AN EFFICIENT ALGORITHM FOR LATTICE FILTER/PREDICTOR

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ABSTRACT

An efficient method for updating the lattice filter/predictor coefficients using the sign algorithm is introduced. The pertinent coefficients are updated using only the signs of the estimation errors at each stage. This method requires less number of multiplications than other adaptive lattice filter algorithms. The performance of this method is compared with that of the lattice filter using the LMS algorithm for the problems of spectrum estimation and adaptive interference cancellation.

I. INTRODUCTION

In this paper, we will introduce the lattice filter/predictor with the sign algorithm (SA) [1,2] (SA-LAT). In this approach, the pertinent coefficients are updated using only the signs of the estimation errors at each stage. Thus, the SA-LAT method requires less number of multiplications than that of other adaptive lattice filtering algorithms [3,4] and hardware implementation of the proposed algorithm is easier than that of others. The performance of this efficient technique will be demonstrated for applications of adaptive noise cancellation and spectrum estimation of stationary signals.

II. LATTICE FILTER WITH SIGN ALGORITHM

The M-stage symmetric lattice filter structure is shown in Fig. 1, where $e_j(n)$, $w_j(n)$ and $v_j(n)$ are the forward and backward prediction errors (residuals) and the estimated filter outputs at the j-th stage of the filter for $0 \le j \le M$. Also, ρ_j and g_j are the reflection coefficients and the lattice filter coefficients, respectively. Of course, the structure given in Fig. 1, sans the lattice filter coefficients g_j for $0 \le j \le M$ gives the lattice predictor structure. The relevant equations at each stage are given by

$$v_i(n) = v_{i-1}(n) - g_i w_i(n); \quad 0 \le i \le M, \quad (1)$$

$$e_i(n) = e_{i-1}(n) - \rho_i w_i(n-1); \quad 1 \le i \le M, \quad (2)$$

and

$$w_i(n) = w_{i-1}(n-1) - \rho_i e_{i-1}(n); \quad 1 \le i \le M \quad (3)$$

where $v_{-1}(n) = y(n)$ and $w_o(n) = b_o(n) = x(n)$.

In many filtering applications, when either the input signal statistics are not known, or the input signals are nonstationary, we may adaptively estimate ρ_i's and g_i's at each time instant. The LMS algorithm, which was originally proposed to update the tapped delay line (TDL) filter coefficients [5], has been used to update these coefficients [3,4]. The relevant equations are given by

$$\hat{g}_i(n+1) = \hat{g}_i(n) + \tilde{\mu}_i(n) v_i(n) w_i(n); \quad 0 \le i \le M, \quad (4)$$

$$\hat{\rho}_i(n+1) = \hat{\rho}_i(n) + \mu_i(n)[e_i(n) w_{i-1}(n-1) + e_{i-1}(n) w_i(n)]; \quad 1 \le i \le M \quad (5)$$

where $\hat{g}_i(n)$ and $\hat{\rho}_i(n)$ denote estimates of $g_i(n)$ and $\rho_i(n)$ at time n and $\tilde{\mu}_i(n)$ and $\mu_i(n)$ are time varying convergence parameters given by

$$\tilde{\mu}_i(n) = \alpha / \hat{\tilde{\sigma}}^2_{i-1}(n) \quad (6)$$

and

$$\mu_i(n) = \alpha / \hat{\sigma}^2_{i-1}(n) \quad (7)$$

where

$$\hat{\tilde{\sigma}}^2_{i-1}(n) = \beta \hat{\tilde{\sigma}}^2_{i-1}(n-1) + (1-\beta) w^2_{i-1}(n) \quad (8)$$

and

$$\hat{\sigma}^2_{i-1}(n) = \beta \hat{\sigma}^2_{i-1}(n-1) + (1-\beta)[e^2_{i-1}(n) + w^2_{i-1}(n-1)] \quad (9)$$

is similarly an estimate of the backward prediction error power at the (i-1)th stage. Also, α and β are constants such that $0 < \alpha$ and $0 < \beta < 1$ and β is usually chosen to be $1-\alpha$ [6].

