Zero-Bias Anomaly in Disordered Wires

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We calculate the low-energy tunneling density of states $\nu(\epsilon, T)$ of an N-channel disordered wire, taking into account the electron-electron interaction nonperturbatively. The finite scattering rate $1/\tau$ results in a crossover from the Luttinger liquid behavior at higher energies, $\nu \propto \epsilon^{\alpha}$, to the exponential dependence $\nu(\epsilon, T = 0) \propto \exp(-\epsilon^*/\epsilon)$ at low energies, where $\epsilon^* \propto 1/(N\tau)$. At finite temperature T, the tunneling density of states depends on the energy through the dimensionless variable $\epsilon/\sqrt{\epsilon^* T}$. At the Fermi level $\nu(\epsilon = 0, T) \propto \exp(-\sqrt{\epsilon^*/T})$.

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The influence of electron-electron interactions on transport in disordered systems has been extensively investigated for the past two decades [1]. It is well known that the interaction has the strongest effect in low-dimensional systems. Electron tunneling into a one-dimesional conductor is suppressed by interactions even in the absence of disorder. This suppression, which yields vanishing tunneling density of states (TDOS) at the Fermi level, can be described in the framework of the Luttinger liquid theory. The recently discovered carbon nanotubes provide a unique opportunity for studying interaction effects in quantum wires [2–5]. Although the properties of single-wall nanotubes are well described by the Luttinger liquid theory [6], the transport in multiwall nanotubes (MWNT) is still not very well understood. The number of channels, disorder strength, and carrier concentrations in these systems can vary over a wide range and are difficult to control experimentally. Whereas some measurements indicate ballistic electron transport [7], most experiments exhibit diffusive electron motion [5,8–10]. Furthermore, experiments [11] demonstrate a strong suppression of TDOS $\nu(\epsilon)$ near the Fermi level ($\epsilon = 0$). On the other hand, the existing microscopic theory [1] treats the screened Coulomb interaction in the first order of perturbation theory; it provides the result for the correction to the density of states, $\delta\nu(\epsilon) \propto -1/\sqrt{\epsilon}$, which is valid as long as $\delta\nu(\epsilon)$ is small. Clearly, this result of the lowest-order perturbation theory is insufficient for finding the behavior of TDOS $\nu(\epsilon)$ in the limit of $\epsilon \to 0$.

In this paper we present a theory of the zero-bias anomaly in the tunneling density of states in quantum wires. We treat the dynamically screened Coulomb interaction nonperturbatively and allow for an arbitrary value of $\epsilon\tau$, where τ is the elastic momentum relaxation time of electrons, and energy ϵ is measured from the Fermi level. This enables us to describe the crossover from the known Luttinger liquid results [12] valid at higher energies, $\epsilon\tau \gg 1/\sqrt{N}$, to the new low-energy ($\epsilon\tau \ll 1/\sqrt{N}$) behavior of the TDOS:

$$\nu(\epsilon, T) \propto \exp\left\{-\sqrt{\frac{\epsilon^*}{T}}\, F\left(\frac{\epsilon}{\sqrt{\epsilon^* T}}\right)\right\} \tag{1}$$

(hereinafter we use units with $\hbar = k_B = 1$). The characteristic energy ϵ^* here depends on the interaction strength g,

$$\epsilon^* = \frac{g}{\pi N \tau}, \quad g = \frac{\pi e^2}{4\bar{\nu}} \ln\frac{d}{R}, \tag{2}$$

on the number of channels N in the quantum wire, and on τ. Here $\bar{\nu}$ is the Fermi velocity averaged over all channels, and $d \gg R$ is the distance at which the electric field is shielded [13] say, by conducting electrodes surrounding the wire. The scaling function $F(x)$ and its asymptotics at $x \ll 1$ (regime considered first in Ref. [14]) and $x \gg 1$, are presented in Fig. 1. The result (1) applies at sufficiently low temperatures and energies, when the value of the exponent in this equation is large. Note that according to Eq. (1), at finite T the characteristic scale for the energy dependence of TDOS is given not by T, but by a much larger value $\sqrt{\epsilon^* T}$.

