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Quark-bag model with low-energy pion interactions. I. Theory

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A standard method for restoring chiral symmetry in the bag model is to introduce an explicit external pion field. Questions of the consistency and compatibility of this method with the assumptions of the static-cavity approximation of the bag model are discussed. An approximate version of the model is justified. It is argued that consistency requires treating pion-induced quark-pair creation and annihilation at the bag surface as a zeroth-order process in pion-coupling perturbation theory. An approximation treating these pairs as an inward extension of the pion field is discussed. The resulting model gives an improved value of g_A.

I. INTRODUCTION

When quarks are confined as in the static bag model,[1,2] it is inevitable that chiral symmetry is broken. The symmetry is restored if interactions with pions are taken into account.[3-8] In the MIT bag model[1] the confined quark field $q(x)$ satisfies the boundary conditions

$$in_\mu \gamma^\mu q(x) = q(x) , \qquad (1.1a)$$

$$\tfrac{1}{2} n \cdot \partial(\bar{q}q) = B \qquad (1.1b)$$

on the surface S of the bag. Here n_μ is the unit inward normal to the surface. The condition (1.1a) is equivalent to requiring that the quark have an infinite mass outside the bag.[1] Both of these conditions break chiral symmetry. Thus the axial-vector current based on the confined quarks,

$$\vec{A}_{\mu Q}(x) = \tfrac{1}{2} {:}\bar{q}(x)\gamma_5\gamma_\mu \vec{\tau} q(x){:}\theta_V(x) , \qquad (1.2)$$

where $\theta_V(x)$ vanishes outside the confinement volume and is one inside, is not conserved for zero-mass quarks, but satisfies

$$\partial^\mu \vec{A}_{\mu Q}(x) = \frac{i}{2} : \bar{q}(x)\gamma_5 \vec{\tau} q(x) : \delta_S(x) , \qquad (1.3)$$

where $n_\mu \delta_S(x) = \partial_\mu \theta_V(x)$. The by-now standard method for restoring chiral symmetry in the bag model involves introducing an auxiliary massless pion field.[4-8] The essence of this approach is that if a pion contribution to the total axial-vector current is included,

$$\vec{A}_\mu(x) = \vec{A}_{\mu Q}(x) - f \partial_\mu \vec{\pi}(x) , \qquad (1.4)$$

the total current is conserved, if

$$0 = \partial^\mu \vec{A}_\mu(x) = \partial^\mu \vec{A}_{\mu Q}(x) - f \partial^2 \vec{\pi}(x) . \qquad (1.5)$$

Here and throughout f is the pion decay constant ($f \approx 93$ MeV). In effect the divergence of the quark contribution becomes a source for the massless pion field at the bag surface.

There is, of course, already a pion in the bag model, appearing as a massless quark-antiquark state.[9] It is believed that pion-bag interactions should emerge naturally in the original unadorned bag model when surface fluctuations (bag fission, etc.) are taken into account. However, despite some progress in treating bag-bag interactions we are still far from a dynamical theory of relativistic interactions.[10,11] Thus it is attractive to explore approximations starting from the static cavity model. In the case of low-energy pion interactions PCAC (partial conservation of axial-vector current) comes to our aid as long as we are willing to introduce a pion field. We shall assume, however, that the pion field appears only as an approximation to a bag pion, that a theory with an explicit pion field is inherently redundant, and that we shall consequently need to take particular care to avoid double counting. For the sake of definiteness and consistency chiral symmetry should be built into a modified bag Lagrangian, versions of which are Chodos and Thorn's linear σ model[4] and Jaffe's nonlinear σ model "hybrid chiral bag."[8] Although these Lagrangians have been discussed as classical field theories we shall treat them as quantum field theories (in the static cavity approximation).

Versions of chirally symmetric bag models have been used in perturbation theory to study low-energy pion-nucleon scattering[12] and, more specifically, the renormalization of various hadron masses[5,6,8] and a calculation of the pion-nucleon-Δ coupling constants.[4,5] It may seem that a phenomenology of low-energy bag-pion interactions is well in hand. However, basic questions of procedure have not yet received adequate discussion: Where is the pion field? Should it exist only outside the bag? How does one formulate perturbation theory in the pion-bag coupling constant? In addition there are questions of consistency: How does one reconcile the existence of two pions in the theory? How does the pion affect the size of the bag through a chirally symmetric version of the nonlinear boundary condition (1.1b)?

We take up these questions in Secs. III and IV after reviewing the basic chirally symmetric bag model in Sec. II. We regard the external pion field as an approximation to the amplitude for finding the c.m. of a composite pion at a given spacetime point. This interpretation is consistent with the Donoghue-Johnson[9] treatment of the c.m. motion of the pion bag, wherein the static cavity boundary serves as an artificial barrier to the free motion of a composite pion. The chirally symmetric eigenstate must not be so constrained—hence the necessity in a chirally symmetric theory of allowing the pion to appear at positions other than within the confines of the static cavity. This interpretation also requires that the pion be able to move freely in some sense across the bag boundary. It is preferable to represent the pion in terms of quarks for distances comparable to the bag size, whereas an elementary-field description should be adequate at larger distances. Thus a nucleon serves as a source for a pion field, but that field must in turn give rise freely to quark-antiquark pairs inside the original nucleon. We propose an approximate treatment of the effect of these additional pairs by extending the definition of the pion field into the bag interior. Chodos and Thorn also suggested this approach, but for different reasons.[4] This approximation forms the basis for a practical perturbative model of bag-pion coupling. Miller, Thomas, and Thêberge have studied an identical model, but they do not insist upon an interpretation of the internal pion field as a reflection of excited quark-antiquark pairs, nor do they require a rigorous treatment of the nonlinear boundary condition.[12] The model is applied in a determination of the masses, couplings, and other parameters for the low-lying nonstrange hadrons in the following paper (hereafter referred to as II).[13] Although many of these parameters have already been evaluated in piecemeal fashion in hybrid chiral models,[5,6,12,14] we attempt a complete treatment, including corrections for the c.m. motion.

