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Absence of (1,0) supersymmetry anomaly in world-sheet gauge theories: A purely cohomological proof

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A purely cohomological proof is given for the absence of the (1,0) supersymmetry anomaly in gauge theories on a world sheet. In particular, it is shown that generalized cohomological approaches to anomalies in supersymmetric gauge theory, either formulated in whole superconnection space or only in the Wess-Zumino-gauge surface, yield results which agree with those obtained by noncohomological field-theoretical methods. We argue that the success of the cohomological arguments implies that there should be a generalization of (family) index theorem in the supersymmetric cases.

Anomalies in quantum theory have played a ubiquitous role in particle physics, not only in unified gauge theories (the standard model and grand unified theories) but also in string theories. In the latter, it is generally believed that space-time physics has its origin in $D=2$ dimensional world-sheet physics, although the connection is only partially understood. An important link between them has been provided by anomalies in the $D=2$ quantum field theory which describes the propagation of a string in certain space-time backgrounds. For example, the well-known critical dimension and modular invariance of a string theory[1] are both actually anomaly-free conditions for the $D=2$ field theory. The discovery of any new anomaly would impose extra constraints on string-model construction and would provide new insight into space-time physics.

Since supersymmetry turns out to be an important ingredient for the construction of realistic string models, it is interesting to see whether or not there is an anomaly for it. Although no supersymmetry anomaly has ever been found in all presently known models, a deep understanding of this absence is still in demand. Several approaches to anomalies in, say, supersymmetric gauge theories have been suggested and worked out in higher dimensions ($D\geq 4$) (Refs. 2–4). But no two results look alike, so usually we do not know whether or not they are consistent with each other. A principle for comparing the results in different approaches has been suggested by us some time ago[5] (see also Zumino[6] and Hwang[7]); nevertheless it is difficult to practice it in $D\geq 4$ except for a few cases. In $D=2$, the situation is much simpler: the results[7–9] do not look very involved. So we feel it is possible to directly compare various approaches by applying our method,[5] and we expect to gain some insight into the problem from such comparison.

Here we report on a recent study on (1,0) world-sheet supersymmetric gauge theories which have been lately explored in the construction of consistent string models.[10] Our study confirms again that there is no supersymmetry anomaly in these theories and, therefore, no constraint on string-model construction arises from it. Our discussion also verifies the consistency of generalized cohomological arguments with other noncohomological field-theoretical methods in the supersymmetric cases. This not only shows the validity of the cohomological approaches, but also implies that there should be a generalization of the (family) index theorem in the space of superconnections. Whether there exists such a generalization is an interesting open problem. Our argument in favor of a positive answer is the following.

The absence of a supersymmetry anomaly seems to be a quite universal rule. If so, there must be a deep reason for it. It might be related to the fact that a superspace (or supermanifold) is always topologically trivial in the fermionic directions,[11] and this topological triviality might be somehow carried over to a supersymmetry orbit in the space of superconnections. If this is true, a generalization of the index theorem or family index theorem might underlie the absence of the supersymmetry anomaly. On the other hand, it is well known that in nonsupersymmetrical gauge theories the validity of usual cohomology argument for chiral gauge anomaly[12] is assured by the family index theorem in the space of connections.[13] Without the latter, the usual cohomology argument only provides us with a solution to the Wess-Zumino consistency condition,[14] but we are not sure that this solution is the right one, since solutions to the consistency condition may not be unique. The family index theorem guarantees that the solution provided by the usual cohomology argument is indeed the right one. The existence of this theorem may have been indicated by the fact that the result from a perturbative calculation[15] or path-integral derivation[16] coincides with the solution provided by the usual cohomology argument. Namely this coincidence of the results obtained by apparently different methods should not be considered as purely accidental; there must be something deep and fundamental hidden behind it. Now we can repeat this line of reasoning for the supersymmetric cases: if once again a generalized cohomology argument can reproduce the results obtained by other field-theoretical methods, there should be a sort of generalized family index theorem which makes the generalized cohomology argument valid. This is why we are interested in a new proof of the known absence of a (1,0) world-sheet supersymmetry anomaly[8] from a *cohomological* point of view.

In this paper, we restrict ourselves to gauge theories with (1,0) global supersymmetry. [Our treatment applies also to (p,q) supersymmetric theories which can be written in terms of (1,0) superfields by projecting into (1,0) superspace.] We hope to discuss the locally supersymmetric cases (supergravity) in later publications. Results of this study have been briefly announced in Ref. 17.

