The Problem and Treatment of DC Offsets in FDTD Simulations

Cynthia M. Furse, *Senior Member, IEEE*, Daniel H. Roper, Dale N. Buechler, *Member, IEEE*, Douglas A. Christensen, and Carl H. Durney, *Fellow, IEEE*

***Abstract*—This paper discusses the causes of and some solutions to the commonly observed problem of dc field offsets in finite-difference time-domain (FDTD) simulations. DC electric and magnetic field offsets are shown to be valid calculated responses of the modeled systems, resulting from interaction between the turn-on characteristics of the source and the properties of the models. The dc offsets may be avoided in the time domain by tailoring the source waveforms or in the frequency domain by post-processing the FDTD output.**

***Index Terms*—Finite-difference time-domain (FDTD) methods.**

I. Introduction

In recent years, the finite-difference time-domain (FDTD) method has become a popular tool for solving problems involving Maxwell's equations [1]. Although the method is versatile and conceptually straightforward, an FDTD simulation must be carefully designed in order to yield meaningful results. In particular, if a highly conducting model has a closed current path, physically correct dc currents that do not decay appreciably with time may be induced by the time-varying source fields. These nonoscillating currents produce a constant magnetic field in the near field of the object, which may lead to interpretation errors in both the time and frequency domains [2].

II. Example of DC Offset for an Infinite Circular Metal Cylinder

A dc magnetic field offset can be observed in the results of an FDTD analysis of a perfectly conducting infinite circular cylinder illuminated by a plane wave. The cylinder is 7.5 cm (20 Yee cells) in diameter, modeled with a square cell size of 0.375 cm ($\lambda/20$ at 4 GHz) in a two-dimensional (2-D) model space of 56×56 cells. Each time step is 6.25 ps. Fig. 1 shows time histories of the calculated fields at a point four cells in front of the cylinder (for a frontally incident plane wave source). For case (a), the time dependence of the electric field of the source is a raised cosine pulse with a 2 V/m peak and a 4-GHz bandwidth (half-width half-maximum); for case (b) the source is an unramped 4-GHz continuous wave (CW), with 1 V/m peak.

Manuscript received August 21,1998; revised May 4, 2000.

C. M. Furse is with the Department of Electrical and Computer Engineering, Utah State University, Logan, UT 84322–4120 USA.

D. H. Roper is with Hughes Aircraft Company, Los Angeles, CA 90059 USA.

D. N. Buechler is with the College of Engineering and Applied Science, University of Wisconsin, Milwaukee, WI 53201–0784 USA.

D. A. Christensen and C. H. Durney are with both the Department of Electrical Engineering and the Department of Bioengineering, University of Utah, Salt Lake City, UT 84112 USA.

Publisher Item Identifier S 0018-926X(00)07705-X.

