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## Role of spin-orbit coupling in the metal-insulator transition in two-dimensional systems

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We propose an experiment, which would allow us to pinpoint the role of spin-orbit coupling in the metalnonmetal transition observed in a number of two-dimensional systems at low densities. Namely, we demonstrate that in a parallel magnetic field the interplay between the spin-orbit coupling and the Zeeman splitting leads to a characteristic anisotropy of resistivity *with respect to the direction of the in-plane magnetic field*. Though our analytic calculation is done in the deeply insulating regime, the anisotropy is expected to persist far beyond that regime.

In a recent paper<sup>1</sup> an interesting experimental observation was reported. It was demonstrated that the period of beats of the Shubnikov-de Haas oscillations in a two-dimensional hole system is strongly correlated with the zero-magneticfield temperature dependence of the resistivity. The beats of the Shubnikov-de Haas oscillations have their origin in the splitting of the spin subbands in a zero magnetic field.<sup>2,3</sup> The authors of Ref. 1 were able to tune the zero-field splitting by changing the gate voltage. They observed that, while in the absence of the subband splitting, the zero-magnetic-field resistivity was temperature independent below T=0.7 K, a pronounced rise (by 5 percent) in resistivity with temperature emerged in the interval  $0.2 \text{ K} \le T \le 0.7 \text{ K}$  at the maximal subband splitting, indicating a metalliclike behavior. This close correlation suggests that it is a mechanism causing the spin subband splitting that plays an important role in the crossover from the metalliclike to the insulatinglike temperature dependence of resistivity with decreasing carrier density (the metal-nonmetal transition). This transition by now has been experimentally observed in a number of different twodimensional electron<sup>4-7</sup> and hole<sup>8-12</sup> systems. By challenging the commonly accepted concepts, it has attracted a lot of theoretical interest and attempts to identify the underlying mechanism. Possible relevance of zero-field splitting to the transition was first conjectured in Ref. 13. The evidence presented in Ref. 1 about the importance of the subband splitting for metalliclike behavior of resistivity is further supported by the very recent data reported in Ref. 14.

Another important feature of the metal-nonmetal transition, which might also provide a clue for the understanding of its origin, is that the metallic phase is destroyed by a relatively weak parallel magnetic field.<sup>12,15–20</sup> At the same time, no quenching of the metallic phase in a parallel magnetic field was observed in an SiGe hole gas,<sup>21</sup> in which the strain, caused by the lattice mismatch, splits the light and heavy holes.

As far as the theory is concerned, the role of the parallel magnetic field was previously accounted for exclusively through the Zeeman energy, which either alters the exchange interactions (and, thus, electron-ion binding energy<sup>22,23</sup>) or suppresses the liquid phase,<sup>24</sup> or affects the transmittancy of the point contact between the phase-coherent regions.<sup>25</sup>

It is appealing to combine the observations<sup>1,14</sup> of the subband splitting in zero field and the results<sup>12,15–20</sup> in a parallel magnetic field within a single picture. The spin-orbit (SO) coupling appears to be a promising candidate for such a unifying mechanism. Indeed, on one hand, it is known to lead to spin subband splitting. On the other hand, a parallel magnetic field, though not affecting the orbital in-plane motion, destroys the SO coupling and, thus, suppresses the intersubband transitions. The possible importance of these transitions was emphasized in Ref. 14. Their suppression with increasing magnetic field is caused by the fact that the corresponding subband wave functions become orthogonal for all wave vectors.

At the present moment there is no consensus in the literature about the role of the SO coupling. Several authors<sup>26-28</sup>

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FIG. 1. Azimuthal positions of the in-plane magnetic field,  $\phi_{\rm B}$ , and of the wave vector of electron,  $\phi_{\rm k}$ , are shown schematically.

have explored the role of the SO coupling as a possible source of the metalliclike behavior, by considering noninteracting two-dimensional system and including the SO terms into the calculation of the weak-localization corrections. At the same time, the majority of theoretical works,<sup>22–25,29–37</sup> stimulated by the experimental observation of the transition, disregarded the SO coupling.<sup>38</sup>

To pinpoint the role of the SO coupling in the metalnonmetal transition, it seems important to find a qualitative effect which exists only in the presence of the SO coupling. Such an effect is proposed in the present paper. We show that an interplay between the SO coupling and the Zeeman splitting gives rise to a characteristic anisotropy of resistivity with respect to the direction of the parallel magnetic field. Obviously, the Zeeman splitting alone cannot induce any anisotropy. To demonstrate the effect, we consider the deeply insulating regime, where the physical picture of transport is transparent.

