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Enhancing superconductivity: Magnetic impurities and their quenching by magnetic fields

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Abstract. – Magnetic fields and magnetic impurities are each known to suppress superconductivity. However, as the field quenches (i.e. polarizes) the impurities, rich consequences, including field-enhanced superconductivity, can emerge when both effects are present. For the case of superconducting wires and thin films, this field-spin interplay is investigated via the Eilenberger-Usadel scheme. Non-monotonic dependence of the critical current on the field (and therefore field-enhanced superconductivity) is found to be possible, even in parameter regimes in which the critical temperature decreases monotonically with increasing field. The present work complements that of Kharitonov and Feigel'man, which predicts non-monotonic behavior of the critical temperature.

Introduction. – In their classic work, Abrikosov and Gor'kov [1] predicted that unpolarized, uncorrelated magnetic impurities suppress of superconductivity, due to the de-pairing effects associated with the spin-exchange scattering of electrons by magnetic impurities. Among their results is the reduction, with increasing magnetic impurity concentration, of the superconducting critical temperature T_{c}, along with the possibility of "gapless" superconductivity in an intermediate regime of magnetic-impurity concentrations. The latter regime is realized when the concentration of the impurities is large enough to eliminate the gap but not large enough to destroy superconductivity altogether. Not long after the work of Abrikosov and Gor'kov, it was recognized that other de-pairing mechanisms, such as those involving the coupling of the orbital and spin degrees of freedom of the electrons to a magnetic field, can lead to equivalent suppressions of superconductivity, including gapless regimes [2–5].

Conventional wisdom holds that magnetic fields and magnetic moments each tend to suppress superconductivity (see, e.g., Ref. [6]). Therefore, it seems natural to suspect that any increase in a magnetic field, applied to a superconductor containing magnetic impurities, would lead to additional suppression of the superconductivity. However, very recently, Kharitonov and Feigel'man [7] have predicted the existence of a regime in which, by contrast, an increase in the magnetic field applied to a superconductor containing magnetic impurities leads to a critical temperature that first increases with magnetic field, but eventually behaves more conventionally, decreasing with the magnetic field and ultimately vanishing at a critical value of the field. Even more strikingly, they have predicted that, over a certain range of concentrations of magnetic impurities, a magnetic field can actually induce superconductivity out of the normal state.

The Kharitonov-Feigel'man treatment focuses on determining the critical temperature by determining the linear instability of the normal state. The purpose of the present Letter is to address properties of the superconducting state itself, most notably the critical current and its dependence on temperature and the externally applied magnetic field. The approach that we shall take is to derive the (transport-like) Eilenberger-Usadel equations [9, 10], by starting from the Gor'kov equations. We account for the following effects: potential and spin-orbit scattering of electrons from non-magnetic impurities, and spin-exchange scattering from magnetic impurities, along with orbital and Zeeman effects of the magnetic field. In addition to obtaining the critical current, we shall recover the Kharitonov-Feigel'man prediction for the critical temperature, as well as the dependence of the order parameter on temperature and applied field. In particular, we shall show that not only are there reasonable parameter regimes in which both the critical current and the transition temperature vary non-monotonically with increasing magnetic field, but also there are reasonable parameter regimes in which only the low-temperature critical current is non-monotonic even though the critical temperature behaves monotonically with field. The present theory can be used to explain certain recent experiments on superconducting wires [8].

Before describing the technical development, we pause to give a physical picture of the relevant de-pairing mechanisms. First, consider the effects of magnetic impurities. These cause spin-exchange scattering of the electrons (including both spin-flip and non-spin-flip terms, relative to a given spin quantization axis), and therefore lead to the breaking of Cooper pairs [1]. Now consider the effects of magnetic fields. The vector potential (due to the applied field) scrambles the relative phases of the partners of a Cooper pair, as they move diffusively in the presence of impurity scattering (viz. the orbital effect), which suppresses superconductivity [2, 3]. On the other hand, the field polarizes the magnetic impurity spins, which decreases the rate of exchange scattering (because the spin-flip term is suppressed more strongly than the non-spin-flip term is enhanced), thus diminishing this contribution to de-pairing [7]. In addition, the Zeeman effect associated with the effective field (coming from the applied field and the impurity spins) splits the energy of the up and down spins in the Cooper pair, thus tending to suppress superconductivity [6]. We note that strong spin-orbit scattering tends to weaken the de-pairing caused by the Zeeman effect [5]. Thus we see that the magnetic field produces competing tendencies: it causes de-pairing via the orbital and Zeeman effects, but it mollifies the de-pairing caused by magnetic impurities. This competition can manifest itself through the non-monotonic behavior of observables such as the critical temperature and critical current. In order for the manifestation to be observable, the magnetic field needs to be present throughout the samples, the scenario being readily accessible in wires and thin films.