II. REVIEW OF THE BAG MODEL WITH CHIRAL SYMMETRY

It is convenient to start with the nonlinear model of Jaffe[8] although other nonlinear models, or the original linear σ model of Chodos and Thorn,[4] would serve as well. Jaffe's results are summarized in Eqs. (2.1)–(2.8) below. Since the pion field is to be regarded as an approximation for a composite quark-antiquark state, the field is introduced only outside the bag. We use the explicit quark-antiquark form inside.[5,7,8] The action is (omitting gluons)

$$A=\int_V d^4x\left(\frac{i}{2}\bar{q}\gamma\cdot\overleftrightarrow{\partial}q-B\right)+\int_{\bar{V}}d^4x\,\tfrac{1}{2}(D_\mu\vec{\pi})^2 -\tfrac{1}{2}\int_S d^3x\,\bar{q}e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q\,, \tag{2.1}$$

where the bag surface S separates the inside (V) and outside ($\bar{V}$) of the bag, and where

$$D_{\mu ij}\equiv\left[\frac{\sin x}{x}\delta_{ij}+\left(1-\frac{\sin x}{x}\right)\hat{\pi}_i\hat{\pi}_j\right]\partial_\mu\,, \tag{2.2}$$

$x=|\vec{\pi}|/f$, and $\hat{\pi}=\vec{\pi}/|\vec{\pi}|$. The action is invariant under the usual nonlinear chiral transformations

$$\delta q=\frac{i}{2}\vec{\theta}\cdot\vec{\tau}\gamma_5 q\,,\qquad \delta\vec{\pi}=-f[\vec{\theta}-(1-x\cot x)\hat{\pi}\times(\vec{\theta}\times\hat{\pi})]. \tag{2.3}$$

The equations of motion are

$$\partial^2\pi_i-\partial^\mu\left[\left(1-\frac{\sin 2x}{2x}\right)(\delta_{ij}-\hat{\pi}_i\hat{\pi}_j)\partial_\mu\pi_j\right]=0\quad\text{in }\bar{V}\,, \tag{2.4a}$$

$$i\partial_\mu\gamma^\mu q=0\quad\text{in }V\,, \tag{2.4b}$$

$$in_\mu\gamma^\mu q=(e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f})q\quad\text{on }S\,, \tag{2.4c}$$

$$n\cdot\partial\pi_i=-\left[\delta_{ij}-\left(1-\frac{2x}{\sin 2x}\right)(\delta_{ij}-\hat{\pi}_i\hat{\pi}_j)\right]\times\frac{i}{2f}\bar{q}\tau_j\gamma_5 e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q\quad\text{on }S\,. \tag{2.4d}$$

It is useful to note that the boundary condition (2.4c) at S above implies that

$$\bar{q}e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q=0\quad\text{on }S\,. \tag{2.5}$$

The nonlinear boundary condition follows from requiring that the action be stationary with respect to variations in S. Direct variation of S gives

$$\tfrac{1}{2}(n\cdot\partial)(\bar{q}e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q)=B-\tfrac{1}{2}(D_\mu\vec{\pi})^2\,. \tag{2.6}$$

The conserved axial-vector current is

$$\vec{A}_\mu(x)=\vec{A}_{\mu Q}(x)-f\left[\frac{\sin 2x}{2x}\partial_\mu\vec{\pi}+\left(1-\frac{\sin 2x}{2x}\right)\hat{\pi}\hat{\pi}\cdot\partial_\mu\vec{\pi}\right]\theta_{\bar{V}}\,, \tag{2.7}$$

where

$$\vec{A}_{\mu Q}(x)=\tfrac{1}{2}\bar{q}\gamma_\mu\gamma_5\vec{\tau}q\theta_V\,. \tag{2.8}$$

The equations of motion and boundary condtions (2.4) assure that $\vec{A}_\mu(x)$ is conserved.

We wish to consider a perturbation theory based on a formal expansion in inverse powers of the constant f. To second order the action (2.1) becomes

$$A = A_0 + A_1/f + A_2/f^2 ,$$

where

$$A_0 = \int_V d^4x\left(\frac{i}{2}\bar{q}\gamma\cdot\overleftrightarrow{\partial}q - B\right) - \tfrac{1}{2}\int_{\bar{V}} d^4x(\partial_\mu\vec{\pi})^2 - \tfrac{1}{2}\int_S d^3x\,\bar{q}q ,$$

$$A_1/f = -\tfrac{1}{2}\int_S d^3x\,\bar{q}e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q , \quad (2.9)$$

$$A_2/f^2 = -\tfrac{1}{2}\int_{\bar{V}} d^4x[(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2/3f^2 - x^2/3(\partial_\mu\vec{\pi})^2] + \tfrac{1}{4}\int_S d^3x\,\bar{q}qx^2 .$$

