Fernando earthquake and ending with the 1994's North Bridge earthquake. These data include the measured period in the longitudinal and transverse directions. They showed that codes formulas underestimate fundamental period of moment resisting frames, especially structures more than 60 stories. According to these results, they proposed an alternative period-height formula, Equation 2, with limiting the estimated period by upper bound. ASCE 7-10 prescribed the formula which proposed by Goel and Chopra (1977) in order to estimate the approximated fundamental period of moment resisting frames [7, 8].

$$T = C_t \times H^X \quad (2)$$

In which  $C_t$ : regression coefficient (0.075 for RC MRFs);  $X= 0.9$  for RC MRFs;  $H$ : height of building.

It is allowed to calculate the fundamental period by using rational analysis but the calculated period should not exceed  $C_a \cdot T$ ; where "T" is the approximated fundamental period determined by Equation 2 and " $C_a$ " is the magnification factor shown in Table 1

**Table 1. Coefficient for upper limit on calculated period**

Design spectral Response Acceleration (SD1)	Coefficient $C_a$
$\geq 0.4$	1.4
0.3	1.4
0.2	1.5
0.15	1.6
0.1	1.7
$\leq 0.05$	1.7

### 3. Measured and Analytical Fundamental Periods

The instrumentation of buildings provides important information. It increases the knowledge about the actual behaviour of building during earthquakes. The data can be used to verify formulas analytically. A lot of researches have been carried out to determine the vibration properties of buildings from their recorded motions during earthquakes and to evaluate the current analytical modelling with the performance of instrumented buildings.

Hart et al. evaluated the response of reinforced concrete structure with 10 stories located in California during Whittier earthquake. The main lateral resisting system was moment resisting frames. The recorded fundamental period from the strong motion was 1.1 sec. A structural analysis was carried by using ETABS 2015 [9, 10]. The analysis was carried out for three different degrees of cracking. Un-cracked gross section, cracked beams only and fully cracked (beams and columns). Hart et al. (1989) found that the case with fully cracked best correlated with the recorded period during earthquake strong motion with measured period 1.17 sec [9].

Anderson and Bertero (1977) studied the seismic performance of the San Bruno office building. The lateral resisting force system was three ductile moment resisting frames in the longitudinal direction and two ductile moment resisting frames in the transverse direction. The fundamental periods from the Loma Prieta earthquake were 1.05 and 0.85 sec. for the longitudinal and transverse directions, respectively. The identified initial periods due to low amplitude accelerations were 0.71 and 0.57 sec. for the same directions. It was observed that these initial periods correlated well with ambient vibration test results of 0.58 and 0.71 sec; respectively. Undertaking a three dimensional analytical modelling for the building using ETABS 2015 [10] software, Anderson and Bertero (1977) found that the results were in good agreement with the ambient vibration tests when cracking was not considered in modelling, and it was close to the values obtained from strong motion records, Loma Prieta earthquake records, when the rigidity of the columns was decreased to 50%. The estimated fundamental periods by finite element software were 1.05 and 0.85 sec; respectively [10, 11].

Mirtaheri and Salehi (2018) have been attempt to make a comprehensive examination between the fundamental period obtained from ambient vibration tests and those obtained from code provisions and finite element analysis [12]. ISC-2014 [13] and ASCE 7-10 [8] are the codes utilized in their study. Their study was carried out on 17 reinforced concrete buildings. Their study was carried out on 17 reinforced concrete buildings, lateral system of buildings is varied between moment resisting concrete buildings and dual system. Based on the results provided by the investigations general results were achieved: first result, the fundamental period obtained from the ISC-2014 [13] have, on average, a difference of about 18%–38% with those obtained from the tests. This amount of difference may be due to the fact that the empirical relationships which have been suggested in the ISC-2014 [13] to calculate the fundamental period are not sufficiently accurate and need to be reviewed. Second result, the average difference between the fundamental period obtained from the ASCE 7-10 [8] and those obtained from the tests ranges from 16% to 54% for the various lateral resisting systems. Same as before, this can indicate that the empirical relationships of the ASCE 7-10 [8] suggested to calculate the fundamental period are also not accurate enough.

### 4. Comparison between Numerical Analyses Results and Code Formulas

Al-Balhawi and Zhang (2017) studied MRF systems designed under gravity and wind loadings. The parameters considered include the number of storeys, the number and length of bays, plan configurations, mechanical properties of infill walls, and the presence of openings in the un-cracked and cracked infill wall. They found that periods of vibration of bare frames reasonably agrees with codes formulas by disregarding contributions of infills' stiffness towards the structural systems. On the other hand, the proposed formulas for RC MRF buildings with un-cracked infills agree well with most cited experimentally based formulas and some numerically based ones [14].

Young and Adeli (2016) investigated fundamental periods of eccentrically braced frame (EBF) structures with varying geometric irregularities. A total of 12 EBFs are designed and analyzed. Based on the results obtained from vibration theory. They found that a 3-variable power model which is able to account for irregularities resulted in a better fit to the Rayleigh data than equations which were dependent on height only. They also compared the resulted periods of vibration with available measured period data and found that their analytical results are closer to the measured periods than current codes equations [15].

Asteris et al. (2016) investigated the results of a large-scale analytical study on the parameters that affect the fundamental period of reinforced concrete structure. They studied parameters were number of storeys, the number of spans, the span length, the infill wall panel stiffness and the percentage of openings within the infill panel. The results were shown to fit better the data than codes equation, having a high correlation factor and a low mean square error and can adequately estimate the fundamental period of masonry-infilled RC buildings [16].

Asteris et al. (2016) used artificial neural networks (ANNs) to predict the fundamental period of infilled reinforced concrete (RC) structures. For the training and the validation of the ANN, a large data set is used based on a detailed investigation of the parameters that affect the fundamental period of RC structures. To comparison of the predicted

values with analytical ones indicates the potential of using ANNs for the prediction of the fundamental period of infilled RC frame structures taking into account the crucial parameters that influence its value [17].

### 5. The Applied Element Method

The structure is modeled in AEM by creating small elements that are connected together. The two elements are then connected through a group of normal and shear springs that represent the stress and strain around each element Tagel-Din and Meguro (2000) [18]. The main feature of this software is analyzing the model in all stages until it is totally collapsed. It is able also to track behavior of elements during separation, contact and collision. In AEM the connecting springs are representing the tool which is used to gather the elements together. Each spring is actually 3 springs; 1 spring for normal and 2 springs for shear deformation. To model the concrete in compression the Maekawa compression model as shown in Figure 1(a) is adopted. To model concrete in tension, its response is elastic until the formation of cracks occurred. After cracking, it is assumed that concrete subjected to tension loses strength through a softening mechanism and the opened cracks can be represented by a loss of elastic stiffness. To model concrete in shear, shear stress-strain relation remains linear until it reaches the cracking point. After cracking, shear stress descends down as shown in Figure 1(b). The level of drop in shear stresses depends on the interlock of aggregate particles on the two sides of crack and friction at the crack surface. For reinforcement springs, Figure 2 shows the model, presented by Menegotto and Pinto (1973) for cyclic loading of reinforcement steel bars used in AEM [19].

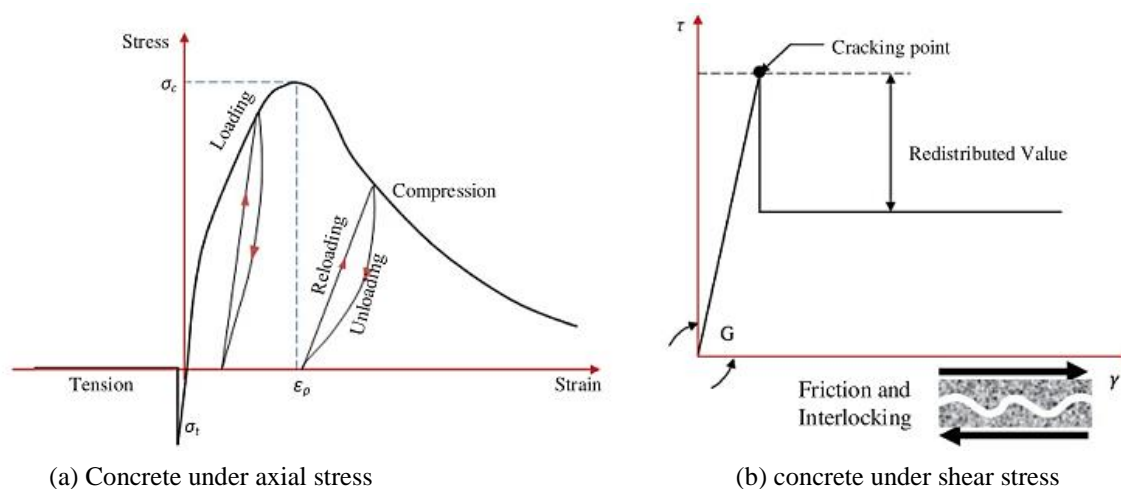


Figure 1. Stress strain curve due to relative displacement

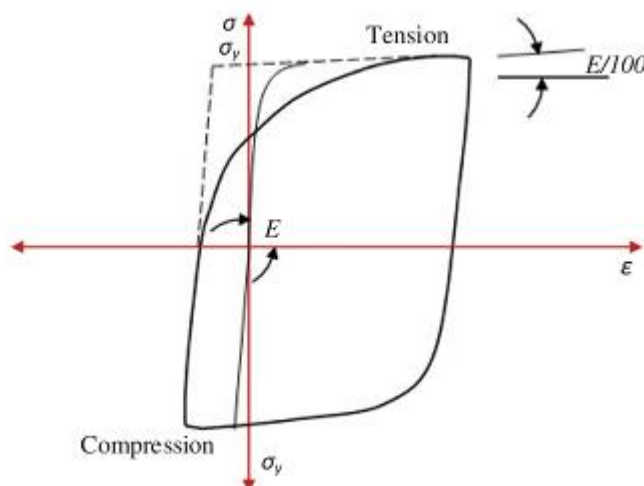


Figure 2. Stress strain curve for steel springs due to relative displacement

### 6. Case Study

#### 6.1. Description of the Studied Structure

Studied structure is a typical 10-storey building. The structure consists of four bays in both directions, the typical bay width is five meters in the both directions. The height of each storey is three meters. The columns are assumed to be fixed at their base. Figure 3 shows general layout of the studied building.

