BLIND IDENTIFICATION OF BILINEAR SYSTEMS

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ABSTRACT

This paper is concerned with the blind identification of bilinear systems excited by higher-order white noise. Unlike prior work that restricted the bilinear system model to simple forms and required the excitation to be Gaussian distributed, the results of this paper are applicable to a more general class of bilinear systems and for the case when the excitation is non-Gaussian. We describe an estimation procedure for the computation of the system parameters using output cumulants of order less than four.

1. INTRODUCTION

We consider bilinear systems of the form

$$y(n) = \sum_{i=1}^{K_a} a(i)y(n-i) + \sum_{i=0}^{K_b} b(i)u(n-i) + \sum_{j=1}^{K_{cu}} \sum_{i=j}^{K_{cy}} c(i,j)y(n-i)u(n-j) + v(n) \quad (1)$$

where $y(n)$ is the output of the system, $u(n)$ its input and $v(n)$ the measurement noise. We assume that $b(0) = 1$, $c(K_{cu}, K_{cu}) \neq 0$, $K_{cu} \geq K_b$, and C, the matrix of coefficients $c(i,j)$, is lower triangular. We assume also that the measurement noise $v(n)$ belongs to a Gaussian white process with zero mean value and is independent of the input, and that the input signal $u(n)$ is a higher-order white process that has zero mean value and is not necessarily Gaussian distributed. The cumulants of $u(n)$ are given by

$$\mathrm{cum}[u(n), u(n-l_1), \cdots, u(n-l_{k-1})] = \gamma_k \delta(l_1, \cdots, l_{k-1}) \quad (2)$$

where $\delta(l_1, \cdots, l_{k-1})$ is the $(k-1)$ dimensional unit impulse signal and γ_k denotes the signal intensity of order k. The coefficients $a(i)$, $b(i)$ and $c(i,j)$ are such that $y(n)$ is a stationary process. Sufficient conditions for the stationarity of bilinear processes are derived in [1, 4, 5]. The blind identification problem addressed in this paper is formulated as follows. Given the system orders K_a, K_b, K_{cu} and K_{cy} and the output statistics, determine the system parameters $a(i)$, $b(i)$ and $c(i,j)$, as well as the higher-order signal intensities γ_2, γ_3, γ_4 and γ_5 associated with the input signal $u(n)$.

Closed form expressions that relate measurable statistics of the output signal to the parameters that must be estimated are available only for a very restricted class of nonlinear system models [6, 7]. Consequently, the most common approach to estimating the parameters of the model is to resort to some form of numerical search algorithm that operates in an iterative manner [8]. This paper describes a set of equations that relate parameters of the system in (1) to cumulants of the output signal up to order four, and then provides a direct method for estimating the parameters based on these relationships.

The rest of this paper is organized as follows. The next section describes the blind estimation procedure. The estimation is performed in a sequential manner. The coefficients $a(i)$ are estimated first, and the estimated parameter values are employed to find the remaining parameters. Because of space limitations, we only describe the methodology employed in our approach. Complete derivations will be presented in [3]. The concluding remarks are made in Section 3.

2. BLIND ESTIMATION OF THE PARAMETERS

2.1. Estimation of $a(i)$

The first stage of the blind estimation algorithm involves the computation of the $a(i)$ parameters through the output covariance $c_y^{(2)}(l) = \mathrm{cum}[y(n), y(n-l)]$. It is relatively straightforward to show that the covariance function of the output signal satisfies the following results for sufficiently large values of the lag:

Proposition 1. Let $L > K = \max(K_b, K_{cu})$. Then, the output covariance sequence satisfies the autoregressive model

$$c_y^{(2)}(L) = \sum_{i=1}^{K_a} a(i) c_y^{(2)}(L-i). \quad (3)$$

Proposition 1 states that the output covariance sequence behaves in a manner that is identical to the covariance function of an autoregressive signal for sufficiently large values of the lag l. This property enables the computation of the $a(i)$ parameters with the aid of the second-order statistics and a linear system Toeplitz solver such as a variant of the Levinson algorithm [2].