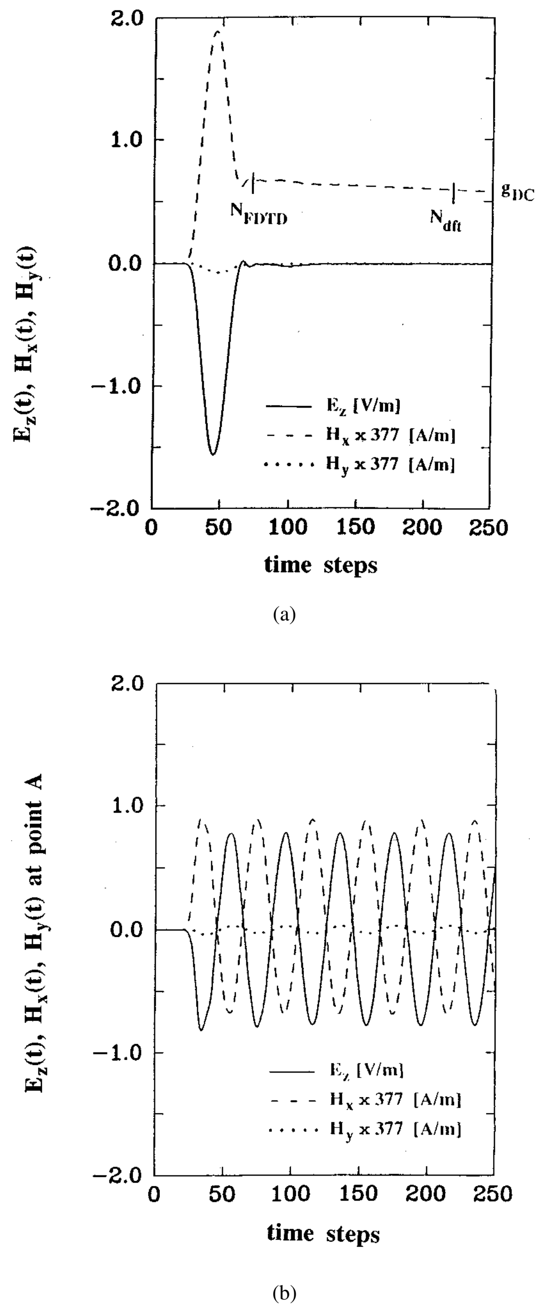

Fig. 1. The fields calculated by FDTD at a point one-fifth of a wavelength in front of a perfectly conducting circular cylinder illuminated from the front by a TM-polarized plane wave with time history of (a) raised cosine pulse and (b) single frequency (CW) unramped sine wave. The coordinate system is oriented such that the x-axis is in a direction tangential to the cylinder's surface at the front, and the y-axis is normal to the front surface. The z-axis is parallel to the axis of the cylinder.

0018–926X/00$10.00 © 2000 IEEE

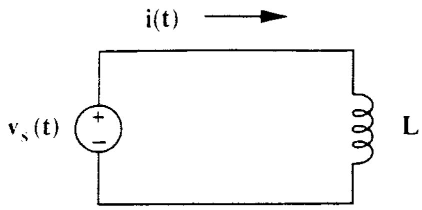

Fig. 2. Circuit model of an ideal voltage source across an inductor used to explain dc-offset phenomena in lossless FDTD simulations.

A dc offset in the tangential magnetic field (H_x) is observed for the cylinder illuminated by a TM-polarized plane wave [in which the incident electric field is oriented in the axial (z) direction]. This offset occurs with both pulsed and CW TM-polarized excitation, as shown in Fig. 1(a) and (b). The offset is not observed in any field component for TE-polarized excitation nor for TM polarized waves incident on a nonconducting cylinder. The reason is that the TM-polarized plane wave induces a permanent axial dc current in the perfectly conducting cylinder, but such an axial current will not be set up with a TE-polarized source, and will not survive in an imperfectly conducting object.

III. Circuit Theory Analog to DC Offsets

To illustrate how dc offsets may be induced, consider a simple circuit composed of an ideal voltage source exciting an inductive load as shown in Fig. 2. The current through the inductor is given by

$$i(t) = \frac{1}{L}\int_0^t v_s(\tau)\,d\tau. \tag{1}$$

For sine excitation $v_s(t) = u(t)\sin(\omega t)$, (1) gives

$$i(t) = \frac{1}{\omega L} - \frac{1}{\omega L}\cos(\omega t). \tag{2}$$

Assuming the initial condition $i(0) = 0$, (2) shows that a dc offset is present in the circuit response. Further analysis shows that the dc offset is not required when loss is present in the system. It is reasonable to expect that similar phenomena may occur in FDTD simulations. The relationship between the circuit in Fig. 2 and the TM-illuminated cylinder is demonstrated in the next section.

IV. Avoiding DC Offsets in the Time Domain

For pulsed excitations, the dc offset may be avoided by using a bipolar pulse with equal positive and negative values. This causes the dc offset that is established by the positive portion of the pulse to be removed by the negative portion of the pulse.

