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p-p' Strings in M(atrix) Theory

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Abstract

We study the off-diagonal blocks in the M(atrix) model that are supposed to correspond to open strings stretched between a Dp-brane and a Dp'-brane. It is shown that the spectrum, including the quantum numbers, of the zero modes in the off-diagonal blocks can be determined from the index theorem and unbroken supersymmetry, and indeed reproduces string theory predictions for p-p' strings. Previously the matrix description of a longitudinal fivebrane needed to introduce extra degrees of freedom corresponding to 0-4 strings by hand. We show that they are naturally associated with the off-diagonal zero modes, and the supersymmetry transformation laws and low energy effective action postulated for them are now derivable from the M(atrix) theory.

1 Introduction

D(irichlet)-branes have many faces. In string theory, they arise as nonperturbative dynamic objects, allowing strings to end on them and carrying R(amond)-R(amond) charge [1]. In the conformal field theory formulation, a Dp-brane is a p-dimensional hyperplane in target space on which strings satisfy the Dirichlet boundary conditions [2]. In the low energy field theory limit (supergravity), it appears as a soliton-like background with nontrivial R-R antisymmetric tensor field, solving classical equations of motion (see a recent review [3] and references therein). The low energy dynamics of parallel D-branes, due to strings stretched between them, can be described by a dimensionally reduced supersymmetric Yang-Mills theory on their world volume [4], which happens to describe a quantum space in the sense of non-commutative geometry [5].

In the M(atrix) model [6] for M theory, which is conjectured to unify all known perturbative string theories, the D0-branes are treated as fundamental microscopic degrees of freedom. The SYM quantum mechanics, which was originally thought to be the low-energy theory of N D0-branes, is promoted in the large N limit to the status of the fundamental light-cone dynamics of M theory. As dimensionally reduced $U(N)$ SYM theory, its field content matches the lowest modes of open strings ending on D0-branes. Thus, in M(atrix) theory, everything else appears as a collective (bound) state of D0-branes. In particular, a multiple parallel D-brane background is realized as a block-diagonal matrix [6], each block represented by a topologically nontrivial gauge field configuration [7, 8] on a D-brane volume. In this paper we study the dynamics of D-branes by introducing and examining off-diagonal blocks, that are supposed to correspond to strings stretched between D-branes. One of the advantages of the M(atrix) theory is that it provides a unifying framework for explicitly dealing with both D-brane backgrounds and strings stretched between them.

Previously Berkooz and Douglas [9] have considered the background of a longitudinal M5-brane, which wraps around the (invisible) 11-th direction that defines the light-cone to give rise to a D4-brane in IIA language. They bypassed the question of explicitly representing the D4-brane in matrix form, but rather proposed a modified M(atrix) theory by introducing by hand additional dynamical variables that are supposed to correspond to the massless modes of open strings stretched between the D4-brane and the D0-branes (called 0-4 strings). It was shown that integrating out the extra variables leads to the correct gravitational field of an M5-brane. Later Dijkgraaf, Verlinde and Verlinde [10] showed that if one integrates out the off-diagonal blocks in

the $U(N)$ matrix fields with two diagonal blocks for a D4-brane and a D0-brane respectively, one can also recover the gravitational field of a longitudinal M5-brane. Based on this result, one may be tempted to identify the extra fields introduced in Ref. [9] with the above-mentioned off-diagonal blocks. However, there is a mismatch for the quantum numbers: the extra bosonic field in Ref. [9] is a spinor of the $SO(4)$ in the 4-brane directions, in accordance with string theory [11], while the bosonic off-diagonal block is an $SO(4)$ vector. Resolving this puzzle was part of the motivation for this paper.

Another related, unsettled issue is how to obtain the 32 additional fermions in the heterotic matrix theory, which is the M(atrix) theory compactified on S^1/Z_2. First it was suggested to add these fermions by hand to cancel anomalies in the 1+1 dimensional field theory [12, 13, 14]. Later Horava [15] proposed that they are zero modes of the off-diagonal blocks that correspond to 0-8 strings. However, there is a puzzle of why these fermions are invariant under surviving supersymmetries. A better understanding of the 0-8 strings in the M(atrix) theory should help resolve this problem.

In this paper we study the spectrum of the off-diagonal blocks in M(atrix) theory that are supposed to correspond to p-p' strings in the background of a Dp-brane and a Dp'-brane. In particular we show that the spectrum of zero modes for the off-diagonal blocks matches the massless spectrum of p-p' strings. Since the string theory results about p-p' string spectrum are most directly seen in the Neveu-Schwarz-Ramond formalism, while the M(atrix) description of type IIA theory [17, 18, 19] is in the Green-Schwarz formalism, it is nontrivial to check if their predictions agree. Moreover, note that D-brane charges and supersymmetry do not give a complete characterization for parallel D-brane configurations in M(atrix) theory. The study of the zero-modes of off-diagonal blocks will provide more information on proper identification of D-brane backgrounds, and on their dynamical behavior as well, such as R-R charge and stability etc.

In this paper we will refer to configurations in M theory by their names in the IIA theory that is related to the M theory through compactification of the (invisible) eleventh dimension. Hence a D0-brane is a Kaluza-Klein mode of a graviton, a D2-brane an M-membrane, a D4-brane a longitudinal M5-brane [20]. It is unclear what D6 and D8-branes in IIA really correspond to in M theory, but they are needed to give various D-branes under compactifications.

We will review related results in string theory in Sec.2 and M(atrix) description of D-brane configurations in Sec.3. In Sec.4 we derive the equations of motion for the bosonic and fermionic zero modes of the off-diagonal blocks, which we will use to find

the zero modes, and explain how to derive their supersymmetry transformations and low-energy effective action, for 0-2, 0-4, 0-6 and 0-8 strings respectively in Sec.5-7. Sec. 6 also includes a discussion on the application of the off-diagonal zero modes to the matrix description [9] of longitudinal fivebranes. More discussions on the physical implications of our results can be found in Sec. 7 and in Sec. 8 .

