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A NEW CARRIER FREQUENCY ESTIMATOR FOR MODEM SIGNALS

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ABSTRACT

A novel carrier frequency estimation scheme for a relatively broad class of voiceband data signals (modem signals) is presented in this paper. The class of signals being studied includes different types of phase-shift-keyed (PSK) and frequency-shift-keyed (FSK) signals. The frequency estimates are obtained by averaging the estimates of the derivative of the instantaneous signal phase after discarding the estimates at the baud boundaries. Experimental results illustrate that the frequency estimates obtained from our scheme are unbiased and have small variances. Results also show that the carrier frequency estimation scheme presented here is superior to two other techniques in terms of both the mean and variance of the carrier frequency estimates.

I. INTRODUCTION

In synchronous detection, knowledge of the carrier frequency is needed. Usually the carrier frequency is not known a priori and must be estimated. The conventional approach to the problem of carrier frequency estimation for double-side-band suppressed carrier signals is to use phase-locked loop methods [1]. However, in systems where automatic tuning is required, the phase-locked loop methods cannot be applied. Another example where such system will not work is in voiceband data signal compression algorithms that employ quantization of the baseband signals [2, 3]. In such systems, the input passband signals are first converted to their baseband equivalent before quantization is performed. In general, the types of modulation or carrier frequency are not known to the data compression system. A frequency estimation scheme that does not require any a priori information about the input signals is vital for such systems.

Many researchers have studied the problem of estimating the frequency of a sinusoid embedded in noise. Given a time-limited signal of the form

$$x(t) = A\cos(wt + \varnothing) + n(t); \qquad t1 \le t \le t2, \tag{1}$$

the problem is to estimate one or more of the unknown parameters A, w and ø. Here A, w and ø are the amplitude, frequency and phase, respectively, of the sinusoid and they are, in general, unknown. The maximum-likelihood method for estimating w from discrete-time samples of a single tone has been examined by Rife and Boorstyn [4]. Tretter [5] has studied the estimation of the frequency of a noisy sinusoid using linear regression. The frequency estimates obtained by this method has been shown to attain the Cramer-Rao lower bound. Another very simple approach is to count the number of zero crossings of the signal in a specified interval and estimate the sinusoidal frequency as half the number of zero crossings per second. Tufts and Kumaresan [6], Palmer [7], and Slepian [8] have also studied the problem of estimating the frequency of a sinusoidal signal embedded in noise. The above is a small sample of the literature on frequency estimation. For an extensive list of references, see Tufts and Kumaresan [6].

Because of the time varying and possibly discontinuous nature of the phase, the above approaches cannot be used without modification to accurately estimate the carrier frequency of a modem signal. In the approach proposed here, the carrier frequency is found as the average of the derivative of the instantaneous phase of the signal. Due to the phase jumps, the estimates of the derivative of the phase at the baud boundaries are not necessarily related to the carrier frequency. Before the actual frequency estimate is computed, these aberrant estimates of the phase derivatives are removed by examining the first difference of the estimates of the derivative of the phase. The estimation is done on contiguous nonoverlapping segments of the signal. The rest of the paper is organized as follows. In Section II, the new frequency estimation algorithm is discussed in more detail. Experimental results and concluding remarks are presented in Section III.

II. A NEW CARRIER FREQUENCY ESTIMATOR

A block diagram of the proposed carrier frequency estimation system is shown in Fig. 1. To see how the system works, consider a time-limited modem signal which is represented by

$$s(t) = m(t)\cos[2\pi f_c t + p(t)] + n(t); \qquad t1 \le t \le t2 \tag{2}$$

where m(t), f_c, p(t) and n(t) denote the pulse shape, the carrier frequency, the information-bearing phase signal and an additive noise component, respectively. Note that (2) includes the class of phase-shift-keyed (PSK) and frequency-shift-keyed (FSK) signals.

