Resonant above-threshold ionization at quantized laser intensities

J. Gao,[1] Dong-Sheng Guo,[2] and Yong-Shi Wu[3,*]
[1]Department of Electrical Engineering, University of Illinois, Urbana, Illinois 61801
[2]Department of Physics, Southern University and A&M College, Baton Rouge, Louisiana 70813
[3]School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, New Jersey 08540
(Received 29 January 1998; revised manuscript received 23 February 1999; published 7 March 2000)

We argue that quantum electrodynamics dictates resonance phenomena in multiphoton processes as the laser intensity varies. A perturbation theory is developed in which the coupling between an electron and the second quantized laser mode is treated nonperturbatively. As an example, we predict that the above-threshold ionization rate can exhibit resonance at intensities with integer ponderomotive parameter. Such quantum effects may be exploited to calibrate laser intensities.

PACS number(s): 32.80.Wr

INTRODUCTION

In the literature on multiphoton processes of charged particles, the strong laser field is usually treated as a nondynamic, classical background [1,2]. However, according to quantum electrodynamics (QED), the fundamental theory for electromagnetic interactions, the radiation field is composed of dynamical oscillator modes, specified by their wave vector and frequency. All the modes have discrete levels in terms of photons with definite energy and momentum. Being dynamical, the photon number in the laser mode interacting with a charged particle fluctuates due to (intensity-dependent) stimulated interactions. Though the fluctuations are extremely tiny compared to the total photon number, at appropriate intensities the *absolute magnitude* of their energy can be *in resonance with* the equally tiny level spacing of the laser mode. When this happens, one expects to see resonance phenomena in intensity dependence due to the quantum nature of the radiation field.

As an example, let us examine the above-threshold ionization (ATI) of neutral atoms, say hydrogen or xenon, in the focus of a monochromatic, elliptically polarized single-mode laser (with wave vector $\mathbf{k}_0$). Even if the photon energy $\hbar\omega_0$ is merely a fraction of the ionization energy E_b, at high laser intensity a bound electron can absorb simultaneously a number of, say ten to twenty, photons to become ionized with kinetic energy appreciably higher than the threshold value. Before the photoelectron exits from the focus, it has very strong stimulated interactions with the laser. Previously in the Keldysh-Faisal-Reiss theory [3] for ATI, the states of the ionized electron were described by the Volkov states in a classical plane wave background [4]. However, according to QED, before the ionized electron leaves the focus, the intermediate states should be described by eigenstates of the combined electron-laser-mode system, called quantum field Volkov states (QFVS) [5,6].

In contrast to ordinary Volkov states, QFVS incorporates the reaction of the laser mode to the electron, i.e., the fluctuations in photon number due to stimulated emission and absorption. These fluctuations lead to an extra contribution to the total energy and momentum lying on the light cone [5,6] (also see below):

$$U_p = u_p\omega_0, \qquad \mathbf{P}_p = u_p\mathbf{k}_0. \tag{1}$$

Here $u_p \equiv 2\pi e^2 I/m_e\hbar c\omega_0^3$ (with I laser intensity, m_e electron mass); we call it the ponderomotive parameter, since U_p can be identified with the ponderomotive energy for an electron in the light field. We suggest to interpret U_p and $\mathbf{P}_p$ as arising from a fluctuating laser photon cloud that dresses the electron, and to identify u_p as the average number of laser photons in the dressing cloud (a distinctive concept of QED). Then it is natural to interpret $\mathbf{P}_p$ as the ponderomotive momentum accompanying the ponderomotive energy [7]. An exit process for the photoelectron to leave the laser field is naturally included in the transition matrix derived by Guo, Åberg, and Crasemann (GAC) [8] [see Eq. (13) in this paper] from the standard formal theory of scattering [9]. An adiabatic switching on and off of the interacting field is assumed in the scattering theory. In the transition matrix element of GAC, the Volkov states, in the same energy level of the initial and the final states, act as intermediate states; while an electron-photon plane wave acts as the final state. Upon exiting from the laser field, the photoelectron has to undress the dressing photon cloud, with both energy and momentum conserved. GAC's exit process obtained firm experimental verification in standing-wave multiphoton ionization, which was well-known as the half Kapitza-Dirac effect, performed by Bucksbaum *et al.* [12,13]. But in the single-mode multiphoton ionization case, GAC's transition matrix element encountered a serious difficulty. With the single-mode assumption, the conservation laws would forbid processes unless u_p is an interger, anticipating a resonance phenomenon [8].