The model. – We take for the impurity-free part of the Hamiltonian the BCS mean-field form [6, 11]:

$$H_0 = -\int dr\, \frac{1}{2m}\psi_\alpha^\dagger\Big(\nabla - \frac{ie}{c}A\Big)^2\psi_\alpha + \frac{V_0}{2}\int dr\, \Big(\langle\psi_\alpha^\dagger\psi_\beta^\dagger\rangle\psi_\beta\psi_\alpha + \psi_\alpha^\dagger\psi_\beta^\dagger\langle\psi_\beta\psi_\alpha\rangle\Big) - \mu\int dr\, \psi_\alpha^\dagger\psi_\alpha, \tag{1}$$

where $\psi_\alpha^\dagger(r)$ creates an electron having mass m,charge e, position r and spin projection α, A is the vector potential, c is the speed of light, μ is the chemical potential, and V_0 is the pairing interaction. Throughout this Letter we shall put $\hbar = 1$ and $k_B = 1$. Assuming the superconducting pairing is spin-singlet, we may introduce the complex order parameter Δ, via

$$-V_0\langle\psi_\alpha\psi_\beta\rangle = i\sigma^y_{\alpha\beta}\Delta, \qquad\qquad V_0\langle\psi_\alpha\psi_\beta\rangle = i\sigma^y_{\alpha\beta}\Delta^*, \tag{2}$$

where $\sigma^{x,y,z}_{\alpha\beta}$ are the Pauli matrices. We assume that the electrons undergo potential and spin-exchange scattering from the magnetic impurities located at a set of random positions $\{x_i\}$, in addition to undergoing spin-orbit scattering from an independent set of impurities or defects located at an independent set of random positions $\{y_j\}$, as well as being Zeeman coupled to the applied field:

$$H_{\rm int} = \int dr\, \psi^\dagger_\alpha V_{\alpha\beta}\psi_\beta, \tag{3a}$$

with $V_{\alpha\beta}$ being given by

$$V_{\alpha\beta} = \sum\nolimits_i \left\{u_1(r-x_i)\delta_{\alpha\beta} + u_2(r-x_i)\vec{S}_i\cdot\vec{\sigma}_{\alpha\beta}\right\} + \sum\nolimits_j \left\{\vec{\nabla}v_{so}(r-y_j)\cdot\left(\vec{\sigma}_{\alpha\beta}\times\vec{p}\right)\right\} + \mu_B B\,\sigma^z_{\alpha\beta}, \tag{3b}$$

where $\vec{S}_i$ is the spin of the i-th magnetic impurity and where, for simplicity, we have attributed the potential scattering solely to the magnetic impurities. We could have included potential scattering from the spin-orbit scattering centers, as well as potential scattering from a third, independent set of impurities. However, to do so would not change our conclusions, save for the simple rescaling of the mean-free time. We note that cross terms, i.e. those involving distinct interactions, can be ignored when evaluating self-energy [5, 7]. Furthermore, we shall assume that the Kondo temperature is much lower than the temperature we are interested in.

The impurity spins interact with the applied magnetic field through their own Zeeman term:

$$H_{\rm Z} = -\omega_s S^z \tag{4}$$

where $\omega_s \equiv g_s\mu_B B$, and g_s is the impurity-spin g-factor. Thus, the impurity spins are not treated as static but rather have their own dynamics, induced by the applied magnetic field. We shall approximate the dynamics of the impurity spins as being governed solely by the applied field, ignoring any influence on them of the electrons. Then, as the impurity spins are in thermal equilibrium, we may take the Matsubara correlators for a single spin to be