In the case of low noise levels, the additive noise component n(t) can be modeled as an extra phase component $p_n(t)$ [5], as

$$s(t) \approx m(t)\cos[2\pi f_c t + p(t) + p_n(t)]; \qquad t1 \le t \le t2. \tag{3}$$

Taking the the Hilbert transform of this signal, we have

$$s_h(t) = m(t)\sin[2\pi f_c t + p(t) + p_n(t)]; \qquad t1 \le t \le t2. \tag{4}$$

Define

$$XR(t) = \begin{cases} s(t+t1); & 0 \le t < t2-t1 \\ XR[2(t2-t1)-t]; & t2-t1 \le t \le 2(t2-t1), \end{cases} \tag{5a}$$

$$XI(t) = \begin{cases} -s_h(t+t1); & 0 \le t < t2-t1 \\ -XI[2(t2-t1)-t]; & t2-t1 \le t \le 2(t2-t1), \end{cases} \tag{5b}$$

and

$$X(t) = XR(t) + jXI(t) \tag{5c}$$

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What we are trying to do here is to create a complex signal X(t) whose real part XR(t) is even, and whose imaginary part XI(t) is odd (i.e., X(t) is the Fourier transform of some real signal x(w)).

Let F{x(w)} and F^{-1}{X(t)} denote the Fourier transform of x(w) and the inverse Fourier transform of X(t), respectively. They are defined as

$$X(t) = \int_{-\infty}^{\infty} x(w) e^{-jwt} \, dw \tag{6a}$$

$$x(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(t) e^{jwt} \, dt. \tag{6b}$$

The inverse Fourier transform, x(w), of (5c) is a real signal and the phase of its Fourier transform is given by

$$p_x(t) = \tan^{-1} \frac{XI(t)}{XR(t)}$$

$$= \begin{cases} -[2\pi f_c t + p(t) + p_n(t)]; & t1 \le t < t2 \\ 2\pi f_c t + p(t) + p_n(t); & t2 \le t \le 2t2\text{-}t1 \end{cases} \tag{7}$$

The derivative of $p_x(t)$ can be obtained from X(t)=F{x(w)} and X'(t)=F{-jwx(w)} as [9]

$$p_x'(t) = \frac{XR(t)XI'(t) - XI(t)XR'(t)}{XR^2(t) + XI^2(t)}$$

$$= \begin{cases} -[2\pi f_c + p'(t) + p_n'(t)]; & t1 \le t < t2 \\ 2\pi f_c + p'(t) + p_n'(t); & t2 \le t \le 2t2\text{-}t1 \end{cases} \tag{8}$$

where XR'(t) and XI'(t) are the real and imaginary parts of X'(t), respectively.

Equation (8) gives us the carrier frequency except for the terms p'(t) and $p_n'(t)$. Assuming that p(t) and $p_n(t)$ have zero mean, which implies that p'(t) and $p_n'(t)$ also have zero mean, an unbiased estimate of the carrier frequency can be obtained by averaging $p_x'(t)$ over t and normalizing with 2π. However, the estimates of the phase derivatives at the baud boundaries may be erroneous because of the possible discontinuities of the signal phase at these samples. One way to deal with the aberrant estimates of the phase derivatives is to estimate the location of the baud boundaries and then discard the estimates of the phase derivatives at these points. This requires synchronization of the symbols as well as information about the baud rate.A simple modification to the above scheme will eliminate the need for symbol synchronization at the cost of a very small degradation in the performance. In this method, the phase derivative estimates are first lowpass filtered, then the aberrant estimates of the phase derivative estimates at the baud boundaries are removed by examining the first difference of the lowpass filtered derivative estimates and the carrier frequency estimate is computed as the average of the remaining phase derivative values.

The motivation for this scheme is seen from the case in which the input signal is a sine wave. In the case that the signal is a pure sine wave the instantaneous phase is a straight line (i.e., the derivative is just a constant). If this sine wave is embedded in noise, the frequency estimate obtained as the slope of the straight line that best fits the phase has a variance that approaches the Cramer-Rao lower bound [5].

III. EXPERIMENTAL RESULTS

Several experiments were conducted to study the performance of the carrier frequency estimation algorithm proposed here. The set of signals used in the experiments are tabulated in Table 1. The results presented in Table 2 are the average behavior of the method over 80 independent estimates, each of which was made using 1024 data samples. Results of the "optimal" method as well as the modified approach are presented in this table. The normalized variance of the frequency estimate f_e shown in Table 2 is defined as

$$NVAR(f_e) = Var(f_e)/E^2(f_e), \tag{9}$$

where $Var(f_e)$ is the estimation variance and $E(f_e)$ is the expected value of the frequency estimate.