It is well known that the convergence speed of the above adaptive lattice predictor/filter is superior to that of the TDL counterparts. Now, the problem of interest is to update the reflection and lattice filter coefficients with less number of multiplications than required in (4)-(9). Recently, the sign algorithm has been proposed to update the TDL coefficients [1,2]. While the SA requires fewer computations than the LMS algorithm, convergence of the TDL filter coefficients computed using the SA is slower than that of the adaptive TDL filter using the LMS algorithm [1,2]. To overcome this problem, we propose to use the SA to update the lattice filter and reflection coefficients. The pertinent equations are

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$$\hat{g}_i(n+1) = \hat{g}_i(n) + \tilde{\mu}_i(n)\ \mathrm{sign}[v_i(n)]w_i(n); \quad 0 \le i \le M \qquad (10)$$

$$\hat{\rho}_i(n+1) = \hat{\rho}_i(n) + \mu_i(n)[\mathrm{sign}\{e_i(n)\}w_{i-1}(n-1) + \mathrm{sign}\{w_i(n)\}e_{i-1}(n)]; \quad 1 \le i \le M \qquad (11)$$

where $\tilde{\mu}_i(n)$ and $\mu_i(n)$ are now given by

$$\tilde{\mu}_i(n) = \alpha/\tilde{\gamma}_{i-1}(n) \qquad (12)$$

and

$$\mu_i(n) = \alpha/\hat{\gamma}_{i-1}(n) \qquad (13)$$

with

$$\tilde{\gamma}_{i-1}(n) = \beta\tilde{\gamma}_{i-1}(n-1) + (1-\beta)|w_{i-1}(n)| \qquad (14)$$

and

$$\hat{\gamma}_{i-1}(n) = \beta\hat{\gamma}_{i-1}(n-1) + (1-\beta)\{|e_{i-1}(n)| + |W_{I-1}(N-1)|\}. \qquad (15)$$

In (10)-(15)

$$\mathrm{sign}(x) = \begin{cases} 1 & \text{if } x \ge 0 \\ -1 & \text{otherwise} \end{cases} \qquad (16)$$

and $|(\cdot)|$ denotes absolute value of $(\cdot)$.

Note that if α and β are chosen such that multiplications by α and β can be replaced by bit shifting operations, we need only one division for updating each of the coefficients, whereas the LMS-LAT structure requires four multiplications and one division for updating each of the reflection coefficients and two multiplications and one division for updating each lattice filter coefficient. Also, if we do not use time-varying convergence parameters in (10)-(15) the SA-LAT structure needs no multiplication or division at all.

III. AR SPECTRUM ESTIMATION

Suppose that we are given the autocorrelation function $R(m)$ for $|m| \le M$, of a wide sense stationary time series $\{x(n)\}$, the maximum entropy spectrum of $\{x(n)\}$ is given by [7]

$$\hat{S}(f) = P_M \left| 1 + \sum_{m=1}^{M} a_m e^{-j2\pi fm} \right|^{-2} \qquad (17)$$

where the autoregressive (AR) parameters $\{a_m\} = \{a_1, a_2, \ldots, a_M\}$ are obtained as the solution to the matrix equation

$$\begin{bmatrix} R(0) & R(1) & R(2) & \cdots & R(M) \\ R(1) & R(0) & R(1) & \cdots & R(M-1) \\ \vdots & & & & \vdots \\ R(M) & R(M-1) & \cdots & \cdots & R(0) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} P_M \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (18)$$

and

$$P_M = \sum_{m=0}^{M} a_m R(m) \quad \text{for } a_0 = 1 \qquad (19)$$

is the prediction error power of the optimum predictor of order M designed to best approximate the current sample value $x(n)$ as the sum of the weighted past samples. The reflection coefficients are related to the AR parameters through the Levinson-Durbin relations [12] given by