$$\langle T_\tau S^+(\tau_1)S^-(\tau_2)\rangle = T\sum\nolimits_{\omega'} D^{+-}_{\omega'}\, e^{-i\omega'(\tau_1-\tau_2)}, \tag{5a}$$

$$\langle T_\tau S^-(\tau_1)S^+(\tau_2)\rangle = T\sum\nolimits_{\omega'} D^{-+}_{\omega'}\, e^{-i\omega'(\tau_1-\tau_2)}, \tag{5b}$$

$$\langle T_\tau S^z(\tau_1)S^z(\tau_2)\rangle = d^z = \overline{(S^z)^2}, \tag{5c}$$

where ω' ($\equiv 2\pi nT$) is a bosonic Matsubara frequency, $\overline{\cdots}$ denotes a thermal average, and

$$D^{+-}_{\omega'} \equiv 2\overline{S^z}/(-i\omega'+\omega_s), \qquad\qquad D^{-+}_{\omega'} \equiv 2\overline{S^z}/(+i\omega'+\omega_s). \tag{6}$$

We shall ignore correlations between distinct impurity spins, as their effects are of the second order in the impurity concentration.

To facilitate the forthcoming analysis, we define the Nambu-Gor'kov four-component spinor (see, e.g., Refs. [5, 12]) via

$$\Psi^\dagger(x) \equiv \Big(\psi^\dagger_\uparrow(r,\tau), \psi^\dagger_\downarrow(r,\tau), \psi_\uparrow(r,\tau), \psi_\downarrow(r,\tau)\Big). \tag{7}$$

Then, the electron-sector Green functions are defined in the standard way via

$$\mathcal{G}_{ij}(1:2) \equiv -\langle T_\tau \Psi_i(1)\Psi^\dagger_j(2)\rangle \equiv \begin{pmatrix} \hat{G}(1:2) & \hat{F}(1:2) \\ \hat{F}^\dagger(1:2) & \hat{G}^\dagger(1:2)\end{pmatrix}, \tag{8}$$

where $\hat{G}$, $\hat{G}^\dagger$, $\hat{F}$, and $\hat{F}^\dagger$ are each two-by-two matrices (as indicated by the ˆ symbol), being the normal and anomalous Green functions, respectively. As the pairing is assumed to be singlet, $\hat{F}$ is off-diagonal whereas $\hat{G}$ is diagonal.

Eilenberger-Usadel equations. – The critical temperature and critical current are two of the most readily observable quantities. As they can be readily obtained from the Eilenberger and Usadel equations, we shall focus on these formalisms. A detailed derivation will be presented elsewhere. The procedure is first to derive Eilenberger equations [9], and then, assuming the dirty limit, to obtain the Usadel equations. The self-consistency equation between the anomalous Green function and the order parameter naturally leads, in the small order-parameter limit, to an equation determining the critical temperature. Moreover, solving the resulting transport-like equations, together with the self-consistency equation, gives the transport current, and this, when maximized over superfluid velocity, yields the critical current.

To implement this procedure, one first derives the equations of motion for $\mathcal{G}$ (viz. the Gor'kov equations). By suitably subtracting these equations from one another one arrives at a form amenable to a semiclassical analysis, for which the rapidly and slowly varying parts in the Green function (corresponding to the dependence on the relative and center-of-mass coordinates of a Cooper pair, respectively) can be separated. Next, one treats the interaction Hamiltonian as an insertion in the self-energy, which leads to a new set of semi-classical Gor'kov equations. These equations are still too complicated to use effectively, but they can be simplified to the so-called Eilenberger equations [9,13–15] (at the expense of losing detailed information about excitations) by introducing the energy-integrated Green functions,

$$\hat{g}(\omega,k,R) \equiv \frac{i}{\pi}\int d\xi_k\, \hat{G}(\omega,k,R), \quad \hat{f}(\omega,k,R) \equiv \frac{1}{\pi}\int d\xi_k\, \hat{F}(\omega,k,R), \tag{9}$$

and similarly for $\hat{g}^\dagger(\omega,k,R)$ and $\hat{f}^\dagger(\omega,k,R)$. Here, ω is the fermionic frequency Fourier conjugate to the relative time, k is the relative momentum conjugate to the relative coordinate, and R is the center-of-mass coordinate. (We shall consider stationary processes, so we have dropped any dependence on the center-of-mass time.) However, the resulting equations do not determine g's and f's uniquely, and they need to be supplemented by additional normalization conditions [9, 13–15],