We choose the simplest form for the spin-orbit Hamiltonian:<sup>39,40</sup>

$$\hat{H}_{SO} = \alpha \mathbf{k} \cdot (\boldsymbol{\sigma} \times \hat{\mathbf{z}}). \tag{1}$$

Here  $\alpha$  is the SO coupling constant, **k** is the wave vector,  $\hat{\mathbf{z}}$  is the unit vector normal to the 2D plane,  $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$  are the Pauli matrices. In the presence of the parallel magnetic field, the single particle Hamiltonian can be written as

$$\hat{H} = \frac{\hbar^2 k^2}{2m} + \alpha \mathbf{k} \cdot (\boldsymbol{\sigma} \times \hat{\boldsymbol{z}}) + g \mu_B \boldsymbol{\sigma} \cdot \mathbf{B}$$
$$= \begin{pmatrix} \frac{\hbar^2 k^2}{2m} & \frac{\Delta_Z}{2} e^{-i\phi_B} - i \alpha k e^{-i\phi_k} \\ \frac{\Delta_Z}{2} e^{i\phi_B + i \alpha k e^{i\phi_k}} & \frac{\hbar^2 k^2}{2m} \end{pmatrix}, \quad (2)$$

where *m* is the effective mass, *g* and  $\mu_B$  are the *g* factor and the Bohr magneton, respectively;  $\Delta_Z = 2g \mu_B B$  is the Zeeman splitting;  $\phi_{\mathbf{B}}$  and  $\phi_{\mathbf{k}}$  are, correspondingly, the azimuthal angles of magnetic field **B** (Fig. 1) and the wave vector **k**. The energy spectrum of the Hamiltonian Eq. (2) is given by

$$E_{\pm}(\mathbf{k}) = \frac{\hbar^2 k^2}{2m} \pm \frac{1}{2} \sqrt{\Delta_Z^2 + 4\alpha^2 k^2 + 4\alpha k \Delta_Z \sin(\phi_{\mathbf{B}} - \phi_{\mathbf{k}})}.$$
(3)

Note that the spectrum is anisotropic only if both  $\Delta_Z$  and  $\alpha$  are nonzero.

The standard procedure for the calculation of the hopping conductance is the following.<sup>41</sup> We denote with  $P_{12}$  the hopping probability between the localized states 1 and 2. The logarithm of  $P_{12}$  represents the sum of two terms

$$\ln P_{12} = -\frac{\varepsilon_{12}}{T} - \ln |G(\mathbf{R})|^2, \qquad (4)$$

where the first term originates from the activation;  $\varepsilon_{12}$  is the activation energy,<sup>41</sup> and *T* is the temperature. The second term in Eq. (4) describes the overlap of the wave functions of the localized states centered at points  $\mathbf{R}_1$  and  $\mathbf{R}_2$ , so that  $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$ . In Eq. (4) we use the fact that within the prefactor the overlap integral coincides with the Green function  $G(\mathbf{R})$ . For the matrix Hamiltonian Eq. (2), the Green function is also a matrix,

$$\hat{G}(\mathbf{R}) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{e^{i\mathbf{k}\cdot\mathbf{R}}}{E - \hat{H}(\mathbf{k})}.$$
(5)

By projecting onto the eigenspace of Hamiltonian Eq. (2), the above expression can be presented as

$$\hat{G}(\mathbf{R}) = \int \frac{dkkd\phi_{\mathbf{k}}}{(2\pi)^2} e^{ikR\cos(\phi_{\mathbf{k}} - \phi_{\mathbf{R}})} \\ \times \left[ \frac{\hat{P}_{+}(\mathbf{k})}{E - E_{+}(\mathbf{k})} + \frac{\hat{P}_{-}(\mathbf{k})}{E - E_{-}(\mathbf{k})} \right], \tag{6}$$

where the projection operators  $P_{\pm}(\mathbf{k})$  are defined as

$$\hat{P}_{+}(\mathbf{k}) = \frac{1}{2} \begin{pmatrix} 1 & O(\mathbf{k}) \\ O^{*}(\mathbf{k}) & 1 \end{pmatrix}, \quad \hat{P}_{-}(\mathbf{k}) = 1 - \hat{P}_{+}(\mathbf{k}),$$
(7)

where  $O^*(k)$  is the complex conjugate of O(k), which is defined as

$$O(k) = \frac{(\Delta z/2)\exp(-i\phi_{\mathbf{B}}) - i\alpha k \exp(-i\phi_{\mathbf{k}})}{E_{+}(\mathbf{k}) - E_{-}(\mathbf{k})}.$$
 (8)

