Remarks on noncommutative open string theory: *V* duality and holography

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In this paper we study the interplay of electric and magnetic backgrounds in determining the decoupling limit of coincident *D* branes towards a noncommutative Yang-Mills (NCYM) or open string (NCOS) theory. No decoupling limit has been found for the NCYM model with space-time noncommutativity. It is suggested that there is a new duality, which we call *V duality*, which acts on NCOS theory with both space-space and space-time noncommutativity, resulting from decoupling in Lorentz-boost related backgrounds. We also show that the holographic correspondence, previously suggested by Li and Wu, between the NCYM model and its supergravity dual can be generalized to NCOS theory as well.

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I. INTRODUCTION

A noncommutative space or space time is one with noncommuting coordinates, satisfying

$$[x^{\mu},x^{\nu}]=i\theta^{\mu\nu} \quad \mu,\nu=0,1,2,\ldots, \tag{1}$$

where $\theta^{\mu\nu}$ are antisymmetric and real parameters of dimension length squared. A field theory on such a space can be formulated using a representation, in which the coordinates x^{μ} are the same as usual, but the product of any two fields of x^{μ} is deformed to the Moyal star product:

$$f*g(x)=\exp(i/2)\theta^{\mu\nu}\partial_{\mu}^{x}\partial_{\nu}^{y}f(x)g(y)|_{y=x}, \tag{2}$$

while the commutator in Eq. (1) is understood as the Moyal brackets with respect to the star product:

$$[x^{\mu},x^{\nu}]\equiv x^{\mu}*x^{\nu}-x^{\nu}*x^{\mu}. \tag{3}$$

Recently it has been shown that Yang-Mills theory (or open string theory) on such noncommutative space (or space time), which we will abbreviate as NCYM (or NCOS) theory, arises naturally in string or M(atrix) theory on coincident *D*-brane world-volume in antisymmetric tensor backgrounds in certain scaling limits [decoupling or discrete light cone quantization limits] [1–5]. (In order to obtain a nontrivial theory defined only on the brane world volume, these scaling limits require that in addition to the usual $\alpha'\to 0$ limit, certain components of the closed-string metric and/or those of the background parallel to the brane world volume should also be scaled in appropriate way. For details, see Refs. [3–5].) This strongly suggests that space-space or even space-time noncommutativity could be a general feature of the unified theory of quantum gravity at a generic point inside the moduli space of string or M theory. Though perhaps not every noncommutative field (or string) theory is a consistent quantum theory on its own, there is a belief that noncommutative field (or string) theories that can arise as effective limits in fundamental string theory should be consistent quantum theory on their own. Up to now, only NCYM theory with space-space noncommutativity and NCOS theory with space-time noncommutativity have been obtained by taking certain decoupling limits in string theory. It is important to clarify whether there exist decoupling limits in string theory with backgrounds that lead to either NCYM or NCOS with both space-space and space-time noncommutativity. Namely, one wants to know how big the moduli space is for NCYM and NCOS that can arise from string theory.

Constant bulk *B* background in string theory in topologically trivial space time can be gauged away, while inducing constant gauge field background on the *D*-brane world volume. One might wonder whether the nature, electric or magnetic, of the gauge background would affect the scaling limit that decouples the theory on the *D* branes from closed strings in the surrounding bulk. From the Born-Infeld action it is known that on the *D* brane no electric field can be stronger than a critical electric field, while the same is not true for magnetic fields. Indeed recent careful re-examination of the decoupling limits shows that though the decoupling limit in a magnetic background always results in NCYM with only space-space noncommutativity [3], in an electric background the decoupling limit becomes different and leads to NCOS with only space-time noncommutativity [4–6]. Moreover, theory with space-time noncommutativity is expected to behave very differently from one with only space-space noncommutativity. Recently, whether a field theory with space-time noncommutativity is unitary, has been questioned in the literature [7,8]. All these inspire the following questions: What would happen if there are both electric and magnetic backgrounds? Could an NCYM with space-time noncommu-

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tativity, or an NCOS with both space-space and space-time noncommutativity, arise in favorable situations?

