Nonlinear Voltage Dependence of the Shot Noise in Mesoscopic Degenerate Conductors with Strong Electron-Electron Scattering

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It is shown that measurements of zero-frequency shot noise can provide information on electron-electron interaction, because the strong interaction results in the nonlinear voltage dependence of the shot noise in metallic wires. This is due to the fact that the Wiedemann-Franz law is no longer valid in the case of considerable electron-electron interaction. The deviations from this law increase the noise power and make it strongly dependent on the ratio of electron-electron and electron-impurity scattering rates.

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Current fluctuations in nonequilibrium mesoscopic diffusive conductors proportional to the average current $\bar{I}$ manifest the discreteness of charge carriers and are usually addressed as the shot noise (for a recent review of the subject see [1]). The effect depends on the length of the conductor L, as well as the electron-impurity l_{ei} and the electron-electron l_{ee} scattering mean free lengths. In order to observe shot noise, the electron-phonon mean free length l_{ep} has to be the largest scale of problem. This means that inelastic effects of electron-lattice thermalization can be disregarded. Otherwise, at $l_{ep} < L$, the noise power $S = 2\int dt\, \overline{\delta I(t)\delta I(0)}$ just approaches the Johnson-Nyquist equilibrium value $4GT$, where G is the conductance and T is the lattice temperature.

Different regimes of shot noise exist.

(i) Diffusive regime, $l_{ei} \ll L \ll l_{ee}$.—In this case the effects of electron-electron (e-e) scattering are negligible. The energy of each electron is conserved during its diffusive motion through a conductor as soon as the electron-impurity scattering is elastic. The electronic distribution function satisfies the diffusion equation and has a two-step shape $f(\epsilon, x) = (1/2 - x/L)f_0(\epsilon - eV/2) + (1/2 + x/L)f_0(\epsilon + eV/2)$, where f_0 is the Fermi-Dirac function, and $\pm eV/2$ are the shifts of chemical potentials of the left and right reservoirs, under bias voltage V. The noise power in this regime is known to be $1/3$ of the Poissonian value $2eI$ [2,3].

(ii) Hot-electron regime, $l_{ei} \ll l_{ee} \ll L$.—The e-e scattering is still small in a sense that all transport processes are governed by the impurity scattering. However, e-e scattering is already efficient enough to smear the two-step partition function and thermalize electrons to the local Fermi-Dirac distribution with some effective temperature profile $T_e(x) \sim 0.3$ eV $\gg T$, that is to be found from the equation for energy transfer. The noise power, now simply representing the average Johnson-Nyquist noise in the conductor with inhomogeneous temperature distribution $T_e(x)$, is equal to $\sqrt{3}/4$ instead of the $1/3$ value [4,5]. The crossover from the diffusive to the hot-electron regime has been experimentally confirmed [6,7].

These values of shot noise are *universal* [8,9]; i.e., they do not depend on the applied voltage, shape of the conductor, anisotropy, distribution, and concentration of impurities, etc. It has been noted [9] that the universality of the hot-electron shot noise has its origin in the Wiedemann-Franz law for impurity scattering (proportionality of the thermal conductivity to the current conductivity, see Ref. [10]).

(iii) $l_{ee} < l_{ei} \ll L$.—This regime has not been studied yet. It was pointed out that the electron distribution function becomes Fermi-Dirac at relatively weak e-e scattering and does not change with its further increase [11]. Therefore, in order to obtain information about e-e scattering the finite frequency noise power has been studied, which is sensitive to the details of Coulomb screening and e-e interaction [11,12].

However, the zero-frequency noise power becomes dependent on the details of e-e scattering if the applied voltage is high enough. Shot noise power in mesoscopic wires is a fruit of both the energy transport and the charge transport. As soon as even normal processes of e-e scattering affect considerably the thermal conductivity (violating the Wiedemann-Franz law) they must result in a change of the shot noise power. Here we show that this is really the case. Despite the fact that the electron distribution function is of the Fermi-Dirac form regardless of the ratio between l_{ee} and l_{ei}, the temperature profile $T_e(x)$ is very sensitive to this ratio. The shot noise power then acquires nonlinear dependence on the applied voltage and might even exceed the Poissonian value. It becomes a possible probe of e-e interaction, as it happens, e.g., in the case of nondegenerate mesoscopic conductors (although because of completely different reason) [13–15].

