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DEVELOPMENT OF A MULTICOMPONENT-MULTISIZE LIBERATION MODEL

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ABSTRACT

This paper is concerned with the development of a kinetic model for mineral liberation by grinding. Existing population balance size reduction models are extended to include two minerals with an arbitrary number of locked particle fractions. The validity of this approach to liberation modelling is demonstrated for the ball mill grinding of a copper ore in batch laboratory and continuous pilot plant mills.

Keywords
Population balance; size reduction; liberation; grinding model

INTRODUCTION

Liberation of valuable minerals by comminution prior to concentration may be the most important series of process steps occurring in a mineral process plant. Although significant advances have been made in terms of modelling and characterization of both size reduction and concentration processes during the past few decades, the study of liberation, by contrast, has received relatively little attention. The principle reasons for this inattention appear to be the tedious experimentation required and the lack of a methodology for the quantitative interpretation of the resulting data. Since particle mineralogical composition as well as size is important in concentration processes, a liberation model is essential if the simulation of integrated ore dressing plants is to ever be developed.

The first documented attempt to develop a liberation model was reported by Gaudin [1]. He proposed a model to explain "liberation by size reduction" in a binary mineral system consisting of a regular array of elements. Later Wiegel [2,3] randomised the arrangement of the mineral grains in the Gaudin model, thus yielding a more realistic description of the liberation process. This liberation model has also been combined with a simple batch grinding model through the use of "directional coefficients", which give the quantity of liberated mineral grains produced by the reduction of large locked particles [4]. Instead of an actual physical model for mineral liberation, other investigators have outlined methods which use computerized image processing systems for characterizing the extent of liberation in multiphase minerals. King [5] developed a model based on the linear intercept length through the mineral grains of an unbroken ore sample. These data are then adjusted mathematically and transformed into a liberation model. This model has been recently discussed and evaluated by Finch and Petruck [6].

In another approach special attention has also been given to the determination and use of specific surface area of mineral intergrowth for assessing the degree of liberation [7]. Recently, using the Monte Carlo simulation and assuming random fracture, Klimpel and Austin [8] developed a liberation model for a coal-ash system and for a synthetic ore of polystyrene, pyrite and quartz system.

Andrews and Mika [9] combined liberation with size reduction kinetics in a comprehensive model for liberation in a batch mill. This work has been discussed recently by several investigators [10,11]. In more recent work, using a similar approach, Ruebush, Herbst and Rajamani [12] developed a

simplified three component size reduction-mineral liberation model. The later model combines both liberation and size reduction into one set of kinetic equations which is simple enough to admit a closed form analytical solution. A simulator, called SIMPLANT, which uses this size reduction-mineral liberation model, was also developed to predict the performance of an integrated grinding and flotation circuit. Most recently, a parameter estimation program called ESTILIB was developed by Peterson [13] to reduce the number of experiments that are needed to find the large number of model parameters from liberation data. Further, a new experimental technique which involves an image analysis system that can be used to measure the particle liberation quantitatively was developed at University of Utah [14-16].

In this paper, extensions of the three component size reduction-mineral liberation model are considered. Volumetric grade distributions estimated from linear grade measurements are used to estimate the model parameters. Further, a copper ore is used to evaluate this type of model from batch grinding and pilot plant open circuit tests.

MODEL DEVELOPMENT

This section describes the work done to extend standard population balance models for size reduction to account for liberation kinetics involving mixtures of minerals. For single component size reduction a size discretized equation involving selection functions and breakage functions for each size class is used. The linear size-discretized model for the breakage kinetics of a single component in a batch mill is obtained by dividing the assembly being ground into N narrow size intervals, (X_i, X_{i+1}), i = 1, 2, --N. A mass balance for the material in the i-th size interval at time t yields for i = 1, 2, --N:

$$\frac{d\,Hm_i(t)}{dt} = -S_i\,Hm_i(t) + \sum_{j=1}^{i-1} b_{ij} S_j\,Hm_j(t) \qquad (1)$$

Here, $m_i(t)$ is the mass fraction of material in the i-th size interval and H is the total mass of material (hold-up) being ground. The size discretized selection function, S_i denotes the fractional rate at which material is broken out of the i-th size interval. The size discretized breakage function, b_{ij}, represents the fraction of the primary breakage product generated from the j-th size interval that appears in the i-th size interval.

This concept can be extended to the grinding of a multicomponent mixture. For a binary mineral system, M classes of particles can be represented based on mineralogical composition: a free A type particle (a valuable mineral), a free B type particle (gangue), and M-2 sets of locked AB particles. When free particles break, only small free particles of the same mineral species are produced. On the other hand, when locked particles break, they can produce free A, free B, and all M-2 types of locked particles. Figure 1 schematically illustrates the concept of grinding with respect to both size and mineralogical composition.

Using Eq.(1) and the above description, a set of similar equations for liberation/size reduction can be written for each of the M classes of particles, i = 1, 2,--,N:

For free A-type particles:

$$\frac{d\,Hm_i^A(t)}{dt} = -S_i^A\,Hm_i^A(t) + \sum_{j=1}^{i-1} b_{ij}^{A-A}\,S_j^A\,Hm_j^A(t) + \sum_{k=1}^{M-2}\sum_{j=1}^{i-1} b_{ij}^{(AB)_k-A} S_j^{(AB)_k}\,Hm_j^{(AB)_k}(t) \qquad (2.1)$$

