Mapping Gregory Bateson's Epistemology to Nonlinear Dynamic Systems Theory: Dynamic Form and Hierarchies of Knowledge

Thomas E. Malloy[1*] and Gary C. Jensen[2]

[1]University of Utah, Department of Psychology, 380 S. 1530 E., Room 502, Salt Lake City, UT 84112-025; malloy@pscyh.utah.edu.

[2]University of Utah, Department of Psychology, 380 S. 1530 E., Room 502, Salt Lake City, UT 84112-025; jensen@pscyh.utah.edu.

[*]Correspondence can be directed to this author, as the primary contact.

Abstract

Gregory Bateson construes mental process as the flow and transform of differences in a system whether the system be a single human or a complex ecology. Stuart Kauffman uses NK Boolean systems to model the self-organization of order in biological evolution. Because the Boolean base (0, 1) maps to Bateson's idea of difference, we are able explore new implications of Bateson's epistemology using a Boolean system. This paper will map Bateson's difference-based epistemology to nonlinear dynamic systems theory (NDS); more specifically we will use a Boolean simulation model (E42) to examine and extend his deep epistemological insight that the relations between double (multiple) descriptions generate new knowledge where, following Bateson's definition of mental process, a "description" is a specific flow of differences in a network. This connects Bateson to mathematical developments in NDS theory and makes explicit implications derived from Bateson's work. We will present perceptual demonstrations of how the relations between double descriptions generate knowledge in two very different realms: Form perception and hierarchies of knowledge. In the first realm, we will propose a perceptual model in which dynamic visual form self-organizes from the phase relations between two such descriptions. Using Java applets generated by the freely available, open-source E42 simulation software, we will demonstrate perceptually how dynamic form perception emerges from the phase relations between what can be called systemic processes (the flow of differences in the system itself) and representational processes (the flow of differences that generate perceptual experience of the system's behavior). Moreover the relations between systemic and representational processes will be of two kinds: visual forms that code fundamental characteristics of the system itself versus visual forms that arise solely from the relationship of systemic process and representational process; the latter are not map-able to any characteristics of the system *per se*. We will call the first kind of form "fundamental forms" and the second "derivative forms." Regarding the second realm, Bateson proposes that differences themselves differ and that categorizing the differences in differences produces a hierarchy of knowledge. We will demonstrate that taking differences in differences in the flow of differences in a Boolean system results in perceptual hierarchies in visual perception. In this second realm, the first description is defined as any flow of differences in a system while the second description is defined as the flow of differences that are generated by taking the differences in the differences in the first flow. The perceptual hierarchies (in the context of the Boolean model) will allow us to define precisely the distinction between ideas about the evolutionary processes that generate the emergence of biological forms in evolution and ideas about the mental processes that generate the hierarchies we use to categorize those biological forms (e.g. Chordata, Aves, Corvidae, Ravens).

If you have hard copy, all the links can be found by searching **www.pysch.utah.edu/dynamic_systems** (there is an underscore between dynamic and systems). Preview dynamic form applets at:

www.psych.utah.edu/dynamic_systems/exemplar1
at: www.psych.utah.edu/dynamic_systems/exemplar2
and at: www.psych.utah.edu/dynamic_systems/exemplar3
Certain browsers in combination with certain operating systems require a Java plugin. (Click here to Get Java plugin or go to http://java.com/en/)
Keywords: Epistemology; Mental Hierarchies; Form Perception; Bateson, Dynamic Systems.

Introduction

Gregory Bateson (e.g., 2002, p. 85ff; 2000, p. 457-460) construed knowledge to be the propagation of "difference" in a complex network, noting (2000 p. 460) that "the transform of difference traveling in a circuit is an elementary idea." In a Boolean system the idea of difference is coded as 0 and 1. Kauffman (1993) developed NK Boolean computer simulations as a way explore how the structure of genomes might self-organize into emergent form (see Kauffman, 1995, p. 76 for a simple, concrete example). The N in NK Boolean systems refers to the number (N) of abstract entities called nodes; in Kauffman's simulations N was very large, as high as 100,000 (1995, p. 83). The K refers to the number of inputs (from other nodes in the network) that each node has. Kauffman states that "While this [a Boolean model] is surely an idealization, we can extend it to networks of genes and their products interacting with one another in enormous webs of regulatory circuitry," (1995, p. 99). This reasoning will parallel our own, below, when we speak about epistemological process as grounded in neurology; the Boolean model is not meant to model neurology except in the most idealized and abstract way. The Boolean model is intended to model Bateson's verbal statements about differences flowing in networks as the basis of knowing. Another connection between the Boolean models developed by Kauffman for evolutionary process and Bateson's ideas is that Bateson includes genetic activity as a part of mental process, stating that "the phenomena we call thought, evolution, ecology, life, learning and the like occur only in systems that satisfy these criteria," (2002, p. 86). Mind and nature are, as his book title (Bateson 2002) states, a necessary unity.

A major conceptual thrust of nonlinear dynamic systems theory in general and Kauffman in particular is that such systems self-organize. In this paper we will examine coupled systems which are a rather general subset of dynamic systems. A coupled system is one that has multiple (at least two) processes and that the outputs of some processes are the inputs for other processes (in what Bateson calls a "circular chain of causality," 2002, p. 86ff). To speak clearly of the implications coupled nonlinear systems requires a distinction in levels of analysis. At one level of analysis, we will refer to the coupled processes as "generating processes" (akin to Holland, 1998); in our discussions the generating processes will be the N nodes. At a higher level of analysis is some phenomenon that emerges from the the interactions of these generating processes, In this paper this phenomenon will by visual form. For example, in a seminal paper Turing (1952) referred to process by which forms (e.g., camouflage patterns in an animal's fur) self-organize from the interactions of coupled nonlinear processes as "morphogenesis." This self-organization of form is a pivotal idea and leads to new approaches in many theoretical realms. Kauffman and Turing both propose that biological form emerges spontaneously (see p. 75, Kauffman, 1995 and Turing, 1952) from the interaction of nonlinear processes in coupled systems. Kauffman's work led him to conclude that "sources of order in the biosphere will now include [natural] selection and self-organization." In Kauffman's view, natural selection is seen to act on those forms that self-organize from the coupled interactions of genes, selecting against some of them and favoring others. This rescues natural selection from having to take vast random walks to arrive at the great diversity of form in biology since form will self-organize spontaneously from genetic interactions; but those forms survive or do not survive based on the processes of natural selection.

