A CAPILLARY NETWORK MODEL FOR FILTER CAKE BASED ON PORE STRUCTURE ANALYSIS

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ABSTRACT

Dewatering of fine coal by continuous filtration involves filter cake formation and removal of surface moisture by drawing air through the capillaries of the cake. In order to gain a better understanding of the complex transport phenomena that occur in the filter cake, analysis of the effect of three-dimensional pore geometry on the effective transport properties of the filter cake is necessary. This paper provides information on the techniques and methodology necessary to provide a detailed three-dimensional analysis of a completely interconnected porous system. In addition, a conceptual capillary network model based on a 3-D interconnected porous system is proposed.

INTRODUCTION

Prediction of the behavior of heterogeneous systems based on geometrical structure is of importance in many fields of applied science and technology, including fluid flow in a filter cake during dewatering by fine coal filtration. The physical laws that govern the equilibrium and flow of several fluids in filter cake (one kind of porous medium), at the pore level, are simple and well known. In practice, however, only the global physical properties of the system are known at best. Unfortunately, because of the complexity of the pore geometry, it is difficult to predict macroscopic (effective) properties from microscopic (pore level) properties. In general, there is no linear or non-linear rule for combination of the effective physical properties from the microscopic scale which can be used to predict the macroscopic scale properties. In this regard, it is essential to introduce appropriate techniques and models to describe the physical properties of macroscopic heterogeneous systems.

Almost all theory related to transport phenomena in porous media lead to macroscopic laws applicable to systems whose dimensions are large compared with the dimensions of pores. These macroscopic pore structure parameters represent average behavior of a sample containing many pores. The important macroscopic pore structure parameters are porosity, permeability, specific surface area, formation resistivity factor and the breakthrough capillary pressure. A review of the definitions and measurement techniques for these macroscopic parameters is given by Dullien[1]. Generally speaking, the macroscopic quantity of interest is more or less influenced by the microscopic properties of the pore structure and is obtained by a spatial integration of the local field. The overall pore fraction and pore size distribution are the two of the most important microscopic parameters frequently used for model development to obtain fundamental relationships between pore structure and the effective transport coefficients.

Most present methods to characterize the pore microstructure and its interconnected network rely on the microscopic observation of a series of thin or polished sections of the porous media (in our case filter cake). These data sets are then used to reconstruct and to display the three-dimensional image of the porous system with the help of advanced computer graphic techniques. Complete analysis of the 3-D porous system from serial sections is a tedious and time consuming process. In addition, for a completely interconnected porous system, pore size distribution is not a well defined parameter. To illustrate the nature of an interconnected porous structure, surface rendering of the three-dimensional image of a packed bed of irregularly shaped coal particles is shown in the left-handside of Figure 1 as established from sequential scans by x-ray CT (computed tomography). The right-hand side of Figure 1 shows one slice from a cross section of the packed bed of irregularly shaped particles. A detailed description of the interconnected type of pore structure based on the concepts of "connectivity", "percolation" and "tortuosity", needs to be developed by computed tomography in order to establish fundamental relationships between this kind of complex random pore structure and the corresponding effective transport coefficients.

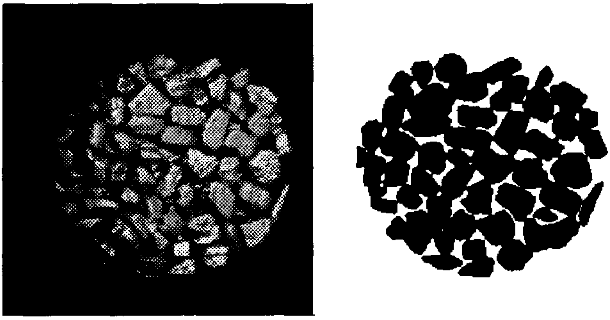


Fig. 1. Completed interconnected pore structure of a packed bed of coal particles.

As the resolution and the techniques for 3-D geometric analysis have advanced in the last decade, it is now possible to map in detail the pore structure in three-dimensional digital space. For modeling of transport phenomena, it is expected that the complex pore structure of randomly organized filter cake must be described in three dimensions with micrometer resolution. In this regard, three-dimensional x-ray microtomography offers a unique imaging capability. Spatial resolution on the order of 1 micron and 15 microns can be achieved with the use of synchrotron radiation[2,3] and conventional microfocus x-ray generators,[4,5] respectively. This paper will provides information on how to use the x-ray microtomography technique to characterize interconnected pore structures based on the concepts of "connectivity", "percolation" and "tortuosity". In addition, the requirements for model development to describe fundamental relationships between complex randomly organized pore structures and their effective transport coefficients is addressed.

