

Adaptive phase transform processors for time delay estimation^{a)}

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This paper introduces two recursive realizations of the phase transform (PHAT) processor for time-delay estimation (TDE), using a simple one-pole low-pass filter and the least-mean-square (LMS) adaptive filter, respectively. It is shown that these adaptive methods are capable of tracking time-varying delay functions which correspond to moving sources or receivers, and are very effective in reducing the effect of interfering tonals which must be generated by the target as jamming signals to mask its movement. The performances of these methods are compared with those of other existing adaptive TDE algorithms via computer simulations.

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INTRODUCTION

The problem of estimating the time difference of arrival of the same signal at two spatially separated sensors arises in a variety of applications of sonar, radar, acoustics, geophysics, and biomedical engineering where we need to locate the signal source.¹⁻⁵

Of interest in this paper are passive systems, in which, unlike the active systems, the source signal strength cannot be controlled. However, their covertness can be advantageous, since passive systems do not rely on self-generated energy that is reflected off the source or target. An important example of such systems is a passive sonar system which receives the signals generated by a source, possibly corrupted by noise, at an array of spatially separated sensors. It is well known¹ that the location of the source can be determined if the time delays between the arrival times of the signal at three sensors are available.

We consider the two-sensor time delay estimation (TDE) problem, where the signals received at the two sensors are given by

$$x_1(k) = s(k) + w_1(k) + p(k) \quad (1a)$$

and

$$x_2(k) = s(k - D) + w_2(k) + p(k - \tilde{D}), \quad (1b)$$

where k is the discrete time index, $s(k)$ is the source signal, $w_1(k)$ and $w_2(k)$ are the additive noises at sensors 1 and 2, $p(k)$ denotes interfering tonals which might be generated by a target as a jamming signal to mask its movement, and D and \tilde{D} are delay parameters associated with the signal and interfering tonals, respectively. Also, it is assumed that the source signal $s(k)$ and additive noises $w_1(k)$ and $w_2(k)$ are mutually uncorrelated random processes with zero mean.

Most approaches for TDE have been shown to be related through generalized cross correlation (GCC) methods which involve prefiltering the received signals and estimating the time delay as the time lag where the cross correlation function of the prefiltered signals

$$R_{12}^{(g)}(m) = F^{-1}\{W^{(g)}(f)e^{j\theta_{12}(f)}\}, \quad |m| < M \quad (2)$$

is maximum.⁶ In (2), $F^{-1}\{\cdot\}$ denotes the inverse Fourier transform of $\{\cdot\}$, $W^{(g)}(f)$ is a weighting function in the frequency domain that is determined by the prefilters, and $\theta_{12}(f)$ is the phase function of the cross-power density spectrum (cross-PDS) of $x_1(k)$ and $x_2(k)$. That is,

$$e^{j\theta_{12}(f)} = [G_{12}(f)]/|G_{12}(f)|, \quad (3)$$

where $G_{12}(f)$ is the cross-PDS of $x_1(k)$ and $x_2(k)$. If there are no interfering tonals in the received signals [i.e., $p(k) = 0$ in (1)], the phase function in (3) is given by $\theta_{12}(f) = 2\pi f D$, which means that the phase function is directly proportional to the delay parameter D . The frequency domain weighting functions of the GCC methods of interest in this paper are summarized below:

$$W^{(B)}(f) = |G_{12}(f)|; \quad (4a)$$

BCC (basic cross correlation) method,²

$$W^{(R)}(f) = |G_{12}(f)| |G_{22}(f)|; \quad (4b)$$

Roth processor,⁷

$$W^{(P)}(f) = |G_{12}(f)|/|G_{12}(f)| = 1; \quad (4c)$$

PHAT (phase transform).²

Recently, the BCC method and the Roth processor have been realized using a simple one-pole low-pass filter⁸⁻¹⁰ and the LMS adaptive filter,¹⁰⁻¹⁴ respectively. The main advantages of these recursive time-domain implementations are that they track time-varying delay functions and also avoid the difficulties encountered in spectral estimation with finite record lengths.

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The phase transform processor was proposed as an *ad hoc* method to reduce the effect of strong tonals by uniformly weighting the phase function $e^{j\theta_{12}(f)}$ in the entire frequency band.² The purpose of this paper is to introduce two recursive methods which realize the PHAT processor. In these adaptive techniques the relevant GCC functions are updated using a simple one-pole low-pass filter⁸⁻¹⁰ and the LMS adaptive filter,¹⁰⁻¹⁶ respectively.

