A novel basis for interpreting recent acceleration of anthropogenic carbon dioxide emissions

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A variety of strategies have been employed to reduce CO_2 emissions. However, recent evidence points to CO_2 emissions rising at rates that are consistent with the "worst-case" of possible scenarios. This paper seeks to explain the driving forces behind recent CO_2 trajectories from a more simple perspective than has previously been considered. At its most fundamental level, the global economy is shown to behave as a form of thermodynamic heat engine. Much like biological organisms, it consumes available energy from its environment in order to offset spontaneous decay into waste, and it grows by doing thermodynamic work. A 35–year continuous period of available data, covering half of total growth, is shown to support this conclusion: globally, financial value has been related to the consumption rate of primary energy supplies through a fixed factor of 9.7±0.3 milliwatts per inflation-adjusted 1990 US dollar. The implication of this new result is that any climate policy aimed at accelerating energy efficiency gains should accelerate the production of global economic value, but with the counter-intuitive side-effect of accelerating global energy consumption and CO_2 waste emission rates. Thus, effective policies aimed at achieving stabilization of emissions should not focus on improving energy efficiency but rather on a rapid shift of the global primary energy mix to alternative resources, currently at a rate of about 300 GW per year.

Introduction

There is now broad acceptance among the public of a need to reduce household and industrial CO_2 emissions, preferably without excessive harm to their livelihoods. With this cause in mind, engineers and scientists have over the past couple of decades developed a variety of recipes for some vision of a "soft landing". For example, the International Panel on Climate Change (IPCC) Special Report on Emissions Scenarios (SRES) [1] has provided a wide range of timelines for CO_2 emissions, each designed to show how a given set of policy decisions might correspond to a particular atmospheric CO_2 trajectory. SRES models are highly sophisticated, and contain numerous interactive components, each designed to reflect a realistic range of societal dynamic behavior. On the basis of these and similar models, international treaties and economic instruments have been both developed and implemented, each aimed at staving the rise in atmospheric CO_2. Indisputably, this progress is a considerable societal achievement.

Nonetheless, recent evidence indicates that global CO_2 emissions growth has not slowed, but rather has continued to accelerate, matching or even exceeding the "worst-case" of the SRES scenarios [2]. On the face of it, that we have not yet done better is a bit perplexing. Is this simply a matter of absence of sufficient political and individual will? The intent of this contribution is to explore a different possibility. I will try to show from a more fundamental, thermodynamic perspective that there may have been strong natural constraints on our ability to slow future CO_2 emissions growth.

Basic identities

At its most basic level, the anthropogenic CO_2 emissions rate E is related to the rate a at which primary energy is made available to the human economy through c, the quantity of non-recyclable carbon dioxide emitted per primary energy unit consumed. Thus

$$E = ca \qquad [1]$$

Society consumes energy in order to facilitate economic production of the value in goods and services P. The factor relating the two quantities is termed the "energy productivity" f, thereby leading to the identity

$$P = fa \qquad [2]$$

Total production may be expressed in terms of current price (or "nominal") currency, but is often adjusted so that it is expressed in fixed year (i.e. inflation-adjusted or "real") currency. As long as f and P are self-consistent in terms of currency, Eqs. 1 and 2 indicate that the instantaneous relationship between production and emissions is

$$E = \frac{c}{f}P \qquad [3]$$

For the purpose of tractability, the IPCC SRES models all adopt a framework known as the Kaya Identity to express the primary drivers of growth in E [3]. This framework extends the relationship in Eq. 3 by referencing CO_2 emissions to human population p. Thus,

$$E = p \times g \times i \times c \qquad [4]$$

where g represents the real economic production per person and $i = 1/f$ represents the energy intensity of real economic production. Although the Kaya Identity is purely a diagnostic expression, it is still considered useful to express the Kaya Identity in a prognostic form as it helps dictate how policy decisions, when applied to each of its component terms, might be used to constrain CO_2 emissions growth [2, 3]

$$\frac{d\ln E}{dt} = \frac{d\ln p}{dt} + \frac{d\ln g}{dt} - \frac{d\ln f}{dt} + \frac{d\ln c}{dt} \qquad [5]$$

The Kaya Identity makes a clear statement that CO_2 emissions may be reduced through technological solutions, either by increasing the energy efficiency of economic production (increasing f) or by shifting consumption towards energy resources that emit less carbon dioxide (decreasing c) [4, 1, 5, 6].

However, there is some recognition that the picture may not be as simple as the Kaya Identity suggests. In an 1865 exposition on energy economics, W. S. Jevons was emphatic that the recent introduction of energy efficient steam engines had increased rather than decreased coal consumption because it had made the cost of steam-powered coal extraction cheaper, and thus more attractive [7, 8, 9, 10]. Recently, interest in the "rebound" and "backfire" effects (or Jevon's Paradox) has been revived, both politically [11] and scientifically [12, 13, 14, 15, 16, 17, 18] . But even though the phenomenon is potentially an important component of the CO_2 emissions problem, there remains no general consensus on the total magnitude of rebound at macroeconomic scales [10].

A thermodynamic emissions growth model

Thermodynamic basis. In order to clarify the role of energy and efficiency in the anthropogenic CO_2 emissions problem, and whether backfire exists, it is worth starting from first principles. Perhaps the most fundamental of physical laws is the "Second Law" of thermodynamics, which requires that all systems, even those that are living, exist through a spontaneous conversion of environmental potential energy into some less available form [19, 20, 21].