PORE STRUCTURE MODELS

Two pore structure models, namely packing of particles and capillary networks, have been used to mimic porous media. In the first approach, the pore structure is generally modeled based on arrays of spherical particles. Filling and emptying of the pore space can be calculated as a function of the capillary pressure only in the case of some relatively simple arrangement. Qualitative information, such as capillary hysteresis, irreducible wetting phase, and residual nonwetting phase saturation, can be obtained with the first approach.[1] Such information is suited primarily for capillary pressure calculations for packed beds of smooth spheres. In the second approach, termed capillary networks, the pore structure is modeled by arrays of capillary tubes. As shown in Figure 2(a), the interconnected and irregularly sized space is formed inside a filter cake of fine coal particles. An approximate description of the pore geometry is given in Figure 2(b). Two capillary tubes of differing diameter, branching from a common stem, diverge and come together again.

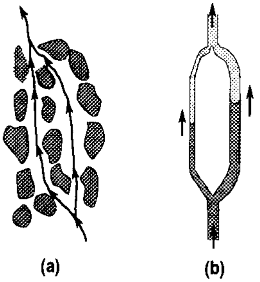
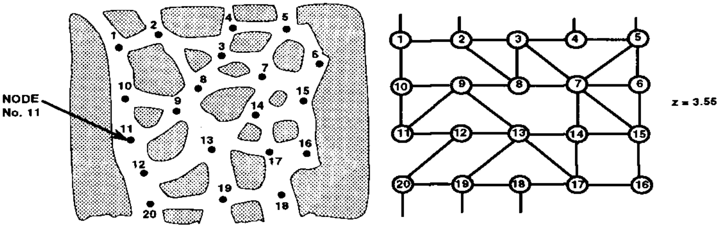


Fig. 2. Representation of porous medium as effective capillary tubes for capillary network modeling.

The pore structure of filter cake consists of an interconnected three-dimensional network of voids, pores or capillaries. This three-dimensional network usually has irregular geometry, with different shape and size of capillary segments distributed over the network in irregular fashion. In this regard, it is logical to model capillary pressure curves and other transport properties of filter cake with the help of network models of the pore structure. For example, as shown in Figure 3, a network model is used to represent the 2-dimensional pore structure of a porous medium. The left-handside of Figure 3 shows the two-dimensional view of pores Fig.



3. Symbolic representation of the interconnections in a two-dimensional porous medium. [1]

whose intersections, the nodes, are numbered. The symbolic graph of this pore structure can be described in the form of a network of bonds and nodes as shown in the right-handside of Figure 3. The coordination number, one of the important parameters for the interconnection of pore structure, is defined as the average number of bonds meeting at each node[6]. In the case of system presented in Figure 3, the coordinate number of the pore structure is 3.55. It should noted that only the interconnectedness of the pore structure is shown in this figure. The other parameters of pore geometry such as, dimension and orientation of the pores is not represented in this case. The use of network models for the purpose of capillary pressure characterization studies appears to have been introduced by Fatt.[7] Capillary pressure is a basic parameter in the study of the behavior of porous media containing two or more immisible fluid phases. For a meniscus in a conical capillary, as shown in Figure 4, the capillary pressure can be defined by the following equation:

$$P_c = P_2 - P_1 = (2\sigma|\cos(\theta + \phi)|) / R \qquad (1)$$

which follows directly from Laplace's equation and the geometry of the meniscus shown in Figure 4. Note that $R / |\cos(\theta + \phi)|$ is the mean radius of curvature of the meniscus. The pressure difference between fluid 2 and fluid 1 is defined as the capillary pressure. By combining the calculation for two-dimensional networks with a variety of tube radius distributions, Fatt found that, in general, the curves resembled capillary pressure curves obtained with sandstone samples.

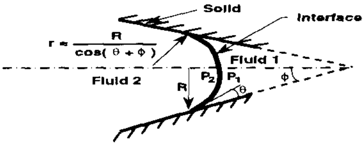


Fig. 4. Meniscus in a conical capillary.