The present paper will address the problem of the interplay of constant electric and magnetic backgrounds in determining the decoupling limit towards a noncommutative theory on the D brane world volume. To simplify, we will restrict ourselves to the case of the $D3$ brane(s). Generalizing to other Dp branes should be straightforward. We will consider two special cases, in which the electric and magnetic backgrounds are either *parallel* or *perpendicular* to each other. It is known that the endpoints of an open string behave like (opposite) charges on the $D3$ brane, and the motion of a charge in the above two background configurations is very different. So we expect that there should be important differences between the decoupling limits in the above two cases. As we will show in Sec. II, in either case *no* decoupling limit can be found to lead to an *NCYM with space-time* noncommutativity. On the other hand, in Sec. III we will show that in favorable situations an appropriate decoupling limit may result in *NCOS with both space-space and space-time* noncommutativity.

In electrodynamics it is known that Lorentz boosts act on constant electromagnetic backgrounds. Through the decoupling limit the latter, in turn, affects the noncommutativity parameters that define the resulting NCOS. Thus, the NCOS that result from Lorentz-boost related backgrounds should be equivalent to each other, describing the same decoupled D-brane system. We will call this exact equivalence among NCOS *V duality*, which can be viewed as the fingerprint of the antecedent Lorentz-boost action surviving the decoupling limit. In Sec. IV we will identify some orbits of V duality in the moduli space of NCOS (with both space-space and space-time noncommutativity).

Previously Li and one of us [9] have shown that there is a running holographic correspondence between NCYM and its gravity dual. Namely, the radial dependence of the profile of Neveu-Schwarz–Neveu-Schwarz (NSNS) fields in the gravity dual of an NCYM can be derived from the Seiberg-Witten relations [3] between close string moduli and open string moduli, provided that the string tension is running with a simply prescribed dependence on the energy scale, which is identified with the radial coordinate by the well-known ultraviolet-infrared (UV-IR) relation [10]. We will show in Sec. V that the Li-Wu holography argument can be generalized to NCOS, though with a different prescription for the running string tension.

II. DECOUPLING LIMIT FOR NCYM

In this section, we concentrate on the decoupling limit for NCYM, when the gauge background $B_{\mu\nu}$ on a flat Dp brane world volume (with a constant metric $g_{\mu\nu}$) has both electric and magnetic components. For definiteness, we consider the case with $p=3$. To be specific, we restrict ourselves to the special cases when the electric and magnetic fields are either perpendicular or parallel to each other. The generalization to the most general configuration should be straightforward.

A constant B-background on the D brane does not affect the equations of motion for open strings, while it changes the open string boundary conditions to

$$g_{\mu\nu}\partial_n X^\nu + 2\pi\alpha' B_{\mu\nu}\partial_s X^\nu|_{\partial\Sigma} = 0, \tag{4}$$

where the operators ∂_n and ∂_s are the derivatives normal and tangential to the worldsheet boundaries $\partial\Sigma$. For the disk topology, the propagator along the boundary is known to be [11,12]

$$\langle x^\mu(\tau) x^\nu(0)\rangle = -\alpha' G^{\mu\nu}\ln(\tau^2) + i\frac{\theta^{\mu\nu}}{2}\varepsilon(\tau). \tag{5}$$

As emphasized by Seiberg and Witten in Ref. [3], the physics behind these equations is that the moduli ($G_{\mu\nu}$, $\theta^{\mu\nu}$, G_s) seen by open string ends on the D brane are very different from those ($g_{\mu\nu}$, $B_{\mu\nu}$, g_s) seen by close strings; they are related by the following elegant relations [3]

$$G_{\mu\nu} = g_{\mu\nu} - (2\pi\alpha')^2 (Bg^{-1}B)_{\mu\nu}, \tag{6}$$

$$G^{\mu\nu} = \left(\frac{1}{g+2\pi\alpha' B}\right)_S^{\mu\nu}, \tag{7}$$

$$\theta^{\mu\nu} = 2\pi\alpha' \left(\frac{1}{g+2\pi\alpha' B}\right)_A^{\mu\nu}, \tag{8}$$

$$G_s = g_s\left(\frac{\det G_{\mu\nu}}{\det(g_{\mu\nu}+2\pi\alpha' B_{\mu\nu})}\right)^{1/2}, \tag{9}$$

where $()_S$ and $()_A$ denote, respectively, the symmetric and antisymmetric parts, and G_s, g_s the open string and closed string coupling [13,14].