Take for the moment a fairly general case, and consider some subjectively defined system illustrated in Fig. 1. The system exists at

Gibbs energy potential $G_{eq} + \delta G$, and interacts with its immediate environment through a system-specific boundary held at temperature T_s and pressure p, while radiating energy to a colder universe. In general, the Gibbs energy potential of matter is given by $\sum_i n_i \mu_i (T, p)$, where n_i refers to the number of the species i with a temperature and pressure specific chemical potential $\mu_i (T, p)$ [22]. The existence of a "free" energy potential δG enables the system to spontaneously transfer internal energy to its environment as inaccessible "heat", driving the system back towards its zero-potential equilibrium level $G_{eq} (T_s, p)$. If T_s and p do not change during this process, the reduction in the available potential is necessarily due to a loss of matter or conversion of matter into a less available, lower chemical potential form (i.e. $d \left(\sum_i n_i \mu_i \right) < 0$) [23].

Equally, however, the existence of a system potential δG might be defined according to its capacity to offset "heat" losses by making environmental energy available to the system at rate $a = \alpha \delta G$, where α is some system specific rate coefficient. If δG stays constant, a exactly offsets losses of "heat" to the environment; if a exceeds this "heat" loss then the system potential increases at rate $d (\delta G) / dt = w$, where w is the net "work" done by the system in order to accumulate high potential matter energy from its environment. Effectively, this represents a type of heat engine, which differs from a more familiar conception only in frame of reference: rather than work adding to the available potential of some external body, work is referenced here from the standpoint of the system itself: work acts to expand δG at rate $d (\delta G) / dt = w = \epsilon a$ where $\epsilon = w/a$ is the engine's thermodynamic efficiency. Since the energy input to the system is $a = \alpha \delta G$, what this then characterizes is a positive feedback loop, such that for any given state of δG

$$\frac{da}{dt} = \alpha w = \alpha \epsilon a = \eta a \qquad [6]$$

where η is the efficiency of the feedback on a. Note how η in this reciprocal form of heat engine is related to the more familiar heat engine efficiency ϵ through the system-specific rate coefficient α.

An example that might be particularly easy to relate to could be the growth of a young child. The child consumes the accessible energy contained in food in proportion to her size. This rate of consumption $a = \alpha \delta G$ – perhaps about 50 Watts – enables her to do "work" w with energy efficiency ϵ in order to incorporate water and nutrients into her structure. Of course, the consumption of food also leaves a variety of forms of inaccessible waste energy or "heat" at rate $a - w$. The child maintains homeostasis while the thermal component of this "heat" radiates quickly to space at a relatively cold local planetary blackbody temperature of about 255 K. Thermodynamically, "heat" can also be material, and arise from reduction of the chemical potential of the nutrients into a lower potential form, including, for example, CO_2. "Heat" is produced regardless, but if $w > 0$, the girl grows logarithmically at a rate $\eta = d \ln a / dt$. Of course, absent sufficient nutrition, $w < 0$, and the energy feedback η is negative. But, assuming the child reaches maturity, there becomes a balance between consumption and waste, and η tends to zero. There are two important aspects here: first, the child's current consumption a reflects a past trajectory in η; and, second, the greater the child's energy efficiency ϵ, the higher the energy feedback rate η, and the more rapidly her energy consumption a increases.

Application to global civilization. The following discussion takes the above thermodynamic framework and applies the relationship given by Eq. 6 to the human system as a whole. While a variety of past efforts have also been directed at applying thermodynamic principles to the human system [16, 24, 25, 26, 27, 28], this work might be distinguished by appearing particularly simple and falsifiable.

Only global quantities are considered here, without explicit reference to specific nations or economic sectors. This approach has the advantage of simplicity and relevance. In the case of the problem of anthropogenically induced global warming, through atmospheric mixing carbon dioxide concentrations are nearly equivalent in all locations; also, through trading in international markets the valuation of a given economic unit of currency is everywhere identical. Considered from a global perspective, details in atmospheric mixing and economic trade become unimportant.

To summarize the general thermodynamic basis, if a system maintains a fixed temperature and pressure and is in radiative contact with a cold reservoir, its internal potential energy spontaneously grows only by doing thermodynamic work to incorporate matter from its environment. The level of Gibbs potential that can be maintained against decay is proportional to its energy consumption rate, as consumed energy is continuously converted to a less available, lower potential form in accordance with the Second Law.

Extending this physical principle to the human system, suppose that primary energy consumption a (units power) is what supports a human system potential δG, leaving a variety of forms of waste. A testable hypothesis can then be made that civilization implicitly assigns inflation-adjusted (or real) monetary value to this *rate* of energy consumption. Value is not derived from energy itself, as has also been suggested [29, 30], but rather from energy per time, or power. If all current exothermic processes supporting civilization suddenly ceased, and a equaled zero, all civilization would become worthless, as it could no longer sustain a non-equilibrium level of Gibbs potential energy $\delta G = \alpha a$. So, for example, the potential energy in oil combustion is valuable only to the extent it is available to civilization, not at all if it burns wastefully in the desert, and only then in its chemical, not nuclear bonds.

So, from this thermodynamic perspective, economic value might be non-human, and include working animals, roads, computers and communications; or it might be human, including the production capacity of active bodies and brains. What is important is only that all elements of value operate in synergy, working together as a single energy-consuming organism to help access primary energy supplies [28].