Table 1. Test signal set.

Signal type	Baud rate	Bit error probability		
cfsk (continuous phase FSK)	1200 baud	0	10^{-2}	10^{-4}
cbpsk (conventional binary PSK)	1200 baud	0	10^{-2}	10^{-4}
dbpsk (differentially encoded binary PSK)	1200 baud	0	10^{-2}	10^{-4}
dqpsk (differentially encoded quadrature PSK)	1200 baud	0	10^{-2}	10^{-4}
dopsk (differentially encoded octal PSK)	1600 baud	0	10^{-2}	10^{-4}

Note that the difference between the two methods is only in the way the aberrant estimates of the phase derivative are removed. In the first method, only the derivative values at the baud boundaries can be removed if phase jumps are detected. In

Table 2. Comparison of the "optimal" and modified schemes

Signal type	Exact value (Hz.)	Average (Hz.) "Optimal" method	Average (Hz.) Modified method	Normalized Variance $\times 10^{-4}$ "Optimal" method	Normalized Variance $\times 10^{-4}$ Modified method
(Noise free)					
cfsk	1700	1704	1705	3.20	3.43
cbpsk	1650	1670	1670	0.75	0.77
dbpsk	1650	1651	1651	0.19	0.20
dqpsk	1800	1808	1809	0.89	0.88
dopsk	1800	1807	1806	2.49	2.53
(Noisy : bit error probability = 10^{-4})					
cfsk	1700	1701	1702	4.60	4.65
cbpsk	1650	1693	1695	4.47	4.49
dbpsk	1650	1671	1674	2.48	2.49
dqpsk	1800	1803	1805	1.23	1.25
dopsk	1800	1806	1805	2.51	2.53
(Noisy : bit error probability = 10^{-2})					
cfsk	1700	1710	1712	5.21	5.27
cbpsk	1650	1698	1700	8.87	8.92
dbpsk	1650	1682	1684	7.89	7.93
dqpsk	1800	1811	1810	1.83	1.91
dopsk	1800	1803	1804	3.00	3.05

the latter method, besides those derivative estimates at the baud boundaries, the phase derivative estimates inside the bauds might be removed if the absolute value of the first difference of the lowpass filtered phase derivative estimates exceeds some threshold.

Intuitively, the first method has the advantage that the phase derivative values that are not used in estimating the carrier frequency are only those at the baud boundaries if phase jumps are detected. As a result, removal of the phase derivatives within a baud cannot happen, and the frequency estimates are obtained from as many samples as possible. However, this method requires additional information about the baud boundaries which involves baud rate estimation and symbol synchronization; implying that this scheme is much more complex than the latter method. An advantage that the modified scheme has is that it is more robust to errors in synchronization since it does not require information about the locations of the baud boundaries.

It is clear from the results in Table 2 that the modified method performs almost as good as the "optimal" method. The results presented in the rest of the paper is based on the modified algorithm.

Several further remarks can be made concerning the results of Table 2. (1) For the noise free signals, the estimated carrier frequency of the cbpsk signal is biased by about 1.21%. This bias is possibly due to the fact that this particular test signal is not band limited; and as a consequence, the Hilbert transformation in the frequency estimation system cannot be done exactly. (2) Even though the dqpsk and dopsk signals are more complex than other tested signals, results obtained for these signals are better than those of other signals in the noisy cases in terms of the bias. This is because these signals have higher signals-to-noise ratios (SNR) than other signals for the same probability of bit error. (3) One of the assumptions made in developing the carrier frequency estimation scheme is that the additive noise component is small enough so that it can be modeled by an additive phase component. This is not the case for the simple signals (cfsk,cbpsk,dbpsk,dqpsk) when the bit-error probability is 10^{-4} or 10^{-2}. This may be a reason why the frequency estimates for these signals are biased in the noisy cases. However, the bias is only about 3.0% for the worst case that corresponds to the cbpsk signal with the SNR ≈ -3.9 dB. (4) Two implicit assumptions were made while developing the above algorithm: (a) the phase sequence has zero mean and (b) the noise level is small enough so that the additive noise can be modeled as an extra additive phase component. If the first assumption is violated (i.e., if the bit streams are such that the resulting phase sequence has a nonzero mean), the frequency estimate will be biased. Experimental results presented in Table 2 were done using noisy signals having a much higher noise content than what can be expected in real life situations, and yet the results are still very good. As a result, it is reasonable to believe that the frequency estimation scheme proposed here will work very well in practical applications.

