Neutralino with the right cold dark matter abundance in (almost) any supersymmetric model

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(Received 29 March 2006; published 7 July 2006)

We consider nonstandard cosmological models in which the late decay of a scalar field ϕ reheats the Universe to a low reheating temperature, between 5 MeV and the standard freeze-out temperature of neutralinos of mass m_χ. We point out that in these models all neutralinos with standard density $\Omega_{\text{std}} \gtrsim 10^{-5}(100 \text{ GeV}/m_\chi)$ can have the density of cold dark matter, provided the right combination of the following two parameters can be achieved in the high-energy theory: the reheating temperature, and the ratio of the number of neutralinos produced per ϕ decay over the ϕ field mass. We present the ranges of these parameters where a combination of thermal and nonthermal neutralino production leads to the desired density, as functions of Ω_{std} and m_χ.

In supersymmetric models, the lightest supersymmetric particle (LSP), usually a neutralino χ, is a good cold dark matter candidate. The cosmological density of the neutralino is a function of the supersymmetric model parameters, and it has been computed theoretically to high precision. Requiring the LSP to have the measured dark matter density constrains the models considerably to very narrow regions in parameter space.

The standard computation of the relic density relies on the assumptions that the entropy of matter and radiation is conserved, that neutralinos are produced thermally and were in thermal and chemical equilibrium before they decoupled. The decoupling, or freeze-out, temperature, i.e. the temperature after which their number practically does not change, is $T_{\text{f.o.}} \simeq m_\chi/20$, where m_χ is the neutralino mass. The standard neutralino relic density Ω_{std} obtained in this way can be larger or smaller than the measured density of cold dark matter $\Omega_{\text{cdm}} = 0.113 \pm 0.009h^{-2}$ [1].

However, there are nonstandard cosmological models in which the assumptions mentioned above do not hold. These include models with moduli decay [2], Q-ball decay [3], and thermal inflation [4]. In all of these models there is a late episode of entropy production and nonthermal production of the LSP in particle decays is possible.

We concentrate on cosmological models in which the late decay of a scalar field ϕ reheats the Universe to a low reheating temperature T_{RH}, which can be smaller than $T_{\text{f.o.}}$ without spoiling primordial nucleosynthesis [5] (5 MeV $\lesssim T_{\text{RH}} \lesssim T_{\text{f.o.}}$). Such scalar fields are common in superstring models where they appear as moduli fields. The decay of ϕ into radiation increases the entropy, diluting the neutralino number density. The decay of ϕ into supersymmetric particles, which eventually decay into neutralinos, increases the neutralino number density. We denote by b the net number of neutralinos produced on average per ϕ decay.

Both thermal and nonthermal production mechanisms have been discussed in the literature [2,6–13]. Several supersymmetric models with particular combinations of T_{RH}, b, and m_ϕ have been studied [2,7–13]. Ref. [7] assumed that one neutralino is produced per ϕ decay ($b \simeq 1$), and concluded that in the MSSM the nonthermal neutralino production leads to an overabundance of relic neutralinos by several orders of magnitude. Also the thermalization of χ's produced in decays was discussed in Ref. [7]. References [8,9,12] studied the thermal production of LSPs during the decay of a scalar field with $b = 0$, and mentioned the possibility of $b \neq 0$ but did not study it. Reference [2] considered purely nonthermal production. Reference [13] is closest to our work, in that both thermal and nonthermal production were considered, but the general strategy to rescue models with too low or too high standard relic density remained, in our opinion, unclear.

In spite of all the above work, no coherent overview of the issue, which is remarkably easy to understand, has to our knowledge been laid down. This is what we intend to provide in this letter. A detailed analysis of the consequences in the Minimal Supersymmetric Standard Model, including Minimal Supergravity, Anomaly Mediated Supersymmetry Breaking, and Split Supersymmetry, is presented in Ref. [14].