The free fields for which A_0 is stationary satisfy the usual conditions

$$i\gamma\cdot\partial q = 0 \text{ in } V, \quad i\gamma\cdot nq = q \text{ on } S , \qquad \partial^2\pi = 0 \text{ in } V, \quad n\cdot\partial\pi = 0 \text{ on } S , \quad (2.10)$$

and may be used as a basis for a quantum perturbation theory. In the usual static-cavity approximation, the equations (2.10) are solved for the static-cavity eigenmodes, and the fields are quantized accordingly:

$$q = \sum_{n=0}^{\infty}[q_n(x)e^{-i\omega_n t}b_n + q_n^c(x)e^{i\omega_n t}d_n^\dagger] , \qquad \vec{\pi} = \int k^2dk \sum_{lm}\frac{\pi_{lmk}(x)}{[2E_l(k)]^{1/2}}(e^{-iE_l(k)t}\vec{a}_{klm} + \text{H.c.}) . \quad (2.11)$$

The pion field couples to the quarks only at the surface of the bag. It gives rise to excitations of the quarks as well as producing quark-antiquark pairs. The second-order term in the action is peculiar to the particular formulation of a chirally symmetric action. Other nonlinear models give a different second-order term. However, the first-order terms are common to all models. Thus more model-independent results must be of first order in perturbation theory or of second order in cases in which A_2/f^2 has no effect.

It is convenient in discussing the spectrum of perturbed states in the static-cavity approximation to separate the Hamiltonian in first order as follows:

$$H = H_0 + H_I ,$$

where

$$H_0 = \sum_{n=0}^{\infty}\omega_n(b_n^\dagger b_n + d_n^\dagger d_n) + \sum_{m=0}^{\infty}E_m\vec{a}_m^\dagger\vec{a}_m , \qquad H_I = :\int_S d^2x\,\tfrac{1}{2}\bar{q}q: + \tfrac{1}{2}\int_S d^2x:\bar{q}i\vec{\tau}\cdot\vec{\pi}\gamma_5/fq: . \quad (2.12)$$

The first term in the expression for the interaction Hamiltonian may seem superfluous, since the linear boundary condition (2.10) implies that $\bar{q}_n q_m = 0$ for eigenfunctions in the free-cavity expansion for q (2.11). However, we show in the Appendix that its expectation value on a perturbed eigenstate does not vanish if we define the value on the bag surface as a limit of the values inside the bag. Indeed the expectation value of H_I itself vanishes on the perturbed eigenstates.

As a preliminary illustration of perturbation theory with the Hamiltonian (2.12) we calculate the shift to order $1/f^2$ in the energy of the spherical nucleon. In lowest order it is a three-quark state with quarks in the lowest orbital with wave function

$$q_0 = \frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0(\omega r)U \\ i\sigma\cdot\hat{r}j_1(\omega r)U \end{pmatrix} , \quad (2.13)$$

where $R^3N^2 = \Omega^2/[1 - j_0(\Omega)^2]$, $\omega = \Omega/R$, U is a two-component spinor, and $\Omega \approx 2.04$ so that $j_0(\Omega) = j_1(\Omega)$ according to (2.10).[2] To second order the action A_2 does not contribute to the energy of the state.[15] Thus the energy shift is found from H_I above, and it is

$$\Delta E = \sum_\mu \frac{\langle 0|H_I|\mu\rangle\langle\mu|H_I|0\rangle}{E_0 - E_\mu} , \quad (2.14)$$

where $|\mu\rangle$ is a state containing one pion and three quarks, one of which could be excited. However, we shall neglect terms in which the quark is excited, allowing only spin and isospin rearrangements of the ground state. The unperturbed state $|0\rangle$ and the unexcited state $|\mu\rangle$ are annihilated by the first term in H_I. Therefore, it does not contribute to the energy shift in this order, and we are correct in omitting terms of higher order in H_I in (2.14). Since the intermediate state contains unexcited quark states, the energy shift (2.14) can be found by solving the classical equations

$$\nabla^2\vec{\pi}_\nu = 0 \text{ in } \bar{V},$$

$$\hat{r}\cdot\nabla\vec{\pi}_\nu = \frac{i}{2f}\langle 0|\bar{q}\gamma_5\vec{\tau}q|\nu\rangle \text{ on } S , \quad (2.15)$$

$$\Delta E = -\tfrac{1}{2}\int_{\bar{V}}(\nabla\vec{\pi}_\nu)^2d^3x ,$$

where ν refers only to the quark degrees of freedom in $|\mu\rangle$. To be more specific, we have

$$\vec{\pi} = \sum_{i=1}^{3}\vec{\sigma}_i\cdot\hat{r}\vec{\tau}_i c/r^2 , \quad r > R \qquad c = \frac{3g_A\lambda}{40\pi f} , \quad (2.16)$$

where $\lambda = \tfrac{3}{2}$ and

$$g_A = \frac{5}{9}\frac{\Omega}{\Omega - 1} \tag{2.17}$$

is the nucleon axial-vector charge of the original model.[2] The nucleon axial-vector charge in this model is λg_A.[8] In terms of these constants,

$$\Delta E = -\frac{9}{100}\frac{\lambda g_A^{\,2}}{8\pi f^2 R^3}\left\langle \sum_{i,j} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \right\rangle, \tag{2.18}$$

where σ_i, etc., is an operator acting on the spin of the ith quark. The intermediate states $|\nu\rangle$ in this case consist of nucleons and Δ states of various spins and charges. The expression for the energy shift is valid for the Δ as well as the nucleon. The expectation value $\langle \sum \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \rangle$ is 57 for the nucleon and 33 for the Δ. Barnhill and Halprin[6] advocate omitting the terms with $i=j$ as corresponding to a renormalization of quark mass and cavity energy. Doing so makes it impossible to separate the πNN coupling from the $\pi N\Delta$, etc., and is undesirable for our purposes.