For CW computations, we have also found (in all our test cases) that the offset may be eliminated by multiplying the sinusoidal excitation by an appropriate ramp function. An example is $v_s(t) = r(t)\sin(wt)$, where $r(t)$ is either a linear ramp or a raised cosine (RC) ramp given by

$$r(t) = \begin{cases} 0, & t < 0 \\ 0.5[1 - \cos(\omega t/2\alpha)], & 0 \le t \le \alpha T \\ 1, & t > \alpha T \end{cases} \tag{3}$$

where T is the period of the sine function, and α is the number of sine wave cycles during the ramp duration αT. This excitation has the desirable properties that both the function and its first derivative start at zero and are continuous for all values of α. The choice of ramp function can be evaluated by the simple circuit analogy of an ideal voltage source across an inductor. For the linear ramp this gives

$$i(t) = \frac{1}{\omega L}\left(\frac{\sin(2\pi\alpha)}{2\pi\alpha}\right) - \frac{1}{\omega L}\cos(\omega t) \tag{4a}$$

and for the RC ramp

$$i(t) = \frac{1}{2\omega L}\left(\frac{1 + \cos(2\pi\alpha)}{1 - 4\alpha^2}\right) - \frac{1}{\omega L}\cos(\omega t). \tag{4b}$$

The first terms in (4a) and (4b) give the value of the dc current offset for each excitation. Note that although the excitation function and its first derivative start at zero and are continuous, the dc offset is zero only for particular values of α. Fig. 3 shows these dc values for the linear and RC ramps, normalized to the magnitude of the offset produced by a step function $u(t)$ as a function of the ramp duration parameter α. As expected, the magnitude of the offset decreases as the ramps get longer. The RC ramp produces less offset than the linear ramp for values of α greater than about 1.5. Also, the dc offset is identically zero for certain values of α.

To test the similarity of this circuit model to the results for the TM-illuminated perfectly conducting cylinder, the FDTD simulations for the cylinder were repeated using the linear- and RC-ramped sine excitations. The normalized magnitude of the resulting dc offset in the circumferential magnetic field component are superimposed as dots in Fig. 3. There is excellent agreement (less than 5% difference) between the FDTD data and the results for the inductive circuit of Fig. 2; the change in sign of the offset is also predicted correctly. The results demonstrate a strong similarity between these two models, and show clearly how the dc offset may be controlled by the choice of excitation ramp function in FDTD simulations. A dc offset will not, of course, persist when a resistive loss is added to either model.

V. Removal of DC Offsets in the Frequency Domain

For CW excitation, the dc offset appears in the zero-frequency term only, so it is easily separated from the higher frequency terms in the frequency domain. For pulsed simulations, the FDTD simulation is stopped when the output pulse converges (after N_{FDTD} time steps), and this is often before the discrete Fourier transform (DFT) summation has been completed (N_{dft} time steps). Without a dc offset this does not cause any problems; the fields are assumed to be zero after the simulation is stopped, and the remaining terms in the DFT summation are zero. However, when the pulse has a dc offset [as shown in Fig. 1(a)], stopping the summation before it is

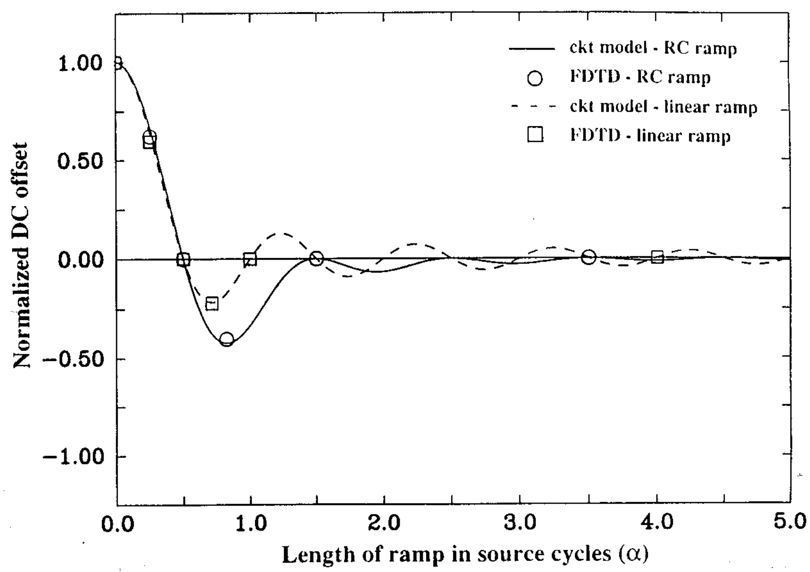

Fig. 3. Variation of normalized magnitude of the dc offset of magnetic field caused by linear and raised-cosine ramp envelopes on a $\sin(\omega t)$ source. Values are plotted as functions of the ramp duration parameter a. Solid and dashed lines are for the inductive circuit model of Fig. 2. Discrete points are FDTD results for the TM-illuminated perfectly conducting cylinder.

