Conjecture concerning the modes of excitation of the quark-gluon plasma

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It is a widely held belief that at temperatures much higher than the confinement scale of quantum chromodynamics (QCD), quarks and gluons become free, giving rise to a new form of matter, called the quark-gluon plasma. It is conjectured here that the characterization of the plasma as a free or weakly interacting gas of quarks and gluons is valid only for short distances and short time scales of the order $1/T$, but that at scales larger than $1/g^2T$ (where g^2 is the running QCD coupling) the plasma exhibits confining features similar to that of the low-temperature hadronic phase. The confining features are manifest in the long-range, i.e., long-wavelength, low-frequency, modes of the plasma. To examine the long-range real-time response of the plasma goes beyond the capabilities of current lattice-gauge-theory techniques. However, some properties of these modes can be determined indirectly. An attempt is made to characterize the long-range modes of excitation by examining the static high-temperature limit, focusing upon the static screening lengths of colored and neutral local operators. Since g^2 is not small at temperatures likely to be accessible in heavy-ion collisions, the nonperturbative effects associated with vestiges of confinement are likely to be important in the phenomenological analysis of measurements made at accelerators.

I. INTRODUCTION

It is natural to assume that asymptotic freedom permits us to use perturbation theory to describe the properties of matter at temperatures well above the confinement scale $\Lambda_{\rm QCD}$, since the temperature T sets a scale for interactions. However, it has been known for some time that there are problems with infrared divergences in high-temperature quantum-chromodynamic (QCD) perturbation theory. A straightforward attempt to calculate the "magnetic mass" of the gluon perturbatively encounters infrared divergences that prevent an orderly summation of diagrams.[1–3] Furthermore, in the Yang-Mills theory of purely gluonic matter, the Wilson-loop expectation value has an "area-law" behavior for loops with a strictly spatial orientation at low as well as high temperatures, a behavior usually associated with nonperturbative confinement.

To study the possible nonperturbative or confining features of the plasma, it is necessary, of course, to avoid using perturbation theory. The approach adopted here draws upon what is known or can be known through direct numerical studies of the Euclidean functional-integral formulation of the theory, paying particular attention to the stable or nearly stable modes of excitation of the plasma. These excitations are associated with narrow peaks at low frequency ω and low wave number k in the spectral function ρ_{AB} of pairs of local operators A and B:

$$iS_{AB}(k,\omega)=\int_{-\infty}^{\infty} d^3x \int_{-\infty}^{\infty} dt\, e^{i\mathbf{k}\cdot\mathbf{x}-\omega t} \times[\langle A(\mathbf{x},t)B(0,0)\rangle - \langle A(0,0)\rangle\langle B(0,0)\rangle] , \tag{1.1a}$$

$$\rho_{AB}(k,\omega)=(1\mp e^{-\beta\omega})S_{AB}(k,\omega) . \tag{1.1b}$$

The operation $\langle\ \rangle$ denotes an average on the Gibbs ensemble. The upper sign is for bosons, lower for fermions. For the relativistic quantum-electrodynamic (QED) plasma these excitations are the quasielectron, the plasmon, the photon, and, at long wavelengths, the various hydrodynamic modes, namely, the photon and viscous-damping and thermal-conduction modes. The peaks in ρ are narrow typically at low k where dissipation is weak. These are the modes of a nonconfining gauge theory. If QCD were to be nonconfining at high temperature, then it would be expected to have a similar assortment of excitations. I suggest below, however, that this need not be the case for QCD. Instead the long-wavelength excitations could be color singlets and "hadronic" in character just as they are at zero temperature.

As an electron moves through a QED plasma it polarizes the surrounding medium, with the result that on a scale much longer than the Debye screening length no net current flow is associated with its passage. It is important to be able to distinguish this screening phenomenon from a confining phenomenon in QCD, which would also lead to a cancellation of the color current of a moving quark. To this end one may compare the propagator of the quark operator in a suitable gauge with the propagator of an operator for a color-singlet meson containing that quark. In the case of confinement both propagators have a large-distance decay controlled by the same mass constant. In the case of screening they do not. It is essential to choose a suitable definition of the quark propagator for the purposes of carrying out this test. The "confinement" test is described in Sec. II. This test is nothing but a rephrasing of the area-law test for spacelike Wilson loops, but it is helpful to phrase it in terms of the quark propagator itself.

The term "screening" has also been used to characterize the response of the zero-temperature vacuum and high-temperature plasma to the introduction of magnetic strings.[4,5] This "magnetic screening" is compatible with the term "confinement" as it is used here. See the Appendix.

To study the modes of excitation of the plasma, it would obviously be most desirable to carry out the difficult, full, nonperturbative analysis of the real-time response and construct the spectral function (1.1) directly. Such a goal lies beyond the capabilities of current lattice-gauge-theory techniques. For the present, therefore, we must content ourselves with studying the imaginary-time response, for which a nonperturbative numerical analysis in lattice-gauge theory is possible. In imaginary time, correlation functions are measured over a time scale of maximum interval $\Delta \operatorname{Im} t = 1/T$. Fourier analysis yields information about Green's functions at discrete imaginary frequencies.[6] In principle an analytic continuation would result in a complete knowledge of the spectral function for real ω. However, in practical calculations statistical errors and a limited lattice size make the continuation impossible. Thus, studying only the imaginary-time response imposes a severe limitation upon the available information.