Estimation of the $c(i,j)$ coefficients relies on the third and fourth order cumulant sequences

$$c_y^{(3)}(l_1,l_2) = \mathrm{cum}[y(n), y(n-l_1), y(n-l_2)] \qquad (4)$$

and

$$c_y^{(4)}(l_1,l_2,l_3) = \mathrm{cum}[y(n), y(n-l_1), y(n-l_2), y(n-l_3)]. \quad (5)$$

The behavior of the cumulants of order 3 or 4 when all arguments l_1, l_2 and l_3 are large is easy to predict. Similar to the results in Proposition 1, we can show that $c_y^{(3)}(l_1,l_2)$ and $c_y^{(4)}(l_1,l_2,l_3)$ satisfy the same autoregressive model:

$$c_y^{(3)}(L_1,L_2) = \sum_{i=1}^{K_a} a(i) c_y^{(3)}(L_1-i, L_2-i); \qquad L_1, L_2 > K \qquad (6)$$

and

$$c_y^{(4)}(L_1,L_2,L_3) = \sum_{i=1}^{K_a} a(i) c_y^{(4)}(L_1-i, L_2-i, L_3-i); \qquad L_1, L_2, L_3 > K. \quad (7)$$

2.2. Estimation of the Last Column of C

To estimate the remaining parameters $b(i)$ and $c(i,j)$ requires the derivation of relationships that relate the cumulants for smaller lag values through these coefficients. To derive such relationships in a manageable fashion we hold one of the arguments at a large value and analyze the cumulants as the remaining lag values fall in the range $0 \le l \le K$. The following result allows the estimation of the last column of the coefficient matrix C.

Proposition 2. Let

$$D_2(l) = -\sum_{i=0}^{K_a} a(i) c_y^{(2)}(l-i) \qquad a(0) = -1, \qquad (8)$$

$$D_3(l_1,l_2) = -\sum_{i=0}^{K_a} a(i) c_y^{(3)}(l_1-i, l_2-i) \qquad (9)$$

and

$$D_4(l_1,l_2,l_3) = -\sum_{i=0}^{K_a} a(i) c_y^{(4)}(l_1-i, l_2-i, l_3-i). \quad (10)$$

Then

$$D_3(K,L) = \gamma_2 \sum_{i=K_{cu}}^{K_{cy}} c(i,K_{cu}) c_y^{(2)}(L-i) \qquad (11)$$

and

$$D_4(K,K,L) = 2\gamma_2 \sum_{i=K_{cu}}^{K_{cy}} c(i,K_{cu}) c_y^{(3)}(K-i, L-i) + \frac{\gamma_3}{\gamma_2} D_3(K,L). \qquad (12)$$

The quantities $D_m(\cdot)$ can be computed from the output cumulants since the parameters $a(i)$ have already been computed. Application of (11) for $K_{cy} - K_{cu} + 1$ different values of L enables us to determine the last column of C scaled by the input variance γ_2. Furthermore, evaluation of (12) at a value of L such that $D_3(K,L) \ne 0$, enables us to determine the ratio γ_3/γ_2.

2.3. Estimation of $b(K)$

We analyze the statistics of the output signal sequentially, starting with lag values of K, and then descending to smaller lag values. The following expressions provide useful information for lag values equal to K.

$$D_2(K) = \gamma_2 b(K) + \bar{y}\gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) + \gamma_3 c(K,K) \qquad (13)$$

$$D_3(K,K) = \gamma_3 b(K) + \bar{y}\gamma_3 \sum_{i=K}^{K_{cy}} c(i,K) + \gamma_4 c(K,K) + 2\gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(2)}(K-i) \qquad (14)$$

$$D_4(K,K,K) = \gamma_4 b(K) + \bar{y}\gamma_4 \sum_{i=K}^{K_{cy}} c(i,K) + \gamma_5 c(K,K) + 3\gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(3)}(K-i, K-i) + 3\gamma_3 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(2)}(K-i) \qquad (15)$$

In the above expressions $\bar{y}$ denotes the mean value of $y(n)$. Equation (13) yields $\gamma_2 b(K)$. Equations (14) and (15) provide γ_4/γ_2 and γ_5/γ_2.

