THEORY OF NEGATIVE-MASS CYCLOTRON RESONANCE

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In recent communications[1,2] Dousmanis et al. suggest that the re-entrant energy contours of the heavy-hole bands in Ge and Si[3] could contribute a negative resistivity component to the overall resistivity of these materials, possibly related to a nonequilibrium density of carriers in these re-entrant states. Such a phenomenon could be observed under cyclotron resonance conditions (H_{dc} along [100] axis, electric field transverse to H_{dc}) as an emission spectrum centered about the cyclotron frequency of these states, or as a reduction in the strength of the background absorption at that frequency.

We have examined this idea in some detail, and find that except under very restrictive conditions, these states will in general add a normal, positive resistivity contribution to the material independently of their population. Therefore, indisputable experimental evidence of any emissive properties at the cyclotron frequency of the carriers in these "dimple" states will be shown to reveal more about the nature of conduction in the valence band of Ge and Si than merely the existence of these re-entrant energy contours, or deviations from thermal equilibrium, or a combination of these.

As an introduction to the analysis consider the expression for the integrated absorption coefficient A, calculated for a hypothetical system of carriers of constant, isotropic effective mass m^*. Under crossed dc magnetic and rf electric fields, the carriers will be stimulated by the latter to make transitions between Landau levels (denoted by quantum number n). We obtain

$$A = (\pi e^2/2m^*)\sum(n+1)[f(n,\mathfrak{k}) - f(n+1,\mathfrak{k})]$$
$$= (\pi e^2/2m^*)\sum f(n,\mathfrak{k}) = \pi e^2 N/2m^*. \tag{1}$$

The sum in Eq. (1) is over all quantum numbers, only n changing in a transition. The result we obtain is a sum rule independent of the magnetic field, namely, that the integrated absorption depends only upon the total density of carriers, N.

Were it possible to isolate carriers in a region of opposite band curvature where the energy is given by $E = E_0 - p^2/2m^*$, such that over the whole range where carriers exist the effective mass $-m^*$ is constant, isotropic, and <u>negative</u>, the integrated absorption coefficient would be

$$A = (\pi e^2/2|m^*|)\sum(n+1)[f(n+1,\mathfrak{k}) - f(n,\mathfrak{k})]$$
$$= -\pi e^2 N/2|m^*|. \tag{2}$$

This system of negative masses is emissive, just as the system of positive masses was absorptive. Now, it might be argued that the sign of the mass determines whether a system of particles is emissive or absorptive. We believe this point of view to be unsound. Our arguments, to be developed below, will indicate that in the case of distributions which can be characterized by a function of energy alone, emission will occur only if there is a maximum energy cutoff on the states of interest (such as E_0 in the case of negative masses discussed above) combined with an unbalanced "inverted" distribution. These arguments do not hold when the distribution is anisotropic, as is the case for the NEMAG (negative-effective mass amplifiers and generators) suggested by Krömer.[4] However, in experiments where holes are optically excited, thermal scattering of the holes along a surface of constant energy is presumably rapid enough that the population of states along such a contour may be taken to be constant. We idealize the experiments of Dousmanis <u>et al</u>. by means of the following assumptions:

1. Holes are excited up to the optical phonon energy E_{opt}. (Those created at higher energies will rapidly emit optical phonons and fall to this energy or below.)

2. A negative transverse mass $-m_T$ and positive longitudinal mass m_L will be assumed constant for carriers within narrow cones of half-angle α whose axes lie along the [100] axes of Ge and Si. Beyond this cone, the heavy holes under consideration have a positive effective mass m_P, whose magnitude is sufficiently different from m_T that the conductivity at frequencies near $\omega_T = eH/m_T c$ will be determined largely by the properties of the carriers within the cones. We shall consider the possibility that m_T, m_L, or α vary with energy.

