ADAPTIVE REALIZATIONS OF THE PHASE TRANSFORM FOR TIME DELAY ESTIMATION

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ABSTRACT

This paper introduces two recursive realizations of the Phase Transform (PHAT) processor for time delay estimation (TDE), using a simple one-pole lowpass filter and the least mean square (LMS) adaptive filter, respectively. It is shown that these adaptive methods are very effective in reducing the effect of interfering tonals. The performances of these methods are compared with those of other existing adaptive TDE algorithms via computer simulations.

1. INTRODUCTION

We consider the two-sensor time delay estimation (TDE) model given by

$$x_1(k) = s(k) + w_1(k) + p(k) \qquad (1a)$$

and $x_2(k) = s(k-D) + w_2(k) + p(k-\tilde{D})$ (1b)

where k is the discrete time index, $s(k)$ is the source signal, $w_1(k)$ and $w_2(k)$ are additive noises at sensors-1 and -2, respectively, $p(k)$ denotes interfering tonals which might be generated by a target as a jamming signal to mask its movement, and D and $\tilde{D}$ are delay parameters associated with the source signal and the interfering tonals, respectively. Also, it is assumed that the source signal $s(k)$ and the additive noises $w_1(k)$ and $w_2(k)$ are mutually uncorrelated random processes with zero mean.

Most approaches for TDE have been shown to be related through the generalized crosscorrelation (GCC) methods which involve prefiltering the received signals and estimating the time delay as the time lag where the crosscorrelation function of the prefiltered signals is maximum [1]. The relevant crosscorrelation functions are called the GCC functions [1] which can be expressed as

$$R_{12}^{(g)}(m)=F^{-1}\{W^{(g)}(f)e^{j\theta_{12}(f)}\}, \quad |m|\leq M \qquad (2)$$

where $F^{-1}\{\cdot\}$ denotes the inverse Fourier transform (IFT) of $\{\cdot\}$, $W^{(g)}(f)$ is a weighting function in the frequency domain determined by the prefilters, and $\theta_{12}(f)$ is the phase function of the cross power density spectrum (cross-PDS) of $x_1(k)$ and $x_2(k)$. That is,

$$e^{j\theta_{12}(f)} = G_{12}(f)/|G_{12}(f)| \qquad (3)$$

where $G_{12}(f)$ is the cross-PDS of $x_1(k)$ and $x_2(k)$. The frequency domain weighting functions of the GCC methods of interest in this paper, viz., the basic crosscorrelation (BCC) processor [1], the Roth processor [2], and the Phase transform (PHAT) processor [1], are summarized in (4a), (4b), and (4c), respectively.

$$W^{(B)}(f) = |G_{12}(f)| \qquad (4a)$$

$$W^{(R)}(f) = |G_{12}(f)|/G_{22}(f) \qquad (4b)$$

$$W^{(P)}(f) = |G_{12}(f)|/|G_{12}(f)| = 1 \qquad (4c)$$

Recently, the BCC and the Roth processors have been realized recursively, using a single-pole lowpass filter [3] and the LMS adaptive filter [4-7], respectively.

From (2) and (4a), the GCC function of the BCC method is given by

$$R_{12}^{(B)}(m)=F^{-1}\{G_{12}(f)\}=C_{12}(m), \quad |m|\leq M \qquad (5a)$$

where $C_{12}(m) = E\{x_1(k)x_2(k+m)\}$ (5b)

and $E\{\cdot\}$ is the statistical expectation of $\{\cdot\}$.