To account for nonzero ATI rate at $u_p \neq$ integer, in this paper we develop a theory for QED at high laser photon density, by including nonlaser radiation modes and treat their couplings to the electron as perturbation. With the help of emission of a nonlaser photon, now it is easy to balance both energy and momentum at noninteger u_p. However, at intensity with integer u_p, the energy U_p of the intermediate dressing photon cloud matches the level spacing of the laser

*On leave from Department of Physics, University of Utah, Salt Lake City, UT 84112.

mode. Thus we predict that the ATI ionization rate should exhibit resonance peaks at quantized intensities with integer $u_p=N$:

$$I=NI_0\equiv N\frac{\hbar m_e c\omega_0^3}{2\pi e^2}, \tag{2}$$

with $N\geqslant 1$ an integer. Note that I_0 is proportional to the cube of the laser frequency ω_0. This and similar intensity dependent, resonating quantum effects of the light field in other multiphoton processes may be experimentally exploited to calibrate laser intensities in approriate range.

PERTURBATION THEORY FOR QED

To properly deal with photon number fluctuations, we need to second quantize the radiation field, but still treat the electron quantum mechanically, ignoring pair production, vacuum polarization, and other relativistic corrections for the electron as well, if the laser intensity is not too high.

In the Schrödinger picture, the Hamiltonian of the electron-radiation system is (with $\hbar=c=1$)

$$H=\frac{1}{2m_e}[-i\nabla-e\mathbf{A}(\mathbf{r})]^2+\sum_k \omega_k N_k, \tag{3}$$

with $N_k=a_k^\dagger a_k+1/2$. Here the photon field operator is given by the time-independent vector potential in the radiation gauge $(\nabla\cdot\mathbf{A}=0)$:

$$\mathbf{A}(\mathbf{r})=\sum_k \mathbf{A}_k(\mathbf{r})\equiv\sum_k g_k(\boldsymbol{\epsilon}_k a_k e^{i\mathbf{k}\cdot\mathbf{r}}+\text{H.c.}), \tag{4}$$

with k labeling the photon modes, including the wave vector $\mathbf{k}$ and transverse polarizations described by $\boldsymbol{\epsilon}$:

$$\boldsymbol{\epsilon}=[\boldsymbol{\epsilon}_x\cos(\xi/2)+i\boldsymbol{\epsilon}_y\sin(\xi/2)]e^{i\Theta/2}. \tag{5}$$

Here $g_k=(2\omega_{\mathbf{k}}V_\gamma)^{-1/2}$, with $\omega_k=|\mathbf{k}|$, and V_γ the normalization volume of the radiation field. a_k and $a_k^\dagger$ are photon annihilation and creation operators.

Now let us separate the laser modes, say a single mode labeled by k_0, from other photon modes: $\mathbf{A}=\mathbf{A}_{k_0}+\mathbf{A}'$, and try to first treat the electron-laser-mode interactions nonperturbatively, then add the coupling of the electron to nonlaser modes as perturbation. Thus, we are led to split $H=H_0+V+V'$, with

$$H_0=\frac{(-i\nabla)^2}{2m_e}+\omega_0 N_0+\sum_{k'\neq k_0}\omega' N',$$

$$V=-\frac{e}{m_e}\mathbf{A}_{k_0}(\mathbf{r})\cdot(-i\nabla)+\frac{e^2\mathbf{A}_{k_0}(\mathbf{r})^2}{2m_e}, \tag{6}$$

$$V'=-\frac{e}{m_e}\mathbf{A}'(\mathbf{r})\cdot(-i\nabla)+\frac{e^2\mathbf{A}_{k_0}(\mathbf{r})\cdot\mathbf{A}'(\mathbf{r})}{m_e},$$

with the $\mathbf{A}'^2$ term neglected.