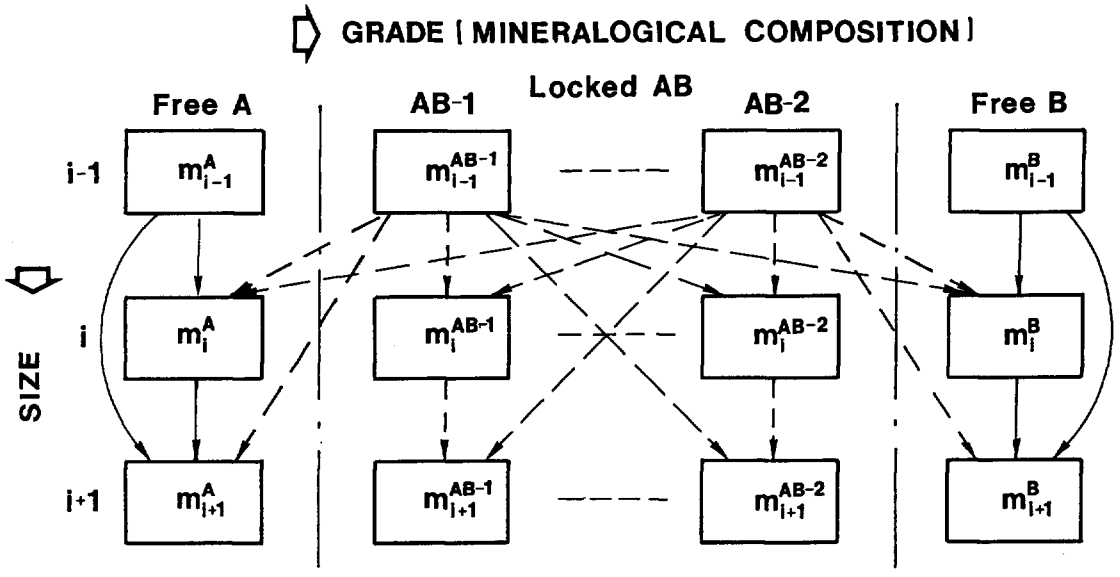

Fig.1 Particle identity change due to size reduction

For free B-type particles:

$$\frac{d\,Hm_i^B(t)}{dt} = -S_i^B\ Hm_i^B(t) + \sum_{j=1}^{i-1} b_{ij}^{B-B}\ S_j^B\ Hm_j^B(t)$$

$$+ \sum_{k=1}^{M-2} \sum_{j=1}^{i-1} b_{ij}^{(AB)_k-B} S_j^{(AB)_k}\ Hm_j^{(AB)_k}(t) \qquad (2.2)$$

For locked $(AB)_k$ type, k = 1, 2, ...M-2

$$\frac{d\,Hm_i^{(AB)_k}(t)}{dt} = -S_i^{(AB)_k}\ Hm_i^{(AB)_k}(t)$$

$$+ \sum_{1=1}^{M-2} \sum_{j=1}^{i-1} b_{ij}^{(AB)_k-(AB)_1} S_j^{(AB)_1}\ Hm_j^{(AB)_1}(t) \qquad (2.k)$$

where H is constant in a batch ball mill. The superscript in Eq.(2) indicates the type of particle to which the parameter applies, and in the case of the b_{ij}, the type of breakage that is occurring. For instance:

$$b_{ij}^{(AB)_k-A}$$

applies to locked AB particles in the k class (composition) of size j breaking to liberated A of size i. The mass fractions are defined so that at any time t:

$$\sum_{i=1}^{N} \left[m_i^{A}(t) + m_i^{B}(t) + \sum_{k=1}^{M-2} m_i^{(AB)_k}(t) \right] = 1 \qquad (3)$$

Constraints are imposed upon the breakage functions in order to satisfy the mass conservation equations for the mass of material and the mass of each component. These constraints take the form:

$$\sum_{i=j+1}^{N} b_{ij}^{A-A} = 1 \quad \text{for each } j,\ 1 \leq j \leq N-1 \qquad (4.1)$$

$$\sum_{i=j+1}^{N} b_{ij}^{B-B} = 1 \quad \text{for each } j,\ 1 \leq j \leq N-1 \qquad (4.2)$$

and for locked $(AB)_k$ type k = 1, 2, ..., M-2

$$\sum_{i=j+1}^{N} \left[b_{ij}^{(AB)_k-A} + b_{ij}^{(AB)_k-B} + \sum_{1=1}^{M-2} b_{ij}^{(AB)_k-(AB)_1} \right] = 1$$

For each j, $1 \leq j \leq N-1$ (4.k)

When the selection and breakage function are independent of both size consist in ball mill and time, the model is said to be linear with constant coefficients. This will be the case considered here. Under such conditions, a set of MxN simultaneous linear differential equations derived from Eqs. (2.1), (2.2) and (2.k), can be represented as a single matrix equation of the form:

$$\frac{d\underline{m}(t)}{dt} = -[\underline{\underline{I}} - \underline{\underline{B}}]\,\underline{\underline{S}}\,\underline{m}(t) = \underline{\underline{A}}\,\underline{m}(t) \qquad (5)$$

Figure 2 shows all the matrices of dimensions MN x MN. Finally, Figure 3 illustrates how these matrices are combined to form the overall matrix "$\underline{\underline{A}}$".

$$\underline{m} = \begin{bmatrix} \underline{m}^{AB_1} \\ \vdots \\ \underline{m}^{AB_{M-2}} \\ \underline{m}^{A} \\ \underline{m}^{B} \end{bmatrix}_{MN \times 1} \qquad \underline{\underline{I}} = \begin{bmatrix} \underline{\underline{I}} & & \underline{\underline{0}} \\ & \underline{\underline{I}} & \\ & \ddots & \\ \underline{\underline{0}} & & \underline{\underline{I}} \end{bmatrix}_{MN \times MN}$$

$$\underline{\underline{B}} = \begin{bmatrix} \underline{\underline{B}}^{AB_1-AB} & \cdots & \underline{\underline{B}}^{AB_{M-2}-AB_1} & \underline{\underline{0}} & \underline{\underline{0}} \\ \vdots & & \vdots & \vdots & \vdots \\ \underline{\underline{B}}^{AB_1-AB_{M-2}} & \cdots & \underline{\underline{B}}^{AB_{M-2}-AB_{M-2}} & \underline{\underline{0}} & \cdot \\ \underline{\underline{B}}^{AB_1-A} & \cdot & \cdot & \underline{\underline{B}}^{A-A} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB_1-B} & \cdot & \cdot & \underline{\underline{0}} & \underline{\underline{B}}^{B-B} \end{bmatrix}_{MN \times MN} \qquad \underline{\underline{S}} = \begin{bmatrix} \underline{\underline{S}}^{AB_1} & & & \\ & \underline{\underline{S}}^{AB_2} & & \underline{\underline{0}} \\ & & \ddots & \\ & & & \underline{\underline{S}}^{AB_{M-2}} \\ & & & & \underline{\underline{S}}^{A} \\ \underline{\underline{0}} & & & & & \underline{\underline{S}}^{B} \end{bmatrix}_{MN \times MN}$$