Although this calculation is axiomatically complete and consistent based on the action (2.1) we shall argue in the next section that further analysis of the significance of the pion field reveals that the calculation is incomplete to this order in $1/f$, and so it must be modified. The proposed modification sets $\lambda = 1$.

III. THE PION BAG VS THE PION FIELD

We discuss in this section the consequences of one interpretation of the static-cavity pion state in a theory with an elementary pion field. An interpretation rather than an analysis of fact is necessary here, since we do not propose here to identify the origin of the spontaneous breakdown of chiral symmetry, a phenomenon that is presumably connected with the mechanism that gives rise to quark and gluon confinement. These questions go beyond the static cavity approximation to the bag model—even beyond the bag model itself.

In the absence of an external pion field there is a stable low-lying quark-antiquark bag state that is a candidate for the pion. It can be arranged to have zero mass in the chiral limit.[9,16] Donoghue and Johnson[9] propose that the static-cavity eigenstates be regarded as localized wave packets of the true momentum eigenstates in analogy with states in a nonrelativistic shell model. Thus the pion-bag state is regarded as the superposition

$$|\pi, \text{bag}\rangle = \int d^3p\, \phi(p)\, |\pi, \vec{p}\rangle\,. \tag{3.1}$$

Donoghue and Johnson obtain an expression for the wave function by calculating the bag-to-vacuum matrix element of the quark component of the axial-vector current (1.2):

$$\langle 0, \text{bag}|A_{\mu Q}^{(+)}(x)|\pi^-, \text{bag}\rangle = \sqrt{2} f \int d^3p\, e^{ip\cdot x} p_\mu \phi(p)\,. \tag{3.2}$$

The right-hand side follows from (3.1) and the identification of $\vec{A}_{\mu Q}(x)$ as the axial-vector current. The matrix element is readily calculated from the wave function (2.13) giving

$$f\partial^0\phi(x) \approx \sqrt{N_c}\, iN^2[j_0^{\,2}(\omega r) - j_1^{\,2}(\omega r)]e^{-2i\omega t}\theta(R - r)\,, \tag{3.3a}$$

$$f\nabla\phi(x) \approx 2\sqrt{N_c} N^2 j_0(\omega r) j_1(\omega r)\hat{r} e^{-2i\omega t}\theta(R - r)\,, \tag{3.3b}$$

where $N_c = 3$ is the number of colors. From the normalization of $\phi(p)$ f is found to be

$$f \approx 0.5/R_\pi\,, \tag{3.4}$$

where R_π is the radius of the pion bag. With a recent value[16] $R_\pi = 3.8\ \text{GeV}^{-1}$, $f = 135$ MeV, about 40% higher than the experimental value. This is not bad in view of the approximations involved in defining $\phi(p)$.

The Donoghue-Johnson interpretation gives a new meaning to the static cavity: It serves not only to confine the quarks in their relative separation, but it also provides a mechanism for localizing the center of mass of the state. If the pion is indeed the Goldstone boson of broken chiral symmetry, then the pion should behave in the absence of other hadrons as a free massless particle.[17] The mere existence of a localized pion eigenstate in the theory implies a breakdown of chiral symmetry. Clearly this particular problem is associated with the static-cavity approximation and not with the original Lorentz-covariant formulation of the model. But we do not have a satisfactory quantum theory for the bag in three dimensions; therefore, we work within the approximation. Chiral symmetry is also lost in this approximation by the confinement of the quarks, as discussed in the beginning of Sec. I. We suggest that introducing the pion field serves not only to restore chiral symmetry at the bag surface, it should also in a sense restore translational invariance to the static pion-bag state. We say "should" because it is not obvious that the action (2.1) does the job. However, a "correct" formulation of the theory should, in fact, do it, and the theory defined by (2.1) is at least suggestive of a restoration of translational invariance to the pion as discussed below.

We note that in perturbation theory, when the pion field couples to the pion-bag state, the quark and antiquark may annihilate to produce the external field. If we accept the wave-packet interpretation of the static-cavity eigenstate, restoration of translational invariance produces a somewhat similar effect: The spreading of the wave packet gives rise to a finite amplitude for finding the c.m. of the composite pion outside the confines of the original cavity. The spreading of the wave packet should occur freely, independently of the smallness of the coupling $1/f$. Indeed, if we examine the annihilation amplitude, we find that it is proportional to (3.3b), which determines f in the Donoghue-Johnson scheme. Thus, although the annihilation graph is counted formally as first order in $1/f$, it is actually of zeroth order. We must calculate the pair annihilation and creation processes to all orders in the interaction Hamiltonian to determine the effect of the external field upon the pion-bag state. We note that the annihilation and creation amplitudes differ from the emission and absorption amplitudes used to calculate the self-energy of the nucleon in Sec. II, in that an extra factor $\sqrt{N_c}$ appears in the former amplitudes. In effect $f \propto \sqrt{N_c}$ and the expansion in powers of $1/f$ is formally an expansion in powers of $1/\sqrt{N_c}$.

Although pion emission and absorption in the nucleon may still be treated in perturbation theory, it is evident that the pion field, once emitted, can give rise to additional quark-antiquark pairs in the nucleon bag before being absorbed, as illustrated in Fig. 1(b). Just as in the case of the pion bag, these processes must be treated to all orders in the interaction Hamiltonian, since they are of zeroth order in $1/f$.