We leave biologists to decide about the processes that produce order and form in evolution and return to a Batesonian epistemology and to the emergence of visual form from coupled generating processes in a system characterized by mental process (see Bateson, 2002, chapter 4 for heuristic criteria for defining mental process). Particularly we are interested in how visual form and knowledge in general emerges from the interaction of lower-level processes which we will discuss in terms of double description in the next section. While we leave evolution to biologists, we point out, consistent with Bateson's approach, that what we are going to say about human visual form perception is deeply grounded in at least one modern approach to biological evolution via Kauffman's Boolean systems. For our purposes we don't use Kauffman's programs but have developed our own Boolean simulation program, E42.

Double Description. A fundamental aspect of Bateson's epistemology (2002, p. 27) is the metaphorical distinction between map and territory: A map we use to navigate around Sonoma, CA is not Sonoma, CA itself. In the same way a sentient being's knowledge of a thing is *not* the thing itself (*ding an sich*, Bateson, 2000, p.460). Moreover, in a Batesonian framework maps (knowledge) are of their very nature flows of difference; what gets onto a map from the territory is a difference (2000, p. 457). A summary of the above points as a visual representation of the map/territory relations can be seen by clicking here. Given that foundation, knowledge emerges from the relationship between two or more descriptions of the territory. One example offered by Bateson (2002, p. 64ff) is that the immediate perceptual experience of spatial depth emerges from the relationship between the two slightly different descriptions of the two eyes in binocular vision. To state this in more general terms, a mental system has multiple of flows of difference (multiple descriptions) of the territory it is mapping (see Bateson, 2002, chapters 3 and 5) and it is from the *relationships* among these multiple flows of difference that what we call knowledge arises. This paper will focus on how this succinct Batesonian idea leads to deep and complex ramifications in visual form perception. The core idea is double description; it is the idea that knowledge arises from the relations between two descriptions, that is, between two flows of differences in a network. We will use this idea twice in this paper in two slightly different ways. The first will generate a model of dynamic form perception; the second will generate hierarchical categories of visual forms.

Availability of Interactive Materials. All simulations and applets are open source and freely available. If you have hard copy only, all the links can be found on **www.pysch.utah.edu/dynamic_systems** (there is an underscore between dynamic and systems). An HTML version of this paper with all figures is also available at that url. Click on "Page Contents" and look for tutorials or other materials. The applets for dynamic form are at:
www.psych.utah.edu/dynamic_systems/exemplar1
www.psych.utah.edu/dynamic_systems/exemplar2
www.psych.utah.edu/dynamic_systems/exemplar3
Certain browsers in combination with certain operating systems require a Java plugin. (Click here to Get Java plugin or go to http://java.com/en/)

The Emergence of Visual Form in a Boolean Network

We will not present the mathematical details of Boolean systems in this paper; the mathematics (truth table logic) is simple and straightforward but takes a fair amount of space and some effort on the part of readers not already familiar with the ideas. Details can be found on an informal online tutorial (click here) and in Malloy, Jensen and Song (2005). That said, we will present a brief overview of the major conceptual points of Boolean systems particularly addressing how the flow of Boolean process can be represented as visual form. Suppose we have a very simple Boolean system that has only four nodes (N=4) each node taking input from two other nodes (K=2). Thus this is an N=4, K=2 Boolean system. At any point in time (called iteration T) each of the four nodes will be in one of its two states; these two states can be named ON, OFF, or 0,1, or WHITE, BLACK, or whatever we wish. To describe the system's state at time T we define a state vector, **S**(T). The state vector, **S**(T), is an ordered list of the states of the four nodes from the first to last. Thus if the first node is ON, the second OFF, the third OFF, and the fourth ON at time T, then we can write **S**(T) = {1001}. For convenience in publishing, we have written **S**(T) as a row vector but the reader is asked to imagine it as a column vector (with the left most value in the row vector, which represents the state of the first node, being the top value in the column vector).

A Boolean system iterates in discrete moments of time; that is, at every moment in time each node looks at its two inputs (K=2) and using a logical truth table (that defines various logical relations among the inputs such as AND, OR, XOR, etc.) decides if it will be ON or Off on the next iteration (T+1). Put another way, the logical relations among the inputs (the states of other nodes at moment T) determine if a particular node is ON or OFF on the next iteration T+1 (for more details click on the tutorial). Thus at T+1 the system will move to another state vector; in the small system shown in the tutorial **S**(T+1) must equal {1101}. Please note that we are skipping the derivation by which **S**(T) = {1001} moves to **S**(T+1) = {1101}. What is important is that the system is deterministic, that there is a derivation by which we can demonstrate how a state vector at T must move another state vector at T+1 based on truth tables which determine how each state vector at T+1 flows from the previous state vector at T. Thus for example in the tutorial if **S**(1) = {1001} then **S**(2) must equal {1101}, **S**(3) = {1111}, **S**(4) = {1011}, **S**(5} = {1001}, **S**(6) = {1101}, etc. Look at that sequence carefully; you will notice that **S**(1) = **S**(5) and **S**(2) = **S**(6), etc. That is, as the system generates state vectors across time the state vectors repeat themselves in the same order every four iterations; since this flow of differences is deterministic it will cycle endless through same state vectors. This is called an attractor cycle. Vectors are mathematically one-dimensional and so the flow of state vectors is like a line or edge or boundary moving through time. Note that the nodes are not things but sets of logical relations between the states of other nodes; a node is simply a truth table for deciding what its own state will be based on the relations between the states of other nodes. The system, simply as a consequence of its logical relations, self-organizes into emergent structural characteristics, most notably attractor cycles. The emergent structure of the example derived in the tutorial is given in Figure 1 which shows that this system self-organizes into three basins. A basin consists of an attractor cycle along with all the tributaries (transients) that flow into the attractor. For example, the attractor cycle for Basin 1 in Figure 1 is the endless repeating sequence of state vectors {1001} ==> {1101} ==> {1111} ==> {1011} which we labeled **S**(1), **S**(2), **S**(3), **S**(4) above. The other vectors in Basin 1 (e.g., {0111}), if they ever do occur pass directly into the attractor cycle and are never repeated, thus the name transient or tributary. It will be useful to define a variable L for later use. L is the length (in number of iterations) of the attractor cycle. In this example shown in Figure 1, L = 4 for the attractor cycles of basins 1 and 2 and L=1 for the attractor cycle of basin 3. Here we return to the distinction of levels we discussed above. In this case the coupled generating processes are the logical relations defining the nodes and the higher level phenomenon is the basin structure shown in Figure 1.