To express this hypothesis mathematically, the global consumption of available primary energy by civilization a is related to its total value through a constant factor λ. It is from this cornerstone result that other conclusions will follow. Thus,

$$a = \lambda C \qquad [7]$$

A thermodynamically based expression for the production rate P of value is obtained through division of Eq. 6 by $\lambda = a/C$, or equivalently, taking the first derivative of Eq. 7. This yields the following relations

$$P = \frac{1}{\lambda} \frac{da}{dt} = \frac{\eta}{\lambda} a = \frac{\alpha}{\lambda} w \qquad [8]$$

$$\frac{dC}{dt} \equiv P = \eta C \qquad [9]$$

These relationships imply, rather intuitively, that current global value has been acquired through a history of doing thermodynamic work w (units power)

$$C(t) = \int_0^t P(t') \, dt' = \frac{\alpha}{\lambda} \int_0^t w(t') \, dt' \qquad [10]$$

So, thermodynamically speaking, P (units currency per time) and C (units currency) are, respectively, representations of work and capacity to facilitate consumption of available primary energy. Fiscally, P represents global real, or inflation-adjusted, economic production (commonly termed gross domestic product, GDP), and C is a very general representation of global economic capital. The value of $\eta = \alpha w / a$ that characterizes the speed of the feedback loop driving this reciprocal heat engine can be considered to be the real rate of return on investment P in capital C

Admittedly, expressing global capital thermodynamically, as a simple integral of real production, might appear somewhat unorthodox by normal economic standards. However, the two approaches in fact bear some strong similarities. See supporting information (SI) *Comparison with traditional economic models*, for a discussion of how differences can be resolved by considering the thermodynamic meaning of the distinction between real (inflation-adjusted) and nominal value. Also, It should not be surprising that a model of identical form is often used to model growth of vegetation, where vegetative "capital" C refers not to money but instead to biomass [32, 33]. Plants too consume energy in order to offset decay and do work in order to grow.

Another important similarity between plants and the global economy is that among their waste "heat" by-products is CO_2. Unlike plants, though, the global economy derives most of its CO_2 from fossil-carbon, in which case it is not recyclable, and accumulates in the atmosphere. A schematic illustrating the economic relationship between energy consumption and waste carbon dioxide emissions is shown in Fig. 2. The model illustrates how the combination of Eqs. 1, 7 and 9 imply that CO_2 emissions can be represented simply through

$$E(t) = \lambda c C = \lambda c \int_0^t P(t')\,dt' \qquad [11]$$

Present-day emissions are determined by past accumulation of real economic production and the current carbonization of energy of the energy supply.

So what has been described above is a thermodynamic growth model, that is organic, and relates readily to the global economy. Most importantly, it can easily be falsified using global data for energy consumption and economic production [31, 35]. The following section provides a test of whether the thermodynamic model illustrated by Figure 2 indeed provides a valid framework for interpreting recent economic and CO_2 emissions growth.

Framework evaluation

Establishing the validity of the prognostic solution for CO_2 emissions given by Eq. 11, and the economic growth solution represented by Eq. 9, rests on whether there exists a constant of proportionality λ that relates the consumption of energy a to a monetary representation C (Eq. 7), where C is the accumulation of real economic production P over history:

$$\lambda(t) = \frac{a(t)}{C(t)} = \frac{a(t)}{\int_0^t P(t')\,dt'} = const. \qquad [12]$$

This study uses global statistics for the years 1970 to 2004, as it is only for this 35 year interval that global records are available for the combination of energy production, carbon dioxide emissions and economic production (expressed here in fixed 1990 US dollars) [See SI *Data Sources* for details].

Comparison of economic capital and primary energy consumption. There are no explicit records for the global worth of economic capital C, especially as it is defined here, so the quantity must be estimated. Strictly speaking, because C represents a time-integral, its calculation requires yearly records of real global economic production P starting from the beginning of civilization. While such records for P are not available, more sporadic estimates have been ascertained for select years over the past two millennia [34], and these can be combined with more recent annual records [35] to create a two-millenia yearly time-series in P [See SI *Estimation of economic capital*]. Gross World Product estimates in 1990 market exchange rate dollars are available for each year since 1970 [35]. Long-term but intermittent historical estimates are available for the years 1 to 1992 CE [34]. The latter data set is expressed in Geary Khamis purchasing power parity (PPP) 1990 US dollars, and is first adjusted to 1990 market exchange rate dollars, and then mapped onto a yearly grid to provide a continuous inflation-adjusted record in P for the period 1 to 2004 CE. Economic capital C is then simply the running integral of P over time, as shown in Fig. 3.

To determine whether λ is indeed constant, the above historical estimates for global economic capital C can now be compared with measured statistics for global primary energy consumption a between 1970 and 2004 [31]. Figure 4 shows that, between 1970 and 2004, economic capital and energy production both approximately doubled; they grew at an average rate of 1.8% per year, from 820 trillion to 1515 trillion 1990 US dollars, and from 227 to 467 exajoules, respectively. Despite doubling, the ratio of the two quantities $\lambda(t) = a/C$ remained nearly constant over the period. On average, the ratio was 0.306 exajoules per trillion 1990 US dollars per year, with just 3% standard deviation from the mean. This variability is small, sufficiently so that it plausibly reflects errors or noise in historical estimates of C or a. For example, new primary energy production (what is measured) only reflects new primary energy consumption (what is relevant) in the average, not the instant. So the simplest interpretation of the analysis is that the value expressed in a single fixed-year 1990 US dollar is tied to continuous primary energy consumption through a constant coefficient of 9.7 mW per dollar. Corrected for autocorrelation in the time-series, the observational uncertainty in this value at the 95% confidence level is just ± 0.3 mW per dollar.

In principle, it should be possible to derive an equivalent value for λ from the ratio of the growth of primary energy consumption da/dt to economic production $P = dC/dt$, as shown in Fig. 2. This could be seen as an alternative illustration of the validity of the model. The advantage of the previously introduced comparison is that a and C are integrators of historical growth, and therefore considerably less noisy than their respective time derivatives da/dt and dC/dt. And, as expected, this noisiness is reflected in the ratio $\lambda = (da/dt)/(dC/dt)$. The mean value of λ calculated from the time derivatives is 12.1 mW per 1990 US dollar, but with an autocorrelation-corrected 95% confidence in the mean of ± 4.0 mW per 1990 US dollar. So, although the mean value of $\lambda = (da/dt)/(dC/dt)$ is 25% higher than the value found for $\lambda = a/C$, the difference is not in fact statistically significant.

