Pseudospin Ferromagnetism in Double-Quantum-Wire Systems

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We propose that a pseudospin ferromagnetic (i.e., interwire coherent) state can exist in a system of two parallel wires of finite width in the presence of a perpendicular magnetic field. This novel quantum many-body state appears when the interwire distance decreases below a certain critical value which depends on the magnetic field. We determine the phase boundary of the ferromagnetic phase by analyzing the softening of the spin-mode velocity using the bosonization approach. We also discuss the signatures of this state in tunneling and Coulomb drag experiments.

Ferromagnetism (FM) in low dimensional itinerant electronic systems is one of the most interesting subjects in condensed matter physics. As early as the 1960s Lieb and Mattis [1] (LM) have proved that a ferromagnetic state cannot exist in one-dimensional (1D) system if the electron-electron interaction is spin or velocity independent and symmetric with respect to the interchange of electron coordinates. Therefore, possible candidates for 1D FM must involve some nontrivial modification in the band structure and interaction to avoid the restrictions of LM's theorem. Most of the examples proposed in the literature [2] rely on some highly degenerate flat bands (or at least systems with the divergent density of states) and can be understood as a generalization of Hund's rule [3]. The only exception appears to be a model of finite range hopping with a negative tunneling energy [4].

From the experimental point of view, however, physical realization of the 1D FM in thermodynamical limit is still absent to the best of our knowledge. In two dimensions (2D), some of the most intriguing ferromagnetic systems are the quantum Hall (QH) bilayers at the total filling factor one. In these systems the flat band structure is provided by the magnetic field (Landau levels) and clear experimental evidence of the 2D pseudospin ferromagnetism (PSFM, with the pseudospin being the layer index) has been observed in the tunneling [5] and drag experiments [6] several years after theoretical proposals [7].

In this Letter we propose a realistic *one-dimensional* system which should exhibit a pseudospin ferromagnetic order. The system consists of two *finite-width* quantum wires with a magnetic field applied perpendicular to the wire surface; see Fig. 1(a). Because of the perpendicular magnetic field, single-electron states as well as the effective mass and Coulomb interaction are strongly modified, leading to the softening of the spin-mode velocity when the interwire distance is smaller than a critical value, d_c. The system then becomes an easy-plane PSFM state due to the appearance of interwire coherence (IWC), which should manifest itself in the appearance of the resonant peak in the tunneling conductance at small bias voltages. We also

calculate the drag resistance of such 1D PSFM states within the mean-field approximation and demonstrate that the drag resistance first increases and scales with the longitudinal size as the magnetic field is increased (or the interwire distance is decreased) toward the phase transition boundary and then becomes dramatically reduced (i.e., not scaled with the size) when entering the PSFM state. The proposed 1D PSFM transition should be experimentally accessible by the present or near future semiconductor technology.

The double wire system we consider is aligned in the y direction, Fig. 1(a), and centered at $x = 0$ and $z = \pm d/2$. Electrons are confined by a parabolic potential, $\frac{1}{2} m \omega_0^2 x^2$, in the x direction, and their motion in z direction is quenched. Using the Landau gauge, the single particle Hamiltonian of momentum k in each wire can be derived to be

$$H_0 = -\frac{1}{2m} \partial_x^2 + \frac{1}{2} m \tilde{\omega}^2 (x - x_0)^2 + \frac{k^2}{2m^*}, \qquad (1)$$

where $m^* = m(\omega_c^2 + \omega_0^2)/\omega_0^2$ is the renormalized electron mass, $\tilde{\omega} = \sqrt{\omega_0^2 + \omega_c^2}$ is the Landau level splitting, and $x_0 = l_0^2 k$ is the guiding center coordinate with $l_0 = \sqrt{\omega_c/m(\omega_c^2 + \omega_0^2)}$ being the magnetic length. $\omega_c = eB/mc$ is the bare cyclotron frequency. The wave functions and energy spectrum of Eq. (1) are similar to the standard QH
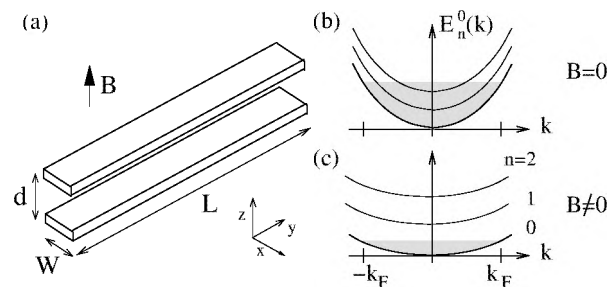