To further study the performance of the carrier frequency estimator of this paper, we compared it with two other competing techniques. The first method estimated the carrier frequency as half the number of zero crossings of the modem signals per second. In the second approach, the instantaneous phase of the modem signals as given by (7) is first unwrapped, and a straight line is fitted through the unwrapped phase using linear regression. The carrier frequency is then estimated as the slope of the line. The results of the experiments are shown in Tables 3,4 and 5.

Several remarks can be made about the results in Tables 3, 4 and 5. (1) In general, the new carrier frequency estimation scheme is superior to the other two methods in terms of both the mean and the variance. (2) Linear regression fails to give accurate frequency estimates for all signals except for the cfsk

Table 3. Comparison of three schemes for carrier frequency estimation (block size = 128).

Signal type	Exact value	Average: New method	Average: Zero crossing	Average: Linear reg.	Normalized Variance $\times 10^{-4}$: New method	Normalized Variance $\times 10^{-4}$: Zero crossing	Normalized Variance $\times 10^{-4}$: Linear reg.
(Noise free)							
cfsk	1700	1705	1690	1704	28.64	28.36	31.41
cbpsk	1650	1671	1664	1617	6.56	29.03	33.38
dbpsk	1650	1651	1660	1346	1.30	25.77	29.23
dqpsk	1800	1809	1817	1673	6.74	26.60	23.77
dopsk	1800	1806	1790	1706	19.69	29.86	32.83
(Noisy : bit error probability = 10^{-4})							
cfsk	1700	1701	1695	1699	34.87	30.02	31.24
cbpsk	1650	1695	1745	1508	38.53	36.31	57.37
dbpsk	1650	1674	1708	1487	25.38	39.93	57.40
dqpsk	1800	1805	1816	1687	10.64	24.44	25.45
dopsk	1800	1805	1798	1705	20.25	27.07	35.93
(Noisy : bit error probability = 10^{-2})							
cfsk	1700	1710	1732	1649	40.23	33.77	37.96
cbpsk	1650	1791	1841	1430	84.82	50.15	116.74
dbpsk	1650	1733	1770	1455	54.71	52.44	73.72
dqpsk	1800	1806	1829	1654	19.40	28.28	30.10
dopsk	1800	1804	1800	1694	21.76	27.01	37.63

signals. This is to be expected since this method was formulated for the frequency estimation of sinusoids embedded in noise. The big difference between modem signals and sinusoids is that modem signals can have discontinuous phase. In (4.11), p(t) is assumed to have zero mean. One could argue that linear regression would give unbiased estimates. This is true if the data length is long enough and if the phase is unwrapped successfully. The phase jumps of the modem signals due to the information bearing signal p(t) are very large (especially for binary PSK signals) causing the phase unwrapping algorithm to fail on these signals except for the cfsk signals where there is no phase jump. As a consequence, carrier frequency estimation using linear regression works for cfsk signals but not for PSK signals. (3) In the zero crossing method, since the number of zero crossing is counted for a given block of data it tends to be sensitive to the level of noise in the signal. Under the assumption that the noise and the phase signal p(t) have zero mean, this method should produce unbiased estimates if the noise level is not too large. The variance of the frequency estimates obtained from this method is found to be higher than the new frequency estimation scheme in most cases (note that a smaller value of the normalized variance does not always imply a smaller value of the variance since the normalized variance is defined as the ratio of the variance of the estimates over the average of the estimates; thus both the average and the normalized variance need to be examined at the same time when comparing different entries of Tables 3-5.). Nevertheless, this method still yields satisfactory results. (4) Under the assumption that the components p'(t) and $p_n'(t)$ are independent random processes and the two processes are independent from each other, it can be shown [9] that the variance of the frequency estimate obtained from the sample mean is inversely proportional to 1/N, where N is the length of the data block for which each estimate is obtained. This implies that the new frequency estimation method presented in this section is a consistent estimator. (5) It should be noted again that the SNR values of the noise corrupted signals (cfsk,cbpsk,dbpsk) are very low, and the results obtained are still acceptable. It is reasonable to believe that the new carrier frequency estimation scheme proposed here will work well with real world signals for which the SNR values are high.