2 Review of p-p' Strings

In this section we briefly review the results in string theory on p-p' strings [11]. First we consider an open string connecting a Dp-brane and a Dp'-brane parallel to each other. Since we are using II A language, both p and p' are even integers. Assuming that $p' \geq p$. In directions $0, 1, \cdots, p$, where the two D-branes overlap, the bosonic fields X have Neumann boundary conditions on both ends. In directions $p+1, \cdots, p'$, they have Dirichlet boundary condition on the p-brane and Neumann condition on the p'-brane. In the rest directions $p'+1, \cdots, 9$, the open string has Dirichlet conditions on both ends.

There will be unbroken supersymmetries for a system of parallel Dp-branes and Dp'-branes, if and only if the number, ν, of the directions in which the bosonic sector has DN or ND boundary conditions is 0,4 or 8. (Note that $\nu = p' - p$ for a parallel Dp- and Dp'-brane.)

The Ramond sector of the p-p' string has the same kind of boundary conditions as the bosonic part. It always offers a massless fermionic $SO(1, 9-(p'-p))$ Weyl spinor (after GSO projection) for the directions with NN or DD boundary conditions. The NS sector has the opposite kind of boundary conditions. Only when $(p'-p) = 4$ will there be a massless $SO(p'-p)$ bosonic Weyl spinor for the directions with ND or DN boundary conditions.

Since we can always use T-duality to switch a Dp-brane to a D0-brane, we only need to consider four types of open strings: the 0-2, 0-4, 0-6 and 0-8 strings. In summary, the massless spectrum for a 0-p string consists of only a fermionic $SO(1, 9-p)$ Weyl spinor, except that when $p = 4$ there is in addition a bosonic $SO(4)$ Weyl spinor. Below we are going to verify this spectrum of massless fermionic and bosonic modes in M(atrix) theory. (Though it is amusing to note that in M(atrix) theory, the bosonic off-diagonal blocks that are supposed to correspond to 0-4 strings are $SO(4)$ vectors!)

3 M(atrix) Description of D-Brane Configurations

The action of the M(atrix) model is [6]

$$S = \int dt Tr(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\bar{\Psi}\Gamma^{\mu}[X_{\mu}, \Psi]), \tag{1}$$

where $\mu, \nu = 0, 1, \cdots, 9$, $F_{\mu\nu} = [X_{\mu}, X_{\nu}]$ and $X_0 = -iD_0 = -i(\frac{\partial}{\partial t} + A_0)$. X_{μ} and Ψ_{α} are Hermitian $N \times N$ matrices. The dynamical and kinematical SUSY transformations are respectively [7]

$$\delta X_{\mu} = i\bar{\epsilon}\Gamma_{\mu}\Psi, \quad \mu = 0, 1, \cdots, 9, \tag{2}$$

$$\delta\Psi = (D_0 X_i)\Gamma^{0i}\epsilon + \frac{i}{2}[X_i, X_j]\Gamma^{ij}\epsilon, \quad i, j = 1, 2, \cdots, 9, \tag{3}$$

and

$$\delta X_{\mu} = 0, \quad \delta\Psi = \tilde{\epsilon}, \tag{4}$$

each with 16 generators.

The configuration of a Dp-brane in M(atrix) theory is given by big (infinite dimensional) matrices giving the appropriate p-brane charge [7]: [1]

$$tr(\epsilon_{\mu_1,\cdots,\mu_p}[X_{\mu_1}, X_{\mu_2}]\cdots[X_{\mu_{p-1}}, X_{\mu_p}]). \tag{5}$$

We can choose the X's to satisfy

$$[X_{2n-1}, X_{2n}] = F_{(2n-1)(2n)} \tag{6}$$

for $n = 1, 2, \cdots, p/2$ with $F_{\mu\nu}$ being constant $K \times K$ matrices. The fermionic partner is taken to be zero.

There are two ways to realize this physical setting in the M(atrix) theory. Take the D2-brane as an example. One way is to set $X_1 = R_1 P$, $X_2 = R_2 Q$, where $[P, Q] = i2\pi/N$ [6]. P and Q can in turn be realized as $P = -i(2\pi/N)\frac{\partial}{\partial\sigma}$ and $Q = -\sigma$ through an angle parameter $\sigma \in [0, 2\pi)$. Another way is to first compactify the M(atrix) model on a torus with radii $R_i, i = 1, 2$, and then take the limit $R_i \to \infty$ if one wishes. A Dp-brane configuration corresponds to a gauge field configuration with certain topological charge [7] (the k-th Chern character $Q_k = \int trF^k$ for $k = p/2$.) on the dual torus which becomes infinitesimal in the large radii limit. The X matrices in the p-brane directions

[1] Only the 2-brane and 4-brane charges are defined in the SUSY algebra. The 6 and 8-brane charges are extrapolations of those.

become ($(-i)$ times) the covariant derivatives. For a D2-brane it can be taken as, say, $X_1 = -i2\pi R_1 \frac{\partial}{\partial\sigma_1}$ and $X_2 = -i2\pi R_2 \frac{\partial}{\partial\sigma_2} - R_2\sigma_1$.

For our purpose the difference between the two descriptions is only a scaling in the derivatives. For simplicity in notation we choose to use the latter description in this paper.

A static Dp-brane configuration (6) preserves half of the total SUSY if and only if the F's are proportional to the unit matrix, in which case 16 linear combinations of the dynamical and kinematical SUSY are preserved [7]. These states contain D0, D2,..., D$(p-2)$-branes in addition to the Dp-brane. The kinematical SUSY (4) is never preserved by itself. The condition for part of the dynamical SUSY to be preserved is

$$F_{12} - \sum_{i=2}^{p/2} \varepsilon_i F_{(2i-1)(2i)} = 0 \tag{7}$$

for some $\varepsilon_i = \pm 1$. It preserves $1/2^{(p/2-1)}$ of the dynamical SUSY parametrized by ϵ satisfying

$$\Gamma^{12}\Gamma^{(2i-1)(2i)}\epsilon = \varepsilon_i\epsilon, \quad i = 2, 3, \cdots, p/2. \tag{8}$$

Because $tr(F_{12}^2) \neq 0$ it follows from (7) that any D6 or D8-brane configuration with unbroken dynamical SUSY must always include D4-branes. A discussion on general bound states from the low energy D-brane point of view can be found in [21].