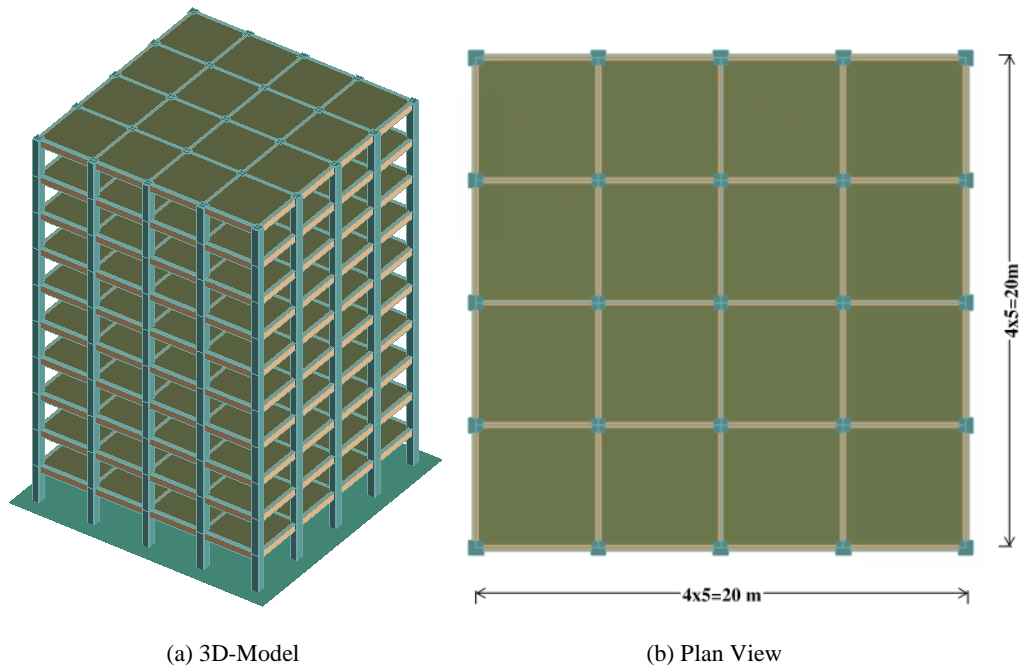


Figure 3. General Layout of the studied building

**6.2. Slab Dimension**

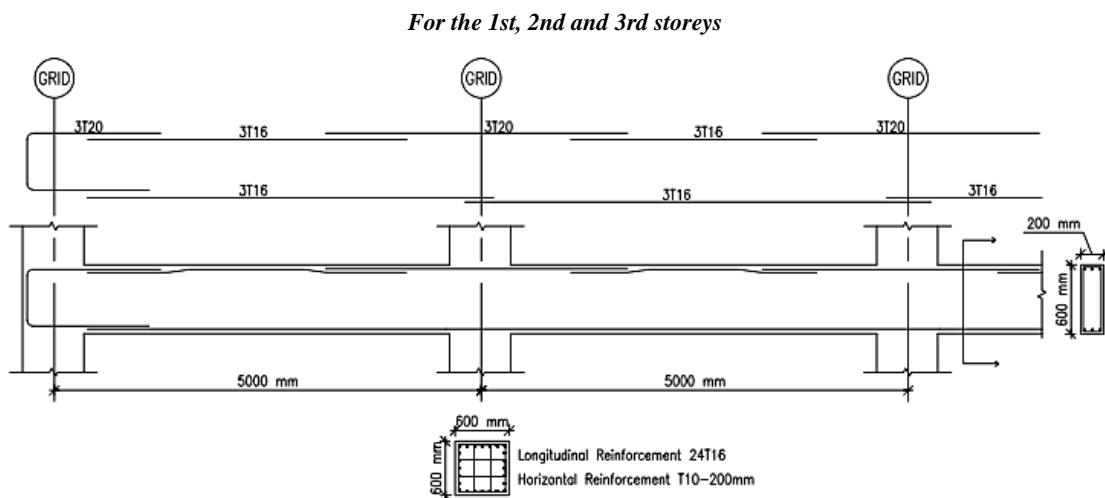
The studied cases were designed according to BS EN 1992-1 [20]. So, according to these guidelines the thickness of solid slab with span 5 m is 150 mm. Cube compressive strength concrete ( $F_{cu}$ ) is 30 MPa and yield strength of steel reinforcement ( $F_y$ ) is 360 MPa.

**6.3. Gravity and Seismic Loads**

Analysis software calculates own weight of members, by multiplying the specific weight by volume of element. The flooring cover, equivalent uniform wall load and uniform live load are assumed to be 2.5, 3.0 and 2.5  $\text{KN}/\text{m}^2$ , respectively. For seismic loads, seismic zone is assumed to be 5B with peak ground acceleration=0.3g. Soil type is assumed to be type C with soil factor=1.5 and the response modification factor equals to 6.0.

**6.4. Elements Reinforcement**

Figure 4 shows the reinforcement steel bars that are used according to design of the three sections, one at both ends and the third one at the middle, due to gravity loads and seismic loads straining actions:



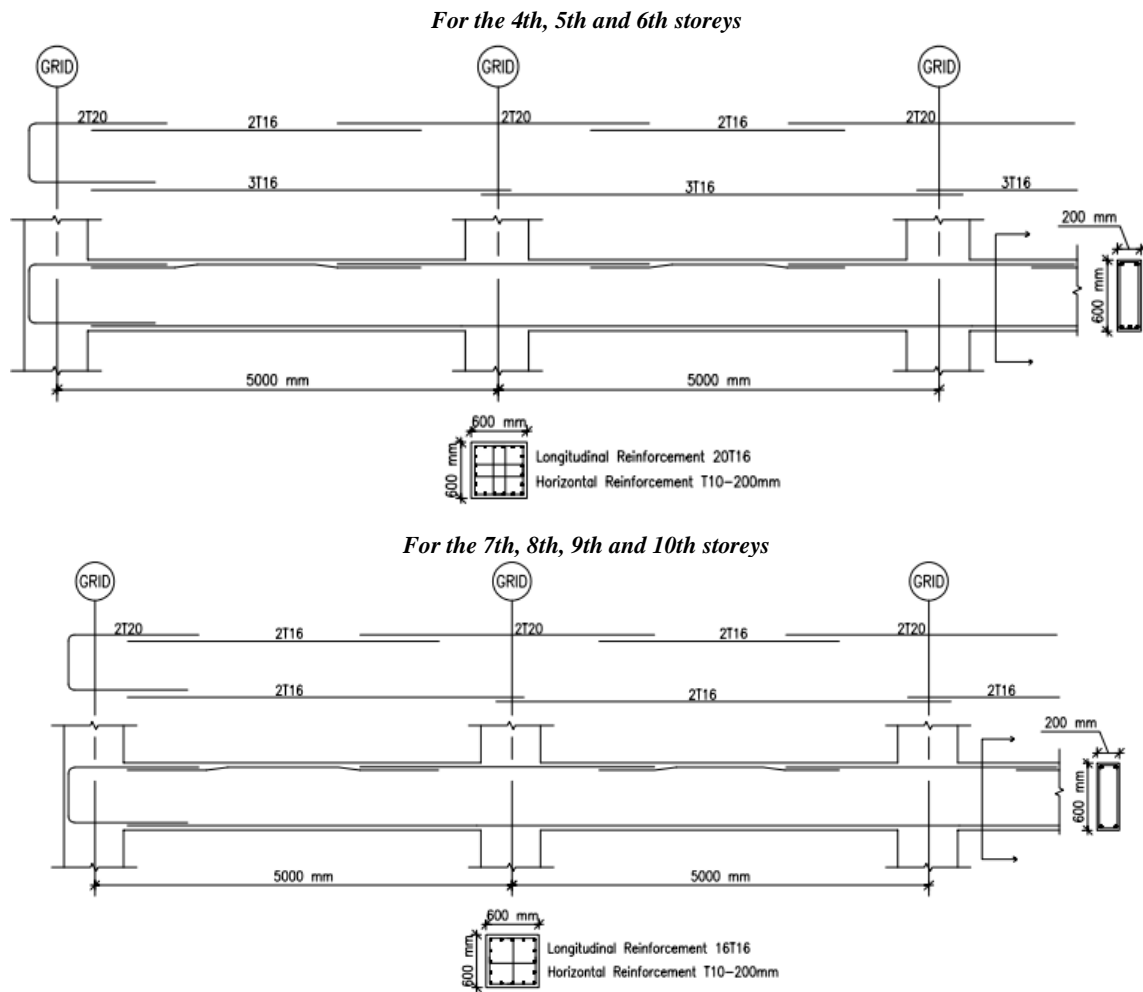


Figure 4. Reinforcement details of Beams and Columns (Base case)

### 7. Description of Case Studies

In this study, effect of building height, length to width ratio (L/W), columns size and area of structure on fundamental period was considered. The study is divided to four groups with four cases for each group, where each group represents the effect of these parameters on the fundamental period. For example, the 1<sup>st</sup> group includes the effect of building height. Each case has a designation beginning with MRF followed by 1, 2, 3 or 4 representing 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> group, respectively; Following is a numerical designation indicating the case number. For example, MRF-1-3 represents that this structure is the third case in the first group. Tables 2 shows the Varied and constant parameters of studied cases.