Fig.2 Definition and structure of the matrices used in Equation 4

$$\underline{\underline{A}} = -[\underline{\underline{I}} - \underline{\underline{B}}]\underline{\underline{S}} = \begin{bmatrix} -[\underline{\underline{I}} - \underline{\underline{B}}^{AB_1\text{-}AB_1}]\underline{\underline{S}}^{AB_1} & \cdots\cdots & \underline{\underline{B}}^{AB_{M-2}\text{-}AB_1}\underline{\underline{S}}^{AB_{M-2}} & \underline{\underline{0}} & \underline{\underline{0}} \\ \vdots & & \vdots & \vdots & \vdots \\ \underline{\underline{B}}^{AB_1\text{-}AB_{M-2}}\underline{\underline{S}}^{AB_1} & \cdots & -[\underline{\underline{I}} - \underline{\underline{B}}^{AB_{M-2}\text{-}AB_{M-2}}]\underline{\underline{S}}^{AB_{M-2}} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB_1\text{-}A}\underline{\underline{S}}^{AB_1} & \cdots & & -[\underline{\underline{I}} - \underline{\underline{B}}^{A\text{-}A}]\underline{\underline{S}}^{A} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB_1\text{-}B}\underline{\underline{S}}^{AB_1} & \cdots & & \underline{\underline{0}} & -[\underline{\underline{I}} - \underline{\underline{B}}^{B\text{-}B}]\underline{\underline{S}}^{B} \end{bmatrix}$$

Fig.3 Combination of the matrices of Figure 2

SOLUTION OF MODEL EQUATIONS

For a batch mill with initial size distribution $\underline{m}_{BATCH}(0)$, a formal analytical solution to Eq.(5) can be obtained using standard matrix techniques, i.e.,

$$\underline{m}_{BATCH}(t) = \exp\{\underline{\underline{A}}\, t\}\, \underline{m}_{BATCH}(0) \tag{6}$$

The matrix exponential appearing in Eq.(6) is not in general easily evaluated computationally. Two cases, involving one locked class and then more than one locked class, are treated below.

Case I: Three Component System

For a three component system, there is only one locked class. In this case progeny locked particles will be considered to have the same mineralogical composition as the parent locked particles. Then, the matrix $\underline{\underline{A}}$ of Eq. (5) is lower triangular in form as shown in Figure 4. A more convenient solution arises for this case making a similarity transformation on matrix $\underline{\underline{A}}$ of Eq.(5) such that:

$$\underline{\underline{A}} = \underline{\underline{T}}\, \underline{\underline{\Lambda}}\, \underline{\underline{T}}^{-1} \tag{7}$$

Here $\underline{\underline{T}}$ is the matrix of eigenvectors of $\underline{\underline{A}}$ and $\underline{\underline{\Lambda}}$ is a diagonal matrix of eigenvalues of $\underline{\underline{A}}$. In order to use this transformation, it is necessary that a set of 3N linearly independent eigenvectors be identified for the $\underline{\underline{A}}$ matrix. Based on the study by Ruebush [12], three linearly independent eigenvectors do exist for the zero-valued eigenvalue of multiplicity three. Therefore, the analytical solution of Eq.(6), for three component system, can be rewritten as:

$$\underline{m}_{BATCH}(t) = \underline{\underline{T}}\, \underline{\underline{J}}\,(t)\, \underline{\underline{T}}^{-1}\, \underline{m}_{BATCH}(0) \tag{8}$$

The matrix $\underline{\underline{J}}$ (t) is a diagonal matrix which is equal to exp $\{\underline{\underline{\Lambda}}t\}$.

$$\underline{\underline{A}} = -[\underline{\underline{I}} - \underline{\underline{B}}]\,\underline{\underline{S}} = \begin{bmatrix} -[\underline{\underline{I}} - \underline{\underline{B}}^{AB\text{-}AB}]\,\underline{\underline{S}}^{AB} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB\ A}\underline{\underline{S}}^{AB} & -[\underline{\underline{I}} - \underline{\underline{B}}^{A\text{-}A}]\underline{\underline{S}}^{A} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB\ B}\underline{\underline{S}}^{AB} & \underline{\underline{0}} & -[\underline{\underline{I}} - \underline{\underline{B}}^{B\text{-}B}]\underline{\underline{S}}^{B} \end{bmatrix}$$

Fig.4 A matrix of Equation 5 for three component case

Case II: Four Component System

If the number of locked fractions is greater than one, the matrix $\underline{\underline{A}}$ of Eq.(5) is no longer lower triangular. The solution of equation (6) can either be obtained numerically or with special manipulation of $\underline{\underline{A}}$. The special manipulation of $\underline{\underline{A}}$ for a four component system is given as follows:

The matrix $\underline{\underline{A}}$ can be written as:

$$\underline{\underline{A}} = -(\underline{\underline{I}}-\underline{\underline{B}})\underline{\underline{S}} = \begin{bmatrix} -[\underline{\underline{I}}-\underline{\underline{B}}^{AB_1-AB_1}]\underline{\underline{S}}^{AB_1} & \underline{\underline{B}}^{AB_2-AB_1}\underline{\underline{S}}^{AB_2} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB_1-AB_2}\underline{\underline{S}}^{AB_1} & -[\underline{\underline{I}}-\underline{\underline{B}}^{AB_2-AB_2}]\underline{\underline{S}}^{AB_2} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB_1-A}\underline{\underline{S}}^{AB_1} & \underline{\underline{B}}^{AB_2-A}\underline{\underline{S}}^{AB_2} & -[\underline{\underline{I}}-\underline{\underline{B}}^{A-A}]\underline{\underline{S}}^{A} & \underline{\underline{0}} \\ \underline{\underline{B}}^{AB_1B}\underline{\underline{S}}^{AB_1} & \underline{\underline{B}}^{AB_2-B}\underline{\underline{S}}^{AB_2} & \underline{\underline{0}} & -[\underline{\underline{I}}-\underline{\underline{B}}^{B-B}]\underline{\underline{S}}^{B} \end{bmatrix}$$

With the use of an elementary transformation, column 2 of the $\underline{\underline{A}}$ matrix can be multiplied by:

$$\underline{\underline{B}}^{AB_2-AB_1}\,[\underline{\underline{I}}-\underline{\underline{B}}^{AB_2-AB_2}]^{-1}$$

and added to column 1.