It has been shown [3] that the cross-correlation function of $x_1(k)$ and $x_2(k)$ can be estimated using a bank of one-pole lowpass filters as

$$\hat{C}_{12}(m,k)=\beta\hat{C}_{12}(m,k-1) + (1-\beta)x_1(k)x_2(k+m), \quad |m|\leq M \qquad (6)$$

where $\hat{C}_{12}(m,k)$ denotes an estimate of $C_{12}(m)$ at time k and $0 < \beta < 1$ controls the time constant of the lowpass filter approximated as [3]

$$\tau_A \cong 1/(1-\beta) \text{ samples.} \qquad (7)$$

Taking the Fourier transform (FT) of $\hat{C}_{12}(m,k)$ with respect to m yields an estimate of the cross-PDS of $x_1(k)$ and $x_2(k)$ at time k. That is,

$$\hat{G}_{12}(f,k) = F\{\hat{C}_{12}(m,k)\} \qquad (8)$$

where $F\{\cdot\}$ represents the FT of $\{\cdot\}$.

Comparing (5a), (6) and (8) we can see that (6) realizes the BCC processor in a recursive way. (6) has been used to implement the BCC method [3] and this approach has been referred to as ABCTDE (adaptive basic crosscorrelation for TDE) algorithm.

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From (2) and (4b), the GCC function of the Roth processor is given by

$$R_{12}^{(R)}(m) = F^{-1}\{\frac{G_{12}(f)}{G_{22}(f)}\}, \quad |m| \leq M. \tag{9}$$

It is known that $R_{12}^{(R)}(m)$ represents the impulse response function, $\hat{h}_{12}(m)$, of the optimum (Wiener) filter which best approximates $x_1(k)$ as a weighted sum of $x_2(k-m)$ for $|m| \leq M$. The LMS adaptive filter algorithm [8] has been used to update the optimum filter coefficients recursively [4-7]. The algorithm may be summarized as

$$\hat{h}_{12}(m,k+1)=\hat{h}_{12}(m,k)+\mu e(k)x_2(k-m), \quad |m| \leq M \tag{10a}$$

where

$$e(k) = x_1(k) - \sum_{m=-M}^{M} \hat{h}_{12}(m,k)x_2(k-m). \tag{10b}$$

In (10a), μ controls the convergence rate and stability of the adaptive filter. The time constant of the LMS adaptive filter can be approximated as [8]

$$\tau_L \cong 1/\mu\sigma_2^2 \tag{11}$$

where σ_2^2 is the variance of $x_2(k)$. We can see from (9) and (10a) that taking the FT of $h_{12}(m,k)$ yields an estimate of $G_{12}(f)/G_{22}(f)$ at time k. That is,

$$\hat{H}_{12}(f,k) \triangleq F\{\hat{h}_{12}(m,k)\}, \quad |m| \leq M \tag{12a}$$

$$= \widehat{G_{12}(f,k)/G_{22}(f,k)} \tag{12b}$$

From (9), (12a) and (12b) we can see that the response function of the LMS adaptive filter is an estimate of the GCC function of the Roth processor. This approach has been referred to as the LMSTDE (LMS for TDE) algorithm [4-7].

2. ADAPTIVE REALIZATIONS OF PHAT

From (2) and (4c) the GCC function of the PHAT is given by

$$R_{12}^{(P)}(m) = F^{-1}\{G_{12}(f)/|G_{12}(f)|\}, \quad |m| \leq M$$

$$= F^{-1}\{e^{j\theta_{12}(f)}\}. \tag{13}$$

In many passive sonar signal processing problems, the received signals often include strong tonals (see (1)). Computing the cross-correlation function of $x_1(k)$ and $x_2(k)$ in (1a) and (1b), we have

$$C_{12}(m) = C_{ss}(m-D) + C_{pp}(m-\tilde{D}) \tag{14}$$

where $C_{ss}(m)$ and $C_{pp}(m)$ are the autocorrelation functions of s(k) and p(k), respectively. When tonals are present in the received signals the crosscorrelation function $C_{12}(m)$ might give its peak at incorrect places to yield erratic delay estimates since the crosscorrelation functions of periodic signals are also periodic.

The PHAT is an ad hoc method to reduce the effect of strong tonals by uniformly weighting the phase function in (3) in the entire frequency band [1]. Two adaptive implementations of the PHAT processor are presented next.