In Sec. I, adaptive realizations of the BCC and the Roth processors are briefly summarized, while Sec. II is devoted to the PHAT processor and its adaptive implementations. Experimental results and conclusions are presented in Secs. III and IV, respectively.

I. SOME THEORETICAL BACKGROUND

From (2) and (4a), the GCC function of the BCC method is given by the cross correlation function of the received signals without prefiltering. That is,

$$R_{12}^{(B)}(m) = F^{-1}\{G_{12}(f)\} = C_{12}(m), \quad |m| < M, \quad (5a)$$

where

$$C_{12}(m) = E\{x_1(k)x_2(k+m)\}, \quad (5b)$$

and $E\{\cdot\}$ denotes the statistical expectation of $\{\cdot\}$.

It has been shown⁸⁻¹⁰ that the cross correlation function of $x_1(k)$ and $x_2(k)$ can be estimated using a bank of simple one-pole low-pass filters as

$$\hat{C}_{12}(m,k) = \beta \hat{C}_{12}(m,k-1) + (1-\beta)x_1(k)x_2(k+m), \quad |m| < M, \quad (6a)$$

where $\hat{C}_{12}(m,k)$ denotes an estimate of $C_{12}(m,k)$ in (5b) at time k and $0 < \beta < 1$ controls the time constant of the low-pass filter whose transfer function is given by

$$A(z) = (1-\beta)/(1-\beta z^{-1}) \quad (6b)$$

when $x_1(k)x_2(k+m)$ is applied as its input. The time constant of the above low-pass filter can be approximated as^{9,10}

$$\tau_A \cong 1/(1-\beta) \text{ samples.} \quad (7)$$

From (5a)-(6a), we can see that taking the Fourier transform (FT) of $\hat{C}_{12}(m,k)$ with respect to m yields an estimate of the cross-PDS of $x_1(k)$ and $x_2(k)$ at time k . That is,

$$\hat{G}_{12}(f,k) = F\{\hat{C}_{12}(m,k)\}, \quad (8)$$

where $F\{\cdot\}$ represents the FT of $\{\cdot\}$ with respect to m .

The cross correlation function estimate $\hat{C}_{12}(m,k)$ in (6a) has been used to estimate the time delay parameters,⁸⁻¹⁰ and the approach has been referred to as the ABCTDE (adaptive basic cross correlation for TDE) algorithm.¹⁰ From (5a), (6a), and (8), we can see that the ABCTDE algorithm realizes the BCC method in a recursive way.

From (2) and (4b), the GCC function of the Roth processor is given by

$$R_{12}^{(R)}(m) = F^{-1}\{[G_{12}(f)]/[G_{22}(f)]\}, \quad |m| < M. \quad (9)$$

It is known that $R_{12}^{(R)}(m)$ represents the impulse response function $h_{12}(m)$ of the optimum (Weiner) filter which best

approximates $x_1(k)$ as a weighted sum of $x_2(k-m)$ for $|m| < M$.

A class of adaptive filter algorithms has been developed to recursively update the optimum filter coefficients.¹⁵⁻¹⁷ In this paper, we restrict our interest to the LMS adaptive filter^{15,16} since it is computationally very simple but still very effective. The LMS adaptive filter algorithm updates the filter coefficients $h_{12}(m,k)$ to minimize the mean-squared error $E\{e^2(k)\}$ in Fig. 1, where $x_1(k)$ and $x_2(k)$ are applied as primary and reference inputs, respectively, and the M -sample delay is introduced to $x_1(k)$ to make the system causal. The LMS algorithm is summarized in the following:

$$\hat{h}_{12}(m,k+1) = \hat{h}_{12}(m,k) + 2\mu e(k)x_2(k-m), \quad |m| < M, \quad (10a)$$

where

$$e(k) = x_1(k) - \sum_{m=-M}^M \hat{h}_{12}(m,k)x_2(k-m). \quad (10b)$$

In (10a), μ controls the convergence rate and stability of the adaptive filter. The time constant of the LMS adaptive filter can be approximated as^{15,16}

$$\tau \cong 1/2\mu\sigma_2^2, \quad (11)$$

where σ_2^2 is the variance of $x_2(k)$. From (9) and (10a), we can see that taking the Fourier transform of $\hat{h}_{12}(m,k)$ with respect to m yields

$$\hat{H}(f,k) \cong F\{\hat{h}_{12}(m,k)\}, \quad |m| < M \quad (12a)$$

$$= \left\{ \frac{G_{12}(f,k)}{G_{22}(f,k)} \right\}, \quad (12b)$$

which is an estimate of $G_{12}(f)/G_{22}(f)$ in (9) at time k .