For an electron in the laser field, we choose H_0+V as the unperturbed Hamiltonian. The eigenstates of N' are simply the Fock states for the nonlaser mode. For the electron-laser-mode subsystem, almost exact eigenstates has been obtained before [5,6], which are labeled by a momentum $\mathbf{p}$ and an integer n, denoted as $\Psi^0_{\mathbf{p}n}$. They are the nonrelativstic limit of the exact solutions [5] to the Dirac equation coupled to the quantized laser mode. They form a complete, orthogonal set of states, called quantized field Volkov states (QFVS), which are the QED analog of the classical Volkov states [4]. Their nonrelativistic limit is verified [10] to satisfy the Schrödinger-like equation. In practice, we need only to consider their large photon-number limit, $n\to\infty$, $g_{k_0}\to 0$ and $\sqrt{n}g_{k_0}\to\Lambda$, with the QFVS simplified to

$$\Psi^0_{\mathbf{P}n}=V_e^{-1/2}\sum_{j\geqslant -n}\exp\{i[\mathbf{P}+(z-j)\mathbf{k}_0]\cdot\mathbf{r}\}$$
$$\times\mathcal{J}_j(\eta,\zeta_{\mathbf{P}},\phi_{\mathbf{P}})^*\exp\{-ij\phi_{\mathbf{P}}\}|n+j\rangle, \tag{7}$$

where there is no dipole approximation involved. Here $u_p\equiv e^2\Lambda^2/m_e\omega_0$ is the ponderomotive parameter, $|n\rangle$ a laser-mode Fock state and

$$\eta=\frac{1}{2}u_p\cos\xi,\quad \zeta_{\mathbf{P}}=\frac{2|e|\Lambda}{m_e\omega_0}|\mathbf{P}\cdot\boldsymbol{\epsilon}|,$$

$$\phi_{\mathbf{P}}=\tan^{-1}\left(\frac{\mathbf{P}_y}{\mathbf{P}_x}\tan\frac{\xi}{2}\right)(+\pi,\ \text{if } \mathbf{P}_y\langle 0).$$

The $\mathcal{J}_j$ is compounded from Bessel functions J_m:

$$\mathcal{J}_j(\zeta_{\mathbf{P}},\eta,\phi_{\mathbf{P}})=\sum_{m=-\infty}^{\infty}J_m(\eta)J_{-j-2m}(\zeta_{\mathbf{P}})e^{2im\phi_{\mathbf{P}}}. \tag{8}$$

The energy and momentum $(\hat{\mathbf{p}}_0=-i\nabla+N_{k_0}\mathbf{k}_0)$ eigenvalues of the QFVS are given by, respectively,

$$E_0(\mathbf{P},n)=\mathbf{P}^2/2m_e+(n+1/2)\omega_0+u_p\omega_0,$$
$$\mathbf{p}_0(\mathbf{P},n)=\mathbf{P}+(n+1/2)\mathbf{k}_0+u_p\mathbf{k}_0. \tag{9}$$

The QFVS is a superposition of Fock states in the laser mode with different photon number; this implies that the electron in the laser field is dressed by a coherent photon cloud which has a nonzero component in each Fock state specified by photon surplus (or deficit) j. By intepreting the first two terms in Eq. (9) as contributions from the electron and the background photons, each being on shell, it is natural to identify the third term or the ponderomotive energy and momentum given by Eq. (1), as arising from the dressing photon cloud.

By using the QFVS as unperturbed states, we can develop a perturbation theory for the electron-radiation system, in which the electron-nonlaser-mode coupling V' is treated as perturbation. Then the eigenstate for an electron in the laser field is the perturbed QFVS, $\Psi_{\mathbf{P}n,n'}=|\mathbf{P}n,n'\rangle+|\mathbf{P}n,n'\rangle'$, with

$$|\mathbf{P}n,n'\rangle' = \sum_{\tilde{\mathbf{P}},\tilde{n},\tilde{n}'} |\tilde{\mathbf{P}}\tilde{n},\tilde{n}'\rangle \frac{\langle \tilde{\mathbf{P}}\tilde{n},\tilde{n}'|V'|\mathbf{P}n,n'\rangle}{\mathcal{E}(\mathbf{P}n,n') - \mathcal{E}(\tilde{\mathbf{P}}\tilde{n},\tilde{n}')}, \quad (10)$$

where $|\mathbf{P}n,n'\rangle = \Psi^0_{\mathbf{P}n}|n'\rangle$, with $|n'\rangle$ a Fock state in a nonlaser mode; $\mathcal{E}(\mathbf{P}n,n') = E_0(\mathbf{P},n) + (n'+1/2)\omega'$. Note that there is no energy shift up to first order.