Observed growth. To summarize, I have shown that, to within a narrow margin of uncertainty, there is a constant value parameter $\lambda = 9.7 \pm 0.3$ milliwatts per dollar that can be used to relate global primary energy consumption to the summation of global economic production over history. As a basis of argument, this is the central result of this work; it is a validation of the initial thermodynamically supported hypothesis that, on a global level, the historical accumulation of economic value, when it is adjusted for inflation, is implicitly a financial representation of the capacity to facilitate global primary energy consumption. Among the many waste "heat" by-products that result from this thermodynamic growth engine is CO_2, whose growth is modified also by the carbon content of the available energy supply c.

Time series for the years 1970 to 2004 are given in Figures 4 and 5 for the variables η, c, P, a, C and E. Table 1 summarizes average observed growth rates over this period, and compares these to theoretically expected results based on the easily derived prognostic forms for these parameters. While the comparisons between the model and observed rates provide no additional support for the model, as they merely stem from showing that λ is constant, they do illustrate the anticipated consistency among prognostic solutions and measurements, based only on "exogenous", observed rates of change in the economic energy feedback (or the innovation rate) $d\ln\eta/dt$ and the CO_2 emission intensity of energy (or the carbonization rate) $d\ln c/dt$.

Of course, a fully prognostic model would also provide equations for $d\ln\eta/dt$ and $d\ln c/dt$. While developing a model for the evolution of c and η is a topic of contemporary economic research [3, 36], it is beyond the scope of this study. A possible solution might come from considering the seemingly universal scaling laws that have been derived for the growth rates of species and cities [37, 38, 39].

An interesting result that can be derived from Eq. 11 using the values for λ in Fig. 4 and c in Fig. 5 is that the "carbon footprint" of civilization in recent decades reflects a simple relationship between the rate of global carbon emissions E and the quantity of global capital C. The coefficient is $\lambda c = 5.2 \pm 0.2$ MtC per year, per trillion 1990 US dollars. What follows is a discussion of what this correspondence implies for climate change mitigation.

Discussion

Drivers of emissions growth. The framework normally used for interpreting the actions needed to limit CO_2 emissions growth, The Kaya Identity (Eq. 5), treats changes in population p, per capita production g, technology $i = 1/f$, and energy carbonization c as the primary independent parameters driving growth [4]. Even more sophisticated SRES models appeal to this guide [1].

The thermodynamically based framework introduced here, by contrast, portrays the energy feedback η as what drives emissions growth (Table 1). To see how the two frameworks are related, consider that substitution of Eq. 2 into Eq. 7 leads to $P = \lambda f C$. An equivalent expression given by Eq. 9 is $P = \eta C$, in which case from the definition of η in Eq. 6

$$\eta \equiv \alpha\epsilon \equiv \lambda f \qquad [13]$$

In effect, the thermodynamic efficiency ϵ, economic productivity f, and the economic feedback η are each related, the latter two only through system specific constants α and λ.

To show the implications of this relationship between energy productivity and the energy feedback, I now equate the Kaya (Eq. 5) and thermodynamic (Table 1) prognostic expressions for CO_2 emissions growth. Both expressions employ exogenous expressions for carbonization growth $d\ln c/dt$, which leaves a comparison for growth in energy consumption $d\ln a/dt$.

$$\frac{d\ln p}{dt} + \frac{d\ln g}{dt} - \frac{d\ln f}{dt} = \lambda f \qquad [14]$$

Now suppose that some climate policy causes energy productivity growth $d\ln f/dt$ to be faster than expected from normal "spontaneous" change [40]. Eq. 14 implies that, for any given state of f, such gains require accelerated growth of some combination of population p and standard of living g – society praises innovation for good reason. Moreover, because f is now growing faster, there is accelerated growth of $\eta \equiv \lambda f$, thermodynamically requiring accelerated growth of energy consumption and emissions – backfire or Jevon's Paradox is intrinsic to the economic system [16].

This principle can be illustrated by comparing predictions of energy consumption growth using the thermodynamically based, zeroth order growth model on the right hand side of Eq. 14 with the first order Kaya-based expression on the left.

Fig. 6 shows a time-series of comparisons with observations of forecast energy growth rates using both methods. Forecasts are for 10-years, calculated for each of the years 1975 to 1994. Kaya Identity based predictions are derived from measured trends from the prior 5 years; and thermodynamically based predictions from the current state of η. To give an example, a Kaya-based 1980 forecast for energy consumption growth between 1980 and 1990 is based on persistence in observed growth of p, g, and f over the period 1975 to 1980 (Eq. 5); alternatively, if a thermodynamically-based model is used, forecasted growth rates are estimated based only on the 1980 value for $\eta \equiv \lambda f = P/C$ (hence zeroth order). While a forecast period of 10 years is arbitrary, it is sufficiently long to allow for deviations from persistence in trends, but it is short enough that Kaya Identity based predictions do not deviate significantly from more sophisticated SRES simulations [2].

Fig. 6 shows that forecasts made using the thermodynamic model introduced here, despite it being only zeroth order, are more accurate in predicting the future than assuming simple persistence of recent trends, by about a factor of 3. The standard deviation in their respective errors is 0.4 % per year compared to 1.4 % per year.