From (9), (12a), and (12b), we can see that the impulse response function of the LMS adaptive filter is an estimate of the GCC function of the Roth processor. This approach has been referred to as the LMSTDE (LMS for TDE) algorithm.^{10,13,14}

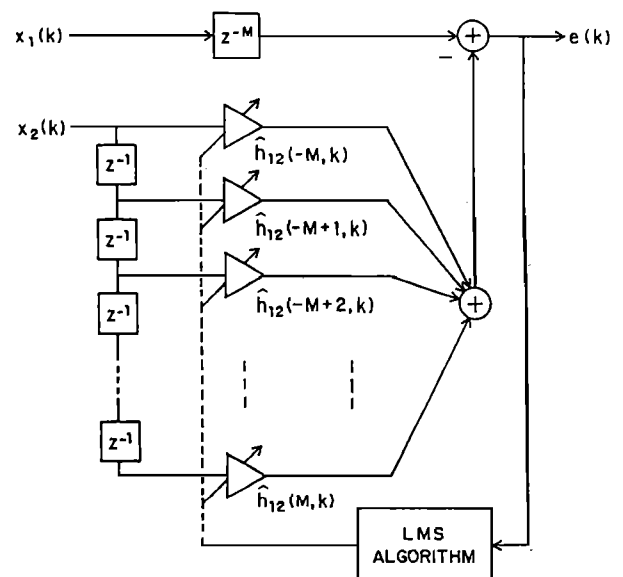


FIG. 1. Block diagram of the LMS adaptive filter algorithm.

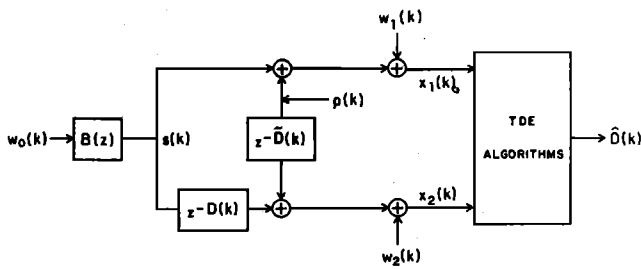


FIG. 2. Block diagram for generating signals for simulations.

II. THE PHASE TRANSFORM PROCESSOR AND ITS ADAPTIVE IMPLEMENTATIONS

The phase transform processor was proposed as an *ad hoc* method to obtain a clear indication of the peak and to remove the effect of interfering tonals of the pertinent GCC function by weighting the phase function in (3) uniformly over the entire frequency band. Thus, from (2) and (4c), the GCC function of the PHAT processor is given by

$$R_{12}^{(P)}(m) = F^{-1}\{[G_{12}(f)]/|G_{12}(f)|\}, \quad |m| < M$$

$$= F^{-1}\{e^{j\theta_{12}(f)}\}. \quad (13)$$

Introducing a time index k in (13) yields the time-varying GCC function of PHAT as

$$R_{12}^{(P)}(m, k) = F^{-1}\{[G_{12}(f, k)]/|G_{12}(f, k)|\}$$

$$= F^{-1}\{e^{j\theta_{12}(f, k)}\}. \quad (14)$$

Now, using (8) and (12b), the time-varying GCC function of the PHAT can be estimated using the one-pole low-pass filter in (6a) and the LMS adaptive filter algorithm in (10a) and (10b) as follows:

$$\hat{R}_{12}^{(P1)}(m, k) = F^{-1}\left\{\frac{\hat{G}_{12}(f, k)}{|\hat{G}_{12}(f, k)|}\right\}, \quad |m| < M \quad (15a)$$

and

$$\hat{R}_{12}^{(P2)}(m, k) = F^{-1}\left\{\frac{\hat{H}_{12}(f, k)}{|\hat{H}_{12}(f, k)|}\right\}$$