$$a_{kk} = -\rho_k; \qquad k=1,2,\ldots,M \qquad (20)$$

$$a_{ki} = a_{k-1,i} + a_{kk}a_{k-1,k-i}; \quad i=1,2,\ldots,k-1 \qquad (21)$$

and

$$a_i = a_{M,i}; \quad i=1,2,\ldots,M. \qquad (22)$$

Also, P_M is given by the recursive relations

$$P_k = (1 - |\rho_k|^2)P_{k-1}; \quad k=1,2,3,\ldots,M. \qquad (23)$$

where $P_0 = R(0)$. However, in many cases, we may only need to know the relative magnitude of the spectrum and in such cases, we may arbitrarily set P_M as 1 and avoid the computations in (23).

Now, we will compare the spectrum estimates obtained by computing the AR parameters using (20)-(22) after substituting the estimates $\rho_k(n)$ obtained from (11) for the SA-LAT predictor and those obtained from (5) for the LMS-LAT predictor for the reflection coefficients in (20), when the input to the lattice predictor is $\{x(n)\}$. That is, at any time n, we will estimate the spectrum as

$$\hat{S}(f,n) = \left| 1 + \sum_{m=1}^{M} \hat{a}_m(n) e^{-j2\pi fm} \right|^{-2} \qquad (24)$$

where unit prediction error power is assumed for all n.

The simulation example concerns the convergence properties and the steady state characteristics of the LMS-LAT and SA-LAT predictors. The problem here is to estimate the AR parameters of the signals obtained as the output of a filter given by

$$H(z) = 1/\{1-z^{-1} + 0.8z^{-2}\} \qquad (25)$$

when the input is zero mean, white, Gaussian stationary signals with unit variance. The relevant MEM spectrum is estimated using (24) at 1000-th iteration. Thirty independent simulations were run using 1000 samples each and three stages for both the SA-LAT and LMS-LAT predictors. Also, in each case $\alpha = 0.0025$ and $\beta = 0.9975$. The results presented in Fig. 2 are the averages of all the 30 simulations. In Figs. 2(a) and (c), the estimates of a_1 and a_2 for $n = 0,1,2,\cdots,1000$ are plotted for the SA-LAT and LMS-LAT predictors. Also, Figs. 2(b) and (d) compare the ensemble variances (in dB's) of the thirty estimates of a_1 and a_2 for both the methods. From these results, we can observe that in this example, the SA-LAT predictor converges faster than the LMS-LAT predictor and that the variance of the estimated coefficients for both the methods are more or less the same in the steady state. However, in the transient state, the LMS-LAT performs better from an estimation variance point of view. Therefore, we would expect no degradiation in performance for spectrum estimation of stationary signals for large n. However, there may be some degradation in performance in a non-stationary environment.

The thirty spectrum estimates (in dB's) obtained using the AR parameters obtained at the 1000-th iteration are plotted together in

Fig. 3(a) for the LMS-LAT approach and in Fig. 3(b) for the SA-LAT method. We can see that the SA-LAT approach yields more consistent estimates than the LMS-LAT method, even though we need substantially less number of computation than for the LMS-LAT predictor for computing the reflection coefficients.

IV. ADAPTIVE NOISE CANCELLATION

The block diagram for the setup used in the simulation in this section is shown in Fig. 4. The example considered is one of removing multiple sinusoidal interferences when the desired signal is a simulated electrocardiographic (ECG) signal. In this example, the additive noise signal $\tilde{r}(n)$ and the reference input r(n) are given by

$$\tilde{r}(n) = 0.2 \sin(120\pi/750) + 0.2 \sin(260\pi/750 + 10^0) \qquad (26)$$

and

$$r(n) = 0.5 \sin(120\pi/750 + 20^0) + 0.15 \sin(260\pi/750 + 45^0), \qquad (27)$$

respectively. The simulated ECG signal s(n) and the primary input d(n) are displayed in Figs. 5a and b. Now, the problem is to estimate $\tilde{r}(n)$ as a weighted sum of r(n-i), $0<i<M$, so that we can remove the sinusoidal components from d(n).