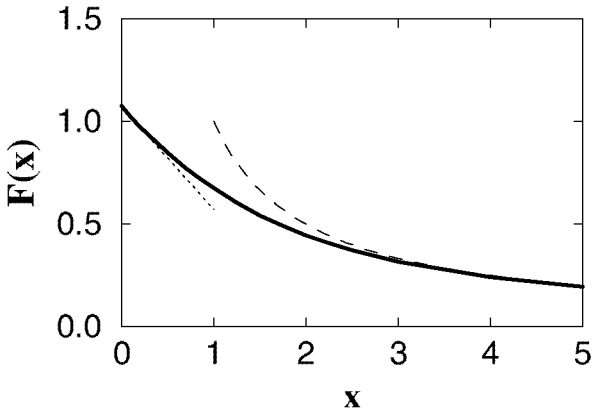

FIG. 1. The scaling function $F(x)$ and its asymptotics: $F(x) = 1.07 - x/2$ for $x \ll 1$ (dotted line), and $F(x) \sim 1/x$ for $x \gg 1$ (dashed line).

In the absence of shielding Eq. (1) is somewhat modified, as the interaction parameter g becomes a weak function of temperature and energy. We briefly discuss this case at the end of the paper; see Eqs. (19)–(21).

Having in mind experimental applications, we focus our discussion on carbon nanotubes. A typical MWNT consists of several (up to ten) graphite monolayer sheets rolled concentrically into cylinders. At zero doping they can be either metallic or semiconducting, depending on the helical arrangement of the carbon hexagons. The electron band structure of a single carbon nanotube [15] has two points in the Brillouin zone with the Dirac-like spectrum $\epsilon_k = v(k^2 + k_\perp^2)^{1/2}$; the velocity here is $v \simeq 8 \cdot 10^7$ cm/s, and the transverse momentum $k_\perp = n/R$ is quantized due to the periodic boundary conditions around the circumference, $-k_F R \le n \le k_F R$. The number of conducting subbands around each Dirac point $N = 2k_F R$ is determined by the radius of the nanotube and by the doping level, $\mu = vk_F$ ($\sim$0.5 eV). A typical radius of the outermost shell is of the order of 10 nm. Each subband has its own Fermi velocity $v_n = v\sqrt{1 - n^2/k_F^2 R^2}$ and momentum $k_F v_n/v$ along the cylinder axis. The electrons are scattered between different subbands within the same tube by impurities, lattice imperfections, and by the incommensurate lattice potential of the neighboring tubes. We consider the experimentally relevant case of the ballistic electron motion around the circumference, $l = v\tau > R$, and concentrate on low-energy, $\epsilon < v/R$, limit. Electron tunneling at higher energies, $\epsilon > vl/R^2$, in the opposite case of a short mean free path, $l < R$, was recently discussed in Ref. [16].

In the measurements of the TDOS the tunneling current propagates through the outermost shell [8] while the intershell tunneling is largely suppressed. The role of the electrons in inner shells is then believed to be reduced merely to the dynamical screening of the Coulomb interaction between the electrons in the outer shell. Little is known about the contribution of innershell electrons to screening since the doping level in the inner shells is difficult to characterize experimentally. Two distinct scenarios can be imagined: (i) the dopants are outside the nanotube, and the doping electrons reside in the outer shell only; (ii) the dopants are distributed uniformly inside the MWNT, which leads to a uniform density of carriers across the shells. Below we concentrate on the first scenario, when the inner shells of the tube may be ignored. The second scenario will be considered elsewhere.

We start with calculating the TDOS in the first order in the screened interaction potential. This calculation follows the well-known route first developed for the case of a diffusive electron motion [17], and extended later [18,19] to the case of an arbitrary value of $\epsilon\tau$. The zero-temperature result can be cast in the familiar [17,20] form,

$$\frac{\delta\nu(\epsilon)}{\nu_0} = \int_\epsilon^\infty d\omega\, \mathcal{V}(\omega). \tag{3a}$$

$$\mathcal{V}(\omega) = \Im \sum_{q_\perp} \int_{-\infty}^{\infty} \frac{dq}{2\pi^2}\, \Gamma^2(\omega,\mathbf{q}) \mathcal{G}^2(\omega,\mathbf{q}) U(\omega,\mathbf{q}). \tag{3b}$$

Here Γ is the impurity-renormalized vertex. Its inverse is given by the usual impurity ladder,

$$\Gamma^{-1}(\omega,\mathbf{q}) \equiv \left\langle \frac{\omega - \mathbf{qv}}{\omega - \mathbf{qv} + i/\tau} \right\rangle = \frac{1}{\pi\nu_0} \sum_n v_n^{-1} \times \frac{\omega(\omega + i/\tau) - (qv_n + q_\perp v_{\perp n})^2}{(\omega + i/\tau)^2 - (qv_n + q_\perp v_{\perp n})^2}. \tag{4}$$