CALCULATION OF THE ATI RATE

Now we apply the above perturbative formalism to ATI. We want to calculate the ionization rate and angular distributions etc., and study their intensity dependence.

Let us start with the following initial state for the electron-radiation system: the electron in a bound state Φ_i, the laser mode in the Fock state $|n_i\rangle$, and the nonlaser modes in the vacuum state (with $n_i'=0$ photons), denoted by $|\Phi_i,n_i,0\rangle$. In the final state of the ATI, denoted as $|\mathbf{P}_f,n_f,n_f'\rangle$, the electron is in a free state with momentum $\mathbf{P}_f$ outside the laser beam, the laser mode in the state $|n_f\rangle$ and at most one, say k', of the nonlaser modes in $|n_f'=1\rangle$ (to first order). All previous treatments did not include the possibility of having a nonlaser photon in the final state, but in QED this allows the photoelectron to emit a photon to balancing energy and momentum upon exiting from the laser field.

To calculate the transition amplitude at the $u_p \neq$ integer case, we include spontaneous-emission modes in the scattering matrix element of GAC [8], where the interaction due to spontaneous-emission modes is only up to the first order in perturbation theory. As usual we ignore the effects of the ion potentials on the ionized electron. With the intermediate states on the energy shell of the system only, according to the GAC's theory, we obtain

$$T_{fi} = \sum_{\mathbf{p},n,n'} \langle \mathbf{P}_f,n_f,n_f'|\Psi_{\mathbf{P}n,n'}\rangle \times \langle \Psi_{\mathbf{P}n,n'}|V+V'|\Phi_i,n_i,0\rangle, \quad (11)$$

where the summation of intermediate states is subject to

$$\mathcal{E}(\mathbf{P}n,n') = \mathcal{E}_i \equiv -E_b + (n_i+\tfrac{1}{2})\omega_0 + \tfrac{1}{2}\omega',$$
$$= \mathcal{E}_f \equiv \mathbf{P}_f^2/2m_e + (n_f+1/2)\omega_0 + (n_f'+1/2)\omega', \quad (12)$$

with E_b the binding energy in the initial state Φ_i, while both n' and n_f' are either 0 or 1, up to first order.

We note that the product structure of the terms in Eq. (11) verifies that the ATI is indeed a two-step process [2]. (1) The electron is first ionized into the laser field, so the intermediate state of the system is a QFVS given by Eqs. (7) and (10) (2) Then it exits out of the laser beam becoming a free electron. Previously no theoretical formalism has accounted for the exiting except [8].

Inspection shows only the following terms are nonzero:

$$T_0 = \sum_{\mathbf{p},n} \langle \mathbf{P}_f,n_f,0|\mathbf{P}n,0\rangle\langle \mathbf{P}n,0|V|\Phi_i,n_i,0\rangle, \quad (13)$$

$$T_1 = \sum_{\mathbf{P},n} \langle \mathbf{P}_f,n_f,1|\mathbf{P}n,1\rangle\langle \mathbf{P}n,1|V'|\Phi_i,n_i,0\rangle,$$

$$T_2 = \sum_{\mathbf{P},n} \langle \mathbf{P}_f,n_f,1|\mathbf{P}n,1\rangle\langle \mathbf{P}n,1|'V|\Phi_i,n_i,0\rangle, \quad (14)$$

$$T_3 = \sum_{\mathbf{P},n} \langle \mathbf{P}_f,n_f,1|\mathbf{P}n,0\rangle'\langle \mathbf{P}n,0|V|\Phi_i,n_i,0\rangle.$$

The zeroth order term T_0 has been calculated years ago [8]. T_1 and T_2, as well as T_0, contribute only at $u_p=$integer, while T_3 contributes at arbitrary u_p. We are interested in the cases with noninteger u_p, so we focus on T_3.