$$= F^{-1}\left\{\left[\frac{\hat{G}_{12}(f, k)}{\hat{G}_{22}(f, k)}\right]\left[\frac{\hat{G}_{12}(f, k)}{\hat{G}_{22}(f, k)}\right]^{-1}\right\},$$

$$|m| < M. \quad (15b)$$

The above approaches in (15a) and (15b) will be referred to as the APHAT-1 and APHAT-2, respectively, when the

time-delay estimate is given by the argument $m = \hat{D}(k)$, where the relevant time-varying GCC functions $\hat{R}_{12}^{(P1)}(m, k)$ and $\hat{R}_{12}^{(P2)}(m, k)$ are maximum.

In many passive sonar signal processing problems, the received signals often include strong tonals $p(k)$ [see (1)]. One of the sources of the periodic components might be the engine or propeller of a target. Another important case of such signals can be encountered when the target transmits narrow-band jamming signals to hide its location and movement. In general, there may be more than one tonal involved. Computing the cross correlation function of $x_1(k)$ and $x_2(k)$ in (1), we have

$$C_{12}(m) = C_{ss}(m - D) + C_{pp}(m - \tilde{D}), \quad (16a)$$

where

$$C_{ss}(m) = E\{s(k)s(k+m)\} \quad (16b)$$

and

$$C_{pp}(m) = E\{p(k)p(k+m)\} \quad (16c)$$

represent the auto correlation functions of $s(k)$ and $p(k)$, respectively. If no periodic components are involved in the received signals, (16a) becomes

$$C_{12}(m) = C_{ss}(m - D) \quad (17)$$

and the time-delay parameter D can be estimated as the argument $m = D$, where $C_{12}(m)$ is maximum. However, in the presence of strong tonals, the cross correlation function $C_{12}(m)$ might yield peaks at several different places to estimate incorrect delay parameters, since the cross correlation functions of periodic signals are also periodic.

The PHAT processor in (13) is rather simple but performs very well in the presence of strong tonals when the source signal is white or broad bandlimited. If we consider the magnitude of the cross-PDS of $x_1(k)$ and $x_2(k)$ in the presence of strong tonals, the spectral components of the periodic signals are given by impulse functions at the relevant frequencies. Thus we see that normalizing the cross-PDS with its magnitude as in (13) or (15) produces an effect of de-emphasizing the strong tonals.

Now, consider the case of $G_{12}(f) = 0$ in some frequency band (i.e., bandlimited source signal). Then the phase function in (3) is undefined in that band and the estimate of the phase is erratic. Thus normalizing the cross-PDS with its magnitude or weighting the phase function uniformly in the entire frequency range introduces errors in estimating the time delay. Therefore, this behavior suggests that the phase

TABLE I. Summary of the parameters used for the simulations.

Case	$B(z)$	$P(k)^a$	$D(k)$	$\tilde{D}(k)$	β	μ
1	$(1 + z^{-1})/2$	0	4	0	0.9998	5×10^{-5}
2	b	0	4	0	0.9998	5×10^{-5}
3	1	0	4	0	0.9999	5×10^{-5}
4	$\frac{z^{-1}}{1 - z^{-1} + 0.8z^{-2}}$	$3P_1(k) + 2P_2(k)$	4	9	0.9999	5.88×10^{-6}
5	1	$2P_1 + P_2(k)$	$-8 + 0.002k$	0	0.99	1.11×10^{-3}
6	1	$3P_1(k) + 2P_2(k)$	$-8 + 0.002k$	$4 - 0.001k$	0.998	1.18×10^{-4}

^a $P_1(k) = \sin(0.46\pi k - 0.5)$ and $P_2(k) = \sin(0.12\pi k - 0.5)$.

^b $B(z)$ for case 2 is the 6th-order Butterworth low-pass filter with cutoff frequency of 0.2 Hz, and sampling frequency of 2 Hz.

function $e^{j\theta_{12}(f)}$ be additionally weighted to compensate for the presence or absence of signal power as in the case of the Roth,⁷ Scot,⁸ and ML (maximum likelihood)² processors. Even though the APHAT algorithms, like the conventional PHAT, have the above problem, it will be shown that they are very effective when the source signal has broad bandwidth and when the received signals contain strong interfering tonals. This property will be demonstrated in the next section via computer simulations.