FIG. 1. (a) Schematic double wire system considered in this Letter. (b) and (c) are single particle energy $E_n^0(k)$ of each wire for $B = 0$ and $B \neq 0$ cases, respectively.

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system [8]: $\psi_{n,k,s}(\mathbf{r}) = L^{-1/2}e^{iky}\varphi_n(x + x_0)\sqrt{\delta(z - sd/2)}$ and $E_n^0(k) = (n + \frac{1}{2})\tilde{\omega} + \frac{k^2}{2m^*}$, where n is the Landau level index and $s = \pm\frac{1}{2}$ is the pseudospin index for the lower or upper wire, $\varphi_n(x) = (\pi^{1/2}2^n n!\tilde{l}_0)^{-1/2}e^{-x^2/2\tilde{l}_0^2}H_n(x/\tilde{l}_0)$ is the nth eigenfunction of a parabolic potential with $\tilde{l}_0 \equiv \sqrt{1/m\tilde{\omega}}$. Throughout this Letter we concentrate on the strong magnetic field (or low electron density) regime so that only the lowest energy level ($n = 0$) is occupied. One can see that the magnetic field enhances the effective mass in the longitudinal (y) direction, leading to a flatband structure with high density of states similar to the Landau level degeneracy in 2D system; see Fig. 1(b).

The interaction Hamiltonian can be derived to be [8]:

$$H_1 = \frac{1}{2\Omega_\perp} \sum_{s_1,s_2,k_1,k_2,\mathbf{q}_\perp} V_{s_1,s_2}(\mathbf{q}_\perp, k_1, k_2)$$

$$\times c^\dagger_{s_1,k_1+q_y/2}c_{s_1,k_1-q_y/2}c^\dagger_{s_2,k_2-q_y/2}c_{s_2,k_2+q_y/2}, \quad (2)$$

where $c_{s,k}(c^\dagger_{s,k})$ are the electron field operators, $\Omega_\perp = LW$ is the wire area, and $V_{s,s'}(\mathbf{q}_\perp, k_1, k_2) = A(\mathbf{q}_\perp)^2 \int\frac{dq_z}{2\pi}V(\mathbf{q}) \times [1 + \delta_{s,-s'}(e^{-iq_zd} - 1)]e^{-iq_x(k_1-k_2)l_0^2}$ is the effective 1D interaction with $V(\mathbf{q})$ being the Coulomb interaction. The form function, $A(\mathbf{q}_\perp) = \exp[-(q_x^2 l_0^2 + q_y^2 l_0^4/\tilde{l}_0^2)/4]$, is obtained by integrating the electron spatial wave function [8]. Because of the presence of magnetic field, the effective 1D interaction, $V_{s,s'}(\mathbf{q}_\perp, k_1, k_2)$, is *not* equivalent to any spin-independent (or velocity-independent) symmetric potential. Thus, in our system, the ferromagnetic state is not inhibited by the LM's theorem.

Starting from Eqs. (1) and (2), one can use the standard bosonization approach to describe the low energy physics near the Fermi points. After neglecting the irrelevant (non-local) terms, we obtain $H = \Sigma_{a=\rho,\sigma}H_a + H_b$, where

$$H_a = \frac{u_a}{2\pi} \int dy[K_a\Pi_a(y)^2 + \frac{1}{K_a}\partial_y\Phi_a(y)^2]. \quad (3)$$

