Time Variation of Newton's Gravitational Constant in Superstring Theories

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(Received 29 May 1986)

The present time variation of coupling constants in superstring theories with currently favorable internal backgrounds critically depends on the shape of the potential for the size of the internal space. If the potential is almost flat, as in perturbation theory to all orders, the value of $\dot{G}/G$ for Newton's gravitational constant is calculable and estimated to be $-1\times10^{-11\pm1}\ \mathrm{yr}^{-1}$. If the potential has a minimum with finite curvature due to unknown nonperturbative effects, $\dot{G}/G$ will become unobservably small. Improvement of the measurement of $\dot{G}/G$ would discriminate between the two situations. Problems with the time variation of other coupling constants are also discussed.

PACS numbers: 12.10.Gq, 04.50.+h, 06.20.Jr, 98.80.Dr

The time variation of fundamental constants may provide a connection between cosmology and particle physics. This idea can be traced back to Dirac,[1] although his original proposal for variation in Newton's gravitational constant G seems not supported by observations.[2]

Very recently superstring theories[3,4] appear to be promising candidates for a consistent quantum theory unifying all known interactions including gravity. They provide a suitable framework for studying the time variation of fundamental constants.[5] The consistency of superstring theories fixes the space-time dimensionality to be ten, six of which form a very small compact manifold K ($\sim10^{-32}$ cm). The metric and other bosonic backgrounds in K are constrained by string-compactification and particle-phenomenology considerations.[6,7] The coupling constants in the four-dimensional world are related to those in ten dimensions by a factor of the inverse volume of K. The cosmology in the more-dimensional universe governs the evolution of the usual three-space as well as that of K and, through the latter, dynamically determines the time variation of coupling constants in four dimensions. Generally in a more-dimensional field-theory approach,[8] quantum effects in K[9] give rise to an effective potential which may fix the size of the internal space R_6 in vacuum and influences its cosmological evolution. But in superstring theories, Witten's nonrenormalization theorem[10] tells us that such a potential for R_6 is flat up to all orders in perturbation theory. So far, the study of nonperturbative supersymmetry-breaking effects,[11,12] including world-sheet instantons, also has failed to produce a potential with a minimum at finite R_6, whose existence is expected by the conventional wisdom.

In this Letter we will show that the time variation of coupling constants critically depends on the shape of this potential. If the potential is flat the present value of $\dot{G}/G$ is calculable; for example, for an open universe

$$(\dot{G}/G)_0=(q_0-13\Omega_0H_0^2t_0^2/8)/t_0, \tag{1}$$

where H_0 is the Hubble constant, t_0 the age of the universe, q_0 the deceleration parameter, and $\Omega_0\equiv8\pi\times G_0\rho_0/3H_0^2$ the density parameter. Here ρ_0 is the density in ordinary three-space and the subscript 0 denotes the present value of the quantity. We estimate $(\dot{G}/G)_0$ to be in the range

$$(\dot{G}/G)_0\approx-1\times10^{-11\pm1}\ \mathrm{yr}^{-1}, \tag{2}$$

which overlaps the present observational upper bound[13]

$$|\dot{G}/G|\leq1\times10^{-11}\ \mathrm{yr}^{-1}. \tag{3}$$

However, if the potential really has a minimum at finite R_6, $(\dot{G}/G)_0$ will be suppressed and become unobservably small. So an improvement on the measurements of $\dot{G}/G$ will give us important information about the shape of the potential. Here we concentrate on $\dot{G}/G$, since theoretically it is independent of the dilation field and experimentally extracting it from data is simple and direct. Some remarks about time variation of other coupling constants in superstring theories are given at the end of the Letter.

We start with the following equations of motion in ten dimensions:

$$R_{AB}-\tfrac{1}{2}g_{AB}R=\tfrac{9}{2}\kappa_{10}^4\phi^{-3/2}(H_{AMN}H_B{}^{MN}-\tfrac{1}{6}g_{AB}H_{MNP}^2)+9\kappa_{10}^4\nabla^M(\phi^{-3/2}H_{APQ}R_{MB}{}^{PQ})$$
$$+\tfrac{9}{8}\phi^{-2}[\partial_A\phi\partial_B\phi-\tfrac{1}{2}g_{AB}(\partial_M\phi)^2]+\tfrac{1}{30}\kappa_{10}^2\phi^{-3/4}(\mathrm{Tr}F_{AM}F_B{}^M-\tfrac{1}{4}g_{AB}\,\mathrm{Tr}F_{MN}^2)$$
$$+\tfrac{1}{2}\kappa_{10}^2\phi^{-3/4}[\tfrac{1}{2}g_{AB}(R_{MNPQ}^2-4R_{MN}^2+R^2)-2RR_{AB}$$
$$+4R_{AM}R_B{}^M+4R_{AMBN}R^{MN}-2R_A{}^{MNP}R_{BMNP}]+\kappa_{10}^2T_{AB}, \quad (4)$$