3. Steady state is described by a distribution function $f(E)$ dependent only on energy. The microwave fields are a small perturbation, whose effects we shall describe by the Boltzmann equation. Thus, under cyclotron resonance conditions, with the magnetic field along the axis of one of the cones, we find a frequency-dependent conductivity given by[5]

$$\sigma(\omega) = \frac{-Ne^2\int_0^{\alpha} d\theta\ \sin^3\theta \int dp\, p^2 v^2 [df(E)/dE](\tau + i\omega\tau^2)/[(\omega_T\tau)^2 + (1+i\omega\tau)^2]}{2\int_0^{\alpha} d\theta\ \sin\theta \int dp\, p^2 f(E)}$$
$$= \frac{-Ne^2\alpha^2 \int dE\, E^{3/2}(df/dE)(\tau + i\omega\tau^2)/[(\omega_T\tau)^2 + (1+i\omega\tau)^2]}{2m_L \int dE\, E^{1/2} f(E)} \tag{3}$$

The second expression follows if $\alpha^2 \ll 1$. The total density of carriers within the cone is taken to be N, and the momentum relaxation time τ obeys $\omega_T\tau \gg 1$ for a well-defined resonance. A dispersion relation valid near the axis of the cone is

$$E = \tfrac{1}{2}p^2(\cos^2\theta/m_L - \sin^2\theta/m_T) \doteq \tfrac{1}{2}p^2/m_L$$
$$= \tfrac{1}{2}v^2(m_L\cos^2\theta - m_T\sin^2\theta) \doteq \tfrac{1}{2}v^2 m_L, \tag{4}$$
$$(\alpha^2 \ll 1)$$

and was used to express the variables of Eq. (3) in terms of the energy.

If ω_T and τ vary slowly with energy compared with $f(E)$, the numerator of Eq. (3) can be integrated by parts, and we obtain

$$\sigma(\omega) = \frac{3Ne^2\alpha^2}{4m_L}\left[\frac{\tau + i\omega\tau^2}{(\omega_T\tau)^2 + (1+i\omega\tau)^2}\right]. \tag{5}$$

It is easily verified that $\text{Re}[\sigma(\omega)] > 0$, so that we have a direct check that under reasonable assumptions the conductivity, and hence the absorption for the system under consideration is positive.

If all the hypotheses we stated are not valid, there will exist possibilities for these negative masses to amplify resonant radiation. We envisage two distinct situations: the first involves a cutoff energy $E_C < E_{\text{opt}}$, such that for $E > E_C$ the shape of the energy contours has changed sufficiently that no particles[6] above this cutoff energy can contribute to the resonance at ω_T. If such is true for the energy band structure of the heavy holes in Ge or Si, emission would be observed at ω_T provided

$$\int_0^{E_C} dE\, E^{3/2}\, df/dE > 0.$$

This last condition is tantamount to an inversion of the population of these low-lying energy levels, which could be achieved by a mechanism such as selective recombination of slow carriers.

If the band structure does not permit such a drastic change in m_T, a second situation is conceivable whereby the distribution function while still of cubic symmetry shows some anisotropy.

If, for example,

$$f = f_1(E) + f_2(|\vec{p}|), \tag{6}$$

those particles distributed according to f_1 will absorb radiation, whereas the contribution of the carriers with negative transverse mass $-m_T$ distributed according to f_2 can be shown to be emissive, and peaked about the frequency ω_T. Whether emission or absorption predominates at that frequency will depend on the relative magnitudes of f_1 and f_2.

In this connection we propose that light may not provide the best mechanism for obtaining a selective excitation of such carriers. A possible alternative excitation technique could utilize microwave impact ionization of impurity centers in p-type Ge by resonant absorption at the cyclotron frequency ω_P of the positive-mass heavy holes. With a double resonant cavity system, the changes in the rf conductivity may then be studied with a low-power probing signal at the resonance frequency of the negative-mass holes, ω_T. Such a technique would result in the presence of positive-charge carriers only, and with proper circular polarization the effects of the negative-mass holes can be separated from those of the positive-mass holes.

In concluding, we express our thanks to colleagues of this laboratory—to Mr. W. Kopka for computational work on the IBM 704 computer, and to Dr. S. Koenig, Dr. R. Landauer, and Dr. E. Adams for their interest and helpful suggestions.

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[1] G. C. Dousmanis, Phys. Rev. Letters 1, 55 (1958).

[2] G. C. Dousmanis et al., Phys. Rev. Letters 1, 404 (1958).

[3] Dresselhaus, Kip, and Kittel, Phys. Rev. 98, 368 (1955).

[4] H. Krömer, Proc. Inst. Radio Engrs. 47, 397 (1959).

[5] For method of derivation, see, for example, D. Mattis and G. Dresselhaus, Phys. Rev. 111, 403 (1958).

[6] This is an idealization. In reality we would not necessarily expect a sharp cutoff and there would be overlap in the spectra of states above and below E_C.