Introducing a time index k in (13) yields the time-varying GCC function for PHAT as

$$R_{12}^{(P)}(m,k)= F^{-1}\{\frac{G_{12}(f,k)}{|G_{12}(f,k)|}\}=F^{-1}\{e^{j\theta_{12}(f,k)}\} \tag{15}$$

Using (8) and (12b) we can estimate $R_{12}^{(P)}(m,k)$ using the one-pole lowpass filter in (6) and the LMS adaptive filter algorithm in (10a) and (10b) as follows:

$$\hat{R}_{12}^{(P1)}(m,k)=F^{-1}\{\frac{\hat{G}_{12}(f,k)}{|\hat{G}_{12}(f,k)|}\}, \quad |m| \leq M \tag{16a}$$

and

$$\hat{R}_{12}^{(P2)}(m,k) = F^{-1}\{\frac{\hat{H}_{12}(f,k)}{|\hat{H}_{12}(f,k)|}\}$$

$$= F^{-1}\{[\widehat{\frac{G_{12}(f,k)}{G_{22}(f,k)}}] \cdot \left|[\widehat{\frac{G_{12}(f,k)}{G_{22}(f,k)}}]\right|^{-1}\}, \quad |m| \leq M. \tag{16b}$$

The approaches in (16a) and (16b) will be referred to as the APHAT-1 (Adaptive PHAT-1) and APHAT-2, respectively, when the time delay estimate is given by the argument m = D(k) where the relevent time-varying GCC functions $\hat{R}_{12}^{(P1)}(m,k)$ and $\hat{R}_{12}^{(P2)}(m,k)$ are maximum.

Now, consider the case of $G_{12}(f) = 0$ in some frequency band (i.e., bandlimited source signal). The phase function in (3) is undefined in this band and the estimates of the phase in (16a) and (16b) are erratic and may result in erroneous time delay estimates. Even though the APHAT algorithms, like the conventional PHAT, have the above problem, it will be shown via computer simulations that they are very effective when the source signal has broad bandwidth and when the received signals contain strong interfering tonals.

3. EXPERIMENTAL RESULTS

In this Section, we will compare the APHAT algorithms with the ABCTDE and LMSTDE algorithms through computer simulations.

The model used for generating the received signals $x_1(k)$ and $x_2(k)$ was the same as in (1a) and (1b). The source signal for cases 1 and 3 was Gaussian white signal, while, for case 2, the source signal was obtained by passing the above signal sampled at 2 Hz through a 6th order Butterworth lowpass filter with the cutoff frequency of 0.2 Hz. For all the simulations, 61 coefficients of $C_{12}(m,k)$ and $h_{12}(m,k)$ were estimated (i.e., M = 30) and 64 point DFT's of these sequences were taken after applying a 61 point hamming window to each of them and padding zeroes to the windowed versions of $C_{12}(m,k)$ and $h_{12}(m,k)$, to obtain $G_{12}(f,k)$ and $\hat{H}_{12}(f,k)$, respectively. The other parameters for the simulations are summarized in Table 1. The estimated GCC functions at k = 8000 for cases 1 and 2 are displayed in Figs. 1 and 2, respectively, where the delay parameters of interest are constant (i.e., D(k) = 4 samples).

The estimated delay function for case 3 is presented in Fig. 3, where the delay function of the source signal linearly increases from -8 to 8 in 8000 samples as indicated by a dotted line, and the delay parameter was computed every 20 samples, starting from k = 80 and ending at k = 8000. For this case, the delay function of the tonals linearly decreased from 4 to -4 in 8000 samples.

The estimated GCC functions in Fig. 1 demonstrate that the APHAT-1 and -2 perform as well as the ABCTDE and LMSTDE algorithms, when the source signal has broad bandwidth. However, since the source signal for case 2 is narrow bandlimited, the APHAT algorithm emphasizes the frequency band where only spectral estimation errors exists, to yield noisy GCC function estimates as shown in Figs. 2(c) and (d).