Let Ω_{std} be the density that neutralinos in a particular model would have with the usually assumed cosmology. An appropriate combination of the following two parameters can bring the relic neutralino density to the desired value Ω_{cdm}: the ratio b/m_ϕ giving the average number of LSPs produced per unit of energy released in each ϕ decay and the reheating temperature T_{RH} (which must be $<T_{\text{f.o.}}$). We assume that the oscillating ϕ field dominates the energy density of the Universe at early times and that at T_{RH} the Universe becomes dominated by the radiation produced in ϕ decays. We follow the usual choice of the

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parameter T_{RH} as the temperature the Universe would attain under the assumption that the ϕ decay and subsequent thermalization are instantaneous

$$\Gamma_\phi = H_{\mathrm{decay}} = \sqrt{\left(\frac{8\pi}{3}\right)\rho_R} = \sqrt{\frac{8}{90}\,\pi^3 g_\star}\,\frac{T_{\mathrm{RH}}^2}{M_P}. \quad (1)$$

Γ_ϕ is the decay width of the ϕ field,

$$\Gamma_\phi \simeq \frac{m_\phi^3}{\Lambda_{\mathrm{eff}}^2}. \quad (2)$$

Notice that Eq. (1) provides the definition of the parameter T_{RH}, even when one does not make the approximation of an instantaneous decay. In our numerical calculations we do not use this approximation, and find that T_{RH} provides a good estimate of the first temperature of the radiation-dominated epoch. If ϕ has nonsuppressed gravitational couplings, as is usually the case for moduli fields, the effective energy scale $\Lambda_{\mathrm{eff}} \simeq M_P$. In models with intermediate scales, Λ_{eff} could be smaller [11]. Thus, with $g_\star \simeq 10$

$$T_{\mathrm{RH}} \simeq 10 \text{ MeV}\left(\frac{m_\phi}{100 \text{ TeV}}\right)^{3/2}\left(\frac{M_P}{\Lambda_{\mathrm{eff}}}\right). \quad (3)$$

In order not to disrupt the predictions of Big Bang nucleosynthesis, $T_{\mathrm{RH}} \gtrsim 5$ MeV [5]. Thus

$$m_\phi \gtrsim 100 \text{ TeV}\left(\frac{\Lambda_{\mathrm{eff}}}{M_P}\right)^{2/3}. \quad (4)$$

The number b of neutralinos produced per ϕ-decay is highly model-dependent. It is determined by the physics of the hidden sector, by the mechanism of supersymmetry breaking, and in superstring-inspired models by the compactification mechanism.

The coupling of the ϕ to the gravitino arises from the term $e^{K/2}\bar{\psi}_\mu \sigma^{\mu\nu}\psi_\nu$, where K is the Kähler potential. If m_ϕ is larger than twice the gravitino mass $m_{3/2}$, the decay mode $\phi \rightarrow \psi_{3/2}\psi_{3/2}$ of the moduli field into two gravitinos is present with branching ratio of order 0.01 (see Refs. [15], which correct previous claims [16] that this branching would be chirally suppressed by a factor $(m_{3/2}/m_\phi)^2$). Even if this decay is kinematically forbidden, the decay of ϕ into its supersymmetric partner and a gravitino may happen as long as $m_\phi > m_{3/2}$ [17]. Gravitinos must then decay rapidly not to disrupt nucleosynthesis (so $m_{3/2} \gtrsim 100$ TeV), and they produce comparable amounts of normal particles and their supersymmetric partners. If $m_\phi \gg m_{3/2}$, the gravitino decays during the radiation-dominated epoch after the decay of the ϕ field (here we do not address this case and we focus on neutralino production during ϕ domination). When m_ϕ and $m_{3/2}$ are of the same order of magnitude, we can consider the gravitino decay as part of the ϕ decay, since they happen almost simultaneously. In this case, depending on how important the direct decay of

ϕ into supersymmetric particles other than the $\psi_{3/2}$ is, b can typically be 0.01–1, but not smaller.

If instead $m_\phi < m_{3/2}$ more possibilities open up. The yield per ϕ decay b can still be of order one but it can also be much smaller.