Spectral reddening. While the current value of the energy feedback $\eta \equiv \lambda f$ is the primary factor determining civilization's growth, it is crucial to recognize that the current value of η has only been arrived at through a long history of accumulations in the rate of change in economic energy feedback $d\ln\eta/dt$ (or the innovation rate). Even the most distant past has had an influence on the present. Expressed in terms of time-series analysis, η is highly "reddened": its variability is slow because it is an integrator of $d\ln\eta/dt$.

One consequence of reddening is that, if the present innovation rate is positive then there is an exponential increase in the efficiency of economic production $\eta \equiv \lambda f$ with characteristic time-scale $\tau_\eta = 1/(d\ln\eta/dt)$. Looking at the prognostic forms of C and a in Table 1, economic capital and energy consumption will then increase super-exponentially (i.e. the exponent of an exponent). In fact, Fig. 3 shows that such super-exponential growth has broadly characterized civilization for over one hundred years, a feature that is already acknowledged for cities [39].

So if maintaining positive innovation rates continues to be possible, society will enjoy super-exponential growth of capital. But, if the carbonization of the energy supply c stays unchanged, growth of CO_2 emissions E will also be super-exponential. The solution for emissions from Table 1, starting at some time $t = 0$, is

$$\frac{E}{E_0} = \exp\left[\eta_0\tau_\eta\left(e^{t/\tau_\eta} - 1\right)\right] \qquad [15]$$

Note, however, that growth condenses to the single exponential form in the limit of $\tau_\eta \gg t$. It is for this reason that there has not in fact been much departure from single exponentiality in more recent emissions growth (Fig. 5). The time-scales for innovation τ_η and decarbonization $\tau_c = -1/(d\ln c/dt)$ over the past three decades have been long – approximately 100 years and 500 years respectively (Table 1).

So global society can change, but only rather slowly. The value of the energy feedback η climbed from 1.4 % per year in 1970 to 2.1 % per year in 2004 (Fig. 5). If energy efficiency continues along its current trajectory of improvement, η will continue to grow. But, without a shift in the fuel mix away from fossil-based supplies, CO_2 emissions will also accelerate, and at a weakly super-exponential rate.

Mitigation. I have shown that energy efficiency gains help the economy but also accelerate CO_2 emissions. Assuming reducing η through economic self-destruction is off the table, what then is an effective and palatable CO_2 emissions mitigation strategy? Can economic growth and CO_2 emissions growth be decoupled? Consider that the expression $d\ln E/dt = \eta + d\ln c/dt$ in Table 1 points towards a non-dimensional stabilization number

$$S = \frac{-d\ln c/dt}{\eta} \qquad [16]$$

for which, if $S \geq 1$, $d\ln E/dt \leq 0$, and emissions are stabilized or declining.

Achieving values of $S \geq 1$ would require that decarbonization be at least as fast as the efficiency of the economy. Thus, for an economic efficiency of 2.1% per year in 2004, and a decarbonization $-d\ln c/dt$ between 1970 and 2004 of approximately 0.2% per year (Fig. 5), stabilization requires acceleration of decarbonization by at least a factor of ten. 2.1% of current energy production corresponds to an annual provision globally of approximately 300 GW of new non-carbon emitting power capacity - approximately one nuclear power plant per day. If economic growth continues to be defined by improvements in energy efficiency such that $d\ln\eta/dt$ is positive, η will increase also, meaning that rates of decarbonization would need to increase correspondingly.

Conclusions

In this paper, I have introduced a framework for the growth of global carbon dioxide emissions that, like the Kaya Identity, is simple, but differs by being based on thermodynamic first principles. Also, unlike more sophisticated SRES models, it requires no appeal to the apparent complexity of the human system. Whether consumption is facilitated by bridges, roads, or people, the human system can be framed in its whole as a reciprocal form of heat engine, that uses all its elements in synergy in order to make environmental energy resources available for its own future consumption. Consumption maintains and grows the internal free energy of the system against spontaneous decay.

A primary conclusion that can be derived from this framework is that inflation-adjusted economic value is an implicit representation of civilization's capacity to sustain energy consumption; three and a half decades of available data, covering half of historical growth, show that the constant of proportionality is 9.7 ± 0.3 mW per fixed, inflation-adjusted 1990 US dollar. A counter-intuitive implication of this result is that any process that increases energy efficiency also accelerates production of value, with the unintended consequence of accelerating the growth of civilization's consumption of energy.

So, unfortunately, it appears that when it comes to mitigating climate change, there will be no free lunch. Political agreements aimed at increasing energy efficiency, while they may have provided society with greater overall wealth, have only contributed to accelerating CO_2 emissions. Barring changes to the carbon content of the energy supply, CO_2 emissions will continue to grow at the same rate as civilization's worth: the coefficient in recent decades has been 5.2 ± 0.2 MtC per year, per trillion 1990 US dollars. And if the value of civilization is to continue to be permitted to grow, successful techniques aimed at stabilizing emissions will be limited to rapid and accelerating decarbonization of the world economy. The task is daunting: at current energy feedback rates of 2.1% per year, the rate of continuous decarbonization that will be required is about 300 GW of non-emitting power production per year.

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Table 1. Summary of average observed and modeled quantities for the period 1970 to 2004.