If all $F_{\mu\nu}$'s in (6) are proportional to the unit matrix, they define a natural complex structure on the dual torus. It can be used to view the dual torus T^p as composed of $p/2$ complex tori T^2. A Dp-brane with unit p-brane charge can be realized by a $U(K)$ gauge field with twisted boundary conditions. This is analogous to how one defines a long string [19, 18] in the conjugacy class of length K. The unit Dp-brane charge means a twisted bundle with the minimal topological charge on each T^2.

An explicit construction of the minimal twisted bundle of the fundamental representation of $U(K)$ is given in Ref.[8]. There the gauge fields can be chosen as $A_1 = 0$ and $A_2 = -i(\sigma_1/2\pi K)\mathbf{1}$, where we use σ_i as the coordinates on T^2 normalized to range between 0 and 2π and $\mathbf{1}$ is the unit matrix. The field strength F_{12} is then $1/2\pi K$. The quasi-periodic boundary conditions on A are [8]:

$$A_\mu(2\pi, \sigma_2) = \Omega_1(\sigma_2)A_\mu(0, \sigma_2)\Omega_1^{-1}(\sigma_2) + \Omega_1(\sigma_2)\partial_\mu\Omega_1^{-1}(\sigma_2), \tag{9}$$

$$A_\mu(\sigma_1, 2\pi) = \Omega_2(\sigma_1)A_\mu(\sigma_1, 0)\Omega_2^{-1}(\sigma_1) + \Omega_2(\sigma_1)\partial_\mu\Omega_2^{-1}(\sigma_1), \tag{10}$$

where Ω_1 and Ω_2 can be chosen as

$$\Omega_1(\sigma_2) = q^{\sigma_2/2\pi}U, \quad \Omega_2(\sigma_1) = V, \tag{11}$$

where $q = e^{i2\pi/K}$, $U_{ij} = q^i \delta_{ij}$ and $V_{ij} = \delta_{i+1,j}$ with $i, j = 0, \cdots, K-1$ $(mod\ K)$. U and V satisfy $UV = q^{-1}VU$. It can be checked that $\Omega_1(2\pi)\Omega_2(0) = \Omega_2(2\pi)\Omega_1(0)$. This is in contrast with the twisted bundle of $SU(K)/\mathbf{Z}_K$ where one has $\Omega_1(2\pi)\Omega_2(0) = \Omega_2(2\pi)\Omega_1(0)Z$ for some element Z in the center $\mathbf{Z}_K$ of $SU(K)$ [22].

The bundle in fundamental representation has the corresponding boundary conditions:

$$\phi(2\pi, \sigma_2) = \Omega_1(\sigma_2)\phi(0, \sigma_2), \quad \phi(\sigma_1, 2\pi) = \Omega_2(\sigma_1)\phi(\sigma_1, 0). \tag{12}$$

Note that consistency of the boundary conditions requires

$$\Omega_1(2\pi)\Omega_2(0) = \Omega_2(2\pi)\Omega_1(0).$$

A section of the bundle has the general form of [8]

$$\phi_j(\sigma_1, \sigma_2) = \sum_{m \in \mathbf{Z}} \hat{\phi}(\sigma_2/2\pi + j + mK) q^{(\sigma_2/2\pi + j + mK)\sigma_1/2\pi}, \quad j = 0, 1, \cdots, K-1, \tag{13}$$

for an arbitrary function $\hat{\phi}$ for which the series converges.

Since this is the D-brane analogue of a long string in the conjugacy class of length K, this gauge field configuration is identified with a single D2-brane instead of K D2-branes. Here K gets interpreted as the longitudinal momentum carried by the single D2-brane, as can be seen by examining its light-cone energy.

It is essential that the gauge group is $U(K)$ instead of $SU(K)$. Although there are twisted $SU(K)/\mathbf{Z}_K$ bundles in the adjoint representation [22] with the same topological charge, there is no corresponding vector bundle in the fundamental representation, because the element Z acts nontrivially on the fundamental representation while it acts trivially on the adjoint. Note that the presence of anything other than the D4-branes introduces off-diagonal blocks in the fundamental representation. Hence, for instance, although one can use two copies of the twisted $SU(2)$ bundle on T^2 with (anti-)self-duality to construct pure D4-brane states preserving half of the dynamical SUSY, at this moment it is unclear how to describe their interaction with other D-branes.

4 Equations of Motion

Consider a D0-brane very close to a Dp-brane. We decompose the matrix fields into the block form:

$$X_\mu = \begin{pmatrix} Z_\mu & y_\mu \\ y_\mu^\dagger & x_\mu \end{pmatrix}; \quad \Psi = \begin{pmatrix} \Theta & \theta \\ \theta^\dagger & \psi \end{pmatrix}, \tag{14}$$

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where Z_μ represents the Dp-brane and x_μ the D0-brane. The generalization to many Dp-branes and D0-branes is straightforward. While the Z's are realized as covariant derivatives, the x's in general can have nontrivial coordinate dependence on the dual torus, but when we take the limit of infinite radii, only coordinate-independent states can have finite energy and remain coupled to the theory. (We allow infinite energy for Z because it is just the energy for the Dp-brane.)

For simplicity we choose the coordinates of spacetime such that $x_\mu = 0$ and $Z_a = 0, a = p+1, \cdots, 9$. The D$p$-brane is parallel to the directions $1, 2, \cdots, p$ and the D0-brane is right on top of it. The diagonal part is taken as the background configuration.

When putting the two D-branes together as in (14) and set the off-diagonal parts to be zeros, one can check easily that part of the supersymmetry is preserved only if p is $0, 4$ or 8.

To count the number of zero modes, or equivalently to count the dimension of the moduli space for this background, it is easier to consider the perturbation of this background and keep only the lowest order terms to obtain linear differential equations for the perturbative fields y and θ. In this way we count the dimension of the tangent space on the moduli space. One may also introduce perturbations in the diagonal blocks for fluctuations on the Dp-brane and deviations of the D0-brane from the origin, but here we are for the time being only interested in the off-diagonal blocks y and θ since they represent the p-p' strings. The perturbations of the diagonal blocks can be studied in the same way we study the off-diagonal part. To the lowest order in perturbation, the perturbative diagonal and off-diagonal parts are not correlated, hence we can treat the off-diagonal ones alone.