At high temperatures it has been shown that at macroscopic scales the $(3+1)$-dimensional gauge theory reduces to a three-dimensional Euclidean gauge theory.[1,7–9] To be more specific, the Green's functions for k and ω much less than T in the $(3+1)$-dimensional theory can be computed as if the theory were a zero-temperature Euclidean three-dimensional pure Yang-Mills theory, augmented by a minimally coupled color-octet scalar field (the vestige of A_0). The coupling constant of the three-dimensional theory is $g_3{}^2 = g^2 T$, which also sets the scale for confinement. It has been shown by Nadkharni[9] that the dimensional reduction is valid to the one-loop level in perturbation theory. This dimensional reduction is illustrated in strong-coupling lattice-gauge theory in Sec. III.

Because the three-dimensional Yang-Mills theory is confining, its spectrum must consist of color-singlet analogs of glueballs. These states of the three-dimensional theory give some information about the modes of excitation of the $(3+1)$-dimensional theory. In particular, if a plasma mode has a dispersion relation given by

$$f(k,\omega) = 0 \ , \qquad (1.2)$$

then, as shown in Sec. IV, masses M_3 of the glueballs in the three-dimensional theory correspond to roots of the equation

$$f(\pm i M_3, 0) = 0 \ , \qquad (1.3)$$

if the roots exist.

In quantum electrodynamics the various nonhydrodynamic modes, namely, the quasielectron, plasmon, and transverse electromagnetic waves, all have the property that the roots (1.3) for imaginary wave number correspond one-to-one with the roots of (1.2) for real wave number and nearly real frequency—i.e., the two-dimensional surface in complex k and ω defined by (1.2) intersects the imaginary k axis at $\omega = 0$. For example the plasmon in QED is associated with Debye screening. It is conjectured that this correspondence is also found in QCD, but that only color-singlet modes occur there. If so, then the long-range (distances of the order $1/g^2T$) modes of the plasma must be "confined" color-singlet modes in order to coincide with the spectrum of the confined static three-dimensional theory at $\omega = 0$. This conjecture is offered as the simplest possibility that permits a reconciliation of the long-range confining characteristics of the three-dimensional theory and what is known about the validity of perturbation theory in the $(3+1)$-dimensional theory. There are other possibilities. A low-lying color-nonsinglet mode of the plasma could have a dispersion relation for which no root to (1.3) exists for $M_3 \lesssim O(T)$, the ultraviolet cutoff for the three-dimensional theory. In that case the confined modes with masses M_3 could become unstable with respect to decay into color-nonsinglet modes. The possibility of such an instability was suggested by d'Hoker with reference to the QCD plasmon.[10] A further possibility is that a color-nonsinglet mode could satisfy (1.3), but decouple from all local operators at $\omega = 0$. Indeed, this behavior is found for the color-singlet hydrodynamic modes.

As a consequence of this conjecture, the following characterization of the high-temperature plasma ($T \gg \Lambda_{\text{QCD}}$) emerges, as illustrated in Fig. 1. At distances and times less than $O(1/T)$, perturbation theory is valid and a partonlike description of the plasma as a gas of quarks and gluons is most economical. At distances and times of the order $1/gT$, perturbation theory with screening may be used, and it is economical to treat the plasma as a gas of quasiquarks, plasmons, and dressed tranverse gluons. At distances and times of order $1/g^2T$ nonperturbative confining effects become important. The plasma is described as a fluid of color-singlet excitations. Finally at distances much greater than $1/g^2T$, hydrodynamic modes are important. Since at thermal equilibrium the temperature determines the dominant momentum for an excitation, the high-temperature plasma is, to a good approximation, a weakly interacting gas of quarks and gluons. However, this description fails to the extent

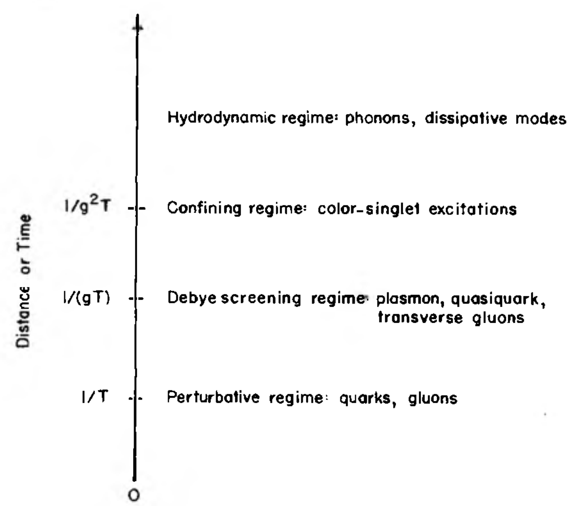

FIG. 1. Scales of the quark-gluon plasma for $T \gg \Lambda_{\text{QCD}}$.

it relies on quasifree particle momenta of the order g^2T, which are subject to confinement. For temperatures $T \approx \Lambda_{\rm QCD}$, g^2 is of order one and the corrections are expected to be considerable.

