ADAPTIVE REALIZATION OF A MAXIMUM LIKELIHOOD TIME DELAY ESTIMATOR

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ABSTRACT

This paper presents an adaptive maximum likelihood method for estimating the time difference of arrival of a source signal at two spatially separate sensors. It is well-known that the maximum likelihood technique achieves the Cramer-Rao lower bound for time delay estimation error for certain signal conditions. The α-β tracker is a heuristic mechanism that is heavily used in target tracking applications. In this work, we combine an adaptive realization of the maximum likelihood time delay estimator with the α-β tracker to obtain significant improvement in the performance of the tracker. Experimental results showing 2 to 8 dB improvement in the mean-square estimation error over the conventional α-β tracker for various signal-to-noise ratios are also included in the paper.

1. INTRODUCTION

This paper presents an adaptive maximum likelihood method for estimating the time difference of arrival of a source signal at two spatially separate sensors. The signals arriving at the two sensors from a target are modeled in this work as

$$x_1(n) = s(n) + \eta_1(n) \tag{1}$$

and

$$x_2(n) = s(n - D(n)) + \eta_2(n), \tag{2}$$

where $s(n)$ is the source signal and $\eta_1(n)$ and $\eta_2(n)$ are noise components that have zero-mean values and are uncorrelated with each other and the source signal. The component of the source signal arriving at the second sensor is delayed by an amount $D(n)$. We assume in this work that the amplitudes of the signals arriving at the two sensors are approximately the same. Note that the delay can vary with time, especially when the receiver array or the target is moving.

The basic approach for estimating the time delay is to estimate the cross-correlation function of the two signals and then to find the time lag at which this estimate peaks [1]. Using the model for the received signals in (1) and (2) and assuming stationary source signals with constant delay D, we see that the cross-correlation function $R_{12}(m)$ is given by

$$\begin{aligned} R_{12}(m) &= \mathcal{E}\{x_1(n)x_2(n-m)\} \\ &= \mathcal{E}\{s(n)s(n+D-m)\} \\ &= R_{ss}(m-D)\ , \end{aligned} \tag{3}$$

where $\mathcal{E}\{\cdot\}$ denotes the statistical expectation of $\{\cdot\}$ and $R_{ss}(\cdot)$ is the autocorrelation function of $s(n)$. Since the autocorrelation function peaks at zero lag, we can see that the time delay can be estimated by finding the location of the peak of $R_{12}(m)$.

The generalized cross-correlation (GCC) method [1, 6] of estimating the time delays involves smoothing the cross-correlation function using certain weighting functions in the frequency domain. Let $G_{12}(f)$ denote the cross-spectral density function of $x_1(n)$ and $x_2(n)$. Then, the generalized cross-correlation function of $x_1(n)$ and $x_2(n)$ is defined as

$$R_{12}^{(G)}(f) = F^{-1}\{W(f)G_{12}(f)\}\ , \tag{4}$$

where $W(f)$ is a weighting function and $F^{-1}\{\cdot\}$ denotes the inverse-Fourier transform of $\{\cdot\}$. The weighting function is selected to reduce the effect of noise on the estimates. The maximum likelihood method for finding the estimate of the time-delay between two spatially separate sensors was developed in [2, 3, 6]. This method was found to achieve the Cramer-Rao lower bound for Gaussian input statistics. The weighting function $W^{(M)}(f)$ for this estimator is

$$W^{(M)}(f) = \frac{|\gamma_{12}(f)|^2}{|G_{12}(f)|(1 - |\gamma_{12}(f)|^2)}\ , \tag{5}$$

where $\gamma_{12}(f)$ is the complex coherence function of $x_1(n)$ and $x_2(n)$ defined as

$$\gamma_{12}(f) = \frac{G_{12}(f)}{\sqrt{G_{11}(f)G_{22}(f)}}. \tag{6}$$

The generalized cross-correlation method with this maximum likelihood weighting function is refered to as the ML-GCC method in this paper.