completed gives erroneous results. An efficient way to handle complete the summation is to divide the Fourier sum into two summations

$$G(k\Delta f) = \Delta t \left\{ \sum_{n=1}^{N_{\rm FDTD}} g(n\Delta t) \exp\left[\frac{-j2\pi k(n-1)}{N_{\rm dft}}\right] + g_{\rm dc} \sum_{n=N_{\rm FDTD}+1}^{N_{\rm dft}} \exp\left[\frac{-j2\pi k(n-1)}{N_{\rm dft}}\right] \right\}. \quad (5)$$

The first term in (5) is the summation over the time-varying portion of the pulse, $g(n\Delta t)$, up to the time the FDTD simulation is completed, $N_{\rm FDTD}$. The second term is the summation over the dc portion of the pulse $g_{\rm dc}$ up to time the DFT summation has been completed $N_{\rm dft}$.

The second term can be written as a finite geometric series and summed to get

$$G(k\Delta f) = \Delta t \left\{ \sum_{n=1}^{N_{\rm FDTD}} g(n\Delta t) \exp\left[\frac{-j2\pi k(n-1)}{N_{\rm dft}}\right] + g_{\rm dc} \frac{1 - \exp\left[\frac{-j2\pi k N_{\rm FDTD}}{N_{\rm dft}}\right]}{1 - \exp\left[\frac{-j2\pi k}{N_{\rm dft}}\right]} \right\}. \quad (6)$$

The first summation is updated along with the FDTD simulation. The second term, which represents the summation over the dc portion of the pulse, is now a single term and is subtracted after the simulation is completed.

VI. Conclusion

This paper has demonstrated the occurrence of dc offsets in FDTD simulations. These offsets are shown to be due to the physical response of the modeled system to certain sources. These offsets may be avoided in the time domain by tailoring the incident waveform, for example, by using a bipolar pulse for broad-band simulations or a ramped sine wave for CW simulations. DC offsets may also be filtered from frequency-domain data by post processing the FDTD output.

References

[1] A. Taflove, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Boston, MA: Artech House, 1995.

[2] C. M. Furse, S. P. Mathur, and O. P. Gandhi, "Improvements to the finite-difference time-domain method for calculating the radar cross section of a perfectly conducting target," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 919–927, 1990.

Cynthia M. Furse (S'85–M'95–SM'99) was born in Stillwater, ME, in 1963. She received the B.S.degree in electrical engineering, with a mathematics minor, in 1985, the M.S. degree in electrical engineering, in 1988, and the Ph.D. degree in electrical engineering, in 1994, all from University of Utah, Salt Lake City.

She was a Research Assistant, an Associate Professor, and an Assistant Professor at the University of Utah from 1988 to 1997, studying the application of the finite-difference time-domain (FDTD) method to simulation of bioelectromagnetic interactions with the human body, including cellular telephones and high-voltage power lines. She is currently an Assistant Professor at Utah State University, Logan, where her research interests include high-frequency applications in bioelectromagnetics, geophysics, and advanced sensing systems. She teaches electromagnetics, wireless communication, computational electromagnetics, microwave engineering, and antenna design.

Dr. Furse has been a National Science Foundation Computation and Information Science and Engineering (NSF CISE) Graduate Fellow, an IEEE Microwave Theory and Techniques Graduate Fellow, and Professor of the Year 2000 for the Utah State University College of Engineering.

Daniel H. Roper received the B.S. and M.S. degrees in electrical engineering from the University of Utah, Salt Lake City, in 1990 and 1992, respectively.

From 1989 to 1993, he was with the University of Utah working on finite-difference time-domain analysis for many projects in the areas of medical and industrial heating, microwave components, and high-power multimode waveguides. In 1993, he joined Hughes Aircraft Company, El Segundo, CA, and worked on active antennas, microwave integrated circuits, and monolithic microwave integrated circuits (MMICs). Since 1995, he has been a Senior Payload Engineer with Hughes Space and Communications Company. His current work-related interests are in design of spacecraft antennas and payload systems for broad-band satellite applications.