For closed strings, the simplest world-sheet supersymmetry is (1,0) supersymmetry.[18] For our notation we are going to use the flat superspace used by Ovrut and his collaborators.[19] (For other references on the same subject, see Ref. 20. Although the notations in the first paper of Ref. 20 have more indices, they are convenient for keeping track of the Lorentz transformation property.) The superspace coordinates are $z^M=(x^{\pm},\theta)$ where θ is a Weyl-Majorana spinor. The (1,0) supercharge is $Q=i(\partial/\partial\theta-i\theta\partial/\partial x^+)$, satisfying $\{Q,Q\}=2i\partial/\partial x^+$. The supercovariant derivatives are given by $D_M=(D_\pm,D_\theta)$; $D_\pm=\partial/\partial x^\pm$, $D_\theta=\partial/\partial\theta+i\theta\partial/\partial x^+$. In (1,0) superspace, one can define scalar superfields $\Phi^a(z)=\varphi^a(x)+\theta\lambda^a(x)$ and spinor superfields $\Psi^a(x)=\psi^a(x)+\theta F^a(x)$; they form, respectively, some representations of the group G. Also one can introduce the gauge-potential super one form: $A=e^C A_C^a(iT^a)\equiv e^C A_C(z)$ where $e^A\equiv dz^M e_M^A$ are frame super one-forms. With appropriate constraints imposed and the Bianchi identities solved, only the fields χ_θ^a, $V_\pm^a$, and χ_-^a in the following equations are independent components:

$$A_\theta^a=i(\chi_\theta^a+\theta V_+^a),\quad A_-^a=V_-^a+i\theta(\chi_-^a+D_-\chi_\theta^a)\ . \tag{1}$$

The (1,0) supersymmetry transformation is given by $\delta_S(\epsilon)=i\epsilon Q$ and the generalized gauge transformation for A_C reads

$$\delta_G(\Lambda)A_C=-D_C\Lambda+[A_C,\Lambda]\ , \tag{2}$$

where $\Lambda\equiv\Lambda^a(z)(iT^a)$ is an infinitesimal scalar superfield taking values in the Lie algebra of G. The gauge and supersymmetric invariant action is

$$S=\int d^2x\, d\theta[-\mathrm{Tr}F_{-\theta}\mathcal{D}_\theta F_{-\theta}+\mathcal{D}_\theta\Phi^a\mathcal{D}_-\Phi^a -\Psi^a(\mathcal{D}_\theta)_{ab}\Psi^b]\ , \tag{3}$$

where $F_{-\theta}^a=D_-A_\theta^a-D_\theta A_-^a-[A_-,A_\theta]^a$ and $\mathcal{D}_A=D_A-A_A$. First integrating over the matter superfields Φ^a and Ψ^a, one obtains the effective action

$$\exp(iW_{\text{eff}}[A])=\int D\Phi^a D\Psi^a \times\exp\left[i\int d^2x\, d\theta[\mathcal{D}_\theta\Phi^a\mathcal{D}_-\Phi^a -\Psi^a(\mathcal{D}_\theta)_{ab}\Psi^b]\right]\ . \tag{4}$$

Then the gauge and supersymmetry anomalies are, respectively, defined by

$$A_G[A;\Lambda]=\delta_G(\Lambda)W_{\text{eff}}[A]\ ,$$
$$A_S[A;\epsilon]=\delta_S(\epsilon)W_{\text{eff}}[A]\ . \tag{5}$$

In Ref. 9 the gauge anomaly is obtained by a generalized cohomology argument in (1,0) superspace as follows:

$$A_G[A;\Lambda]=c\int d^2x\, d\theta\,\mathrm{Tr}[\Lambda(D_-A_\theta-D_\theta A_-)] \tag{6}$$

(where c is a constant depending on the representation content of matter superfields) with no mentioning about the supersymmetry anomaly $A_S[A;\epsilon]$. We note that the integrand in Eq. (6) is the e^-e^θ component, as it should be, of $\omega_2^1(A,F)\equiv\mathrm{Tr}[\Lambda(F-A^2)]$. Here $\omega_2^1(A,F)$ is one of the quantities appearing in the usual descent equations[12] in $D=2$, but now with A and F being superforms. Again, it is the descent equation satisfied by $\omega_2^1(A,F)$ which guarantees that the right-hand side of Eq. (6) satisfies the Wess-Zumino consistency condition for the anomaly A_G in superconnection space.[9]

To check this result and prove that it implies the absence of supersymmetry anomaly, we use a technique we invented before,[5] i.e., first study the anomalies in the Wess-Zumino (WZ) gauge in the way Itoyama, Nair, and Ren[4] did in $D\geq 4$ and then compare them with the above $A_G[A;\Lambda]$ [Eq. (6)] by the method we suggested in Ref. 5.