This operation can be expressed as follows:

$$\underline{\underline{A}}^* = \underline{\underline{R}}_1 \times \underline{\underline{A}}$$

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ & & 0 & 0 \\ \text{same as A} & & & 0 \end{bmatrix} = \begin{bmatrix} I & r_{12} & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \\ \underline{\underline{A}} \\ \\ \end{bmatrix}$$

where

$$\underline{\underline{a}}_{11} = -\{[\underline{\underline{I}}-\underline{\underline{B}}^{AB_1-AB_1}]-\underline{\underline{B}}^{AB_2-AB_1}[\underline{\underline{I}}-\underline{\underline{B}}^{AB_2-AB_2}]^{-1}\underline{\underline{B}}^{AB-AB_2}\}\underline{\underline{S}}^{AB_1},$$

$$\underline{\underline{r}}_{12} = \underline{\underline{B}}^{AB_2AB_2}\,[\underline{\underline{I}}-\underline{\underline{B}}^{AB_2-AB_2}]^{-1}$$

Therefore $\underline{\underline{A}} = \underline{\underline{R}}^{-1}\,\underline{\underline{A}}^*$

where $\underline{\underline{R}}^{-1} = \begin{bmatrix} I & \frac{-1}{r_{12}} & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$

Here $\underline{\underline{R}}^{-1}$ and $\underline{\underline{A}}^*$ are upper and lower triangular matrices, respectively. In this case the solution of Eq.(6) can be expressed as:

$$\underline{m}(t) = \exp\{\underline{\underline{A}}\}\ \underline{m}(0)$$
$$= \exp\{\underline{\underline{R}}^{-1}\ \underline{\underline{A}}^*\}\ \underline{m}(0)$$
$$= \exp\{\underline{\underline{R}}^{-1}\}\ \underline{m}(0) + \exp\{\underline{\underline{A}}^*\}\ \underline{m}(0)$$

It is possible to simplify the matrix exponentials by a similarity transformation to give:

$$\underline{\underline{A}}^* = \underline{\underline{T}}\ \underline{\underline{\Lambda}}\ \underline{\underline{T}}^{-1} \qquad \underline{\underline{R}}^{-1} = \underline{\underline{T}}_R\ \underline{\underline{\Lambda}}_R\ \underline{\underline{T}}_R^{-1}$$

$$\underline{m}(t) = \underline{\underline{T}}_R\ \underline{\underline{J}}_R(t)\ \underline{\underline{T}}_R^{-1}\ \underline{m}(0) + \underline{\underline{T}}\ \underline{\underline{J}}(t)\ \underline{\underline{T}}^{-1}\ \underline{m}(0)$$

where $\underline{\underline{T}}_R$, $\underline{\underline{T}}$ = matrices of eigenvector of $\underline{\underline{R}}^{-1}$ and $\underline{\underline{A}}^*$ respectively,
$\underline{\underline{\Lambda}}_R$, $\underline{\underline{\Lambda}}$ = diagonal matrix of eigenvalues of $\underline{\underline{R}}^{-1}$ and $\underline{\underline{A}}^*$ respectively and $\underline{\underline{J}}_R(t)$; $\underline{\underline{J}}(t)$ are equal to $\exp\{\underline{\underline{G}}_R\}$ and $\exp\{\underline{\underline{\Lambda}}\}$.

The recursion formula (as for the elements of the Eigenvector Matrix $\underline{\underline{T}}$) are given in Appendix 1.

For the case of M greater than four components, a similar procedure can be followed.

MODEL VERIFICATION FOR THREE COMPONENT CASE

The general approach followed for model verification involved conducting a series of bench scale wet batch grinding experiments using a copper porphyry ore from which model parameters could be estimated based on volumetric grade distribution. These parameters were then used to calculate model predictions of pilot scale open circuit grinding and compare them to actual open circuit grinding of copper porphyry ore.

Model for the Estimation of Volumetric Grade Distribution

Model parameter estimation should be based on the volumetric grade distribution of the particle population. The measured linear grade distributions must be converted into three-dimensional information in order to predict true mineralogical composition distribution. A model [17] has been developed and tested to convert the one- or two-dimensional results into three dimensional information. For a monosize sample, the relationship between linear grade distribution $f(g_1)$ and volumetric grade distribution $p(g)$ requires information about the transformation function, $H(g_1|g,Nn)$, a conditional probability function. The transformation can be expressed as:

$$f(g_1) = \int_0^1 H(g_1|g,Nn)p(g)dg \qquad (9)$$

The transformation function has been established from computer simulation. On this basis, the transformation equation, Eq.9, has been solved and tested against experimental depth profile measurements for different ore particles at the University of Utah [16, 17].

In practice, particles of specified size were mounted in a resin matrix and the linear grade distribution from a polished section was determined as $f(g_1)$ of Eq.9. Then, volumetric grade distribution, $p(g)$, is estimated by solving the transformation equation. True mineralogical composition distribution of specified size can be calculated from the estimated volumetric grade distribution.

Wet Grinding Batch Test

To determine the selection and breakage function parameters for the copper ore, a series of wet grind batch tests were conducted. Four batch grinding tests were performed: the material was ground for 2 minutes, 4 minutes, 8 minutes and 16 minutes respectively. A ball mill of 10 inch diameter and 11.5 inch length was filled to 50% of its volume with balls having an equilibrium size distribution. The mill is equipped with a Graham variable speed transmission coupled with a BLH torque sensor. The sensor is connected to a

chart recorder to measure the torque in the drive shaft between the mill and the transmission. For all tests the mill was rotated at 60% of its critical speed or 54 rpm. The mill was operated at 70 weight % solids for all of the tests. It was charged with 3422 g of copper ore. Added to this dry material was 1467 ml of water.