Supergravity models with chiral superfields Φ_I are specified in terms of the Kähler potential $K(\Phi_I, \bar{\Phi}_I)$, the superpotential $W(\Phi_I)$, and the gauge kinetic function $f_{\alpha\beta}(\Phi_I)$. Specific relations between the ϕ mass m_ϕ, the gravitino mass $m_{3/2}$, and the gaugino mass $m_{1/2}$ arise as a consequence of the relations $m_{3/2} = \langle e^{K/2}W\rangle$, $m_{1/2} = \langle F^J \partial_J \ln\mathrm{Re}f\rangle$, and $m_\phi = \langle \partial^2 V/\partial\phi^2\rangle$. With appropriate choices of K, W, and f, the hierarchy $m_{3/2} \gtrsim m_\phi \gg m_{1/2}$ may be achieved. Here $V = K_{I\bar{J}}F^I\bar{F}^{\bar{J}} - 3e^K|W|^2 + \frac{1}{2}(\mathrm{Re}f)^{-1}_{\alpha\beta}D^\alpha D^\beta$ is the scalar potential, $F^I = -e^{K/2}K^{I\bar{J}}D_{\bar{J}}\bar{W}$ is the F-term of the chiral superfield Φ_I, $D^\alpha = T^{\alpha IJ}\phi_j D_I W/W$ is the D-term of the vector superfield, $D_I\mathcal{F} = \partial_I \mathcal{F} + \mathcal{F}\partial_I K$ is the Kähler covariant derivative, $\partial_I\mathcal{F} = \partial\mathcal{F}/\partial\Phi_I$, and square brackets denote vacuum values. As far as b is concerned, one finds, for example, $b \simeq O(1)$ when the main ϕ decay mode is through a coupling of the type $h\phi\psi^2$ with a chiral matter supermultiplet ψ in the superpotential W. This leads to comparable decay rates of ϕ into the scalar and fermionic components of ψ (which are supersymmetric partners). On the other hand, it is possible that the ϕ field decays mostly into Higgs fields, or gauge fields (W's, Z's, photons, gluinos). In this case b can be very small 10^{-2}, 10^{-4}, 10^{-6} etc. [2,10]. For example, the coupling of ϕ to the gauge bosons arises from the term $\mathrm{Re}f_{\alpha\beta}F^\alpha_{\mu\nu}F^{\mu\nu\beta}$ and with nonminimal kinetic terms $f_{\alpha\beta}$ may contain ϕ. The ϕ decay width into gauge bosons is then $\Gamma_g \sim \lambda_g m_\phi^3/M_P^2$ with $\lambda_g = \frac{\partial}{\partial\phi} \times \ln\mathrm{Re}f$, while that into gauginos is $\Gamma_{\tilde{g}} \sim \lambda_{\tilde{g}}^2 m_\phi/M_P^2$ with $\lambda_{\tilde{g}} = m_{1/2}\frac{\partial}{\partial\phi}\ln(F^\phi\frac{\partial}{\partial\phi}\mathrm{Re}f)$. Thus in principle the gaugino coupling may be suppressed relative to the coupling to gauge bosons.

Neutralinos are in kinetic equilibrium with the radiation when their scattering rate off relativistic particles is faster than the Hubble expansion rate, $\Gamma^{\mathrm{scatt}} \gtrsim H$. During the epoch in which the Universe is dominated by the decaying ϕ field, H is proportional to T^4 [18]. In fact, in the evolution equation for the radiation energy density (Eq. (5) below with $p = \rho/3$) substitute $\rho \simeq T^4$ and $\rho_\phi \simeq M_P^2 H^2$. Then use $H \sim t^{-1}$, write $T \propto t^\alpha$, match the powers of t in all terms, and determine $\alpha = -(1/4)$. Hence, during the oscillating ϕ dominated epoch $H \propto t^{-1} \propto T^4$ (and $\rho_\phi \propto H^2 \propto T^8$). Since H equals T_{RH}^2/M_P at $T = T_{\mathrm{RH}}$, it is $H \simeq T^4/(T_{\mathrm{RH}}^2 M_P)$. Since $\Gamma^{\mathrm{scatt}} \simeq n_\gamma \sigma_{\mathrm{scatt}} \simeq T^3 \sigma_{\mathrm{scatt}}$, we see that kinetic equilibrium is maintained at low temperatures, $T \lesssim T_{\mathrm{RH}}^2 M_P \sigma_{\mathrm{scatt}}$. This is contrary to the usual radiation-dominated scenario in which the temperature has to be large enough for kinetic equilibrium to be maintained. The origin of this difference is the strong T^4 de-