Dale N. Buechler (S'83–M'86) was born in Inglewood, CA, on May 11, 1962. He received the B.S. and M.S. degrees in electrical engineering, in 1984 and 1986, respectively, and the Ph.D. degree in electrical engineering with a specialty in numerical electromagnetics, in 1997, all from the University of Arizona, Tucson.

From 1984 to 1993, he was involved in clinical research in the Radiation Oncology Departments at the University of Arizona, the University of Wisconsin-Madison, and the University of Utah, Salt Lake City. He is currently an Assistant Professor in the Department of Electrical Engineering and Computer Science, University of Wisconsin-Milwaukee, where his research interests include numerical electromagnetics, medical imaging, and medical instrumentation.

Douglas A. Christensen received the B.S.E.E. degree from Brigham Young University, Provo, UT, in 1962, the M.S. degree from Stanford University, Stanford, CA, in 1963, and the Ph.D. degree from the University of Utah, Salt Lake City, in 1967.

From 1972 to 1974, he held a special National Institute of Health (NIH) Postdoctorate position in biomedical engineering at the University of Washington, Seattle. He has been a Faculty Member at the University of Utah since 1971. He currently holds a joint appointment as Professor of bioengineering and Professor of electrical engineering. He is the author of *Ultrasonic Bioinstrumentation* (New York: Wiley, 1988) and coauthored *Basic Introduction to Bioelectromagnetics* (Boca Raton, FL: CRC, 1999). His major research interests are in the area of waves in biological sensing, including optical biosensors, fluorescent waveguiding immunosensors, numerical modeling of optical waveguides and near-field optical effects, and ultrasonic bioinstrumentation.

Carl H. Durney (F'92) was born in Blackfoot, ID, on April 22, 1931. He received the B.S. degree in electrical engineering from Utah State University in 1958, and the M.S. and Ph.D. degrees in electrical engineering from the University of Utah, Salt Lake City, in 1961 and 1964, respectively.

From 1958 to 1959, he was an Associate Research Engineer with the Boeing Airplane Company, Seattle, WA, where he investigated the use of delay lines in control systems. He has been with the University of Utah since 1963. He is currently Professor Emeritus of electrical engineering and Professor Emeritus of bioengineering there. From 1965 to 1966, he worked in the area of microwave avalanche diode oscillators at Bell Laboratories, Holmdel, NJ, while on leave from the University of Utah. He was a Visiting Professor at the Massachusetts Institute of Technology, Cambridge, doing research in Nuclear Magnetic Resonance (NMR) imaging and hyperthermia for cancer therapy during the 1983–1984 academic year, while on a sabbatical leave from the University of Utah. He was on the editorial board of *Magnetic Resonance Imaging* (1983–1995). At the University of Utah, until he retired in 1997, he taught and did research in electromagnetics, engineering pedagogy, electromagnetic biological effects, and medical applications of electromagnetics.

Dr. Durney is or has been a member of the Bioelectromagnetics Society, Commissions B and K of the International Union of Radio Science (URSI), Sigma Tau, Phi Kappa Phi, Sigma Pi Sigma, Eta Kappa Nu, and the American Society for Engineering Education (ASEE). He served as Vice President (1980–1981) and President (1981–1982) of the Bioelectromagnetics Society. He was a member (1979–1988) and Chairman (1983–1984) of the IEEE Committee on Man and Radiation (COMAR) and a member of the American National Standards Institute C95 Subcommittee IV on Radiation Levels and/or Tolerances with Respect to Personnel (1973–88). He was a member of the editorial board of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES (1977–1997). He was a member of the National Council on Radiation Protection and Measurements from 1990 to 1996. He served as a member of the Peer Review Board on Cellular Telephones (Harvard Center for Risk Analysis) from 1994 to 1997. In 1980, he received the Distinguished Research Award and in 1993 he received the Distinguished Teaching Award from the University of Utah. In 1982, he received the ASEE Western Electric Fund Award for Excellence in Teaching and the Utah Section IEEE Technical Achievement Award. Utah State University named him College of Engineering Distinguished Alumnus in 1983. In 1990, the Utah Engineering Council named him Utah Engineering Educator of the Year. In 1993, the Bioelectromagnetics Society awarded him the d'Arsonval Medal.