The necessity of treating pair creation and annihilation to all orders in the interaction Hamiltonian (2.12) makes it difficult to determine the pion eigenstate in the interacting theory. In fact it is likely that, with a more complete understanding of the nature of the vacuum,[15] the pion eigenstate of this particular theory would be found wanting. Thus we seek a modification to the action (2.1) that retains some of the desirable features of the static cavity approximation and is capable of taking into account pair annihilation and creation processes at the bag surface in a nonperturbative manner. A crude but simple modification extends the pion field into the bag so that it is continuous at the bag surface. The internal field is intended to represent in an approximate manner the quark-antiquark pairs that are produced by the external field. Thus to avoid overcounting in the modified theory we must exclude additional pair creation and annihilation induced by the extended field. Turning off the field external to a cavity, leaving only an internal field, gives an analog "pion" eigenstate in a wave packet somewhat like that of (3.3a). Turning it on allows the wave packet to spread continuously across the cavity wall.

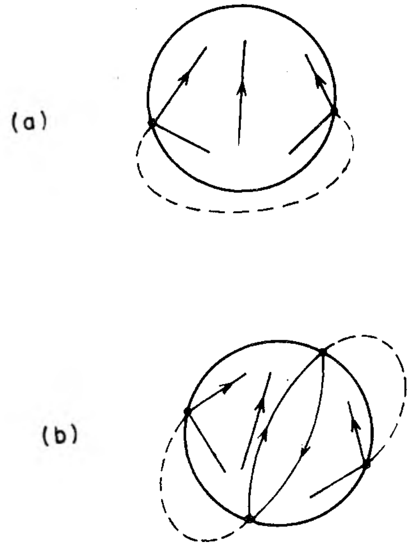

FIG. 1. Perturbation diagram for nucleon self-energy to order $1/f^2$ showing (a) a conventional contribution and (b) a quark-pair contribution induced by an external pion.

It should be reemphasized that the elementary pion field is always to be considered as an approximation to a composite object. Such an approximation should be valid only for low-energy pion interactions for which the pion wavelength is large compared to its bag size.

The revised action (2.1) now becomes[12]

$$A = \int_V d^4x\left(\frac{i}{2}\bar{q}\gamma\cdot\overleftrightarrow{\partial}q - B\right) - \int_{V+\bar{V}} d^4x\,\tfrac{1}{2}(D_\mu\vec{\pi})^2 - \tfrac{1}{2}\int_S d^3x\,\bar{q}e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q\,, \tag{3.5}$$

where it is understood that the pion field is continuous across the bag surface. The equations of motion (2.4) are modified so that (2.4a) now holds in $\bar{V}$ as well as V and (2.4d) now reads

$$n\cdot\partial\pi_i\big|_-^+ = -\left[\delta_{ij} - \left(1-\frac{2x}{\sin 2x}\right)(\delta_{ij}-\pi_i\pi_j)\right] \times\frac{i}{2f}\bar{q}\tau_j\gamma_5 e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q\,, \tag{3.6a}$$

$$\pi_i\big|_-^+ = 0 \quad \text{on } S\,, \tag{3.6b}$$

where the notation $|_-^+$ means the value outside minus the value inside. The nonlinear boundary condition (2.6) becomes

$$\tfrac{1}{2}(n\cdot\partial)(\bar{q}e^{i\vec{\tau}\cdot\vec{\pi}\gamma_5/f}q) = B\,, \tag{3.7}$$

where it is necessary to interpret the derivative acting on $\vec{\pi}$ as the average inside and outside.[4] Because of the different boundary condition (3.6) the strength of the pion field produced by the nucleon in (2.15) is changed. Thus the pion-nucleon coupling constant and axial-vector charge λg_A is changed. The effect is simply to set $\lambda = 1$ [(2.16), (2.18), etc.] as noted at the end of Sec. II. As we shall see this change gives better agreement with the experimental values.[13] Jaffe[8] was first to call attention to the rather large value of the nucleon axial-vector charge that results from excluding the pion field from the bag interior. This problem has plagued subsequent analyses that neglect the effect of quark-pair creation and annihilation induced by the external field, and has led to some ingenious proposals for remedies.[18] The reduction of λ that results from making the pion field approximately continuous across the surface is general and well known (see Ref. 4).

As a matter of convenience, the internally extended pion field is treated as though it were massless in (3.5) and with the physical mass in II. The assignment of a zero mass is justified approximately by Eq. (3.3). Although the equations are not exactly consistent, i.e., the gradient of (3.3a) is not the time derivative of (3.3b), they are consistent in shape and magnitude. Furthermore, the four-divergence of the current defined by (3.3) vanishes, corresponding roughly to the Klein-Gordon equation for ϕ.

IV. CHIRAL NONLINEAR BOUNDARY CONDITION

A necessary consequence of restoring chiral symmetry to the static cavity is the chiral modification [(2.6) and (3.7)] of the nonlinear boundary condition (1.1b). We consider in this section the effect of the chiral nonlinear boundary condition (3.7) on bag states and propose a slight modification in the procedure for static-cavity calculations that improves the stability of the nucleon in pion perturbation theory.