III. EXPERIMENTAL RESULTS

The properties of the APHAT algorithms will be discussed by comparing the performances of the APHAT-1 and -2 processors with those of the ABCTDE^{9,10} and LMSTDE¹⁰⁻¹⁴ algorithms through computer simulations.

The schematic diagram used to generate the received signals $x_1(k)$ and $x_2(k)$ is depicted in Fig. 2, where a white Gaussian random signal $w_0(k)$ is processed through $B(z)$ to generate the source signal $s(k)$. Also, the source signal $s(k)$ and the periodic signal $p(k)$ were passed through time-varying filters with the transfer functions of $e^{-j\omega f D(k)}$ and

$e^{-j2\pi f \bar{D}(k)}$ to generate $s[k - D(k)]$ and $p[k - \bar{D}(k)]$, respectively.¹⁹ Here, $D(k)$ and $\bar{D}(k)$ represent the time-varying delay functions related to the source signal $s(k)$ and interfering tonals $p(k)$, respectively. For all of the simulations, 61 coefficients of $\hat{C}_{12}(m, k)$ and $\hat{h}_{12}(m, k)$ were estimated (i.e., $M = 30$) and a Hamming window function with 61 points was applied before taking the FT of $\hat{C}_{12}(m, k)$ and $\hat{h}_{12}(m, k)$, respectively. For all of the simulations except case 3, the source signals and additive noises were scaled to have unit variances, while the variances of $s(k)$ and $w_i(k)$ are given by 0.1 and 0.9 for case 3(a) (i.e., SNR = 1/9) and 0.0476 and 0.9524 for case 3(b) (i.e., SNR = 1/20). Other parameters for the simulations are summarized in Table I. The estimated GCC functions at $k = 8000$ for cases 1-4 are displayed in Figs. 3-6, where the delay parameter of interest is constant [i.e., $D(k) = 4$ samples]. Also, the estimated delay functions for cases 5 and 6 are presented in Figs. 7 and 8, respectively, where the delay function of the source signal linearly increases from -8 to 8 in 8000 samples as indicated by a dotted line, and the delay parameter was computed every 20 samples, starting from $k = 80$ and ending at $k = 8000$.

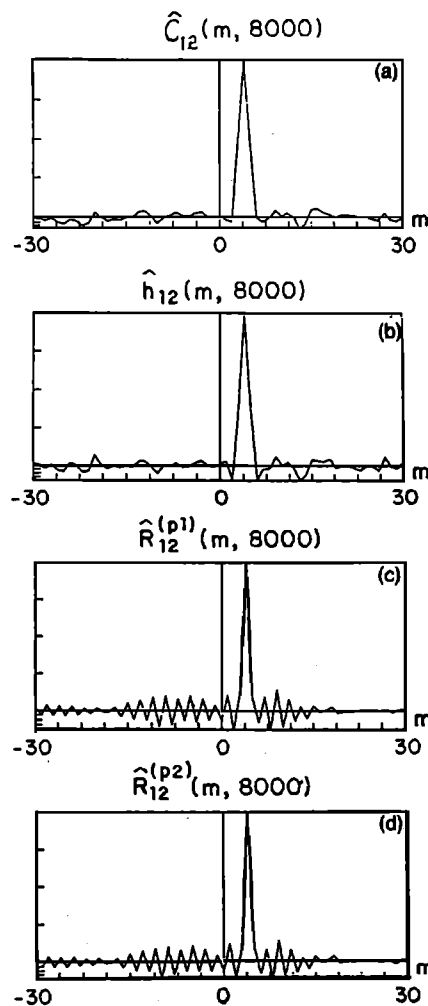


FIG. 3. Estimated GCC functions for broadband low-pass source signal with additive white noise: (a) ABCTDE; (b) LMSTDE; (c) APHAT-1; (d) APHAT-2.