$$\hat{g}^2 + \hat{f}\hat{f}^\dagger = \hat{g}^{\dagger 2} + \hat{f}^\dagger \hat{f} = \hat{1}, \tag{10}$$

as well as the self-consistency equation,

$$\Delta = |g| \sum\nolimits_\omega f_{12}(\omega). \tag{11}$$

In the dirty limit (i.e. $\omega\tau_{\rm tr} \ll G$ and $\Delta\tau_{\rm tr} \ll F$), where $\tau_{\rm tr}$ is the transport relaxation time (which we do not distinguish from the elastic mean-free time), the Eilenberger equations can be simplified further, because, in this limit, the energy-integrated Green functions are almost isotropic in k. This allows one to retain only the two lowest spherical harmonics ($l = 0, 1$), and to regard the $l = 1$ term as a small correction (i.e. $|\check{k}\cdot\vec{F}| \ll |F|$) so that we may write

$$g(\omega,\check{k},R) = G(\omega,R) + \check{k}\cdot\vec{G}(\omega,R), \quad f(\omega,\check{k},R) = F(\omega,R) + \check{k}\cdot\vec{F}(\omega,R), \tag{12}$$

where $\check{k}$ is the unit vector along k. In this nearly-isotropic setting, the normalization conditions simplify to

$$G_{11}^2 = 1 - F_{12}F_{21}^\dagger, \qquad G_{22}^2 = 1 - F_{21}F_{12}^\dagger, \tag{13}$$

and the Eilenberger equations reduce to the celebrated Usadel equations [10] for $F_{12}(\omega,R)$, $F_{21}(\omega,R)$, $F_{12}^\dagger(\omega,R)$, and $F_{21}^\dagger(\omega,R)$.

Application to thin wires and films. – Let us consider a wire (or film) not much thicker than the effective coherence length. In this regime, we may assume that the order parameter has the form $\Delta(R) = \tilde{\Delta} e^{iuR_x}$, where R_x is the coordinate measured along the direction of the current (e.g. for a wire this is along its length) and u is a parameter encoding the velocity of the superflow $\hbar u/2m$. Similarly, we may assume that the semiclassical anomalous Green functions have a similar form:

$$F_{12}(\omega, R) = \tilde{F}_{12}(\omega) e^{iuR_x}, \qquad F_{21}(\omega, R) = \tilde{F}_{21}(\omega) e^{iuR_x}, \tag{14a}$$

$$F^{\dagger}_{12}(\omega, R) = \tilde{F}^{\dagger}_{12}(\omega) e^{-iuR_x}, \qquad F^{\dagger}_{21}(\omega, R) = \tilde{F}^{\dagger}_{21}(\omega) e^{-iuR_x}. \tag{14b}$$

Together with the symmetry amongst $\tilde{F}$'s (i.e. $\tilde{F}^*_{\alpha\beta} = -\tilde{F}^{\dagger}_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta} = -\tilde{F}^*_{\beta\alpha}$) we can reduce the four Usadel equations for $\tilde{F}_{12}$, $\tilde{F}_{21}$, $\tilde{F}^{\dagger}_{12}$, and $\tilde{F}^{\dagger}_{21}$ to one single equation:

$$\Big[\omega + i\delta_B + \frac{T}{2\tau_{\rm B}} \sum_{\omega'} \Big(D^{-+}_{\omega'} G_{22}(\omega-\omega')\Big) + \Big(\frac{d^z}{\tau_{\rm B}} + \frac{\tilde{D}}{2}\Big) G_{11}(\omega) + \frac{1}{3\tau_{\rm so}} G_{22}(\omega)\Big] \frac{\tilde{F}_{12}(\omega)}{\tilde{\Delta}}$$
$$- G_{11}(\omega) = -G_{11}(\omega) \frac{T}{2\tau_{\rm B}} \sum_{\omega'} \Big(D^{-+}_{\omega'} \frac{\tilde{F}^*_{12}(\omega-\omega')}{\tilde{\Delta}^*}\Big) + \frac{1}{3\tau_{\rm so}} G_{11}(\omega) \frac{\tilde{F}^*_{12}(\omega)}{\tilde{\Delta}^*}, \tag{15}$$