The nonlinear boundary condition is found by making the action stationary with respect to the position of the bag surface. It has been interpreted as a condition for balancing the field pressure against the bag pressure B at the surface.[1,2] In the semiclassical static spherical-cavity approximation, minimizing the energy with respect to the radius is equivalent to imposing (3.7) as an equation for the expectation value of the normal-ordered product averaged over the surface.[19]

In perturbation theory the bag energy to order $1/f^2$ is

$$E(R) = E_0(R) + E_1(R)\,, \tag{4.1}$$

where $E_0(R)$ is the energy of the bag without the pion field present and $E_1(R)$ is the correction to order $1/f^2$. In the approximation of SU(6) degeneracy and zero pion mass,

$$E_1(R) = -\beta/R^3f^2\,, \tag{4.2}$$

where β is a dimensionless constant. Now let us suppose that $E_0(R)$ is minimum at $R = R_0$. Expanding about this radius gives

$$E(R) \approx E_0(R_0) + \tfrac{1}{2}(R - R_0)^2 E_0''(R_0) + E_1(R)\,. \tag{4.3}$$

Minimizing the new expression (4.2) gives a new radius R that differs from R_0 by an amount formally of order $1/f^2$:

$$R - R_0 = \delta R = -E_1'(R_0)/E_0''(R_0)\,. \tag{4.4}$$

But the change in $E_0(R)$ in going to the new radius is clearly of order $1/f^4$; the new value of the energy is therefore found to order $1/f^2$ by evaluating (4.1) at R_0:

$$E = E_0(R_0) + E_1(R_0) + O(1/f^4)\,. \tag{4.5}$$

To find the energy to this order we are not required to change R.

Problems arise, however, in a quantitative determination of the minimum (see Sec. II). Although the correction $E_1(R)$ may be small ($\sim 20\%$) when evaluated at R_0, its derivative is generally large; i.e., the pion field exerts a strong inward pressure on the bag. (The effect is reduced somewhat by allowing the pion to pass into the bag interior, but it persists because of the discontinuity in the radial derivative of the pion field at the surface.) The inward pressure can be sufficiently great as to give a large negative value of δR (4.3). As the bag shrinks, perturbation theory also fails, since the dimensionless parameter $1/f^2R^2$ grows. Thus one can hope that higher-order terms come to the rescue but, if they are needed, we cannot proceed with low-order perturbation theory. Perhaps we must forsake perturbation theory,[20] but the qualitative successes of the quark model in describing the light hadrons argue in favor of the perturbative approach. We propose a modification in the standard procedure for static cavity calculations that increases the stability of the bag, i.e., allows values of $|\delta R| \leq 0.15R$.

It is traditional to minimize, not the cavity energy, but the mass, after correcting for c.m. motion.[1,15] The mass is obtained from[13]

$$M(R)^2 = E(R)^2 - P(R)^2\,, \tag{4.6}$$

where, for the n-quark bag with all quarks in the $S_{1/2}$ orbital,

$$P^2(R) \approx n(\overline{x}/R)^2 \tag{4.7}$$

and $\overline{x} \approx 2$. For the nucleon, the correction $E - M$ is about 250 MeV. The effect of including the c.m. correction in the minimization procedure is to reduce the outward pressure of the fields—hence a destabilizing effect. If, however, $E(R)$ were minimized first and the correction applied without further adjustment of R, stability would be increased.

The procedure for correcting for the c.m. motion is not determined by the static-cavity approximation—in fact the purist might insist that it is preferable to minimize E rather than M since it corresponds more accurately to the nonlinear boundary condition. The effect of minimizing before correcting is to produce a slightly larger bag, the size of which reflects more the c.m. wave-packet size rather than the size of the wave packet of relative motion of the quarks. A consequence of the wave-packet interpretation is that all bag parameters must now be corrected for the spreading of a finite wave packet. The procedure, in the spirit of the Donoghue-Johnson method, is straightforward.[9,21] For example, in calculating the charge radius of the proton (excluding the pion for the moment), one evaluates

$$\langle r_p^2\rangle = \langle r_{\text{bag}}^2\rangle - \langle r_{\text{c.m.}}^2\rangle . \tag{4.8}$$

The first quantity on the right is given directly by the static-cavity wave function,[2] and the quantity $\langle r_{\text{c.m.}}^2\rangle$ depends both on kinematic effects, which are small,[9] and on a large positive contribution of the form

$$\tfrac{1}{3}\int_0^R r^2 q^\dagger q \, d^3x , \tag{4.9}$$

where q is the Dirac wave function of a single quark. Thus the true charge radius $\langle r_p^2\rangle$ before correction for the pion cloud is smaller than that given by the static-cavity calculation. Other static or quasistatic parameters can be corrected by similar methods as discussed in II.

V. SUMMARY AND DISCUSSION

We have offered a justification for a phenomenological approach to a chirally symmetric theory of pion-bag coupling that strives for consistency with the current understanding of the meaning of the static-cavity approximation. The pion is approximated at long wavelengths by an elementary field. It interacts with quarks at the bag surface in a manner dictated by chiral symmetry. Although emission and absorption processes may be treated in perturbation theory in inverse powers of the pion decay constant, pair creation and annihilation cannot be treated in this way. We propose, therefore, extending the definition of the field inside the bag to take approximate account of these processes. This extension was also proposed by Chodos and Thorn[4] and Miller, Thomas, and Théberge,[12] but for different reasons. The resulting theory may then be used for perturbative calculations as long as the pair creation and annihilation processes are omitted. A slight modification in the standard procedure for correcting for the c.m. motion of the hadron in the static-cavity approximation allows for a stable nucleon bag to lowest order in the pion coupling and permits a consistent calculation of the mass renormalization in perturbation theory, as described in II.

The approximation of replacing quark-antiquark pairs by an elementary field is crude and deserves further analysis. It would be useful to have a more explicit scheme that allows these pairs to be taken into account.