II. AREA LAWS AND QUARK AND GLUON PROPAGATORS: A TEST FOR DYNAMICAL CONFINEMENT OF THE PLASMA MODES

There is little doubt now that static charges are screened at high temperature in a pure gluon plasma.[11] When quarks are introduced into the statistical ensemble static charges are screened at all temperatures, just as charges are screened at nonzero temperature in QED. In QED an electron passing through a plasma carries with it a polarization cloud that screens it at long distances. In QCD moving colored objects are undoubtedly screened as well. However, the nature of the screening for dynamical excitations may be qualitatively different. Therefore I propose to distinguish between long-range confinement and screening of dynamical charges, since this choice has a bearing on an understanding of the macroscopic composition of the plasma. The question is whether it is possible to produce a long-wavelength quarklike or gluonlike excitation that cannot also be generated by a color-singlet source. In the corresponding case of QED there is a quasielectron excitation that is certainly distinguished from a neutral excitation, even though screening is present. In QED the neutralizing polarization cloud that moves with the electron is not required to involve the transport of a positron, per se, but merely the shifting of the vacuum polarization. In QCD it may happen, by contrast, that the neutralizing quanta are forced to move in such a way that they render the would-be quarklike or gluonlike disturbance indistinguishable from a disturbance initiated by a suitable color-singlet source.

To consider the questions raised above it is necessary to find a suitable definition of a quark or gluon propagator—a definition that can be used as well for the electron in QED.[12] At zero temperature in QCD the construction of a quark or gluon propagator is plagued by an awkward interplay between the gauge dependence and confinement. By contrast, because QED is not confining, there is no confusion about the electron propagator. One might hope that if QCD were not confining at high temperatures, there would be a similar ease in defining propagators for colored objects. In fact, we find that this hope is not realized. Nonetheless, it is possible to choose a particular gauge in which the electron propagator in QED exhibits long-range screening behavior at all temperatures and the quark propagator exhibits long-range confining behavior in the sense used here. Such a result should render the long-range colored propagators thankfully irrelevant.

Let us attempt to construct the fermion propagator in an axial gauge. The asymptotic behavior of the propagator or correlation function at large spatial distances can be analyzed in a theory in which spacelike Wilson loops have an area law in the pure gauge theory, as they may in QCD, and in a theory in which they do not, such as QED. In the former case the asymptotic behavior is controlled by a correlation length corresponding to a singlet "mass." In the latter it is not.

The analysis of the various correlation functions involves, in effect, finding the eigenvalues and eigenstates of the spacewise transfer matrix. This is the transfer matrix that characterizes evolution along a particular spatial direction, say the 3 axis. In the finite-temperature Euclidean path-integral formulation, one may regard the 3 axis as the imaginary-time axis and the 4 axis as one of the spatial axes. The path integral can then be regarded as giving a zero 3-axis-temperature partition function for a Hamiltonian defined on a three-dimensional space with the original 1 and 2 directions intact but with the third spatial dimension (the original 4 axis) reduced to a finite, periodic interval of length $1/T$. The imaginary-time evolution operator for this Hamiltonian for short times is the original spacewise transfer matrix. This Hamiltonian at high T and long-distance scales is approximately that of a $(2+1)$-dimensional gauge theory with an additional scalar field, as discussed in Sec. III below.[9] To the extent that this limiting theory is confining one expects large *spacelike* Wilson loops in the original $(3+1)$-dimensional theory to obey an area law.

Consider the fermion correlation function in QCD or QED,

$$iS(x,y)=\langle \psi(x)\bar{\psi}(y)\rangle \ , \tag{2.1}$$

where the average is over the Gibbs ensemble. Since the operator $\psi(x)$ is gauge dependent, it is necessary to render the expression gauge invariant through the introduction of a string operator that connects x and y. Let the gauge fields be defined in a periodic spatial volume of large but finite extent. Make the string run from x to y for the most part along the 3 axis on a contour D by going to the boundary of the lattice and reappearing at the opposite boundary as shown in Fig. 2. Here we have assumed $x_3 < y_3$. The volume is to be taken to infinity at fixed $|y_3-x_3|$. There is a detour needed to align the positions perpendicular to the 3 axis. It is taken to infinity with the volume. Thus with the string operator

$$C_D(y,x)=P\exp\left[i\int_{xD}^{y}\lambda^a A_\mu^a(z)dz^\mu\right] \ , \tag{2.2}$$

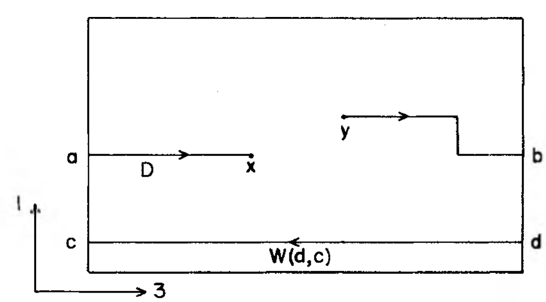

FIG. 2. Position of the string D connecting x and y in a periodic volume and the Polyakov loop $W(d,c)$. The points x and y are allowed to have an offset in the 2 and 4 directions as well.

the expression

$$iS_D(x,y)=\langle C_D(y,x)\psi(x)\bar{\psi}(y)\rangle \quad (2.3)$$

is gauge invariant. Of course a suitable ultraviolet regularization, e.g., a lattice, must be introduced to avoid an ultraviolet divergent self-energy due to the infinitesimal size of the string. To allow for return electric flux in nonconfining theories it is useful to introduce a Polyakov loop operator $W(d,c)$ also shown in Fig. 2 at a large distance from the other points,

$$W(d,c)=\mathrm{Tr}\left[P\exp\left[i\int_c^d \lambda^a A_\mu^a(z)dz^\mu\right]\right], \quad (2.4)$$

so that we have

$$iS'_D(x,y)=\langle W(d,c)C_D(y,x)\psi(x)\bar{\psi}(y)\rangle . \quad (2.5)$$

Next it is useful to remove the "disconnected" part of the correlation function by subtracting its asymptotic value

$$S_{D\mathrm{conn}}(x,y)=S'_D(x,y)-\lim_{|y_3-x_3|\to\infty} S'_D(x,y) . \quad (2.6)$$