Super-Poissonian noise has been reported in a number of mesoscopic systems. In Refs. [16,17] the enhancement of shot noise was experimentally found in resonant-tunneling structures biased in the negative differential

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resistance regions of their I-V characteristics. Magnetic field has been used to pronounce this enhancement [18]. Coulomb interaction effects on the shot noise in resonant quantum wells near an instability threshold were studied in Ref. [19]. The effects of nonlinear I-V characteristics were considered microscopically with the self-consistent Coulomb potential taken into account [20]. In these studies disorder does not play a role.

In the present paper we show how super-Poissonian noise can occur in a disordered metallic wire with strong e-e scattering. Despite the fact that the I-V characteristic of such a wire is still linear, the noise power has a nonlinear voltage dependence. To simplify the problem as much as possible we disregard all umklapp processes. As usual, the convenient starting point is the stationary Boltzmann equation,

$$\mathbf{v} \cdot \nabla f_{\mathbf{p}}(\mathbf{r}) + e\mathbf{E} \cdot \frac{\partial f_{\mathbf{p}}(\mathbf{r})}{\partial \mathbf{p}} = I_{ei}[f_{\mathbf{p}}] + I_{ee}[f_{\mathbf{p}}] + \mathcal{L}_{\mathbf{p}}, \tag{1}$$

for the electron distribution function in the phase space $f_{\mathbf{p}}(\mathbf{r})$, where $I_{ei}[f_{\mathbf{p}}]$ and $I_{ee}[f_{\mathbf{p}}]$ are the electron-impurity and e-e collision integrals, respectively,

$$I_{ei}[f_{\mathbf{p}}] = \int \frac{d^3p'}{(2\pi)^3} w_{ei}\delta(\epsilon_p - \epsilon_{p'})(f_{\mathbf{p}'} - f_{\mathbf{p}}), \tag{2}$$

$$\begin{aligned} I_{ee}[f_{\mathbf{p}}] = \int \frac{2d^3p'}{(2\pi)^3}\frac{2d^3k}{(2\pi)^3} w_{ee}\delta(\epsilon_p + \epsilon_k - \epsilon_{p'} - \epsilon_{k'}) \\ \times [f_{\mathbf{p}'}f_{\mathbf{k}'}(1 - f_{\mathbf{p}})(1 - f_{\mathbf{k}}) \\ - f_{\mathbf{p}}f_{\mathbf{k}}(1 - f_{\mathbf{p}'})(1 - f_{\mathbf{k}'})], \end{aligned} \tag{3}$$

where $\mathbf{p} + \mathbf{p}' = \mathbf{k} + \mathbf{k}'$. The quasimomentum conservation law is exact due to the absence of umklapp processes. The scattering amplitudes of the electron-impurity w_{ei} and e-e interaction w_{ee} are independent of the directions of scattering particles (this restriction is not crucial). The last term in Eq. (1) is the extraneous Langevin source, zero on average $\overline{\mathcal{L}_{\mathbf{p}}} = 0$, with the correlator $\overline{\mathcal{L}_{\mathbf{p}}\mathcal{L}_{\mathbf{k}}}$ defined by the very structure of the collision integrals (2) and (3) [21]. Both electron-impurity and e-e collisions contribute to this correlator. Its exact form is quite cumbersome but will not be needed. By multiplying the kinetic equation (1) by the kinetic energy $\epsilon_p - \mu$, and integrating over the momentum space one gets the energy balance equation,

$$\nabla \cdot \int \frac{2d^3p}{(2\pi)^3}\mathbf{v}(\epsilon_p - \mu)f_{\mathbf{p}}(\mathbf{r}) = e\mathbf{E} \cdot \int \frac{2d^3p}{(2\pi)^3}\mathbf{v}f_{\mathbf{p}}(\mathbf{r}), \tag{4}$$

which simply means that the dissipative energy flow is equal to the Joule heat.