For each batch test, a representative sample was obtained by using a riffle splitter. Representative samples were taken from four size intervals (48x65, 100x150, 150x200 and 200x270 mesh) for mineralogical composition measurement. Mineralogical compositions were determined using microprobe analysis. A sufficiently large number of sectioned particles, suspended in an appropriate molding resin, can be examined for the quantitative identification of the various constituents. During linear scanning of the section of the specimen, the analyst may record the length of the resulting chord through each intercepted section as well as the length of the same chord across the valuable phase appearing in that specific particle section. Therefore, a linear grade may be defined as the ratio between the length of these two chords for each particle. The end result is a probabilistic distribution of this linear grade as $f(g_1)$ of Eq.9. The volumetric grade distributions were then estimated from these linear grade distributions [17]. Both linear grade and volumetric grades were used to estimate the parameters for three-component models. Breakage rates were estimated by non-linear regression using an unpublished program termed "ESTILIB", developed by J.A. Herbst and coworkers at the University of Utah. The resulting specific selection and breakage functions estimates are shown in Figure 5. A different specific selection function of the locked particles estimated from the linear grade distribution is observed as shown in Figure 5.

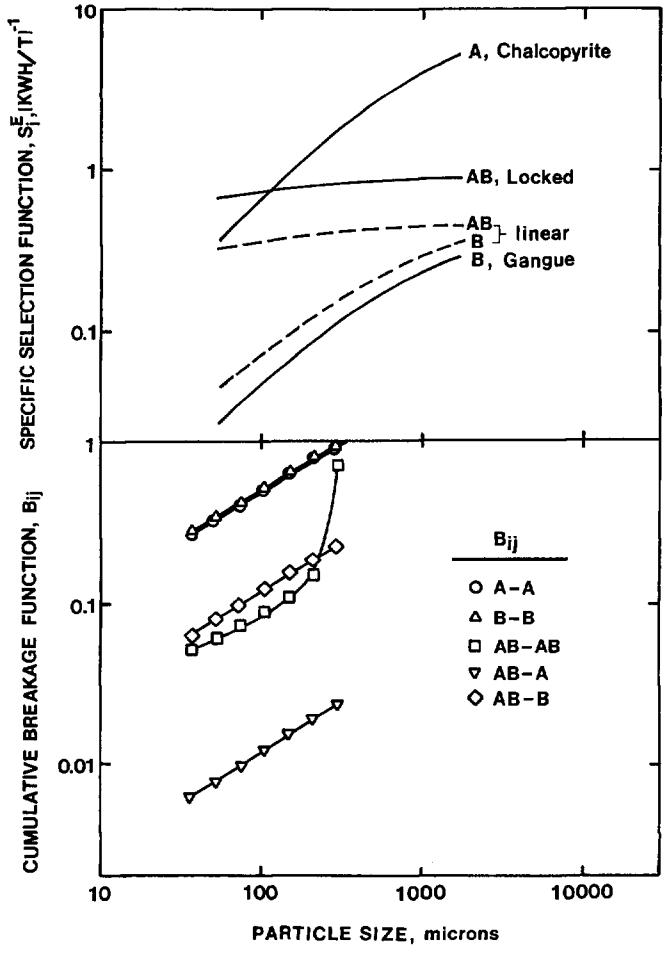

Fig.5 Estimated specific selection and breakage functions for A (Chalcopyrite), B (Gangue) and AB (Locked) components

For batch tests, all grind times were converted to their equivalent specific energy inputs. Plots of cumulative mass fractions for each component (AB, A, and B) at different energy inputs versus particle size are shown in Figures 6, 7 and 8. Depending upon the amount of liberation occurring the total mass of AB particles should remain about the same or decrease, while the total mass of A and B particles should remain the same or increase with grinding time. This is true for all cases as shown in Figures 6 to 8. Using ESTILIB, the best fit parameter where determined. Plots of the best model values are also shown in Figures 6, 7 and 8. The RMS residuals value for this fitting was 1.98×10^{-2}. Generally, the curves fit the data well, although there appears to be more error in the predictions of A and AB values due to the small amount of material in these two composition classes.

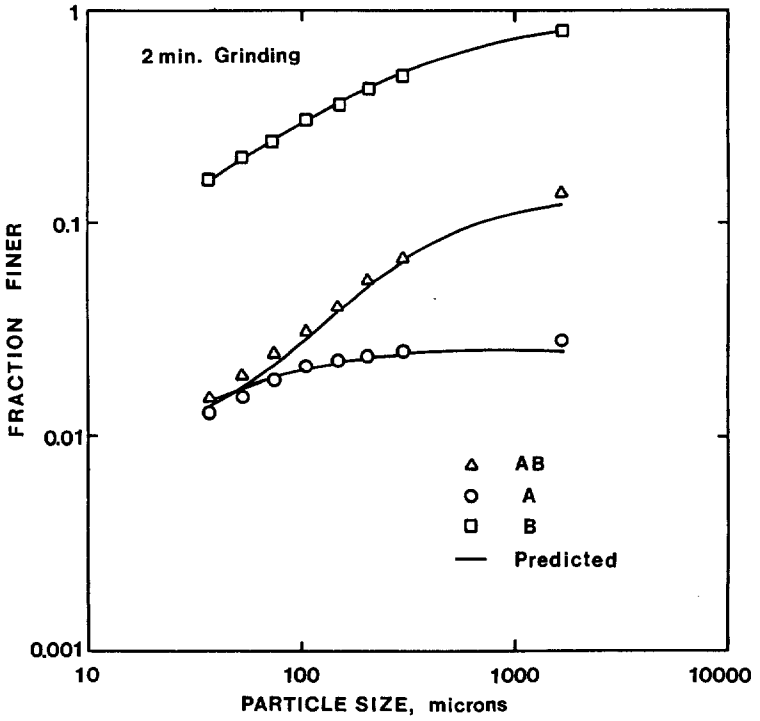

Fig.6 Comparison of experimental and model predicted results of components obtained from 10-inch ball mill test- 2 minutes grinding

Open Circuit Pilot Plant Test

The open circuit pilot plant test work was conducted in the University of Utah's Ore Dressing Laboratory. To conduct an open circuit grinding test, all of the mass flow measurement devices were calibrated. Feed rate to the circuit was controlled using an HP1000 computer. Once steady state conditions were attained, the test was begun. The set-point for the feed streams were 300 lbs/hr for solid and 128.6 lbs/hr for water. Copper ore samples from the open circuit mill were analyzed for size distribution and mineralogical composition as previously mentioned.