The α-β tracker [7, 8] is a heuristic mechanism that is heavily used in target tracking applications. In this paper, we combine an adaptive realization of the maximum likelihood time delay estimator with the α-β tracker to obtain significant improvement in the performance of the tracker. The adaptive realization of the ML-GCC function is similar

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to the method proposed by Youn and Mathews [9], and is described in Section 3. One novel feature of our realization is that the step size of the adaptive filters is updated online based on the input signal-to-noise measurement to provide as close to the best possible performance without manual supervision.

The rest of this paper is organized as follows. The next section discusses the α-β tracker. The third section discusses the adaptive realization of the ML-GCC function. The adaptive maximum likelihood α-β tracker algorithm is developed in Section 4. Experimental results are given in Section 5. The concluding remarks are made in Section 6.

2. THE α-β TRACKER

The α-β tracker [7, 8] is a block adaptive algorithm that attempts to estimate the time delay from the peak of an estimate of the cross-correlation function and tracks the changes in the time delay over time. The α-β tracker has three major components: (1) the cross-correlation function estimation, (2) the asymmetry error estimation, and (3) the parameter update. The cross-correlation function estimation block generates an estimate of the normalized cross-correlation function between the two input signals over a contiguous block of data. The cross-correlation function for the kth block is estimated as

$$\tilde{R}_{12}(m,k) = \frac{1}{N}\sum_{n=0}^{N-1} x_1(kL+n)x_2(kL+n-m) \qquad (7)$$

where N is the number of samples in each block and L represents the offset between the first samples of successive input blocks. Let $\hat{\tau}(k)$ denote the estimate of the time delay for the kth block. Assuming that this estimate is reasonably accurate, we can expect that the normalized cross-correlation function estimate for this block given by

$$\hat{R}_{12}(m,k) = \frac{\tilde{R}_{12}(m,k)}{\sqrt{\tilde{R}_{11}(0,k)\tilde{R}_{22}(0,k)}} , \qquad (8)$$

peaks in the vicinity of $\hat{\tau}(k)$. The asymmetry error

$$\varepsilon(k) = \hat{R}_{12}(\hat{\tau}(k)+1,k) - \hat{R}_{12}(\hat{\tau}(k)-1,k) \qquad (9)$$

provides a measure of the error between the actual time delay $\tau(k)$ and the estimate of the time delay $\hat{\tau}(k)$ [3, 4]. The α-β tracker iteratively estimates the time delay $\hat{\tau}(k)$ and its derivative $\hat{\dot{\tau}}(k)$ as

$$\hat{\dot{\tau}}(k+1) = \hat{\dot{\tau}}(k) + \frac{\beta(k)}{\Delta t}\varepsilon(k) \qquad (10)$$

and

$$\hat{\tau}(k+1) = \hat{\tau}(k) + \Delta t\hat{\dot{\tau}}(k+1) + \alpha(k)\varepsilon(k) , \qquad (11)$$

after every block of N samples. The parameter Δt in the above equations is the time between updates. The parameters $\alpha(k)$ and $\beta(k)$ control the rate of convergence and tracking of the system, and are updated heuristically [7, 8]. The α and β update equations are adapted on the basis of an estimate of the input signal-to-noise ratio (SNR) [7, 8]. The estimate of the SNR is found from the peak of the normalized cross-correlation estimate [7, 8] as

$$\widehat{SNR} = \frac{\hat{R}_{12}(\tilde{\tau}(k),k)}{1-\hat{R}_{12}(\tilde{\tau}(k),k)} , \qquad (12)$$

where $\tilde{\tau}(k)$ is the location of the peak of the normalized cross-correlation function and we have assumed that the SNRs of the two received signals are very close to each other. The cross-correlation function estimate $\hat{R}_{12}(\tilde{\tau}(k),k)$ is found by interpolating from the original estimates $\hat{R}_{12}(m,k)$ with a polynomial interpolation scheme [7, 8].

