Light hadron properties with improved staggered quarks

C. Bernard [a], T. Burch [b], T. DeGrand [c], C. DeTar [d], Steven Gottlieb [e], E.B. Gregory [b], U.M. Heller [f], J. Osborn [d], R. Sugar [g], and D. Toussaint [b]*

[a]Department of Physics, Washington University, St. Louis, MO 63130, USA

[b]Department of Physics, University of Arizona, Tucson, AZ 85721, USA

[c]Physics Department, University of Colorado, Boulder, CO 80309, USA

[d]Physics Department, University of Utah, Salt Lake City, UT 84112, USA

[e]Department of Physics, Indiana University, Bloomington, IN 47405, USA

[f]CSIT, Florida State University, Tallahassee, FL 32306-4120, USA

[g]Department of Physics, University of California, Santa Barbara, CA 93106, USA

Preliminary results from simulations with 2+1 dynamical quark flavors at a lattice spacing of 0.09 fm are combined with earlier results at $a = 0.13$ fm. We examine the approach to the continuum limit and investigate the dependence of the pseudoscalar masses and decay constants as the sea and valence quark masses are separately varied.

The MILC collaboration is engaged in a program of QCD simulations using three flavors of dynamical quarks — two light and one strange. These simulations use the "a^2_{tad}" action for staggered quarks, which removes the tree level order a^2 discretization errors[1]. Results for scaling of hadron masses, for dynamical quark effects on the static quark potential, and for the light hadron spectrum at $a \approx 0.13$ fm have been reported in Refs. [2]. We are continuing these simulations at smaller quark masses and on finer lattices, with $a \approx 0.09$ fm. These new runs allow us to extend our studies of the approach to the continuum limit, and to study some new problems such as exotic hybrid mesons with staggered quarks. Several projects based on lattices from these simulations were reported in other talks at this conference[3].

Before exploring more technical points, we would like to emphasize that the big picture of hadronic physics coming from lattice simulations is encouraging. For example, in Fig. 1 we show some hadron masses from our three flavor $a \approx 0.13$ fm calculations compared to experimental results. In this plot the masses (squared masses for the pseudoscalars) were simply extrapolated linearly to the physical light quark mass, and no continuum extrapolation was made. Here the ρ mass was used to set the scale, and the π mass to set the light quark mass. (We think that the a_1 and b_1 energies are small because of couplings to two meson states, and we are investigating this further.)

A preliminary indication of the effect of nonzero lattice spacing (the continuum extrapolation) is seen in Fig. 2, where the nucleon and vector meson (ρ) masses are plotted in units of r_1. The leading corrections are expected to be proportional to $a^2 g^2$, or about half the size in the 0.09 fm runs as in the 0.13 fm runs. One can see that the corrections are small, but possibly larger in three flavor QCD than in the quenched approximation.

We have used the static quark potential to fix the lattice spacing in our simulations. The potential is also useful in phenomenological models of hadrons. In our earlier simulations we showed

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*Presented by D. Toussaint

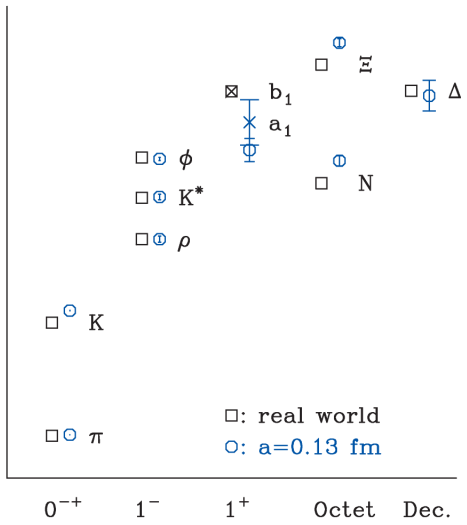

Figure 1. 2+1 flavor lattice spectrum results compared with the real world. These are from simulations with $a \approx 0.13$ fm (no continuum extrapolation), with a simple linear extrapolation to the physical quark mass.

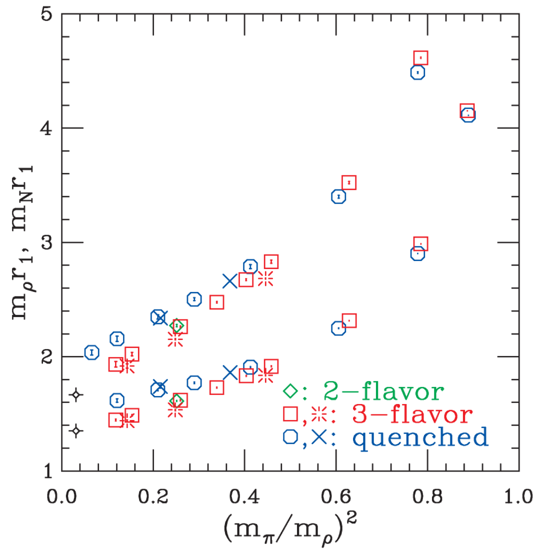

Figure 2. Nucleon and ρ masses in units of r_1 (upper and lower bands, respectively). The open symbols are $a = 0.13$ fm. Bursts and crosses are $a = 0.09$ fm (preliminary). Fancy plusses are physical values, using $r_1 = 0.35$ fm.

clear effects of sea quarks on the shape of this potential. With our 0.09 fm runs in progress, we can begin to investigate the nonzero lattice spacing corrections to this shape. Figure 3 shows one shape parameter, r_0/r_1. (r_0 and r_1 are defined by $r^2F(r) = 1.65$ and 1.00 respectively.) The horizontal axis is basically the dynamical quark mass, with the quenched limit at the right and the physical point at 0.033. The kink at 0.45 occurs because for quark masses larger than this we varied all three quark masses together, while for light quark masses smaller than this the strange quark mass was held fixed. The crosses are preliminary 0.09 fm results. We can see that there is a small upward shift as the lattice spacing is lowered, leading to a very crude estimate of a shift of 0.02 between $a = 0.13$ fm and $a = 0$.