pendence of H on the radiation temperature. Using the known relation $\Omega_{std}h^2 \simeq 10^{-10} \text{ GeV}^{-2}/\langle\sigma v\rangle$, and taking σ_{scatt} of the same order of magnitude as the annihilation cross section $\langle\sigma v\rangle$, gives $T \lesssim (10^6 \text{ MeV}/\Omega_{std}h^2) \times (T_{RH}/\text{MeV})^2$. For the following we need to assume kinetic equilibrium before neutralino production ceases: the right hand side is larger than T_{RH} for $\Omega_{std}h^2 \lesssim 10^6(T_{RH}/\text{MeV})$, so we will safely assume that kinetic equilibrium is reached.

In kinetic equilibrium the equations which describe the evolution of the Universe are:

$$\dot{\rho} = -3H(\rho + p) + \Gamma_\phi\rho_\phi \tag{5}$$

$$\dot{n}_\chi = -3Hn_\chi - \langle\sigma v\rangle(n_\chi^2 - n_{\chi,eq}^2) + \frac{b}{m_\phi}\Gamma_\phi\rho_\phi \tag{6}$$

$$\dot{\rho}_\phi = -3H\rho_\phi - \Gamma_\phi\rho_\phi \tag{7}$$

$$H^2 = \frac{8\pi}{3M_P^2}(\rho + \rho_\phi). \tag{8}$$

Notice that the equations, and thus the results, depend only on the combination b/m_ϕ and not on b and m_ϕ separately. In Eqs. (5)–(8), a dot indicates a time derivative, ρ_ϕ is the energy density in the ϕ field, which is assumed to behave like nonrelativistic matter; ρ and p are the total energy density and pressure of matter and radiation at temperature T, which are assumed to be in kinetic but not necessarily chemical equilibrium; n_χ is the number density of LSPs, and $n_{\chi,eq}$ is its value in chemical equilibrium; finally, $H = \dot{a}/a$ is the Hubble parameter, with a the scale factor. For convenience in the numerical calculations, we used as independent variable $\ln a$ and as dependent variables $Y = n_\chi/s$, $Y_\phi = \rho_\phi/(m_\phi s)$ and T, where $s = (\rho + p - m_\chi n_\chi)/T$ is the entropy density of the matter and radiation. We also used the first principle of thermodynamics in the form $d(\rho a^3) + d(\rho_\phi a^3) + p da^3 = T d(sa^3)$ to rewrite Eq. (5) as

$$\dot{s} = -3Hs + \frac{\Gamma_\phi\rho_\phi}{T}. \tag{9}$$

Initial conditions are specified through the value H_I of the Hubble parameter at the beginning of the ϕ dominated epoch. This amounts to giving the initial energy density $\rho_{\phi,I}$ in the ϕ field, or equivalently the maximum temperature of the radiation T_{max}. Indeed, one has $H_I \simeq \rho_{\phi,I}^{1/2}/M_P \simeq T_{max}^4/(T_{RH}^2 M_P)$. The latter relation can be derived from $\rho_\phi \simeq T^8/T_{RH}^4$ and the consideration that the maximum energy in the radiation equals the initial (maximum) energy $\rho_{\phi,I}$.

If the neutralino reaches chemical equilibrium, it is clear that its final density does not depend on the initial conditions. An approximate condition for reaching chemical

equilibrium is [9] $\langle\sigma v\rangle \gtrsim 10^{-9} \text{ GeV}^{-2}(m_\chi/100 \text{ GeV}) \times (T_{RH}/\text{MeV})^{-2}$. Even without reaching chemical equilibrium, the neutralino density is insensitive to the initial conditions provided the maximum temperature of the radiation $T_{max} \gtrsim m_\chi$ [9].