Insert Figure 1 about here

The important conceptual point is that this basin structure deterministically emerges from the coupled generating processes (the interconnected nodes). If the connections between the nodes are changed, or if the logical relations among the nodes are changed, or if anything is changed, then a different basin structure will emerge. But given the exact generating process, then the system will self-organize into the basin structure that is shown in Figure 1. Given that a basin structure has emerged in a system, it is worth noting that once in a basin, a system will flow to an attractor and then cycle in that attractor endlessly unless it is perturbed from the outside or unless it perturbs itself (supposing that a system has self-perturbing capabilities which most complex systems do). The systems in E42 are self-perturbing but that is a topic not covered here. At a minimum, a perturbation consists of changing the value of one node in one state vector. For example, look on Figure 1 and notice that if you are in Basin 1 and the state vector for some specific moment of time is {1001} and you change the first node from 1 to 0, the resulting state vector will be {0001} which is found in Basin 2. Once in Basin 2, the system will stay until perturbed again. Thus systems can shift basins of attraction through internal or external perturbations.

Now let us turn to representing attractor cycles as visual forms. As we noted we can express the binary distinction as 0,1, or ON, OFF, or BLACK, WHITE. Let us replace 1's in our state vectors with black squares and 0's with white squares. And let us rotate the state vectors from their easily typed row format to columns. Thus the sequence of state vectors mentioned above [{1001}, {1101}, {1111}, {1011}] rotated to be columns and with 1's = BLACK and 0's = WHITE, becomes Panel A in Figure 2. Examine Figure 2, Panel A, and note that if you rotate {1001} and replace 1's with BLACK and 0's with WHITE you will get the first column of Panel A (with the first node at the top). Panel C shows Basin 2 from Figure 1 and Panels B and D show the perceptual effects of starting the visualization at different points in time. Panels E through H show "tiling effects;" that is, if you let the system run through an attractor cycle multiple times and then visualize it, the resulting visual form will have Gestalt characteristics akin to those resulting from tiling a floor or wall.

Insert Figure 2 about here

Returning to the larger conceptual point, flows of difference in a Boolean system, when the differences are expressed as black and white squares, self-organize into temporal visual forms generated by something like an ever-changing line moving through time. These visualizations are conceptually parallel to those made by Turing in his morphogenesis paper; indeed, the tutorial shows how this type of visualization can produce camouflage-like zebra stripes. The basic can be illustrated with Figure 2.

Dynamic Form

Think of sitting on the ocean beach watching the waves roll up on the shore and think of the complexity of the moving patterns as they play across each other making dynamic patterns within dynamic patterns while sheets of water race from the surf up to your feet. These patterns playing across patterns making other patterns is an example of what Bateson called moire patterns (2002, p. 74) which he defined as a combination of two patterns producing a third pattern. As another example think about the flow of a swiftly falling rapids on a river. A river runner (boater or kayaker) standing on the bank looking at the rapids will see standing waves (large and small), holes (generally to be avoided) and small sub-rivers where the river's moire current snakes across the surface and hints at ways a kayak might get through. The boater's success in the rapids depends critically on extracting dynamic patterns and sub-patterns from what at first may seem to be the chaos of the thundering fall of water. This pattern extraction becomes even more complex once the river runner puts in and is sucked into the rapids since the boater will be moving in relation to river's patterns. How are coherent patterns extracted from such complexity? River rapids or autumn leaves shaking in the wind and partially masking a herd of deer moving down a steep mountainside are examples of a kind of form perception that is of central interest to, at the very least, large mammals. But these patterns are difficult to capture in a way that can be easily shared among people. Taking a still photo leaves most of the moving pattern out of the picture so to speak. And words often fail when we use them to point to a subtle aspect of such patterns and we are left with saying, "Look at that cloud that looks like a camel." There is no "camel" (*ding an sich*) in the cloud (and indeed our friend may or may not seen anything in the cloud that resembles a camel). The camel we see in the cloud is a co-construction of the dynamic movement of the cloud and the dynamics of our own perceptual processes. We will use an NK Boolean simulation as a model of Bateson's idea of double description as a way to demonstrate how a living being may co-construct dynamic forms through the relationship between two flows of difference (two descriptions). As a preview, examine Exemplar 3 which demonstrates this approach experientially with wave-like dynamic forms and sets some frames to motivate the details that follow. If necessary for your operating system/browser combination, get the Java plugin. Recent Apple computers using the Safari browser do not require the plugin. For PC's we recommend the Firefox browser with the Java plugin (http://java.com/en/).

Kauffman, as noted above, indicated that his Boolean models were extremely idealized models of genes. In the same way our Boolean dynamic systems are not models of neural activity except in the most idealized way. The simulations are intended to model Bateson's difference-based epistemology. That said, like Kauffman did with genes, we will make a broad parallel to neural activity in order to have a concrete context from which to generate examples. Suppose in this idealized sense, the neurons of the retina can be idealized as as a system of interconnected Boolean nodes. A Boolean node at a particular moment in time is either 0 or 1; the neuron at a particular moment is either not firing or firing. Assuming that neurons are interconnected then their on-off patterns of firing can be construed to self-organize into attractor cycles in some way abstractly related to Boolean systems. In particular we are interested in framing retinal neurons as capable transforming differences in the territory (movement, color, etc.) into a flow of differences in a neural circuit that can reverberate in attractor cycles like those found in Boolean models. We therefore model the retina as a net of interconnected Boolean nodes (again, see the Map/Territory illustrations) which transform differences in territory into differences within the knower. Bateson (2000, p. 460) discusses in detail the distinction between external (may be analogue) and internal flows of difference. Malloy, Jensen and Song (2005) offer a more detailed mapping of Bateson's epistemology to Boolean systems.

