QUANTUM PHASE TRANSITION AND ENGINEERING IN TWO-COMPONENT BEC IN OPTICAL LATTICES

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In this paper we review recent progress in studying quantum phase transitions in one- and two-component Bose-Einstein condensates (BEC) in optical lattices. These phase transitions involve the emergence and disappearance of quantum coherence over whole optical lattice and of linear superposition of macroscopic quantum states. The latter may provide new means to engineer and to manipulate novel macroscopic quantum states and novel coherent atomic beams for quantum information processing, quantum computing etc.

1. Prologue

The celebration of Professor Chen-Ning Yang's Eightieth Anniversary has a particular meaning to Chinese physicists of my age. A whole generation of Chinese physicists like me were inspired to choose physics as a career mainly by the news about the Nobel Prize of Physics in 1957. (Later we became to understand that among the great contributions of Prof. Yang to phyiscs, parity violation is actually *not* the greatest!) The celebration brings me back to the time of my first contact with Prof. Yang. During the early years of the Cultural Revolution (1966-1971), China was completely isolated from the world. This was particularly bad for young physicists who had just graduated from college, like me. When Prof. Yang first visited Beijing in 1971, I was lucky to be in the audience of his lectures in Beijing University. I still clearly remember his two lectures, i.e. the integral definition of the Yang-Mills field and the Bethe ansatz in one-dimensional exactly solvable many-body systems, respectively. The two lectures had great impacts on my later career in theoretical physics. In the first ten years afterwards, I was mainly working on Yang-Mills theory. Since I came to Stony Brook in 1981 from China upon Prof. Yang's invitation, I have got involved with quantum statistical mechanics of fractional statistics (first with anyons and later with exclusons) in lower dimensional systems. Now I firmly believe that Quantum Yang-Baxter Equations will play a significant role in the foundation of string/M theory.

The time allocated for my talk does not allow me to detail the other impacts that Prof. Yang had on my career. Here before presenting a recent work of mine with collaborator, I would like to express my wishes for Prof. Yang's good health, longevity and continuing success in his new career.

By now it is clear that we are in the early stages of an exciting second "quantum revolution", which is bringing us to the intersection of the microscopic quantum and the macroscopic classical worlds. This common meeting ground is rich in new physics and in new technology. The goal of the study is to achieve eventually the engineering and manipulation of atoms and molecules, either individually or collectively, entirely at the quantum level. Among others, BEC as a new form of matter in the quantum regime is a fascinating system full of promises for quantum manipulation and engineering. Below I will describe a recent work of mine with Guang-Hong Chen on quantum phase transitions and appearance of novel macroscopic quantum states in two-component BEC in optical lattices.

2. BEC as a Macroscopic Quantum State

Prof. Yang recently has summarized the three main melodies of the twentieth century physics: *symmetry, quantization and phases*[1]. Of course, the theory of Yang-Mills fields has been one of the successful syntheses of the three themes. Bose-Einstein condensates (BEC) may be viewed as another example of such synthesis: BEC are a collective *quantum* state, originated from *totally symmetric* wave function with respect to permutations of identical atoms of integer spin, in which there is a *phase coherence* over macroscopic distances.

Originally Einstein predicted the existence of BEC, based on the assumption of ideal gas (with interactions between atoms ignored). It was Yang and collaborators[2] in late 1950's who successfully treated the more realistic case of gaseous BEC with weakly interacting atoms. The paper of Byers and Yang[3] in 1961 clarified the importance of being a macroscopic quantum state with phase coherence for the flux quantization in superconductivity, a phenomenon that shares many features with BEC.

The transition in a trap between ordinary Bose gas and BEC occurs at a finite critical temperature T_c. At $T >> T_c$, a dilute gas consisting of identical atoms of integer spin behaves like a classical ideal gas, since two

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neutral atoms most of the time do not interact with each other unless they collide. However, as temperature decreases, the quantum mechanical "thermal de Broglie wavelength" of the atoms increases. At $T = T_c$ the thermal de Broglie wavelength becomes of the same order as the inter-particle distance, the wave packets of the atoms overlap with each other significantly. The whole system becomes, in a sense, a giant matter wave with a coherent phase. The Bose statistics of atoms begins to take effect, and all the atoms want to be in the same quantum state, forming a condensate called BEC. Thus it is clear that the transition to BEC at T_c is controlled by temperature or by thermal fluctuations, so it is a *thermal* phase transition. Below T_c the system remains to be a BEC which, as F. London suggested, exhibits superfluidity.