2.4. Estimation of Other Parameters

The remaining calculations require the definition of the following parameters:

$$S_3(m,L) = -\sum_{l=m}^{K} a(l-m) D_3(l,L), \qquad 1 \le m \le K-1 \qquad (16)$$

and

$$S_4(m,K,L) = -\sum_{l=m}^{K} a(l-m) D_4(l,K,L), \qquad 1 \le m \le K-1. \qquad (17)$$

The estimation of the remaining columns of C relies on expressions of the following form:

$$
\begin{aligned}
& S_4(K-1,K,L) + a(1)\gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(3)}(K-i,L-i) \\
& -\gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(3)}(K-1-i,L-i) = \\
& \sum_{i=1}^{K_{cy}} \Big[D_2(K) c_y^{(3)}(1-i,L-K+1-i) + \\
& \left(D_3(K,K) - \gamma_2 \sum_{n=K}^{K_{cy}} c(n,K) c_y^{(2)}(K-n) \right) \times \\
& \times c_y^{(2)}(L-K+1-i) + \\
& \gamma_2 \sum_{n=K}^{K_{cy}} c(n,K) \times \\
& \times c_y^{(4)}(n-K+1-i,1-i,L-K+1-i) + \\
& +\gamma_3 \sum_{n=K}^{K_{cy}} c(n,K) c_y^{(3)}(n-K+1-i,L-K+1-i) \\
& +\gamma_4 \sum_{n=K}^{K_{cy}} c(n,K) c_y^{(2)}(L-n)\delta(i-1) + \\
& \gamma_2 \sum_{n=K}^{K_{cy}} c(n,K) c_y^{(2)}(L-n) c_y^{(2)}(1-i) + \\
& \bar{y}\gamma_3 \sum_{n=K}^{K_{cy}} c(n,K) c_y^{(2)}(L-n) + \\
& +\gamma_3 \sum_{n=K}^{K_{cy}} c(n,K) c_y^{(3)}(K-n,L-n)\delta(i-1) + \\
& \bar{y}\gamma_2 \sum_{n=K}^{K_{cy}} c(n,K) c_y^{(3)}(K-n,L-n) \Big] c(i,1) + \\
& +\gamma_2 c(K-1,K-1) c_y^{(3)}(1,L-K+1) + \\
& \sum_{i=K}^{K_{cy}} \left(\gamma_2 c(i,K-1) + b(1)\gamma_2 c(i,K) \right) c_y^{(3)}(K-i,L-i) + \\
& + \left(b(1)\frac{\gamma_3}{\gamma_2} + \gamma_2 c(1,1) \right) \gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(2)}(L-i). \qquad (18)
\end{aligned}
$$

The above equation evaluated for different choices of L leads to a linear system of equations with the following groups of unknowns:

1. The first column of C, $c(i,1)$
2. The term $\gamma_2 c(K-1,K-1)$
3. A linear combination of the last two columns of C: $\gamma_2 c(i,K-1) + b(1)\gamma_2 c(i,K)$.
4. The term $b(1)\frac{\gamma_3}{\gamma_2} + \gamma_2 c(1,1)$.

We estimate the four unknown groups of parameters by solving the system of linear equations resulting from (18). In group 4 we have one linear equation with unknowns $b(1)$ and γ_2, since γ_3/γ_2 and $c(1,1)$ are already available from prior calculations. A second equation for $b(1)$ and γ_2 is obtained using an expression similar to (18) for $S_3(K-1,K)$ and given by