ESTILIB was used to simulate open circuit grinding of the copper porphyry ore. The values for selection and breakage functions determined using the batch grinding data were used in ESTILIB along with the size and mineralogical composition of the feed.

The specific selection function values from the batch tests were scaled up for prediction of the pilot scale continuous tests using the relationship [18]:

$$S_i = S_i^E \ (P/H)$$

for all mineralogical components.

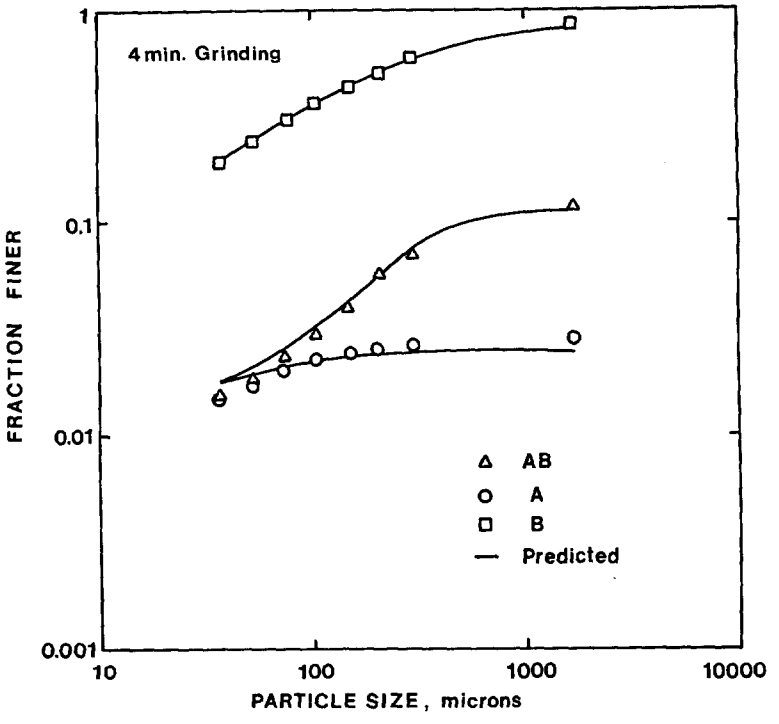

Fig.7 Comparison of experimental and model predicted results of components obtained from 10-inch ball mill test- 4 minutes grinding

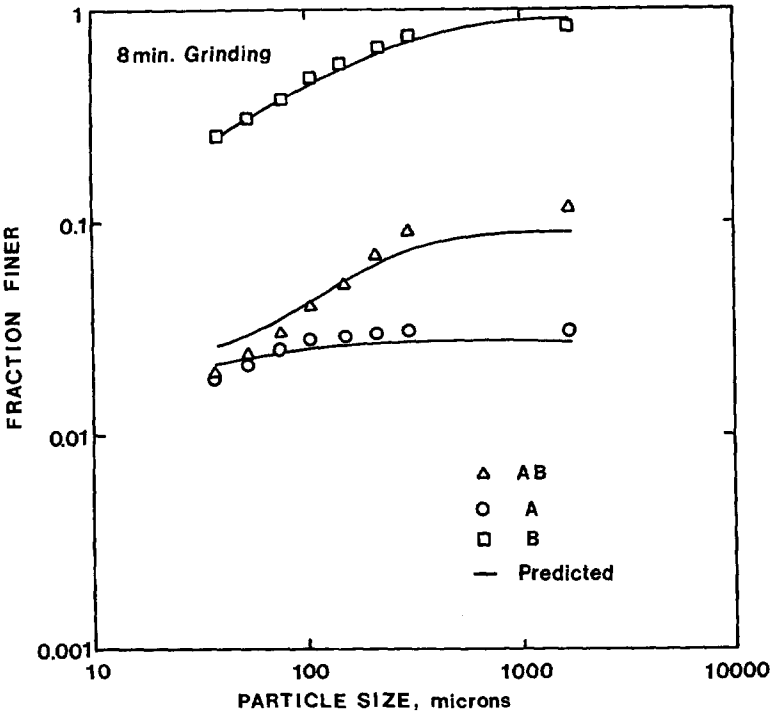

Fig.8 Comparison of experimental and model predicted results of components obtained from 10-inch ball mill test- 8 minutes grinding

Figures 9 and 10 show the pilot plant data in the steady state form of the cumulative mass fraction for each component and the overall cumulative mass fraction as a function of particle size, respectively. Also, shown in these figures are comparisons between the experimental data and model predictions. It is noted that a very good fit is observed between the model predictions and actual mill discharge product.