Here the sum consists of charge ρ and spin σ channels. $H_b \propto \int dy \cos[\sqrt{8}\Phi_\sigma(y)]$ describes the undiagonalizable backward scattering term [9]. Π_a and Φ_a are the bosonic operators satisfying the commutation relation: $[\Phi_a(y), \Pi_{a'}(y')] = i\delta_{a,a'}\delta(y - y')$. The renormalized velocity and Luttinger exponents are

$$u_a = v_F\sqrt{(1 + \tilde{g}_{\theta_a})(1 + \tilde{g}_{\phi_a})}, \qquad K_a = \sqrt{\frac{1 + \tilde{g}_{\theta_a}}{1 + \tilde{g}_{\phi_a}}}, \quad (4)$$

where $\tilde{g}_{\theta_a/\phi_a} = \frac{1}{2\pi v_F}[2g_{4,a} \mp (2g_{2,a} - g_{1,\parallel})]$ and $g_{i,\rho/\sigma} \equiv \frac{1}{2}(g_{i,\parallel} \pm g_{i,\perp})$. Here $g_{4,\parallel/\perp} = \int\frac{dq_x}{2\pi}[V_{I/O}(q_x, 0)]$, $g_{2,\parallel/\perp} = \int\frac{dq_x}{2\pi}V_{I/O}(q_x, 0)\cos(2q_x k_F l_0^2)$, and $g_{1,\parallel/\perp} = \int\frac{dq_x}{2\pi}V_{I/O}(q_x, 2k_F)$ are defined as the usual g-ology interaction in the Luttinger liquid theory [9] with k_F being the Fermi momentum. $V_I(\mathbf{q}_\perp)$ and $V_O(\mathbf{q}_\perp)$ are the intrawire and inter-wire interaction matrix elements, respectively. To simplify calculations we model the screened Coulomb interaction

by using $V(\mathbf{q}) = (4\pi e^2 \lambda_0^2/\epsilon_0)e^{-|\mathbf{q}|^2\lambda_0^2}$ where ϵ_0 is the static dielectric constant and λ_0 is screening length. The qualitative results obtained below should not be sensitive to the details of the screening potential.

The ferromagnetic transition occurs as the spin stiffness, $v_{N,\sigma} = u_\sigma/K_\sigma = v_F(1 + \tilde{g}_{\phi_\sigma})$, becomes zero [10], or $g_{1\parallel} = 2\pi v_F + 2(g_{4,\sigma} + g_{2,\sigma})$. In general, the low energy Luttinger liquid parameters should be renormalized by the backward scattering, H_b, and therefore the phase boundary obtained from the bare Luttinger parameters should be modified also. However, when in PSFM phase, the spin stiffness is negative so that higher order derivatives, like $\partial_y^2\Phi_\sigma$, have to be included to stabilize the system and to give a nonzero spin density, $\rho_s \propto \partial_y\Phi_\sigma$ [10]. As a result, the sine-Gordon backward scattering will oscillate in real space and hence become negligible after averaging in the thermodynamical limit. Therefore for simplicity we may assume that the renormalization effects are not very serious so that the phase boundary of the PSFM state can still be estimated roughly by using the bare Luttinger parameters as stated above. The critical behavior of similar transition has been also discussed very recently [10].

In Fig. 2 we show the calculated critical interwire distance as a function of magnetic field for various single wire electron densities, n_e. PSFM occurs in the large field and small distance regime. At zero distance, $g_{2/4,\sigma} = 0$, and therefore the critical field ($\omega_{c,cr}$) is the minimum field strength for the backward interaction ($g_{1,\parallel}$) to be dominant. On the other hand, in the extremely large field regime, the Fermi velocity approaches zero. The critical distance (d_{cr}) is now determined by the competition between the backward scattering and the forward scattering in the spin channel. For $k_F\lambda_0 = \pi n_e\lambda_0 > 1$ we can obtain the analytic expression of $\omega_{c,cr}$ and d_{cr}: $\omega_{c,cr} \sim \omega_0\sqrt{2r_s^{-1}e^{(2k_F\lambda_0)^2} - 1}$ and $d_{cr} \sim \lambda_0 e^{-(2k_F\lambda_0)^2}$, where $r_s \equiv me^2/\epsilon_0\pi k_F$ is the ratio of the average potential and kinetic energies. We also checked explicitly that in the parameter regime we consider here the (easy-axis) pseudospin polarized state is
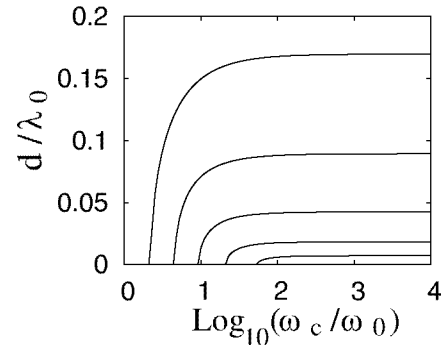