As above, we will not describe the mathematical details of the double Boolean flow of differences that constitute our double description; an easy and informal tutorial is available (click here). But we will present the main ideas. Examine Figure 2 once again. Notice that Panel A and Panel E show the same attractor cycle but that Panel A shows that cycle for four moments of time (iterations) while Panel E shows that cycle across sixteen iterations. Another way to say this is that Panel A shows the system passing only once through its attractor cycle while Panel E shows it passing through the attractor cycle four times (the pattern repeats four times). This is a fundamental question in representation: What is the temporal chunk size? If we have a flow of differences and we want to represent that flow we are required to ask how long a segment of the flow am I to represent? It is as if we have a window (or snapshot) into the flow of process and that window by its nature must have a width. We can show only one iteration through the window or four iterations (Panel A) or sixteen interactions (Panel E), or whatever length we want. But the length of the window into the flow must be specified. We will call the number of iterations in the window W. So Panel A is a window of size 4 (W=4) into the flow of state vectors in Basin 1 and Panel E is window of size sixteen into the same basin (W=16). Panels C and G in Figure 2 show W=4 and W=16 for the flow of differences in the attractor cycle of basin 2. Another way to think of this is that the window is a static snapshot of system's process and that snapshot can be was wide or narrow as we want it to be.

System Process versus Representational Process. Recall that we defined the length, L, of the attractor cycle as the number of iterations it takes to cycle fully through the attractor (that is, for the sequence of state vectors to begin repeating itself); in basins 1 and 2 of the running example L = 4. Note that L is an emergent characteristic of the dynamic system because the attractor cycles themselves are emergent characteristics of the system as argued above. In contrast, W is a characteristic of a second system; the second system represents the emergent characteristics (attractors) of the original system. The original system could exist without any such secondary system to represent it; undoubtedly in nature many systems exist without any intrinsic representation of their process. W arises because we want to have a representational window into the flow of the system's process. In epistemological theory these are very different kinds of variables, L indicating something about the process of the original system itself and W indicating something about the processes by which the emergent dynamics of the original system are represented. In our Boolean simulations that distinction is between mathematical algorithm generating an ongoing systemic process in a computer's CPU and the representation of that processes by painting what is going on in the CPU to a monitor. For purposes of representation to the screen, the E42 program tracks the systemic process (a series of state vectors) for W iterations; then E42 paints those state vectors to the screen (as a snapshot of black and white squares), then it tracks the next W iterations and paints another snapshot over the previous one, and so on. What is seen on the screen is a series of snapshots presented, rapidly, one after the other so that we can see the ongoing flow as the system runs in the CPU. Note that cinematography works by similar principles; movies present the viewer a series of slightly different still frames, briefly and rapidly, to create motion. A major difference between cinematography and our simulations is that movies are recordings motion. Whereas in our simulations nothing is recorded; they are real-time representations of process and real-time interactions with the ongoing process are possible. This is the difference between the passive nature of old media and the interactive nature of new media. On a technical note, most computers calculate Boolean functions faster than monitors can accurately paint so the simulation program has a "Delay" control that allows the user to slow down the speed of the system (and consequently of the painting).

Fundamental Frequencies. In the general case a system can self-organize in a way that has sub-cycles within the attractor cycles (circles within circles). The on-off firing of some nodes fall into patterns that repeat themselves more frequently than the attractor does as a whole. In Figure 2, Panel D, for example, notice that (counting from the top) the first node repeats its on-off pattern every two iterations as does the fourth, bottom, node. But if you look at rows 2 and 3 you will see that nodes 2 and 3 fire only once every four iterations. Nodes 2 and 3 are responsible for an attractor cycle length of L=4. But node 1 (and node 4) fire at a faster frequency so we will use the notation sub-L to describe the short length of their faster frequencies. In the case of these two nodes sub-L=2. Both L and sub-L's emerge as formal characteristics of the system and we will refer to both as the fundamental frequencies of the system.

Modeling Double Description. Now we can describe more precisely the nature of the double description that will produce dynamic form. The first description is the flow of differences within the original system (defined rigorously by a sequence of state vectors) and the second description is a parallel flow of snapshots of the original flow. That is, we have two flows of process (two descriptions), systemic and representational; and visual form arises in the relationship between the two. We will demonstrate that dynamic visual form is a co-construction of systemic and representational flows. Let us expand these idea,emphasizing two points. First, regarding representational process, there will be a series of snapshots of the system's flow of state vectors, each snapshot of length W and each occurring rapidly, in order, one after the other. As noted, this rapidly occurring series of snapshots works like a movie; one still frame is followed quickly by another.. The second point is that we can adjust the length, W, of the snapshots in real time as we go. Doing so, in mathematical terms, adjusts the phase relations between the frequency of the system's attractor cycles and the frequency of the recurring representational process. We will show a simple model in which dynamic form emerges from the the phase relations between systemic and representational frequencies. Let us map this Boolean model to Bateson's epistemology using an idealized notion of the retina. As Bateson points out, somebody could (theoretically) go out and put the on-off responses of individual neurons in the retina onto a piece of paper (2000, p. 460); to keep up with ongoing changes in the retinal system across time such a person would have to draw a series of snapshots and would consequently end up with an ordered stack of papers. This Batesonian thought-experiment maps directly to our series of snapshots.