The proper order for the limits is first to take the transverse dimensions of the volume (1 and 2 directions) to infinity along with the transverse separation between D and $W(d,c)$, then to take the limit $|y_3-x_3|\to\infty$ in the second term above, keeping $|x_3-a_3|$ and $|b_3-y_3|$ fixed. Finally, after taking the difference, it is useful to normalize the result before taking the third dimension to infinity:

$$\bar{S}_{D\mathrm{conn}}(x,y)=S_{D\mathrm{conn}}(x,y)/\langle W(d,c)W(b,a)\rangle . \quad (2.7)$$

What does this axial-gauge fermion-correlation product measure? Consider first the Polyakov loops at zero temperature in QCD. For large $|b_3-a_3|=|d_3-c_3|=L_3$,

$$\langle W(d,c)W(b,a)\rangle \sim \exp(-2m_H L_3) , \quad (2.8)$$

where m_H is the lightest mass of an unphysical color-singlet meson containing one fixed-triplet source. It is easy to understand why this state appears. Close to zero temperature the Euclidean space-time volume can be made symmetric under the interchange of the time and 3 axes. Thus the Polyakov loops give a measure of the free energy of a state containing a pair of fixed-triplet sources at large separation. As $L_3\to\infty$ the temperature vanishes, and the only surviving states of finite mass are color singlets containing antiquarks or quarks that combine with the fixed sources to form color singlets. The world lines of these additional dynamical quarks lie close to the Polyakov loops, thereby assuring that after dynamical fermion degrees of freedom have been considered, no Wilson loops of large area have been formed. In the language of functional integration, the fermion determinant may be expanded as a series of Wilson and Polyakov loops. The gauge action suppresses loop combinations that result in a large-area spacelike loop.

Now consider the spatial Polyakov loops at high temperatures. Assume that the spatially oriented Wilson loops still have an area-law behavior. Then to prevent large-area Wilson loops, it is again necessary for the fermion world lines to lie close to the loop contours. We have

$$\langle W(c,d)W(b,a)\rangle \sim \exp[-2m_H(T)L_3] . \quad (2.9)$$

In Hamiltonian language $m_H(T)$ is the mass of the lowest-lying color-singlet meson containing a fixed-triplet source for QCD defined on a periodic space-time volume with one of the dimensions reduced to $1/T$ and the other two dimensions large.

Next, consider the correlation product $S_D(x,y)$. A fermion is created at y and moves to x. The world lines it may generate are disconnected, if it is possible to divide the volume into a left and right part without cutting a dynamical world line. Otherwise they are connected. The simplest connected world line goes directly from x to y. The simplest disconnected world line runs back along the string from x to y. The operation (2.6) removes the disconnected contributions. Now the simplest surviving connected contribution resembles closely the Polyakov loop except that for a portion of the loop there is a dynamical quark rather than a fixed one, and there is a small detour. Again other dynamical fermion lines must appear so as to suppress the infinite area loops both at low and high temperatures. For large $|y_3-x_3|$ the combinations of fermion world lines appearing between x and y are those of propagating color-singlet mesons with no fixed sources involved. They must contain the propagating quark, however. Let the mass of the lightest such color-singlet meson be $m(T)$. Then the normalized correlation product has the behavior (up to powers of $|y_3-x_3|$),

$$S_{D\mathrm{conn}}(x,y) \underset{|y_3-x_3|\to\infty}{\sim} \mathrm{const}\exp\{-|y_3-x_3|[m(T)-m_H(T)]\} . \quad (2.10)$$

(It is understood that the limit $L_3\to\infty$ is taken before the limit $|y_3-x_3|\to\infty$.) The key observation here is that the asymptotic behavior at large spatial separations is controlled by "masses" of color-singlet states. Notice that in QCD the extra Polyakov loop $W(d,c)$ is not needed because confinement forces the generation of a color-singlet state localized around each loop.

In continuum quantum electrodynamics the result is different. Because Wilson loops do not have an area-law behavior, there is no need to include additional neutralizing fermion world lines in the Polyakov-loop product (2.8) or in the correlation product (2.5). The asymptotic behavior of the normalized axial-gauge correlation product at large spatial separation is still of the form (2.9), but the "masses" have a different interpretation. The quantity $m_H(T)$ is the energy of a point source in a periodic space with one small dimension $1/T$. The quantity $m(T)$ is the mass of an electron in such a space. In neither case is an electrically neutral object required because the theory is not confining.

To summarize, we have proposed a procedure for defining a fermion correlation product that has the property that at large spatial separation its asymptotic form is controlled by neutral states in a "confining" theory, but charged states in a nonconfining theory. A similar procedure can be used to construct correlation products for the gauge field $F^a_{\mu\nu}$ itself. Again in a confining theory such as QCD, color-neutral states control the asymptotic behavior. However, the photon is, of course, neutral and so provides no contrasting result for QED.

Finally, to complete the test for dynamical confinement it is necessary to measure the corresponding masses for all the neutral objects in the plasma. If $m(T)$ happens to be the same as one of these, then the theory is dynamically confining.