Fatt's procedure has been modified by assigning a volume of its own for each node and estimating the dimensions of both nodes and tubes from geometrical considerations for a packed bed of spherical particles.[8]

Percolation processes were first used by Broadbent and Hammersley[9] to describe the flow of a fluid in a random maze. They considered a number of networks in which the connecting channels have a finite probability of being blocked. The question is how to calculate the critical probability below which a fluid starting from one part of the medium would not be able to spread or move indefinitely. This critical probability depends on the dimension and the lattice structure. Based on the fundamental properties of capillary network models along with the concept of percolation, penetration for various networks has been studied.[10] The relationship between capillary pressure and saturation, which is usually referred to as processes of "drainage" or "imbibition", has been studied using a 2-D network and a percolation process.[10] Comparison of the simulated and measured capillary pressure curves for sandstone samples is shown in Figure 5. It is concluded that network simulation of drainage capillary pressure curves when one phase has a strong wetting preference shows a strong promise of becoming a useful predictive tool, approaching the accuracy that can be expected from core analysis. Of course, the agreement will be best when the simulated and real porous media's physical parameters match. In addition to fluid and wettability properties, however, a better representation of the pore structure is required to obtain these parameters.

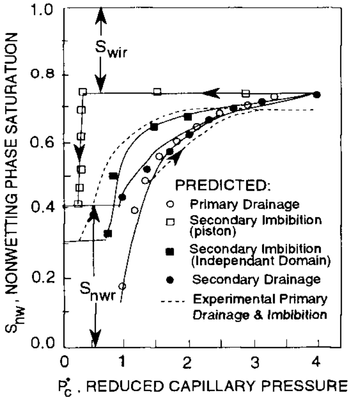


Fig. 5. Comparison of simulated and measured capillary pressure curves of sandstone samples.[10]

CAPILLARY NETWORK MODEL FOR FILTER CAKE

Fluid transport in filter cake during fine coal filtration is a subject of wide interdisciplinary concern. In order to gain a better understanding of the complex transport phenomena that occur in filter cake, a study of the effect of three-dimensional pore geometry on the effective transport properties of the porous media is necessary. Of course, it is known that the agreement will be best if the physical parameters are matched for both model simulation and the actual porous structure. At present, the information from the microscopic pore geometry analysis is not detailed enough to provide an accurate prediction of transport properties for the filter cake from model simulation. Techniques and methodology for a detailed description of the three-dimensional pore structure of a completely interconnected porous system is needed.

It is intended to predict the effective transport parameters from microscopic analysis of the 3-dimensional interconnected pore structure. To achieve this goal, three major obstacles need to be overcome. First, a reliable and accurate technique for 3-D pore geometry analysis must be established. Second, methodology and procedures for the description of the 3-D inter-connected pore structure must be established. Finally, based on this description the last task will be to develop the relationship between the effective transport parameters and the microscopic properties of the pore structure. Of course, model verification is an essential step to confirm any simulation work.

3-D Pore Geometry Analysis by X-Ray Microtomography

The microstructure and the connectivity of the pore space are important to describe fluid flow in filter cake during fine coal filtration. In this regard, characterization of pore structure based on parameters permitting inferences on the fluid balance is of particular interest. The pore structure has to be described by parameters which are of special relevance for the interpretation of fluid transport phenomena. These parameters should be based on directly measured variables of the pore system and not indirect variables (such as those determined empirically from transport processes) valid only for a particular pore structure. In this way fundamental relationships between pore structure and fluid transport at the microstructure level can be described. The ultimate objective of this task is direct access to the three-dimensional interconnected pore structure of the filter cake.

Cone-beam x-ray microtomography[11] offers a unique imaging capability which can produce three-dimensional images of the internal structure of samples with micrometer resolution. Rather than rotating the x-ray source and detectors during data collection, as in medical CT technology, the specimen is rotated. Instead of generating a series of two-dimensional sliced images from one dimensional projectors, a three-dimensional reconstruction image array is created directly from two dimensional projectors. Figure 6 shows a schematic diagram for the cone beam geometry micro-CT system. Details for the description of this micro-CT system and its corresponding reconstruction algorithm can be found in the literature[11].

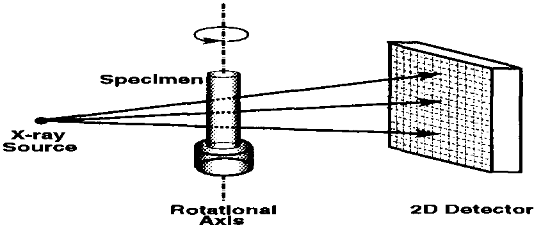


Fig. 6. Schematic diagram of the cone-beam x-ray microtomography system.