In Fig. 1 we show how the neutralino density $\Omega_\chi h^2$ depends on T_{RH} for illustrative values of the parameter $\eta = b(100 \text{ TeV}/m_\phi)$, both for neutralinos which are underdense and which are overdense in usual cosmologies.

The behavior of the relic density as a function of T_{RH} is easy to understand physically. If $\langle\sigma v\rangle$ is large enough so that chemical equilibrium is achieved, the usual thermal production scenario occurs for $T_{RH} > T_{f.o.}$. Neutralino annihilation compensates thermal production until the latter ceases to be effective at $T = T_{f.o.}$. The LSP density is then determined by the condition $\Gamma^{ann} = n\langle\sigma v\rangle \simeq H$ at $T = T_{f.o.}$. Using $H \simeq T^2/M_P$, as appropriate for a radiation-dominated Universe, this leads to $Y_0 \simeq Y_{f.o.} \simeq (n/s)_{f.o.} \simeq (H/s\langle\sigma v\rangle)_{f.o.} \simeq 1/(T_{f.o.}M_P\langle\sigma v\rangle)$. This gives the usual result $\Omega_{std} \simeq (ms_0/\rho_c)Y_0 \simeq (ms_0)/(\rho_c T_{f.o.}M_P\langle\sigma v\rangle) \simeq 2 \times 10^{-10} \text{ GeV}^{-2}/\langle\sigma v\rangle$. Here we used $s_0/\rho_c = 2.8 \times 10^8 \text{ GeV}^{-1}$. When $\langle\sigma v\rangle$ is too small for chemical equilibrium to be achieved, the usual equation does not hold. Notice that this is the case for the smallest $\langle\sigma v\rangle$ in Fig. 1.
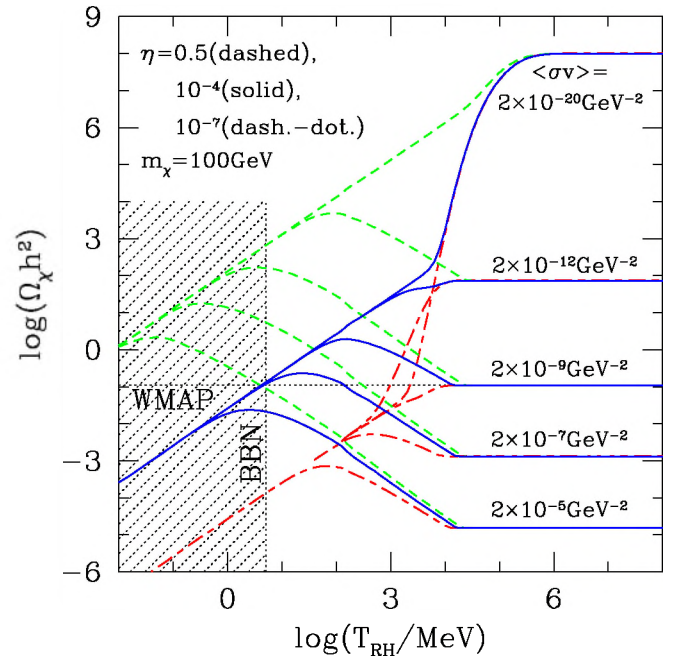

FIG. 1 (color online). Neutralino density $\Omega_\chi h^2$ as a function of the reheating temperature T_{RH} for illustrative values of the ratio $\eta = b(100 \text{ TeV}/m_\phi)$ (the number of neutralinos produced per unit of energy released in ϕ decay, in units of $(100 \text{ TeV})^{-1}$). The curves are insensitive to the initial conditions, i.e. to the value H_I of the Hubble parameter at the beginning of the ϕ dominated epoch.