For the purely magnetic case (with $B_{0i}=0$), the scaling limit that decouples the theory on the D brane from closed strings in the bulk has been analyzed in Ref. [3], and may be summarized as a limit subject to the following conditions: (1) $\alpha'\to 0$, (2) $G_{\mu\nu}$ is finite, and (3) $\theta^{\mu\nu}$ is finite. The decoupling limit results in an NCYM on the D brane world volume. For the purely electric case (with $B_{ij}=0$), the above decoupling limit has been shown [4] not to exist, because of the existence of a critical electric-field strength. In the following, we will carry out an analysis for the case with both electric and magnetic components present in the antisymmetric tensor $B_{\mu\nu}$. The interplay between the electric background (**E**) and the magnetic background (**B**) is worthwhile to explore, since in the presence of both an electric field and a magnetic field the dynamical behavior of a point charge, representing an endpoint of the open string, is known to be very different from the case in either a purely magnetic or a purely electric field. For simplicity, we assume that either $\mathbf{E}\perp\mathbf{B}$ or $\mathbf{E}\|\mathbf{B}$. In this section, we discuss whether a decoupling limit leading to NCYM exists in these two cases.

First, let us consider the case with $B_{01}=E$ and $B_{12}=B$, all other components being zero, namely, the tensor $B_{\mu\nu}$ takes the form (for $\mu,\nu=0,1,2$)

$$B_{\mu\nu}=\begin{pmatrix} 0 & E & 0 \\ -E & 0 & B \\ 0 & -B & 0 \end{pmatrix}. \tag{10}$$

The closed string metric $g_{\mu\nu}$ is taken to be of the diagonal form

$$g_{\mu\nu}=\begin{pmatrix} -g_0 & 0 & 0 \\ 0 & g_1 & 0 \\ 0 & 0 & g_2 \end{pmatrix}. \tag{11}$$

For convenience, we follow Refs. [4,5] to introduce the critical value, E_c, of the electric field

$$E_c=\frac{\sqrt{g_0 g_1}}{2\pi\alpha'}. \tag{12}$$

Substituting Eq. (10) and Eq. (11) into the Seiberg-Witten relations Eqs. (6)–(9), we get

$$G_{\mu\nu}=\begin{pmatrix} -g_0(1-e^2) & 0 & -g_0 eb \\ 0 & g_1(1-e^2)+\dfrac{g_0 g_1}{g_2}b^2 & 0 \\ -g_0 eb & 0 & g_2+g_0 b^2 \end{pmatrix}, \tag{13}$$

$$\theta^{\mu\nu}=\frac{1}{[g_2(1-e^2)+g_0 b^2]E_c}\begin{pmatrix} 0 & g_2 e & 0 \\ -g_2 e & 0 & -g_0 b \\ 0 & g_0 b & 0 \end{pmatrix}, \tag{14}$$

$$G_s=g_s\sqrt{1+\frac{g_0}{g_2}b^2-e^2}, \tag{15}$$

where the dimensionless electric and magnetic-field strength are given by

$$e=\frac{E}{E_c}, \quad b=\frac{B}{E_c}. \tag{16}$$

To get an NCYM, we need to take $\alpha'\to 0$ to decouple massive open string excitations, while keeping the open string moduli $G_{\mu\nu}$, $\theta^{\mu\nu}$, and G_s finite. Inspection of Eqs. (13) and (14) shows that the following conditions provide the only possible solution for the NCYM limit:

(1) $|e|<1$;

(2) $B=bE_c=1/\theta$ finite;

(3) $g_0=1$, $g_1=g_2=g\sim(\alpha')^2$, so that formally E_c is a finite parameter; for later convenience, to normalize open string metric to $G_{11}=G_{22}=1$, one may take $g=(2\pi\alpha' B)^2$;

(4) $g_s\sim\alpha'$ to keep G_s finite.