Apparent Stability and Apparent Motion. We now place these two descriptions in relation to each and define their phase relationship. Figure 3, which has time (iterations) on the horizontal axis and two nodes of a hypothetical Boolean system on the vertical axis, shows what happens when L is not equal to W, specifically when L=4 and W=5. To be concrete, let B (black) indicate that a node "fires" and W (white) indicate that it does not in Figure 3. Notice in Figure 3 that the top node is cycling BWWW every four iterations (it fires once every four iterations) while the bottom node is cycling BWBW. For the whole hypothetical system (of N=2 nodes) L=4, but for node 2 sub-L=2. The vertical cross-hatched bars indicate the divisions between a series of three snapshots (each capturing W = 5 iterations of the system's process). More formally the system itself is cycling with frequency of 4 iterations and the representational process is cycling with a frequency of 5 iterations; the processes are out of phase. As a result of this out-of-phase relationship, for the top node, the position of its single B (fire) appears twice in the first window and then appears to move backwards relative to the frames defined by subsequent two snapshots. This is the same out of phase relation that causes a wagon wheel to turn backwards in movie. It is called apparent motion and is generally described as an illusion. In contrast, within the framework of this model apparent motion is not caste as an illusion but as a process central to form perception. When W = L, that is when the frequencies of system and of the representational process are in phase, the same snapshot will be painted over itself over and over and the dynamic pattern appears to stabilize; this is only apparent stability since the system processes are cycling as fast as they always do. But because system process and representational process are in phase the system's process appears to freeze. Thus such a system can create the appearance of static objects in a dynamic relational world and resolves the paradox of how objects seem static in a world of flow. Moreover this apparent stability will occur if W is any integer multiple of L. Also notice that, excluding integer multiples of 4, if W = 2 or any integer multiple of 2 then node 2 (which has sub-L=2) will freeze even while node 1 keeps moving. Thus adjusting phase relations between systemic and representational process allows the possibility of freezing some parts of a system while allowing other parts to be perceived dynamically. You may confirm for yourself that if L>W features of the system process (such as nodes firing) will apparently move forward rather than backward as they did above when L<W. We will now make these points experientially.

Insert Figure 3 about here

Instructions. If necessary, get the Java plugin. In all the following applets begin by pressing the **Use Delay** radio button and then adjust the **Delay Slider** until your particular computer is painting windows to the screen at about **25 to 35 frames per second** (fps). Adjusting the delay (between iterations of the system) is crucial because different computers paint visualizations on the screen at different speeds; so the Delay slider allows you to adjust your computer to paint windows (frames) to your screen at a known rate; apparent motion research has shown that a particularly useful range is between 25 and 35 fps. The **fps readout** is to the right of the **Stop/Play** control bar. The number of fps is a potent variable and you may set it to any value you want by adjusting the delay between iterations. Keep the fps below 65 since most monitors cannot paint accurately beyond about 65 fps. To adjust the delay between each screen-paint, you may either drag the Delay Slider or, for finer adjustments, single-click on the Delay Slider Bar either above or below the Slider.

Each applet loads to a default basin; perceptual experience with the dynamics of the default basin is discussed on the text accompanying each applet. In the sections below we do not repeat the details of these discussions but simply note the major results. Optionally, on each applet, you may click the **Perturb** button which (almost always) provokes the system into a different basin to explore perceptual experiences in other basins. When you perturb the system into other attractors the particulars of your observations will change but the same general conclusions will usually be verifiable; sometimes the new basin will have characteristics that we are not focusing on here and other, interesting, observations may apply. Remember, these are not old media, movie-like set pieces where everything is known. We can set the system's initial conditions to start in a particular basin that we find interesting. But when you perturb you do so pseudo-randomly and there is no way to know where you might go. Some of these systems have hundreds, even thousands of basins, many of which have not been observed by users before; some have only a few basins. When you perturb a system you may find yourself in a basin that demonstrates something new.

Perceiving fundamental dynamics. The first applet demonstrates how the phase relations between the process flows of the original system and the representational system can freeze (or cause to move together in a coherent way) formal characteristics of the original system such as attractor cycles and attractor sub-cycles. Link to Exemplar 1 (by clicking); the page that pops up includes detailed instructions for adjusting phase relations between two flows of Boolean process. These two flows, in Bateson's terms, are a double description of the same event. One flow is systemic; it is the flow across time (iterations) of the Boolean system as it cycles through an attractor. The other is representational; it captures a given number (W) of iterations of the system and presents them visually to the screen. Adjusting the phase relations between these two descriptions allows you to "extract" the fundamental frequency (L) of the systemic attractor cycles as well as systemic sub-cycles (sub-L). When W is equal to L (or any integer multiple of L) the whole attractor freezes even though the system is running. When W is equal to the sub-L (or an integer multiple of sub-L) you the sub-cycle freezes. Thus you can highlight whole attractors or sub-cycles of attractors by freezing them or by making them move coherently together. In summary, adjusting phase relations between to descriptions allows the extraction of the fundamental frequencies of the system.

Forms Derived from Relationship. Link to the Exemplar 2 applet which will demonstrate how visual forms emerge from the phase relations between two descriptions (flows of difference) that are not in any way technical characteristics of the system *per se*. These "derivative forms" only emerge in the relationship between systemic and representational process. Notice that on the web page that contains the Exemplar 2 applet there is, as well as the dynamic representation provided by the applet, also a static snapshot of the the attractor's ongoing flow. The snapshot does not, *indeed cannot*, show these derivative forms we are referring to here because they emerge dynamically from the phase relations between the ongoing systemic and ongoing representational process. The reader can adjust W to change these phase relations to experience how different derivative forms emerge as the phase relations between systemic and representational processes change. We will discuss this below.

Ambiguous Motion. Exemplar 2 (and even more so Exemplar 1) demonstrates another interesting phenomenon: Ambiguous motion. Set W = 77 and observe the fifth node from the top (lined up with a red hash mark). If you run a mouse arrow or the tip of a pen back and forth along the horizontal line of the 5th node you will note that the motion will change directions. There are many examples of ambiguous static figures (e.g., Necker Cubes); this appears to be a more general case in which motion itself changes orientation. With a little practice users can provoke this change of direction of movement with their eyes alone. With W = 83, complex emergent forms with ambiguous motion can be perceived moving either right to left or left to right; once again, you may require a horizontally moving pointer to observe this phenomenon. Very triking ambiguous motion is shown in Exemplar 1 where whole groups of nodes move together to the right or to the left depending on which way you move the cursor across them.

