# Optical ranging by wavelength multiplexed interferometry

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A new optical technique is described for measurement of absolute distance. The approach is based upon a wavelength multiplexed heterodyne interferometer with FM demodulation. By temporally multiplexing discrete wavelengths in a heterodyne interferometer, a complete elimination of interferometric range ambiguity can be achieved while maintaining the high range sensitivity and resolution of interferometry. The basic theory is presented and an algorithm is described for measurement of range over meter distances with submicrometer resolution. The experimental implementation of the wavelength multiplexed interferometer is described and ranging results with  $2\,\mu\mathrm{m}$  resolution from 20 cm are presented. A scanned three-dimensional map of a surface contour with 3-mm topography is also presented.

## INTRODUCTION

A substantial interest has been demonstrated in the optical measurement of absolute distance by those who work in the areas of machine vision, robotic positioning, and automated inspection. Particularly, the ability to measure range unambiguously with a line-of-sight technique appears to have multiple applications. In machine vision, a great emphasis has been placed upon the extraction of geometrical shape and position from intensity images. A direct line-ofsight measurement of range eliminates much of the need for complicated algorithms and detection schemes for this information extraction. In the area of robotic positioning, the ability to measure absolute range, rather than changes in range, makes possible the multiplexing of multiple range sensors, eliminates the need for calibrating position at startup and allows for measurement of noncontinuous changes in position. For automated inspection, the measurement of absolute distance provides an alternative to mechanical coordinate measuring machines which are typically very time consuming. The ability to rapidly and accurately measure absolute range opens up a host of possibilities in the measurement area.

Research in the area of distance measurement has existed for many years. 1 Many of the approaches taken are based upon the principle of triangulation. If the illumination and observation directions are different, range variation can be mapped into a lateral variation and imaged. Work has also been done in the area of line-of-sight ranging systems, where the power of the illuminating beam is temporally modulated and range is extracted from the propagation delay of the modulated light relative to the reference modulation. These approaches are generally attractive when the required range resolution is on the order of 100  $\mu m$  or greater. In order to achieve range resolution on the order of 1  $\mu$ m without sacrificing bandwidth, coherent interferometric techniques must be employed. In this paper, a new technique is described which is based upon discrete wavelength multiplexed interferometry. This approach provides unambiguous range measurement over distances on the order of meters with submicron accuracy while maintaining large bandwidths. The basic theory is described and experimental results are given. demonstrating the inherent sensitivity and resolution of the multiple wavelength interferometric approach.

# THEORY OF UNAMBIGUOUS MEASUREMENT OF DISTANCE

The output of a two-arm heterodyne interferometer is given by

$$\cos\left[\omega_b t - 2k(z_1 - z_2)\right],\tag{1}$$

where  $\omega_b$  is an acousto-optically generated rf frequency shift of the reference beam relative to the signal beam, k is the optical k vector, and  $z_1$  and  $z_2$  are the lengths of the two arms of the interferometer. It has been assumed that the frequency shift  $w_b$  of the reference beam is very small relative to the frequency of the light itself, so that the k vectors of the two interfering beams are essentially the same. Under these conditions, the interferometer output is an rf carrier of frequency  $\omega_b$  whose phase relative to the drive signal of the frequency shifter is given by

$$\phi = 2k(z_1 - z_2) = 2k\Delta z. \tag{2}$$

This phase can be measured electronically and used to detect changes in the interferometer arm lengths. If one of the arm lengths is kept constant, the phase can be directly related to the distance between some fixed reference point and a reflector which forms the second arm of the interferometer. Measurement of this phase provides submicrometer range resolution, but suffers a significant limitation. Because the optical path difference  $\Delta z$  is measured through a sinusoidal function, there is an inherent ambiguity in the determination of the absolute path difference. The phase can only be identified modulo  $2\pi$ . This ambiguity in phase maps into a range ambiguity. As can be seen in Eq. (2), a  $2\pi$  phase shift will take place each time the arm length difference  $\Delta z$  changes by more than a half wavelength. Therefore, the ambiguity length  $\Delta z_{amb}$  is half of an optical wavelength

$$\Delta z_{\rm amb} = \pi/k = \lambda / 2. \tag{3}$$

This ambiguity limits the application of such a ranging system to measurement of continuous changes in reflector position. Absolute distance cannot be measured directly. This approach has been called fringe counting. It suffers from the fact that any interruption of the beam or discontinuity in the optical path of greater than a half wavelength will introduce ambiguity into the measurement of absolute distance.