Plugging the expression of the matrix fields (14) into the action of the M(atrix) model (1), we find

$$S = \int dt (L_Z + L_x + L_y), \tag{15}$$

where L_Z and L_x are of the same form as (1) except that we replace (X, Ψ) by (Z, Θ) and (x, ψ), respectively. $L_y = L_B + L_F$ with

$$\begin{aligned} L_B &= \frac{1}{2}|(Z_\mu y_\nu - Z_\nu y_\mu + y_\mu x_\nu - y_\nu x_\mu)|^2 + y_\mu^\dagger [Z_\mu, Z_\nu] y_\nu - [x_\mu, x_\nu] y_\mu^\dagger y_\nu \\ &\quad - y_\mu^\dagger y_\nu y_\mu^\dagger y_\nu + \frac{1}{2} y_\mu^\dagger y_\mu y_\nu^\dagger y_\nu + \frac{1}{2} y_\mu^\dagger y_\nu y_\nu^\dagger y_\mu \end{aligned} \tag{16}$$

and

$$L_F = -\theta^\dagger \Gamma^0 \Gamma^\mu (Z_\mu \theta - \theta x_\mu) + \theta^\dagger \Gamma^0 \Gamma^\mu (\Theta y_\mu - y_\mu \psi) + (y_\mu^\dagger \Theta - \psi y_\mu^\dagger) \Gamma^0 \Gamma^\mu \theta. \tag{17}$$

For more than one D0-branes the x's are matrices and we need to take traces for these formulas.

¿From the action one can derive the equations of motion for y and θ. Since the Hamiltonian for a time-independent background in the temporal gauge ($A_0 = 0$) is minimized by time-independent y, we look for time-independent solutions for y and θ. Ignoring the time derivative and higher order terms, we find

$$D^\mu(D_\mu y_\nu - D_\nu y_\mu) + [D_\nu, D_\mu]y^\mu = 0, \qquad \mu, \nu = 1, \cdots, p, \tag{18}$$

$$D^\mu D_\mu y_a = 0, \qquad a = p+1, \cdots, 9, \tag{19}$$

where $D_\mu = iZ_\mu$ are covariant derivatives on the dual torus T^p as $D_\mu = 2\pi R_\mu(\frac{\partial}{\partial\sigma^\mu} + A(\sigma))$, $\mu = 1, \cdots, p$. Eq.(18) has to be supplemented by the gauge-fixing condition

$$D_\mu y^\mu = 0. \tag{20}$$

Using (20), eq.(18) can be written as

$$D_\mu D^\mu y_\nu + 2[D_\nu, D_\mu]y^\mu = 0. \tag{21}$$

The equation of motion for θ is

$$\Gamma^\mu D_\mu \theta = 0. \tag{22}$$

In terms of the covariant exterior derivative d_A, its dual d_A^*, the Hodge dual $*$ and the projection $P = \frac{1}{2}(1 - *)$ (so $P^2 = P$), eqs.(18) and (20) now read

$$d_A^* P d_A y = 0, \tag{23}$$

$$d_A^* y = 0, \tag{24}$$

where $y = y_\mu d\sigma^\mu$. These equations are formally the same as those for the instanton zero modes, which correspond to perturbations of the Z's above. The only difference is that the perturbations of Z is in the adjoint representation of $U(K)$, while y is in the fundamental representation.

Because we are considering the Euclidean torus, the inner product $\langle\cdot|\cdot\rangle$ defined by integration on the torus and the trace of matrices is positive definite. Hence $\langle y|d_A^* P d_A y\rangle = 0$ implies that

$$P d_A y = 0. \tag{25}$$

In addition, eq.(19) implies that $\langle D_\mu y_a | D^\mu y_a\rangle = 0$ and so $D_\mu y_a = 0$, which means that the topological charge vanishes unless $y_a = 0$. Thus we conclude that $y_a = 0$ for $a = p+1, \cdots, 9$.

5 0-2 Strings

Let Z_1 and Z_2 be realized as $U(K)$ covariant derivatives on the dual torus as $Z_i = -iD_i$ with $D_i = \frac{\partial}{\partial \sigma_i} + A_i$ as given in Sec.3 so that

$$[Z_1, Z_2] = if\mathbf{1}, \tag{26}$$

where $f = 2\pi R_1 R_2/K$. For simplicity we are considering unslanted torus with radii $R_1 = R_2 = 1/2\pi$. It is straightforward to generalize to slanted tori with arbitrary radii. Let $z = (\sigma_1 + i\sigma_2)/2\pi$, $\bar{z} = (\sigma_1 - i\sigma_2)/2\pi$ be the complex coordinates on T^2, and let $\mathcal{D} = D_1 - iD_2$ and $\bar{\mathcal{D}} = D_1 + iD_2 = -\mathcal{D}^\dagger$, then $[\mathcal{D}, \bar{\mathcal{D}}] = 2f\mathbf{1}$. It follows that

$$D^2 = \mathcal{D}\bar{\mathcal{D}} - f = \bar{\mathcal{D}}\mathcal{D} + f, \tag{27}$$

where $D^2 = D_1^2 + D_2^2$. Note that the algebra of $\bar{\mathcal{D}}$ and $-\mathcal{D}$ is the canonical commutation relation for annihilation and creation operators scaled by $2f$. Therefore the spectrum of $\mathcal{D}\bar{\mathcal{D}}$ is $\{0, -2f, -4f, \cdots\}$ and the spectrum of D^2 is

$$\{-f, -3f, -5f, \cdots\}. \tag{28}$$

The fermionic zero modes satisfy (22), which gives $(D_1 + \Gamma^1\Gamma^2 D_2)\theta = 0$, so that

$$\mathcal{D}\theta_+ = 0, \quad \bar{\mathcal{D}}\theta_- = 0, \tag{29}$$

where $\theta_\pm$ are the two Weyl components of θ satisfying $i\Gamma^1\Gamma^2\theta_\pm = \pm\theta_\pm$. Because $\langle\theta_+|\bar{\mathcal{D}}\mathcal{D}\theta_+\rangle = \langle\theta|(D^2 - f)\theta\rangle < 0$ for any $\theta_+ \neq 0$, we must have $\theta_+ = 0$. The solution of θ_- is obviously the vacuum state annihilated by $\bar{\mathcal{D}}$.