The results from cases 1 and 2 suggest that the APHAT-1 and -2 are efficient methods to estimate time delays for source signals with broad bandwidth, but may fail to estimate correct delay parameters for narrow bandlimited source signals. Case 3 concerns the problem of estimating time-varying delay functions, which correspond to moving sources or receivers [4-7,9], when the source signals are corrupted by interfering tonals, also with different time-varying delay parameters.

From the estimated delay functions in Fig. 3, we observe that the ABCTDE method estimates the delay function relevant to the interfering tonals ($\tilde{D}(k)$), while the LMSTDE, APHAT-1 and APHAT-2 algorithms track the correct delay parameter. Also, the results show that APHAT-1 and APHAT-2 perform superiorly to the LMSTDE algorithm.

4. CONCLUSIONS

Two adaptive realizations of the PHAT processor for TDE are introduced. The two methods have been referred to as the APHAT-1 and APHAT-2 when a simple one-pole lowpass filter and the LMS adaptive filter, respectively, are used to realize the PHAT recursively. In general, the APHAT-1 is preferred to the APHAT-2 because of its computational simplicity. However, in some applications, the APHAT-2 might be a better choice since the LMS adaptive filter provides useful additional information [8].

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CASE	p(k)†	SNR††	β *	μ *
1	0	1/20	0.9999	$1\text{x}10^{-4}$
2	0	1	0.9998	$1\text{x}10^{-4}$
3	$3p_1(k)+2p_2(k)$	1	0.998	$2.36\text{x}10^{-4}$

† $p_1(k) = \sin(0.46 \cdot 0.5\,\pi)$ and $p_2(k) = \sin(0.12 \cdot 0.5\,\pi)$.

†† Signal to additive white noise ratio.

* The source signals were so scaled as to have the same time constant for each processor.

Table 1. Summary of the parameters used for the simulations.

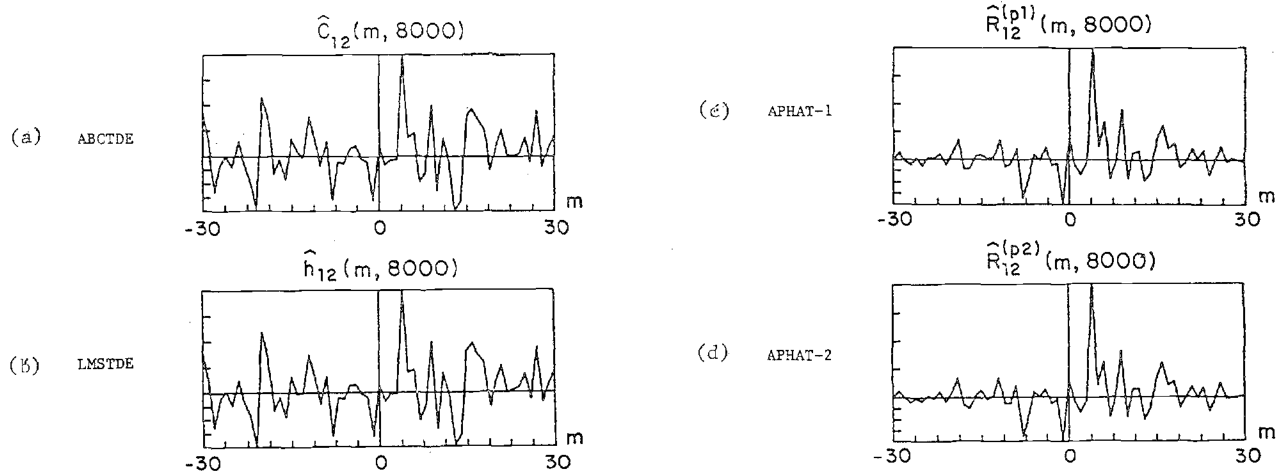

Figure 1. Estimated GCC functions for Case 1.