Parameter	Proposed functional dependence	Observed mean	Model mean
energy efficiency growth	$d\ln\eta/dt$	1.06 %/yr	-
carbonization growth	$d\ln c/dt$	-0.19 %/yr	-
energy feedback	$\eta = \eta_0 \exp\left(\frac{d\ln\eta}{dt}t\right)$	1.83 %/yr	-
energy consumption growth	$d\ln a/dt = \eta$	1.85 %/yr	1.83 %/yr
economic capital growth	$d\ln C/dt = \eta$	1.82 %/yr	1.83 %/yr
economic production growth	$d\ln P/dt = \eta + d\ln\eta/dt$	2.88 %/yr	2.89 %/yr
CO_2 emissions growth	$d\ln E/dt = \eta + d\ln c/dt$	1.65 %/yr	1.64%/yr

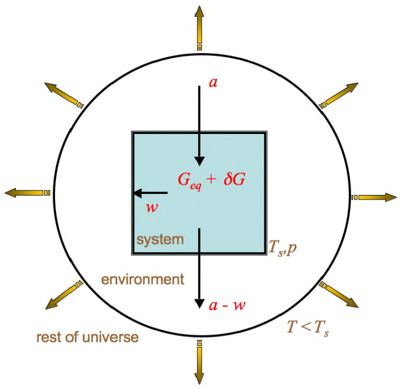

Fig. 1. Illustration of an evolving system at energy potential $\delta G(T_s, p)$, that consumes energy from its environment at rate a. The system expands at rate w while releasing to the environment unavailable waste energy at rate $a - w$. The environmental interface maintains a constant temperature while thermal heat is radiated to a colder universe.

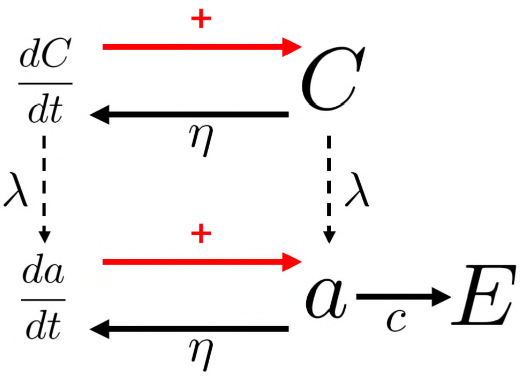

Fig. 2. Schematic diagram of the introduced CO_2 emissions growth model relating Eqs. 1, 6, and 9. Black arrows point in the direction of the product, red arrows in the direction of the integral over time. In fixed-year currency, C is civilization's economic capital (units currency) that produces economic production $P = dC/dt$ at rate η, which in return adds to C. The economic system maps onto economically available energy through a constant of proportionality λ. Energy consumed by civilization a (units power) enables additional energy consumption da/dt at rate η. Energy consumption is related to non-recyclable CO_2 emissions E through the carbon content of fuel as represented by c.

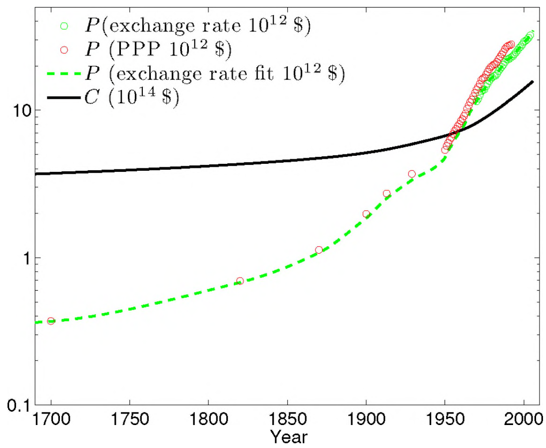

Fig. 3. Estimates of gross world product P in market exchange rate, and purchasing power parity (PPP) 1990 U.S. dollars. Economic capital C represents the accumulated value of market exchange rate gross world product P.

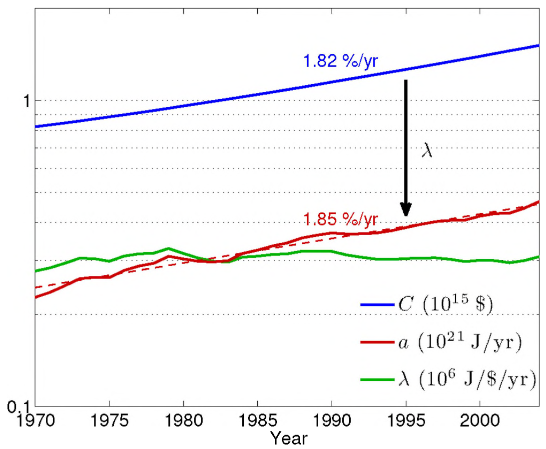

Fig. 4. Trajectories for total infrastructure C and total energy production a during the period 1970 to 2004. The parameter λ represents the ratio a/C. Dashed lines represent a least-squares best fit.

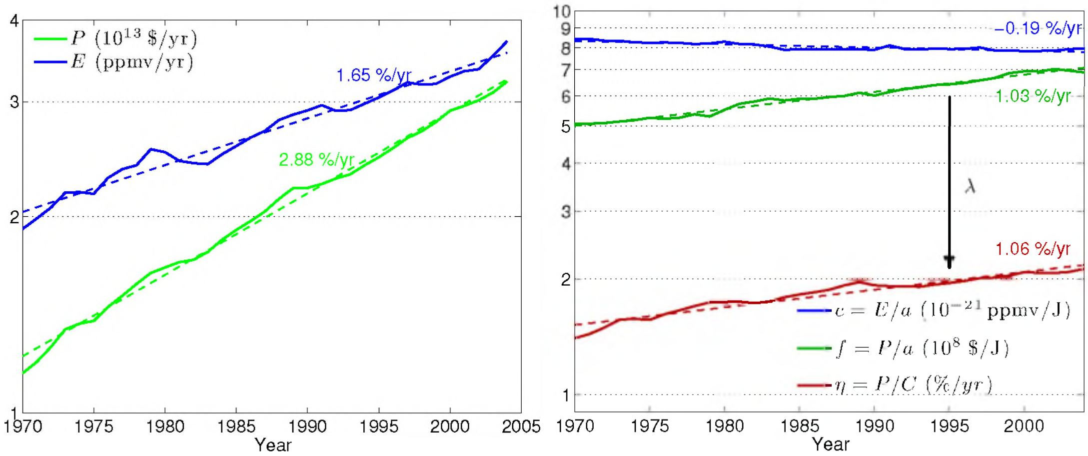

Fig. 5. For the period 1970 to 2004, trajectories in real global world production P and carbon dioxide emissions E (left), and real economic energy efficiency $\eta = \lambda f$ and the carbon dioxide emission intensity of energy $c = E/a$ (right). Here, c represents the increase in atmospheric concentrations of CO_2 E, per unit primary energy consumption a, that would be expected in a well-mixed atmosphere in the absence of terrestrial sink and source terms (1 ppmv CO_2 = 2.13 Gt emitted carbon [41]). Dashed lines represent a least-squares first-order fit.