A Simple Model of Dynamic Form. Above, we have used E42 to demonstrate how changing phase relations between the computational flow in a computer's CPU and the representational flow to a computer's monitor produces interesting perceptual experiences. We now turn to a more speculative and risky venture; we propose one possible model that maps the computer simulation we have just reviewed to a Batesonian difference based epistemology. While we are not modeling neural activity (neural network models do that better), let us begin by talking about the retina in an abstract way as a richly connected network through which differences flow. At this point we re-emphasize the map/territory distinction (Bateson, 2002, p. 27) to make it clear that we do not propose to model any aspect of the territory itself; rather we are modeling Bateson's proposal that what gets onto maps from the territory are differences (2000, p. 457) and that knowledge (2002, chapter 3) emerges in the relationships among multiple descriptions (multiple flows of difference). Note that, while we do not model the territory itself, we do assume that the retina is coupled (i.e., entrained) with systemic processes ongoing in the territory (e.g., Turvey, 1990, p. 942) and consequently that retinal dynamics have a useful relationship of dynamics in the territory.

In this spirit then the retinal image is modeled as a discrete dynamic system coupled to the environment. We also propose that form emerges through phase relations between at least two streams of differences which we call, after Bateson, two descriptions. The first description is the retinal image as it flows toward higher centers. As a second description we propose a parallel, representational flow. Thus we propose that dynamic visual form emerges from phase relations between the flows of the first (retinal) and second (representational) descriptions. Another assumption is necessary: The perceptual system must have some mechanism for adjusting the phase relations between these two descriptions. Such a mechanism would allow the extraction of different dynamic forms. Presumably one part of perceptual learning would be learning to adjust these phase relations in context specific ways that both have utility for a person and correspond to social conventions in a particular context.

A profound insight (e.g., see Varela, Thompson and Rosch's concept of enactment, 1993) in modern thinking is that knowledge in general and representation in particular is not a reflection of (or a photo of, or a tape of) what is "out there" but is more usefully described as something that emergences in the relationship between the processes of the knower and the processes of what is known. Representation is not a passive response to the universe; it is an active co-construction, an enactment in Varela, Thompson and Rosch's terms. Knowledge is neither here nor there, it is neither in the territory nor on the map; it is in the relationship between the two. How do we begin to specify, beyond these provocative words, what such statements might mean? In our proposed model the retina is a (binary) dynamic system whose characteristics (attractors, etc.) are entrained with the dynamic system called the ecological context. Whatever is the appropriate model of the ecology's dynamics might be, following Bateson, what gets onto maps (retina in this case) from the territory are differences; thus we model the retina's response as a Boolean system. The retinal Boolean system already entrained to the ecology also entrains itself with a proposed representational Boolean system. To be as explicitly as possible, in our computer simulations we can specify Boolean process running in the CPU and representational process painting to the screen. We noted in Exemplar 2 and Exemplar 3 that visual forms emerge that are not formal characteristics of CPU's Boolean system but are a co-construction of the CPU Boolean system and the representational process of painting to the screen. Thus we propose that the dynamics of the retina is Boolean-like process that is entrained with the ecology around it and additionally with a representational proceeds such that some visual forms emerge relationally and are not actually characteristics of the retinal dynamics per se and therefore not characteristics of the ecological dynamics. The camel we see in the clouds is a co-construction of the light patterns reflected to our eyes from the dynamically systemic flow of turbulence in the cloud *and* of our retina as a dynamic system *and* of a representational dynamic system. Certainly there is no camel in the clouds; much is derived in the relationships among dynamic systems.

Hierarchies of Knowledge: Differences in Differences

Bateson (2000), pp. 463f) proposes a mental experiment for the reader:

> I have said that what gets from territory to map is transforms of differences and that these (somehow selected) differences are elementary ideas. But there are differences between differences. Every effective difference denotes a demarcation, a line of classification, and all classifications are hierarchic. In other words, differences are themselves to be differentiated and classified. In this context I will only touch lightly on the matter of classes of difference, because to carry the matter further would land us in the problems of *Principia Mathematica*. Let me invite you to a psychological experience, if only to demonstrate the frailty of the human computer. First note that differences in texture are different (a) from differences in color. Now note that differences in size are different (b) from differences in shape. Similarly ratios are different (c) from subtractive differences. Now let me invite you... to define the differences between "different (a)," "different (b)," and "different (c)" in the above paragraph.

In this powerful elaboration of his difference-based epistemology Bateson proposes that taking differences in differences will produce a hierarchical structure differences (and differences are the basis of knowledge). The idea of hierarchic levels is a core insight about epistemology for Bateson (e.g., 2000, pp. 248-253). We propose to use the Boolean simulations as a tool for exploring this insight.

TAO. Malloy, Jensen, and Song (2005) describe a method for differentiating differences in Boolean systems using a function they call TAO because it tracks the differences inherent in change over time. Conceptually their approach is very simple and we will consider a simple example here to outline the logic of that approach. In the N=4 example shown in Figure 1, the attractor cycle for Basin 1 consists of the following state vectors: **S**(1) = {1001}, **S**(2) = {1101}, **S**(3) = {1111}, **S**(4) = {1011}, **S**(5) = **S**(1) = {1001}, ..., recursively, forever. We can find the differences in the pattern of differences seen across time in the state vectors. Examine **S**(1) and **S**(2). As time moves from iteration 1 to iteration 2, notice that the first node's value is the same in **S**(1) and **S**(2), that the second node's value changes from 0 to 1, that the third node's value is the same, and that the fourth node's value is the same. So the pattern of differences in the differences from T(1) to T(2) is {same, different, same, same}. In contrast the pattern of differences in the differences between **S**(2) and **S**(3) is {same, same, different, same}. Before we continue the process let us formalize the notation. Let same = 0 and different = 1; and call the operation we just did in our minds TAO-1 (don't worry about what the "1" means for the moment). We will use TAO-1(1,2) to indicate the differences between **S**(1) and **S**(2). So for the comparison of **S**(1) and **S**(2), we write **TAO-1**(1,2) = {0100}. Similarly we can write **TAO-1**(2,3) = {0010} for the comparison of **S**(2) and **S**(3). If we continue with our thought experiment, comparing **S**(3) and **S**(4) we get **TAO-1**(3,4) = {0100}. Finally an attractor cycle is a circle (so **S**(5) = **S**(1)); therefore we only have to calculate one more TAO to completely analyze all the differences among the differences in the flow of state vectors in this particular vector: **TAO-1**(4,1) = {0010}. To fully record the results of TAO-1 for basin 1, we can take those four vectors and make them the rows of a matrix, which we will call the TAO-1 matrix. We could repeat this analysis for the state vectors of the attractor cycle for Basin 2 (see Figure 2). We won't repeat a similar derivation here for basin 2 but such a derivation would result in four TAO-1 vectors for Basin 2: **TAO-1**(1,2) = {1101}, **TAO-1**(2,3) = {1111}, **TAO-1**(3,4) = {1011}, **TAO-1**(4,1) = {1001}. To fully record TAO-1 results for basin 2, we can take those four vectors and make them the rows of a matrix which we can call the TAO-1 matrix for basin 2.