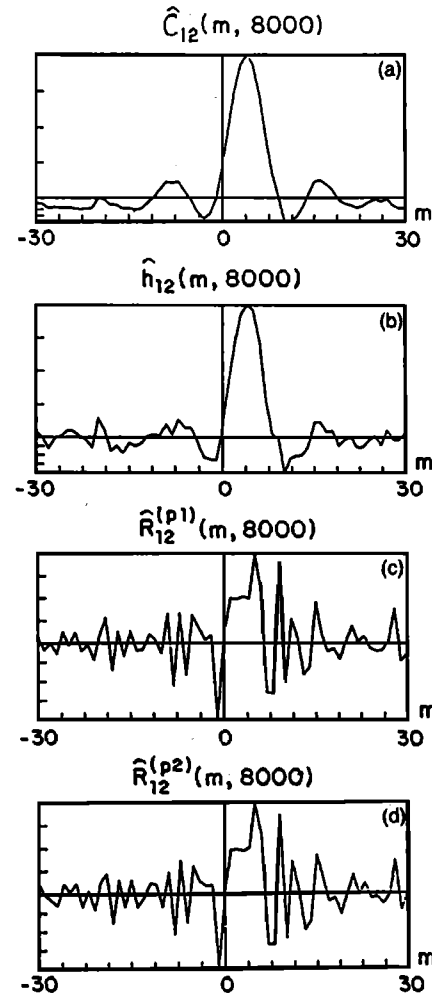


FIG. 4. Estimated GCC functions for narrow-band low-pass source signal with additive white noise: (a) ABCTDE; (b) LMSTDE; (c) APHAT-1; (d) APHAT-2.