This solution is unique up to finite separate rescaling for g_0, g_1, and g_2. It is easy to verify that in this limit

$$\theta^{0i}=0, \quad \theta^{12}=-\theta. \tag{17}$$

Therefore the resulting field theory has only space-space noncommutativity. Though E or e does not affect the noncommutativity parameters $\theta^{\mu\nu}$, it does make the open string metric $G_{\mu\nu}$ nondiagonal, i.e., it makes the x_0- and x_2- axes oblique with respect to open string metric. The appearance of the off-diagonal G_{02} is not surprising: the open string endpoint, behaving like a charge, acquires a drift velocity in the x_2 direction in the present cross-field background with $E_1=E$ and $B_3=-B$.

In this way, we see that the scaling limit of NCYM is incompatible with space-time noncommutativity. This is just right, since field theory with space-time noncommutativity is potentially nonunitary [7,8]. A similar analysis can be done for the case with $\mathbf{E}\|\mathbf{B}$, again resulting in an NCYM with vanishing θ_{0i}.

III. DECOUPLING LIMIT OF NCOS

In this section, we present a new decoupling limit of NCOS to demonstrate the interplay between the electric and magnetic components of the background.

A. The E⊥B case

To achieve this goal, we take in the closed string metric Eq. (11) $g_0=g_1=g$, this leads to corresponding open string moduli by using Eqs. (12)–(15):

$$G_{\mu\nu}=\begin{pmatrix} -g(1-e^2) & 0 & -geb \\ 0 & g(1-e^2)+\dfrac{g^2b^2}{g_2} & 0 \\ -geb & 0 & g_2+gb^2 \end{pmatrix}, \tag{18}$$

$$\theta^{\mu\nu}=\frac{2\pi\alpha'}{g_2 g(1-e^2)+g^2b^2}\begin{pmatrix} 0 & g_2 e & 0 \\ -g_2 e & 0 & -gb \\ 0 & gb & 0 \end{pmatrix}. \tag{19}$$

$$G_s=g_s\sqrt{1-e^2+\frac{gb^2}{g_2}} \tag{20}$$

In taking the decoupling limit for NCOS, α' is kept fixed, while $G_{\mu\nu}$ and $\theta^{\mu\nu}$ have to have a finite limit. To achieve this goal, we introduce the following scaling limit: (1) $e\to 1$, with $g(1-e^2)=2\pi\alpha'/\theta_0$ finite; (2) g_2 is finite; for convenience, we take $g_2=1$; (3) $b\to 0$, with $gb=2\pi\alpha'/\theta_1$ finite.

With this scaling limit, we get the moduli of the resulting NCOS as follows: the metric

$$G_{\mu\nu}=\begin{pmatrix} -\frac{2\pi\alpha'}{\theta_0} & 0 & -\frac{2\pi\alpha'}{\theta_1} \\ 0 & \frac{2\pi\alpha'}{\theta_0}+\left(\frac{2\pi\alpha'}{\theta_1}\right)^2 & 0 \\ -\frac{2\pi\alpha'}{\theta_1} & 0 & 1 \end{pmatrix}, \tag{21}$$

and the noncommutativity matrix

$$\theta^{\mu\nu}=\frac{2\pi\alpha'}{\frac{2\pi\alpha'}{\theta_0}+\left(\frac{2\pi\alpha'}{\theta_1}\right)^2}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -\frac{2\pi\alpha'}{\theta_1} \\ 0 & \frac{2\pi\alpha'}{\theta_1} & 0 \end{pmatrix}. \tag{22}$$

This scaling limit is striking in that it results in NCOS with both space-time and space-space noncommutativity. Note that this is different from the NCYM limit, where space-time noncommutativity cannot result from the decoupling limit. Again, the appearance of nonzero off-diagonal elements $G_{02}=G_{20}$ in the NCOS metric is very natural; the end points of the open string behave like opposite charges which, in a cross field with E_1 and B_3, acquire a drift velocity in the x_2-direction independent of the sign of charges. (This is nothing but the classical picture of the Hall effect in condensed-matter physics.)

In the above scaling limit, the open string coupling G_s vanishes. To have an interacting theory, we can follow Ref. [4] to consider N coincident D branes, so that the effective open string coupling is

$$G_{eff}=NG_s=Ng_s\sqrt{1-e^2}. \tag{23}$$

In the large N limit, if we scale N as

$$N\sim\frac{1}{\sqrt{1-e^2}}, \tag{24}$$

we can keep the effective open string coupling G_{eff} finite.