Table 2. Varied and constant parameters of studied cases

Building Designation	Height (m)	L/W Ratio	Columns size (mm)	Area (m <sup>2</sup> )
MRF-1-1	30	1	600×600	400
MRF-1-2	33	1	600×600	400
MRF-1-3	36	1	600×600	400
MRF-1-4	39	1	600×600	400
MRF-2-2	30	2	600×600	400
MRF-2-3	30	3	600×600	400
MRF-2-4	30	4	600×600	400
MRF-3-2	30	1	800×800	400
MRF-3-3	30	1	1000×1000	400
MRF-3-4	30	1	1200×1200	400
MRF-4-2	30	1	600×600	900
MRF-4-3	30	1	600×600	1600
MRF-4-4	30	1	600×600	2500

## 8. The Stiffness of RC Members in Linear Eigenvalue Analysis

The gross (un-cracked) stiffness is inappropriate in considering cracking of critical elements such as beams which generally cracked under gravity loading only. However, if cracking is not found to have occurred before the design seismic level of excitation (considered unlikely as this level of excitation would with all probability have preceded by a number of lower intensity events), it will occur early on in the response to excitation and thereafter the stiffness will reduce rapidly. Therefore, Codes provided stiffness reduction factors to reflect this reduction in member's stiffness. The two cases of member's stiffness, cracked and gross stiffness, have been applied to the case studies and eigenvalue analysis has been done to find the corresponding fundamental period at each case.

## 9. Time History Records

Each of the studied cases has undergone non-linear dynamic analysis by using Extreme Loading of Structures software (ELS, 2016 [21]). In this study, the design earthquake acceleration equals 0.3g. The ground acceleration is obtained by converting the EC-8 [6] response spectrum into time history function by using SIMQKE program, Gasparini and Vanmarcke (1976) and then this function is imported to (ELS, 2016) [11] to represent the ground motion. The ground acceleration for our case is shown in Figure 5 [22].

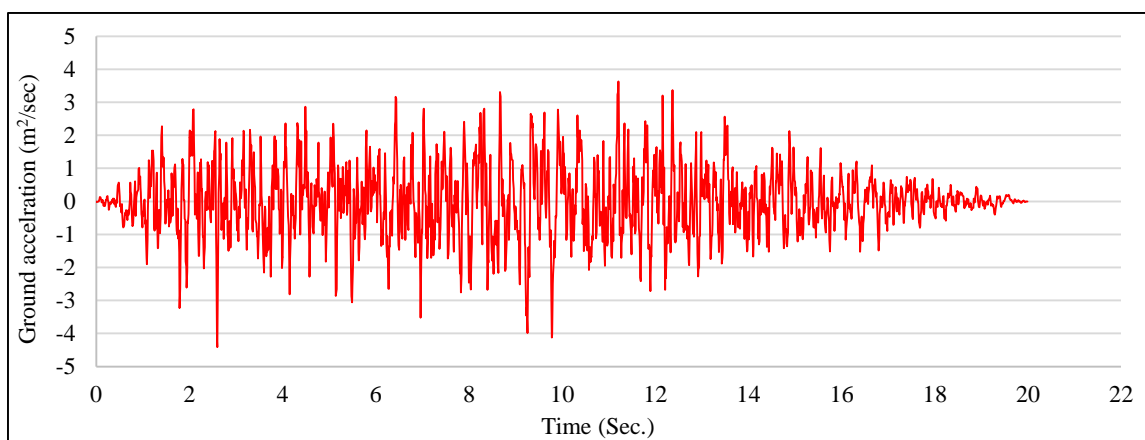


Figure 6. Ground Acceleration time history of earthquakes equals to (0.3g)

## 10. Determining Period of Vibration from Dynamic Non-Linear Analysis

Codes equations were calibrated according to the period at first yield of buildings which obtained from the available data of these buildings during their motions records. The same criteria were followed here in determining the fundamental period. The first time at which reinforcement springs reach yield stress is determined from Time-Springs stresses chart, shown in Figure 8; and so the period which corresponds to the first time at which reinforcement springs stresses is the yield stress in Time- Period chart, shown in Figure 9; is the period at first yield of building. According to Figure 8, the first time at which reinforcement springs stress reaches yield stress is 4.5 sec, which does actually correspond to the period 1.8 sec in Figure 9. Hence, fundamental period at first yield of this building is 1.8 sec.

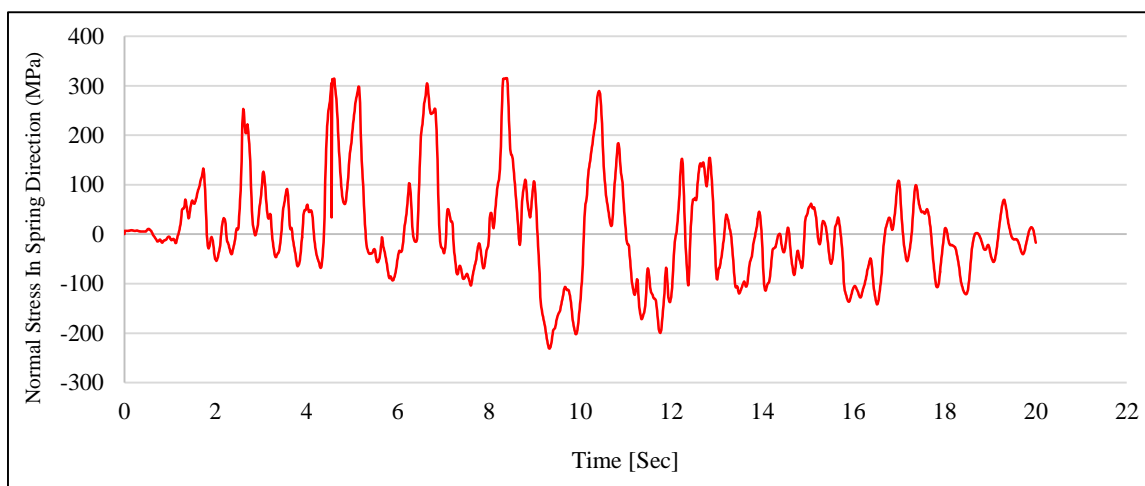


Figure 7. Time-Reinforcement Springs Stresses Chart in ELS

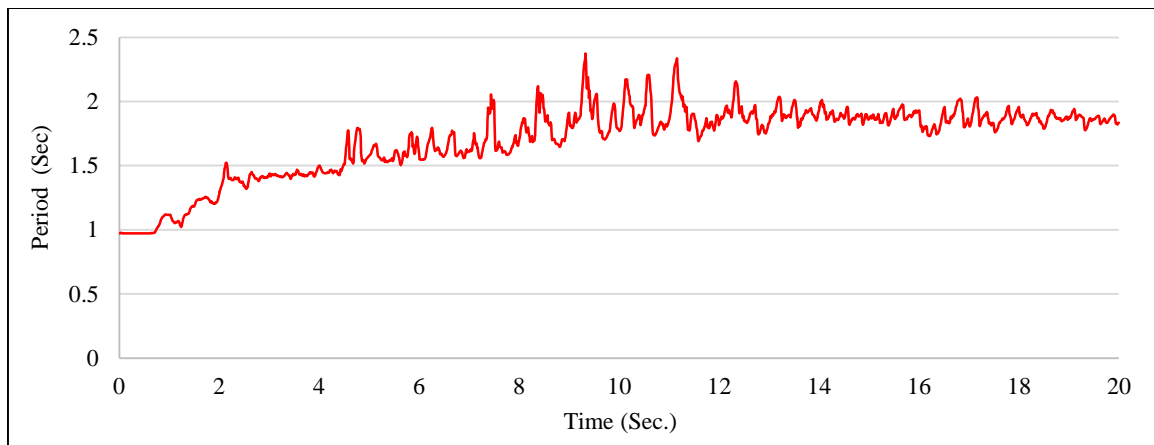


Figure 8. Time-Period Chart in ELS

### 11. Results

For the first group, height of building increased with keeping mass of building constant, therefore lateral stiffness of building decreased and so the fundamental period increased. The estimated fundamental period is much higher than period calculated by codes.