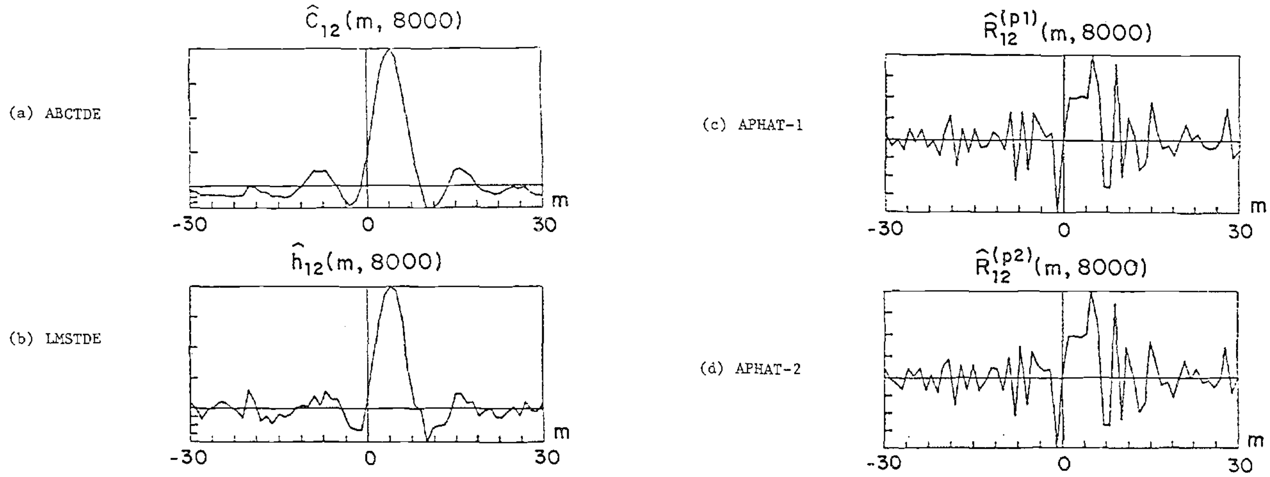

Figure 2. Estimated GCC functions for Case 2.

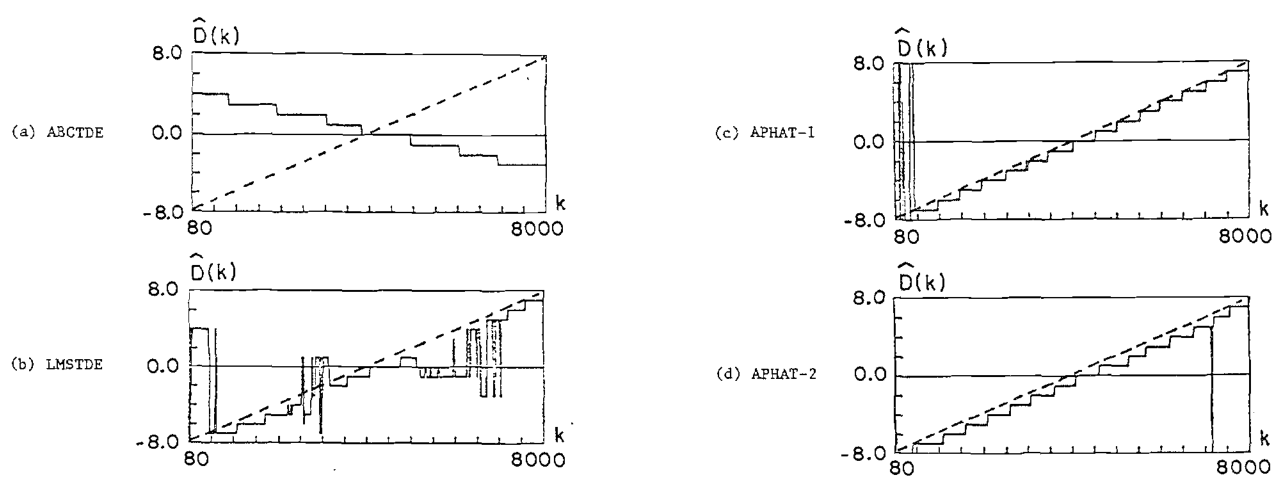

Figure 3. Estimated time varying delay functions for Case 3.