One can easily get the explicit expression of the vacuum as a section of the twisted bundle using the explicit construction in Sec.3. Another way is to note that the equation $\bar{\mathcal{D}}\phi = 0$ has the general solution of $\phi = \exp(-\frac{\pi}{4K}(\bar{z}^2 + 2z\bar{z}))f(z)$, where $f(z)$ is an arbitrary holomorphic function. For ϕ to be a section of the twisted bundle, we need to impose the quasi-periodic boundary conditions on ϕ. One then sees that $f(z)$ is related to the third elliptic theta function ϑ_3 and the solution is

$$\phi_k(\sigma_1, \sigma_2) = \exp\{\pi/K[2i(\sigma_1/2\pi)(\sigma_2/2\pi + k) - (\sigma_2/2\pi + k)^2]\}\vartheta_3(q|\pi(z + ik)), \tag{30}$$

where $q = \exp(-\pi K)$ and ϕ_k $(k = 0, 1, \cdots, K-1)$ gives a section on the vector bundle in the fundamental representation. Applying the creation operator $\mathcal{D}$ to the vacuum one obtains other eigenstates of the operator D^2.

Obviously the zero mode of θ_- is just given by the solution (30). The fermionic zero mode is an $SO(2)$ Weyl spinor with negative chirality.

The equations of motion (21) for y_μ are

$$(D^2 - 2f)y = 0, \quad (D^2 + 2f)\bar{y} = 0, \tag{31}$$

where $y = y_1 + iy_2$, $\bar{y} = y_1 - iy_2$. The constraint (20) is

$$\mathcal{D}y + \bar{\mathcal{D}}\bar{y} = 0. \tag{32}$$

Since the spectrum of D^2 is given by (28), we see that there is no solution for y, $\bar{y}$, hence there is no bosonic zero mode.

6 0-4 Strings

We decompose the 10 dimensional γ-matrices as

$$\Gamma^0 = i\sigma_2 \otimes \mathbf{1} \otimes \mathbf{1}, \tag{33}$$
$$\Gamma^\mu = \sigma_1 \otimes \gamma^\mu \otimes \mathbf{1}, \quad \mu = 1, \cdots, 4, \tag{34}$$
$$\Gamma^a = \sigma_1 \otimes \gamma^5 \otimes \gamma^a, \quad a = 5, \cdots, 9, \tag{35}$$
$$\Gamma^{10} = \sigma_3 \otimes \mathbf{1} \otimes \mathbf{1}, \tag{36}$$

where the σ_i's are the Pauli matrices satisfying $\sigma_1\sigma_2 = i\sigma_3$, the γ^μ's ($\mu = 1, \cdots, 4$) are the γ-matrices for $SO(4)$, the γ^a's ($a = 5, \cdots, 9$) are $SO(5)$ γ-matrices. Corresponding to this decomposition, a 10-dimensional spinor $\theta_{i\alpha\beta}$ has three indices, where $i = \pm$, $\alpha, \beta = 1, \cdots, 4$. For Weyl spinors with positive (negative) chirality one has $i = +$ ($i = -$). Since all spinors in this theory are 10-dimensional Weyl spinors with positive chirality, we will omit the index i in the following. We consider the case where the gauge field for the D4-brane background is self-dual, so that half of the dynamical SUSY with parameter ϵ satisfying

$$\Gamma^1\Gamma^2\Gamma^3\Gamma^4\epsilon = \epsilon \tag{37}$$

is preserved.

The number of zero modes for the case of T^4 for (anti-)self-dual gauge field configurations can be determined using the index theorem [23, 24]. The number of spinorial zero modes is found to be $\alpha_V k$, where α_V is the Dynkin index for the representation V of the fermions, and k is the instanton number. The number of vectorial zero modes is $2\alpha_V k$.

For any $U(K)$, the number of spinorial and vectorial zero modes in the fundamental representation are k and $2k$, respectively. The general formula [23] for spinorial and vectorial zero modes in arbitrary representation V on a 4-manifold M are $\alpha_V k+\frac{1}{8}dimV\tau(M)$ and $2\alpha_V k-\frac{1}{2}dimV(\chi(M)+\tau(M))$, respectively, where $\tau(M)=\frac{1}{96\pi^2}\int\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu l\rho}R^{l\rho}_{\alpha\beta}dv$ is the signature of M and $\chi(M)=\frac{1}{128\pi^2}\int\epsilon^{\mu\nu\alpha\beta}\epsilon^{l\rho\gamma\delta}R_{\mu\nu l\rho}R_{\alpha\beta\gamma\delta}dv$ is the Euler characteristic.

While according to the the index theorem the number of zero modes is independent of the details of (anti-)self-dual gauge field configurations, here we give for example an explicit construction of a twisted bundle for the cases with $R_1R_2=R_3R_4$. On each T^2 factor of T^4, one can construct a twisted $U(K)$ bundle as in Sec.3. When putting them together, we obtain a $U(K^2)$ bundle with unit instanton number: $\frac{1}{8\pi^2}\int tr(F^2)=1$. Unlike the case of twisted $SU(K)/\mathbf{Z}_K$ bundles which can have fractional instanton numbers, for $U(K)$ the instanton number are always integral [21]. A section on the twisted $U(K^2)$ bundle on T^4 has the general form of a linear combination of products of sections on each T^2: $\phi_j(\sigma_1,\sigma_2)\phi_k(\sigma_3,\sigma_4)$, where ϕ is defined by (13) for $j,k=0,1,\cdots,(K-1)$. Indices j and k compose an index for the fundamental representation of $U(K^2)$. In general one can also consider a $U(K_1)$ and $U(K_2)$ bundle on the two T^2 factors, respectively, and obtain a $U(K_1K_2)$ bundle on T^4.

Because the supersymmetry is not completely broken, the solution of fermionic zero modes can be used to obtain the solution of bosonic zero modes. The solution of the fermionic and bosonic zero modes can be obtained explicitly by considering T^4 as $T^2\times T^2$ and using the methods in Sec.5. Let the $SO(4)$ spinor satisfying (22) be denoted by θ^0. It is easy to see that the fermionic zero mode satisfies $i\Gamma^1\Gamma^2\theta^0=i\Gamma^3\Gamma^4\theta^0=-\theta^0$, which implies that θ^0 is of negative chirality as an $SO(4)$ Weyl spinor: $\Gamma^1\Gamma^2\Gamma^3\Gamma^4\theta^0=-\theta^0$. (If the gauge field is anti-self-dual, the zero mode will be a Weyl spinor with positive chirality.) For a single D4-brane there is only one fermionic zero mode, which is given by the product of the solutions (30) on each T^2 factor in T^4.