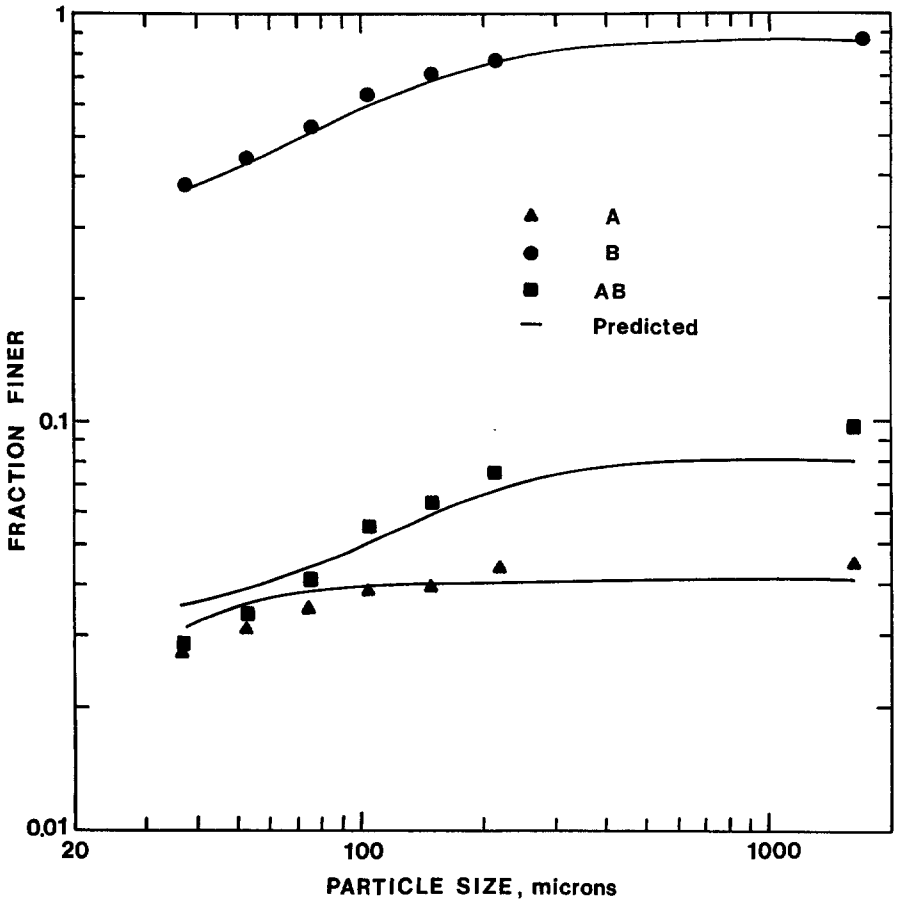

Fig.9 Open circuit pilot plant grind data and prediction based on original model parameters from laboratory scale batch grinding test for different components

CONCLUSIONS

The extended models for size reduction-mineral liberation that were developed in this paper are suitable to provide more information for grinding process analysis and prediction. For three component case, ESTILIB, a program for grinding-liberation model simulation/parameter estimation, made it possible to determine the model parameters from a minimum of experimental data.

The experimental verification of the grinding-liberation model copper ore grinding was found to be successful. The discharge product from an open circuit mill was accurately predicted from data gathered in a laboratory scale batch mill test. This result suggests that further scale-up to commercial operations may also be possible. Additional work is necessary to make the grinding-liberation model more general. This future work includes the development of mathematical solution and parameter estimation schemes for the case of more than one class of locked particles.

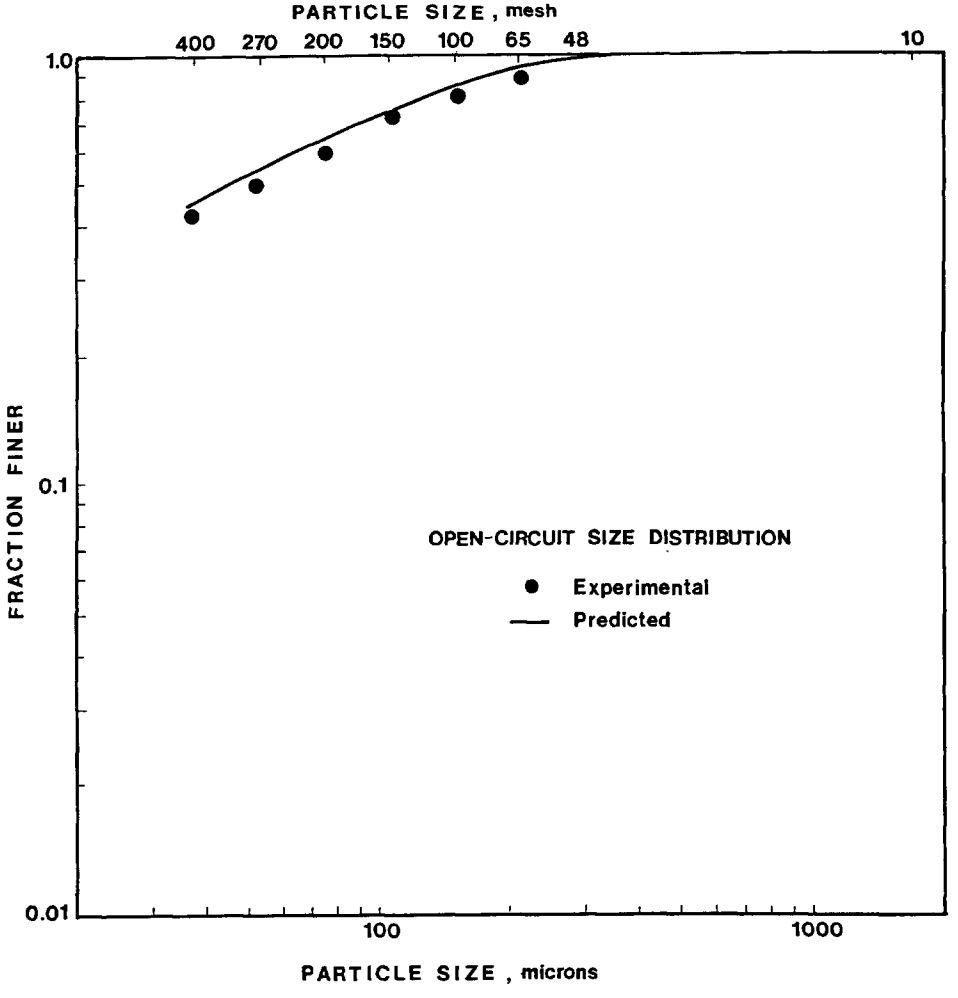

Fig.10 Open circuit pilot plant grinding data and prediction based on original model parameters from laboratory scale batch grinding test for overall mass fraction

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APPENDIX 1

Recursion formulae for the elements of the Eigenvector Matrix $\underline{\underline{T}}$ for Four Component System

(i) Partition

$$\underline{\underline{A}}^{*} = \begin{bmatrix} (I) & 0 & 0 & 0 \\ (V) & (II) & 0 & 0 \\ (VI) & (VIII) & (III) & 0 \\ (VII) & (IX) & 0 & (IV) \end{bmatrix}$$

(ii) (II) $\begin{bmatrix} i=N+1,.....2N \\ j=N+1,.....2N, \end{bmatrix}$ (III) $\begin{bmatrix} i=2N+1,.....3N \\ j=2N+1,.....3N, \end{bmatrix}$ (IV) $\begin{bmatrix} i=3N+1.....4N \\ j=3N+1.....4N \end{bmatrix}$

For (II),(III),(IV)

$$T_{ij} = \begin{cases} 0 & i<j \\ 1 & i=j \\ \sum_{k=j}^{i-1} \frac{b_{ik}\, S_k}{S_i - S_j} T_{kj} & i>j \end{cases}$$