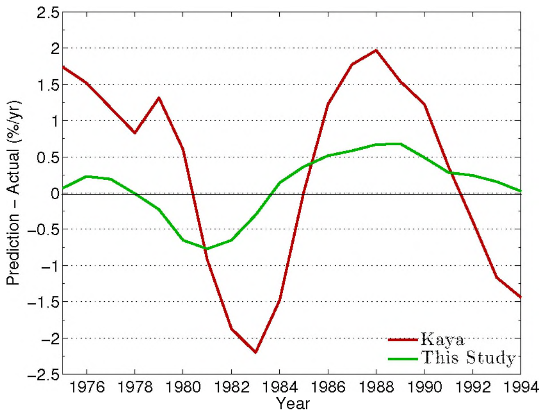

Fig. 6. Difference from reality in predicted growth rates in global energy consumption a derived using persistence in trends (red) and the zeroth-order model presented here (green). The basis for calculation of trend persistance is the previous five years, and the zeroth-order model is based on current year calculations of the energy efficiency feedback η. Forecasts are compared with observation over the following 10 years.

Supporting Information

1 Comparison with traditional economic models

In traditional economic studies, capital appreciation comes from two components, a fraction s representing a savings, or investment; and a fraction $(1-s)$ representing private and government "consumption". Models represent the nominal growth in "capital" K (units currency) as the difference between the portion s of production P (units currency per time) that is a savings or investment, and capital depreciation at rate γ

$$\frac{dK}{dt} = (P-W) - \gamma K = sP - \gamma K \tag{1}$$

where individual and government consumption is represented by $W = (1-s)P$.

In return, according to some functional form, labor L (units worker hours) employs capital K (units value) to generate further production P. For the sake of illustration, a commonly used representation is the Cobb-Douglas production function

$$P = AK^{\alpha}L^{1-\alpha} \tag{2}$$

where A, the "total factor productivity", is a compensating factor designed to account for any residual unaccounted for by K and L. The exponent α is empirically determined. The Solow

Growth Model [1] expresses the prognostic form for Eq. 2 as

$$\frac{d\ln P}{dt} = \frac{d\ln A}{dt} + \alpha\frac{d\ln K}{dt} + (1-\alpha)\frac{d\ln L}{dt} \tag{3}$$

Commonly, the term $d\ln A/dt$ is interpreted to represent technological progress.

There have been criticisms raised of the Solow Model because it makes no explicit reference to natural resources [2, 3]. One suggested remedy is to incorporate primary energy consumption into Eq. 2 as a complement to labor or capital [4, 5], in which case

$$P = (A_K K)^{\alpha}(A_L L)^{\beta}(A_a a)^{1-\alpha-\beta} \tag{4}$$

where, again, a is energy consumption, α and β are empirically determined, and the subscripts for A refer to respective technological progress.

For comparison, the production function introduced in this paper is

$$P = \eta C \tag{5}$$

where C is a more explicitly thermodynamic expression of capital than K, and η is the energy efficiency representing a rate of return due to thermodynamic work by the system on the system. It was argued here that, since $a = \lambda C$

$$P = \frac{\eta}{\lambda}a \tag{6}$$

in which case Eqs. 5 and 6 can be considered to be a radical simplification of Eq. 4. The representation of economic capital C employed here is a substitution of the combination of traditionally defined capital K and labor L in Eq. 4, such that $\alpha = 1$ and $\beta = 0$, and $A_K = \eta$. Alternatively, since C is itself only a monetary representation of the rate of primary energy consumption a, $\alpha = \beta = 0$, and $A_a = \eta/\lambda$. A particular advantage of the diagnostic relations for P introduced here, over more standard formulations, is that the equations are dimensionally self-consistent, and do not appeal to non-integer exponents α and β of dimensional terms (such as L and K), as fitted to a specific set of circumstances, and with no certain application to different economic regimes.

Nonetheless, in terms of a prognostic growth model, both approaches have important mathematical similarities. In the model introduced here $d\ln P/dt = \eta + d\ln\eta/dt$. Likewise, the Solow model (Eq. 3) also describes the growth of production as a sum of rates. In addition, both representations incorporate a representation of innovation, which in the neo-classical Solow model is $d\ln A/dt$ and is more explicitly defined here to be $d\ln\eta/dt$.

While certainly, from traditional economic perspectives, the simplifications employed here might seem overly extreme, they do nonetheless appeal to much the same phenomena, albeit from a different framework. For example, the growth equation for economic capital introduced here, $d\ln C/dt = \eta$ might seem to leave no room for what is normally considered to be consumption W or depreciation γK (as in Eq. 1). Certainly, some portion of economic production must be consumed, at least in order to maintain economic capital against depreciation or decay: buildings crumble; bodies must be maintained; old technology becomes obsolete; as does past acquisition of human skills and knowledge.