The general solution of fermionic zero modes can then be written as $\theta_{\dot\rho\beta}=\theta^0_{\dot\rho}\chi_\beta$ (and $\theta_{\rho\beta}=0$), where $\dot\rho=1,2$ (ρ) is the index for an $SO(4)$ Weyl spinor with negative (positive) chirality and χ is an $SO(5)$ spinor for the directions $5,\cdots,9$. Thus χ represents the massless fermionic mode from the Ramond sector of the 0-4 string.

Since the equations of motion for y and θ are supersymmetric, the bosonic zero mode can be obtained by SUSY transformation [25] as

$$y^\mu=iv^\dagger_\rho\gamma^\mu_{\rho\dot\rho}\theta^0_{\dot\rho}, \tag{38}$$

where v_ρ is an $SO(4)$ Weyl spinor with positive chirality. This comes from the SUSY

transformation of y: $\delta y_\mu = i\bar{\epsilon}\Gamma_\mu\theta$. When one replaces in this transformation $\theta_{\alpha\beta}$ by the zero mode $\theta^0_{\dot{\rho}}$, δy will satisfy the equations of motion of y for any ϵ_ρ in the SUSY preserved by the background (37). It follows that the y given by the above expression is a zero mode of y. Since θ^0 is a function (bosonic), v is a bosonic variable. It matches the massless bosonic field from the NS sector of the 0-4 string. Here it is amusing to see how supersymmetry dictates the zero modes of a field y in vector representation to be described by a variable v in spinor representation. The index theorem [23] assures us that these are all the zero modes in the theory, giving precisely the massless spectrum of 0-4 strings. The supersymmetry transformation between χ and v is induced from the SUSY transformation between θ and y by factoring out the common factor of θ^0. Up to first order perturbation, the SUSY transformation of θ is: $\delta\theta = \frac{1}{2}(D_A y_B - D_B y_A)\Gamma^{AB}\epsilon$, where $A, B = 0, 1, \cdots, 9$. Using (25), (19) and (37), one finds [2]

$$\delta v_\rho = \chi^\dagger_\alpha \epsilon_{\rho\alpha}, \tag{39}$$

$$\delta\chi_\alpha = 2i(D_0 v^\dagger_\rho)\epsilon_{\rho\alpha}. \tag{40}$$

The instanton connection lies in $SU(2)_R \in SO(4)$ which is supposed to be the global R-symmetry for the action of 0-4 strings. Field v carries the fundamental index of $SU(2)_R$. Let τ^i denote generators of the R-symmetry group. There are two possible $SU(2)_R$ invariant D-terms, $\sum_i |v^+\tau^i v|^2$ and $|v^+v|^2$. The two terms are different when there are more than one D0-branes, in which case only the first is actually present in the action [11]. These D-terms are expected to arise from the F^2 term in the Super Yang-Mills theory. Upon expanding this term in y one finds $tr|y_\mu y^+_\nu - y_\nu y^+_\mu|^2$ and $|y^+_\mu y_\nu - y^+_\nu y_\mu|^2$. For a given instanton background, since $SU(2)_R$ is broken explicitly, these terms do not give those $SU(2)_R$ invariant D-terms. Only after averaging over the moduli space does one expect that the symmetry $SU(2)_R$ is restored. However, we do not know how to rule out the $U(1)$ D-term $|v^+v|^2$.

The above discussion easily generalizes to the case of instanton number k. There are $2k$ zero modes for y_μ, and can be interpreted as the fundamental of $U(k) \times SU(2)_R$, where $U(k)$ is the gauge group associated to k coincident D4-branes.

In ref. [9], an action describing M(atrix) theory of a longitudinal 5-brane is proposed. Since a longitudinal 5-brane in M-theory corresponds to a D4-brane in type IIA string theory, some extra dynamical variables corresponding to 0-4 strings were needed and were introduced by hand. Their quantum numbers are exactly the same as those of the variables v and χ that we have discussed above. Thus, it is natural to identify

[2] Our notation is slightly different from that of Ref.[9].

the additional variables introduced by Berkooz and Douglas [9] with the degrees of freedom associated with the off-diagonal zero modes. We have verified that indeed the action of the latter naturally derives from the fundamental M(atrix) model action, and it agrees with the action postulated in ref. [9], with a possible $U(1)$ D-term as we mentioned above. (In the derivation, the coefficient of each term in the action is determined by an integral of a product of the zero mode solutions θ^0. We have not been able to calculate all coefficients; presumbly they are uniquely determined by the surviving supersymmetry.)

In addition to the variables v and χ, the action in ref. [9] has included also fields describing fluctuations of the longitudinal fivebrane background, which in our approach correspond to fluctuations residing in the diagonal blocks. In principle one can consider fluctuations of all blocks in the matrix fields for a given background, and then solve the exact (nonlinear) equations of motion. The parameters analogous to v and χ above for the general solutions correspond to the massless modes of the whole system of (p'-p)-branes. In the above we have only solved the linearized equations of motion for the off-diagonal blocks. The supersymmetry derived from our solutions will only hold to the lowest order in perturbation. If one solves the exact nonlinear equations of motion, one should be able to derive the exact SUSY transformation among the zero mode parameters.

In the above we have only considered the case with vanishing distance between the D0-brane and the D4-brane. When we pull the D0-brane away from the D4-brane, the zero modes will gain masses proportional to the distance. But we expect that the number and representation of the lowest energy modes will remain the same as the zero modes. The proposal of Ref.[9] contains only the lowest energy modes and therefore should be viewed as a low energy effective theory.

7 0-6 Strings and 0-8 Strings

The case of 0-6 strings and 0-8 strings can be studied in a similar fashion as the 0-2 and 0-4 strings. To generalize the consideration for 0-2 and 0-4 strings to 0-p strings for $p = 2, 4, 6, 8$, we choose the gauge field configuration for the Dp-brane to be $p/2$ copies of the gauge field configuration on T^2 described in Sec.5, that is,

$$[Z_{2i-1}, Z_{2i}] = if\mathbf{1}, \quad i = 1, \cdots, p/2, \tag{41}$$

where $f = 1/2\pi K$. This defines a twisted $U(K^{p/2})$ bundle with unit p-brane charge: $\frac{1}{k!(2\pi)^k}\int tr(F^k) = 1$ for $k = p/2$.