$$
\begin{aligned}
& S_3(K-1,K) + a(1)\gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(2)}(K-i) - \\
& \gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) \sum_{n=1}^{k_{cy}} c(n,1) c_y^{(3)}(i-K+1-n,1-n) - \\
& \gamma_3 \sum_{i=K}^{K_{cy}} c(i,K) \sum_{n=1}^{k_{cy}} c(n,1) c_y^{(2)}(i-K+1-n) - \\
& \gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(2)}(K-1-i) - \\
& \gamma_5 c(K,K) c(1,1) - \gamma_3 c(K,K) \sum_{i=1}^{K_{cy}} c(i,1) c_y^{(2)}(1-i) - \\
& \gamma_4 c(K,K)\bar{y} \sum_{i=1}^{K_{cy}} c(i,1) - \gamma_2 c(K-1,K-1) c_y^{(2)}(1) - \\
& - \sum_{i=K}^{K_{cy}} \left(\gamma_2 c(i,K-1) + b(1)\gamma_2 c(i,K) \right) c_y^{(2)}(K-i) \\
& -c(1,1)\gamma_3 \sum_{i=K}^{K_{cy}} c(i,K) c_y^{(2)}(K-i) - \\
& -\bar{y} \sum_{i=1}^{K_{cy}} c(i,1)\gamma_2 \sum_{n=k}^{k_{cy}} c(n,K) c_y^{(2)}(K-n) \\
& -c(1,1)\gamma_4 \left(b(K) + \bar{y} \sum_{i=K}^{K_{cy}} c(i,K) \right) - \\
& - \sum_{i=1}^{K_{cy}} c(i,1)\gamma_2 \left(b(K) + \bar{y} \sum_{n=K}^{K_{cy}} c(n,K) \right) c_y^{(2)}(1-i) - \\
& \bar{y}\gamma_3 \left(b(K) + \bar{y} \sum_{i=K}^{K_{cy}} c(i,K) \right) \sum_{n=1}^{K_{cy}} c(n,1) \\
& = b(1) \left[\gamma_4 c(K,K) + \gamma_3 \left(b(K) + \bar{y} \sum_{i=K}^{K_{cy}} c(i,K) \right) \right] + \\
& \gamma_2 \left[3\gamma_3 c(K,K) c(1,1) + c(1,1)\gamma_2 \left(b(K) + \bar{y} \sum_{i=K}^{K_{cy}} c(i,K) \right) \right] \\
& \qquad (19)
\end{aligned}
$$

The determinant of the above system is

$$
c(1,1)c(K,K)\left(\gamma_4 - \frac{3\gamma_3^2}{\gamma_2}\right)
$$

Provided that $c(1,1) \neq 0$, $c(K,K) \neq 0$ and $\gamma_4 \neq 3\gamma_3^2/\gamma_2$, $b(1)$ and γ_2 are uniquely determined. It is shown in [3] that the above assumption can be relaxed if we use cumulants of order 5. Having determined $b(1)$ and γ_2 we return to groups 2 and 3 and compute the $K-1$ columns of C. Finally $b(K-1)$ is obtained from $S_2(K-1)$ as

$$\begin{aligned}
&b(K-1)\gamma_2 = \\
&S_2(K-1) - \gamma_3 c(K-1,K-1) - \bar{y}\gamma_2 c(K-1,K-1) \\
&-\bar{y}\sum_{i=K}^{K_{cy}} (\gamma_2 c(i,K-1) + b(1)\gamma_2 c(i,K)) \\
&-D_2(K)\bar{y}\sum_{i=1}^{K_{cy}} c(i,1) \\
&-\gamma_2 \sum_{i=K}^{K_{cy}} c(i,K) \sum_{n=1}^{k_{cy}} c(n,1) c_y^{(2)}(i-K+1-n) \\
&-c(1,1)\left[\gamma_3\left(b(K) + \bar{y}\sum_{i=K}^{K_{cy}} c(i,K)\right) + \gamma_4 c(K,K)\right] \\
&-b(1)\left(\gamma_2 b(K) + \gamma_3 c(K,K)\right) - \gamma_2\gamma_2 c(K,K)c(1,1). \quad (20)
\end{aligned}$$

In summary, (18)-(20) lead to the estimation of the first column of C, $c(i,1)$, the $(K-1)$th column of C, $c(i,K-1)$, $b(1)$, $b(K-1)$ and γ_2. The remaining parameters can be determined in a similar manner through successive evaluation of $S_4(l,K,L)$, $S_3(l,K)$ and $S_2(l)$ for $l = K-2, K-3, \cdots$.

3. CONCLUDING REMARKS

This paper has dealt with the blind identification of bilinear systems from measurements of the output signals. The parameters are determined via a sequence of linear systems involving cumulants up to order four. Unlike prior work that restricted the bilinear system model to simple forms and required the excitation to be Gaussian distributed [6, 7], the results of this paper are applicable to a more general class of bilinear systems and for the case when the excitation is non-Gaussian.

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