The heuristic argument employed for updating $\alpha(k)$ and $\beta(k)$ is that when the SNR is large, the algorithm should have a small effective integration time which allows tracking of higher bearing rate targets. The parameter $\beta(k)$ is updated as

$$\beta(k) = \beta_o g_K(K(k)) g_\varepsilon(\varepsilon(k),K(k)) , \qquad (13)$$

where β_o is a weighting constant, $K(k)$ is a smoothed estimate of the value of the peak of the normalized cross-correlation function $\hat{R}_{12}(\tilde{\tau}(k),k)$, $\varepsilon(k)$ is a smoothed estimate of the asymmetry error, and $g_K(K(k))$ and $g_\varepsilon(\varepsilon(k),K(k))$ are gain parameters. Both $K(k)$ and $\varepsilon(k)$ are smoothed with a single pole filter. The gain $g_K(K(k))$ is given by

$$g_K(K(k)) = \sqrt{\frac{K(k)}{K_o}} , \qquad (14)$$

where K_o is the value of the peak of the normalized cross-correlation function for the lowest useful SNR expected. The gain $g_\varepsilon(\varepsilon(k),K(k))$ is given by

$$g_\varepsilon(\varepsilon(k),K(k)) = 1 + \frac{3|\varepsilon(k)|}{\sqrt{100K_o^2 + K^2(k)}} . \qquad (15)$$

The sequence $\alpha(k)$ is a function of $\beta(k)$ and $K(k)$ and is given by

$$\alpha(k) = \sqrt{\frac{2\beta(k)}{K(k)}} - \frac{\beta(k)}{2} . \qquad (16)$$

The details of the derivations as well as the heuristics of the above equations are described in [7, 8]. Our experience is that this method, in spite of its heuristic nature, outperforms almost all adaptive time delay estimation techniques available in the literature for the types of signals we have tested the procedures on.

3. THE ADAPTIVE ML-GCC FUNCTION ESTIMATION

The adaptive realization of the ML-GCC function [9] involves the use of two normalized LMS adaptive filters. The first filter estimates $x_1(n)$ using $x_2(n)$ and the second estimates $x_2(n)$ using $x_1(n)$. Let $\hat{h}_{12}(m,n)$ and $\hat{h}_{21}(m,n)$ represent the coefficients of the two adaptive filters at time n. These coefficients are updated as [4]

$$\hat{\mathbf{h}}_{ij}(n+1) = \hat{\mathbf{h}}_{ij}(n) + \mu(n)e_{ij}(n)\frac{\mathbf{X}_j(n)}{||\mathbf{X}_j(n)||^2} , \qquad (17)$$

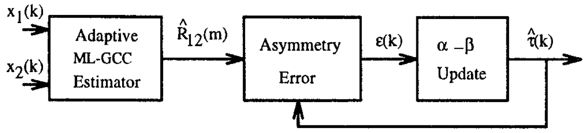

Figure 1. Block diagram of the adaptive ML algorithm.

where $\mu(n)$ is the step size sequence that controls the speed of convergence and steady-state characteristics of the adaptive filter, $\hat{\mathbf{h}}_{ij}(n)$ is the vector of the adaptive filter coefficients given by $\hat{\mathbf{h}}_{ij}(n) = [\hat{h}_{ij}(-M,n), \hat{h}_{ij}(-M+1,n), \ldots, \hat{h}_{ij}(M,n)]^T$, and $\mathbf{X}_j(n)$ is the input vector defined as $\mathbf{X}_j(n) = [x_j(n+M), x_j(n+M-1), \ldots, x_j(n-M)]^T$. The error $e_{ij}(n)$ between the desired signal $x_i(n)$ and the input block $\mathbf{X}_j(n)$ is given by