To solve Eq. (1) we make the substitution [22]

$$f_{\mathbf{p}}(\mathbf{r}) = f_0(\xi) + \mathbf{v} \cdot \mathbf{q}(\xi)\frac{\partial f_0}{\partial \xi}T_e^{-1}, \tag{5}$$

where $f_0(\xi)$ is the Fermi-Dirac function of the energy variable $\xi = (\epsilon_p - \mu)/T_e$, with the effective temperature $T_e(\mathbf{r})$. (Throughout the paper we use the units such that $\hbar = k_B = 1$.) The (yet unknown) functions $T_e(\mathbf{r})$ and $\mathbf{q}(\xi)$ determine the nonequilibrium distribution of electrons near the Fermi surface. It is convenient to write the function $\mathbf{q}(\xi) = \mathbf{q}_s(\xi) + \mathbf{q}_a(\xi)$ as the sum of even function $\mathbf{q}_s(-\xi) = \mathbf{q}_s(\xi)$ and odd function $\mathbf{q}_a(-\xi) = -\mathbf{q}_a(\xi)$, respectively. In the leading order in T_e/μ, the symmetric function $\mathbf{q}_s(\xi)$ determines the electric current while the antisymmetric one $\mathbf{q}_a(\xi)$ gives the dissipative heat flow. Now the energy balance equation (4) takes the form

$$\nabla \cdot T_e \int_{-\infty}^{\infty} d\xi\,\frac{\partial f_0}{\partial \xi}\,\xi\mathbf{q}_a(\xi) = e\mathbf{E} \cdot \int_{-\infty}^{\infty} d\xi\,\frac{\partial f_0}{\partial \xi}\,\mathbf{q}_s(\xi), \tag{6}$$

as soon as we neglect terms of order T_e/μ. The integrals in Eq. (6) are expanded over the infinite energy axis due to the fast exponential decay of the factor $\partial f_0/\partial \xi$. The kinetic equation (1) splits upon the substitution (5) into two independent integral equations for the even and odd functions, respectively,

$$-e\mathbf{E} = \tau_{ei}^{-1}\mathbf{q}_s(\xi) + \int_{-\infty}^{\infty} d\eta\,K(\xi,\eta)[\mathbf{q}_s(\xi) - \mathbf{q}_s(\eta)], \tag{7}$$

$$\begin{aligned} \xi\nabla T_e = \tau_{ei}^{-1}\mathbf{q}_a(\xi) \\ + \int_{-\infty}^{\infty} d\eta\,K(\xi,\eta)[\mathbf{q}_a(\xi) - \tfrac{1}{3}\mathbf{q}_a(\eta)]. \end{aligned} \tag{8}$$

Here $\tau_{ei}^{-1} = w_{ei}p_F m/2\pi^2$ is the electron-impurity collision rate, and the kernel function is given by

$$K(\xi,\eta) = \frac{m^3 w_{ee}T_e^2}{2\pi^4}\frac{(e^{-\xi} + 1)(\eta - \xi)}{(e^{-\eta} + 1)(e^{\eta-\xi} - 1)}.$$

To derive the last terms in Eqs. (7) and (8) one has to perform integrations in the collision integral (3) with respect to the angle and energy variables (see Ref. [22] for details). The solution of Eq. (7) is simply given by the constant,

$$\mathbf{q}_s(\xi) = -e\tau_{ei}\mathbf{E}. \tag{9}$$

The independence of this solution of the details of e-e interaction reflects the fact that the electric conductivity in the absence of umklapp processes is determined only by the impurity scattering.

The exact solution of Eq. (8) is much more complicated and can be obtained by the method of Ref. [23], by which the thermal conductivity of a clean Fermi liquid was found. The brief outline of the method is as follows. By making the Fourier transform of the integral equation (8) one gets an inhomogeneous second-order differential equation for the function $\mathbf{g}(k) = \int d\xi\,e^{-ik\xi}\mathbf{q}_a(\xi)/(\pi\cosh[\xi/2])$,

$$\begin{aligned} \frac{d^2\mathbf{g}}{dk^2} + \left(\frac{2}{3\cosh^2[\pi k]} - \gamma\right)\mathbf{g} = \frac{8i\pi^4 p_F^3}{m^3 w_{ee}T_e} \\ \times \frac{\sinh[\pi k]}{\cosh^2[\pi k]}\nabla T_e, \end{aligned} \tag{10}$$

where the parameter γ of the relative strength of e-e interaction is introduced, $\gamma = 1 + 4\pi^2/(\tau_{ei} m^3 w_{ee} T_e^2)$. The function $\mathbf{g}(k)$ is found by using the expansion over the eigenfunctions of the homogeneous equation (10), namely over the Jacobi polynomials $g_n = \zeta^{\sqrt{\gamma}/2}\sqrt{1-\zeta}\, P_n^{(\sqrt{\gamma},1/2)}(1-2\zeta)$, with $\zeta = 1/\cosh^2[\pi k]$. For the thermal conductivity, defined as usual as