Recursive TAO. Since the output of TAO is a vector of differences (0's and 1's), TAO can be applied recursively to its own output. For example for the analysis of Basin 1 above, **TAO-1**(1,2) = {0100} and **TAO-1**(2,3) = {0010}. Remember that, like the state vectors, the TAO vectors are ordered and correspond to the four nodes of this system. Examine the differences among the four ordered positions in those two vectors; you will get another vector, {0110}, assuming same = 0 and different =1. We formalize this as **TAO-2**. We can also recursively apply TAO to the output of **TAO-2** to get **TAO-3**, and so on. Because **TAO-1** examines changes (in state vectors) over time it is a discrete form of the first derivative; **TAO-2** in turn examines changes in the changes over time (second derivative). We won't go over the details here, but we can write **TAO-2** and **TAO-3** matrices as we did with **TAO-1**. More details as well as a historical perspective can be found in Malloy, Bostic St Clair, and Grinder (2005) and informal descriptions can be found here.

Sorting Attractors using TAO Matrix equality. The four **TAO-1** vectors for Basin 1, [{0100}, {0010}, {0100}, {0010}] can be collated together as the four rows of a TAO matrix for Basin 1. Similarly the four TAO-1 vectors for Basin 2, [{1101}, {1111}, {1011}, {1001}], can also be placed in a **TAO-1** matrix for Basin 2. In the same way, we can create TAO-2 and TAO-3 matrices for each basin separately. Two matrices are equal if and only if every value of every corresponding element in the two matrices are equal. We can sort the attractor cycles for basins into categories based on TAO-1 matrices, placing only those attractors in the same category that have TAO-1 matrices that are equal. This can be done for TAO-2 matrix equality, for TAO-3 matrix equality and so on. Please note that we have not gone over this process in detail; indeed, the small four node example we have used is too simple to provide an example of TAO matrices that are equal. Our intention is to lay out the logic of the analysis we have done. The simple idea is that we can calculate difference in the flow of differences in not just one but several attractor cycles; then we can look for attractor cycles whose flow of differences do not differ. Such equivalence in the flow of differences will produce categories of attractors which, when visualized, we appear similar to each other. A hierarchy of perceptual organization will emerge if we apply TAO recursively, taking differences in differences in differences, as we will demonstrate below.

We now have a way to operationalize Bateson's thought experiment (see quote above) within the Boolean model. We have already discussed how to generate visual forms from attractor cycles. We can use TAO to examine the flow of differences in various attractor cycles and find the differences in differences in differences between attractor cycles recursively as many times as we want. This creates matrices of differences in differences at each level of recursion for each attractor cycle in a system. We can sort the visual forms generated by attractor cycles based on whether their TAO-1 (or TAO-2 or TAO-3) matrices are equal. What could possibly come of this? Frankly we backed into this process partly for other reasons and didn't' expect it to show much. The results surprised us.

In this study three realms of description converge on the concept of hierarchical levels in knowledge: Batesonian epistemology, a Boolean model, and human perceptual experience. We've described relevant aspects of Batesonian epistemology and outlined a Boolean model; now let's turn to perceptual experience. Figure 4 shows visual forms derived from nine attractor basins found in a small N=36 Boolean system. The bottom row (TAO-1) places these basins in categories based on TAO-1 matrix equality. That is, basin patterns 76 and 60 are placed in TAO-1 cat 2 because they have identical TAO-1 matrices; this means that TAO found the differences in the differences (between state vectors) to be identical for forms in this category. Similarly the three basins (89, 74, 95) in TAO-1 cat 4 all have TAO-1 matrices that are equal to each other (but not equal to TAO-1 matrices for basins in other categories). The striking thing (and here is where you check your own perceptual experience) is that the three basins in cat 4 are more similar in human judgment to each other than they are to patterns in other categories. The same is true for cat 2. Notice that for the bottom row, there are six categories, four of them are singletons, having only one instance.

The second row up from the bottom of Figure 4 categorizes visual forms based on equality of TAO-2 matrices. This is a higher level of abstraction in the sense that TAO-2 is the second derivative and looks for changes in the changes detected by TAO-1. These are complex visual judgments but notice that the forms that are collated in TAO-2 cat 1 do go together and those that are placed in TAO-2 cat 2 also go together. This means that within categories the differences in the differences in the differences are identical within each category. In contrast, between categories the differences in differences are different. We are starting to see in perceptual experience something like the hierarchy of differences proposed by Bateson in his thought experiment. The two singletons, basins 67 and 78, are distinct from the other nine patterns and from each other. The third row from the bottom of Figure 4 categorizes visual forms based on the equality of TAO-3 matrices (equality of differences in differences in differences in differences). We now have only three categories, and, while you can find distinctions among the patterns in each of those categories, if you had to place all nine visual forms into three categories, how else would you do it? The question is serious; we are creating a perceptual hierarchy in a Boolean simulation based on Bateson's proposal that taking differences in differences produces a hierarchy of knowledge (in this case perceptual judgments). Finally the top row shows that at the level of TAO-4, all nine forms have identical matrices.