In passing, we emphasize that the decoupling conditions (1) and (3) imply that the ratio between the electric and magnetic-field strength is greater than 1. In other words, in our decoupling scheme, the magnetic field is held to a finite value. (In fact, the parameter θ_1 is just $1/B$.) One may wonder what will be the NCOS scaling limit if one assumes $|B|>|E|$. The answer is that in this case, we do not have a consistent NCOS limit; rather we should take the NCYM limit, just as we have discussed in the last section.

B. The E∥B case

In this section, we study the other special case where the electric field is parallel to the magnetic field. The motivation is to show once more that the magnetic effects can survive the scaling limit for NCOS, resulting in space-space noncommutativity. To do so, we choose the closed string metric $g_{\mu\nu}$ and antisymmetric tensor field $B_{\mu\nu}$ as

$$g_{\mu\nu}=\begin{pmatrix} -g & 0 & 0 & 0 \\ 0 & g & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{25}$$

$$B_{\mu\nu}=\begin{pmatrix} 0 & E & 0 & 0 \\ -E & 0 & 0 & 0 \\ 0 & 0 & 0 & B \\ 0 & 0 & -B & 0 \end{pmatrix}. \tag{26}$$

Again, by using Seiberg-Witten relations Eqs. (6)–(9), we get the open string moduli

$$G_{\mu\nu}=\begin{pmatrix} -g(1-e^2) & 0 & 0 & 0 \\ 0 & g(1-e^2) & 0 & 0 \\ 0 & 0 & 1+g^2b^2 & 0 \\ 0 & 0 & 0 & 1+g^2b^2 \end{pmatrix}, \tag{27}$$

$$G^{\mu\nu}=\begin{pmatrix} -\frac{1}{g(1-e^2)} & 0 & 0 & 0 \\ 0 & \frac{1}{g(1-e^2)} & 0 & 0 \\ 0 & 0 & \frac{1}{1+g^2b^2} & 0 \\ 0 & 0 & 0 & \frac{1}{1+g^2b^2} \end{pmatrix}, \tag{28}$$

$$\theta^{\mu\nu}=2\pi\alpha'\begin{pmatrix} 0 & -\frac{e}{g(1-e^2)} & 0 & 0 \\ \frac{e}{g(1-e^2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{gb}{1+g^2b^2} \\ 0 & 0 & \frac{gb}{1+g^2b^2} & 0 \end{pmatrix}, \tag{29}$$

$$G_s=g_s\sqrt{1-e^2}\sqrt{1+g^2b^2}. \tag{30}$$

Here we adopted the same conventions for E_c, e, and b as in the previous section. From the above open string moduli, we see that the same decoupling limit as that in $\mathbf{E}\perp\mathbf{B}$ case can be applied. We also get the NCOS with both space-time and space-space noncommutativity. In contrast to the $\mathbf{E}\perp\mathbf{B}$ case, the effects of the magnetic field is to increase the effective open string coupling constant G_s by a factor of $\sqrt{1+g^2b^2}$ after we take the large N limit, without inducing a drift motion in other directions.

The most general configuration of $B_{\mu\nu}$ can be considered as a superposition of the two cases we have discussed, with $\mathbf{E}\perp\mathbf{B}$ and $\mathbf{E}\|\mathbf{B}$, respectively. So we conclude that in general, by the decoupling procedure, we can obtain NCYM with only space-space noncommutativity, or NCOS with both space-time and space-space noncommutativity.