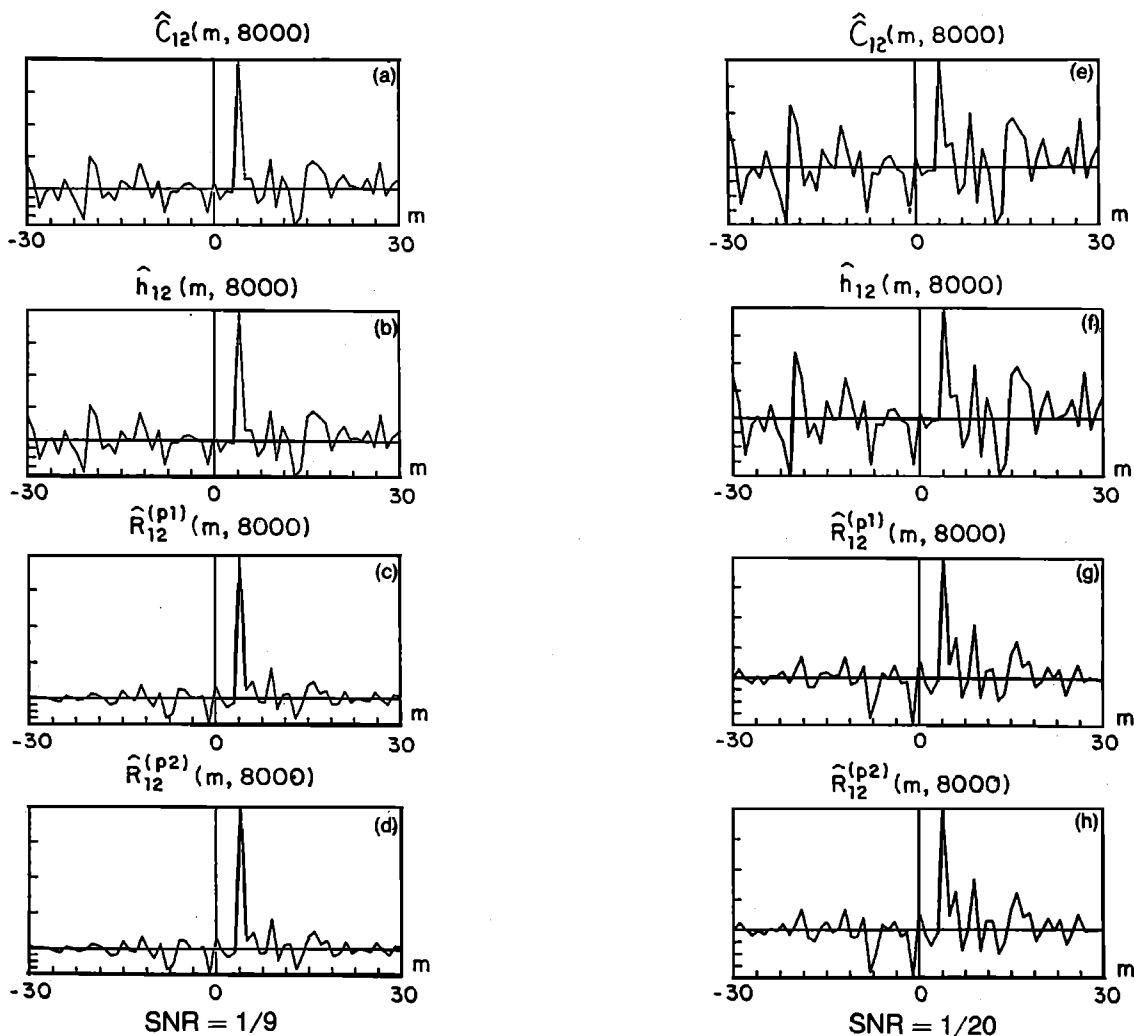


FIG. 5. Estimated GCC functions for white source signal with additive white noise: (a) ABCTDE; (b) LMSTDE; (c) APHAT-1; (d) APHAT-2; (e) ABCTDE; (f) LMSTDE; (g) APHAT-1; (h) APHAT-2.

IV. DISCUSSION

Cases 1 and 2: The estimated GCC functions in Fig. 3 demonstrate that the APHAT-1 and -2 algorithms perform as well as the ABCTDE and LMSTDE algorithms do, when the source signal has broad bandwidth. However, since the source signal for case 2 is narrow bandlimited, the phase information outside the frequency band of the source signal is not related to the time delay, but is given by a random-phase function. Therefore, uniformly weighting the phase function in the entire frequency range as in APHAT-1 and -2 results in emphasizing the frequency band where only spectral estimation errors exist, to yield noisy GCC function estimates as shown in Fig. 4(c) and (d).

The results for cases 1 and 2 suggest that the APHAT-1 and -2 are efficient methods to estimate time delay for the source signals with broad bandwidth, but fail to estimate correct delay parameter for narrow bandlimited source signals.

Case 3: The relevant GCC functions for the four adaptive time-delay estimation algorithms are displayed in Fig. 5 when the source signals are white and for two different SNR's (i.e., 1/9 and 1/20). These results show that the per-

formances of the APHAT-1 and -2 are as good as those of the others. Here, the less noisy GCC function estimates for the APHAT-1 and -2 are due to the Hamming window functions applied before taking the Fourier transform of $\hat{C}_{12}(m, k)$ and $\hat{h}_{12}(m, k)$, respectively.

Case 4: The signals used in this set of simulations were obtained by passing white Gaussian signals through a second-order bandpass filter and then corrupting the output with interfering tonals as well as additive white noises, and the delay parameter of the source signal is given by $D(k) = \text{four samples}$. The result in Fig. 6(a) shows that the GCC function for the ABCTDE algorithm is maximum at $m = 0$, which is the delay parameter relevant to interfering tonals. Similarly, the GCC function in Fig. 6(b) for the LMSTDE algorithm peaks at an incorrect position, even though the effect of the tonals is less than that of the ABCTDE algorithm. However, the GCC function estimates of the APHAT-1 and -2 are maximum at $m = 4$, and yield the correct time-delay estimate. We notice that the APHAT-2 performs better than the APHAT-1. This is because the periodic components have been already de-emphasized and the bandlimited source signal is whitened in the process of LMS adaptive filtering.

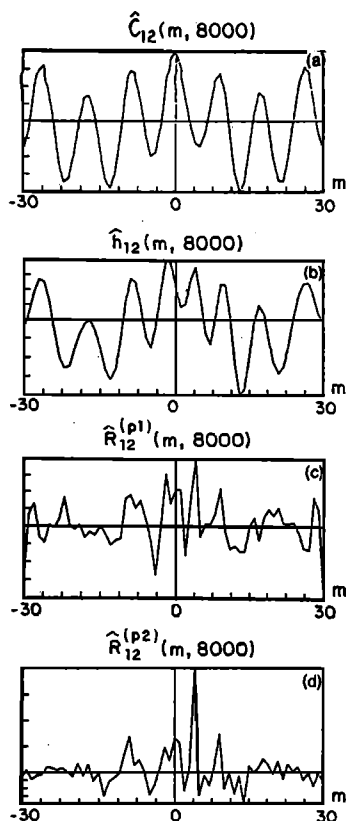


FIG. 6. Estimated GCC functions for bandpass source signal with additive white noise and interfering tonals: (a) ABCTDE; (b) LMSTDE; (c) APHAT-1; (d) APHAT-2.