FIG. 2. Calculated critical interlayer distance, d_c, as a function of magnetic field ($\propto \omega_c$). Electron density in an individual wire, n_e, is $0.6, 0.7, \ldots, 1.0 \times 10^5$ cm^{-1} from top to bottom. Here $\lambda_0 = 500$ Å and $\omega_0 = 0.05$ meV.

always energetically unfavorable compared to the (easy-plane) pseudospin ferromagnetic phase.

We now discuss how such a PSFM phase can be observed in realistic experiments. In this phase the system has quasi long-range order characterized by the presence of a Goldstone mode. Tunneling spectroscopy used in the QH bilayers [5] can be also applied to the present system. We expect a strong enhancement of the tunneling conductance at small voltage bias when the system enters the PSFM state. Another approach to demonstrating the 1D PSFM in the double wire system is to perform the Coulomb drag experiments. Such experiments have been done on 2D [6] and 1D [11] semiconductor heterostructures in recent years, and the drag resistance, R_d, is a direct measure of the interwire interaction [12]. If no magnetic field or interwire coherence, the drag resistance behaves differently in the two different regimes: in the perturbative regime R_d vanishes in low temperature limit ($R_d \propto T^2 \ll e^2/\hbar$) [12,13]; in the strong interaction regime, however, the backward scattering between the two wires becomes relevant [14] and opens a gap $\tilde{\Delta}$ in the energy spectrum, corresponding to the formation of a locked charge density wave phase (LCDW) with a divergent drag resistivity $R_d \propto \exp(\tilde{\Delta}/T)$ in low temperature regime.

To analyze the drag resistance in the presence of interwire coherence, it is useful to employ the Hartree-Fock (HF) approximation. This approach neglects long wavelength fluctuations present in 1D systems, but we expect these fluctuations give rise only to small corrections in the drag resistance deep inside the PSFM phase. The HF Hamiltonian then can be easily diagonalized by transforming the electron operators into the symmetric ($c_{\uparrow,k}^{\dagger} + c_{\downarrow,k}^{\dagger}$) and the antisymmetric ($c_{\uparrow,k}^{\dagger} - c_{\downarrow,k}^{\dagger}$) channels with the eigenenergies, $E_k^{\pm} = k^2/2m^* + \Sigma_k \mp \Delta_k - W_0$. Here Σ_k and Δ_k are the intrawire self-energy and the IWC gap, respectively, and W_0 is the shift of the band energy in response to the reconstruction of the ground state due to coupling to leads; see Figs. 3(b) and 3(c). For simplicity, in our calculation we neglect the momentum dependence of Σ_k and Δ_k and approximate them by their values at $k = 0$. Within this approximation, we obtain (at zero temperature):

$$\Sigma_0 \sim \frac{V_1 n_{\text{coh}}}{4}(1 + e^{-(d/2\lambda_0)^2}) - \frac{V_1}{8\sqrt{\pi}\lambda_y}, \quad (5)$$

$$\Delta_0 \sim \frac{V_1 e^{-(d/2\lambda_0)^2}}{8\sqrt{\pi}\lambda_y}, \quad (6)$$