After a lengthy calculation, introducing $j=n_i-n$, $j'=n_f-\tilde{n}$ and $q=n_i-n_f$, we finally obtain

$$\begin{aligned} T_3 = {} & \frac{e}{m_e} V_e^{-1/2} \Phi_i(\mathbf{P}_f - q\mathbf{k}_0 + \mathbf{k}') e^{iq\Theta/2} g' \sum_{j'} \frac{j-u_p}{u_p-j'} \\ & \times \mathcal{J}_{j'}(\zeta_{\mathbf{P}_f},\eta,\phi_{\mathbf{P}_f})^* e^{-ij'\phi_{\mathbf{P}_f}} \mathcal{J}_j(\zeta_{\mathbf{P}_f+\mathbf{k}'},\eta,\phi_{\mathbf{P}_f+\mathbf{k}'}) \\ & \times e^{ij\phi_{\mathbf{P}_f+\mathbf{k}'}} \{ -[\mathbf{P}_f + (j-q-u_p)\mathbf{k}_0] \cdot \boldsymbol{\epsilon}'^* \\ & \times J_{q-j+j'}(\zeta_{\mathbf{k}'}) e^{i(q-j+j')\phi_{\mathbf{k}'}} \\ & + e\Lambda \boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}'^* J_{q-j+j'+1}(\zeta_{\mathbf{k}'}) e^{i(q-j+j'+1)\phi_{\mathbf{k}'} + i\Theta/2} \\ & + e\Lambda \boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}'^* J_{q-j+j'-1}(\zeta_{\mathbf{k}'}) e^{i(q-j+j'-1)\phi_{\mathbf{k}'} - i\Theta/2} \}. \end{aligned} \quad (15)$$

A careful study shows that the kinetic energy difference for the photoelectron before and after exiting out of the light field is of the order of relativistic corrections. Therefore, we neglect the kinetic energy difference under the nonrelativistic conditions. Thus energy conservation implies a quasidiscrete spectrum, the usual ATI peaks, for the free photoelectron:

$$\omega' \approx [u_p - (j-q)]\omega_0,$$
$$\mathbf{P}_f^2/2m_e \approx j\omega_0 - E_b - u_p\omega_0 \geqslant 0. \quad (16)$$

The physical interpretation is clear: the electron is ionized by absorbing j photons simultaneously and, upon exiting out of the laser field, completely shakes off its ponderomotive energy (or the dressing photon cloud), by emitting j-q laser photons and a nonlaser photon with the remaining ponderomotive energy.

We express the energy delta function $\delta(\mathcal{E}_i - \mathcal{E}_f)$ as

$$\left(\frac{m_e}{2\omega_0}\right)^{1/2} \frac{\delta[P_f - (2m_e\omega_0)^{1/2}(q-\epsilon_b-\nu)^{1/2}]}{(q-\epsilon_b-\nu)^{1/2}}, \quad (17)$$

where $\epsilon_b \equiv E_B/\omega_0$, $\nu \equiv \omega'/\omega_0 = u_p - j + q$, and $P_f = |\mathbf{P}_f|$. Then one obtains the total ATI rate by

$$W = \int_6 \frac{V_e V_\gamma}{(2\pi)^6} |T_3|^2 2\pi\delta(\mathcal{E}_i - \mathcal{E}_f) d^3\mathbf{P}_f d^3\mathbf{k}', \quad (18)$$

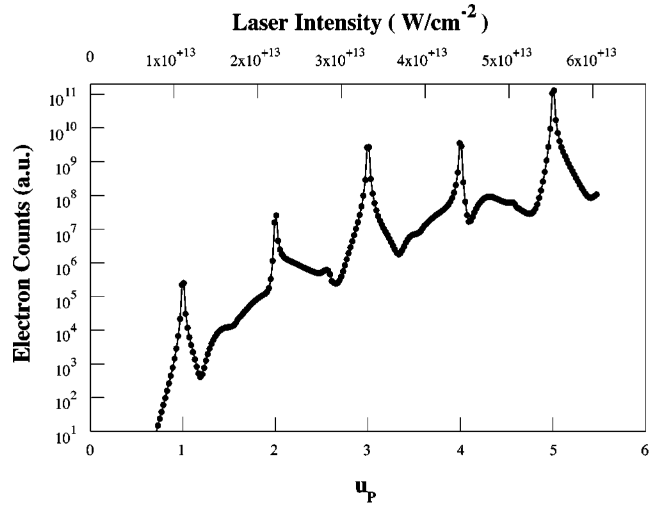

FIG. 1. Total ATI photoelectron count, collected in the laser polarization direction vs laser intensity for xenon in a single-mode, linearly polarized laser at 1064 nm.