The product of Green functions averaged over the Fermi surface [abbreviated as $\mathcal{G}^2$ in Eq. (3)] equals

$$\mathcal{G}^2(\omega,\mathbf{q}) \equiv \left\langle \frac{1}{(\omega - \mathbf{qv} + i/\tau)^2} \right\rangle = \frac{1}{\pi\nu_0} \sum_n v_n^{-1} \times \frac{(\omega + i/\tau)^2 + (qv_n + q_\perp v_{\perp n})^2}{[(\omega + i/\tau)^2 - (qv_n + q_\perp v_{\perp n})^2]^2}, \tag{5}$$

where $\mathbf{qv} = qv_n + q_\perp v_{\perp n}$ and $v_{\perp n} = vn/(k_F R)$ is the transverse velocity in the nth band, $\nu_0 = \sum_n (\pi v_n)^{-1}$ is the total density of states in the outermost shell (the summation accounts also for both spin directions and the presence of two Dirac points in the Brillouin zone).

The function $U(\omega,\mathbf{q})$ in Eq. (3b) represents the dynamically screened Coulomb interaction of electrons and is given by

$$U(\omega,\mathbf{q}) = \frac{V(\mathbf{q})}{1 - V(\mathbf{q})\Pi(\omega,\mathbf{q})}. \tag{6}$$

Within the assumptions of our model, the polarization operator here,

$$\Pi(\omega,q,q_m) = \nu_0 \Gamma(\omega,\mathbf{q}) \left\langle \frac{\mathbf{qv}}{\omega - \mathbf{qv} + i/\tau} \right\rangle, \tag{7}$$

is provided by the outer-shell electrons. The bare Coulomb potential in Eq. (6) is

$$V(q,q_m) = \frac{2e^2}{\pi} \int_0^\pi d\phi K_0\left(2qR \sin\frac{\phi}{2}\right) \cos m\phi, \tag{8}$$

where $K_0(x)$ is the modified Bessel function, and we used the fact that the momenta along the circumference of the tube are quantized and given by $q_m = m/R$.

To consider the low-energy behavior of $\delta\nu(\epsilon)$, we will need only the long-range limit, $qR \ll 1$, of Eqs. (6)–(8); we find for the bare interaction

$$V(q,q_m) = \begin{cases} \frac{e^2}{|m|}, & m \neq 0, \\ e^2 \ln[\min(\frac{d^2}{R^2}, \frac{1}{q^2R^2})], & m = 0. \end{cases} \tag{9}$$

The integrals in Eqs. (3a) and (3b) are dominated by the region of high frequencies $\omega \gg qv$, where the dynamically screened Coulomb interaction (6) has plasmon poles, and can be rewritten as

$$\mathcal{V}(\omega) = \Im \sum_m \int_{-\infty}^{\infty} \frac{dq}{2\pi^2} \frac{(\omega + i/\tau)V(q,q_m)}{\omega[\omega(\omega + i/\tau) - \omega_m^2(q)]}. \tag{10}$$

Here $\omega_m(q)$ denotes the frequency of the plasmon excitations which in the long wave length limit, $qd \ll 1$, is given by the following equation:

$$\omega_m^2(q) = \begin{cases} e^2\nu_0 q^2 v_\parallel^2 \ln[\min(\frac{d^2}{R^2}, \frac{1}{q^2R^2})], & m = 0, \\ \frac{e^2\nu_0}{|m|}(q^2v_\parallel^2 + \frac{v_\perp^2 m^2}{R^2}), & m \neq 0. \end{cases} \tag{11}$$

In Eq. (11) we introduced the average squares of the longitudinal, $v_\parallel^2$, and transverse, $v_\perp^2$, electron velocities

$$v_\parallel^2 = \frac{\sum_n v_n}{\sum_n v_n^{-1}}, \quad v_\perp^2 = \frac{v^2 \sum_n v_n^{-1} n^2}{(k_F R)^2 \sum_n v_n^{-1}}.$$

For $\epsilon > v_\perp/R$ in Eq. (3a), we can approximate the sum over m in Eq. (10) by an integral recovering the ballistic counterpart [19] of the two-dimensional diffusive correction discussed by Egger and Gogolin [16] for short-range interaction. However, in contrast to their conclusions, for lower energies $\epsilon < v_\perp/R$ the contribution of $m \neq 0$ terms becomes energy independent for both short-range and Coulomb interaction, in the latter case because of the gaps in plasmon spectra. The $m = 0$ plasmon, on the other hand, is gapless. Its contribution to Eq. (10) depends on ϵ, and has a singularity at $\epsilon \to 0$. Therefore, to study the energy dependence of DOS at low energies we neglect the nonsingular contribution of the $m \neq 0$ modes and retain only the $m = 0$ term in Eq. (10). The expression (11) for $m = 0$ ceases to be correct at frequencies larger that $\sqrt{N}\bar{v}/R$ which correspond to plasmons with wavelength of the order of the tube radius R, representing the obvious ultraviolet cutoff for one-dimensional effects. Performing the integral over the momenta in Eq. (10), we obtain