The relatively small quark masses accessible in simulations with staggered quarks allow us to clearly see the effects of chiral logarithms. As Sharpe and Shoresh have emphasized[4], partially quenched calculations can be used to determine parameters in the chiral lagrangian that describes the interactions of low energy pions. Figures 4 and 5 show partially quenched pseudoscalar masses and decay constants as a function of valence quark mass for several values of the sea quark mass. Curvature from chiral logarithms is clearly visible, as is the effect of varying the sea quark mass. Fitting this data will require taking into account the effects of the remaining flavor symmetry breaking, as discussed by Claude Bernard in these proceedings[5].

Acknowledgments

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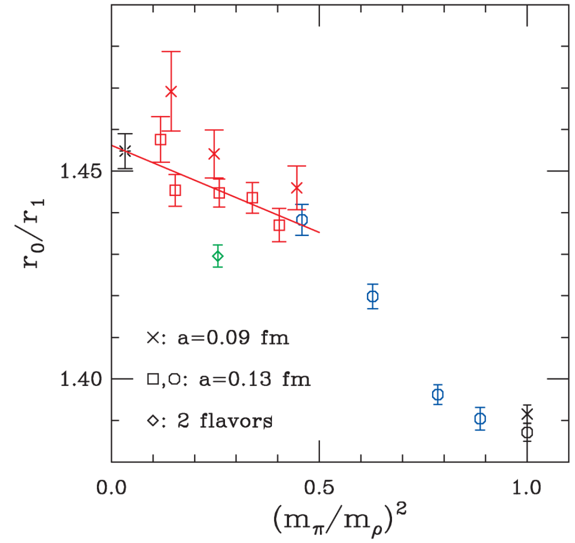

Figure 3. A shape parameter for the static quark potential. Octagons and squares are 0.13 fm runs for three equal masses and 2+1 flavors respectively. The crosses are preliminary 0.09 fm results. The burst is an extrapolation to the physical value. of $(m_\pi/m_\rho)^2$.

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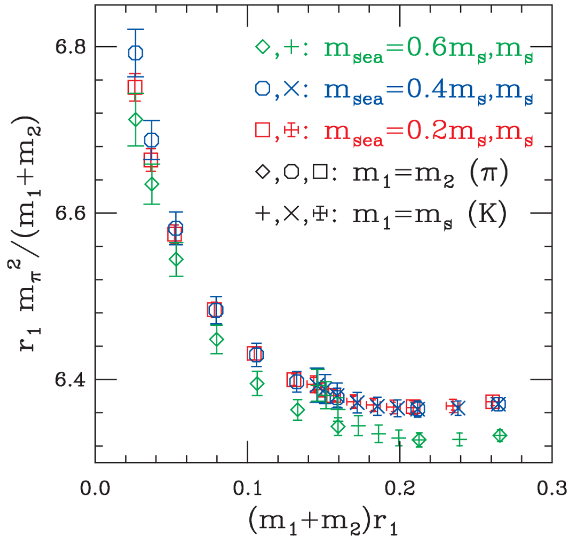

Figure 4. Partially quenched $M_{PS}^2/(m_1+m_2)$ for $a \approx 0.13$ fm. m_1 and m_2 are valence quark masses. Light sea quark masses are 0.2, 0.4 and 0.6 m_s.

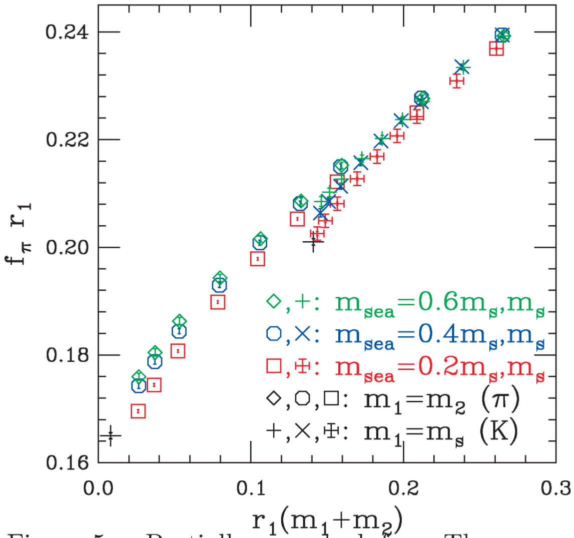

Figure 5. Partially quenched f_{PS}. These were obtained from point-point propagators, with $a \approx 0.13$ fm. The "+" marks the physical f_π and f_K, using $r_1 = 0.35$ fm.