We have given some attention to the chiral nonlinear boundary condition, which follows in an axiomatic way from the chirally symmetric action. Although one may criticize the hybrid chiral model as being rather *ad hoc*, and question the validity of the boundary condition, we believe it is necessary to take it as seriously as the chiral linear boundary condition that gives the coupling to the pion. Both are needed in order to restore chiral symmetry to the static-cavity approximation.

The suggested modification in the procedure for correcting the bag mass for c.m. motion helps to make perturbation theory consistent. This result is desirable, since higher-order terms, particularly those involving the pion self-coupling, are model dependent. An additional benefit is that it is more in the spirit of the Donoghue-Johnson interpretation of the static cavity as a localized hadronic state. Corrections to the static parameters are cleaner if the bag radius reflects the geometry of the wave packet, rather than some partially corrected geometry. These corrections are discussed in II.

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APPENDIX

We consider here the manner in which the linear boundary condition on the quark fields, expressed in the form (2.5) is satisfied in quantum perturbation theory. To first order in $1/f$ it takes the form

$$\bar{q}q + \bar{q}i\vec{\tau}\cdot\vec{\pi}\gamma_5/fq = 0 , \tag{A1}$$

As an introduction we consider the problem in the classical perturbation theory[8] of a quark coupling to an external pion field with Hamiltonian

$$H=\int_V d^3x\left[q^\dagger\left(\frac{-i\alpha\cdot\overleftrightarrow{\nabla}}{2}\right)q\right]$$
$$+\tfrac{1}{2}\int_S d^2x(\bar{q}q+\bar{q}i\gamma_5\vec{\tau}\cdot\vec{\pi}/fq)\,. \qquad \text{(A2)}$$

The unperturbed eigenfunctions satisfy

$$-i\alpha\cdot\nabla q_n=\omega_n q_n \text{ in } V;\quad -i\gamma\cdot\hat{n}q=q \text{ on } S\,. \qquad \text{(A3)}$$

The perturbed eigenfunctions satisfy

$$-i\alpha\cdot\nabla q=\omega q \text{ in } V\,, \qquad \text{(A4a)}$$
$$-i\gamma\cdot\hat{n}q=(1+i\gamma_5\vec{\tau}\cdot\vec{\pi}/f)q \text{ on } S\,. \qquad \text{(A4b)}$$

Writing $q=q_0+\delta q$ and substituting into (A2), we have to first order in δq,

$$E=\omega_0-\tfrac{1}{2}\int_S(\bar{q}_0\delta q+\overline{\delta q}q_0)d^2x$$
$$+\tfrac{1}{2}\int_S(\bar{q}_0\delta q+\overline{\delta q}q_0)d^2x$$
$$+\tfrac{1}{2}\int_S\bar{q}_0 i\gamma_5\vec{\tau}\cdot\vec{\pi}q_0/f\,d^2x\,. \qquad \text{(A5)}$$

Thus there is a cancellation between the surface term from the quark kinetic energy in H and the $\bar{q}q$ term.[8] The linear boundary condition (A4b) or (A1) on the other hand requires that the second term in H vanish. Thus E has the form $\omega_0+\delta\omega-\delta\omega+\delta\omega$ in (A5). The expression for δq can be found in perturbation theory by using a Green's-function technique. Writing

$$G_\omega(x,x')=\sum_n\frac{q_n(x)q_n^\dagger(x')}{\omega_n-\omega}\,, \qquad \text{(A6)}$$

where the sum includes both positive- and negative-energy states, we find, by standard methods,

$$q(x')=-\int_S d^2x\,G_\omega(x',x)\tfrac{1}{2}\gamma_0 i\gamma_5\vec{\tau}\cdot\vec{\pi}/fq(x)\,, \qquad \text{(A7)}$$

or to first order in $1/f$,

$$\delta q=\sum_{n\neq 0}\frac{q_n(x)}{\omega_0-\omega}\int d^2x'\bar{q}_n(x')\tfrac{1}{2}i\gamma_5\vec{\tau}\cdot\vec{\pi}/fq_0(x)\,. \qquad \text{(A8)}$$

Since the solution (A8) was constructed explicitly to satisfy (A4b) to lowest order in $1/f$, $q_0+\delta q$ must satisfy (A1).

We now proceed with the quantum perturbation theory of (2.12) using the illustrative example of the nucleon of Sec. II. We wish to show that (A1) is satisfied as an equation for the normal-ordered expectation value on the perturbed eigenstate. Since in this case the pion field is produced from the quarks, it is necessary to work to second order in $1/f^2$. We want to evaluate

$$F=\langle :\bar{q}q:+:\bar{q}i\vec{\tau}\cdot\vec{\pi}\gamma_5/fq:\rangle$$
$$=\left[\sum_{\mu,\lambda\neq 0}\frac{\langle 0|:\bar{q}q:|\lambda\rangle\langle\lambda|H_I|\mu\rangle\langle\mu|H_I|0\rangle}{(E_0-E_\lambda)(E_0-E_\mu)}+\text{c.c.}\right]$$
$$+\sum_{\mu,\kappa\neq 0}\frac{\langle 0|H_I|\kappa\rangle\langle\kappa|:\bar{q}q:|\mu\rangle\langle\mu|H_I|0\rangle}{(E_0-E_\kappa)(E_0-E_\mu)}$$
$$+\left[\sum_\mu\frac{\langle 0|:\bar{q}i\vec{\tau}\cdot\vec{\pi}\gamma_5/fq:|\mu\rangle\langle\mu|H_I|0\rangle}{E_0-E_\mu}+\text{c.c.}\right]\,, \qquad \text{(A9)}$$