For second group, in this group length to width ratio increased from 1 to 4, as this ratio increased with constant mass the lateral stiffness of building increased and so fundamental period decreased. The estimated fundamental periods through eigenvalue analysis with cracked stiffness become much closer to period calculated by codes as L/W increased and periods at first yield of building become in the range between estimated periods by codes and their upper bound.

For the third group, size of columns increased with keeping mass of building constant, therefore lateral stiffness of building increased and so the fundamental period decreased. The estimated fundamental periods are higher than period calculated by codes but they are close to estimated fundamental periods through eigenvalue analysis with cracked stiffness.

For the fourth group, area of building increased with keeping lateral stiffness of building constant, therefore total mass of building increased and so fundamental period increased. The estimated fundamental period is much higher than period calculated by codes.

#### 11.1. First Group

The results of this group are shown in Figures 10 and 11. This group clarify the effect of building height on the fundamental period.

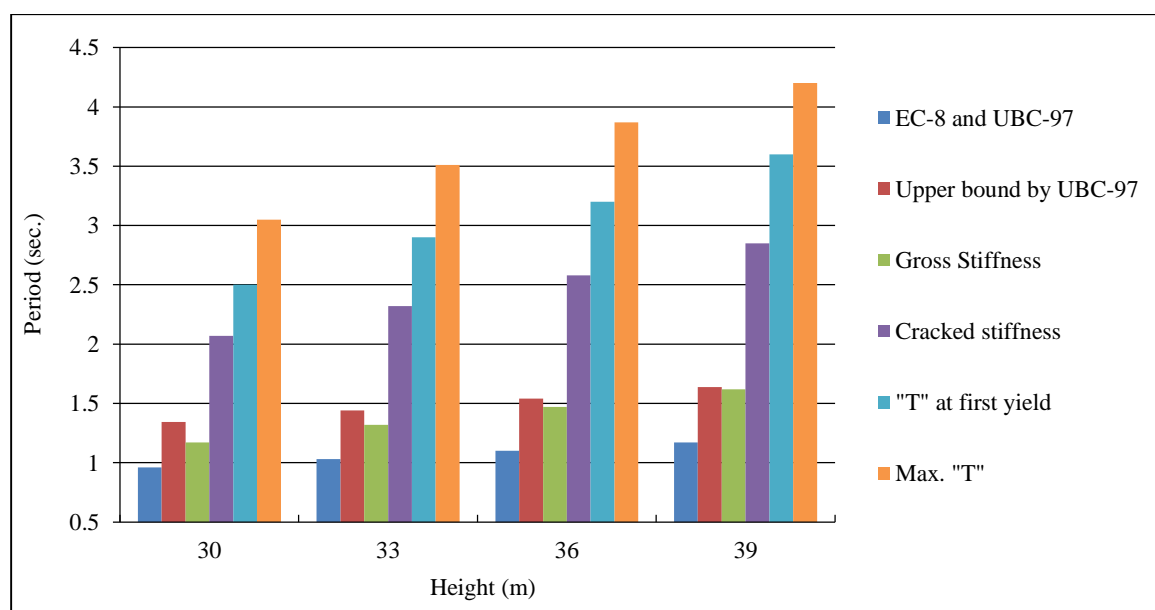


Figure 9. Comparison of Period-Height for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, EC-8 and UBC-97



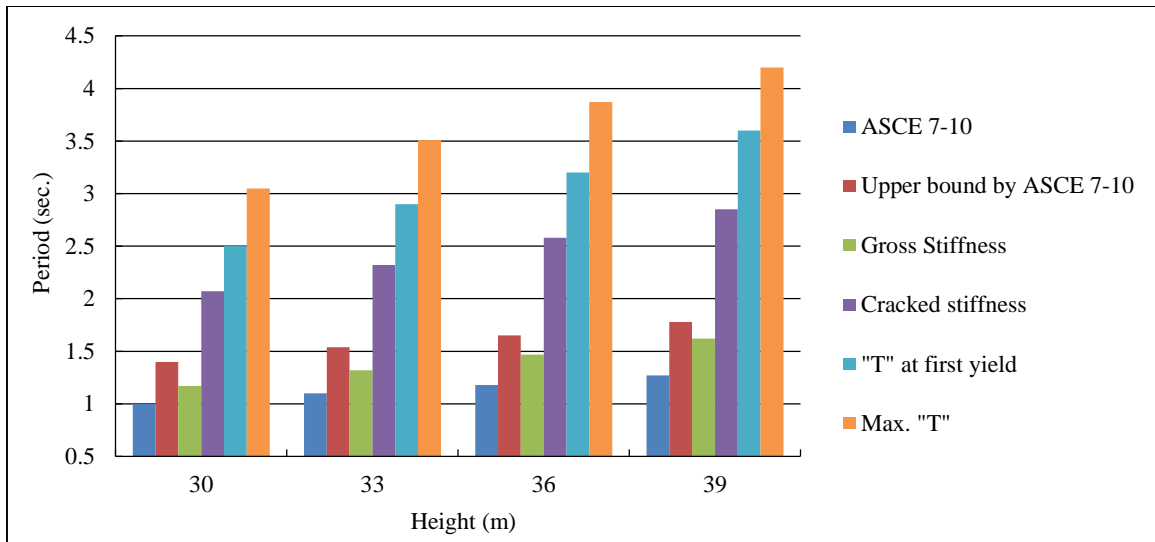


Figure 10. Comparison of Period-Height for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, ASCE 7-10

11.2. Second Group

The results of this group are shown in Figures 12 and 13. This group clarify the effect of length to width ratio of building on the fundamental period.

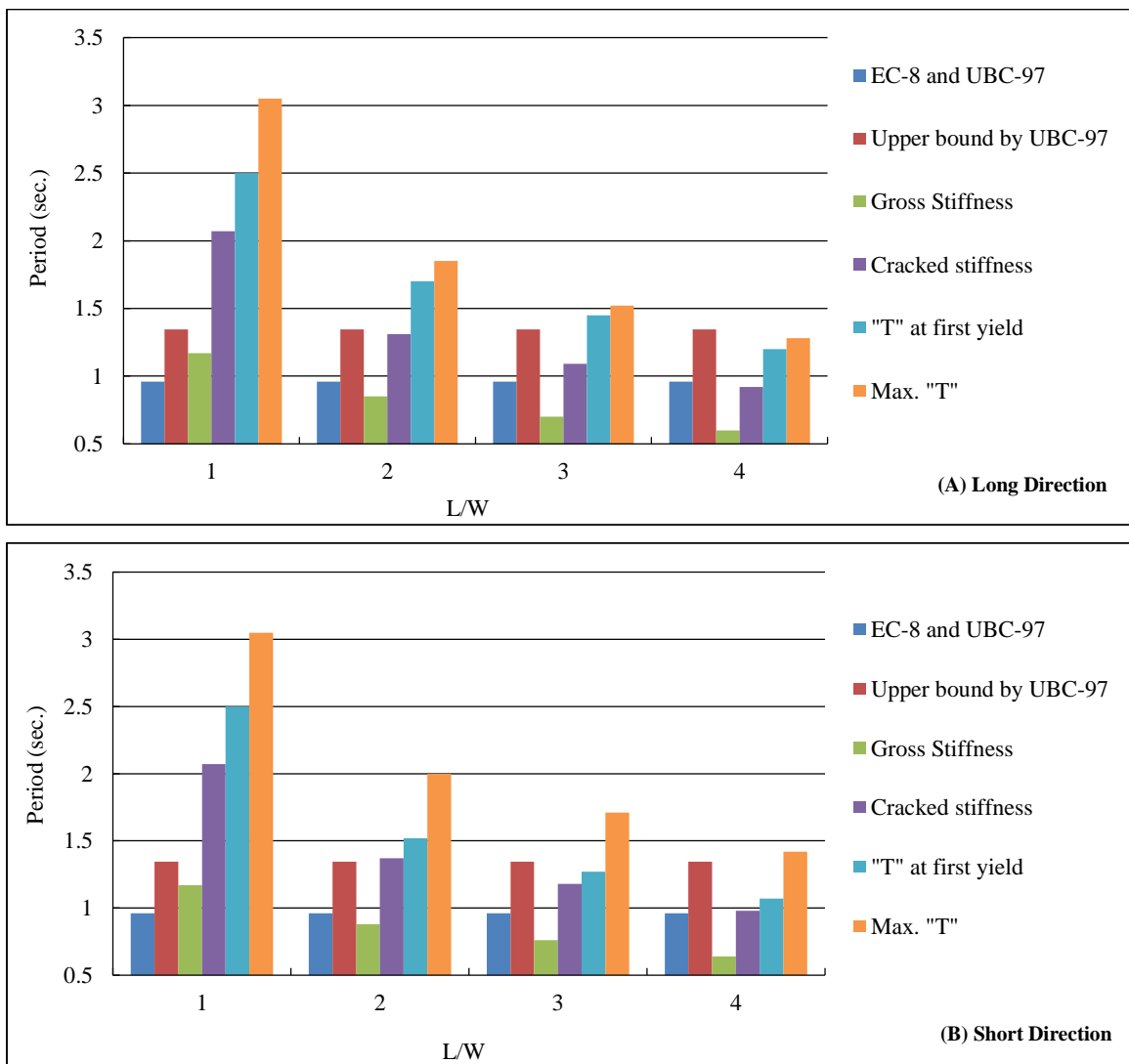


Figure 11. Comparison of Period-L/W in the two directions for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, EC-8 and UBC-97