$$\nabla_M(\phi^{-3/2}H^{MNP})=0, \quad (5)$$

$$D_M(\phi^{-3/4}F^{MPa})+9\kappa_{10}^2(\phi^{-3/2}F_{MN}{}^aH^{MNP})=0, \quad (6)$$

$$6\nabla_M(\phi^{-2}\partial^M\phi)+6\phi^{-3}(\partial_M\phi)^2+6\kappa_{10}^4\phi^{-5/2}H_{MNP}^2+\kappa_{10}^2\phi^{-7/4}[\tfrac{1}{30}\mathrm{Tr}F_{MN}^2-(R_{MNPQ}^2-4R_{MN}^2+R^2)]=0, \quad (7)$$

where $A,B,M,N=0,1,\ldots,9$; g_{AB}, ϕ, $F_{MN}{}^a$, and H_{MNP} are the metric, dilation, Yang-Mills, and Kalb-Ramond strengths. T_{AB} is the thermal energy-momentum tensor; we have neglected the effects of matter on other bosonic backgrounds. κ_{10}^2 is the gravitational constant in ten dimensions. These equations can be derived from the following action in the field-theory limit of superstring theories[14]:

$$S=\int d^{10}x\sqrt{-g}\left\{\frac{1}{2\kappa_{10}^2}R-\frac{3}{4}\kappa_{10}^2\phi^{-3/2}H_{MNP}^2-\frac{9}{16}\frac{1}{\kappa_{10}^2}(\phi^{-1}\partial_M\phi)^2\right.$$
$$\left.-\frac{1}{4}\phi^{-3/4}\left[\frac{1}{30}\mathrm{Tr}F_{MN}^2-(R_{MNPQ}^2-4R_{MN}^2+R^2)\right]+\mathcal{L}_f\right\}. \quad (8)$$

In addition, the following Bianchi identity has to be satisfied[15]:

$$dH=\mathrm{tr}R\wedge R-\tfrac{1}{30}\mathrm{Tr}F\wedge F. \quad (9)$$

We assume that the cosmological metric is of the form

$$g_{MN}=\mathrm{diag}[-1,R_3^2(t)\tilde{g}_{ij}(x),R_6^2(t)\tilde{g}_{mn}(y)], \quad (10)$$

where $i,j=1,2,3$; $m,n=4,\ldots,9$; $R_3(t)$ and $R_6(t)$ are the scale factors. $\tilde{g}_{ij}(x)$ is assumed to be maximally symmetric in three-space. For $\tilde{g}_{mn}(y)$ and other bosonic backgrounds we will adopt the following *Ansatz:*

$$\tilde{g}_{mn}(y) \text{ is Calabi-Yau}, \quad (11a)$$

$$H_{MNP}=0 \quad (M,N,P=0,\ldots,9), \quad (11b)$$

$$F_{MN}{}^{\alpha\beta}=\tilde{R}_{mn}{}^{\alpha\beta} \text{ if } (M,N)=(m,n),$$
$$=0 \text{ otherwise}, \quad (11c)$$

$$\phi=\text{const}, \quad (11d)$$

where α,β are internal-space *vielbein* indices. It is the same as the static vacuum configuration of Ref. 6 except for the metric (10). One can alter Eq. (11b) such that both the internal H_{MNP}[16,17] and an appropriate gluino condensate become nonvanishing and their contributions to the cosmological constant cancel.[17] Alternatively, (11a) can be relaxed: $\tilde{g}_{mn}(y)$ is Ricci flat. These changes would not affect the following discussion of $R_3(t)$, $R_6(t)$, and $\dot{G}/G$, but supersymmetry in four dimensions would be broken. The key observation[5,18] is that the *Ansätze* (11a)–(11d) make Eqs. (5), (6), and (9) satisfied in the time-dependent case (10), as in the static case.[6] We also assume

$$T_{AB}=\mathrm{diag}(\rho,p\tilde{g}_{ij},p'\tilde{g}_{mn}). \quad (12)$$

In the matter-dominant era, $p=p'=0$.[19] The conservation of T_{AB} gives

$$\rho(t)R_3^3(t)R_6^6(t)=\text{const}. \quad (13)$$

[We have normalized $\tilde{g}_{mn}(y)$ such that $\int d^6y\times(\det\tilde{g})^{1/2}=1$.]