In the present situation, the WZ gauge is given by $\chi_\theta=0$. So the physical components are $(V_\pm^a,\chi_-^a)$. The usual gauge transformation in the WZ gauge is

$$\delta_g(\alpha)V_\pm^a=-\partial_\pm\alpha^a+[V_\pm,\alpha]^a\ ,$$
$$\delta_g(\alpha)\chi_-^a=[\chi_-,\alpha]^a\ , \tag{7}$$

where the parameters $\alpha(x)\equiv\alpha^a(x)(iT^a)$ depend on $x^\pm$ only. The supersymmetry transformation in the WZ gauge is given by

$$\delta_s(\epsilon)V_+^a=0,\quad \delta_s(\epsilon)V_-^a=-i\epsilon\chi_-^a,\quad \delta_s(\epsilon)\chi_-^a=-\epsilon F_{+-}^a\ . \tag{8}$$

[We have used little g and s to distinguish the (usual) gauge transformation (7) and the (combined) supersymmetry transformation (8) in the WZ gauge from the (generalized) gauge transformation (2) and the (usual or genuine) supersymmetry transformation in superspace which are labeled by capital G and S.] It is easy to work out the commutation relations among these transformations as follows:

$$[\delta_s(\rho),\delta_s(\epsilon)]=\delta_g(2i\epsilon\rho V_+)+2i\epsilon\rho D_+\ ,$$
$$[\delta_g(\alpha),\delta_g(\beta)]=\delta_g([\alpha,\beta])\ ,$$
$$[\delta_g(\alpha),\delta_s(\epsilon)]=0\ . \tag{9}$$

Therefore, the consistency conditions for anomalies in the WZ gauge read

$$\delta_s(\rho)A_s[\tilde{A};\epsilon]-\delta_s(\epsilon)A_s[\tilde{A};\rho]=A_g[\tilde{A};2i\epsilon\rho V_+]\ , \tag{10a}$$

$$\delta_g(\alpha)A_g[\tilde{A};\beta]-\delta_g(\beta)A_g[\tilde{A};\beta]=A_g[\tilde{A};[\alpha,\beta]]\ , \tag{10b}$$

$$\delta_g(\alpha)A_s[\tilde{A};\epsilon]-\delta_s(\epsilon)A_g[\tilde{A};\alpha]=0\ , \tag{10c}$$

where $\tilde{A}=A|_{\chi_\theta=0}$ is the superconnection in the WZ gauge and

$$\begin{aligned} A_g[\tilde{A};\alpha]&=\delta_g(\alpha)W_{\rm eff}[\tilde{A}]\ ,\\ A_s[\tilde{A};\epsilon]&=\delta_s(\epsilon)W_{\rm eff}[\tilde{A}] \end{aligned} \tag{11}$$

are, respectively, the gauge and supersymmetry anomaly in the WZ gauge.

To find the solution to this set of consistency conditions we note that the gauge anomaly $A_g[\tilde{A};\alpha]$ must be the same as usual:

$$A_g[\tilde{A};\alpha]=c\int d^2x\,{\rm Tr}[\alpha(\partial_- V_+ - \partial_+ V_-)] \tag{12}$$

since in the WZ gauge the gauge transformation $\delta_g(\alpha)$ [Eq. (7)] is the same as usual. It is well known that Eq. (12) satisfies the consistency condition (10b). Given this, it is not difficult to find a unique $A_s[\tilde{A};\epsilon]$ from the consistency conditions (10):

$$A_s[\tilde{A};\epsilon]=ic\int d^2x\,{\rm Tr}(\epsilon\chi_- V_+)\ . \tag{13}$$

Actually from Eqs. (10c) and (12) one has

$$\begin{aligned} \delta_g(\alpha)A_s[\tilde{A};\epsilon]&=ic\int d^2x\,{\rm Tr}[\alpha\partial_+(\epsilon\chi_-)]\\ &=-ic\int d^2x\,{\rm Tr}[(\partial_+\alpha)\epsilon\chi_-]\ . \end{aligned} \tag{14}$$

Since among the fields $V_\pm$ and χ_- only V_+ will give rise to a term $\partial_+\alpha$ under gauge transformation (7), $A_s[\tilde{A};\epsilon]$ must contain at least a term such as the right-hand side of Eq. (13). Furthermore, it turns out that this term alone satisfies both conditions (10a) and (10c).