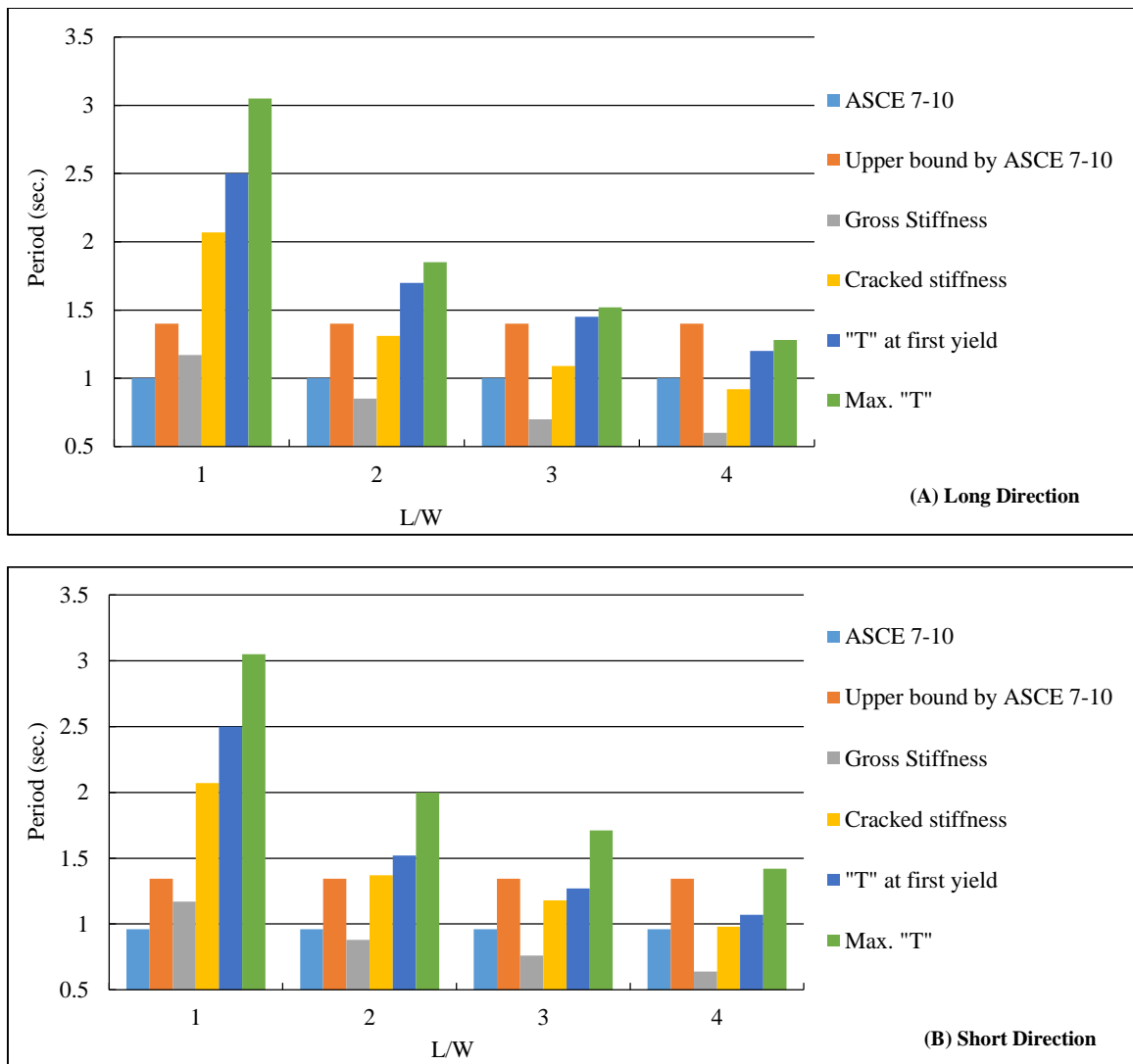


Figure 12. Comparison of Period- L/W in the two directions for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, ASCE 7-10

### 11.3. Third Group

The results of this group are shown in Figures 14 and 15. This group clarify the effect of columns size on the fundamental period.

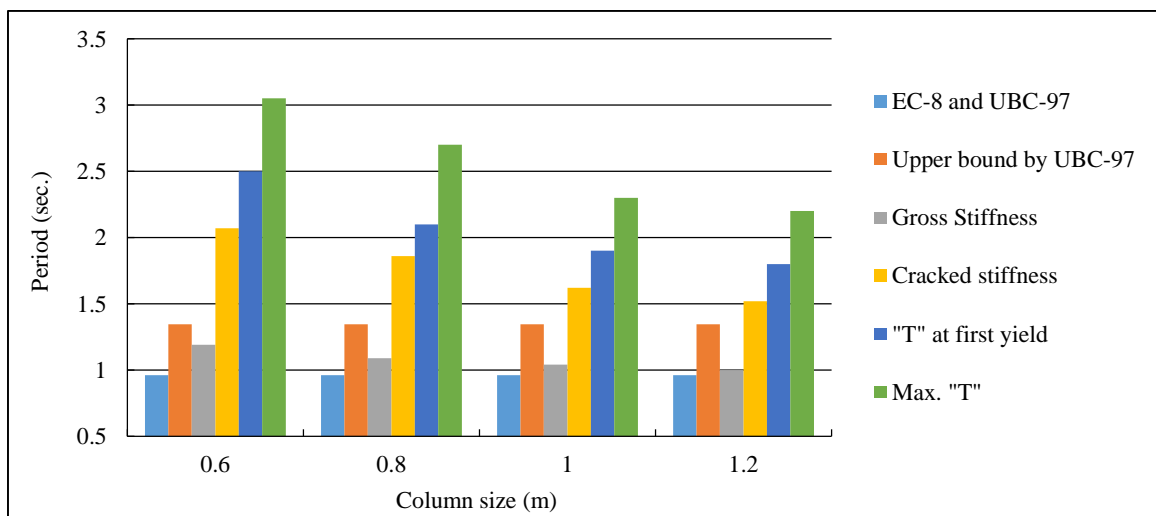


Figure 13. Comparison of Period-Column size for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, EC-8 and UBC-97

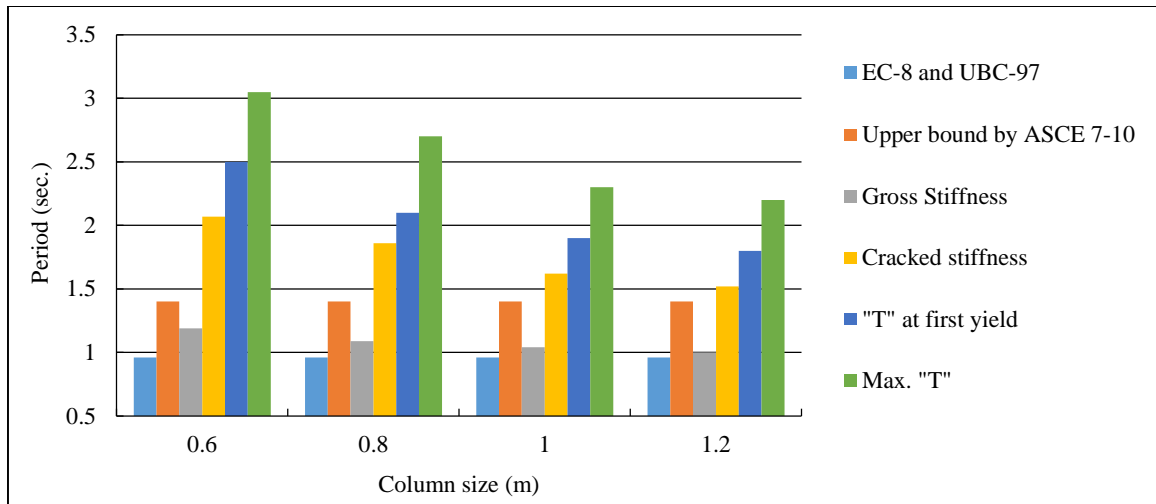


Figure 14. Comparison of Period- Column size for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, ASCE 7-10

11.4. Fourth Group

The results of this group are shown in Figures 16 and 17. This group clarify the effect of area of building on the fundamental period.