where $V_1 \equiv e^2\lambda_0/\epsilon_0\lambda_x$, $\lambda_x \equiv \sqrt{\lambda_0^2 + l_0^2/4}$, and $\lambda_y \equiv \sqrt{\lambda_0^2 + l_0^4/l_0^2}$. $n_{\text{coh}} = 2n_e$ is the total electron density of *both* wires in the coherent regime. We note that due to interwire Coulomb interaction, n_{coh} can be different from the electron density in the incoherent 1D reservoir, $2n_{\text{res}}$. Therefore we define $\eta = n_{\text{coh}}/2n_{\text{res}}$ to be their ratio for the convenience of later discussion and its value will be determined later. In the above equations, we have assumed that all electrons fall into symmetric band. This is justified
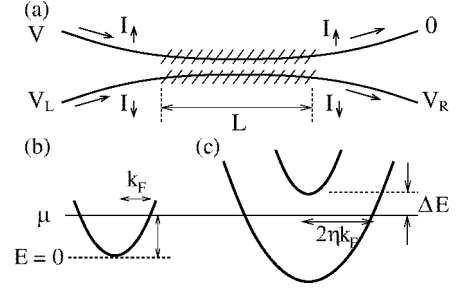

FIG. 3. (a) Typical setup for conductance experiment of the double wire system, where the two wires interact in the middle regime ($0 < y < L$) and are connected to ideal 1D reservoir in the left- ($y < 0$) and right- ($y > L$) hand sides. The upper (active) wire is biased by a voltage V, while the lower (passive) wire is biased by V_R and V_L with currents $I_{\uparrow/\downarrow}$ in the two wires, respectively. (b) and (c) are the band energy for electrons in the incoherent reservoirs and in the coherent double wire regime, respectively. The upper and lower bands in (b) are for the antisymmetric and symmetric bands, respectively.

because the bottom of the antisymmetric band can be shown to be above the chemical potential by $\Delta E = 2\Delta_0 - 4\eta^2 E_F > 0$, when the magnetic field is large enough. (Here $E_F = \frac{k_F^2}{2m^*} \propto B^{-2}$ is the Fermi energy in the incoherent 1D reservoir.)

To calculate the drag resistance in a typical experimental setup, Fig. 3(a), we first note that the drag resistance [$R_d = (V_R - V_L)/I_{\uparrow}$ for $I_{\downarrow} = 0$] can be expressed through the conductance of symmetric currents [$G_+ = I_{\uparrow}/V$ for $V_L = V$, $V_R = 0$, and hence $I_{\uparrow} = I_{\downarrow}$] and the conductance of antisymmetric currents [$G_- = I_{\uparrow}/V$ for $V_L = 0$, $V_R = V$, and hence $I_{\uparrow} = -I_{\downarrow}$], according to: $R_d = G_-^{-1} - G_+^{-1}$. The symmetric and antisymmetric conductances, $G_{\pm}$, in the presence of interwire coherence at temperature T can be easily derived to be [15],

$$\left.\begin{array}{c}G_+ \\ G_-\end{array}\right\} = \frac{e^2}{16\pi T}\int\frac{dE}{\cosh^2(\frac{E-E_F}{2T})}\left\{\begin{array}{c}|t_s|^2, \\ 1 - \text{Re}(r_s r_a^*),\end{array}\right. \quad (7)$$

where $t_{s/a}$ and $r_{s/a}$ are the transition and reflection coefficients for the symmetric or antisymmetric channels, respectively. For simplicity we assume that Δ_0 is constant for $0 < y < L$ and vanishes outside this interval [the shaded area of Fig. 3(a)]. We then obtain

$$\left.\begin{array}{c}t_s \\ r_s\end{array}\right\} = \frac{1}{D}\left\{\begin{array}{c}2ik\kappa_s e^{-ikL}, \\ (k^2 - \kappa_s^2)\sin(\kappa_s L),\end{array}\right. \quad (8)$$

where $D = (k^2 + \kappa_s^2)\sin(\kappa_s L) + 2ik\kappa_s\cos(\kappa_s L)$ and $\kappa_s = \sqrt{k^2 + (2\eta^2 - 1)k_F^2}$ with $E = k^2/2m^*$. r_a is also given by Eq. (8), replacing $\kappa_s \to i\kappa_a$, where $\kappa_a = \sqrt{2\xi k_F^2 - (4\eta^2 - 1)k_F^2 - k^2}$ and $\xi = \Delta_0/E_F$.