Now we come to the key point of our proof. It is well known that the supersymmetry transformation (8) which preserves the WZ gauge is actually the genuine supersymmetry transformation $\delta_S(\epsilon)=i\epsilon Q$ followed by a compensating generalized gauge transformation (2) with $\Lambda=-i\theta\epsilon V_+$: namely,

$$\delta_s(\epsilon)=\delta_S(\epsilon)+\delta_G(\Lambda=-i\theta\epsilon V_+)\ . \tag{15}$$

[The second transformation is needed since $\delta_S(\epsilon)$ does not generally preserve the WZ gauge.] Applying both sides on $W_{\rm eff}[\tilde{A}]$ one obtains an important cohomological identity:

$$A_s[\tilde{A};\epsilon]=A_S[A;\epsilon]|_{A=\tilde{A}}+A_G[A;\Lambda=-i\theta\epsilon V_+]|_{A=\tilde{A}} \tag{16}$$

(up to possible coboundary terms if the two sides are from different approaches). We emphasize that although this equation looks apparently like a trivial consequence of Eq. (15), it is true only when $W_{\rm eff}[A]$ is a *single-valued* function(al) in the space of superconnections, a point (or an assumption) essential to the cohomological argument. Consider an infinitesimal triangle with vertices at $\tilde{A}$, $\tilde{A}+\delta_s(\epsilon)\tilde{A}$, and $\tilde{A}+\delta_S(\epsilon)\tilde{A}$. The cohomological meaning of Eq. (16) is that the one-cochain $\delta_1 W_{\rm eff}$ along the supersymmetry orbit on the Wess-Zumino-gauge surface [from $\tilde{A}$ to $\tilde{A}+\delta_s(\epsilon)\tilde{A}$] is the same as the sum of the one-cochains $\delta_2 W_{\rm eff}$ and $\delta_3 W_{\rm eff}$ along the other two sides of the triangle, i.e., along the genuine supersymmetry orbit [from $\tilde{A}$ to $\tilde{A}+\delta_S(\epsilon)\tilde{A}$] and along the compensating generalized gauge orbit [from $\tilde{A}+\delta_S(\epsilon)\tilde{A}$ back to $\tilde{A}+\delta_s(\epsilon)\tilde{A}$]. In other words, Eq. (16) is a special case of the cohomological statement that the coboundary of $W_{\rm eff}[A]$, namely, $\delta W_{\rm eff}$, is a one-cocycle in the space of superconnections.

Now let us compare our result Eq. (13) with Eq. (6) which was obtained in Ref. 9. It is easy to recognize that if we substitute $\Lambda=-i\theta\epsilon V_+$ and $A=\tilde{A}$ in Eq. (6) we obtain exactly the right-hand side of Eq. (13):

$$A_s[\tilde{A};\epsilon]=A_G[A;\Lambda=-i\theta\epsilon V_+]|_{A=\tilde{A}}\ . \tag{17}$$

Thus, our cohomological identity (16) tells us that

$$A_S[\tilde{A};\epsilon]|_{A=\tilde{A}}=0 \tag{18}$$

(up to coboundary terms). This means that, for $\tilde{A}$ in the WZ gauge, there is no genuine (1,0) supersymmetry anomaly. However, the WZ gauge can be reached from a generic superconnection A by an appropriate generalized transformation (2), and the latter gives rise to an anomaly $A_G[A;\Lambda]$ which is supersymmetrically invariant in the space of superconnections. Thus, by applying the cohomological statement that $\delta W_{\rm eff}$ is a one-cocycle to an infinitesimal parallelogram formed by A, $\tilde{A}$, $\tilde{A}+\delta_S(\epsilon)\tilde{A}$, $A+\delta_S(\epsilon)A$ [where $\tilde{A}$ is obtained from A by a generalized gauge transformation (2)], we can conclude that

$$A_S[A;\epsilon]=0\ . \tag{19}$$

Thus, there is no genuine (1,0) supersymmetry anomaly in superconnection space.