We focus our attention on the first copy of T^2. Let $y = y_1 + iy_2$ and $\bar{y} = y_1 - iy_2$. The equations of motion for them are $(D^2 - 2f)y = 0$ and $(D^2 + 2f)\bar{y} = 0$, where $D^2 = \sum_{\mu=1}^{p} D_\mu^2$ for a Dp-brane. Since the spectrum of $D_1^2 + D_2^2$ is shown to be $\{-f, -3f, -5f, \cdots\}$ in Sec.5, the spectrum of $(D^2 + 2f)y$ is $\{-(p/2-2)f, -p/2f, \cdots\}$ and the spectrum of $(D^2 - 2f)$ is purely negative for any p. It then follows that there is a zero mode for y only if $p = 4$.

The equation of motion for the fermionic mode is decomposed into $p/2$ equations for a Dp-brane: $(D_{2i-1} + \Gamma^{2i-1}\Gamma^{2i}D_{2i})\theta = 0$, $i = 1, \cdots, p/2$. Obviously the solution of θ is simply the product of the solution (30) for each copy of T^2 and it is of negative chirality on each T^2 so that $\Gamma^1 \cdots \Gamma^p \theta = i^{p/2}\theta$. The index theorem [26]

$$ind(E, D) = (-1)^{m(m+1)/2} \int_M ch(\oplus_r (-1)^r E_r) \left. \frac{Td(TM^{\mathbf{C}})}{e(TM)} \right|_{vol} \tag{42}$$

can be used to show that there is only one fermionic zero mode if one can show that there is no zero mode of the opposite chirality: $\Gamma^1 \cdots \Gamma^p \theta = -i^{p/2}\theta$. Indeed one can consider the spectrum of the Dirac operator squared $(\Gamma^\mu D_\mu)^2 = D^2 + \Gamma^{\mu\nu}[D_\mu, D_\nu]$. The spectrum of D^2 is given above and the spectrum of the second term is $\{-(p/2)f, -(p/2-2)f, \cdots, (p/2)f\}$. It follows that the zero mode must have negative chirality on each T^2.

The result is therefore that for a 0-p string there is always a single fermionic zero mode and there is no bosonic zero mode except for the 0-4 string. This is in agreement with the results of string theory.

In Sec.6 we showed that the SUSY property of the zero modes of a 0-4 string follows from that of the off-diagonal blocks. The SUSY transformation of the zero mode for a 0-8 string can also be derived from the SUSY of SYM. Now let us show that the bosonic zero modes derived from the fermionic zero modes using the SUSY transformation as in Sec.6 merely vanish. Note that the $SO(1,9)$ symmetry is decomposed into $SO(1,1) \times SO(4) \times SO(4)$ for the 0-8 string, where the D8-brane has two D4-branes with it. The Γ matrices can be taken as in Sec.6. A 10-dimensional spinor $\theta_{\pm\alpha\beta}$ has three indices corresponding to the three factors of orthogonal group. The SUSY preserved by the D8-brane background is parametrized by ϵ with positive or negative chirality on both factors of $SO(4)$; and the zero mode of θ has negative chirality for both $SO(4)$. Since a given Γ-matrix can change the chirality of only one of the two copies of $SO(4)$, the SUSY transformation $\delta y_\mu = i\bar{\epsilon}\Gamma_\mu\theta$ vanishes for θ being the zero mode and does not give nontrivial solutions to y.

It is easy to see that the fermionic zero mode is given by $\theta_{+\dot{\sigma}\dot{\rho}} = \chi_+ \lambda^0_{\dot{\sigma}\dot{\rho}}$, where the λ^0 is the zero mode solution on T^8. The SUSY transformation of the fermionic zero

mode is trivial ($\delta\chi = 0$) because all y's vanish. This agrees with the proposal of Horava [15] to interpret the zero modes as the extra fermions needed in the heterotic matrix theory [27, 12, 14].

If the gauge field configuration for a D4-brane is not (anti-)self-dual, it is found that [24] the configuration is not stable because of the existence of negative energy states in the perturbation of the gauge fields. Therefore all states tend to decay into an (anti-)self-dual state with the same topological charge. In our consideration of the off-diagonal blocks y, the spectrum of the operator $(-D^2 \pm 2f)/2$ corresponds to the energy of states on the 0-p strings. For the 0-2 string the lowest energy of y is $-f < 0$ and it signifies the instability of the system. This is consistent with the fact that the 0-brane tends to distribute uniformly over the D2-brane [28] to form a bound state. For D4-branes corresponding to (anti-)self-dual configurations the lowest energy of y is 0, but otherwise there would be states with negative energy equal to $-|f_1 - f_2|$ where $F_{12} = if_1$ and $F_{34} = if_2$. In general for a 0-p string the lowest energy is the minimum of $\{(\sum_{i=1}^{p/2} f_i - 2f_j)/2 \mid j = 1, \cdots, p/2\}$. While there are D2-branes inside the D4, D6 and D8-brane configurations we considered, the interaction between the Dp-brane and D0-brane includes the attraction from the D2-brane and repulsion from the D6. (The D0-brane is marginally bound to a pure D4-brane.) [29] If the lowest energy is positive, zero or negative, it means that the configuration is stable, marginally stable or unstable, respectively. In the cases of D6 and D8-branes, the negative modes are due to the D2-branes inside the higher branes. Take D6-brane as an example. Let $f_i > 0$ and $f_1 = f_2$, then there is a D4-brane wrapping around the first two tori. If $f_3 > 2f_1$, there is a negative mode of energy $2f_1 - f_3$. Apparently, the attractive force due to the D2-brane on the third torus overcomes the repulsive force of the D6-brane.