Elimination of the interferometric ambiguity can be

achieved, however, by the use of more than one discrete wavelength in the measurement.<sup>2-5</sup> This can be accomplished by measuring the interferometer output phase difference for two wavelengths

$$\phi_1 = 2k_1 \Delta z,\tag{4}$$

$$\phi_2 = 2k_2 \Delta z,\tag{5}$$

$$\Delta \phi = 2\Delta k(\Delta z). \tag{6}$$

This is illustrated in Fig. 1. It can be seen that the difference phase varies more slowly with range than the phase at either wavelength. The rate at which the difference phase varies is reduced by a factor of  $(\Delta k/k)$ . The path length change required to obtain a  $2\pi$  phase change now corresponds to

$$\Delta z_{\rm amb} = \frac{\pi}{\Delta k} = \left(\frac{k}{\Delta k}\right) \frac{\lambda}{2}.\tag{7}$$

A comparison of Eqs. (3) and (7) indicates that the ambiguity length has been increased by a factor of  $(k/\Delta k)$ . Note that if the shift in optical k vector is very small, the ambiguity length can be made very large.

When the unambiguous range of an interferometer is extended in the manner described above, the resolution  $\Delta z_{\min}$  with which the range can be measured is reduced. This is true because the measurement of range is limited by the resolution with which the phase of the interferometer output can be measured. This phase resolution  $\Delta \phi_{\min}$  is determined by the signal-to-noise ratio of the measurement system. Equation (8) relates the phase resolution to range resolution

$$\Delta z_{\min} = \Delta \phi_{\min} / \Delta k. \tag{8}$$

As can be seen from Eqs. (7) and (8), both the ambiguity length  $\Delta z_{\rm amb}$  and the range resolution  $\Delta z_{\rm min}$  are inversely proportional to  $\Delta k$ . Therefore, any increase in the ambiguity length also increases the uncertainty in the range measurement for a given signal-to-noise ratio.

The relationship between the depth resolution and the ambiguity length need not limit the resolution of a system with a finite signal-to-noise ratio, however. Making several measurements of a given distance with different k-vector shifts can provide high depth resolution without ambiguity in the following way. Beginning with a small k-vector shift  $\Delta k$ , an unambiguous measurement can be made with gross

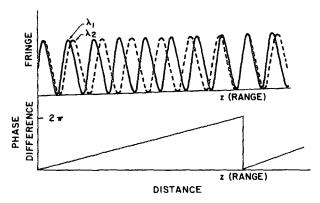


FIG. 1. Illustration of the variation of phase difference with distance for two distinct wavelengths.

depth resolution. If a second measurement is made with a larger wavelength shift, such that the new ambiguity length  $\Delta z_{\rm amb}$  is larger than the previous uncertainty in depth  $\Delta z_{\rm min}$ , a further refinement of the depth can be obtained without ambiguity. This process can continue until the depth resolution is limited only by the maximum k-vector shift realizable and the signal-to-noise ratio. This is graphically represented in Fig. 2.

The number of measurements N required to obtain an unambiguous range measurement with resolution  $\Delta z_{\min}$  over a range of  $\Delta z_{\max}$  is given by the following expression:

$$\left(\frac{\text{Signal}}{\text{Noise}}\right)^N = \frac{\Delta z_{\text{max}}}{\Delta z_{\text{min}}}.$$
(9)

To illustrate, if the maximum distance to be measured is less than 1 m, two wavelengths can be chosen to provide an ambiguity length of 1 m. Under this condition, a single measurement of the distance would require a signal-to-noise ratio of 1 000 000 to obtain a 1  $\mu$ m depth resolution. If two measurements are made, with properly chosen wavelengths pairs, the required signal-to-noise ratio will be 1000. If three measurements are made, the required signal-to-noise ratio will be only 100. It is clear that there is great advantage in making several measurements with different pairs of wavelengths, since the gain in the signal to noise requirement far exceeds the increase in measurement time.