When the distance *R* is much larger than the localization radius,  $a_0$ , the integral over  $\phi_{\mathbf{k}}$  is determined by a narrow interval  $|\phi_{\mathbf{k}} - \phi_{\mathbf{R}}| \sim (kR)^{-1/2} \ll 1$ . This allows us to replace  $\phi_{\mathbf{k}}$  by  $\phi_{\mathbf{R}}$  in the square brackets and perform the angular integration. Then we obtain

$$\hat{G}(\mathbf{R}) = \sqrt{\frac{2\pi}{iR}} \int_0^\infty \frac{dk \sqrt{k}}{(2\pi)^2} e^{ikR} \\ \times \left[ \frac{\hat{P}_+(k,\phi_{\mathbf{R}})}{E - E_+(k,\phi_{\mathbf{R}})} + \frac{\hat{P}_-(k,\phi_{\mathbf{R}})}{E - E_-(k,\phi_{\mathbf{R}})} \right].$$
(9)

The next step of the integration is also standard. Namely, for large *R*, the  $\hat{G}(\mathbf{R})$  is determined by the poles of the integrand. However, in the case under consideration, the equation  $E_{\pm}(k) = E$  leads to a fourth-order algebraic equation. To simplify the calculations we will restrict ourselves to the strongly localized regime  $|E| \ge m \alpha^2 / \hbar^2$ . In this case the poles can be found by the successive approximations. In the zero-order approximation, we get the standard result k $= ik_0$ , where  $k_0$  is defined as

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$$k_0 = a_0^{-1} = \frac{\sqrt{2m|E|}}{\hbar} \tag{10}$$

In the first order approximation, we have  $k = ik_0 + k_1$ , where  $k_1$  is given by

$$k_1 = \pm i \frac{m\alpha}{\hbar^2} \sqrt{\Delta_1^2 - 1 + 2i\Delta_1 \sin(\phi_B - \phi_R)}, \qquad (11)$$

where the dimensionless Zeeman splitting  $\Delta_1$  is defined as

$$\Delta_1 = \Delta_Z / 2\alpha k_0. \tag{12}$$

Within this approximation, the long-distance asymptotics of the Green function is

$$\hat{G}(R) \propto e^{-R/a(\phi_{\mathbf{B}},\phi_{\mathbf{R}})},\tag{13}$$

where the decay length is given by

$$a(\phi_{\mathbf{B}}, \phi_{\mathbf{R}})^{-1} = k_0 \bigg( 1 - \frac{m\alpha}{\hbar^2 k_0} \operatorname{Re} \sqrt{\Delta_1^2 - 1 + 2i\Delta_1 \sin(\phi_{\mathbf{B}} - \phi_{\mathbf{R}})} \bigg).$$
(14)

In the last equation it is assumed that the real part, Re(...), has a positive sign. Our main observation is that the decay length and, concommitantly, the probability of hopping are anisotropic, when the parallel magnetic field and the SO coupling are present simultaneously. By evaluating the real part in Eq. (14) we obtain

$$a(\phi_{\mathbf{B}}, \phi_{\mathbf{R}})^{-1} = k_0 \left( 1 - \frac{m\alpha}{\sqrt{2}\hbar^2 k_0} \times \sqrt{\Delta_1^2 - 1 + \sqrt{1 + \Delta_1^4 - 2\Delta_1^2 \cos 2(\phi_{\mathbf{B}} - \phi_{\mathbf{R}})}} \right).$$
(15)

To characterize the anisotropy quantitatively, we introduce the perpendicular decay length  $a_{\perp} = a[\phi_{\mathbf{B}} - \phi_{\mathbf{R}} = \pm (\pi/2)]$ and the parallel decay length  $a_{\parallel} = a(\phi_{\mathbf{B}} = \phi_{\mathbf{R}})$ . Then a quantitative measure of the anisotropy can be defined as

$$\frac{a_{\perp} - a_{\parallel}}{a_0} = \frac{m\alpha}{\hbar^2 k_0} f(\Delta_1), \qquad (16)$$

where the function f(x) is given by

$$f(x) = x - (x^2 - 1)^{1/2} \theta(x - 1), \qquad (17)$$

where  $\theta(x)$  is the step function. Recall that Eqs. (16) and (17) were derived under the assumption that the Fermi level lies low enough  $(E < 0, |E| \ge m \alpha^2 / \hbar^2)$ . The latter condition ensures that  $|k_1| \le k_0$ . It can be rewritten in the form  $m\alpha \le \hbar^2 k_0$ . As is seen from Eq. (16), under this condition the magnitude of anisotropy is small. The magnetic field dependence of the anisotropy corresponds to  $\Delta_1 = 1$  and it vanishes both in strong and weak magnetic fields. The theory of



FIG. 2. Magnetic field dependence of the anisotropy of the decay length.

