The MIMO Transmission Equation

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Introduction

Multiple-input, multiple-output (MIMO) antenna systems such as the one depicted in Fig. 1 offer capacity benefits over their single-input, single-output (SISO) counterparts [1], thus attracting considerable current research. MIMO performance depends on a wide range of parameters [1] including radiation efficiency, correlation [2], mutual coupling [3], matching efficiency and polarization misalignment [4]. No single simulation method has been described that includes each of these effects so critical to handset array designs. This work synthesizes a comprehensive model to incorporate each of these effects. In order to manage the complexity of such a model, the MIMO Transmission Equation is introduced—similar to the well-known Friis Transmission Equation.

Comprehensive capacity simulations

Capacity, the principle metric of MIMO systems, expresses the maximum rate at which information can be reliably transferred in a system and is a function of the channel matrix, H, in Fig. 1. Assuming a narrowband scenario, H expresses the relationship between the transmit voltage vector, $x = [x_1, \ldots, x_m]$, applied to m transmit antennas and the receive vector, $y = [y_1, \ldots, y_m]$, at n receive antennas:

$$y = Hx. \tag{1}$$

Without significant feedback from the receiver, the transmitter evenly divides power over m transmitters [1]. One is generally only interested in a statistical measure of this distribution, such as the average or ergodic capacity, C_E:

$$C_E = E\left\{ log_2 \left| I + \frac{P_T}{m\sigma^2} HH^H \right| \right\}, \tag{2}$$

where I is the identity matrix, the transmitted signal power is P_T, the noise variance is σ^2, H^H represents the Hermitian of the channel matrix, H, and the nonsubscripted $E\{\}$ represents the expectation operator.

The effect of antenna efficiency on the channel matrix can be included as follows. Given i^{th} receive- and j^{th} transmit-antenna embedded radiation efficiencies, $e_{cdr,i}$ and $e_{cdt,j}$, [5], one may represent the voltage relationships of (1) as:

$$x=\underbrace{\underbrace{\begin{bmatrix}\sqrt{e_{cdr,1}} & 0 & \dots & 0\\ 0 & \sqrt{e_{cdr,2}} & \cdots & 0\\ & \vdots & & \\ 0 & 0 & \cdots & \sqrt{e_{cdr,n}}\end{bmatrix}}_{E_{cdr}} H_L \underbrace{\begin{bmatrix}\sqrt{e_{cdt,1}} & 0 & \dots & 0\\ 0 & \sqrt{e_{cdt,2}} & \cdots & 0\\ & \vdots & & \\ 0 & 0 & \cdots & \sqrt{e_{cdt,m}}\end{bmatrix}}_{E_{cdt}}}_{H} y, \tag{3}$$

where H_L is a lossless channel matrix and H is expanded to include channel losses. E_{cdr} and E_{cdt} are matrices of the antenna radiation efficiencies. Essentially the same form of matrix multiplication can be used to include other losses in the channel, such as absorption by a user's body. Similarly, the average directivity and polarization alignment losses can also be computed on a per-element basis, yielding diagonal matrices D_R and P from Fig. 1.