With the above *Ansätze* and assumptions and neglecting terms of order $\kappa_{10}^2/t^2\propto(t_p/t)^2$ (where $t_p\sim10^{-43}$ sec is the Planck time), the dilation Eq. (7) is satisfied and the Einstein equations (4) become

$$\frac{\ddot{R}_3}{R_3}+2\frac{\ddot{R}_6}{R_6}=-\frac{7}{24}\frac{\kappa_{10}^2\rho_0}{R_6^6}\left[\frac{R_3(t_0)}{R_3(t)}\right]^3, \quad (14)$$

$$\frac{2k}{R_3^2}+\frac{\ddot{R}_3}{R_3}+\frac{2\dot{R}_3^2}{R_3^2}+6\frac{\dot{R}_3\dot{R}_6}{R_3R_6}=\frac{1}{8}\frac{\kappa_{10}^2\rho_0}{R_6^6}\left[\frac{R_3(t_0)}{R_3(t)}\right]^3, \quad (15)$$

$$\frac{\ddot{R}_6}{R_6}+5\frac{\dot{R}_6^2}{R_6^2}+3\frac{\dot{R}_3\dot{R}_6}{R_3R_6}=\frac{1}{8}\frac{\kappa_{10}^2\rho_0}{R_6^6}\left[\frac{R_3(t_0)}{R_3(t)}\right]^3, \quad (16)$$

where we have used $R=\frac{1}{4}\kappa_{10}^2\rho$; k is the Robertson-Walker parameter.

For an open universe ($k=-1$), in the large-t limit we have the following asymptotic solution,[5] which is stable under perturbations:

$$R_3(t)=t+c, \quad R_6(t)=R_{60}=\text{const}. \quad (17)$$

Now we assume that ρ_0 can be treated as a small quantity. Define $r_3(t)$ and $r_6(t)$ by

$$R_3(t)=(t+c)[1+r_3(t)],$$
$$R_6(t)=R_{60}[1+r_6(t)], \tag{18}$$

and treat them as small quantities. Up to first order, Eqs. (14)-(16) are reduced to linear differential equations for $r_3(t)$ and $r_6(t)$ with $\kappa_4^2(t)=8\pi G(t)=\kappa_{10}^2/R_6^6(t)$ replaced by $\kappa_4^2(t_0)\equiv 8\pi G_0$. The solution is

$$r_3(t)=-\frac{5}{8}\Omega_0 H_0^2 t_0^3\frac{\ln(ht)}{t}-\frac{b}{t^2}, \tag{19a}$$

$$r_6(t)=-\frac{3}{8}\Omega_0 H_0^2 t_0^3\frac{1}{t}+\frac{b}{6t^2}, \tag{19b}$$

where h and b are integration constants and we have made the change $t+c\rightarrow t$. From $q_0\equiv -(R_3\ddot{R}_3/\dot{R}_3)_0$ one can determine b. The final result for $(\dot{G}/G)_0 = -6\dot{r}_6(t_0)$ is given by Eq. (1). The parameter t_0 may differ from the age of the universe by a factor of order 1 which we will neglect.

Astronomical observations have produced quite diverse values for the cosmological parameters Ω_0, H_0, q_0, and t_0. (For details, see Rowan-Robinson.[20]) Using the most "satisfactory" set of parameters recommended by Ref. 20, i.e., $(\Omega_0, q_0, H_0)=(0.05, 0.025, 67\text{ km sec}^{-1}\text{ Mpc}^{-1})$ and $t_0=1.6\times10^{10}$ yr, we obtain

$$(\dot{G}/G)_0=-3.6\times10^{-12}\text{ yr}^{-1}. \tag{20}$$

The extreme sets are given by $(\Omega_0, q_0, H_0)=(0.05, -0.925, 100\text{ km sec}^{-1}\text{ Mpc}^{-1})$ and $(1, 0.5, 40\text{ km sec}^{-1}\text{ Mpc}^{-1})$. Correspondingly,

$$(\dot{G}/G)_0=-7.1\times10^{-11}\text{ yr}^{-1} \quad\text{or}\quad -1.2\times10^{-11}\text{ yr}^{-1}. \tag{21}$$

Thus we estimate the range for $(\dot{G}/G)_0$ as given by Eq. (2).

Rigorously speaking, if Ω_0 is close to 1, the above perturbation calculation breaks down. One needs to use a computer for solving Eqs. (14)-(16), but this would not change the estimation (2). The same is expected to be true for $k=0$ or $k=+1$ cases. The key point here is that Eq. (16) with $\rho_0\neq0$ does not allow $\dot{R}_6=0$. Thus $\dot{G}/G=-6\dot{R}_6/R_6\neq0$. Since t_0 is the only relevant cosmological time scale, $(\dot{G}/G)_0$ must be proportional to $1/t_0$ with a coefficient of order unity or, probably, one to two orders lower.