We also observe that Eq. (12) can be obtained as a special case of Eq. (6) as follows:

$$A_g[\tilde{A};\alpha]=A_G[A;\Lambda=\alpha]|_{A=\tilde{A}}\ . \tag{20}$$

Therefore both Eqs. (12) and (13) can be derived from Eqs. (6) and (19). This verifies the consistency of the cohomological approach to anomalies in whole superconnection space by Ovrut and his co-workers[8] with an approach to anomalies in the WZ gauge by Itoyama, Nair, and Ren.[4]

An astute reader may point out that there is one part of our above proof which seems not cohomological in nature: namely, the derivation of the solution (13) for supersymmetry anomaly in the WZ gauge. Now let us show that if one pleases, he or she can rederive Eq. (13) by a cohomological ansatz on the Wess-Zumino-gauge surface. We modify slightly the approach of Borona, Pasti, and Tonin,[3] which was originally suggested for $D\geq 4$ cases, and apply it to the Wess-Zumino-gauge surface ($\chi_\theta=0$) in $D=2$ (1,0) superconnection space. Denote the exterior differential operator in (1,0) superspace by

$$d\equiv e^A D_A=(e^+D_+ + e^- D_-)+e^\theta D_\theta\equiv\hat{d}+D\ , \tag{21}$$

where e^A are frame super one-form and $\hat{d}$ is the differential operator with respect to $x^\pm$. Then the field-strength super two-form $\tilde{F}$ for the superconnection one-form $\tilde{A}$ in the WZ gauge is given by $\tilde{F}=d\tilde{A}+\tilde{A}^2$. Now let us introduce the exterior differential operator $\tilde{\delta}$ in the (usual) gauge orbit of $\tilde{A}$ in the WZ gauge, which acts on the parameters t_i of the gauge functions $\alpha(x;t_1,\ldots)$ in transformations (7):

$$\tilde{\delta}\tilde{A}=-\hat{d}v-[\tilde{A},v],\quad \tilde{\delta}v=-v^2 . \tag{22}$$

Then we identify, following Borona, Pasti, and Tonin,[3]

$$\begin{aligned}\delta_g\left[\int_x\cdots\right]&=\int d^2x\,\tilde{\delta}(\cdots),\\ \delta_s\left[\int_x\cdots\right]&=\int d^2x\,i_\epsilon D(\cdots)\end{aligned} \tag{23}$$

as the coboundary operators along the gauge and supersymmetry orbits, respectively, for a local functional of $\tilde{A}$ and $\tilde{F}$ in the WZ gauge. Here the definition of i_ϵ which acts on a super m-form

$$\psi=e^{A_1}\cdots e^{A_m}\psi_{A_m\cdots A_1} \tag{24}$$

is given by

$$i_\epsilon\psi=m\epsilon^{A_1}e^{A_2}\cdots e^{A_m}\psi_{A_m\cdots A_1} \tag{25}$$

with $\epsilon^A=(0,0,\epsilon)$. δ_g and δ_s satisfy the relations

$$\delta_s^2=\delta_g^2=\delta_s\delta_g+\delta_g\delta_s=0 . \tag{26}$$

Therefore, for the gauge and supersymmetry anomalies in the WZ gauge

$$A_g=\delta_g W_{\text{eff}}[\tilde{A}],\quad A_s=\delta_s W_{\text{eff}}[\tilde{A}] \tag{27}$$

we have the consistency conditions

$$\delta_s A_s=0,\quad \delta_s A_g+\delta_g A_s=0,\quad \delta_g A_g=0 , \tag{28}$$

which is the cohomological form of our previous consistency conditions (10).

To obtain the solution to Eq. (28), we follow a cohomological procedure, which can be viewed as another generalization of the usual one[12] to the supersymmetric case. Let us consider $\text{Tr}\tilde{F}^2$. Here Tr acts on the matrix (or Lie algebra) part as before. Similar to the nonsupersymmetric case, one has

$$\text{Tr}\tilde{F}^2=d\omega_3^0(\tilde{A},\tilde{F}),\quad \omega_3^0(\tilde{A},\tilde{F})=\text{Tr}(\tilde{A}\tilde{F}-\tfrac{1}{3}\tilde{A}^3) . \tag{29}$$