For (V) $\begin{bmatrix} i=N+1,\ldots,2N \\ j=1,2,\ldots N \end{bmatrix}$

$$T_{ij} = \begin{cases} 0 & i-N<j \\ 0 & i-N=j \\ \sum_{k=j}^{i-N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} & i-N>j \text{ and } j>i-N-2 \\ \sum_{k=j}^{i-N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} + \sum_{k=j+1}^{i-N-1} \frac{b_{i,k+N}\,S_{k+N}}{S_i-S_j}\,T_{k+N,j} & i-N>j \;\&\; j<i-N-2 \end{cases}$$

(VI) $\begin{bmatrix} i=2N+1,\ldots.3N \\ j=1,\ldots.N \end{bmatrix}$

$$T_{ij} = \begin{cases} 0 & i-2N<j \\ 0 & i-2N=j \\ \sum_{k=j}^{i-2N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} & i-2N>j \text{ and } j>i-2N-2 \\ \sum_{k=j}^{i-2N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} + \sum_{k=j+1}^{i-2N-1} \frac{b_{i,k+2N}\,S_{k+2N}}{S_i-S_j}\,T_{k+2N,j} & i-2N>j \;\&\; j<i-2N-2 \end{cases}$$

(VII) $\begin{bmatrix} i=3N+1,\ldots.,4N \\ j=1,2,\ldots.,N \end{bmatrix}$

$$T_{ij} = \begin{cases} 0 & i-3N<j \\ 0 & i-3N=j \\ \sum_{k=j}^{i-3N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} & i-3N>j \text{ and } j>i-3N-2 \\ \sum_{k=j}^{i-3N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} + \sum_{k=j+1}^{i-3N-1} \frac{b_{i,k+3N}\,S_{k+3N}}{S_i-S_j}\,T_{k+3N,j} & i-3N>j \text{ and } j<i-3N-2 \end{cases}$$

(VIII) $\begin{bmatrix} i=2N+1,\ 2N+2,\ldots.3N \\ j=N+1,\ N+2,\ldots.2N \end{bmatrix}$

$$T_{ij} = \begin{cases} 0 & i-N<j \\ 0 & i-N=j \\ \sum_{k=j}^{i-N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} & i-N>j \text{ and } j>i-N-2 \\ \sum_{k=j}^{i-N-1} \frac{b_{ik}\,S_k}{S_i-S_j}\,T_{kj} + \sum_{k-j+1}^{i-N-1} \frac{b_{i,k+N}\,S_{k+N}}{S_i-S_j}\,T_{k+N,j} & i-N>j \text{ and } j<i-N-2 \end{cases}$$

$$(IX)\quad \begin{cases} i=3N+1,\ 3N+2.....,4N \\ j=N+1,N+2,.....,2N \end{cases}$$

$$Tij = \begin{cases} 0 & i-2N<j \\ 0 & i-2N=j \\ \sum_{k=j}^{i-2N-1} \frac{b_{ik}\ S_k}{S_i-S_j}\ T_{kj} & i-2N>j \text{ and } j>i-2N-2 \\ \sum_{k=j}^{i-2N-1} \frac{b_{ik}\ S_k}{S_i-S_j}\ T_{kj} + \sum_{k=j+1}^{i-2N-1} \frac{b_{i,k+2N}\ S_{k+2N}}{S_i-S_j}\ T_{k+2N,j} & i-2N>j \text{ and } j<i-2N-2 \end{cases}$$

(iii) For Partition Section (I)

$$R = -\ [I-B^{AB1-AB1}] - B^{AB2-AB1}[I-B^{AB2-AB2}]^{-1} B^{AB1-AB2}\ S^{AB1}$$

$$I-B^{AB2-AB2} = \begin{cases} 0 & i<j \\ 1 & i=j \\ b_{ij}^{AB2-AB2} & i>j \end{cases} \quad \text{is low triangular matrix}$$

Let $[I-B^{AB2-AB2}]^{-1} = (b_{ij}^{\alpha})$

$$\text{then } b_{ij}^{\alpha} = -\sum_{k=j}^{i-1} b_{ik}^{AB2-AB2}\ b_{jk} \Big/ b_{ii}^{AB2-AB2} \qquad i>j$$

$$[I-B^{AB2-AB2}]^{-1} = (b_{ij}^{\alpha}) = \begin{cases} 0 & i<j \\ 1 & i=j \\ -\sum_{k=j}^{i-1} b_{ik}^{AB2-AB2} b_{kj}^{\alpha} & i>j \end{cases}$$

$$\text{Since } B^{AB2-AB1} = \begin{cases} 0 & i<j \\ b_{ij}^{AB2-AB1} & i>j \end{cases} \quad \text{And } B^{AB1-AB2} = \begin{cases} 0 & i<j \\ b_{ij}^{AB1-AB2} & i>j \end{cases}$$

$$B^{AB2-AB1}\ [I-B^{AB2-AB2}]^{-1} B^{AB1-AB2} = \begin{cases} 0 & i<j \\ \sum_{k=j}^{i-1} \left[b_{ik}^{AB_2AB_1} \sum_{m=j}^{k-1} \left(-\sum_{l=m}^{k-1} b_{kl}^{AB2-AB2}\ b_{lm}^{\alpha}\right) b_{mj}^{AB1-AB2} \right] \end{cases}$$

$$\text{or} \quad b_{ij}^{\#} = -\sum_{k=j}^{i-1} \sum_{m=j}^{k-1} \sum_{l=m}^{k-1} b_{ik}^{AB2-AB1}\ b_{kl}^{AB2-AB2}\ b_{lm}^{\alpha}\ b_{mj}^{AB1-AB2} \qquad i>j$$

$$R = \begin{cases} 0 & i<j \\ 1 & i=j \\ (b_{ij}^{AB1-AB1} + b_{ij}^{\#}) = b_{ij}^{*} & i>j \end{cases}$$

For section (I)

$$\begin{cases} i=1,---N \\ j=1,---N \end{cases}$$

$$T_{ij} = \begin{cases} 0 & i<j \\ 1 & i=j \\ \sum_{k=j}^{i-1} \frac{b_{ik}^{*}\ S_k}{s_i-s_j}\ T_{kj} & i>j \end{cases}$$