Table 4. Comparison of three schemes for carrier frequency estimation (block size = 512).

Signal type	Exact value	Average: New method	Average: Zero crossing	Average: Linear reg.	Normalized Variance x10^{-4}: New method	Normalized Variance x10^{-4}: Zero crossing	Normalized Variance x10^{-4}: Linear reg.
(Noise free)							
cfsk	1700	1705	1697	1617	7.68	7.59	9.61
cbpsk	1650	1671	1671	1671	1.55	5.60	7.86
dbpsk	1650	1651	1673	1349	0.32	3.43	8.56
dqpsk	1800	1809	1829	1664	1.64	5.69	5.19
dopsk	1800	1806	1807	1709	4.40	7.60	7.86
(Noisy : bit error probability = 10^{-4})							
cfsk	1700	1701	1701	1697	9.86	7.49	9.86
cbpsk	1650	1695	1739	1505	8.73	6.86	12.40
dbpsk	1650	1674	1729	1491	7.42	9.93	11.90
dqpsk	1800	1805	1820	1682	2.69	5.67	6.07
dopsk	1800	1805	1811	1709	4.43	6.71	8.82
(Noisy : bit error probability = 10^{-2})							
cfsk	1700	1710	1735	1654	11.32	8.79	11.44
cbpsk	1650	1791	1840	1420	20.41	12.94	23.66
dbpsk	1650	1733	1789	1458	14.56	13.33	16.62
dqpsk	1800	1806	1833	1649	5.41	6.53	9.08
dopsk	1800	1804	1814	1698	4.67	6.95	9.18

Table 5. Comparison of three schemes for carrier frequency estimation (block size = 2048).

Signal type	Exact value	Average: New method	Average: Zero crossing	Average: Linear reg.	Normalized Variance x10^{-4}: New method	Normalized Variance x10^{-4}: Zero crossing	Normalized Variance x10^{-4}: Linear reg.
(Noise free)							
cfsk	1700	1705	1698	1701	1.28	1.28	1.39
cbpsk	1650	1671	1674	1615	0.48	1.02	2.07
dbpsk	1650	1651	1677	1351	0.09	1.06	1.74
dqpsk	1800	1809	1831	1693	0.51	1.68	1.65
dopsk	1800	1806	1810	1703	1.30	1.83	1.68
(Noisy : bit error probability = 10^{-4})							
cfsk	1700	1701	1702	1701	1.67	1.20	1.74
cbpsk	1650	1695	1741	1509	1.79	1.46	3.44
dbpsk	1650	1674	1731	1493	1.17	2.68	3.02
dqpsk	1800	1805	1822	1689	6.48	1.58	2.11
dopsk	1800	1805	1813	1703	1.46	2.10	1.49
(Noisy : bit error probability = 10^{-2})							
cfsk	1700	1710	1737	1511	1.75	1.53	2.17
cbpsk	1650	1791	1842	1421	3.69	3.17	5.99
dbpsk	1650	1733	1792	1452	2.44	3.13	4.32
dqpsk	1800	1806	1835	1662	1.43	1.30	2.17
dopsk	1800	1804	1816	1700	1.65	1.81	1.87

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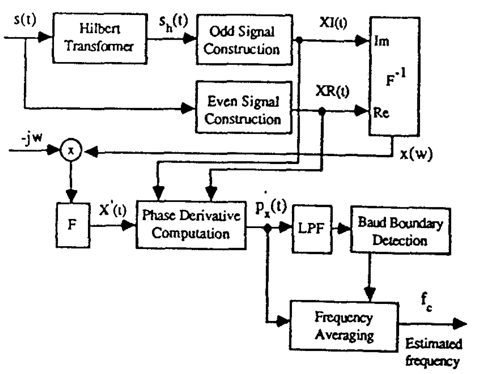

Figure 1. Block diagram of the carrier frequency estimation system