3. Quantum Phase Transitions for BEC in Optical Lattice

The case in an optical standing wave field, or an optical lattice, formed by a number of counter-propagating laser beams is very different. Even at $T = 0$, a phase transition from superfluid BEC to an insulator can happen by changing, say, the optical intensity. This is because the atoms, though neutral, interact with the optical standing wave through their induced electric dipole moment (the ac Stark effect). When the intensity of the laser beams is low, namely the optical period potential is weak, the atoms behave like Bloch waves, extending over the whole lattice like a superfluid. Let us tune the intensity of the laser beams to increase the strength of the optical lattice, then atoms tend to stay near the bottom of the valleys, i.e. the sites of the optical lattice, with a decreasing probability for tunneling between neighboring valleys. In this way, the BEC gets squeezed in optical lattice. If the (short-range) interactions between atoms are repulsive, then atoms do not feel happy to be in the same valley, and the system becomes *strongly correlated* due to the interactions between atoms in a narrow valley.

Thus there is an interesting competition between inter-valley tunneling and intra-valley (or on-site) repulsive interactions: Tunneling makes atoms move from site to site, in favor of a superfluid BEC phase, at absolute zero, spreading over the whole lattice. On the other hand, the on-site repulsive interactions tend to forbid the motion to a neighboring site if that increases on-site potential energy. Therefore, when the on-site energy is big enough, the atoms do not move from site to site, and the system is in an insulating phase, called the Mott insulator, to distinguish it from the usual band insulator. Theoretically such a competition can be described[4] by a

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lattice boson-Hubbard model[5]. By increasing the intensity of the optical standing wave, one can reduce the tunneling probability and, at the same time, increase the on-site energy, because of the shrinking of the atomic wave function in a valley. In this way, we expect a phase transition from superfluid BEC to an insulating phase when the optical potential become sufficiently strong, as indeed predicted by the boson Hubbard model. In the insulating phase, atoms are restricted to individual valleys and can no longer hop between different valleys. Therefore, the macroscopic quantum coherence across the system, that exists in the superfluid phase, is lost in the insulating phase. Namely accompanying the phase transition from BEC to (Mott) insulator is the loss of macroscopic quantum coherence, and the transition in the opposite way is associated with the emergence of macroscopic quantum coherence.

This phase transition is controlled by the intensity of laser beams that produce the optical lattice, not by temperature. It may happen even at absolute zero, driven by change in quantum fluctuations. Such a phase transition is called a *quantum* phase transition (QPT). In condensed matter theory, this is a hot topic in the frontier of strongly correlated systems. But normally in condensed matter systems it is hard to tune non-thermal parameters such as the on-site interactions. Also the presence of disorder either makes the observation of QPT very difficult or completely changes the behavior near transition. Free of these problems, the BEC in optical lattices is a clean system to experimentally observe and study QPT[6].

4. Two-component BEC in Optical Lattice

For the purpose of quantum engineering, it is better to have more components in BEC to manipulate. In the case at hand, we consider atoms with same constituents but in, say, two different internal states, labeled as different species A and B. Suppose the atomic polarizability, or the induced electric dipole moments, for the two states are of *opposite* sign. Then atoms A and B will see different optical potentials with opposite sign; the valleys of the latter form two penetrating sub-lattices, labeled with the same letters A and B, shifted relative to each other by a quarter of the wave length of laser beams. Thus the system consists of two components with different species A and B, each preferring to stay on its own sub-lattice.

Such a two-component system was previously suggested in ref. [4]. It was treated analytically the first time in the literature by Chen and me[7]. We have been able to solve the ground state of a two-component boson-Hubbard model that describes this situation. And we predicted that by tuning the parameters of the optical standing wave and the density of atoms of each component, it is possible to make atoms in species A or B to be either in the BEC phase or in the Mott phase at will, and to make a transition between BEC and Mott phases for each component separately. A new piece of physics here is that there may be interactions between atoms of species A and B; we have shown that the pertinent interaction parameters can be exploited to adjust the phase boundaries. This gives us more flexibility for quantum manipulation.