For $T_{RH} < T_{f.o.}$, there are four different ways in which the density Ωh^2 depends on T_{RH}:

(1) Thermal production without chemical equilibrium. In this case $\Omega_\chi \propto T_{RH}^7$ (e.g. steepest part of the $\eta = 10^{-7}$, $\langle \sigma v \rangle = 2 \times 10^{-20}$ GeV^{-2} line). The relic density was estimated in Ref. [8]:

$$\frac{\Omega_\chi}{\Omega_{cdm}} \simeq \frac{\langle \sigma v \rangle}{10^{-16} \, \text{GeV}^{-2}} \left(\frac{100 \, \text{GeV}}{m_\chi}\right)^5 \left(\frac{T_{RH}}{\text{GeV}}\right)^7 \left(\frac{10}{g_\star}\right)^{3/2}. \tag{10}$$

This matches our numerical calculation for $\langle \sigma v \rangle = 2 \times 10^{-20}$ GeV^{-2}.

(2) Thermal production with chemical equilibrium. In this case $\Omega_\chi \propto T_{RH}^4$ (e.g. steepest part of the $\eta = 10^{-7}$, $\langle \sigma v \rangle = 2 \times 10^{-12}$ GeV^{-2} line). The neutralino freezes out while the universe is dominated by the ϕ field. Its freeze-out density is larger than usual, but it is diluted by entropy production from ϕ decays. The new freeze-out temperature $T_{f.o.}^{NEW}$ is determined by solving $n\langle \sigma v \rangle \simeq H$ at $T = T_{f.o.}^{NEW}$. Using the relations between H, a, and T in the decaying-ϕ dominated Universe, one finds [9,18]

$$\Omega_\chi \simeq \frac{T_{RH}^3 T_{f.o.}}{(T_{f.o.}^{NEW})^4} \Omega_{std}. \tag{11}$$

Our numerical results indicate a slope closer to T_{RH}^4, perhaps due to the change in $T_{f.o.}$.

(3) Nonthermal production without chemical equilibrium. Here $\Omega_\chi \propto T_{RH}$ (e.g. leftmost part of each line). Nonthermal production is not compensated by annihilation. The production of neutralinos is purely nonthermal and the relic density depends on η. It can be estimated analytically as follows. For each superpartner produced, at least one LSP will remain at the end of a chain of decays (due to R-parity conservation), and thus $n_\chi \simeq b n_\phi$. Here $n_\phi = \rho_\phi / m_\phi$. At the time of ϕ-decay $\rho_\chi \simeq m_\chi b \rho_\phi / m_\phi \simeq T_{RH}^4$, and the entropy is $s \simeq T_{RH}^3$. Hence $\rho_\phi / s \simeq T_{RH}$ and $Y_0 = Y_{decay} \simeq b T_{RH}/m_\phi$. It follows that,

$$\frac{\Omega_\chi}{\Omega_{cdm}} \simeq 2 \times 10^3 \eta \left(\frac{m_\chi}{100 \, \text{GeV}}\right)\left(\frac{T_{RH}}{\text{MeV}}\right) \tag{12}$$

(4) Nonthermal production with chemical equilibrium. In this case $\Omega \propto T_{RH}^{-1}$ (e.g. central part of the $\eta = 0.5$, $\langle \sigma v \rangle = 2 \times 10^{-5}$ GeV^{-2} line). Here annihilation compensates for the nonthermal production of neutralinos until the nonthermal production ceases at $T = T_{RH}$. The condition for determining the relic density is $\Gamma^{ann} \simeq \Gamma_\phi$ at $T = T_{RH}$. This leads to $Y_0 \simeq Y_{RH} \simeq \Gamma_\phi/(s_{RH}\langle \sigma v \rangle) \simeq 1/(T_{RH} M_P \langle \sigma v \rangle)$. From here it follows that