while the angular distribution for a given ATI peak is

$$\left.\frac{d^4W}{d^2\Omega_{\mathbf{P}_f}d^2\Omega_{\mathbf{k}'}}\right|_j=\frac{e^2\omega_0^{9/2}}{(2m_e)^{1/2}(2\pi)^5}(j-\epsilon_b-u_p)^{1/2}$$

$$\times(j-u_p)^2\sum_q\,(u_p-j+q)$$

$$\times|\Phi_i(\mathbf{P}_f-q\mathbf{k}_0+\mathbf{k}')|^2|\mathcal{J}_q(\mathbf{P}_f,\mathbf{k}')|^2, \quad (19)$$

where $\mathcal{J}_q(\mathbf{P}_f,\mathbf{k}')$ is defined as

$$\mathcal{J}_q(\mathbf{P}_f,\mathbf{k}')^*\equiv\frac{1}{\omega_0}\mathcal{J}_j(\zeta_{\mathbf{P}_f+\mathbf{k}'},\eta,\phi_{\mathbf{P}_f+\mathbf{k}'})e^{ij\phi_{\mathbf{P}_f+\mathbf{k}'}}$$

$$\times\sum_{j'}\frac{1}{u_p-j'}\mathcal{J}_{j'}(\zeta_{\mathbf{P}_f},\eta,\phi_{\mathbf{P}_f})^*e^{-ij'\phi_{\mathbf{P}_f}}$$

$$\times\{-[\mathbf{P}_f+(j-q-u_p)\mathbf{k}]\cdot\epsilon'^*$$

$$\times J_{q-j+j'}(\zeta_{\mathbf{k}'})e^{i(q-j+j')\phi_{\mathbf{k}'}}$$

$$+e\Lambda\epsilon^*\cdot\epsilon'^*J_{q-j+j'+1}(\zeta_{\mathbf{k}'})$$

$$\times e^{i(q-j+j'+1)\phi_{\mathbf{k}'}+i\Theta/2}$$

$$+e\Lambda\epsilon\cdot\epsilon'^*J_{q-j+j'-1}(\zeta_{\mathbf{k}'})$$

$$\times e^{i(q-j+j'-1)\phi_{\mathbf{k}'}-i\Theta/2}\}. \quad (20)$$

RESONANT ATI RATE VERSUS INTENSITY

One sees from Eq. (15) that the amplitude T_3 becomes very large if u_p is sufficiently close to an integer, because then one of the terms in the sum can have a very small denominator $j'-u_p$. Thus, we predict the emergence of resonances in ATI rate or angualr distribution at quantized intensities given by Eq. (2). As example, in Fig. 1 we show the numerical result for the total photoelectron counts collected in the direction of polarization [i.e., Eq. (19)] summed over j's and integrated over the direction of $\mathbf{k}'$, for xenon in a linearly polarized laser beam with wavelength 1064 nm. It clearly demonstrates the resonances at intensties that are integral multiple of $I_0=1.10\times10^{13}\ \mathrm{W\,cm^{-2}}$.

Such intensities have been experimentally available for more than a decade. Why did the resonances not show up in previous data? We note that the widths of the resonances in Fig. 1 are rather narrow, so they could have been smeared out by the spatial and temporal intensity inhomogeneities in the laser focus and particularly by the instability in intensity from (laser) burst to burst. The test might be a great challenge to experimentalists.

How come these resonances have evaded the correspondence principle argument, which is usually used to justify the classical description of the laser field? This is because the Plack constant in the real world is finite, the correspondence principle argument is merely an approximation that can break down in certain situations even with high photon density. We have found one such situation, i.e., at intensities corresponding to integral ponderomotive parameter. Away from these intensities, we expect to see a smooth background grossly dictated by the classical field picture, as indeed shown in our Fig. 1.

We have used the Fock states as the basis for the laser mode. If one uses Glauber's coherent states to describe the initial and final states of the laser field, the ATI amplitude can be easily derived by superposing our amplitudes. This gives rise to a spread in background photon number n_i and n_f. But the corresponding spread in u_p is expected to be very small. So our prediction of the ATI resonances is unaffected.