This test is so far restricted to the static correlation lengths that control the large-distance behavior of the correlation product. It is important to extend the test to real time. A dramatic test of the absence of long-range quark and gluon propagation would be to define a propagator for charged sources such that for a confining theory the singularities in k and real ω always coincided with the singularities of the propagator of some color-singlet operator up to shifts coming from $m_H(T)$, whereas in a nonconfining theory they did not. The real-time test is beyond the scope of the present work.

III. DIMENSIONAL REDUCTION AT HIGH TEMPERATURE ON THE LATTICE

Here we illustrate dimensional reduction at high temperature for strong-coupling SU(3) lattice-gauge theory with the fermion scheme of Wilson.[13] In this approximation the partition function is

$$Z=\int [dU\,d\bar{\eta}\,d\eta]\exp[S_G(U)+S_F(\bar{\eta},\eta,U)] \ , \tag{3.1}$$

where the integration is (as usual) over the Haar measure for SU(3) link matrices U_{x_ρ} (x labels the Euclidean space-time lattice point on an $N_\tau \times N_s{}^3$ lattice and $\rho=1,2,3,4$ is the direction from that site) and over the Berezin measure for the fermion Grassmann variables η_x and $\bar{\eta}_x$. In the strong-coupling high-temperature limit[14] we take a lattice with $N_\tau=1$, i.e., only one step in the time direction, and let $\tau=\beta_t=1/T$, the lattice constant in the time direction, be smaller than a, the lattice constant in the space direction. For this anisotropic lattice

$$S_G(U)=\frac{2}{g^2}\sum_{\mathbf{x}}\left[\frac{a}{\tau}\sum_{i=1}^{3}\mathrm{Re\,Tr}(U_{\mathbf{x},4i})+\frac{\tau}{a}\sum_{i<j}\mathrm{Re\,Tr}U_{\mathbf{x},ij}\right] , \tag{3.2}$$

where, because $N_\tau=1$, the space-time plaquette is

$$U_{\mathbf{x},4i}=U_{\mathbf{x}4}U_{\mathbf{x}i}U^\dagger_{\mathbf{x}+\hat{\mathbf{i}},4}U^\dagger_{\mathbf{x}i} \tag{3.3}$$

and the space-space plaquette is, as usual,

$$U_{\mathbf{x},ij}=U_{\mathbf{x}i}U_{\mathbf{x}+\hat{\mathbf{i}},j}U^\dagger_{\mathbf{x}+\hat{\mathbf{j}},i}U^\dagger_{\mathbf{x}j} \ . \tag{3.4}$$

The fermion action at zero chemical potential is

$$S_F=(m\tau+1+3\tau/a)\sum_{\mathbf{x}}\bar{\eta}_{\mathbf{x}}\eta_{\mathbf{x}} -\frac{\tau}{2a}\sum_{\mathbf{x},i}[\bar{\eta}_{\mathbf{x}+\hat{\mathbf{i}}}(1+\gamma_i)U^\dagger_{\mathbf{x}i}\eta_{\mathbf{x}}+\bar{\eta}_{\mathbf{x}}U_{\mathbf{x}i}(1-\gamma_i)\eta_{\mathbf{x}+\hat{\mathbf{i}}}] +\tfrac{1}{2}\sum_{\mathbf{x}}[\bar{\eta}_{\mathbf{x}}(1+\gamma_4)U^\dagger_{\mathbf{x}4}\eta_{\mathbf{x}}+\bar{\eta}_{\mathbf{x}}U_{\mathbf{x}4}(1-\gamma_4)\eta_{\mathbf{x}}] \ . \tag{3.5}$$

The third term in (3.5) coming from timeward hopping gives rise to a mass term when $N_\tau=1$ by virtue of the fermion antiperiodic boundary condition. For free fermions the mass is $O(T)$ relative to the three-dimensional hopping term.

At high temperatures it is well known that the timelike links $U_{\mathbf{x}4}$ cluster around the unit matrix.[11] Thus we may expand

$$U_{\mathbf{x}4}\approx 1+i\lambda^a\phi^a_{\mathbf{x}}-\frac{(\lambda^a\phi^a_{\mathbf{x}})^2}{2!} \ , \tag{3.6}$$

where the $\{\lambda^a\}$ are the usual eight generators of SU(3) rotations and $\phi^a_{\mathbf{x}}$ is a color-octet scalar field defined on the three-dimensional lattice. The space-time plaquette then satisfies

$$\mathrm{Re\,Tr}U_{\mathbf{x},4i}\approx 3-[\phi^a_{\mathbf{x}}-D_{ab}(U_{\mathbf{x}i})\phi^b_{\mathbf{x}+i}]^2 \ , \tag{3.7}$$

where $D_{ab}(U)$ is the adjoint representation of the SU(3) matrix U.