At zero temperature the conductance and hence the drag resistance exhibit periodic dependence on the number of electrons. At intermediate temperatures, $v_F/L \ll T \ll E_F$, these oscillations are smeared out yielding

$$R_d = R_0 \left[\frac{(1 + 2\eta)(2\xi - 4\eta^2 - 1)}{2\xi + 2\eta(1 - 2\eta)} - \frac{1 + 4\eta^2}{2\eta} \right], \quad (9)$$

where $R_0 \equiv 2\pi/e^2$. In Fig. 4 we show the calculated drag resistance as a function of $\xi = \Delta_0/E_F$. It is negative when ξ is small, but becomes positive with increasing ξ and eventually saturates at $(1 - 1/2\eta)R_0$.

Finally we determine the electron deplation, η, by equating the chemical potential inside the coherent regime and the chemical potential in the incoherent regime within the HF approximation. Unlike in 3D material, the electron depletion in the present 1D double wire system cannot be compensated by the possible dipole layer formation at the junction points and therefore we expect η can be appreciably smaller than 1. In small d limit, we obtain [15]

$$\eta = \frac{1}{2} + \left(\frac{d}{\lambda_0} \right)^2 \frac{(1 - 1/8\lambda_y k_F)[1 + (\omega_c/\omega_0)^2]}{16[1 + (\omega_c/\omega_0)^2 + \lambda_x/4\lambda_0 r_s]}, \quad (10)$$

where $k_F = \pi n_{\text{res}}$ is determined by the electron density in the reservoir. In the inset of Fig. 4, we plot the drag resistance as a function of magnetic field at a given interwire distance and electron density n_{res}. We note that a finite drag resistance (R_d does not scale with the wire length at $T = 0$) is a signature of the coherent state. The origin of this effect is the indistinguishibility of electrons flowing in the active and passive wires ($\langle c_\uparrow^\dagger c_\downarrow \rangle \neq 0$). A similar phenomenon has already been observed in the 2D QH bilayer systems [6].

As mentioned above, without the magnetic field and interwire coherence, the ground state of the double wire system is predicted to be a LCDW for long-range Coulomb interaction with an infinite drag resistance at zero temperature. R_d calculated in this scenario always *increases* as the interwire distance decreases, due to the enhancement of interwire interaction. However, as we have shown in this Letter, when a strong magnetic field is applied, a finite R_d that does not scale with the wire length is expected to be observed when entering the PSFM phase. The combination
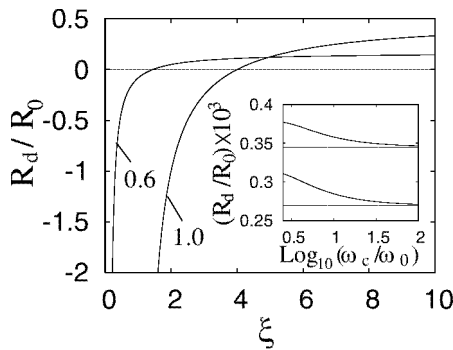
of the above two results leads to the following overall description of the drag resistance: when the interwire distance is decreased from a large value (or the magnetic field is increased from zero) the low temperature drag resistance should first increase and reach a maximum value around the phase boundary (Fig. 2) and then begin to decrease to almost zero due to IWC when entering the PSFM phase. Such nontrivial behavior of drag resistance could indicate a formation of 1D pseudospin ferromagnetism in small interwire distance or large magnetic fields.

To summarize, we have shown that in the presence of a strong magnetic field the electronic system can become (pseudospin) ferromagnetic in the double-quantum-wire system. We further demonstrate that the low temperature drag resistance has a nonmonotonic behavior near the phase transition boundary, which should become observable in the present or near future experiments.

FIG. 4. Drag conductance as a function of $\xi = \Delta_0/E_F$, following Eq. (9). Results for two electron densities, η, are shown together. Inset: drag conductance as a function of magnetic field for $d = 0.08\lambda_0$. $n_{\text{res}} = 0.6$ and 0.7×10^5 cm^{-1} for the lower and upper curves, respectively.

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