The reconstruction region of a standard cubic sample is typically about 8x8x6 mm. Reconstruction points in standard mesh are 50 microns apart; a typical reconstruction set consists of 200x200x120 points. Resolution limitations largely result from the finite resolution of the image intensifier. Because the resolution is determined primarily by the detector system, it is possible to obtain better absolute spatial resolution for specimens with smaller cross-sectional dimensions by increasing the geometric magnification, accomplished by decreasing the distance between specimen and x-ray source. The spatial boundary between pore and solid phase can be easily established due to the large difference in densities. Threshold techniques can be used to differentiate and classify the original 3-D density data.

Model Development of the Capillary Network using a Skeletonization Process

Although we can directly access the 3-D pore structure of the porous media with the use of x-ray microtomography, a fundamental difficulty is how to describe the pore structure. In general, the range of pore sizes is very great in most porous media and the geometrical properties of the void system are not easy to define as mentioned previously. Thus there is a need for the development of an overall concept for the description of the pore structure. In this regard, it is necessary to develop a capillary network from the 3-D digital map obtained from x-ray microtomography.

To characterize the 3-D interconnected pore structure, the term "pore size" is not easily defined. These open or interconnected pores can not be simply regarded as discrete pseudo-particles. Since the capillary network will be used to mimic the real filter cake, a skeletonization techniques[12] should be suitable to build the necessary network and to interpret the linked pore space from the resulting 3-D digital map using x-ray microtomography. Originally, skeletonization (in a plane) denotes a process which transforms a 2-D object into a 1-D graph-like structure, comparable to a stick figure. It is thereby essential that the skeleton retains the original connectivity of the shape. The skeletonization process provides both the radius information and structure of the object space. This information of 3-D interconnected pore space provides a tool to describe important properties of the completed pore structure and can be used for the capillary network simulation by percolation theory.

Two approaches, distance transformation (DT)[13] and discrete Voronoi skeletons (DVS)[14], can be used for algorithm development of the 3-D skeletonization process. The corresponding length and radius of the skeleton segment of the graph can be used as the attributes of the capillary tube segment for the 3-D interconnected pore structure. Furthermore, this information can be used to more accurately represent probabilities for the capillary network model simulation using percolating theory as described in the following.

Simulation of the Transport Parameters of Porous Media using Percolation Theory

The percolation concept is useful when flow problems are considered. In fact, percolation processes were first used to describe the flow of a fluid in a random maze[9]. Until early 1970, percolation theory had been emphasized in the study of critical phenomena. In the late seventies, research on percolation theory was extended to other fields, such as polymer gelatin, dilute magnetic domain, communication networks, ...etc. As mentioned previously, it is logical to model capillary pressure curves and other transport properties of porous media with the help of network models of the pore structure. The detailed 3-D capillary network of the pore structure can be used to simulate the capillary pressure curve and other transport properties of the filter cake. First, based on the skeletization, the radius information and the length of the bond of the capillary network can be determined. Then, the stagewise penetration process can be simulated based on the opening sequence of the size of pore radius and the length of the connected bond. "Breakthrough" is set as the penetration reaches the opposite site of the network. In this manner, the capillary pressure curve and saturation can be simulated. Details of the percolation technique can be found in the literature.[1,10]

The main concern for the simulation of the 3-D percolation process will be on the capacity of computer memory. An effective algorithm will be needed such that the percolation simulation can be performed based on the sequence of layer by layer for a huge 3-D network. In this regard, an algorithm such as that developed by Hoshen and Kopelman[15] can be used for the percolation simulation.

SUMMARY

In order to gain a better understanding of the complex transport phenomena that occur in a filter cake, study of the effect of 3-dimensional pore geometry on the effective transport properties of the filter cake is necessary. The conceptual development of a capillary network model for filter cake based on pore structure analysis has been presented. The proposed model should provide a detailed basis for the analysis of fluid flow in filter cake during the fine coal filtration process. Procedures for the determination of a detailed 3-D interconnected pore structure from x-ray microtomography measurements must be established. A skeletonization process will provide better understanding and allow for the interpretation of the interconnected pore structure. Finally, the capillary network and percolation process will provide the basis for model prediction and allow for the correlation of transport properties of porous media at various scales. Knowledge of multiphase flow in porous media as will be obtained from the model described should provide important information for practical applications in the design of improved filtration processes.

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