$$\mathcal{V}(\omega) = -\left(\frac{e^2 \ln[\min(\frac{d}{R}, \frac{\sqrt{N}v}{R\omega})]}{2\pi N\bar{v}}\right)^{1/2} \mathrm{Re}\frac{\sqrt{\omega + \frac{i}{\tau}}}{\omega^{3/2}}, \tag{12}$$

where $\bar{v} = \sum_n v_n/N$ is the average Fermi velocity. The two distinct regions of the frequency dependence here, $\omega \gg 1/\tau$ and $\omega \ll 1/\tau$, define two domains for the energy-dependent correction to the TDOS. Substituting $\mathcal{V}(\omega)$ into Eq. (3a), and assuming that $\bar{v}\tau > d/\sqrt{N}$, we obtain

$$\frac{\delta\nu(\epsilon)}{\nu_0} = -\frac{1}{\pi}\sqrt{\frac{2g}{N}} \begin{cases} \sqrt{\frac{2}{\epsilon\tau}} + \ln\lambda\tau, & \epsilon < 1/\tau, \\ \ln(\lambda/\epsilon), & \epsilon > 1/\tau, \end{cases} \tag{13}$$

where g is defined in Eq. (2), and $\lambda = \bar{v}\sqrt{N}/(R^2 d)^{1/3}$. The first term of the low-energy asymptotic here is familiar from the zero-bias anomaly theory of Altshuler and Aronov [1], while the second term represents the omitted in [1] high-frequency ($\omega \gg 1/\tau$) contribution to the integral Eq. (3). The behavior of $\delta\nu$ at $\epsilon \gg 1/\tau$ corresponds to the ballistic electron motion.

The perturbative expressions diverge at the Fermi level ($\epsilon \to 0$). To calculate the tunneling DOS to all orders in the interaction constant at small ϵ, we make use of the phase approximation for the fluctuating potential induced by the electron-electron interaction [14]. The nonperturbative expression for the density of states can be cast in the form equivalent to the one derived in Ref. [21],

$$\frac{\nu(\epsilon,T)}{\nu_0} = T\cosh\frac{\epsilon}{2T}\int_{-\infty}^{\infty} dt \frac{\cos\epsilon t}{\cosh\pi Tt} \times \exp\left\{\int_0^\infty d\omega\, \mathcal{V}(\omega)\frac{\cosh\frac{\omega}{2T} - \cos\omega t}{\sinh\frac{\omega}{2T}}\right\}. \tag{14}$$

In the region of validity of Eq. (14), the time integral can be evaluated within the saddle-point approximation. As the saddle point lies on the imaginary axis in the interval $0 \le -it < 1/2T$, the denominator in Eq. (14) is always a slowly varying function and does not contribute to the saddle-point exponent.

We consider first the ballistic regime $\epsilon > \sqrt{\epsilon^*/\tau}$. It allows us to reproduce the conventional Luttinger liquid results, so for brevity we mention here only the limit $T \to 0$. Evaluating the integrand of Eq. (14), we find

$$\nu(\epsilon) \sim \nu_0\left(\frac{\epsilon}{\lambda\sqrt{\epsilon^*\tau}}\right)^\alpha; \quad \alpha = \frac{1}{\pi}\sqrt{\frac{2g}{N}}. \tag{15}$$

In the diffusive regime $\epsilon < \sqrt{\epsilon^*/\tau}$, the main contribution to the TDOS comes from the small frequencies, where one can reduce Eq. (12) to $\mathcal{V}(\omega) = \sqrt{\epsilon^*/\pi\omega^3}$. Furthermore, if $\epsilon \ll 1/N\tau$ and $T \ll g/\pi N\tau$, the saddle-point approximation is again applicable for the evaluation of the time integral in Eq. (14). One can easily check that at the saddle point the $\cosh\pi Tt$ function in Eq. (14) can be replaced by 1, and therefore the density of states satisfies the scaling form

$$\nu(\epsilon,T) \sim \frac{\nu_0}{(\lambda\tau)^\alpha}\exp\left\{-\sqrt{\frac{\epsilon^*}{T}}F\left(\frac{\epsilon}{\sqrt{\epsilon^*T}}\right)\right\}. \tag{16}$$