The estimated ECG signals for $2000<n<3000$ (the system is assumed to have reached steady state in this range) is plotted as Figs. 5c and d when SA-LAT and LMS-LAT filters, respectively were used. The same convergence parameters $\alpha = 0.005$ and $\beta = 0.995$ and 3 lattice filter stages were used for both the methods. Comparing Figs. 5c and d, we can see that the performance of the SA-LAT filter is only slightly worse than that of the LMS-LAT filter. In view of the large computational savings obtained by the SA-LAT approach, this small degradation in performance is certainly acceptable.

V. CONCLUSIONS

The adaptive lattice filter whose coefficients were updated by the SA was introduced in this paper. Its effectiveness for estimating the spectra of stationary signals and adaptive interference cancellation was demonstrated through simulation examples. For the stationary AR model signals used in this paper, the SA-LAT predictor converged faster than the LMS-LAT predictor, even though it required less number of computations. Even though both the methods yield more or less the same mean squared error in the steady state, the LMS-LAT predictor performs better in the transient state from an estimation variance point of view.

The performance of the SA-LAT filter for interference noise cancellation was only slightly worse than that of the LMS-LAT filter, in spite of the fairly large reduction in computational load. The faster convergence of the SA-LAT predictor in the first example is contrary to the results reported in [1,2] where SA and LMS algorithm are used to update the TDL filter coefficients. Thus, further study is required to understand the properties of the SA-LAT structure, and it is being pursued at this time.

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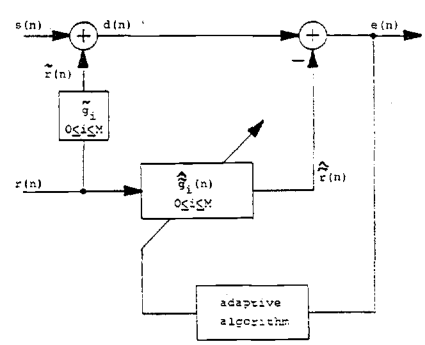