Our ATI rate diverges at exactly integral u_p. This problem is easy to remedy by including an imaginary part (a finite width) in the QFVS energy $\mathcal{E}(\widetilde{\mathbf{P}}\widetilde{n},0)$ in Eq. (10), which arises from possible decay through spontaneous emission of nonlaser photons via the coupling V'. A more thorough treatment of ATI also requires including the atomic potential and intermediate bound states, which we have ignored. Because of no good reason to believe these interaction effects could completely wash out the resonances we have predicted, we leave their study to future research.

OTHER INTENSITY-DEPENDENT QUANTUM EFFECTS

Our argument for the resonance effects in the electron-laser system is very general, based only on the intensity dependent stimulated interactions and the discrete photon structure of the laser mode. So we expect to see them in other multiphoton processes, and our approach to QED at high laser photon density is applicable as well.

One example is a slow electron transversing a single-mode laser beam. Classically, the ponderomotive energy acts as an effective repulsive potential, so at high intensities the electron can hardly get into the laser beam. But according to our argument, the stimulated electron-laser interactions will give rise to a photon cloud dressing the electron, which can be resonant with the laser mode. So we predict that when the laser intensity is close to the quantized values NI_0, there will be resonance peaks for the penetration probability for slow electrons transversing the laser beam. Our perturbation theory is applicable to make quantitative predictions.

It is easy to generalize our approach to more than one laser modes, since the corresponding QFVS have been obtained before [11]. For example, one may consider electrons scattered by a standing wave formed by two laser modes. Previously, Bucksbaum *et al.* [12] has experimentally discovered a dramatic peak splitting in the angular distribution of the scattered electron. This has been theoretically explained in Ref. [13] using the QFVS states, which could not deal with the angular region inside the splitting angle. Our perturbation theory can be employed to deal with the angular region in between the peaks, and is expected to reveal a characteristic variation in the peak separation as I/I_0 changes near an integer. These and similar intensity-dependent quantum effects of the light field, if experimentally verified, would be used to provide a natural calibration of the laser intensity and to generate photoelectrons with higher efficiency.

ACKNOWLEDGMENTS

J.G. thanks Professor J. G. Eden for support and discussions. D.S.G. was supported in part by NSF Grant No. PHY-9603083, Y.S.W. by NSF Grant No. PHY-9601277, and a grant from the Monell Foundation.

[1] F.H.M. Faisal, *Theory of Multiphoton Processes* (Plenum, New York, 1987).

[2] R.R. Freeman and P.H. Bucksbaum, J. Phys. B **24**, 325 (1991).

[3] L.V. Keldysh, Sov. Phys. JETP **20**, 1307 (1965); F.H.M. Faisal, J. Phys. B **6**, L89 (1973); H.R. Reiss, Phys. Rev. A **22**, 1786 (1980).

[4] D.M. Volkov, Z. Phys. **94**, 250 (1935).

[5] D.-S. Guo and T. Åberg, J. Phys. A **21**, 4577 (1988); J. Phys. B **24**, 349 (1991).

[6] D.-S. Guo, Phys. Rev. A **42**, 4302 (1990); D.-S. Guo and G.W.F. Drake, J. Phys. A **25**, 3383 (1992).

[7] For a classical derivation of the ponderomotive momentum, see J. Gao, D. Bagayoko, and D.-S. Guo, Can. J. Phys. **76**, 87 (1998).

[8] D.-S. Guo, T. Åberg, and B. Crasemann, Phys. Rev. A **40**, 4997 (1989).

[9] M. Gell-Mann and M.L. Goldberg, Phys. Rev. **91**, 398 (1953).

[10] D.S. Guo, R.R. Freeman, and Y.S. Wu, Phys. Rev. A **58**, 521 (1998).

[11] D.-S. Guo and G.W.F. Drake, J. Phys. A **25**, 5377 (1992).

[12] P.H. Bucksbaum, D.W. Schumacher, and M. Bashkansky, Phys. Rev. Lett. **61**, 1162 (1988).

[13] D.-S. Guo and G.W.F. Drake, Phys. Rev. A **45**, 6622 (1992).