Ignoring irrelevant constants and rescaling both $\phi^a_{\mathbf{x}}$ and $\eta_{\mathbf{x}}$ gives a revised action

$$S_G+S_F\to S_G(\phi,U)+S_F(\bar{\eta},\eta,U,\phi) \ , \tag{3.8}$$

where

$$S_G(\phi,U)=-\tfrac{1}{2}\sum_{\mathbf{x}}[\phi^a_{\mathbf{x}}-D_{ab}(U_{\mathbf{x}i})\phi^b_{\mathbf{x}+\hat{\mathbf{i}}}]^2 +\frac{2\tau}{g^2a}\sum_{\mathbf{x}}\sum_{i<j}\mathrm{Re\,Tr}U_{\mathbf{x},ij} \ , \tag{3.8a}$$

$$S_F=(ma+2a/\tau+3)\sum_{\mathbf{x}}\bar{\eta}_{\mathbf{x}}\eta_{\mathbf{x}}-\frac{ag^2}{2\tau}\sum_{\mathbf{x}}\bar{\eta}_{\mathbf{x}}(\lambda^a\phi^a)^2\eta_{\mathbf{x}} -\frac{ag}{\tau}\sum_{\mathbf{x}}\bar{\eta}_{\mathbf{x}}i\gamma_4\lambda^a\phi^a\eta_{\mathbf{x}} -\tfrac{1}{2}\sum_{\mathbf{x}i}[\bar{\eta}_{\mathbf{x}+\hat{\mathbf{i}}}(1+\gamma_i)U^\dagger_{\mathbf{x}i}\eta_{\mathbf{x}}+\bar{\eta}_{\mathbf{x}}U_{\mathbf{x}i}(1-\gamma_i)\eta_{\mathbf{x}+\hat{\mathbf{i}}}] \ . \tag{3.8b}$$

In this form the theory is a three-dimensional Euclidean version of QCD augmented by a color-octet scalar field coupled covariantly to the Yang-Mills field. The square of the Yang-Mills coupling constant is obtained from the inverse of the coefficient of the second term of (3.8a), i.e.,

$$g_3{}^2=g^2T \ . \tag{3.9}$$

This three-dimensional lattice theory corresponds to what has been proposed for the continuum theory.[9] The choice $N_\tau=1$ is thought to give a good approximation to the partition function of the Hamiltonian lattice theory only when $1/T=\tau\ll a$.[14] Thus the temperature gives the scale of the ultraviolet (UV) cutoff of the three-dimensional theory. With this approximation we are limited to studying correlations at distances much larger than T^{-1}. In attempting to do perturbation theory with the three-dimensional theory one discovers that the self-energy of the scalar field is potentially UV divergent, but is regulated by the lattice. In this way the scalar field acquires a mass of order eT in quantum electrodynamics and gives rise to Debye screening and the plasmon.

As a consequence of this dimensional reduction, the spacewise transfer matrix of the $(3+1)$-dimensional theory at high temperatures becomes the usual imaginary-time transfer matrix of the Euclidean three-dimensional theory. The static correlation lengths of the $(3+1)$-dimensional theory at high temperature are then inverses of the discrete masses of the dimensionally reduced theory. Therefore the spectrum of the three-dimensional theory is of interest. We close this section by speculating about the spectrum of the theory consisting of QCD_3 coupled to an adjoint scalar field. For the purposes of this discussion let us use A_μ^a for $\mu=1,2,3$ to denote the three-dimensional vector field and ϕ^a to denote the scalar field. The three-dimensional SU(2) Yang-Mills theory was studied numerically by d'Hoker, who presented evidence for confinement.[10] His numerical evidence for the existence of a mass gap was not decisive, however. Let us suppose, nonetheless, that his conclusions are correct and that a mass gap occurs, as it does in QCD_{3+1}. It would correspond to the lightest glueball of the theory, presumably with valence composition AA. Since the coupling constant of the theory (3.9) has a dimension, the glueball mass would necessarily conform to this scale, giving

$$M_{3AA}=O(g^2T) . \tag{3.10}$$

With the scalar field included one could entertain the possibility that a Higgs phenomenon destroys confinement. As a consequence large spatially oriented Wilson loops would follow a perimeter law. This possibility can also be checked numerically. For what follows we assume that no such breakdown of confinement occurs. In this case we expect the scalar fields to bind with the vector fields to form further glueballs of valence composition $\phi\phi$ and ϕA, etc. Since the UV mass renormalization discussed above gives the scalar field an effective mass of $O(gT)$, for $g^2 \ll 1$ the scalar particles would have masses larger than the confinement scale and would bind as color-octet analogs of charmonium and states of bare charm. Then we would have

$$M_{3\phi A}\approx M_{3\phi\phi}\approx O(gT) . \tag{3.11}$$

Thus at distances of order $1/g^2T$ the color plasmon ϕ would be color neutral, carrying at least one vector field A with it. At moderate temperatures it may mix strongly with the AA glueballs.

With quarks also included other hadronic analog states such as mesons and baryons could be constructed. However, they are not as reliably studied in this approximation, since their bare masses in the reduced theory are intrinsically of the same size as the UV cutoff. The binding of quarks to form color singlets is nonetheless to be expected as a consequence of the analysis of the quark propagator in Sec. II above. They should mix with the glueball states.

IV. ANALYTICAL CONNECTION BETWEEN STATIC CORRELATION LENGTHS AND DYNAMICAL MODES

Having discussed a possible spectrum for the three-dimensional theory and therefore for the spacewise transfer matrix at high temperature, we turn now to a brief discussion of the connection between the discrete spectrum of the transfer matrix and the dynamical modes of the plasma. We first summarize some notation and formulas of finite-temperature linear-response theory. The retarded momentum-space propagator for a pair of local operators A and B is given conventionally by

$$i\Gamma_R(\mathbf{k},\omega)=\int d^3x \int_0^\infty dt\, e^{i\omega t-i\mathbf{k}\cdot\mathbf{x}} \times\langle [A(x,t),B(0,0)]_\mp\rangle , \tag{4.1}$$

where the sign denotes a commutator for bosons or anticommutator for fermions. The propagator is in turn related to its spectral function through