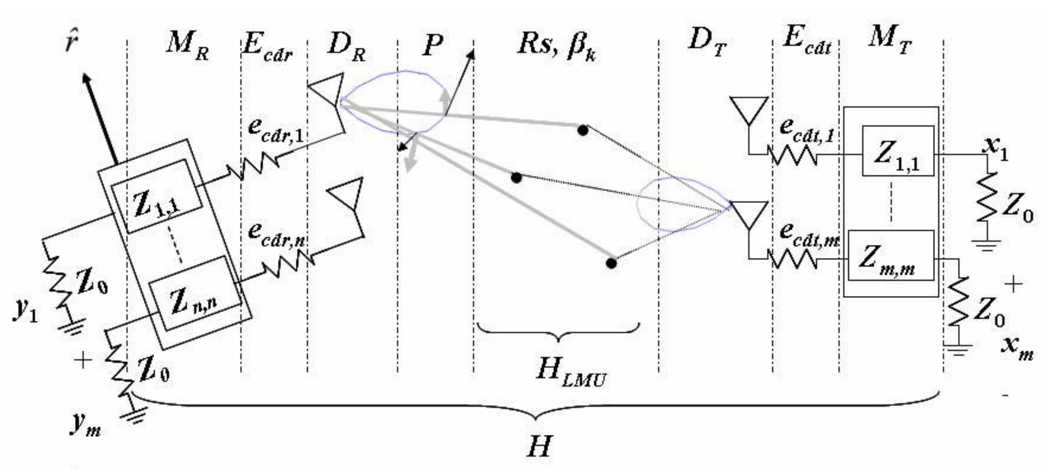

Fig. 1 A general MIMO system model. M_R is the n x n impedance matrix describing the receive antenna array with efficiencies E_{cdr}, $e_{cdr,i}$ described at (3). R_s is the spatial correlation of the signals impinging on the receiver—traditionally including the directivity and polarization effects expressed above as D_R and P. Corresponding matrices for the transmit array are subscripted with a T or t. $\hat{r}$ represents the orientation of the receiver. Grouping designator, H_{LMU}, represents a lossless, matched, uncoupled channel matrix and H represents a complete system-channel matrix.

To comprehensively model H, one may combine (18), (37) and (38) from [3] and add missing effects. We start with characteristic impedance, Z_0, scattering parameters of the unloaded transmit and receive arrays, S_{TT} and S_{RR}, and channel scattering matrix, S_{RT}. Following [3], we let S_{11} and S_{21} represent a selected matching and transmission circuit and use far-field patterns for the i^{th} receive antenna, $E_i^R(AOA)$, and a trans-impedance form [3] for the j^{th} transmit antenna, e_j^T (AOD), as a function of angle-of-arrival and -departure (AOA) and (AOD). The dependence of capacity on receive array orientation is included by making the dependence of the receive gain pattern on the orientation of the receiver, $\hat{r}$, explicit as $E_i^R(AOA,\hat{r})$ and accounting for polarization loss as the dot product between this quantity and the unit vector describing the polarization of the impinging signal, $\hat{p}_T$ [5]. The influence of the channel on channel-system capacity is expressed as a summation of N_p plane waves where the k^{th} plane wave has complex gain (path loss and phase shift) β_k, and angles of arrival and departure, AOA_k and AOD_k. Receive and transmit antenna efficiencies, E_{cdr} and E_{cdt}, are incorporated from (3). Thus, the channel matrix, H, can be expressed as:

$$H = \underbrace{L\,\underbrace{S_{21}(I - S_{RR}S_{11})^{-1}}_{matching}}_{M_R}\underbrace{E_{cdr}}_{\substack{rad\\ eff}}\underbrace{Z_0^{1/2}\left(I + \frac{Z_{RR}}{Z_0}\right)^{-1}\underbrace{\left[\frac{1}{Z_0}\sum_{k=1}^{N_p}\underbrace{E_i^R(AOA_k,\hat{r})}_{directivity}\cdot\underbrace{\hat{p}_T}_{\substack{pol.\\ loss}}\underbrace{\beta_k}_{\substack{path\\ loss}}\underbrace{e_j^T(AOD_k)}_{directivity}\right]}_{S_{RT}}(I - S_{TT})}_{H_{DP}}\underbrace{E_{cdt}}_{\substack{rad\\ eff}}M_T$$

$$= M_R E_{cdr} H_{DP} E_{cdt} M_T\,, \tag{4}$$

where the grouping designators M_R, M_T, E_{cdr}, E_{cdt} and H in (4) correspond to those in Fig. 1 and L is a loss term with $LL^H = I\text{-}S_{RR}S_{RR}^H$ to account for antenna effective area. The form of (4) begins to show the power of defining the interrelationship of channel and antenna effects in a simple linear form. Single-input, single-output (SISO) systems can be represented by the Friis power transmission equation [5]:

$$P_r = \underbrace{\left(1 - |\Gamma_r|^2\right)}_{\substack{match\\ efficiency}}\underbrace{e_{cdr}}_{\substack{radiation\\ efficiency}}\underbrace{D_r(AOA)}_{directivity}\underbrace{|\hat{p}_r \cdot \hat{p}_t|}_{PLF}\underbrace{\left(\frac{\lambda}{4\pi R}\right)^2}_{\substack{path\\ loss}}\underbrace{D_t(AOD)}_{directivity}\underbrace{e_{cdt}}_{\substack{rad\\ efficiency}}\underbrace{\left(1 - |\Gamma_t|^2\right)}_{\substack{match\\ efficiency}}P_t, \tag{5}$$

where receiver descriptors are power, P_r, antenna reflection coefficient, Γ_r, radiation efficiency, e_{cdr}, directivity, D_r(AOA) and unit polarization vector, $\hat{p}_r$, of the gain pattern in the direction of the angle of arrival, AOA. Corresponding terms are designated with a subscript "t" for transmit parameters. λ is the wavelength at the carrier frequency, and R is the separation of the two antennas in a line-of-sight (LOS) configuration. Additional loss terms such as loss in the human body, atmospheric attenuation, etc. can also be added to this equation. In a SISO system, $P_r = yy^H$ and (4) reduces to (5). Otherwise, the Friis equation does not accommodate multiple impinging signals of varying path length, path loss, angular spread and phase shift.

Decomposing the capacity budget

The singular value decomposition $HH^H = U\Lambda_H U^H$ helps indicate when (2) can be considerably simplified:

$$C_E \approx E\left\{log_2\left|\frac{P_T}{m\sigma^2}HH^H\right|\right\} \quad when\ \frac{P_T}{m\sigma^2}\lambda_i >> 1. \tag{6}$$

Thus, if the condition $\lambda_i P_T/(m\sigma^2) >> 1$ holds for all eigenvalues, $\lambda_1 \ldots \lambda_n$, of HH^H, one can ignore the contribution of I in (2). Then, recognizing the concavity of log

$|A|$ and applying Jensen's inequality, Loyka points out that the expected value of the capacity function given in (7.10) is bounded by the capacity function applied to the expected value of its argument [2]. That is if $E\{H_{LMU}H_{LMU}^{H}\} = R_{LMU}$, where R_{LMU} is the signal correlation matrix, then the corresponding term in (7.10) is bounded as $E\{\log_2|H_{LMU}H_{LMU}^{H}|\} \leq \log_2|R_{LMU}|$. The bound can be shown to be relatively tight over a wide range of antenna arrays [4]. This bound and the removal of I as previously discussed, allows (6) to be decomposed to:

$$C_E \approx \overbrace{n\,log_2\left(\frac{P_T}{\sigma^2}\right)}^{\substack{SNR,N\\ pathloss}} + \overbrace{log_2\prod_{i=1}^{n} e_{cdr,i}}^{efficiency} + \overbrace{log_2\left|M_R M_R^H\right|}^{matching} + \overbrace{log_2\prod_{i=1}^{n} PLF_i}^{polarization} + \overbrace{log_2\prod_{i=1}^{n} D_i}^{\substack{distributed\\ directivity}} + \overbrace{log_2\left|\frac{R_{LMU}}{m}\right|}^{\substack{channel\\ correlation}}, \quad (7)$$

where PLF_i and D_i represent the polarization loss factor and directivity of the i^{th} receive antenna averaged over the angle-of-arrival distribution function (compare to [4]). When (7) is valid—see (6), it offers advantages familiar to users of the Friis equation. It illustrates how matching, radiation efficiency, SNR, directivity, and polarization at the receiver independently contribute to a system "capacity budget." Each antenna's log-efficiency values are additive in the capacity budget.

Conclusions

Just as the Friis equation easily summarizes disparate contributions to a SISO power budget, the MIMO Transmission Equation decomposes the system capacity budget into its individual contributors. Its accuracy allows for incremental design iterations without excessive measurement campaigns. Indeed, its comprehensive nature allows for conclusions to be drawn about arrays with different element counts, types, orientations, radiation efficiencies and matching circuits on the basis of measurements involving very canonical, e.g. dipole arrays. When arrays differ in multiple parameters, the MIMO Transmission Equation offers both the correct metric and the correct weight to evaluate each variation.

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