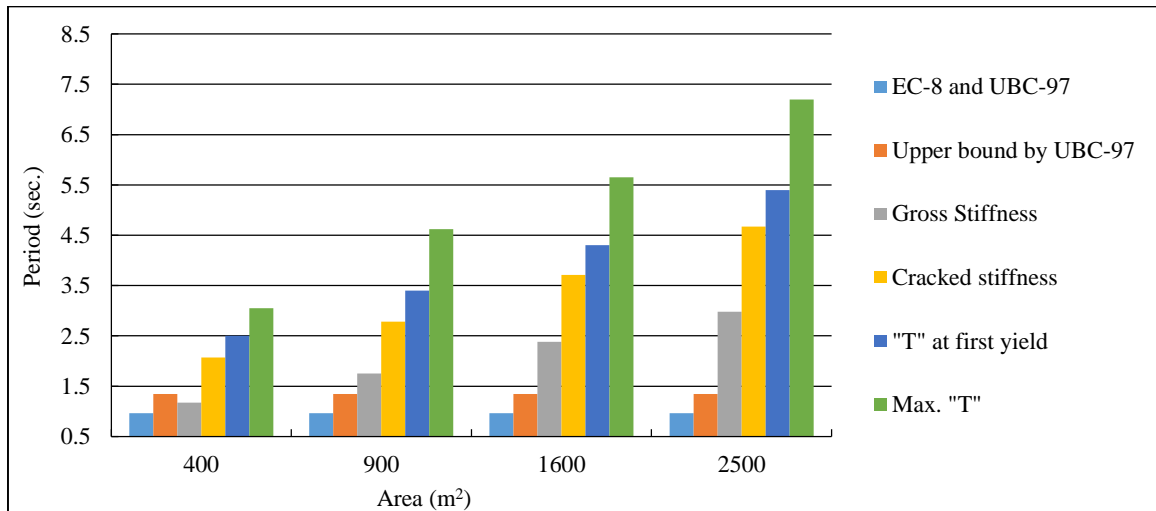


Figure 15. Comparison of Period-Area for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, EC-8 and UBC-97

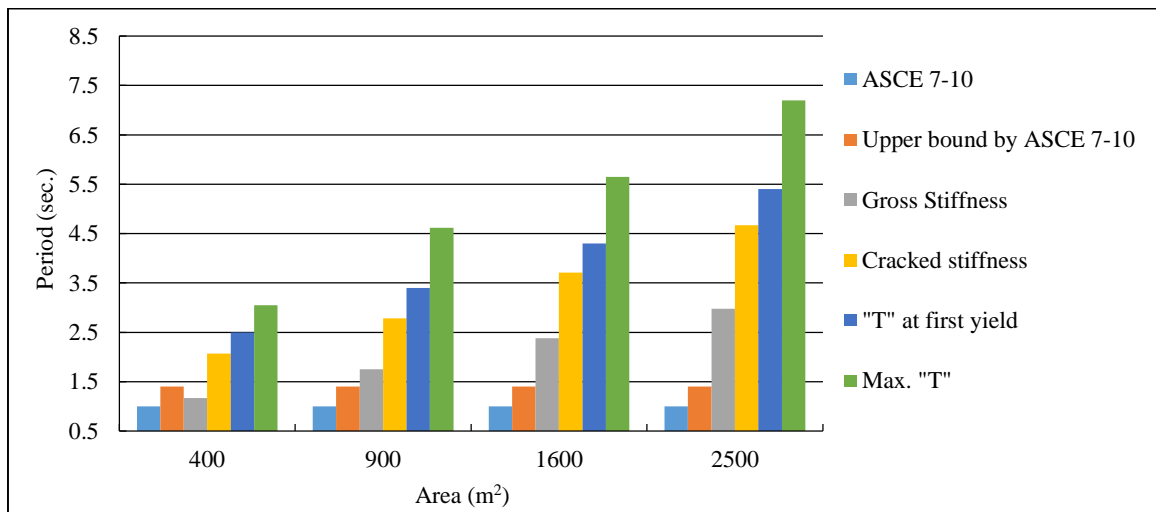


Figure 16. Comparison of Period-Area for Period at First Yield of building, Period at the Level of Significant Yield (Max. T), Cracked Eigenvalue Analysis, Gross Stiffness Eigen value Analysis, ASCE 7-10

### 12. Regression Analysis

Firstly, a separate equation is derived for each group as shown in Figures 18 to 20. Then, the derived equations and fundamental period of base case are used to obtain modification factors  $C_h, C_r, C_k$  and  $C_a$ . The following form give in Equation 3 is used in creating equation to estimate fundamental period of vibration based on the results of gross stiffness eigenvalue analysis, cracked stiffness eigenvalue analysis and non-linear dynamic analysis.

$$T = T_0 * C_h * C_r * C_k * C_a \tag{3}$$

Where;  $T_0$ : Fundamental Period of base Case;  $C_h$ : The result of dividing period-height relationship by  $T_0$ ;  $C_r$ : The result of dividing period-L/W ratio relationship by  $T_0$ ;  $C_k$ : The result of dividing period-column length relationship by  $T_0$ ;  $C_a$ : The result of dividing period-plan area relationship by  $T_0$ .

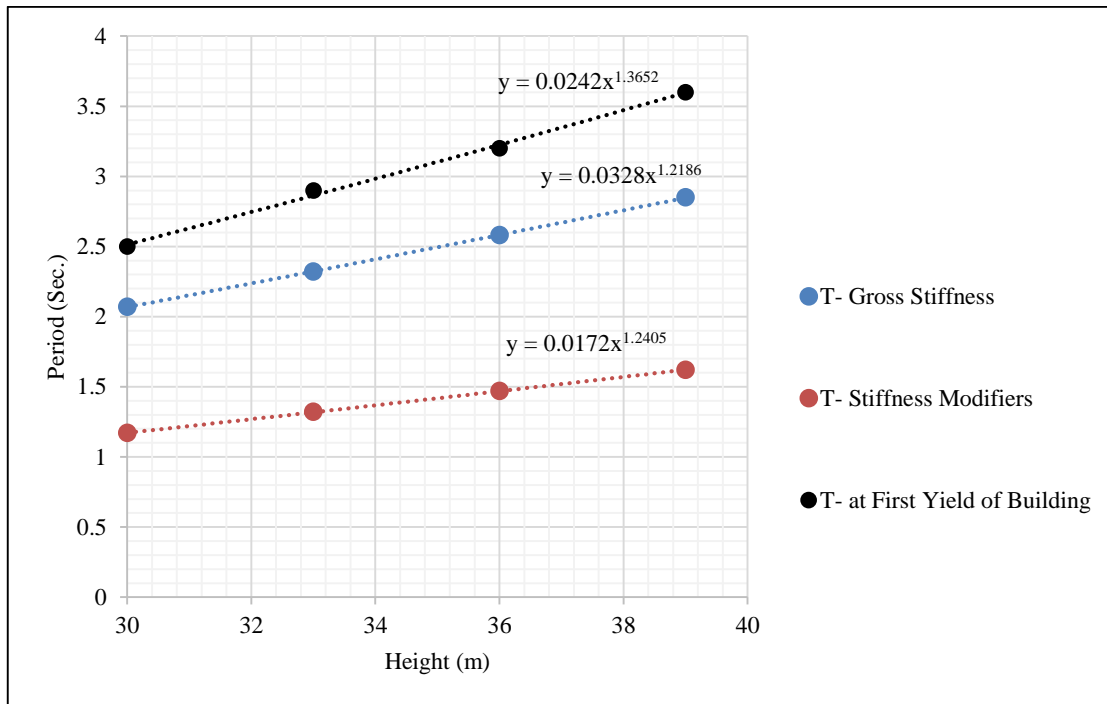
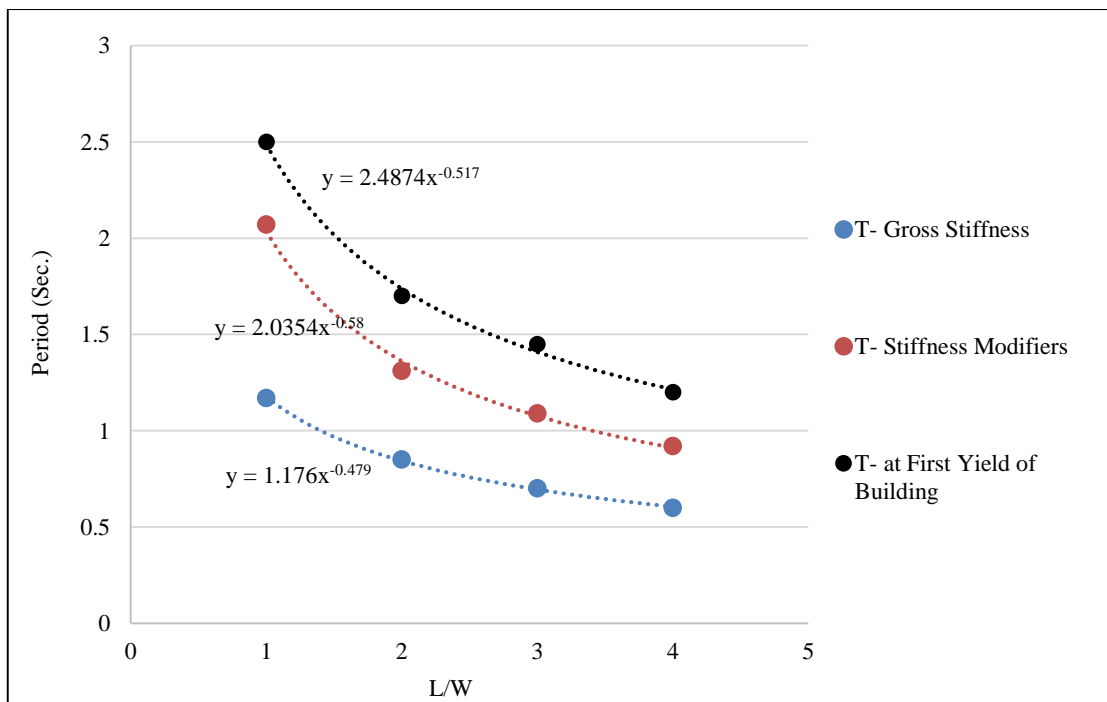
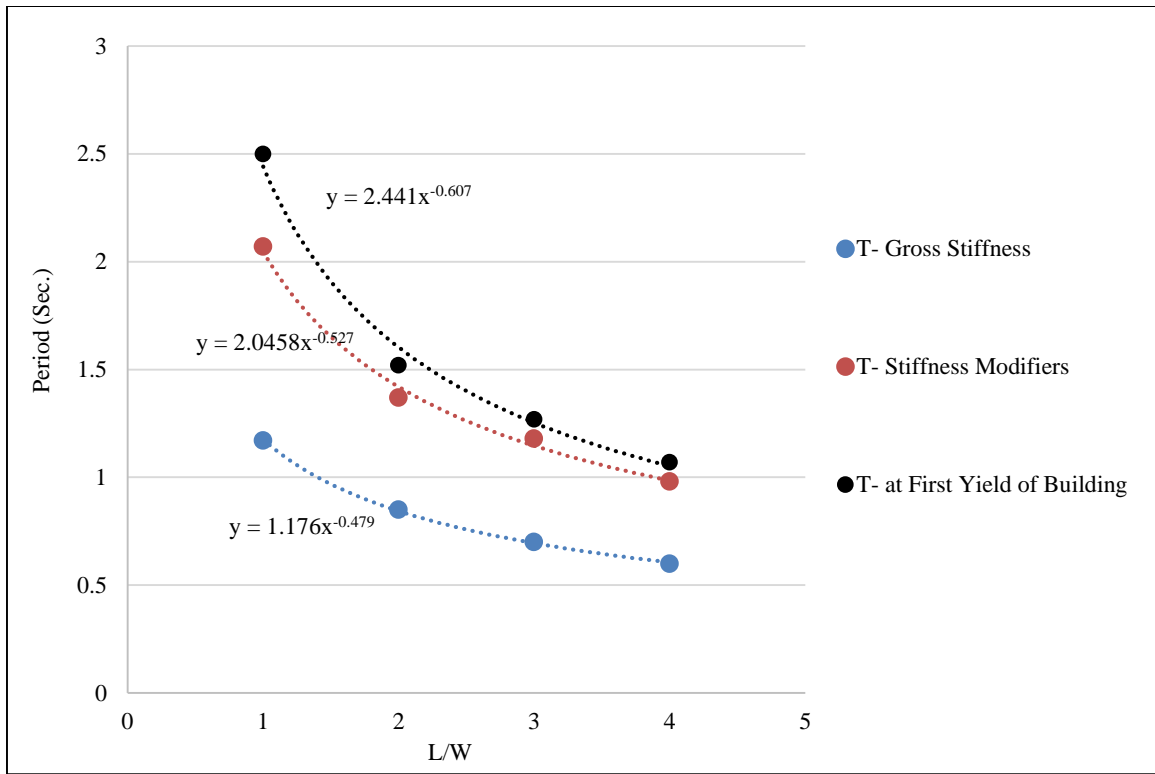