in which $\delta_B \equiv \mu_B B + n_i u_2(0)\overline{S^z}$, $\tilde{D} \equiv D\langle\langle(u - 2eA/c)^2\rangle\rangle$ with the London gauge chosen and $\langle\langle\cdots\rangle\rangle$ defining a spatial average over the sample thickness, $D \equiv v_F^2 \tau_{\rm tr}/3$ is the diffusion constant. The spin-exchange and spin-orbit scattering times, $\tau_{\rm B}$ and $\tau_{\rm so}$, are defined via the Fermi surface averages

$$\frac{1}{2\tau_{\rm B}} \equiv N_0 n_i \pi \int \frac{d^2\check{k}'}{4\pi} |u_2|^2, \qquad \frac{1}{2\tau_{\rm so}} \equiv N_0 n_{\rm so} \pi \int \frac{d^2\check{k}'}{4\pi} |v_{\rm so}|^2\, p_F^2 |\check{k} \times \check{k}'|^2. \tag{16}$$

Here, N_0 is the (single-spin) density of electronic states at the Fermi surface, n_i is the concentration of magnetic impurities, $n_{\rm so}$ is the concentration of spin-orbit scatterers, and $p_F = mv_F$ is the Fermi momentum. The normalization condition then becomes

$$\tilde{G}_{11}(\omega) = \mathrm{sgn}(\omega)[1 - \tilde{F}^2_{12}(\omega)]^{1/2}, \qquad \tilde{G}_{22}(\omega) = \mathrm{sgn}(\omega)[1 - \tilde{F}^{*2}_{12}(\omega)]^{1/2} = \tilde{G}^*_{11}(\omega). \tag{17}$$

Furthermore, the self-consistency condition (11) becomes

$$\tilde{\Delta} \ln\big(T_{C0}/T\big) = \pi T \sum\nolimits_{\omega} \Big(\big(\tilde{\Delta}/|\omega|\big) - \tilde{F}_{12}(\omega)\Big), \tag{18}$$

in which we have exchanged the coupling constant g for T_{C0}, i.e., the critical temperature of the superconductor in the absence of magnetic impurities and fields.

In the limit of strong spin-orbit scattering (i.e. $\tau_{\rm so} \ll 1/\omega$ and $\tau_{\rm B}$), the imaginary part of Eq. (15) is simplified to

$$\big[\delta_B + \frac{T}{2\tau_{\rm B}} \mathrm{Im} \sum\nolimits_{\omega'} D^{-+}_{\omega'}\, G(\omega-\omega')\big] \mathrm{Re}\, C + \frac{2}{3\tau_{\rm so}} \mathrm{Im}(GC) = 0, \tag{19a}$$

and the real part is rewritten as

$$\omega\, \mathrm{Re}\, C + \tfrac{T}{2\tau_{\rm B}} \mathrm{Re} \sum\nolimits_{\omega'} \big[D^{-+}(\omega')\, G(\omega-\omega')\, C(\omega) + G^*(\omega)\, D^{-+}(\omega')\, C^*(\omega-\omega')\big]$$
$$- \big[\delta_B + \tfrac{T}{2\tau_{\rm B}} \mathrm{Im} \sum\nolimits_{\omega'} D^{-+}_{\omega'}\, G(\omega-\omega')\big] \mathrm{Im}\, C + \Big(\tfrac{d^z}{\tau_{\rm B}} + \tfrac{\tilde{D}}{2}\Big) \mathrm{Re}(GC) = \mathrm{Re}\, G, \tag{19b}$$

where $C \equiv \tilde{F}_{12}/\tilde{\Delta}$, $G \equiv G_{11}$, and the argument ω is implied for all Green function, except where stated otherwise. Next, we take the advantage of the simplification that follows by restricting our attention to the weak-coupling limit, in which $\tilde{F}_{12}(\omega) \ll 1$. Then, eliminating G in Eq. (19) using Eqs. (17), and expanding to third order in powers of $\tilde{F}$, one arrives at an equation for $\tilde{F}$ that is readily amenable to numerical treatment. The quantitative results that we now draw are based on this strategy. ([1])