$$e_{ij}(n) = x_i(n) - \hat{\mathbf{h}}_{ij}^T(n)\mathbf{X}_j(n). \tag{18}$$

It was shown in [9] that

$$|\hat{\gamma}_{12}(f,n)|^2 = |\hat{H}_{12}(f,n)\hat{H}_{21}(f,n)| \tag{19}$$

represents an estimate of the magnitude-squared coherence (MSC) function of the input signals at time n. In (19), $\hat{H}_{ij}(f,n)$ represents the discrete-time Fourier transform (dtFt) of the NLMS filter coefficients $\hat{h}_{ij}(m,n)$ at time n. The ML-GCC function may now be estimated as

$$R_{12}^{(M)}(m,n) = F^{-1}\left\{\frac{|\hat{\gamma}_{12}(f,n)|^2}{(1-|\hat{\gamma}_{12}(f,n)|^2)}\frac{\hat{H}_{12}(f,n)}{|\hat{H}_{12}(f,n)|}\right\}. \tag{20}$$

4. THE ADAPTIVE ML α-β TRACKER

The adaptive ML α-β tracker combines the ML-GCC function estimation algorithm with the α-β tracker algorithm. The system first finds an adaptive estimate of the ML-GCC function as described in the previous section.

The estimate of the ML-GCC function is obtained using (19) and (20) once every L samples. The peak location of the ML-GCC function estimate $\tilde{\tau}(k)$ for the kth iteration of the α-β tracker is found using an iterative bisection algorithm [5] that finds the zero of

$$\epsilon(k) = \hat{R}_{12}^{(M)}(\tilde{\tau}(k)+1, kL) - \hat{R}_{12}^{(M)}(\tilde{\tau}(k)-1, kL). \tag{21}$$

Instead of using the estimate of the time delay error given by (9), our method computes the estimate of the error as

$$\varepsilon(k) = \tilde{\tau}(k) - \hat{\tau}(k). \tag{22}$$

This estimate of the error is used in place of the asymmetry error in the α and β update equations (10) and (11). The sequences $\alpha(k)$ and $\beta(k)$ are still updated using (13) and (16). A block diagram of the system is shown in Figure 1.

One major difficulty in implementing this algorithm is in the selection of the step size for the adaptive filters. The problem of choosing an optimal step size value or an appropriate step size sequence is not easy since there are no analytical results that relate the step size to the performance of the time delay estimators. In stationary operating environments, a smaller step size results in a better steady-state estimate, but slower adaptation. A larger step size conversely results in a poorer steady-state estimate, but faster adaptation. In stationary environments, one desires an initially large step size and then a smaller step size after the transient stage. A non-stationary environment requires that the value of step size be large enough to track the changes in the operating environment, yet small enough that the estimate is accurate.

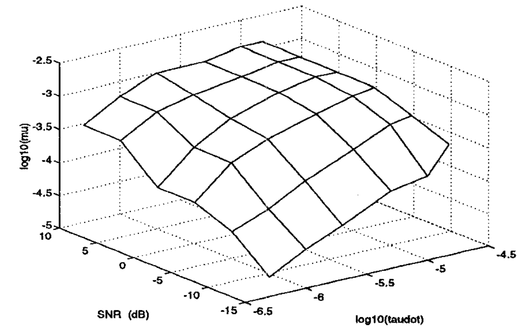

Figure 2. A three-dimensional plot of the optimal step size μ as a function of SNR and $\dot{\tau}$.

Our system employs an empirically derived step-size update algorithm for the adaptive filters. A large number of experiments were conducted to find the value of the step size that minimizes the mean-square error (MSE) between the estimate of the time delay and the actual time delay for a large range of SNRs and rates of variation of the time delay. The different rates of change in the actual time delay were simulated using linearly varying time delay functions with different slopes. For each set of SNR and $\dot{\tau}$, simulations were performed for a large set of step sizes in the range $[4.0\times10^{-5}, 5.0\times10^{-3}]$. The step size resulting in the smallest average MSE in the time delay estimate was tabulated for each of the representative environments. A three-dimensional plot of the "optimal" μ value for the different SNR and rate of change in the time delay ("taudot") are shown in Figure 2. The ML adaptive algorithm uses the tabulated experimental information by choosing the step size that provided the best performance for the estimate of the SNR and the rate of change of the time delay function. The SNR was estimated using the technique from (12) on the same block of L samples using the estimate of the cross-correlation function given in (7) and (8) separately from the estimation of the ML-GCC function. The estimate of the rate of change in the time delay is given in the difference equation (10). The step size parameter is kept constant between updates of the α-β tracker.