Boundary Conditions. These hierarchies of visual form emerge in all cases where the attractor cycle length, L, is a power of 2 (L = 2, 4, 8, 16, ...); that is, the hierarchies of form emerge when L is a power of the number of states in the Boolean base. When L is not equal to a power of 2 other, interesting, phenomena emerge; these phenomena are not discussed in this paper. The boundary condition for the emergence of hierarchies that L equal a power of 2.

The Relation of Mental Hierarchies to the Emergence of Biological Ecologies. As we did with dynamic form in the above discussion we would like to hazard a conceptual leap by using the Boolean simulation of hierarchies to model epistemological phenomena. In this case we model Bateson's suggestion that hierarchies of difference emerge from taking differences in differences. To be concrete, suppose on a hike in the desert we encounter a raven. In standard biological taxonomy we can mentally place that raven in a hierarchy (Anamilia, Chordata, Aves, Corvidae, Raven). Having introduced our friend the raven, we want to make a distinction between epistemology and ontology. Kauffman and Turing when they referred to self-organizing form were referring to the coming into being of ontological form, actual beings, the actual raven in our example. Evolutionary theory in general addresses the coming into being of the existential beings we experience around us. In contrast, biological taxonomies and similar hierarchies are mental processes operating on those beings. Let us map those thoughts about biological beings to the visual forms that are generated by our Boolean attractor cycles. We liken forms themselves in Figure 4 to existential beings (ravens, jays, hawks, etc.). The forms *per se* came into being through processes identical to those proposed by Kauffman for evolution: Those forms emerged into being (as it were) on the computer screen through the coupled nonlinear generating processes of the Boolean system. In other words, the forms themselves are produced in a way that parallels Kauffman's ideas about the emergence of order in biology. In contrast, the hierarchies Figure 4 are produced in a way that parallels Bateson's ideas about how taking differences in differences produces classifications that are hierarchic. Note that in Figure 4 the forms themselves are repeated at each higher categorical level; the hierarchy *does not* consist of different (somehow more abstract) forms at each higher level but rather of *the same forms categorized in more abstract ways* at higher levels. The hierarchy in Figure 4 results from the mental operations of taking differences in differences (ala Bateson). The highest level (TAO_4 equality) in Figure 4 might map to high order levels in a biological taxonomy such as chordata where all chordata (such as birds, mammals, fish, reptiles, etc), no matter how different, are put into the same category. The TAO-3 category 1 might correspond to aves, where birds, however different, are all grouped together, and so on. Humans can perceive the similarity relations in the hierarchic categories shown in Figure 4 even if they might have categorized those forms slightly differently; in a similar way, they can perceive the plausibility of biological taxonomies even though they are capable of categorizing life forms differently (as different cultures do). Our proposal is that the visual forms come into being through Kauffman-like (ontological) processes and are categorized by Bateson-like epistemological processes. This is a crucial distinction; biological beings come into being by evolutionary and morphogenetic process; after the ontological fact they are classified by mental processes. The raven who a few hours from now might well be cleverly unzipping our day pack and eating our lunch while we are away doing something else, came into being however it came into being. The glory of evolutionary theory is that it is a way to describe such wondrous coming into being. The glory of epistemology is that it is way of describing how we can taxonomize the life that has come into being. And the raven, as it observes us, does it too have mental processes that generate taxonomies, and if so, where do we fit in its hierarchies?

Final Remarks

A core aspect of Bateson's epistemology is that what gets from the territory onto mental maps are differences; and the transforms of differences as they flow through a network are the foundation of knowing. Moreover, new, derivative knowledge (such as depth perception, Bateson, 2002, p. 64) results from the relationship between double (or multiple) descriptions (flows of differences). Bateson's arguments are largely verbal descriptions of process, e.g., how binocular vision might produce depth perception. We have operationalized those ideas using Stuart Kauffman's (1993) NK Boolean model as a way of exploring Bateson's verbal arguments using the power of dynamic systems mathematics in the form of Boolean simulations. Our results are not just consistent with Bateson's ideas but extend them in ways that allow us to talk about how dynamic form perception might work and how hierarchies might emerge through the mental process of taking differences in differences.

The key epistemological concept is the multiple flows of difference that act, in Bateson's terms, as multiple descriptions. We have examined how the relationships among such flows generates knowledge. In the case of dynamic form we have examined how the phase relations between systemic and representational processes generate forms that exist not in the two flows themselves but in the relations between the flows. Some the forms that emerge from these relations are characteristics of the systemic dynamics *per se* (where the system *per se* is defined as the Boolean computations or, in terms of the model's extrapolation, the retina). Other of these forms are not characteristics of the system itself but emerge from the process of representing the system

(Exemplar 2 and Exemplar 3).

In terms of hierarchies of difference the two descriptions we have defined are, first, the system's flow as it cycles through attractors and, second, TAO, process of finding differences in the system's flow of differences. Perceptual hierarchies emerge from the relations among these two descriptions.

One risk of precise models such as ours is that they can over specify and trivialize important insights. It is our contention that this is not the case here. The precise models led to interesting results that can be (however speculatively) generalized back to large and important issues.

References

Bateson, G. (2000). *Steps to an ecology of mind*. Chicago: University of Chicago Press. (Originally published by Ballantine, 1972.)

Bateson, G. (2002). *Mind and nature: A necessary unity*. Cresskill, NJ: Hampton Press. (Originally published by Bantam, 1979.)

Holland, J. H. (1998). *Emergence: From chaos to order*. Perseus Books, Cambridge, MA.

Kauffman, S. A. (1993). *The origins of order: Self-organization and selection in evolution*. Oxford: Oxford University Press.

Kauffman, S. A. (1995). At home in the universe: *The search for laws of self-organization and complexity*. Oxford: Oxford University Press.

Malloy, T. E., Bostic St Clair, C. and Grinder, J. (2005). "Steps to an ecology of emergence," *Cybernetics & Human Knowing*.

Malloy, T. E., Jensen, G. C., and Song, T. (2005) Mapping knowledge to Boolean dynamic systems in Bateson's epistemology. *Nonlinear Dynamics, Psychology, and Life Sciences, 9*, 37-60.

Turing, A. M. (1952). The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, *237*, 37-72.

Varela, F. J., Thompson, E., & Rosch, E. (1993). *The embodied mind*. Cambridge, MA: MIT Press.