Results for the critical temperature. – These can be obtained in the standard way, i.e., by (i) setting $u = 0$ and expanding Eqs. (19) to linear order in $\tilde{F}$ (at fixed $\tilde{\Delta}$), and (ii) setting $\tilde{\Delta} \to 0$ and applying the self-consistency condition. Step (i) yields

$$\Big[|\omega| + \tilde{\Gamma}_\omega + \frac{D}{2}\Big\langle\!\Big\langle\Big(\frac{2eA}{c}\Big)^2\Big\rangle\!\Big\rangle + \frac{3\tau_{\rm so}}{2}\delta'_B(\omega)^2\Big]\,{\rm Re}\,C(\omega) \approx 1 - \frac{T}{\tau_{\rm B}}\sum_{\omega'}\frac{\omega_s\overline{S^z}}{\omega'^2+\omega_s^2}\,{\rm Re}\,C(\omega-\omega'), \quad (20a)$$

where

$$\delta'_B(\omega) \equiv \delta_B - \frac{T}{\tau_{\rm B}}\sum_{\omega_c > |\omega'| > |\omega|}\frac{2|\omega'|\overline{S^z}}{\omega'^2+\omega_s^2}, \quad (20b)$$

in which a cutoff ω_c has been imposed on ω', and

$$\tilde{\Gamma}_\omega \equiv \frac{d^z}{\tau_{\rm B}} + \frac{T}{\tau_{\rm B}}\sum_{|\omega'|<|\omega|}\frac{\omega_s\overline{S^z}}{\omega'^2+\omega_s^2}. \quad (21)$$

This is essentially the Cooperon equation in the strong spin-orbit scattering limit, first derived by Kharitonov and Feigel'man [7], up to an inconsequential renormalization of δ_B.

Step (ii) involves solving the implicit equation

$$\ln\frac{T_{C0}}{T} = \pi T\sum_\omega\left[\frac{1}{|\omega|} - \frac{1}{2}\Big(C(\omega) + C^*(\omega)\Big)\right], \quad (22)$$

the solution of which is $T = T_C$.

Figure 1 shows the dependence of the critical temperature of wires or thin films on the (parallel) magnetic field for several values of magnetic impurity concentration. Note the qualitative features first obtained by Kharitonov and Feigel'man [7]: starting at low concentrations of magnetic impurities, the critical temperature decreases monotonically with the applied magnetic field. For larger concentrations, a marked non-monotonicity develops, and for yet larger concentrations, a regime is found in which the magnetic field first induces superconductivity but ultimately destroys it. The physical picture behind this is the competition mentioned in the Introduction: first, by polarizing the magnetic impurities the magnetic field suppresses their pair-breaking effect. At yet larger fields, this enhancing tendency saturates, and is then overwhelmed by the pair-breaking tendency of the orbital coupling to the magnetic field.

Results for the critical current density. – To obtain the critical current density j_c, we first determine the current density (average over the sample thickness) from the solution of the Usadel equation via

$$j(u) = 2eN_0\pi DT\sum\nolimits_\omega {\rm Re}\Big(\tilde{F}_{12}^2(\omega)\big[u - \frac{2e}{c}\langle\!\langle A\rangle\!\rangle\big]\Big), \quad (23)$$

([1])We note that, simplifications associated with the strong spin-orbit scattering assumption and the power series expansion in $\tilde{F}$ are only necessary to ease the numerical calculations. Our conclusions are not sensitive to these simplifications in the parameter regimes considered in Figs. 1 and 2.