IV. *V* DUALITY OF NCOS

In the previous section, in the case with $\mathbf{E}\perp\mathbf{B}$, we have managed to get a decoupling limit that leads to NCOS with both space-space and space-time noncommutativity, provided that $|\mathbf{E}|$ is greater than $|\mathbf{B}|$. In electrodynamics it is known that in this case, by a Lorentz boost, one can go to a favorable inertial frame in which the electromagnetic background becomes purely electric. If we start with this frame, the decoupling limit will give us an NCOS with only space-time noncommutativity. Before the decoupling limit, our string theory is known to have Lorentz symmetry, which allows us to transform the gauge-field background on the *D*-brane world volume without changing the physics. So the above argument implies that the NCOS theory with both space-space and space-time noncommutativity, that we obtained in the previous section for the case with $\mathbf{E}\perp\mathbf{B}$ and $|\mathbf{E}|>|\mathbf{B}|$, should be equivalent to an NCOS with only space-time noncommutativity. More generally, this argument suggests that NCOS theories resulting from electromagnetic backgrounds on the *D* brane that are related by Lorentz boosts, should be equivalent to each other. This is a duality among NCOS with different open string moduli, and it is related to Lorentz boosts depending on the relative *velocity* of the inertial frames. We call it *V duality*, so that alphabetically it follows the S, T, and U dualities we have had already.

An immediate question is how V duality acts on the open string moduli of NCOS? Now let us try to determine the orbit of the V-duality action in the moduli space of NCOS that we obtained in the last section. Let us start with two inertial frames K and K' on the world volume of $D3$ branes, with K' moving relative to K in the x_2 direction with velocity v. Suppose the antisymmetric tensor field $B_{\mu\nu}$ in K is purely electric:

$$B_{\mu\nu}=\begin{pmatrix} 0 & E & 0 & 0 \\ -E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{31}$$

Then the corresponding $B'_{\mu\nu}$ in K' has the form

$$B'_{\mu\nu}=\begin{pmatrix} 0 & E' & 0 & 0 \\ -E' & 0 & -B' & 0 \\ 0 & B' & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{32}$$

To relate E' and B' with E, we need to know the transformation between K and K'. Note that we have taken the metric in both K and K' to be

$$ds^2=-g(dx_0^2-dx_1^2)+(dx_2^2+dx_3^2). \tag{33}$$

To make this metric invariant, the transformation should be the following ‘‘adapted’’ Lorentz one:

$$x_2'=\gamma(x_2-v\sqrt{g}x_0), \tag{34}$$

$$x_0'=\gamma\left(x_0-\frac{v}{\sqrt{g}}x_2\right), \tag{35}$$

where $\gamma=1/\sqrt{1-v^2}$. It is easy to check that the transformed metric is

$$-g'_{00}=g'_{11}=g. \tag{36}$$

Using the invariance of the two form $F=\frac{1}{2}B_{\mu\nu}dx^{\mu}\wedge dx^{\nu}$, we get the transformed $B'_{\mu\nu}$ in K' as

$$E'=\gamma E, \quad B'=\frac{\gamma v}{\sqrt{g}}E. \tag{37}$$

Note that in both K and K', the definition of $E_c=g/2\pi\alpha'$ is *the same*. Thus we have the transformation law for dimensionless electric and magnetic fields:

$$e'\equiv\frac{E'}{E_c}=\gamma e, \quad b'\equiv\frac{B'}{E_c}=\frac{\gamma v}{\sqrt{g}}e. \tag{38}$$

To study the V-duality of NCOS, now we need to take the decoupling limit for NCOS in both frame K and K'. In frame K, the decoupling limit dictates

$$g(1-e^2)=\frac{2\pi\alpha'}{\theta_0}. \tag{39}$$

Correspondingly, in frame K' we have

$$g(1-e'^2)=\frac{2\pi\alpha'}{\theta'_0}, \tag{40}$$

$$gb'=\frac{2\pi\alpha'}{\theta'_1}. \tag{41}$$

Thus we can establish the following relation by using Eq. (41)

$$\gamma v\sqrt{g-\frac{2\pi\alpha'}{\theta_0}}=\frac{2\pi\alpha'}{\theta'_1}. \tag{42}$$

The decoupling limit is the one in which

$$e\rightarrow 1, \quad g\rightarrow\infty. \tag{43}$$

So taking the decoupling limit reduces Eq. (42) to

$$v\sqrt{g}=\frac{2\pi\alpha'}{\theta'_1}. \tag{44}$$

Therefore, we conclude that the boost velocity $v\rightarrow 0$.