Here the function

$$F(x) = \int_0^\infty dy \frac{\cosh\frac{y}{2} - \cosh yz_s(x)}{\sqrt{\pi}y^{3/2}\sinh\frac{y}{2}} - x[\tfrac{1}{2} - z_s(x)] \tag{17}$$

is determined by the value of the integrand in Eq. (14) at the saddle point $t_s \equiv iz_s/T$. The dependence of z_s on the ratio $\epsilon/\sqrt{\epsilon^*T} \equiv x$ is given by equation

$$x = \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{dy\,\sinh z_s y}{\sqrt{y}\sinh y/2}. \tag{18}$$

Numerical solution of this parameter-free equation and the subsequent evaluation of the integral in Eq. (17) yields the graph of scaling function $F(x)$ plotted in Fig. 1 and its asymptotics given in the figure caption. Note that the preexponential factor in Eq. (16), which we omitted in Eq. (1) for the sake of brevity, provides the proper matching of the results (15) and (16) obtained in the energy domains $\epsilon \gg \sqrt{\epsilon^*/\tau}$ and $\epsilon \ll \sqrt{\epsilon^*/\tau}$, respectively.

We derived our main results, Eqs. (16)–(18), assuming that the Coulomb interaction is shielded at distances

$\sim d$. In the absence of shielding, the long-range nature of the interaction potential leads to a stronger [22] than predicted by Eq. (15) suppression of the TDOS in the ballistic regime, $\ln[\nu(\epsilon)/\nu_0] \propto -[\ln(\epsilon/\epsilon^*)]^{3/2}$. At lower energies (corresponding to the diffusive regime), the effect of the long-range potential can be accounted for by replacing the parameter ϵ^* of Eq. (2) with the logarithmic function of energy and temperature,

$$\epsilon^* \to \epsilon^*(\epsilon, T) = \epsilon_0 \ln \frac{\bar{v}/R}{\max(\epsilon, \sqrt{T\epsilon_0})}, \quad \epsilon_0 \equiv \frac{e^2}{4\bar{v}N\tau}. \tag{19}$$

After the definition of ϵ^* is adjusted to reflect this replacement, we can use Eqs. (16)–(18) again. In the limit of low temperatures, $T \ll \epsilon^2/\epsilon_0$, we find

$$\nu(\epsilon) \propto \exp\left(-\frac{\epsilon_0}{\epsilon} \ln \frac{\bar{v}}{\epsilon R}\right). \tag{20}$$

The suppression of the TDOS near the Fermi surface ($\epsilon \ll \sqrt{T\epsilon_0}$) at finite temperatures is given by

$$\nu(T) \propto \exp\left[-1.07\sqrt{\frac{\epsilon_0}{T}} \ln^{1/2}\left(\frac{\bar{v}}{R\sqrt{T\epsilon_0}}\right)\right]. \tag{21}$$

The origin of the strong suppression of the TDOS, see Eqs. (15), (16), (20), and (21), lies in the redistribution of charge of the tunneling electron along the wire. This process is impeded by the finite propagation time of plasmons which enable the charge spreading. The requirement that the wavelength of the relevant plasmons $\bar{v}\sqrt{N}/\epsilon$ be shorter that the length of the wire L imposes the lower energy limit for the applicability of the present theory. For smaller energies, $\epsilon < \bar{v}\sqrt{N}/L$, the tunneling DOS of the wire depends on the impedance of the leads attached to the segment [23]. To observe a sizable suppression of TDOS, the nanotube must be sufficiently long, and therefore have high intrinsic resistance. We find that in order to reach the strong diffusive renormalization of TDOS anomaly ($\epsilon, T \ll \epsilon^*$) the total resistance of the segment should be made larger than $(h/e^2)\sqrt{N/g}$. This condition on the overall segment length does not invalidate the employed method which assumes the diffusive motion of electrons. Indeed, the characteristic frequencies of the plasmons involved are high enough for the weak localization's corrections to be ignored in the calculation of TDOS. In order to avoid localization effects in the dc transport measurement, the two junctions needed for the TDOS measurement should be attached to the MWNT within a distance shorter than the localization length from each other.

We presented above the derivation of TDOS only for the case of a relatively long mean free path l, exceeding the radius R of a MWNT, but our main result (1) is valid for any relation between l and R. The only additional feature appearing in the case $l \ll R$, is the intermediate range of energies studied in Ref. [16], where the TDOS behaves as in a two-dimensional disordered conductor.

In summary, we have obtained the tunneling density of states in a disordered quasi-one-dimensional conductor. Our results are nonperturbative in the electron-electron interaction, and cover both the diffusive and ballistic regimes of the electron motion. In contrast to the two-dimensional case [21,24], the nonperturbative results (14) and (20) are not given by a simple exponentiation of the first-order interaction correction (13).

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