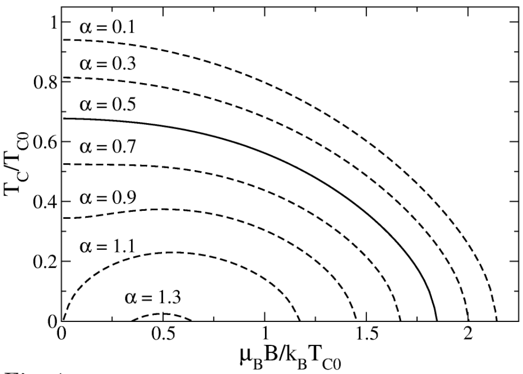

Fig. 1

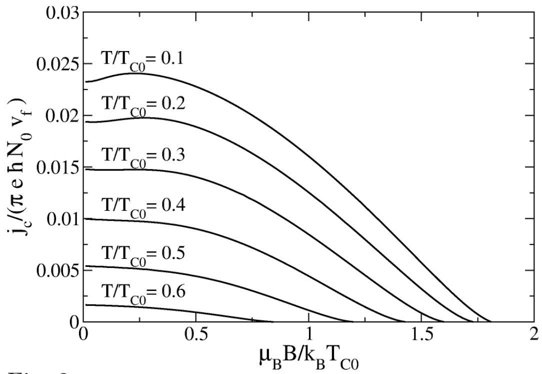

Fig. 2

Fig. 1 – Critical temperature vs. (parallel) magnetic field for a range of exchange scattering strengths characterized by the dimensionless parameter $\alpha \equiv \hbar/(k_B T_{C0}\tau_{\mathrm{B}})$. The strength for potential scattering is characterized by parameter $\hbar/(k_B T_{C0}\tau_{\mathrm{tr}}) = 10000.0$, and that for the spin-orbit scattering is by $\hbar/(k_B T_{C0}\tau_{\mathrm{so}}) = 1000.0$; the sample thickness is $d = 90.0\,\hbar/p_F$, where p_F is the Fermi momentum; the impurity gyromagnetic ratio is chosen to be $g_s = 2.0$; and the typical scale of the exchange energy u_2 in Eq. (3b) is taken to be $E_F/7.5$, where E_F is the Fermi energy.

Fig. 2 – Critical current vs. (parallel) magnetic field at several values of temperature, with the strength of the exchange scattering set to be $\alpha = 0.5$ (corresponding to the solid line in Fig. 1), and all other parameters being the same as used in Fig. 1.

and then maximize $j(u)$ with respect to u. In the previous section, we have seen that, over a certain range of magnetic impurity concentrations, T_C displays an upturn with field at small fields, but eventually decreases. Not surprisingly, our calculations show that such non-monotonic behavior is also reflected in the critical current.

Perhaps more interestingly, however, we have also found that for small concentrations of magnetic impurities, although the critical temperature displays *no* non-monotonicity with the field, the critical current *does* exhibit non-monotonicity, at least for lower temperatures. This phenomenon, which is exemplified in Fig. 2, sets magnetic impurities apart from other depairing mechanisms. The reason why the critical current shows non-monotonicity more readily than the critical temperature does is that the former can be measured at lower temperatures, at which the impurities are more strongly polarized by the field.

Conclusion and outlook. – We address the issue of superconductivity, allowing for the simultaneous effects of magnetic fields and magnetic impurity scattering, as well as spin-orbit impurity scattering. In particular, we investigate the outcome of the two competing roles that the magnetic field plays: first as a quencher of magnetic impurity pair-breaking, and second as pair-breaker in its own right. Thus, although sufficiently strong magnetic fields inevitably destroy superconductivity, the interplay between its two effects can, at lower field-strengths, lead to the enhancement of superconductivity, as first predicted by Kharitonov and Feigel'man via an analysis of the superconducting transition temperature. In the present Letter, we adopt the Eilenberger-Usadel semiclassical approach, and are thus able to recover the results of Kharitonov and Feigel'man, which concern the temperature at which the normal state becomes unstable with respect to the formation of superconductivity; but we are also able to address the properties of the superconducting state itself. In particular, our approach allows us to compute the critical current and specifically, its dependence on magnetic field

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and temperature.

We have found that any non-monotonicity in the field-dependence of the critical temperature is always accompanied by the non-monotonicity of the field-dependence of the critical current. However, we have also found that for a wide range of physically reasonable values of the parameters the critical current exhibits non-monotonic behavior with field at lower temperatures, even though there is no such behavior in the critical temperature.

Especially for small samples, for which thermal fluctuations can smear the transition to the superconducting state over a rather broad range of temperatures, the critical current is expected to provide a more robust signature of the enhancement of superconductivity, as it can be measured at arbitrarily low temperatures. In addition, the critical currents can be measured over a range of temperatures, and can thus provide rather stringent tests of any theoretical models. Recent experiments measuring the critical temperatures and critical currents of superconducting MoGe and Nb nanowires show behavior consistent with the predictions of the present Letter, inasmuch as they display monotonically varying critical temperatures but non-monotonically varying critical currents [8].

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