On the other hand, the boost transformation Eq. (40) leads to

$$\begin{aligned}\frac{2\pi\alpha'}{\theta'_0}&=g(1-e'^2)=g(1-e^2)+ge^2(1-\gamma^2)\\&=g(1-e^2)-\frac{gv^2e^2}{1-v^2}\rightarrow\frac{2\pi\alpha'}{\theta_0}-\left(\frac{2\pi\alpha'}{\theta'_1}\right)^2,\end{aligned} \tag{45}$$

where the arrow ''$\rightarrow$'' means taking the decoupling limit. Thus, we have proved the V-duality action for NCOS

$$\frac{2\pi\alpha'}{\theta_0}=\frac{2\pi\alpha'}{\theta'_0}+\left(\frac{2\pi\alpha'}{\theta'_1}\right)^2. \tag{46}$$

More generally, with other noncommutativity parameters vanishing, the following gives us an invariant under V-duality action:

$$\frac{2\pi\alpha'}{\theta_0}+\left(\frac{2\pi\alpha'}{\theta_1}\right)^2=\text{invariant}. \tag{47}$$

This invariance gives us some *orbits* for V duality. The displacements on an orbit are determined by the action of group elements. This invariance can be viewed as a descendant of the Lorentz invariance with a boost parameter $v\rightarrow 0$, and this is the signal of the Galilean group. Therefore, we suggest that the V duality should be characterized by a Galilean group or its deformation. The invariant (47) is for one of its Abelian subgroup.

V. HOLOGRAPHY IN NCOS

Previously in Ref. [9] a holographic correspondence between NCYM and its supergravity dual was suggested. Namely, the radial profile of the on-shell close string moduli (string-frame metric, NSNS B tensor and dilaton) in the supergravity dual of an NCYM can be easily derived through the Seiberg-Witten relations [3] between close string moduli and open string moduli, provided a simple ansatz for the running string tension as the function of the energy scale is assumed. In this section, we generalize this link between holography and noncommutativity to NCOS.

For convenience of making a contrast between NCYM and NCOS, we first briefly recall the case of NCYM. Suppose only $B_{23}\neq 0$ on a stack of D3 branes. The central suggestion made in Ref. [9] is that in the supergravity dual *the UV limit (from the NCYM perspective)* $u\rightarrow\infty$ *is identified with the NCYM ''scaling limit'' or ''decoupling limit''* in Ref. [3]. In this limit, α' should approach zero, as in the Anti–de Sitter–Conformal–Field–Theory (AdS/CFT) correspondence [15]. To implement this, the overall factor R^2u^2, appeared in the 4d geometry along $D3$ branes, is interpreted as a running string tension

$$\alpha'_{run}=\frac{1}{R^2u^2}, \tag{48}$$

which obviously runs to zero in the UV limit. Note that the manner it approaches zero compared to g_{22} and g_{33} agrees with the NCYM scaling limit taken in Ref. [3]. The holographic correspondence suggested in Ref. [9] is that the *radial profiles* of the on-shell NSNS fields in the gravity dual should *satisfy* the Seiberg-Witten relations Eqs. (6), (8), and (9), with α' being replaced by the *running* α'_{run} given by Eq. (48) and with *constant (unrenormalized)* open string moduli.

In Ref. [9], the same holographic correspondence was shown to hold for all cases in which decoupling leads to an NCYM with *space-space* noncommutativity and with gravity dual known. These include high-dimensional Dp branes in a magnetic background[1] and Euclidean $D3$ branes in a self-dual B background. In the following, we would like to examine whether a similar holographic correspondence holds as well between NCOS (with *space-time* noncommutativity) and its gravity dual, despite that the NCOS limit is very different from the NCYM limit.

Let us consider the case with only $B_{01}\neq 0$. In this case, because of the existence of a critical electric field on the $D3$ branes, to decouple the closed strings, one can no longer take $\alpha' \to 0$. Instead, α' is fixed, leading to an NCOS. Certainly, the above ansatz Eq. (48) for the running string tension α'_{run} should *no longer* hold. We will see that indeed an appropriate modification of the ansatz exists, so that the above holographic correspondence remains to hold for NCOS.