$$\kappa \nabla T_e = -\frac{p_F^2}{3m\pi^2}\int_{-\infty}^{\infty} d\xi\, \frac{\partial f_0}{\partial \xi}\, \xi \mathbf{q}_a(\xi)\,. \qquad (11)$$

we get the following expression:

$$\kappa(T_e) = \frac{\pi^2 p_F^3}{3m^4 w_{ee} T_e}\sum_{n=0}^{\infty}\frac{2\lambda_n + 1/2}{\lambda_n(2\lambda_n+1) - 1/3} \times \frac{\Gamma^2(\lambda_n)\Gamma(\lambda_n + 1 + \sqrt{\gamma}/2)\Gamma(n+3/2)}{\Gamma^2(\lambda_n + 3/2)\Gamma(\lambda_n + 1/2 + \sqrt{\gamma}/2)n!}, \qquad (12)$$

where $\lambda_n = n + \sqrt{\gamma}/2 + 1/2$, and $\Gamma(x)$ is the Gamma function. When the concentration of impurities decreases, $\gamma \to 1$, the thermal conductivity (12) approaches the known expression of the clean limit [23], while in the case of weak e-e scattering, $\gamma \to \infty$, the conventional Wiedemann-Franz law [10] is recovered,

$$\kappa = \begin{cases} \frac{\pi^2 p_F^3}{3m^4 w_{ee} T_e} A, & A = 0.78,\ \gamma \to 1\,, \\ \frac{p_F^3 \tau_{ei} T_e}{9m} \equiv \kappa_0, & \gamma \to \infty\,. \end{cases} \qquad (13)$$

The general expression (12) is the complicated function of temperature. In what follows, we will use the simple interpolation formula built upon the asymptotic properties (13):

$$\kappa(T_e) = \frac{\kappa_0(T_e)}{1 + \beta(T_e)}, \qquad (14)$$

where the ratio of the e-e and electron-impurity scattering rates is defined as $\beta(T_e) \equiv \tau_{ei}/\tau_{ee}(T_e) = \tau_{ei} m^3 w_{ee} T_e^2/3A\pi^2$. Figure 1 demonstrates excellent agreement of this interpolation formula with the exact dependence (12).

Substituting Eqs. (9) and (14) into the energy balance equation (6), we obtain the equation for the electron temperature profile,

$$\frac{d}{dx}\left(\frac{T_e}{1+\beta(T_e)}\frac{dT_e}{dx}\right) = -\frac{3}{\pi^2}(eE)^2. \qquad (15)$$

We assume that a conductor of length L is in a contact with two reservoirs at equilibrium with zero temperature $T_e(\pm L/2) = 0$. The solution of Eq. (15) gives the effective temperature profile inside the conductor,

$$T_e(x) = \frac{eV}{\sqrt{\beta(eV)}}\sqrt{\exp\left[\frac{3\beta(eV)}{4\pi^2}\left(1 - \frac{4x^2}{L^2}\right)\right] - 1}\,. \qquad (16)$$

To get the expression for the shot noise power it is sufficient to note that locally (at any given point x) the principal term in the electronic distribution (5) is equilibriumlike

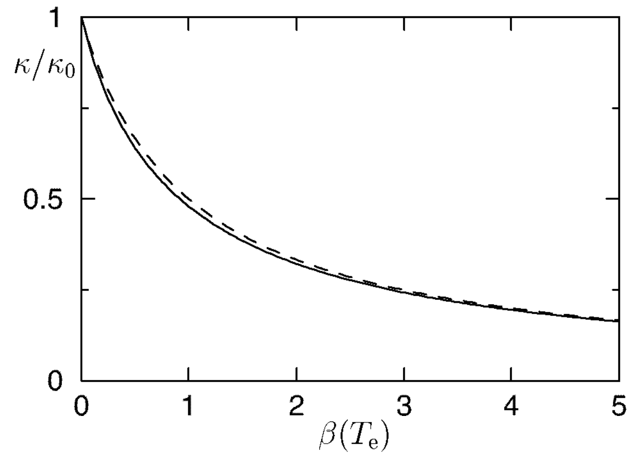

FIG. 1. Deviation of the thermal conductivity (12) from the Wiedemann-Franz law plotted versus $\beta(T_e) = \tau_{ei}/\tau_{ee}(T_e)$. The dashed line shows the approximation by the interpolation formula (14).