Figure 17. Period- Height Relationship Obtained from Gross Stiffness Eigenvalue Analysis, Cracked Stiffness Eigenvalue Analysis and Non-Linear Dynamic Analysis



(A) Long Direction



(B) Short Direction

Figure 18. Period- L/W Ratio Relationship in the two directions Obtained from Gross Stiffness Eigenvalue Analysis, Cracked Stiffness Eigenvalue Analysis and Non-Linear Dynamic Analysis

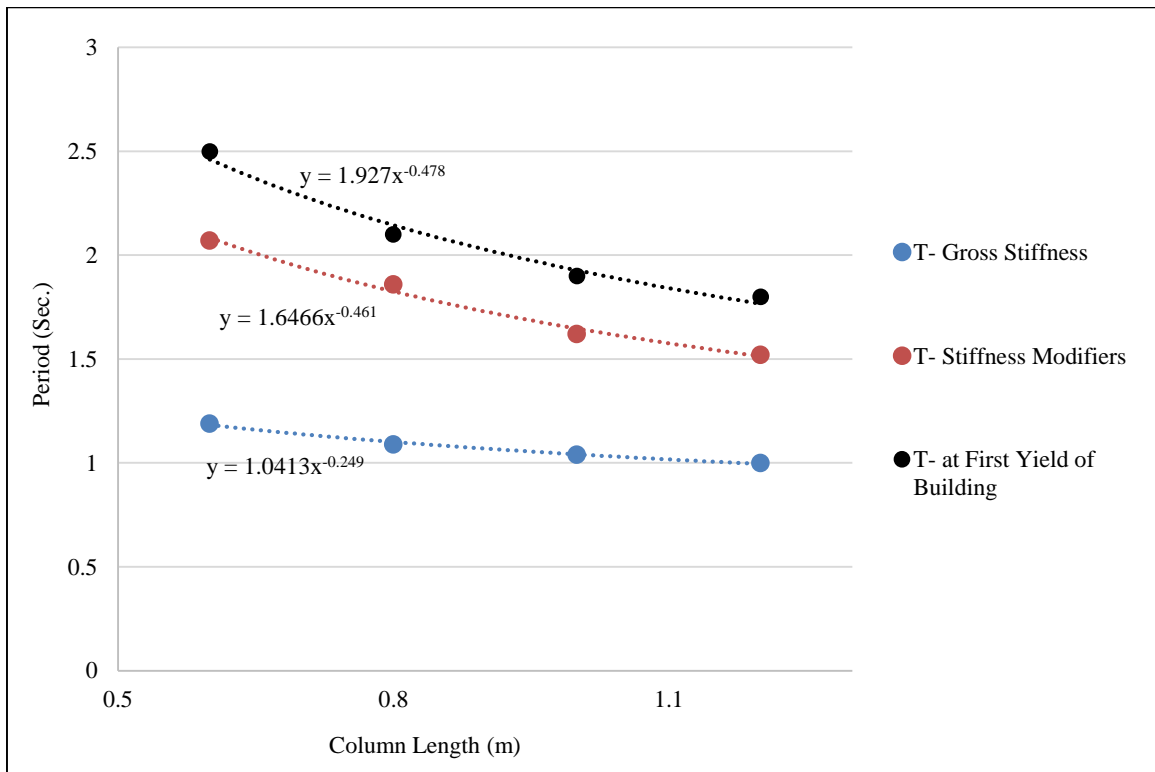
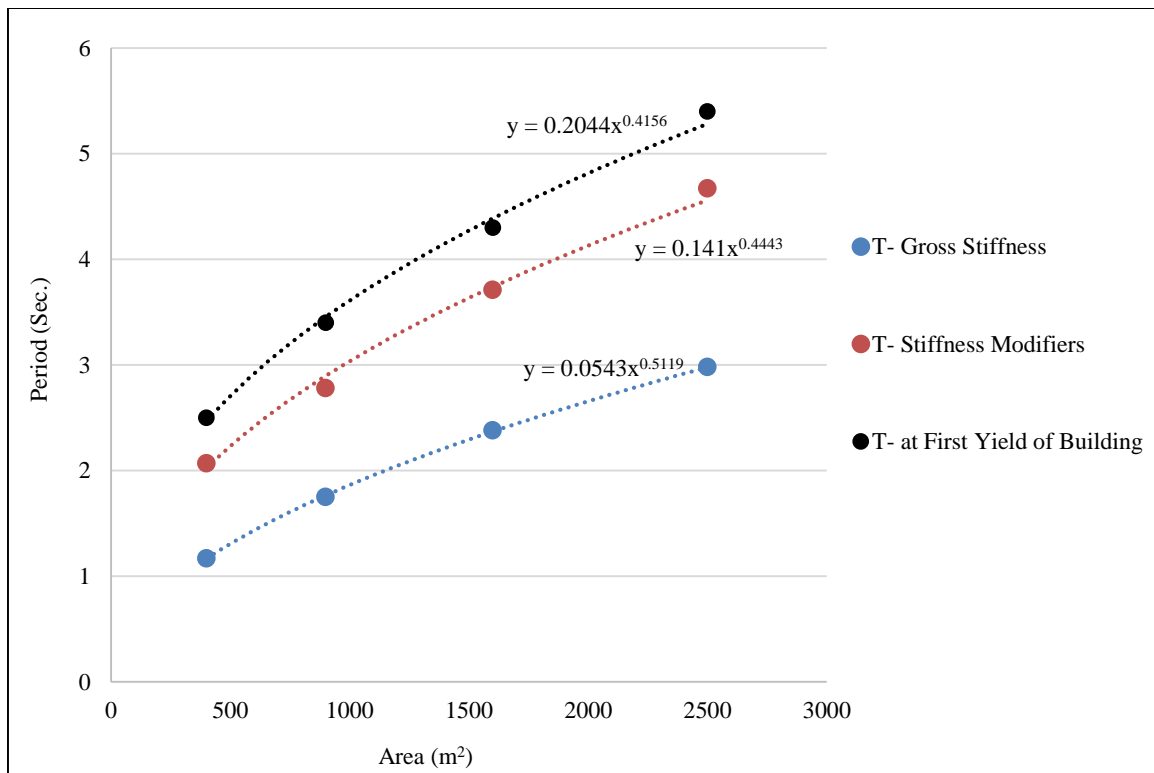