Figure 4. Block diagram for simulations in Section IV

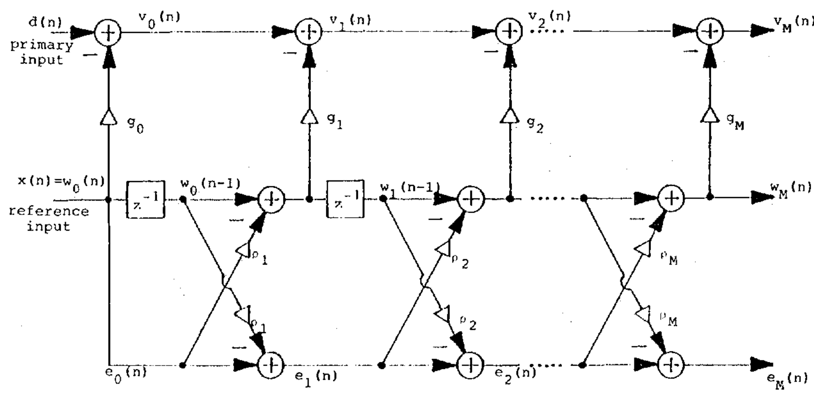

Figure 1. M-stage symmetric lattice filter structure

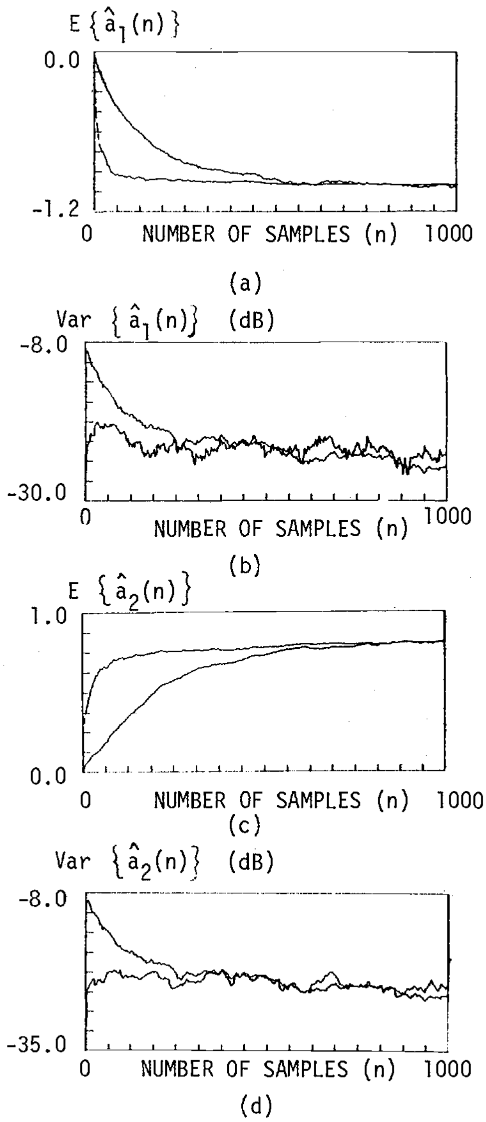

Figure 2. Ensemble averages of the estimated AR parameters and estimation variances in dB

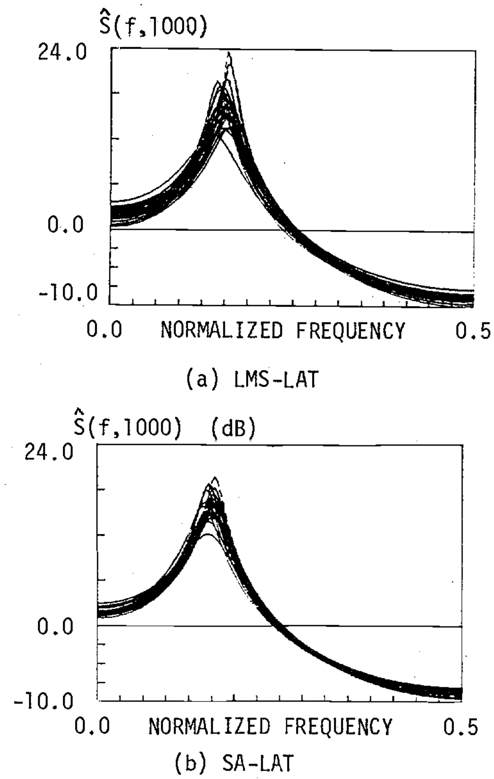

Figure 3. Thirty AR spectrum estimates using the LMS-LAT and SA-LAT predictors

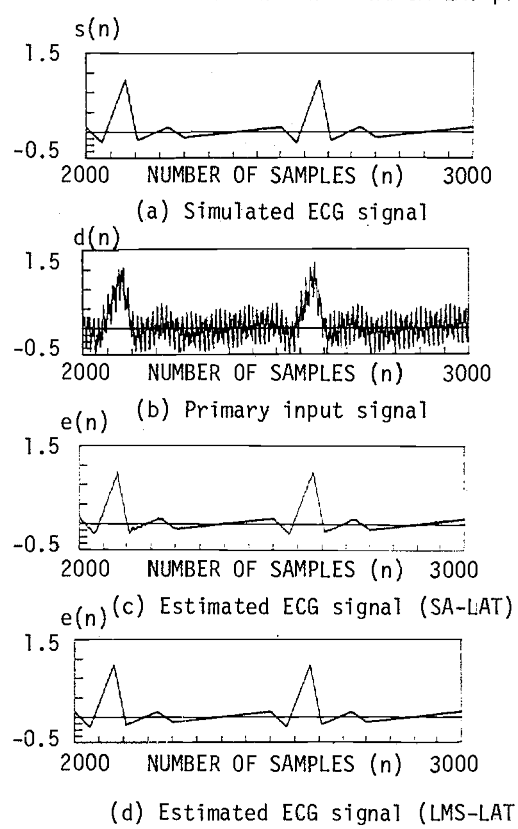

Figure 5. Results of simulation in Section IV