$$\Gamma_R(\mathbf{k},\omega)=\lim_{\epsilon\to 0}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\rho_{AB}(k,\omega')}{\omega-\omega'+i\epsilon} , \tag{4.2}$$

and ρ_{AB} is related in turn to S_{AB} of (1.1a) through (1.1b). Finally, the imaginary-time propagator is given by

$$S_{nAB}(\mathbf{k})=\int_0^\beta d\tau \int d^3x\, e^{i\omega_n\tau}e^{-i\mathbf{k}\cdot\mathbf{x}} \times[\langle A(x,-i\tau)B(0,0)\rangle -\langle A(0,0)\rangle\langle B(0,0)\rangle] \tag{4.3}$$

for discrete frequency $\omega_n=2\pi n/T$ for bosons and $(2n+1)\pi/T$ for fermions. For $n\geq 0$ the imaginary-time propagator is just the analytic continuation of $\Gamma_R(k,\omega)$:

$$S_{nAB}(\mathbf{k})=\Gamma_R(\mathbf{k},i\omega_n) . \tag{4.4}$$

Let us suppose that for boson operators A and B, $\rho_{AB}(\mathbf{k},\omega)$ has a narrow peak at a low-frequency ω and wave-number $\mathbf{k}$, such that $\Gamma_R(\mathbf{k},\omega)$ has a pole in $\mathbf{k}$ and ω corresponding to a dynamical mode of excitation of the plasma. Because of rotational symmetry and parity, the location of the pole is given by

$$f(k,\omega)=0 , \tag{4.5}$$

where $k=|\mathbf{k}|$. It follows that $S_{0AB}(\mathbf{k})$ has a pole in k at $k=\pm iM$ such that

$$f(\pm iM,0)=0 . \tag{4.6}$$

Such a pole implies that the time-averaged correlation between boson operators A and B, defined by

$$S_{AB}(\mathbf{x})=\langle \bar{A}(\mathbf{x})\bar{B}(0)\rangle-\langle\bar{A}(0)\rangle\langle\bar{B}(0)\rangle , \tag{4.7}$$

where, for a general local operator O,

$$\bar{O}(\mathbf{x})=\frac{1}{\beta}\int_0^\beta d\tau\, O(\mathbf{x},-i\tau) , \tag{4.8}$$

has an asymptotic contribution

$$S_{AB}(\mathbf{x}) \underset{|\mathbf{x}|\to\infty}{\sim} \text{const}\exp(-M|\mathbf{x}|) . \tag{4.9}$$

The lowest such M dominates the asymptotic behavior and determines the inverse static correlation length

$$\lambda^{-1}(T)=M(T) . \tag{4.10}$$

It also determines the gap in the spectrum of the spacewise transfer matrix.

As an illustration, let us suppose that to a good approximation

$$f(k,\omega)=\omega^2-s^2k^2-\Delta^2=0 \quad (4.11)$$

is the dispersion relation for a dynamical mode. The gap for real-time excitations is Δ. The static correlation length is then

$$\lambda^{-1}=M=\Delta/s\ . \quad (4.12)$$

Thus the static correlation lengths are determined from the dispersion function for the dynamical modes by an analytic continuation to zero frequency.

Hydrodynamic modes such as phonons have the typical property that $\omega\to 0$ as $k\to 0$. Thus one might expect the static correlation length to be infinite for any fluid with a phonon. However, they decouple from local operators at zero frequency and so are not found in (4.9). For example, for nearly all fluids, the photon contribution to the spectral function of the energy density is given at low k by

$$\rho_{\epsilon\epsilon}(\mathbf{k},\omega)=\pi C_V Tk[\delta(\omega-c_s k)-\delta(\omega+c_s k)]\ , \quad (4.13)$$

where C_V is the specific heat and c_s is the speed of sound.[15] The decoupling at $k=0$ is evident.

V. CONCLUSIONS AND DISCUSSION

Based on the likelihood that the spacewise transfer matrix for QCD at high temperature has a color-singlet spectrum, it has been conjectured that all the low-lying dynamical modes of the quark-gluon plasma consist of color-singlet excitations. They would be dynamically confined in the sense that excitations generated by local colored sources would be indistinguishable from excitations generated by local color-singlet sources. The confinement scale is expected to be of order $1/g^2T$, where g^2 is the running QCD coupling. Other plasma scales are indicated in Fig. 1. Since g^2 is small at very high temperature, the confinement scale is much larger than the typical wavelength of particles at high temperatures. Thus the high-temperature plasma is, to a good approximation, a gas of quasifree quarks and gluons. However, quanta with momenta of the order g^2T are subject to nonperturbative confining effects. They contribute to the thermodynamic potential Ω roughly in proportion to their share of the phase-space volume, inside a sphere in momentum space of radius g^2T, i.e., a correction

$$\Delta\Omega\approx O(g^6)T^4\ . \quad (5.1)$$

At high temperatures $\Delta\Omega$ is rather small. However, at the deconfinement temperature, g^2 is close to one and the corrections are probably considerable.