Now assume that there is an effective potential term for R_6 due to unknown nonperturbative quantum effects in the action (8). If the potential is flat near R_{60}, the result is the same as given above. If the potential has a minimum for finite R_6, R_{60} must be located there. To first order, it adds a term $\mu^2 r_6(t)$ to the left-hand side of Eq. (16), where the mass μ is determined by the curvature of the potential at R_{60}. If we assume $(\mu t)^2 \gg 1$, then

$$r_6(t)=-\tfrac{3}{8}\Omega_0 H_0^2 t_0^3\frac{1}{\mu t^3}+t^{-3/2}A\cos(\mu t+\delta). \tag{19c}$$

The second term is oscillatory and vanishes after being averaged over the period $2\pi/\mu$. The first term, compared to that in Eq. (19b), is suppressed by the factor $(\mu t_0)^{-2}=[(10^{-32}\text{ eV})/\mu]^2$. So a very tiny mass μ would make $(\dot{G}/G)_0$ in this case unobservably small. The conventional wisdom favors a not very small μ, since in four dimensions r_6 represents a Brans-Dicke-type[21] scalar field which would compete with gravitons and would have been observed if it is massless. However, the coupling of this field to matter might be anomalously weak; if so, a flat potential for r_6 is not in conflict with observations.

As for the time variation of particle-physics constants, such as α and strong or weak coupling constants, including the masses of the electron and proton, etc., the following remarks are in order. First, the time dependence of R_6 will lead also to a variation in the grand unification (GU) coupling constant

$$\alpha_{\rm GU}(R_6^{-1})\equiv g_{\rm GU}^2(R_6^{-1})/4\pi=\phi^{3/4}R_6^{-6}/4\pi, \tag{22}$$

which, in turn, gives rise to a variation in almost every coupling constant and mass measured at low energies. This is an important feature of unified string theories, in constrast to the usual assumption made in previous analyses of experimental data that only the quantity considered is varying alone. Second, there is a renormalization-group (RG) running of coupling constants,[22] which relates those measured at low energy μ $(\ll R_6^{-1})$ to $\alpha_{\rm GU}(R_6^{-1})$ calculated from Eq. (22) as follows:

$$\alpha_i^{-1}(\mu)=\alpha_i^{-1}(R_6^{-1})-\frac{1}{\pi}\sum_j C_{ij}\left(\ln\frac{1}{m_j R_6}+\theta(\mu-m_j)\ln\frac{m_j}{\mu}\right), \tag{23}$$

where $i=1$, 2, and 3 correspond to $U(1)_{em}$, $SU(2)_w$, and $SU(3)_c$; the sum is over j = leptons, quarks, gluons, $W^\pm$, etc. The C_{ij} are well-known numbers depending on the spin and group representation of the jth particle. If one neglects the variation of the second term, then

$$\frac{\dot{\alpha}_i(\mu)}{\alpha_i(\mu)}=\frac{\alpha_i(\mu)}{\alpha_{\rm GU}(R_6^{-1})}\frac{\dot{\alpha}_{\rm GU}(R_6^{-1})}{\alpha_{\rm GU}(R_6^{-1})}=\frac{\alpha_i(\mu)}{\alpha_{\rm GU}(R_6^{-1})}\frac{\dot{G}}{G}. \tag{24}$$

For α this is two orders of magnitude lower than $\dot{G}/G$. Thus, though both G and atomic clocks vary in the time defined by the metric (10), the latter probably varies more slowly. Similarly, there is also RG running of particle masses which, however, depends very much on the presence or absence of heavy families.[23] Also, as shown in Eq. (22), the time dependence of the background ϕ, which might arise upon appropriate modification of our *Ansatz*, would lead to an extra contribution to $\dot{\alpha}_{GU}/\alpha_{GU}$. It might be important to include this in considering the variation of, e.g., α over a long period such as 5×10^9 yr, as in some previous determinations of $\dot{\alpha}/\alpha$.[24]

In conclusion, further improvement in measuring $\dot{G}/G$ can discriminate between different shapes of the potential in superstring theories for the size of internal space. If the potential is almost flat or has no minimum for finite R_6, probably we are on the edge of observing $\dot{G}/G$. We encourage that old data be reanalyzed and new experiments be done. Especially, new clever ideas for precise short-time laboratory experiments would be most welcome.

We thank J. Ball, C. DeTar, M. Dine, R. Rohm, S. H. H. Tye, and J. Schwarz for useful discussions. One of us (Y.S.W.) was supported in part by National Science Foundation Grant No. PHY-8405648.

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