Figure 19. Period-Column Length Relationships Obtained from Gross Stiffness Eigenvalue Analysis, Cracked Stiffness Eigenvalue Analysis and Non-Linear Dynamic Analysis



**Figure 20. Period-Area Relationships Obtained from Gross Stiffness Eigenvalue Analysis, Cracked Stiffness Eigenvalue Analysis and Non-Linear Dynamic Analysis**

The derived equations 4 to 6 are given below to estimate the fundamental period of vibration for moment resisting frames bounded by heights from 30 to 39 meters, L/W ratios from 1 to 4, columns length from 600 to 1200mm ,plan areas from 20 × 20 to 50 × 50 m<sup>2</sup> and Peak ground acceleration=0.3a<sub>g</sub>. The three equations are derived from gross eigenvalue analysis, cracked eigenvalue analysis and non-linear dynamic analysis, respectively. Second group results in X and Y directions are nearly equal. Then, one direction was taken into account in deriving these equations. The upper limit of the suggested equation for deriving fundamental period is the multiplication of the three Equations 4, 5 and 6 by factor 1.4, as prescribed in seismic codes.

$$T = 0.00040 * H^{1.24} * R^{-0.48} * K^{-0.25} * A^{0.51} \tag{4}$$

$$T = 0.00077 * H^{1.22} * R^{-0.58} * K^{-0.46} * A^{0.44} \tag{5}$$

$$T = 0.0012 * H^{1.37} * R^{-0.66} * K^{-0.48} * A^{0.42} \tag{6}$$

### 13. Calculation of Base Shear Forces to Show the Interaction between Reduction Factor and the Reduced Period of Vibration

Design codes prescribed a base shear coefficient to be used in design. This base shear coefficient was given as a proportion of the weight which was to be resisted laterally by the structure. EC-8 [6] prescribed the below formulas to calculate ultimate base shear:

$$F_b = (S_d(T) \cdot W \cdot \lambda) / g \tag{7}$$

$$S_d(T) = a_g \cdot \gamma_1 \cdot S \cdot \frac{2.5}{R} \cdot \left[ \frac{T_c}{T} \right] \geq 0.2 \cdot a_g \cdot \gamma_1 \quad T_c \leq T \leq T_D \tag{8}$$

$$S_d(T) = a_g \cdot \gamma_1 \cdot S \cdot \frac{2.5}{R} \cdot \left[ \frac{T_c T_D}{T^2} \right] \geq 0.2 \cdot a_g \cdot \gamma_1 \quad T_D \leq T \leq 4 \text{ sec} \tag{9}$$

Where;  $F_b$ : the ultimate base shear-force;  $W$ : the design weight of structure;  $\lambda$ : correction factor, for the studied cases its value=1.0;  $S_d(T)$ : horizontal design spectrum;  $a_g$ : the designed ground motion;  $\gamma_1$ : the importance factor, in this study its value=1.0;  $R$ : the reduction factor which determined according to structural system;  $S$ ,  $T_c$  and  $T_D$ : Coefficients determined according to soil type, for the studied cases  $T_c = 0.25$ ,  $T_D = 1.2$  and  $S=1.5$ .

Base shear forces were calculated by using the above equations according to the period of vibration calculated from Equation 1 and according to the maximum estimated period by using non-linear dynamic analysis. Base shear values were calculated with considering reduction factor equals to one (R=1) and without considering the minimum value of base shear.

The purpose of calculating the base shear forces with considering the reduction factor equals to one ( $R=1$ ) is to determine the ability of the obtained maximum period of vibration to consider structures non-linearity. If the resulted base shear by using values of obtained period of vibration and according to code's response spectrum and the obtained base shear by ELS [21] are close, this means that the obtained maximum period of vibration can be directly used to estimate inelastic base shear values without taking effect of reduction factor. On the other hand, If the resulted base shear by using values of obtained period of vibration and according to code's response spectrum are much bigger or smaller than the obtained base shear by ELS [21], this means that effect of reduction factors is independent on the effect of the reduced period of vibration. Figure 21 shows the resulted base shear forces for all cases according to the calculated period by Equation 1 and according to the maximum estimated period by using non-linear dynamic analysis.

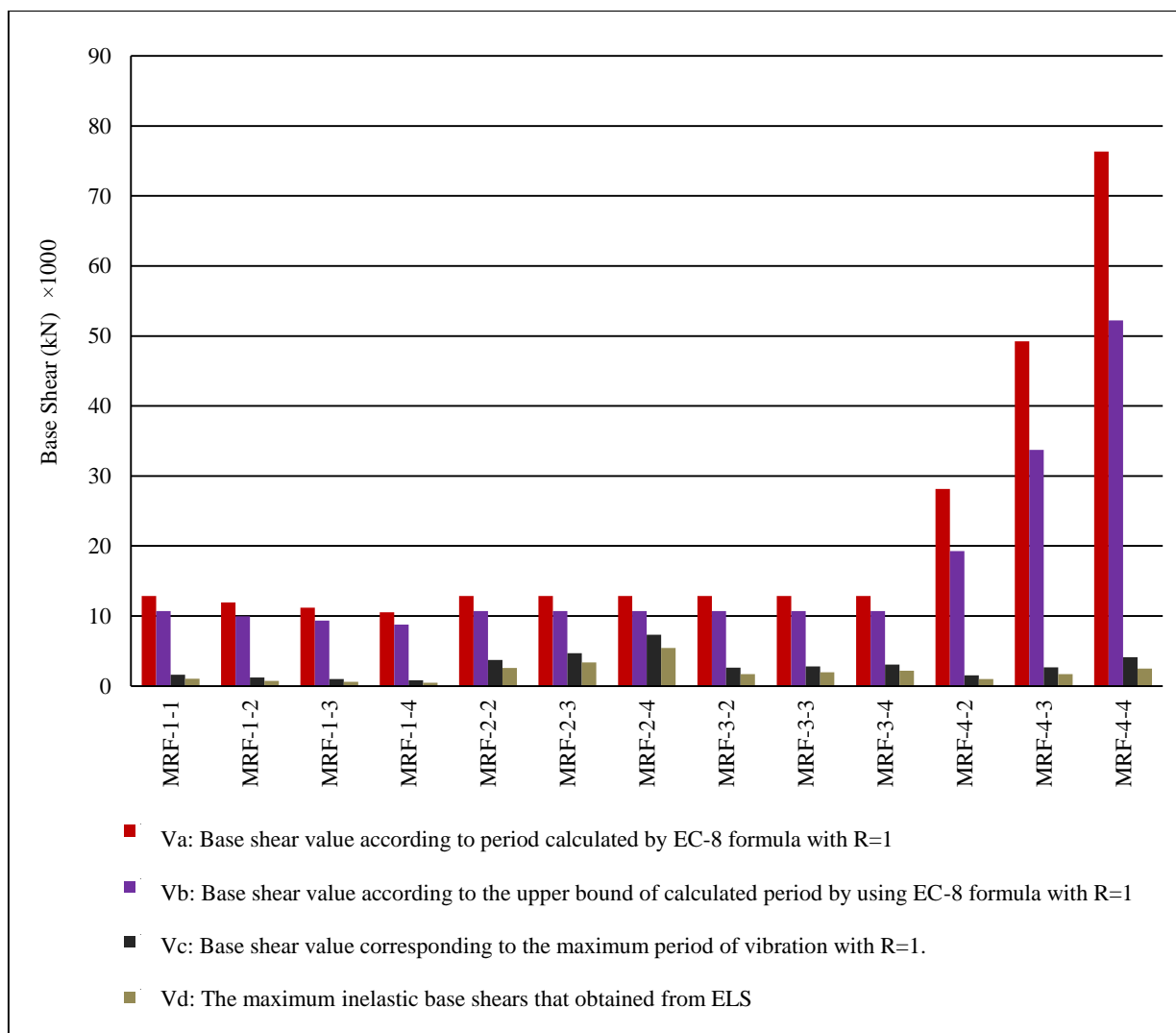


Figure 21. Comparison between the resulted base shear forces for MRFs according to the calculated period by EC-8 and according to the maximum estimated period by using non-linear dynamic analysis. Where;

### 14. Conclusions

Results of the non-linear dynamic analysis are considered to be more realistic as it takes the real effect of cracking into account in estimating stiffness of structures (all members will not be cracked or yielded at the same time). Then, with comparing the results of codes formula with results of nonlinear dynamic analysis the following points are concluded:

- Unless the current form of code formula takes the height of structure into consideration, it gives periods much shorter than the estimated periods. The estimated periods tend to be 3.076 times the code period.
- Area of structure should be considered in code formula for estimating fundamental period of moment resisting frames. The estimated period increased by 116% when the area of structure increased from 400 to 2500 square meters.
- Length to width ratio should be considered in code formula for estimating fundamental period. When the ratio increased from 1 to 4, the estimated period decreased by 62 %.

- Sizes of columns should be considered in code formula for estimating the fundamental period of MRFs. The estimated period decreased by 28% when the size of column increased from 600×600 to 1200×1200.
- As the values of base shear considering maximum period of vibration, at level of significant yield, with modification factor equals to one ( $R=1$ ) is close to the ELS values of base shear, the values of maximum period of vibration can be used as an alternative method to calculate the inelastic base shear value without taking reduction factors in consideration.

## 15. Conflicts of Interest

The authors declare no conflict of interest.

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