and, therefore, the noise is of the Johnson-Nyquist thermal type [4,5]. The noise power is then given by the averaging of the thermal noise over the length of the conductor [24],

$$S = \frac{4G}{L}\int_{-L/2}^{L/2} dx\, T_e(x)\,. \qquad (17)$$

Substituting Eq. (16) into the expression (17) we get the shot noise power,

$$S = 2e\bar{I}F(V)\,, \qquad (18)$$

where the so-called Fano factor (noise suppression factor) is voltage dependent and given by the integral

$$F(V) = \frac{2}{\sqrt{\beta(eV)}}\int_0^1 dz\sqrt{\exp\left[\frac{3\beta(eV)}{4\pi^2}(1-z^2)\right] - 1}\,. \qquad (19)$$

Figure 2 shows the Fano factor as a function of the e-e collision rate.

The shot noise of (18) and (19) is not universal as it depends nonlinearly on the applied voltage. It also becomes sensitive to the geometry of the conductor. The most striking feature of Fig. 2 is the monotonous increase of the shot noise power up to and above its Poissonian value $F = 1$. This is different from the prediction, $F \leq 1$, of the quantum linear statistic theory of the shot noise [25,26]. This is due to the fact that the linear statistic theory does not include effects of inelastic scattering. It is understood, however, that in order to observe super-Poissonian values, very strict conditions should be satisfied $(eV/\mu)^2 p_F l_{ei} \sim 10^2$, making it difficult to achieve such a regime. From the experimental point of view, lower values of $\beta(eV) \leq 10$ are of more interest, at which case e-e interaction contributes a correction to the $\sqrt{3}/4$ noise power, $F = \sqrt{3}/4[1 + 9\beta(eV)/64\pi^2]$. The main reason why the shot noise power becomes nonlinear under strong e-e scattering while the conductance stays Ohmic is due to the fact that e-e interaction conserves the net momentum

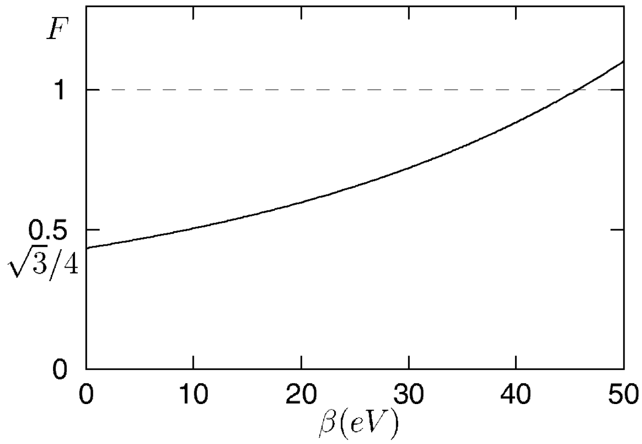

FIG. 2. The Fano factor dependence on the effective ratio of the e-e and electron-impurity scattering $\beta(eV)$ at voltage V. The Fano factor starts with a $\sqrt{3}/4$ value for weak e-e scattering and increases monotonously with its strength.

of particles and therefore does not affect the average current but *does* affect the dissipative energy flow that turns out to be important for current fluctuations.

The calculation presented here essentially assumes that the interference between e-e and impurity scattering [27] is negligible. This is true for sufficiently high effective electron temperatures (i.e., for high voltages), $T_e \gg \tau_{ei}^{-1}$ [28]. In order to observe effects discussed in this paper the mesoscopic conductor has to be prepared sufficiently clean to make the ratio $\beta(eV)$ as large as possible.

To summarize, we presented here a situation where the universality of shot noise is removed by a sufficiently strong e-e interaction and finite voltages. The consideration based on the Boltzmann equation and restricted to the three-dimensional case is given. However, a fully microscopic theory for the shot noise in a strongly interacting system theory is needed, especially for low dimensional systems. As to do this is usually not an easy task, the Monte Carlo simulations could provide helpful insights into the problem.

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