$$\Omega_\chi \simeq (T_{f.o.}/T_{RH})\Omega_{std}. \tag{13}$$

With the help of Fig. 1 and formulas (12) and (13), we can separate the different ranges of η. Notice that in Fig. 1 $m_\chi = 100$ GeV, thus $T_{f.o.} \simeq m_\chi/20 \simeq 5$ GeV. For $T_{RH} > T_{f.o.}$, the standard production mechanism is recovered, thus $\Omega_\chi = \Omega_{std}$ (indicated by the horizontal lines on the right of the Figure). If $T_{RH} < T_{f.o.}$, the value of Ωh^2 depends on η. Overdense neutralinos, i.e. those with $\Omega_{std} > \Omega_{cdm}$ (above the dotted line labeled WMAP), require values of $\eta \lesssim 10^{-4}(100 \text{ GeV}/m_\chi)$ to bring their density to $\Omega_\chi = \Omega_{cdm}$. This bound is derived from the BBN condition $T_{RH} \gtrsim 5$ MeV by taking $\Omega_\chi = \Omega_{cdm}$ in Eq. (12). The condition $T_{RH} \lesssim T_{f.o.}$ in Eq. (12) shows that for a solution $\Omega_\chi = \Omega_{cdm}$ with $\eta \lesssim 10^{-7}(100 \text{ GeV}/m_\chi)^2$ the production must be thermal with entropy dilution (case (1) or (2)), and T_{RH} must be close to $T_{f.o.}$. Notice that in between the two values of η just mentioned the production is purely nonthermal (no chemical equilibrium, namely, case (3)). For all overdense neutralinos, given one value of $\eta \lesssim 10^{-4}(100 \text{ GeV}/m_\chi)$ there is only one value of T_{RH} for which $\Omega_\chi = \Omega_{cdm}$.

Underdense neutralinos, i.e. those with $\Omega_{std} < \Omega_{cdm}$, can have zero, one, or two solutions $\Omega_\chi = \Omega_{cdm}$. There is no solution if Ω_{std} is too low. Neutralinos with $\Omega_{std} \lesssim 10^{-5}(100 \text{ GeV}/m_\chi)$ cannot be brought to Ω_{cdm}, independently of η. This can be seen by imposing the condition $T_{RH} > 5$ MeV and $\Omega_\chi = \Omega_{cdm}$ in Eq. (13). Neutralinos with $\Omega_{cdm} \gtrsim \Omega_{std} \gtrsim 10^{-5}(100 \text{ GeV}/m_\chi)$ cannot be brought to Ω_{cdm} either if $\eta \lesssim 10^{-7}(100 \text{ GeV}/m_\chi)^2 \times (\Omega_{cdm}/\Omega_{std})$. This happens when the nonthermal production is insufficient to increase the density up to Ω_{cdm}, such as for the $\eta = 10^{-7}$, $\langle \sigma v \rangle = 2 \times 10^{-7}$ GeV^{-2} line in Fig. 1. Again using Eqs. (12) and (13) and Fig. 1, one can see that for the same range of densities $\Omega_{cdm} \gtrsim \Omega_{std} \gtrsim 10^{-5}(100 \text{ GeV}/m_\chi)$ there are two solutions for $10^{-7}(100 \text{ GeV}/m_\chi)^2(\Omega_{cdm}/\Omega_{std}) \lesssim \eta \lesssim 10^{-4}(100 \text{ GeV}/m_\chi)$ and a single solution for larger values of η. The two solutions have different values of T_{RH} and are both nonthermal one belonging to case (3) and the other to case (4). The single solution belongs to case (4).

In conclusion the neutralino density can be that of cold dark matter provided $\Omega_{std} \gtrsim 10^{-5}(100 \text{ GeV}/m_\chi)$ and the high-energy theory accomodates the combinations of values of b/m_ϕ and T_{RH} identified in the previous two paragraphs. These conditions may place constraints on high-energy models derived from superstring theories. Considering the possibility of choosing a lower bound $T_{RH\text{-}min}$ on T_{RH} different that 5 MeV, the lower bound on Ω_{std} becomes

$$\Omega_{std} \gtrsim 10^{-5}\left(\frac{100 \text{ GeV}}{m_\chi}\right)\left(\frac{T_{RH\text{-}min}}{5 \text{ MeV}}\right) \tag{14}$$

and the limiting values on b/m_ϕ and T_{RH} would change too, but the qualitative conclusions remain the same.

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