#### **EXPERIMENTAL DESCRIPTION**

The basic principles of multiplexed wavelength interferometry were demonstrated using current and temperature

#### WAVELENGTH MULTIPLEXED INTERFEROMETRY

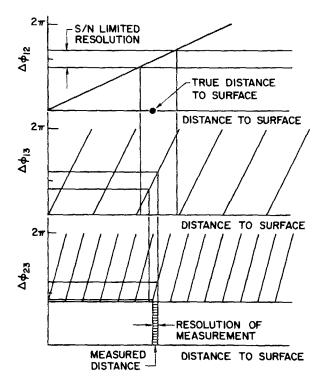


FIG. 2. Illustration of the algorithm for elimination of ambiguity over large distances with finite signal-to-noise ratio.

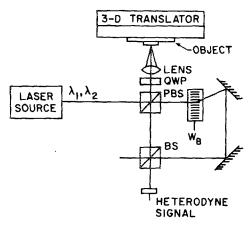


FIG. 3. Experimental arrangement of the heterodyne interferometer and measurement system.

tuned laser diodes for the optical source and the heterodyne interferometer arrangement described in Fig. 3. Two configurations provide a temporally multiplexed wavelength source for input into the heterodyne interferometer. The first consists of a GaAlAs laser diode with a small square wave modulation on the injection bias current. The switching of the injection current between two levels in the laser diode produces a switching of the output between two wavelengths, in addition to some amplitude modulation. The magnitude of the periodic shift from one wavelength to another depends upon the size and frequency of the square wave modulation. The largest wavelength change achieved with the current modulation was 12 Å. This provides an ambiguity length in reflection (double pass) of 280  $\mu$ m. In the second approach, two laser diodes independently controlled by thermoelectric cooling and current are combined in a single-mode optical fiber. Acousto-optic modulators provide the switching necessary to temporally multiplex the two cw laser diode outputs into the fiber, as well as provide isolation for the diodes from light backscattered toward the sources. The smallest ambiguity length achieved with this system was 18  $\mu$ m, which corresponds to a wavelength difference of 20 Å.

The collimated output from the wavelength multiplexed optical source is split into the two arms of the interferometer by a polarizing beam splitter. The signal beam passes through a quarter wave plate and is focused onto the surface of the object. The reflected light returns through the quarter wave plate and passes through the polarizing beam splitter. The reference beam is up shifted in frequency by 80 MHz as it passes through an acousto-optic Bragg cell and is combined with the signal beam in a second beamsplitter. The object is positioned at the focus of the signal beam, and can be translated in three orthogonal directions. The combined signal and reference beams are detected by a PIN photodiode. The heterodyne output at 80 MHz from the photodiode is amplified and mixed with a local oscillator at 80.5 MHz, producing a signal at 500 kHz. This signal is directed to an FM demodulator. The 500-kHz signal is phase modulated when the wavelength in the interferometer is changed. The size of the phase modulation  $\Delta \phi$  is determined by the optical path difference in the two arms of the interferometer

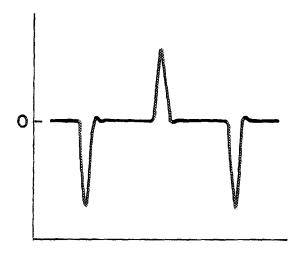


FIG. 4. Output of the FM demodulator as the wavelength in the interferometer is switched three times between  $\lambda_1$  and  $\lambda_2$ . The magnitude of the output peaks are proportional to  $\Delta \phi$ , which can be related to the range.

as given by  $\Delta \phi = 2\Delta k(z_1 - z_2)$ . Since the output of the FM demodulator is proportional to the change of phase, when the wavelength switching takes place, the demodulator puts out a peak proportional to the phase change. Figure 4 contains an oscilloscope trace of the output of the demodulator as the wavelength in the interferometer is switched between  $\lambda_1$  and  $\lambda_2$ . For a given  $\Delta k$ , the peak height from the demodulator will be linearly proportional to the range in the measurement arm of the interferometer over the range of one ambiguity length. As is typical of an FM detection system,

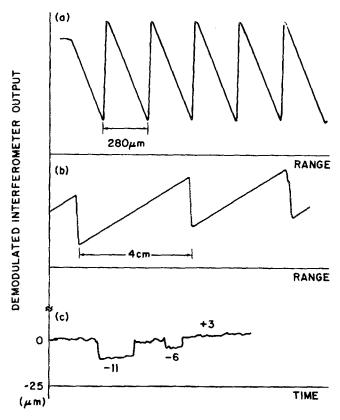


FIG. 5. Range results demonstrating variable ambiguity length for two pairs of optical wavelengths, and the present stability of the ranging system.