We introduce

$$\mathcal{A}=\tilde{A}+v,\quad \Delta=d+\tilde{\delta},\quad \mathcal{F}=\Delta\mathcal{A}+\mathcal{A}^2 \tag{30}$$

and derive from $d^2=\tilde{\delta}^2=d\tilde{\delta}+\tilde{\delta}d=0$ and $\Delta^2=0$ that $\mathcal{F}=\tilde{F}$. Thus,

$$\begin{aligned}\text{Tr}\tilde{F}^2=\text{Tr}\mathcal{F}^2&=\Delta\omega_3^0(\mathcal{A},\mathcal{F})\\ &=(\hat{d}+D+\tilde{\delta})\omega_3^0(\tilde{A}+v,\tilde{F}) .\end{aligned} \tag{31}$$

Now we expand $\omega_3^0(\tilde{A}+v,\tilde{F})$ into terms of different degrees in v, $e^\pm$, and e^θ:

$$\omega_3^0(\tilde{A}+v,\tilde{F})=\sum_{p+q+k=3}\omega_{p,q}^k\,(v,\tilde{A},\tilde{F}) , \tag{32}$$

where $k=$degree of v, $p=$degree of $e^\pm$, $q=$degree of e^θ. Substituting Eq. (32) into Eq. (31) and equating forms with same (p,q,k) degrees, one obtains the descent equations which are the supersymmetric generalization of the usual ones in Ref. 12. However, we point out a special feature of the $D=2$ (1,0) supersymmetrical case: namely, one has the identity

$$\text{Tr}F^2\equiv 0 \quad\text{in } D=2\ (1,0) \text{ superspace} . \tag{33}$$

This follows from the constraint $F_{\theta\theta}=0$ and the Bianchi identities, since they together lead to

$$F_{+\theta}=0,\quad F_{+-}=iD_\theta F_{-\theta} . \tag{34}$$

(Note that in the supersymmetric case *a priori* a super four-form in $D=2$ may not be zero, since $dx^+dx^-d\theta\,d\theta$ does not vanish. Similar identities have been noticed by Borona, Pasti, and Tonin in $D\geq 4$ cases.[3]) Using the identity (33), the descent equations derived from Eqs. (31) and (32) take a simpler form. Among others one has

$$\begin{aligned}&\hat{d}\omega_{1,2}^0+D\omega_{2,1}^0=0 ,\\ &\tilde{\delta}\omega_{1,2}^0+\hat{d}\omega_{1,1}^1+D\omega_{2,0}^1=0 ,\\ &\tilde{\delta}\omega_{2,0}^1+\hat{d}\omega_{1,0}^2=0 .\end{aligned} \tag{35}$$

From these equations we immediately see that if one identifies the anomalies in the WZ gauge by the cohomological ansatz

$$\begin{aligned}A_g[\tilde{A};v]&=c\int_x\omega_{2,0}^1\Big|_{\theta=0} ,\\ A_s[\tilde{A};\epsilon]&=c\int_x i_\epsilon\omega_{2,1}^0\Big|_{\theta=0} ,\end{aligned} \tag{36}$$

then they automatically satisfy the consistency conditions (27). A straightforward calculation shows that

$$A_g[\tilde{A};v]=c\int d^2x\,\text{Tr}[v(D_-V_+-D_+V_-)] , \tag{37}$$

$$A_s[\tilde{A};\epsilon]=ic\int d^2x\,\text{Tr}(\epsilon V_+\chi_-) . \tag{38}$$

Namely, they coincide with Eqs. (12) and (13) we have obtained before; but now they are deduced from a cohomological ansatz (36).

In conclusion, we have applied both a field-theoretical version of Itoyama, Nair, and Ren[4] and a modified version of Borona, Pasti, and Tonin's cohomological approach[3] to study the gauge and supersymmetry anomalies in the WZ gauge in world-sheet (1,0) supersymmetric gauge theories. We find that the results from the two approaches coincide with each other. Also by using a technique we suggested before,[5] we have compared this result with the result obtained by Ovrut and his co-workers[8] with a cohomological procedure in superconnection space; and we confirm the absence of (1,0) supersymmetry in superconnection space, which has been proved in the literature by noncohomological field-theoretical methods.[8] Therefore, we have checked the consistency of the results from several different approaches for anomalies in supersymmetric gauge theory in the $D=2$ (1,0) case. Moreover, as we argued at the beginning of the paper, the coincidence or consistency of these results from field-theoretical and cohomological methods not only verifies the validity of the latter, but also implies the existence, at least in $D=2$, of a generalized (family) index theorem in the space of superconnections which underlies the cohomological approach.

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