5. Engineering New Macroscopic Quantum States

Furthermore, we suggest[8] that a more interesting situation is to turn on an additional *Raman laser* to couple the two internal atomic states A and B. The action of the coupling laser makes atoms continually convert from state A to state B and *vice versa*. After a conversion the atom will see a different optical potential, then it will move toward a neighboring site of the other sub-lattice. The frequent conversion and subsequent atomic motion between the sites of different sub-lattice couple the two components in the system together, and may develop phase coherence *between* the two components. Thus, one should expect to observe more interesting and perhaps more exotic phenomena, due to the *Raman-assisted tunneling between neighboring sub-lattice sites*.

As before, under appropriate conditions, atoms in internal state A and those in B both form superfluid BEC on sub-lattice A and B, respectively. A preliminary analysis[8] done by Chen and me, based on a mean field theory, shows that due to Raman coupling, the order parameter in (or the macroscopic wave function of) the ground state of the system is actually a linear superposition of those for the two superfluid BEC on sub-lattice A and B. In particular, there is a definite phase difference between the two superfluid components (on sub-lattice A and B respectively), implying an additional phase coherence developed between the two superfluid components. The expression for the order parameter of the system is formally similar to that for a superconducting Josephson junction. However, physically this represents a new macroscopic quantum state, which has several characteristic features very different from the case of the Josephson junction.

Of course this state, on one hand, must share some similarities with the Josephson effect. On the other hand, unlike the case of the Josephson junction, the two phase-coherent superfluids are on *inter-penetrating* sub-lattices, rather than in separated spatial regions. Moreover, the atoms in the two coherent superfluids are in different internal states and, therefore, are *not* identical particles, while the Cooper pairs on different sides of a Josephson junction are identical to each other. Therefore, the constituent particles of the new quantum state are, in certain aspects, more like the K_L^0 particles or the neutrinos in neutrino oscillations in particle physics, which are linear superposition of states of *non-identical* particles. In other words, the Raman-assisted tunneling between sub-lattice A and B is accompanied by the conversion between different internal states, which has *no parallel* in the Josephson tunneling. More detailed study of the novel phase-coherent superposition of two *distinct* superfluids will be reported in the near future.

In the original talk we raised the question of whether the new state could be viewed as a Schrödinger cat. Now as of the present writing, we think that in a rigorous sense it is *not*, even though its macroscopic wave function (or order parameter) is indeed a linear superposition of those of two inter-penetrating macroscopic quantum states. However, the new macroscopic quantum state may be better viewed as *a condensate of Schrödinger kittens*, i.e. a condensate of atoms in an internal state that is a linear superposition of two different internal states A and B. When we turn off the trap and the optical lattice, we may expect to get a *novel coherent atomic beam*, in which each atom is a Schrödinger kitten with relative amplitudes and phase *controllable* by adjusting optical parameters and atomic density. In this way, we expect that the two-component BEC in optical lattices provide new means for engineering and manipulating novel coherent atomic beams.

6. Acknowledgement

I thank Prof. H.T. Nieh and the Center of Advanced Study at Tsinghua University in Beijing for inviting me to speak at this Symposium and for their warm hospitality. Also I thank Dr. Guang-Hong Chen for pleasant collaboration. This work was supported in part by the U.S. National Science Foundation under Grant PHY-9907701.

References

1. For an earlier version, go to the website of Prof. C.N. Yang:
   http://insti.physics.sunysb.edu/Physics/faculty.htm#yang
   A recent version is presented by Prof. Yang at the International Conference of Theoretical Physics, UNESCO, Paris (July, 2002); see Proceedings.
2. For an elegant summary of the results, see C.N. Yang, "*Imperfect Bose System*", *Physica* **26**, 549 (1960); and the references therein.

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3. N. Byers and C.N. Yang, *Phys. Rev. Lett.* **7**, 46 (1961).
4. D. Jaksch *et al.*, *Phys. Rev. Lett.* **81**, 3108 (1961).
5. D.S. Fisher and M.P.A. Fisher, *Phys. Rev. Lett.* **61**, 1847 (1988).
6. M. Greiner *et al.*, *Nature* **415**, 39 (2002).
7. G.H. Chen and Y.S. Wu, *Phys. Rev.* **A67**, 013606 (2003).
8. G.H. Chen and Y.S. Wu, in preparation.

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