5. SIMULATION RESULTS

The conventional and adaptive ML α-β trackers were simulated using 30 Monte-Carlo runs with a bandpass filtered Gaussian signal and additive white Gaussian noise at 6 dif-

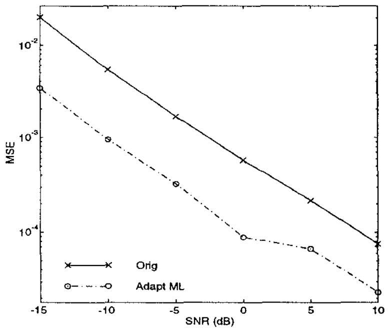

Figure 3. Comparison of the original α-β tracker and the maximum likelihood technique for a constant delay.

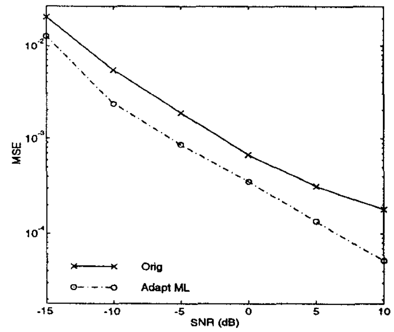

Figure 4. Comparison of the original α-β tracker and the maximum likelihood technique for a linear delay function.

ferent SNR levels. The source signal belonged to a zero-mean Gaussian process with a flat spectrum in the normalized frequency range [0.06,0.25] cycles/sample. The α-β tracker used a block size of 1000 samples with no overlap of the samples. The time-varying parameters of the adaptive ML α-β tracker and the estimate of the time delay were updated every 1000 samples. The two normalized LMS filters each had 51 coefficients. The value for $\beta_o = 7.22 \times 10^{-3}$ was choosen to result in a maximum possible $\beta(k)$ of 0.3. The value for $K_o = 0.0245$ was choosen to match the peak of the cross-correlation function when in the environment with the smallest expected SNR of -16 dB. The experiments were conducted for SNR values ranging from 10 dB to -15 dB in steps of -5 dB. Both a linear and constant delay function were used to evaluate the "steady-state" response of the systems. Each experiment used 390,000 samples of data. The constant delay function used in the experiments was at two sampling periods. The linear delay function started at two sampling periods, and decreased by one sampling period every 100,000 samples. In each experiment the system was initialized to the proper time delay and rate of change of the time delay. The average mean-square error between the estimate of the time delay and the actual time delay was calculated over the 30 runs for the last 300,000 samples. The MSE of the time delay estimates are ploted as a function of the input SNR in Figure 3 for the constant delay and in Figure 4 for the linear delay. The curves labeled as "Orig" refer to the original α-β tracker. The curves labeled as "Adapt ML" refer to the adaptive maximum likelihood realization. Our technique improved the MSE by at least 2 dB and by as much as 5 dB in the non-stationary environment, and it improved the MSE by at least 5 dB and by as much as 8 dB in the stationary environment.

6. CONCLUDING REMARKS

This paper discussed the estimation of the time-difference of arrival of a signal between two spatially separate sensors. A new time-delay estimation algorithm based on the α-β tracking mechanism and the maximum likelihood estimator was presented. Experimental comparisons showed that the new tracker provides significant performance improvement over the conventional α-β tracker. A novel component of the system is an empirically derived algorithm for selecting the step size of the adaptive filters online and without manual supervision.

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