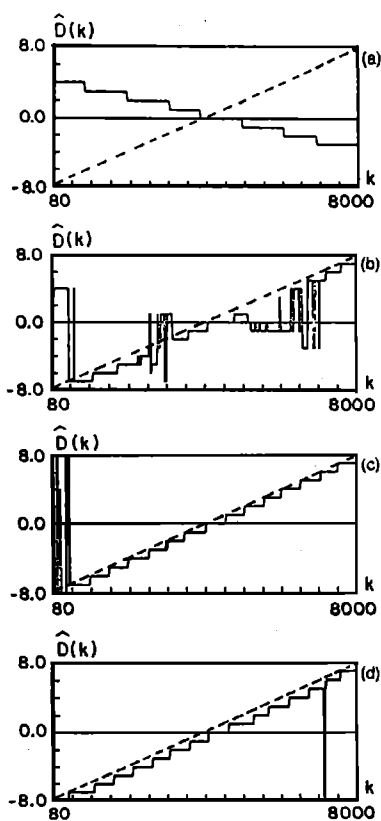


FIG. 8. Estimated time-varying delay functions for white source signal with additive white noise and interfering tonals (case 6): (a) ABCTDE; (b) LMSTDE; (c) APHAT-1; (d) APHAT-2.

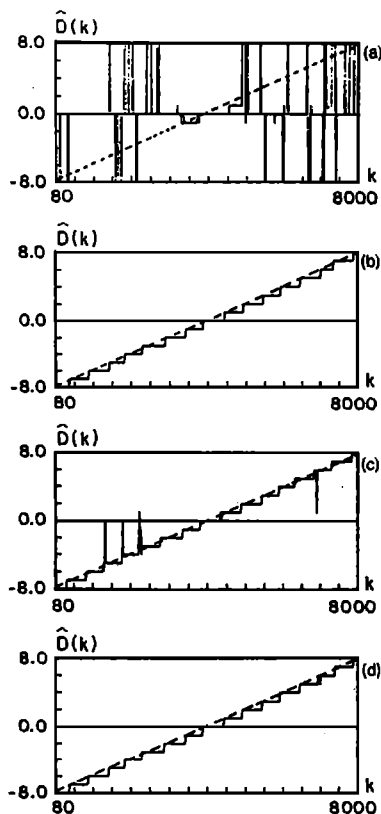


FIG. 7. Estimated time-varying delay functions for white source signal with additive white noise and interfering tonals (case 5): (a) ABCTDE; (b) LMSTDE; (c) APHAT-1; (d) APHAT-2.

From the above results, we see that the APHAT-1 and -2 are effective even when the source signal is narrow bandlimited but still has some power in a wide range of frequency bands.

Cases 5 and 6: The last two sets of simulations concern the problems of estimating time-varying delay functions, which correspond to moving source or receivers.^{10-14,21-23} Here, the delay functions relevant to the source signals linearly increase from -8 to $+8$ in 8000 samples, while those of the interfering tonals are constant [i.e., $\tilde{D}(k) = 0$] and linearly decrease from $+4$ to -4 in 8000 samples; $\tilde{D}(k) = 4 - 0.001k$ for cases 5 and 6, respectively.

From the estimated delay functions in Figs. 7 and 8, we observe that the ABCTDE method estimates the delay function relevant to the interfering tonals [i.e., $\tilde{D}(k)$], see Figs. 7(a) and 8(a)], while the LMSTDE algorithm, APHAT-1, and APHAT-2 track the correct relay parameter relevant to the source signals [i.e., $D(k)$]. Also, the results show that the APHAT-1 and APHAT-2 perform superiorly to the LMSTDE algorithm.

V. CONCLUSIONS

Two adaptive implementations of the phase transform processor, using a bank of simple one-pole low-pass filters

(APHAT-1) and the LMS adaptive filter (APHAT-2), respectively, were introduced. It was demonstrated that these algorithms are more effective than the ABCTDE and LMSTDE algorithms in tracking constant and time-varying delay functions associated with broad bandlimited source signals in the presence of strong tonals. It was also shown that the APHAT algorithms should be used with caution when the source signals are narrow bandlimited.

The APHAT-1 algorithm is attractive because of its computational simplicity. However, as demonstrated in Fig. 6, the APHAT-2 processor may give more accurate time-delay estimates because of the inherent whitening of the source signals in the process of LMS adaptive filtering.

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