where the omitted terms can be shown to vanish to order $1/f^2$. In the example of the computation of the nucleon self-energy in Sec. II, the sum over intermediate states was truncated to include only states with quarks in the ground state, but with a possible rearrangement of spins and isospins. We follow the same approximation here: In the first and last terms in (A9) the states $|\mu\rangle$ are so constrained, and in the second term, either the state $|\kappa\rangle$ or $|\mu\rangle$ is so constrained. The matrix element of H_I between such states, when summed over the pion modes, gave the classical field $\vec{\pi}_\nu$ of (2.15), where ν labels the quark configuration contained in $|\mu\rangle$. The operator $:\bar{q}q:$ on the other hand connects the ground state to excited intermediate states. The structure of the various contributions to F is most easily understood graphically, as illustrated in Fig. 2. In these graphs the solid lines represent quarks and the dashed line, the propagator for the pion. The symbol × represents the operator $:\bar{q}q:$. The symbol ○ represents the operator $:qi\vec{\tau}\cdot\vec{\pi}\gamma_5/fq:$. The bold line denotes a quark that has been excited from the ground state. The thin line is an unexcited quark line. The order of presentation of the graphs in Fig. 2 corresponds to the order

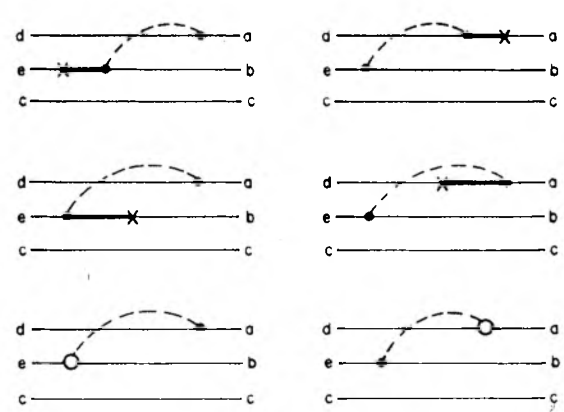

FIG. 2. Diagrammatic representation of contributions to the expectation value F (A9). Symbols are explained in the text.

of terms in (A9) with the second term corresponding to the third and fourth graphs. The letters stand for the quantum numbers of the quarks in a representative term in the expectation value on the three-quark state. We shall now argue that the contributions depicted by the three graphs in each column in Fig. 2 cancel. For the left-hand column they are, explicitly,

$$\sum_{n\neq 0}\frac{\bar{q}_e(x)q_n(x)}{\omega_0-\omega_n}\int d^2x'\bar{q}_n(x')\tfrac{1}{2}i\gamma_5\vec{\tau}\cdot\vec{\pi}_{da}(x')q_b(x') + \sum_n\int d^2x'\bar{q}_e(x')\tfrac{1}{2}i\gamma_5\vec{\tau}\cdot\vec{\pi}_{da}(x')q_n(x')\frac{\bar{q}_n(x)q_b(x)}{\omega_0-\omega_n}+\bar{q}_e(x)i\vec{\tau}\cdot\vec{\pi}_{da}(x)\gamma_5/fq_b(x)\,, \quad \text{(A10)}$$

where $\vec{\pi}_{da}(x)$ is a c-number field satisfying

$$\nabla^2\vec{\pi}_{da}=0 \quad \text{in } \bar{V}\,,$$
$$\hat{n}\cdot\nabla\vec{\pi}_{da}=-\frac{1}{2f}\bar{q}_d\gamma_5\vec{\tau}q_a \quad \text{on } S\,. \quad \text{(A11)}$$

But the expression (A10) may also be written using (A8) as

$$\bar{q}_e\delta q_b+\overline{\delta q_e}q_b+\bar{q}_e i\vec{\tau}\cdot\vec{\pi}_{da}\gamma_5/fq_b\,, \quad \text{(A12)}$$

where δq_b and δq_e are the first-order shifts in the classical eigenfunctions q_b and q_e due to the external field $\vec{\pi}_{da}$. This shift is precisely what is required to meet the boundary condition (A4b). In the form (A1) this boundary condition implies that

$$(\bar{q}_e+\delta\bar{q}_e)(1+i\gamma_5\vec{\tau}\cdot\vec{\pi}_{da}/f)(q_b+\delta q_b)=0 \quad \text{(A13)}$$

and therefore, writing (A13) to the appropriate order in $1/f$, using $\bar{q}_eq_b=0$, we find that (A12) vanishes. Other contributions, including the quark "self-energy" contribution, may be handled in similar fashion. Thus (A1) is satisfied as an equation for the expectation value on the state to order $1/f^2$: Q.E.D.

An important conclusion to be drawn from this analysis is that the operator $:\bar{q}q:$ cannot be set to zero on the surface, even though in the free cavity expansion (2.11) the coefficients of the operators $\bar{q}_nq_m$, etc., all vanish. The problem is caused by the representation of the perturbed state on this basis. The sum over cavity modes converges in such a way that the action of the operator $:\bar{q}q:$ on the surface must be defined as a limit of values obtained inside the bag, in the same sense as in the classical expression (A8). For this reason we have taken pains to include such terms in H_I in (2.12).

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when their parameter σ_0, that plays the role of f here, is written as a function of R and varied with R. Our statement is valid if f is *fixed*, as it should be, as R is varied.

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