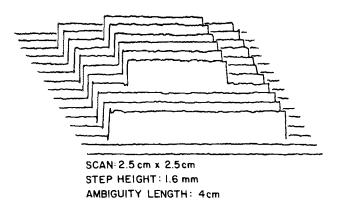


FIG. 6. Range mapping of a three-dimensional object.

any change in the magnitude of the interferometer output has no effect on the range measurement. Therefore, no normalization for reflectivity is necessary. Also, since the carrier frequency is very large (80 MHz), the data rate can be extremely high. The demodulator output is sent to a boxcar averager, where the peak height is detected and the output is displayed on a chart recorder. The object is scanned with respect to the ranging beam in any of the three dimensions by a translation system.

## RESULTS

The capabilities of wavelength multiplexed interferometry were demonstrated two ways. The first measurement was a simple translation of a mirror along the axis of the measurement arm of the interferometer. The boxcar output was recorded as a function of the distance traveled. This was done for two pairs of multiplexed wavelengths from the current tuned laser diode. The results are shown in Fig. 5. The ambiguity lengths were measured to be 280  $\mu$ m and 4 cm. The mirror was also translated over small distances to demonstrate the stability and range resolution. The range stability and resolution appears to be approximately 2  $\mu$ m. The measurements were made from 20 cm, with a 20-kHz demodulator bandwidth. The noise seen in Fig. 5(c), however, was not due to Johnson or optical shot noise. The instability seen in the traces was systemic, and most probably due to wavelength instability caused by feedback into the laser source. With the two independently controlled laser diodes with the 18  $\mu$ m ambiguity length, submicron stability and range resolution was achieved.6 The successive measurement with 4 cm, 280 and 18  $\mu$ m ambiguity lengths provides a means for unambiguous range measurement with submicrometer resolution, over 4-cm distances. The signal-to-noise requirement for such a system is less than  $2 \times 10^2$ , which can be achieved in a 100-MHz bandwidth for milliwatt power levels. The 4 cm ambiguity does not represent any limit on the distance over which the absolute ranging can be done. The coherence length of many laser diodes is larger than 10 m.

To demonstrate the capabilities of the system in an imaging mode, a three-dimensional mirror was constructed by glueing together three flat mirrors of different sizes. The three-dimensional mirror was scanned in the transverse direction to the ranging beam with the wavelengths chosen to obtain a 4-cm ambiguity length. The mirror was rastered in two dimensions, and a range map was made on a chart recorder. The results are shown in Fig. 6. It can be seen that the ambiguity length is larger than the maximum range change on the object, and therefore is capable of making completely unambiguous range measurements.

#### SUMMARY

It has been shown that absolute measurement of range can be accomplished with wavelength multiplexed interferometry. The basic principles have been examined, and a calculation of the required number of measurements necessary to eliminate any ambiguity in the measurement has been displayed. The principles have been implemented in a heterodyne interferometer using an FM detection scheme, and high-resolution ranging has been demonstrated over macroscopic distances.

The importance of wavelength multiplexed interferometry is found in the area of high-resolution range measurement. While alternative techniques lose their capacity for fast, accurate measurement below 100  $\mu$ m, the multiple wavelength approach maintains its high resolution and speed down to submicrometer measurement. With a milliwatt of detected heterodyne power, data rates can exceed the megahertz range. Since switching of wavelengths can be done on a submicrosecond time scale, multiple measurements can be made essentially instantaneously, providing submicrometer resolution over meter distances at speeds consistent with real time imaging. The high speed and accuracy make it a strong candidate for application in many areas of measurement.

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