hopping transport in the systems with anisotropic localization radius is presented in Ref. 41. The principal outcome of this theory is that the anisotropy of the localization radius [and, consequently, the exponential anisotropy of the hopping probability (4)] does not lead to the exponential anisotropy of the hopping resistance. In fact, the exponent of the resistance is the same as for the isotropic hopping with localization radius  $\sqrt{a_{\parallel}a_{\perp}}$ . However, the anisotropy in the Green function manifests itself in the prefactor of the hopping resistance<sup>41</sup>

$$\frac{\rho_{\parallel} - \rho_{\perp}}{\rho_{\parallel} + \rho_{\perp}} = C \frac{a_{\parallel} - a_{\perp}}{a_{\parallel} + a_{\perp}} \approx C \frac{m \alpha}{2\hbar^2 k_0} f(\Delta_1), \qquad (18)$$

where  $C \sim 1$  is the numerical factor, determined by the perturbation theory in the method of invariants for random bond percolation problem.<sup>41</sup> The exact value of the constant *C* depends on the regime of hopping (nearest-neighbor or variable-range hopping). This value will also be different, although still ~1, for variable-range hopping in a strongly interacting system<sup>41</sup> (with large  $r_s$  parameter). This situation is common for experiments on metal-nonmetal transition. We also note that interactions cause the renormalization of the coupling constant  $\alpha$ . For a clean system this renormalization as a function of  $r_s$  was studied in Ref. 42. It was shown that interactions enhance the SO coupling.

The microscopic origin of the SO Hamiltonian Eq. (1) is the asymmetry of the confinement potential. In III-V semiconductor quantum wells there exists another mechanism of the SO coupling, which originates from the absence of the inversion symmetry in the bulk (the Dresselhaus mechanism<sup>43</sup>). Within this mechanism,  $\hat{H}_{SO} = \beta(\sigma_x k_x - \sigma_y k_y)$  (for [001] growth direction). Then the calculation similar to the above leads to the following result for the anisotropic decay length:

$$a(\phi_{\mathbf{B}}, \phi_{\mathbf{R}})^{-1} = k_0 \left( 1 - \frac{m\beta}{\sqrt{2}\hbar^2 k_0} \times \sqrt{\Delta_2^2 - 1 + \sqrt{1 + \Delta_2^4 + 2\Delta_2^2 \cos(\phi_{\mathbf{B}} + \phi_{\mathbf{R}})}} \right),$$
(19)

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where  $\Delta_2$  is related to the Zeeman splitting as

$$\Delta_2 = \Delta_Z / 2\beta k_0. \tag{20}$$

By using Eq. (19) we get for anisotropy

$$\frac{a_{\perp} - a_{\parallel}}{a_0} = -\frac{m\beta}{\hbar^2 k_0} f(\Delta_2), \qquad (21)$$

where the function f is determined by Eq. (17).

In conclusion, we have demonstrated that, due to the SO coupling, the rotation of an in-plane magnetic field with respect to the direction of current should lead to a characteristic angular variation of resistivity with a period  $\pi$ . The anisotropy is maximal for intermediate magnetic fields and vanishes in the weak and the strong-field limits. In the strongly localized regime, considered in the present paper, the magnitude of anisotropy is small. However, as seen from Eqs. (16) and (18), the magnitude of anisotropy should increase as the Fermi level moves up with increasing carrier concentration (since  $k_0$  decreases). So the resistivity is expected to remain anisotropic, perhaps with a modified angular dependence, far beyond the deeply insulating regime. For high enough concentrations the Fermi energy is positive, E>0, so that  $k_0$  defined by Eq. (10) has a meaning of the Fermi momentum. When E exceeds the SO-induced subband splitting,  $E \ge \alpha k_0$ , the anisotropy will be also weak (of the order of  $\alpha k_0 / E = 2m\alpha/\hbar^2 k_0$ ). In contrast to the insulating regime, it will increase with decreasing carrier concentration. If the intersubband scattering governs the metal-nonmetal transition, then the resistivity anisotropy should reach maximum around the critical density.

Finally, let us discuss two possible complications for the experimental observation of the anisotropy in resistivity. Both of them stem from the fact that a realistic twodimensional system has a finite thickness. Firstly, with finite thickness, even a small deviation of the magnetic field direction from the in-plane position would cause a certain anisotropy even without SO coupling. However, in this case, the anisotropy would only increase with increasing magnetic field, while the SO-induced anisotropy should vanish in the strong-field limit. The second effect of the finite thickness is that it causes the anisotropy of the Dresselhaus term with respect to the crystalline axes. As is shown in Ref. 44, the interplay of anisotropic Dresselhaus and isotropic Bychkov-Rashba terms results in the crystalline anisotropy of the resistivity in the weak-localization regime. This effect should be distinguished from the anisotropy with respect to the direction of current predicted in the present paper.

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