Generically for Dp-branes there is a Fock space $\mathcal{H}_i$ for each T^2, where $-\mathcal{D}_i/\sqrt{2f_i}$ and $\bar{\mathcal{D}}_i/\sqrt{2f_i}$ act as the creation and annihilation operators. After imposing the constraint (20), the spectrum of y_μ is found to be

$$\{(\sum_{j=1}^{p/2} f_j - 2f_i)/2,\ (\sum_{j=1}^{p/2}(2n_j+1)f_j + 2f_i)/2 \mid i = 1, \cdots, p/2;\ n_j = 0, 1, 2, \cdots\}. \qquad (43)$$

8 Discussions

In this paper, we have presented a general framework and a systematic analysis for the zero modes in the off-diagonal blocks in M(atrix) theory. More concretely, we have shown how to determine the number of zero modes by index theorem and surviving supersymmetry, and moreover we have determined the quantum numbers of the zero

modes, including the chirality of the fermion zero modes. These quantum numbers are nontrivial, and crucial for us to show the agreement with string theory predictions on open $p-p'$ strings stretching between D-branes, providing one more check for M(atrix) theory. Previously in Refs. [34, 35, 36], in the middle of computing the effective potential between a D0- and Dp-brane, the energy levels of the off-diagonal block have been determined using a slightly different representation for the Dp-brane. But the zero modes were not mentioned and identified, and their quantum numbers were not studied.

Now let us discuss the significance in M(atrix) theory of the zero modes residing in the off-diagonal blocks. First we have shown in Sec. 6 that for the case of a longitudinal fivebrane, the degrees of freedom associated with the off-diagonal zero modes naturally provide the extra degrees of freedom put in by hand by Berkooz and Douglas, ref. [9]. And we have checked that the action they postulated are derivable from the M(atrix) theory action, with a possible D-term. Indeed, in this case, besides the right topological number (or brane charge), the correct counting of zero modes we found in Sec. 6 is crucial for justifying our identification of a longitudinal 5-brane with proper instanton configuration on T^4 rather than on S^4. Also the correct number of zero modes is crucial for a check of the correct tension and R-R charge for the longitudinal 5-brane. It is argued in [9] that upon integrating out 0-4 strings the long range force between a longitudinal 5-brane and a probe supergraviton is generated. If we had a different number of zero modes we would obtain a gravitational field with a different magnitude for the 5-brane. Also as shown in Ref. [9], the R-R charge of a longitudinal 5-brane manifests itself in the Dirac quantization of a membrane moving in its background. By realizing the membrane as a collection of D0-branes, the zero modes on the 0-4 strings would induce fields on the membrane. It is the fermion zero mode χ induced on the membrane that is responsible for generating the Berry phase. In fact, by T-duality the induced zero mode is related to the zero mode on a 0-6 string. Our results in Sec.7 provide the proof for the existence of a single chiral zero mode necessary for the correct Berry phase. Had we had two zero modes, we would have generated twice the correct Berry phase, and therefore twice the R-R charge. As pointed out in Sec.6, in the background of k instantons, there are k fermionic zero modes. The Berry phase is then k times as large, and this signals that there are k units of R-R charge.

Upon compactifying on a 5-torus T^5, instanton strings will appear in the spectrum. These are part of constituents of some 5D black holes [30, 10]. A 5D black hole is described by a long instanton-string carrying momentum. Probing the black hole with a supergraviton, one expects that the corresponding static potential as well as the

velocity dependent force are generated by integrating out the off-diagonal blocks. This is shown to leading order in Ref. [31], where the full $5+1$ massive modes are integrated out. It is an interesting question whether the relevant terms can be generated by integrating out only the zero modes discussed in Sec.6.

It should be interesting to compare our result with the work of Ref.[32]. There a D5-brane is interpreted as an instanton inside 9-branes. The probe is a D1-brane. The 1-5 string sector is constructed with the D-brane technology. A (0,4) sigma model in an instanton background [33] results from integrating out the massive 1-5 strings. There are two differences between the case under discussion and Ref. [32]. First, it is crucial for us to work with T^4, only then we have the correct number of zero modes. Second, the $SU(2)_R$ symmetry in our problem comes from the $SO(4)$ of T^4, while the $SU(2)_R$ of [32] does not act on the gauge field, since the gauge field carries an index transverse to the D5-brane.

Finally, the origin of $p-p$ strings is also easy to see. When $p=2$, the world-volume action is written down [7]. For $p=4$, one can consider zero modes of the fundamental of $SU(2)\times SU(2)\in SU(4)$ in the background instanton number 2 solution with a gauge group $SU(4)$. It is important to embed the instanton to $SU(4)$ rather than to a single $SU(2)$, in order to be able to higgs the off-diagonal strings. By an index theorem, there are 16 real bosonic zero modes. 8 of them are W-bosons, and the other 8 are massive Higgs. The 8-8 strings are discussed in [15].

We have identified the stretched strings between a p-brane and a p'-brane as just the zero modes of off-diagonal blocks; one would like to ask what about the massive modes of p-p' strings in the M(atrix) theory. On one hand, for short open strings these modes, similar to the massive modes of short open 0-0 strings, are simply absent in the M(atrix) model by postulate. (It would be interesting to examine the long strings in M(atrix) theory ending on p p'-branes.) On the other hand, it might be wise to leave the possibility open that these massive modes on short strings and other massive modes, such as KK modes in a higher dimensional super Yang-Mills theory could be physically relevant so that their inclusion is necessary to make the high dimensional theory well-defined in the UV regime. We leave investigation of this issue to the future.

How about the higher modes of the off-diagonal blocks? Could their effects approximate to those of the massive modes of p-p' strings? We do not think so, since the latter is graded by α', while the former is determined by the scale of the background field and the scale of the torus. The modified M(atrix) model in the presence of a longitudinal 5-brane proposed in Ref. [9] should be viewed as a low-energy effective theory of the fundamental M(atrix) model, in which the higher modes of the off-diagonal block

are ignored. Indeed, in this case the zero-modes of the off-diagonal block dominate the low-energy physics, since surviving supersymmetry makes the contributions of the higher modes cancel in the leading order at large distances.

Although in this papers we have used the IIA language for brane names, the above discussions are of M theory nature. It may be amusing to consider an alternative IIA theory which is obtained by compactifying the ninth direction and interchanging the role of the ninth and eleventh directions. What we called D0-branes above become short strings, which are also understood as D0-branes by introducing unit electric flux to the corresponding matrix element [19]. We leave the complete analysis and related topics for the future.

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