This characterization of the long-range composition of the plasma is contrary to the naive picture that the deconfinement of static charges entails the deconfinement of dynamical charges as well. However, as argued in Sec. II a simple deconfinement of dynamical charges giving a QED-like plasma is incompatible with the expectation that spatially oriented Wilson loops in QCD have an area-law behavior. The situation in QCD is more complicated. What is offered here is the next simplest possibility, namely, that the low- and high-temperature phases both have confining characteristics, but at sufficiently high temperatures confinement has an insignificant effect upon the thermodynamic properties. The question then becomes, what is the nature of the phase transition, if any? Although the pure Yang-Mills plasma exhibits deconfinement of static color-triplet charges, dynamical charges may well have a different behavior. With light quarks present there is also the possiblity of a phase transition that leads to the restoration of manifest chiral symmetry. Indeed, if there is a phase transition in light-quark QCD, it may well have more of a chiral character than a deconfinement character. Thus it may be possible that all zero temperature hadrons have analog modes in the high-temperature phase. One should measure the behavior of the gap functions $\Delta(T)$, particularly near the phase transition for all hadronic modes. One would expect that if the phase transition were deconfining, the hadronic modes would be abruptly reduced in number above the critical temperature leaving only a few modes available for flavor and baryon transport and a few hydrodynamic modes. However, if the phase transition were chiral in character, it may happen that most low- and high-temperature modes would be in obvious one-to-one correspondence, but modes sensitive to the pattern of realization of chiral symmetry, such as the pionic mode, would exhibit a significant discontinuity in the gap function $\Delta(T)$ or its derivative at the phase transition.

Although numerical simulations of real-time response in the QCD plasma require as yet undeveloped techniques, measurements of static correlations $\lambda(T)$ are easily possible with current numerical lattice-gauge-theory techniques. Perhaps the behavior of $\lambda(T)$ will give us a hint as to how $\Delta(T)$ behaves. Questions of interest are these:

(1) Is there evidence for a gap? i.e., is $\lambda(T)<\infty$ for all modes?

(2) Can one produce a quarklike or gluonlike mode that is distinguishable from a color-singlet mode?

(3) What are the relative sizes of the correlation lengths for mesonic and baryonic modes in light-quark QCD and glueball modes in pure Yang-Mills QCD? What are the lowest lying nonhydrodynamic excitations in the QCD plasma likely to be? Presumably they would have the largest $\lambda(T)$.

(4) If chiral symmetry becomes manifest, is there evidence in the spectrum, i.e., does $\lambda(T)$ for the pionic mode show a significant change and is there evidence for parity doubling?

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APPENDIX

A strong-coupling analysis shows that the screening of a magnetic string can occur at the same time that spacelike Wilson loops show an area-law behavior. This result is not surprising in the least, but may help to clarify the terminology. Thus what I call confinement of the long-range high-temperature plasma modes, a phenomenon associated with the area law, and leading to the exclusion of colored excitations, is compatible with what others call screening of magnetic strings.[4,5]

Consider a pure SU(2) Yang-Mills lattice action in the Wilson form on a periodic Euclidean space-time. Consider the set of string plaquettes

$$T=\{U_{x12}\,|\,x_1=a_1,x_2=a_2\} \quad \text{(A1)}$$

for constant a_1, a_2. These are a string of plaquettes perpendicular to the 3 axis at fixed $x_1=a_1$, $x_2=a_2$ for all possible x_3 and x_4. The magnetic string is introduced by replacing the usual action by the twisted action

$$S_{\rm tw}=\beta\sum_{P\notin T}U_P-\beta\sum_{P\in T}U_P\ , \quad \text{(A2)}$$

where each of the string plaquettes is multiplied by a nontrivial element of the center of the gauge group, in this case -1 (Ref. 16). The twisted action is interpreted as representing the effect of introducing a static line of magnetic flux passing through the lattice along the 3 direction. The response of the vacuum to the introduction of such a structure is measured by

$$e^{\beta(F_{\rm tw}-F)}=\int[dU_P]\exp S_{\rm tw}\Big/\int[dU_P]\exp S\ . \quad \text{(A3)}$$

Another measure of the effect of the twist is the difference between the average plaquettes in the twisted and untwisted case:

$$\frac{\partial}{\partial\beta}(F_{\rm tw}-F)=N(\langle U_P\rangle_{\rm tw}-\langle U_P\rangle)\ , \quad \text{(A4)}$$

where N is the total number of plaquettes, $\langle U_P\rangle$ is the usual plaquette average, and $\langle U_P\rangle_{\rm tw}$ is the plaquette average *weighted with minus signs* for the string plaquettes and calculated with the twisted action. The twist effect is easily calculated to leading order in the strong-coupling approximation at low as well as high temperatures[17] and is given by

$$\langle U_p\rangle_{\rm tw}-\langle U_p\rangle=-\frac{1}{2N_1N_2}\left|\frac{\beta}{2}\right|^{N_1N_2}, \quad \text{(A5)}$$

where N_1 and N_2 are the number of sites in the 1 and 2 directions. This dependence on the area N_1N_2 is termed the screening of the static magnetic string. It occurs both at low and high temperatures.[4,5] In the same leading-order strong-coupling approximation, the spatially oriented Wilson loops have the behavior

$$W_{nm}\sim\frac{1}{2}\left|\frac{\beta}{2}\right|^{nm} \quad \text{(A6)}$$

at both low and high temperature. This area-law behavior is associated with the confinement of the plasma modes. Thus it is possible for both phenomena to occur in the same theory. Indeed the screening length for the non-Abelian magnetic strings appears to have more in common with the string tension of the spacelike Wilson loops than with the masses $M(T)$ of Eq. (4.10) that are associated with the color-singlet excitations. The scale for all three is expected to be $O(g^2T)$ at high temperature, however.

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