The supergravity dual (with Lorentz signature) with only B_{01} nonvanishing was given in Ref. [5]:

$$ds^2_{str}=H(u)^{-1/2}\left[\frac{u^4}{R^4}H(u)(-dt^2+dx_1^2)+(dx_2^2+dx_3^2)+H(u)(du^2+u^2d\Omega_5^2)\right], \tag{49}$$

$$B_{01}=\frac{1}{2\pi}\frac{u^4}{R^4}, \tag{50}$$

$$e^{2\phi}=g^2\frac{u^4}{R^4}H(u), \tag{51}$$

where we have $\alpha'=1$, and $R=4\pi gN$, $H(u)\equiv 1+R^4/u^4$. (Again, we omit the RR fields.) Recall that in the previous case with $B_{23}\neq 0$, the close string metric g_{ij} (with $i,j=2,3$) shrinks to zero in the UV limit $u\to\infty$. In contrast, in the present case, the close string metric $g_{\mu\nu}$ (with $\mu,\nu=0,1$) goes to infinity in the UV limit, being consistent with the NCOS limit [4,5]. So in the spirit of Ref. [9], we again identify the UV limit $u\to\infty$ with the NCOS ''scaling limit'' or ''decoupling limit,'' assuming the running string tension of the form

$$\alpha'_{run}=H(u)^{1/2}\equiv\left(1+\frac{R^4}{u^4}\right)^{1/2}, \tag{52}$$

which is nothing but the inverse of the overall factor in front of the bracket[2] in the close string metric in the gravity dual (49), in accordance with the same prescription as before for Eq. (48).

Now we want to show that the close string moduli in Eq. (49) can be derived from the Seiberg-Witten relations (6) and (9), in which α' *is replaced by a running one* given by Eq. (52). We introduce the ansatz

$$g_{\mu\nu}=f(u)\eta_{\mu\nu},\quad 2\pi B_{\mu\nu}=h(u)\epsilon_{\mu\nu}, \tag{53}$$

for $\mu,\nu=0,1$, due to the boost symmetry in the (x_0,x_1) plane, and assuming constant open string moduli:

$$G_{\mu\nu}=\eta_{\mu\nu},\quad \theta_{\mu\nu}=2\pi\epsilon_{\mu\nu}. \tag{54}$$

Then two equations in Eq. (6) yield

$$1=f-\frac{h^2H}{f}, \tag{55}$$

$$1=\frac{hH}{f^2-h^2H}, \tag{56}$$

namely,

$$f=f^2-h^2H,\quad f=hH. \tag{57}$$

Solving these two equations, one obtains

$$f(u)=(1-H^{-1})^{-1}=\frac{u^4}{R^4}\left(1+\frac{R^4}{u^4}\right), \tag{58}$$

$$h(u)=\frac{u^4}{R^4}.$$

Similarly, substituting the above solution into the relation (9) with the identification $G_s=g$, one obtains the u-dependent closed string coupling:

$$g_s(u)=g\big(\det(g+\alpha'_{run}B)\big)^{1/2}, \tag{59}$$

or

$$e^{2\phi}=g^2\left(1+\frac{u^4}{R^4}\right). \tag{60}$$

The results (58) and (60) are precisely what appeared in the gravity dual (49), which was previously obtained as a solution to classical equations of motion in IIB supergravity. Note that the closed string coupling approaches unity in the UV limit, in agreement with the decoupling of closed strings for NCOS.

Certainly this derivation adds more evidence to the universality of the link between holography and noncommutativity observed in Ref. [9]. Thus, we have seen that the relations among the closed string moduli and the open string moduli contain much more than we could have imagined. With the appropriate ansatz for the input α_{eff}, they determine the closed string dual of both NCYM and NCOS. This demonstrates a simple and direct connection between holography and noncommutativity, either of which is believed to play a role in the ultimate theoretical structure for quantum gravity.

[1]When $p\neq 3$, the open string (or NCYM) coupling constant is no longer u independent: $G_s^2=g^2u^{(7-p)(p-3)/2}$. But this just means that the open string coupling runs in the same way as in the case when there is no B field, in agreement with the result of Ref. [16] in the large-N limit.

[2]Inside the bracket the transverse metric g_{ij} for $i,j=2,3$ is taken to be δ_{ij}.

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