But it is critical to recognize that the equations derived here are intended to apply only to real, inflation-adjusted production P, and not nominal production $\hat{P}$. To demonstrate, assume inflation is positive, in which case nominal capital $\hat{C}$ grows faster than real capital by some fractional rate of real capital, γ

$$\frac{d\hat{C}}{dt} = \frac{dC}{dt} + \gamma C \tag{7}$$

Since it has been argued here that $dC/dt = P$, this leads to

$$\frac{dC}{dt} = \hat{P} - \gamma C \tag{8}$$

in which case, the source of real capital is nominal production, and the corresponding sink for real capital occurs at rate γ. So, in fact, Eq. 8 does account for depreciation through the term γC, and is thus similar to the depreciation term γK in the standard growth equation for capital (Eq. 1). While depreciation is implicit when the growth equations are expressed in real, inflation-adjusted terms, depreciation is explicit when they are expressed in nominal terms.

It is interesting to see what the capital decay rate γ represents. Again, because $dC/dt = P$, this means Eq. 8 leads to the statement $\hat{P} - P = \gamma C$. Alternatively, when nominal production is expressed in energy consumption co-ordinates through substitution of the expression $a = \lambda C$

$$\hat{P} - P = \gamma C = \frac{\gamma}{\lambda} a \tag{9}$$

Compare this to an equivalent expression derived for real production $P = \frac{\eta}{\lambda} a$. The implication here is that capital decay γC accounts for the difference between nominal and inflation-adjusted production. Also, since $P = \eta C$, the ratio $\gamma/(\gamma + \eta)$ is the fraction of nominal production $\hat{P}$ that does not return itself as a real addition to capital C. The capital depreciation rate γ is an energy barrier that must first be crossed.

What remains is that there is no reference to consumption in the model presented here, as it is defined in the traditional formulation for capital growth given by Eq. 1. An interpretation of this difference is that consumption, while it may not represent an investment in traditional representations of capital K, does nonetheless maintain and contribute to the more thermodynamic expression of economic capital C. To illustrate, real economic production, through the construction of coal mines and power plants, clearly represents an investment in economic capital. A less obvious, although functionally equivalent example, is food. In standard representations, food would be "consumed" by households. However, the combined chemical potential in food $\sum_i n_i \mu_i$ also maintains and improves that household's capacity to further consume energy and do work by supporting its internal potential energy δG. So, the consumption of an ordinary sandwich precisely offsets a body and mind against decay from "heat" loss such that it can continue to consume energy at the same rate it has in the past (in which case the real production rate is zero since $\hat{P} = \gamma C$). The added value of a really good, if more expensive, sandwich is its capacity to facilitate real production and new energy consumption above and beyond decay (in which case real production is greater than zero and $\hat{P} > \gamma C$). The addition to capital may derive either from a heightened sense of well-being or an increased desire to be productive in order to afford such sandwiches. In either case, the increase to the chemical potential δG may be due either to an increase in the person's weight, or a restructuring of the person's brain into a higher chemical potential relative to the environment.

2 Data sources

US Department of Energy statistics for global primary energy production [6] include fossil fuel, hydroelectric, nuclear, geothermal, wind, solar, and biomass sources. It is assumed here that production and consumption rates are, at least on average, equivalent. United Nations time series for world economic production [7] represent the total gross domestic product of all countries, adjusted for inflation and market exchange rates to fixed 1990 US dollars. Statistics for CO_2 emissions are obtained from the Carbon Dioxide Information Analysis Center [8]. Only global quantities are considered because, while energy consumption and economic production can be separated on a regional basis, trade is international, and the by-product CO_2 is well-mixed at global scales.

3 Estimation of economic capital

In general, the motivation for expressing valuation in PPP instead of exchange rate dollars is to account for disparities in product valuation that exist between countries. In PPP dollars, product valuation is equalized according to its apparent contribution to standard of living. Countries with a low standard of living tend to have a relatively high gross domestic product when expressed in PPP rather than market exchange rate dollars because equivalent products and services tend to be less expensive. However, because the focus of this study is energy production and associated CO_2 emissions, rather than national standard of living, it is historical records of market exchange rate valuations that are preferred. Exchange rate measures of production P are assumed to most accurately reflect the total energy costs associated with manifesting products and services in the respective nations where they are consumed.

To account for any discrepancy between PPP and exchange rate estimates in historical records for economic production P, market exchange rate data from 1970 onwards is used to devise a time-dependent correction factor π to be applied to PPP records such that $\pi = \mathrm{PPP}/\mathrm{exchange\,rate}$ (Fig. 3). For the period 1970 to 1992, during which both PPP and market exchange rate estimates of P are available, the fitted value for π is $\pi = 1 + 0.258\exp\left[(t-1998)/73\right]$. This correction factor can be extrapolated and applied to all PPP data between the years 0 and 1969. For the period from 1970 onwards, measured exchange rate values are used. Because the historical estimates of P in PPP dollars are increasingly sparse with distance back in time (e.g. there are only three data points for the period 1 to 1500 CE), the corrected dataset for P is mapped to a yearly distribution using a cubic spline fit.

The corresponding year-by-year estimates of economic capital C represent an accumulation of economic production P over time since 1 CE, i.e. $C(t) = C(1) + \int_1^t P(t')\,dt'$. To estimate a value for $C(1)$, it is assumed that the ratio of population to economic capital in 1 CE. is equivalent to the average value between 1 CE and the threshold of the industrial revolution circa 1700 CE From historical population statistics [9], the associated iterative solution for $C(1)$ is 120 trillion 1990 U.S. dollars. For comparison, the estimated value of C in 2004 CE is 1515 trillion 1990 U.S. dollars (Fig. 3). Although, off-hand, this value for $C(1)$ seems surprisingly high, it is still very small compared to current day values, so the derived value of λ presented in this paper is relatively insensitive to errors in its estimate.

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